Superconducting strings in the classical U(1)×U(1) model

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Introduction

- Witten proposed the model to describe a superconducting string in 80's.

E. Witten, Nucl.Phys.B249, 557–592, 1985.

$$\mathcal{L} = -\frac{1}{4}\sum_{a=1,2} F^{(a)}_{\mu\nu}F^{(a)\mu\nu} + D_\mu\phi^* D^\mu\phi + D_\mu\sigma^* D^\mu\sigma - U$$

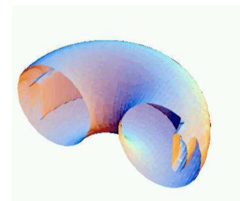
$$F^{(a)}_{\mu\nu} = \partial_\mu A^{(a)}_\nu - \partial_\nu A^{(a)}_\mu$$
$$D^{(a)}_\mu = (\partial_\mu - ig_a A^{(a)}_\mu)$$

$A^{(a)}_\mu$:Gauge fields
ϕ, σ:Complex scalar fields
g_1, g_2:Gauge coupling constants

$$U = \frac{\lambda_\phi}{4}(|\phi|^2 - \eta_\phi^2)^2 + \frac{\lambda_\sigma}{4}(|\sigma|^2 - \eta_\sigma^2)^2 + \beta|\phi|^2|\sigma|^2 - \frac{\lambda_\sigma}{4}\eta_\sigma^4$$

$\eta_\phi, \eta_\sigma, \lambda_\phi, \lambda_\sigma, \beta$:Parameters with positive sine

- It has been found that this model has solitonic solutions such as vortices and vortons.

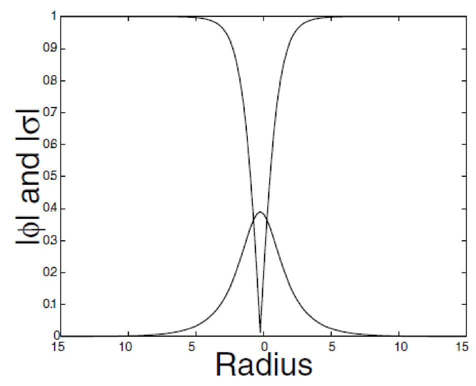


- Several researchers has still studied this model , and it has been used for physical application.

J.Kunz, E.Radu, and B.Subagyo, Gravitating vortons as ring solitons in general relativity, Phys. Rev. D **87**, 104022

J.Garaud, E.Radu, and M.S.Volkov, Stable Cosmic Vortons, Phys. Rev. Lett. **111**,171602

- Strict conditions which ensure the existence of solutions are yet not known even in the case of straight line vortices.



$\lambda_\phi = 1.5, \lambda_\sigma = 10.0, \eta_\phi = 1.0, \eta_\sigma = 0.5, \beta = 1.5$

The condition which they obtained:

$$\beta\eta_\phi^2 - \frac{1}{2}\lambda_\sigma\eta_\sigma^2 < \frac{1}{2}\lambda_\phi\eta_\phi^2$$

Y. Lemperiere,E. P. S. Shellard,
Vorton existence and stability,Phys.Rev.Lett, 91(2003)141601.

The rotationally symmetric solutions

Ungauged model

$$\mathcal{L} = \partial_\mu\phi^*\partial^\mu\phi + \partial_\mu\sigma^*\partial^\mu\sigma - U$$

Ansatz $\phi(\boldsymbol{r}) = \phi(r)e^{im\theta}$ $\quad m$:winding number
$\sigma(\boldsymbol{r}) = \sigma(r)e^{i(\omega t + kz)}$

Boundary conditions $\phi(0) = 0 \quad \phi(\infty) = \eta_\phi$
$\sigma'(0) = 0 \quad \sigma(\infty) = 0$

Equations

$$r\phi'' + \phi' - \frac{m^2}{r}\phi - \frac{\lambda_\phi}{2}r(\phi^2 - \eta_\phi^2)\phi - \beta r\phi\sigma^2 = 0$$
$$r\sigma'' + \sigma' + r(\omega^2 - k^2)\sigma - \frac{\lambda_\sigma}{2}r(\sigma^2 - \eta_\sigma^2)\sigma - \beta r\phi^2\sigma = 0$$

Only σ at the infinity has the problem

Rescaling r : 0∼∞, x : 0∼1 $\qquad r = \dfrac{x}{1-x}$

Asymptotic expansion at the infinity

$$\sigma = \sigma_1(x-1) + \sigma_2(x-1)^2 + \sigma_3(x-1)^3 + \cdots$$

Substitute this expansion for the equation of σ at the infinity

$$\left(k^2 - \omega^2 + \beta\eta_\phi^2 - \frac{1}{2}\eta_\sigma^2\lambda_\sigma\right)\sigma_1$$
$$+\left(k^2 - \omega^2 + \beta\eta_\phi^2 - \frac{1}{2}\eta_\sigma^2\lambda_\sigma\right)(\sigma_1 + \sigma_2)(1-x)$$
$$+\left(\left(\sigma_1^2\lambda_\sigma - \frac{2\beta(\sigma_1^2\beta + m^2)}{\lambda_\phi} - 1\right)\sigma_1 + \left(k^2 - \omega^2 + \beta\eta_\phi^2 - \frac{1}{2}\eta_\sigma^2\lambda_\sigma\right)(\sigma_2 + \sigma_3)\right)(1-x)^2$$
$$+ \cdots = 0$$

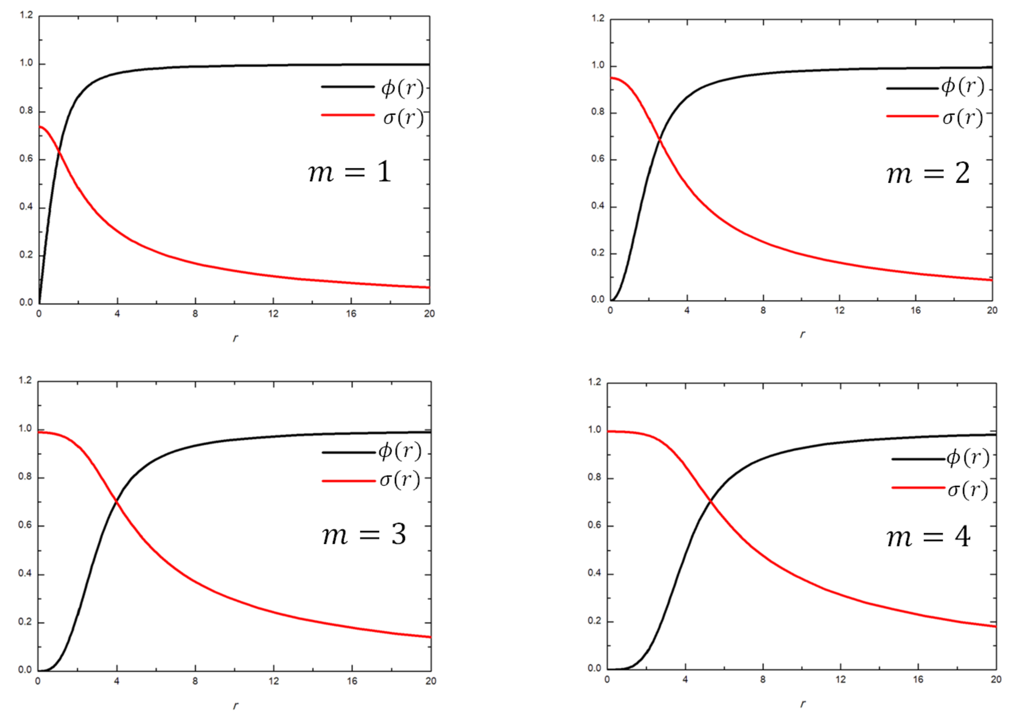
Zeroth order: $\lambda_\sigma = \dfrac{2(k^2 + \beta\eta_\phi^2 - \omega^2)}{\eta_\sigma^2}$

First order: Same as the condition of zeroth order

Second order: $\sigma_1 = -\sqrt{\dfrac{2m^2\beta + \lambda_\phi}{\frac{k^2 - \omega^2 + \beta\eta_\phi^2}{\eta_\sigma^2}\lambda_\phi - 2\beta^2}}$

$$\Rightarrow \frac{k^2 + \beta\eta_\phi^2 - \omega^2}{\eta_\sigma^2}\lambda_\phi > 2\beta^2$$

Higher orders:
Condition of second order ensures their analyticity



$\lambda_\phi = 5.5, \lambda_\sigma = 3.0, \eta_\phi = 1.0, \eta_\sigma = 1.0, \beta = 1.5$

Gauged model

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (D_\mu\phi)^*(D^\mu\phi) + (\partial_\mu\sigma)^*(\partial^\mu\sigma) - U$$

$$D_\mu = (\partial_\mu - igR_\mu)$$
$$F_{\mu\nu} = \partial_\mu R_\nu - \partial_\nu R_\mu$$

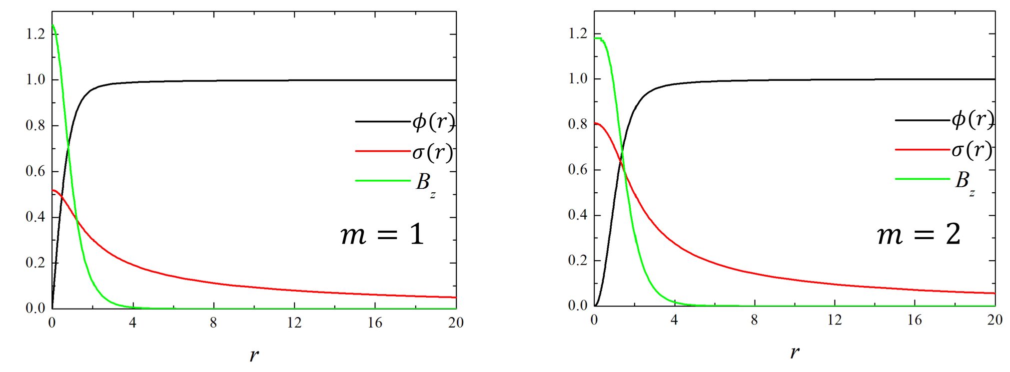
R_μ:Gauge field
g:Gauge coupling constant

Ansatz $\phi(\boldsymbol{r}) = \phi(r)e^{im\theta}$
$\sigma(\boldsymbol{r}) = \sigma(r)e^{i(\omega t + kz)}$
$R_\theta = R(r)$

Boundary consitnios $\phi(0) = 0 \quad \phi(\infty) = \eta_\phi$
$\sigma'(0) = 0 \quad \sigma(\infty) = 0$
$R(0) = 0 \quad R(\infty) = 0$

Equations

$$r\phi'' + \phi' - r\left(\frac{m}{r} - gR\right)^2\phi - \frac{r}{2}\lambda_\phi(\phi^2 - \eta_\phi^2)\phi - r\beta\phi\sigma^2 = 0$$
$$rR'' + R' - \frac{R}{r} + 2rg\left(\frac{m}{r} - gR\right)\phi^2 = 0$$
$$r\sigma'' + \sigma' + r(\omega^2 - k^2)\sigma - \frac{r}{2}\lambda_\sigma(\sigma^2 - \eta_\sigma^2)\sigma - r\beta\phi^2\sigma = 0$$



$\lambda_\phi = 5.5, \lambda_\sigma = 3.0, \eta_\phi = 1.0, \eta_\sigma = 1.0, \beta = 1.5, g = 1.0$

The non-rotationally symmetric solutions

$$\phi(\boldsymbol{r}) = \phi(r)e^{im\theta} \qquad \sigma(\boldsymbol{r}) = \sigma(r)e^{i(\omega t + kz)}$$
$$\Rightarrow \quad \phi(\boldsymbol{r}) = \phi(r,\theta)e^{im\theta} \qquad \sigma(\boldsymbol{r}) = \sigma(r,\theta)e^{i(\omega t + kz)}$$

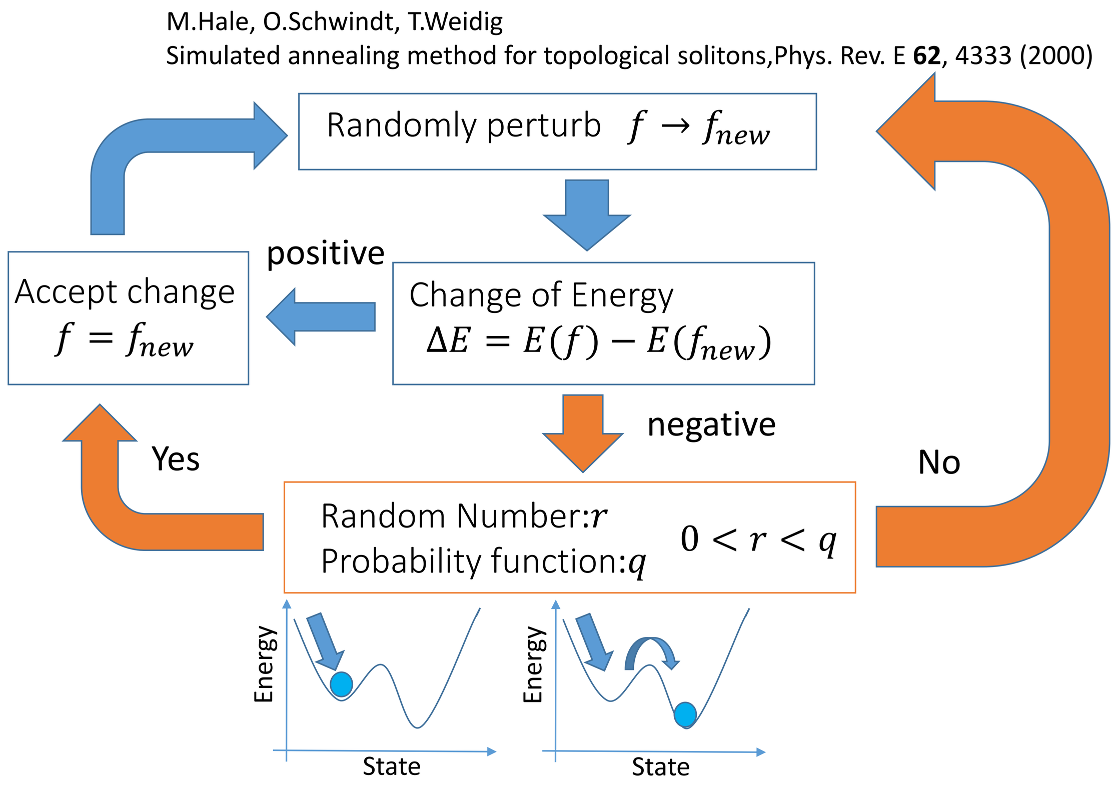
Consider the minimization of the energy on the Cartesian coordinate(x, y)

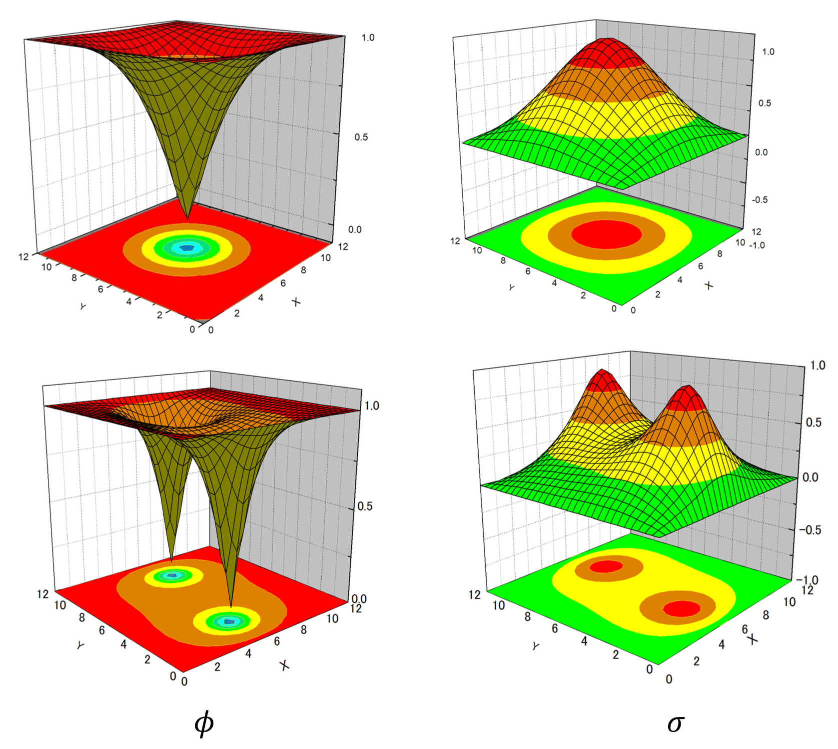
$$\phi \to \phi_R + i\phi_I \qquad \sigma \to \sigma_R + i\sigma_I$$

$$\begin{aligned}\mathcal{H} &= \pi\dot{\sigma} - \mathcal{L} \\ &= \partial_0\sigma\partial^0\sigma^* - \partial_\mu\phi\partial^\mu\phi^* - \partial_\mu\sigma\partial^\mu\sigma^* + U \\ &= (\partial_x\phi_R)^2 + (\partial_x\phi_I)^2 + (\partial_y\phi_R)^2 + (\partial_y\phi_I)^2 \\ &\quad + (\partial_x\sigma_R)^2 + (\partial_x\sigma_I)^2 + (\partial_y\sigma_R)^2 + (\partial_y\sigma_I)^2 + k^2(\sigma_R^2 + \sigma_I^2) + U\end{aligned}$$

Simulated Annealing method

M.Hale, O.Schwindt, T.Weidig
Simulated annealing method for topological solitons,Phys. Rev. E **62**, 4333 (2000)





$\lambda_\phi = 5.5, \lambda_\sigma = 3.0, \eta_\phi = 1.0, \eta_\sigma = 1.0, \beta = 1.5$

A "multi–band" extension of the model

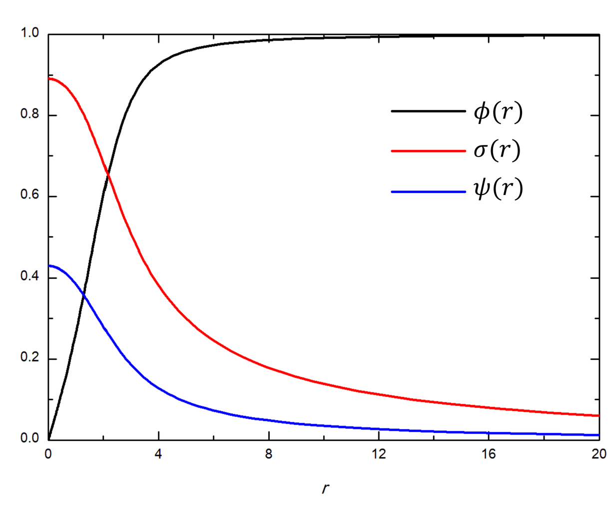
$$\begin{aligned}\mathcal{L} &= (\partial_\mu\phi)^*(\partial^\mu\phi) + (\partial_\mu\sigma)^*(\partial^\mu\sigma) + (\partial_\mu\psi)^*(\partial^\mu\psi) \\ &\quad - \frac{\lambda_\phi}{4}(|\phi|^2 - \eta_\phi^2)^2 - \frac{\lambda_\sigma}{4}(|\sigma|^2 - \eta_\sigma^2)^2 - \frac{\lambda_\psi}{4}(|\psi|^2 - \eta_\psi^2)^2 \\ &\quad - \beta|\phi|^2|\sigma|^2|\psi|^2 - \beta_{12}|\phi|^2|\sigma|^2 - \beta_{13}|\phi|^2|\psi|^2 - \beta_{23}|\sigma|^2|\psi|^2 + \frac{\lambda_\sigma}{4}\eta_\sigma^4 + \frac{\lambda_\psi}{4}\eta_\psi^4\end{aligned}$$

Ansatz
$\phi(\boldsymbol{r}) = \phi(r)e^{im\theta}$
$\sigma(\boldsymbol{r}) = \sigma(r)e^{i(\omega_\sigma t + k_\sigma z)}$
$\psi(\boldsymbol{r}) = \psi(r)e^{i(\omega_\psi t + k_\psi z)}$

Boundary conditions
$\phi(0) = 0 \quad \phi(\infty) = \eta_\phi$
$\sigma'(0) = 0 \quad \sigma(\infty) = 0$
$\psi'(0) = 0 \quad \psi(\infty) = 0$

Equations

$$r\phi'' + \phi' - \frac{m^2}{r}\phi - \frac{\lambda_\phi}{2}r(\phi^2 - \eta_\phi^2)\phi - \beta\phi\sigma^2\psi^2 - \beta_{12}\phi\sigma^2 - \beta_{13}\phi\psi^2 = 0$$
$$r\sigma'' + \sigma' + r(\omega_\sigma^2 - k_\sigma^2)\sigma - \frac{\lambda_\sigma}{2}r(\sigma^2 - \eta_\sigma^2)\sigma - \beta\phi^2\sigma\psi^2 - \beta_{12}\phi^2\sigma - \beta_{23}\sigma\psi^2 = 0$$
$$r\psi'' + \psi' + r(\omega_\psi^2 - k_\psi^2)\psi - \frac{\lambda_\psi}{2}r(\psi^2 - \eta_\psi^2)\psi - \beta\phi^2\sigma^2\psi - \beta_{13}\phi^2\psi - \beta_{23}\sigma^2\psi = 0$$



$\lambda_\phi = 5.5, \lambda_\sigma = 3.0, \lambda_\psi = 2.0, \eta_\phi = 1.0, \eta_\sigma = 1.0, \eta_\psi = 1.0$
$\beta = 1.0, \beta_{12} = 1.5, \beta_{13} = 1.0, \beta_{23} = 0.5$

Summary and further outlooks

For the rotationally symmetric solutions

- the conditions for the parameters which ensure the existence of solutions were obtained by performing the asymptotic expansion.
- the solutions were obtained in both the ungauged model and the gauged model, also with the higher winding number.

For the non-rotationally symmetric solutions

- the new solutions which have multi-center were obtained.
- more new structures may emerge when we take into account the gauge fields and much higher winding number.

For the extension of the model

- the solutions were found in the ungauged model.
- these solutions might be used to describe a complex structure of superconductors such as a gigantic superconductor.