Duality Transformations of GLSM with F-term

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with Shin SASAKI and Masaya YATA (arXiv:1304.4061, 1406.0087, etc.)

GLSM is a powerful tool to investigate non-trivial configurations in string theory :

GLSM = Gauged Linear Sigma Model (2D gauge theory)

Calabi-Yau sigma model and its corresponding CFT

Analysis of (exotic) five-branes

In particular, it is important to study T-duality of their configurations.

(related to stringy “nongeometric” backgrounds)

Images illustrated by YATA

T-duality

might be violated if an F-term exists.

We would like to find a consistent formula to perform T-duality

even in the presence of F-terms.

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- Duality transformations with F-term
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Duality transformations with F-term

in $\mathcal{N} = (2, 2)$ framework

Duality : chiral $\Psi \longleftrightarrow$ twisted chiral Y

Target space geometry is T-dualized under the above transformation.

Duality : chiral $\Psi \longleftrightarrow$ twisted chiral Y

Target space geometry is T-dualized under the above transformation.

- dualize neutral chiral in D-term (ex. torus)

$$\int d^4\theta \, |\Psi|^2 \leftrightarrow \int d^4\theta \left[\frac{1}{2}R^2 - R(Y + \overline{Y})\right] \leftrightarrow -\int d^4\theta \, |Y|^2$$

- dualize charged chiral in D-term (ex. projective space)

$$\int d^4\theta \, |\Psi|^2 \, e^{2V} \leftrightarrow \int d^4\theta \left[e^{2V+R} - R(Y + \overline{Y})\right] \leftrightarrow -\int d^4\theta \, (Y + \overline{Y})\Big[\log(Y + \overline{Y}) - 2V\Big]$$

These are well established.

"Global symmetry" $\Psi + \alpha$ (or $\mathrm{e}^{\mathrm{i}\alpha}\,\Psi$) is preserved in D-term, but broken in F-term.

— Make sense? —

► dualize neutral / charged chiral in D-term and F-term

They can be interpreted as T-duality transformations

under conversion from F-term to D-term with trick(es).

- Dualize neutral chiral in D-term and F-term

$$\mathscr{L}_1 \;=\; \int \mathrm{d}^4\theta \, |\Psi|^2 + \Big\{ \int \mathrm{d}^2\theta \, \Psi \, \mathcal{W}(\Phi) + (\mathrm{h.c.}) \Big\}$$

- Dualize neutral chiral in D-term and F-term

$$\mathscr{L}_1 = \int d^4\theta \, |\Psi|^2 + \Big\{ \int d^2\theta \, \Psi \, \mathcal{W}(\Phi) + (\text{h.c.}) \Big\}$$

1. Convert F-term to D-term via $\mathcal{W} = \overline{D}_+ \overline{D}_- C$:

$$\int d^2\theta \, \Psi \, \mathcal{W} + (\text{h.c.}) = \int d^4\theta \Big[(\Psi + \overline{\Psi})(C + \overline{C}) + (\Psi - \overline{\Psi})(C - \overline{C}) \Big]$$

- Dualize neutral chiral in D-term and F-term

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2. Replace $\Psi \pm \overline{\Psi}$ to auxiliary fields R and $\mathrm{i}S$:

$$\mathscr{L}_2 = \int \mathrm{d}^4\theta \, \Big[\frac{1}{2} R^2 + R(C + \overline{C}) + \mathrm{i}S(C - \overline{C}) - R(Y + \overline{Y}) - \mathrm{i}S(\Upsilon - \overline{\Upsilon}) \Big]$$

Dualize neutral chiral in D-term and F-term

$$\mathscr{L}_1 = \int \mathrm{d}^4\theta\, |\Psi|^2 + \Big\{ \int \mathrm{d}^2\theta\, \Psi\, \mathcal{W}(\Phi) + (\mathrm{h.c.}) \Big\}$$

1. Convert F-term to D-term via $\mathcal{W} = \overline{D}_+ \overline{D}_- C$:

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3. Integrating out R and Υ, we obtain the "dual" system :

$$\mathscr{L}_3 = \int \mathrm{d}^4\theta \Big[-\frac{1}{2} \Big((Y + \overline{Y}) + (C + \overline{C}) \Big)^2 + (\Psi - \overline{\Psi})(C - \overline{C}) \Big]$$

Instead, integrate out Y and $\Upsilon \to \mathscr{L}_1$ appears

- Dualize charged chiral in D-term and F-term

$$\mathscr{L}_4 = \int \mathrm{d}^4\theta \, |\Psi|^2 \, \mathrm{e}^{2V} + \Big\{ \int \mathrm{d}^2\theta \, \Psi \, \mathcal{W}(\Phi) + (\mathrm{h.c.}) \Big\}$$

▶ Dualize charged chiral in D-term and F-term

$$\mathscr{L}_4 = \int \mathrm{d}^4\theta \, |\Psi|^2 \, \mathrm{e}^{2V} + \Big\{ \int \mathrm{d}^2\theta \, \Psi \, \mathcal{W}(\Phi) + (\text{h.c.}) \Big\}$$

1. Convert F-term to D-term via $\mathcal{W} = \overline{D}_+ \overline{D}_- C$:

$$\int \mathrm{d}^2\theta \, \Psi \, \mathcal{W} + (\text{h.c.}) = 2 \int \mathrm{d}^4\theta \, \Big[\Psi C + \overline{\Psi C} \Big]$$

2. Replace $\Psi \pm \overline{\Psi}$ to auxiliary fields R and $\mathrm{i}S$:

$$\mathscr{L}_5 = \int \mathrm{d}^4\theta \, \Big[\mathrm{e}^{2V+R} + 2\Big\{ \mathrm{e}^{\frac{1}{2}(R+\mathrm{i}S)} C + \mathrm{e}^{\frac{1}{2}(R-\mathrm{i}S)} \overline{C} \Big\} - R(Y + \overline{Y}) - \mathrm{i}S(\Upsilon - \overline{\Upsilon}) \Big]$$

3. Integrating out R and Υ, we obtain the “dual” system :

$$\mathscr{L}_6 = \int \mathrm{d}^4\theta \, \Big[-2(Y + \overline{Y}) \log \mathcal{F} + \frac{1}{2} \mathcal{F} \mathcal{T} + 2V(Y + \overline{Y}) \Big]$$

$$\mathcal{F} = -\mathcal{T} + \sqrt{\mathcal{T}^2 + 4(Y + \overline{Y})}, \quad \mathcal{T} = \mathrm{e}^{-V} \Big[\mathrm{e}^{+\frac{1}{2}(\Omega - \overline{\Omega})} C + \mathrm{e}^{-\frac{1}{2}(\Omega - \overline{\Omega})} \overline{C} \Big], \quad \Psi = \mathrm{e}^{\Omega}$$

$\Psi - \overline{\Psi}$ (or $e^{\Omega - \overline{\Omega}}$) still remains after the transformation.

The existence is rather important to complete the Duality transformations.

$\Psi - \overline{\Psi}$ appears as an auxiliary field, which must be removed finally.

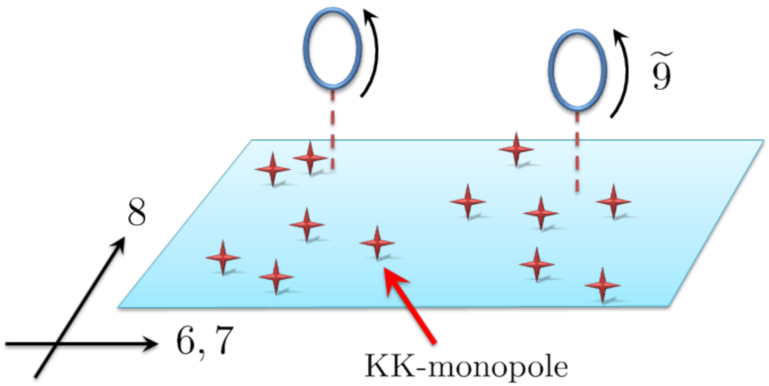
Indeed, the procedure "integrating-out of $\Psi - \overline{\Psi}$" leads to

the correct involution of the dual fields in the system *!*

Examples

1. KK-monopoles (neutral chiral)
2. ALE space (charged chiral)

as $\mathcal{N} = (4,4)$ GLSMs with F-term



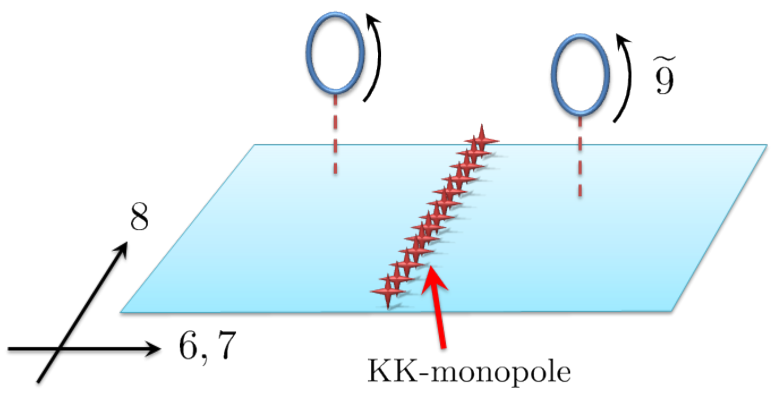
$$
\begin{aligned}
\mathscr{L} \;=\; & \sum_a \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e_a^2} (|\Phi_a|^2 - |\Sigma_a|^2) + |Q_a|^2 \, \mathrm{e}^{-2V_a} + |\widetilde{Q}_a|^2 \, \mathrm{e}^{2V_a} \Big\} \\
& + \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{g^2} |\Psi|^2 + \frac{g^2}{2} (\Gamma + \overline{\Gamma} + 2 \sum_a V_a)^2 \Big\} \\
& + \sum_a \Big\{ \int \mathrm{d}^2\theta \, \Big(\widetilde{Q}_a \Phi_a Q_a + (s_a - \Psi) \Phi_a \Big) + (\text{h.c.}) \Big\} \\
& + \sum_a \Big\{ \int \mathrm{d}^2\widetilde{\theta} \, (t_a \Sigma_a) + (\text{h.c.}) \Big\}
\end{aligned}
$$

Tong (2002)

$(6, 8)$-,$(7, \widetilde{9})$-directions are described by chirals Ψ, Γ

Sasaki and TK; arXiv:1304.4061

Talk by Sasaki and QFT2013 @ YITP



$$
\begin{aligned}
\mathscr{L} &= \sum_a \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e_a^2} (|\Phi_a|^2 - |\Sigma_a|^2) + |Q_a|^2 \, \mathrm{e}^{-2V_a} + |\widetilde{Q}_a|^2 \, \mathrm{e}^{2V_a} \Big\} \\
&\quad + \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{g^2} |\Psi|^2 + \frac{g^2}{2} (\Gamma + \overline{\Gamma} + 2 \sum_a V_a)^2 \Big\} \\
&\quad + \sum_a \Big\{ \int \mathrm{d}^2\theta \, \Big(\widetilde{Q}_a \Phi_a Q_a + (s_a - \Psi) \Phi_a \Big) + (\text{h.c.}) \Big\} \\
&\quad + \sum_a \Big\{ \int \mathrm{d}^2\widetilde{\theta} \, (t_a \Sigma_a) + (\text{h.c.}) \Big\}
\end{aligned}
$$

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$(6,8)$-,$(7,\widetilde{9})$-directions are described by chirals Ψ, Γ

Each position is labelled by FI parameters (s_a, t_a)

Sasaki and TK; arXiv:1304.4061

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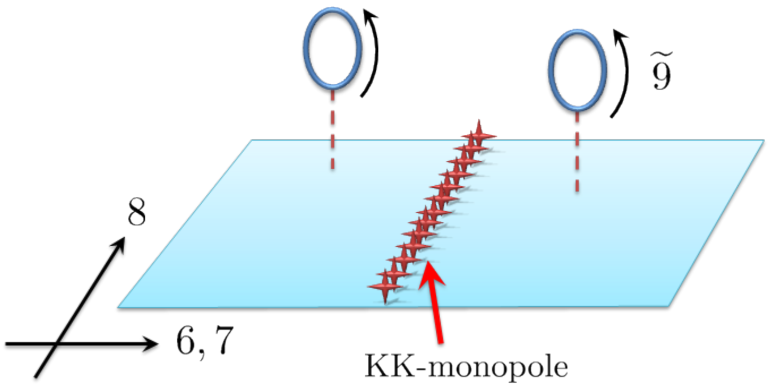


$$\begin{aligned}\mathscr{L} \ &= \sum_a \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e_a^2} (|\Phi_a|^2 - |\Sigma_a|^2) + |Q_a|^2 \, \mathrm{e}^{-2V_a} + |\widetilde{Q}_a|^2 \, \mathrm{e}^{2V_a} \Big\} \\ &\quad + \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{g^2} |\Psi|^2 + \frac{g^2}{2} (\Gamma + \overline{\Gamma} + 2 \sum_a V_a)^2 \Big\} \\ &\quad + \sum_a \Big\{ \int \mathrm{d}^2\theta \, \Big(\widetilde{Q}_a \Phi_a Q_a + (s_a - \Psi) \Phi_a \Big) + (\text{h.c.}) \Big\} \\ &\quad + \sum_a \Big\{ \int \mathrm{d}^2\widetilde{\theta} \, (t_a \Sigma_a) + (\text{h.c.}) \Big\}\end{aligned}$$

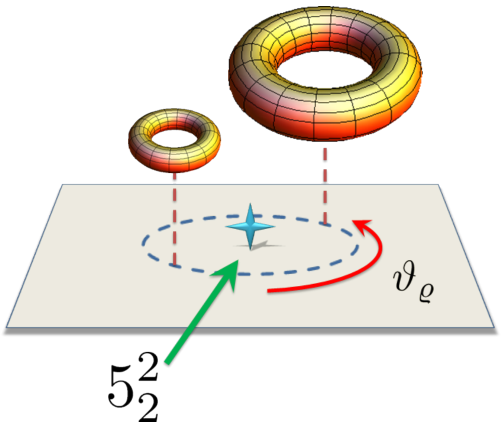
Tong (2002)

$(6,8)$-,$(7,\widetilde{9})$-directions are described by chirals Ψ, Γ

Dualize the neutral chiral $\Psi \to \Xi$: T-duality along 8th-direction

Sasaki and TK; arXiv:1304.4061

Talk by Sasaki and QFT2013 @ YITP



$$
\begin{aligned}
\mathscr{L}' \;=\; & \sum_a \int \mathrm{d}^4\theta \, \Big\{ \frac{1}{e_a^2} (|\Phi_a|^2 - |\Sigma_a|^2) + |Q_a|^2 \, \mathrm{e}^{-2V_a} + |\widetilde{Q}_a|^2 \, \mathrm{e}^{2V_a} \Big\} \\
& + \int \mathrm{d}^4\theta \, \Big\{ - \frac{g^2}{2} (\Xi + \overline{\Xi} - \sum_a (C_a + \overline{C}_a))^2 + \frac{g^2}{2} (\Gamma + \overline{\Gamma} + 2 \sum_a V_a)^2 \Big\} \\
& + \sum_a \Big\{ \int \mathrm{d}^2\theta \, \Big(\widetilde{Q}_a \Phi_a Q_a + s_a \Phi_a \Big) + (\text{h.c.}) \Big\} \\
& + \sum_a \Big\{ \int \mathrm{d}^2\widetilde{\theta} \, (t_a \Sigma_a) + (\text{h.c.}) \Big\} - \int \mathrm{d}^4\theta \, (\Psi - \overline{\Psi}) \sum_a (C_a - \overline{C}_a)
\end{aligned}
$$

$(6, \widetilde{8})$-,$(7, \widetilde{9})$-directions are described by (twisted) chirals Ξ, Γ

Integrate out $\Psi - \overline{\Psi}$: Exotic Five-brane!

Sasaki and TK; arXiv:1304.4061

Talk by Sasaki and QFT2013 @ YITP

$$
\begin{aligned}
\mathscr{L} \;=\; & \int \mathrm{d}^4\theta \, \Big[\frac{1}{e^2} \Big(-|\Sigma|^2 + |\Phi|^2 \Big) + \frac{1}{\widetilde{e}^2} \Big(-|\widetilde{\Sigma}|^2 + |\widetilde{\Phi}|^2 \Big) \Big] \\
& + \int \mathrm{d}^4\theta \, \Big[|A_1|^2 \mathrm{e}^{2V} + |A_2|^2 \mathrm{e}^{-4V - 2\widetilde{V}} + |A_3|^2 \mathrm{e}^{2V} \Big] + \int \mathrm{d}^4\theta \, \Big[|B_1|^2 \mathrm{e}^{-2V} + |B_2|^2 \mathrm{e}^{4V + 2\widetilde{V}} + |B_3|^2 \mathrm{e}^{-2V} \Big] \\
& + \Big\{ \int \mathrm{d}^2\theta \, \Big(\Phi(-A_1 B_1 + 2 A_2 B_2 - A_3 B_3) + \widetilde{\Phi} A_2 B_2 \Big) + (\text{h.c.}) \Big\} \\
& - \Big\{ \int \mathrm{d}^2\widetilde{\theta} \, (t\Sigma) + (\text{h.c.}) \Big\}
\end{aligned}
$$

toric data	(A_1, B_1)	(A_2, B_2)	(A_3, B_3)
$U(1)(V, \Phi)$	$(+1, -1)$	$(-2, +2)$	$(+1, -1)$
$U(1)(\widetilde{V}, \widetilde{\Phi})$	$(0, 0)$	$(-1, +1)$	$(0, 0)$

Yata and TK; arXiv:1406.0087

YATA's Poster (Friday)

$$\begin{aligned}
\mathscr{L} \ &= \ \int \mathrm{d}^4\theta \, \Big[\frac{1}{e^2} \Big(- |\Sigma|^2 + |\Phi|^2 \Big) + \frac{1}{\widetilde{e}^2} \Big(- |\widetilde{\Sigma}|^2 + |\widetilde{\Phi}|^2 \Big) \Big] \\
&\quad + \int \mathrm{d}^4\theta \, \Big[|A_1|^2 \mathrm{e}^{2V} + |A_2|^2 \mathrm{e}^{-4V - 2\widetilde{V}} + |A_3|^2 \mathrm{e}^{2V} \Big] + \int \mathrm{d}^4\theta \, \Big[|B_1|^2 \mathrm{e}^{-2V} + |B_2|^2 \mathrm{e}^{4V + 2\widetilde{V}} + |B_3|^2 \mathrm{e}^{-2V} \Big] \\
&\quad + \Big\{ \int \mathrm{d}^2\theta \, \Big(\Phi(-A_1 B_1 + 2 A_2 B_2 - A_3 B_3) + \widetilde{\Phi} A_2 B_2 \Big) + (\mathrm{h.c.}) \Big\} \\
&\quad - \Big\{ \int \mathrm{d}^2\widetilde{\theta} \, (t \Sigma) + (\mathrm{h.c.}) \Big\}
\end{aligned}$$

$\downarrow$

$$\begin{aligned}
\mathscr{L}_{\mathrm{NLSM}} \ &= \ - \frac{\mathcal{A}^{-1}}{2} (\partial_m \rho)^2 - \frac{\rho^2}{8} \Big\{ (\partial_m \vartheta)^2 + (\partial_m \varphi)^2 \sin^2 \vartheta \Big\} - \frac{\rho^2 \mathcal{A}}{8} \Big\{ \partial_m \psi + (\partial_m \varphi) \cos \vartheta \Big\}^2 \\
\mathcal{A} \ &= \ 1 - \frac{\mathfrak{a}^4}{\rho^4}
\end{aligned}$$

Yata and TK; arXiv:1406.0087

YATA's Poster (Friday)

$$
\begin{aligned}
\mathscr{L} \;=\; & \int \mathrm{d}^4\theta \, \Big[\frac{1}{e^2} \Big(- |\Sigma|^2 + |\Phi|^2 \Big) + \frac{1}{\widetilde{e}^2} \Big(- |\widetilde{\Sigma}|^2 + |\widetilde{\Phi}|^2 \Big) \Big] \\
& + \int \mathrm{d}^4\theta \, \Big[|A_1|^2 \mathrm{e}^{2V} + |A_2|^2 \mathrm{e}^{-4V - 2\widetilde{V}} + |A_3|^2 \mathrm{e}^{2V} \Big] + \int \mathrm{d}^4\theta \, \Big[|B_1|^2 \mathrm{e}^{-2V} + |B_2|^2 \mathrm{e}^{4V + 2\widetilde{V}} + |B_3|^2 \mathrm{e}^{-2V} \Big] \\
& + \Big\{ \int \mathrm{d}^2\theta \, \Big(\Phi(-A_1 B_1 + 2 A_2 B_2 - A_3 B_3) + \widetilde{\Phi} A_2 B_2 \Big) + (\text{h.c.}) \Big\} \\
& - \Big\{ \int \mathrm{d}^2\widetilde{\theta} \, (t\Sigma) + (\text{h.c.}) \Big\}
\end{aligned}
$$

Dualize charged chiral $A_1 \to Y_1$

Yata and TK; arXiv:1406.0087

YATA's Poster (Friday)

$$
\begin{aligned}
\mathscr{L}' \;=\; & \int \mathrm{d}^4\theta \, \Big[\frac{1}{e^2} \Big(- |\Sigma|^2 + |\Phi|^2 \Big) + \frac{1}{\widetilde{e}^2} \Big(- |\widetilde{\Sigma}|^2 + |\widetilde{\Phi}|^2 \Big) \Big] \\
& + \int \mathrm{d}^4\theta \, \Big[|A_2|^2 \mathrm{e}^{-4V - 2\widetilde{V}} + |A_3|^2 \mathrm{e}^{2V} \Big] + \int \mathrm{d}^4\theta \, \Big[|B_1|^2 \mathrm{e}^{-2V} + |B_2|^2 \mathrm{e}^{4V + 2\widetilde{V}} + |B_3|^2 \mathrm{e}^{-2V} \Big] \\
& + \Big\{ \int \mathrm{d}^2\theta \, \Big(\Phi (2 A_2 B_2 - A_3 B_3) + \widetilde{\Phi} A_2 B_2 \Big) + (\mathrm{h.c.}) \Big\} \\
& + \Big\{ \int \mathrm{d}^2\widetilde{\theta} \, (-2 Y_1 - t) \Sigma + (\mathrm{h.c.}) \Big\} + \int \mathrm{d}^4\theta \, \Big[4 (Y_1 + \overline{Y}_1) \log \mathcal{F}_1 + \frac{1}{2} \mathcal{F}_1 \mathcal{T}_1 \Big]
\end{aligned}
$$

Integrate out $\mathrm{e}^{\Omega_1 - \overline{\Omega}_1}$ in $\mathcal{T}_1$ with $A_1 \;=\; \mathrm{e}^{\Omega_1}$: T-dualized configuration

$$
\begin{aligned}
\mathrm{d}s^2 \;=\; & -\frac{\mathcal{A}^{-1}}{2} (\partial_m \rho)^2 - \frac{\rho^2}{8} (\partial_m \vartheta)^2 - \frac{\rho^2}{4} \frac{\mathcal{A} \sin^2 \vartheta}{\mathcal{A} \cos^2 \vartheta + \sin^2 \vartheta} (\partial_m \psi)^2 - \frac{2}{\rho^2} \frac{1}{\mathcal{A} \cos^2 \vartheta + \sin^2 \vartheta} (\partial_m \widetilde{\varphi})^2 \\
& + \frac{\mathcal{A} \cos \vartheta}{\mathcal{A} \cos^2 \vartheta + \sin^2 \vartheta} \, \varepsilon^{mn} \, (\partial_m \widetilde{\varphi})(\partial_n \psi)
\end{aligned}
$$

$\leftarrow$ B-field for 2 units of NS5-branes

Yata and TK; arXiv:1406.0087

YATA's Poster (Friday)

Summary

Summary

✓ Dualized neutral / charged chiral multiplets in D-term and F-term

✓ Applied to $\mathcal{N} = (4,4)$ GLSMs with F-term

✓ Calabi-Yau and its T-duality, revisited

Conifold $\leftrightarrow$ Intersecting NS5-branes

✓ GLSM for exotic 5^2_2-brane with D-branes

Brane construction with string dualities

We would like to see a deeper insight of exotic five-branes

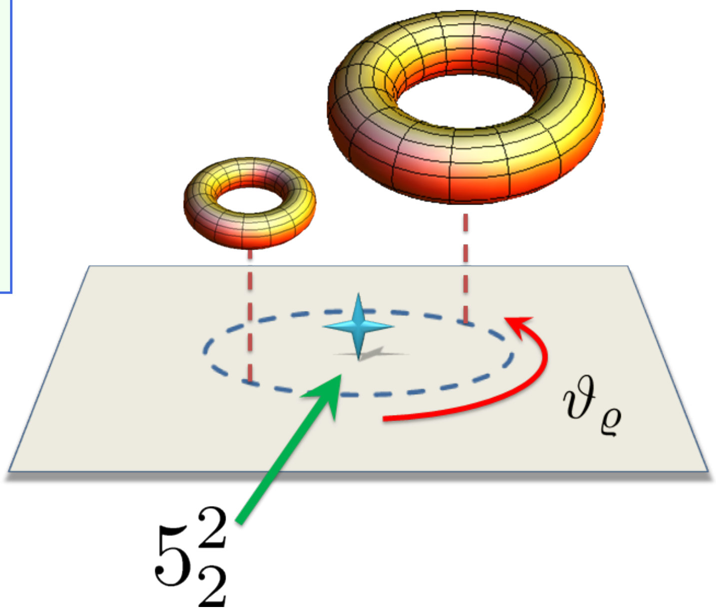
from various viewpoints !

Thanks

Continue to the POSTER Presentation by Masaya YATA

Appendix

$$
\begin{aligned}
ds^2 &= dx^2_{012345} + H\big[d\varrho^2 + \varrho^2(d\vartheta_\varrho)^2\big] + \frac{H}{K}\big[(d\widetilde{x}^8)^2 + (d\widetilde{x}^9)^2\big] \\
B_{89} &= -\frac{\sigma\,\vartheta_\varrho}{K}, \quad e^{2\Phi} = \frac{H}{K}, \quad K = H^2 + (\sigma\vartheta_\varrho)^2 \\
H &= h + \sigma \log\Big(\frac{\mu}{\varrho}\Big)
\end{aligned}
$$



$$\vartheta_\varrho = 0 \quad : \quad G_{88} = G_{99} = \frac{1}{H}$$

$$\vartheta_\varrho = 2\pi \quad : \quad G_{88} = G_{99} = \frac{H}{H^2 + (2\pi\sigma)^2}$$

$$
\begin{aligned}
\mathscr{L} \;=\; & \int \mathrm{d}^4\theta \left[\frac{1}{e^2}\Big(- |\Sigma|^2 + |\Phi|^2\Big) + \frac{1}{\widetilde{e}^2}\Big(- |\widetilde{\Sigma}|^2 + |\widetilde{\Phi}|^2\Big)\right] \\
& + \int \mathrm{d}^4\theta \left[|A_1|^2 \mathrm{e}^{2V} + |A_2|^2 \mathrm{e}^{-4V-2\widetilde{V}} + |A_3|^2 \mathrm{e}^{2V}\right] + \int \mathrm{d}^4\theta \left[|B_1|^2 \mathrm{e}^{-2V} + |B_2|^2 \mathrm{e}^{4V+2\widetilde{V}} + |B_3|^2 \mathrm{e}^{-2V}\right] \\
& + \Big\{ \int \mathrm{d}^2\theta \, \Big(\Phi(-A_1 B_1 + 2A_2 B_2 - A_3 B_3) + \widetilde{\Phi} A_2 B_2\Big) + (\mathrm{h.c.})\Big\} \\
& - \Big\{ \int \mathrm{d}^2\widetilde{\theta} \, (t\Sigma) + (\mathrm{h.c.})\Big\}
\end{aligned}
$$

$$
\begin{aligned}
A_1 &\simeq \frac{r}{2} \cos\frac{\vartheta}{2}\, \mathrm{e}^{\frac{\mathrm{i}}{2}(\psi+\varphi)} & A_3 &\simeq \frac{r}{2} \sin\frac{\vartheta}{2}\, \mathrm{e}^{\frac{\mathrm{i}}{2}(\psi-\varphi)} \\
B_1 &\simeq \frac{r}{2} \sin\frac{\vartheta}{2}\, \mathrm{e}^{\frac{\mathrm{i}}{2}(\psi-\varphi)} & B_3 &\simeq -\frac{r}{2} \cos\frac{\vartheta}{2}\, \mathrm{e}^{\frac{\mathrm{i}}{2}(\psi+\varphi)}
\end{aligned}
$$

$$
\rho^4 = \mathfrak{a}^4 + r^4
$$

$$
\begin{aligned}
\mathscr{L}'_{\mathrm{NLSM}} \;&=\; 2\int \mathrm{d}^4\theta\,(Y_1+\overline{Y}_1)\Big[\log|A_3|^2 - \log|B_1|^2 + \log|B_3|^2\Big] \\
&\quad -t_1\int \mathrm{d}^4\theta\,\log\big(|B_1|^2+|B_3|^2\big) \\
&\quad +\int \mathrm{d}^4\theta\,\sqrt{t_1^2 + \frac{4|B_3|^2}{|B_1|^2}\big(|B_1|^2+|B_3|^2\big)^2} \\
&\quad +t_1\int \mathrm{d}^4\theta\,\log\Big\{t_1^2 + \frac{4|B_3|^2}{|B_1|^2}\big(|B_1|^2+|B_3|^2\big)^2\Big\}
\end{aligned}
$$