Nonrelativistic Nambu-Goldstone modes localized around topological solitons

July 25/2014 Strings and Fields 2014 @ YITP

Muneto Nitta (Keio U.)

Topological Quantum Phenomena in
Condensed Matter with Broken Symmetries

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Michikazu Kobayashi (Kyoto U.),
Daisuke A. Takahashi (RIKEN･Keio U.)

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[1] M.Kobayashi & MN, PTEP:021B01,2014 [arXiv:1307.6632]
[2] M.Kobayashi & MN, arXiv:1402.6826 [hep-th],
[3] M.Kobayashi & MN, Phys.Rev.D arXiv:1403.4031 [hep-th]
[4] D.A.Takahashi & MN, arXiv:1404.7696[cond-mat.quant-gas]
[5] MN, S.Uchino & W.Vinci, arXiv:1311.5408 [hep-th]
[6] D.A.Takahashi,M.Kobayashi &MN, in preparation

Number of Nambu-Goldstone(NG) modes

type-I (A) $\omega \sim k$ # $= N_{\mathrm{I}}$

type-II (B) $\omega \sim k^2$ # $= N_{\mathrm{II}}$

Only type I in relativistic theories

non-relativistic theories

Nielsen-Chadha inequality Nielsen-Chadha('76),Nambu('04)

$$N_{\mathrm{I}} + 2N_{\mathrm{II}} \geq N_{\mathrm{BG}} = \#\,\text{broken generators}$$

$$N_{\mathrm{II}} = N_{\mathrm{BG}} - N_{\mathrm{NG}} = \frac{1}{2}\operatorname{rank}\rho$$

$$\rho_{ij} = \langle \mathrm{GS} | [T_i, T_j] | \mathrm{GS} \rangle$$

WB matrix

Watanabe-Brauner(WB) relation

Nambu ('01,'04)

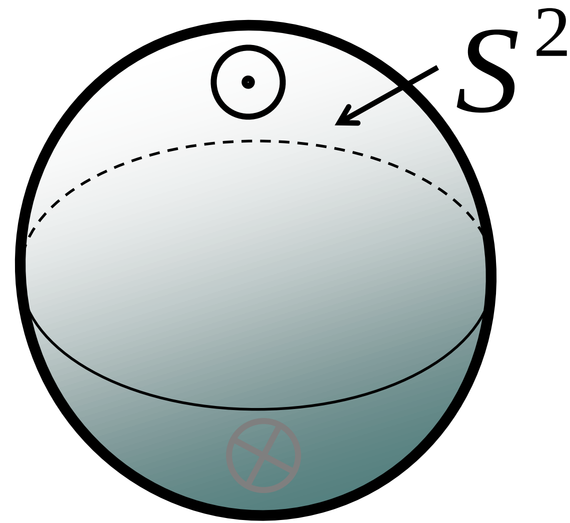
Watanabe-Brauner('11), Watanabe-Murayama('12), Hidaka('12)

Number of Nambu-Goldstone(NG) modes

type-I (A) $\omega \sim k$ $\# = N_{\mathrm{I}}$

type-II (B) $\omega \sim k^2$ $\# = N_{\mathrm{II}}$

example **Heisenberg magnets**



$$SO(3)/SO(2) = S^2 \quad N_{\mathrm{BG}} = 2$$

$$\langle \mathrm{GS} | [S_x, S_y] | \mathrm{GS} \rangle = i \langle \mathrm{GS} | S_z | \mathrm{GS} \rangle$$

= 0 **Anti-ferro** $N_{\mathrm{I}} = 2,\ N_{\mathrm{II}} = 0$ $N_{\mathrm{NG}} = 2$

↑↓↑↓↑↓↑↓

≠ 0 **Ferro** $N_{\mathrm{I}} = 0,\ N_{\mathrm{II}} = 1$ $N_{\mathrm{NG}} = 1$

↑↑↑↑↑↑↑↑

Anti-ferromagnetic Heisenberg model (in continuum limit)

O(3) nonlinear sigma model or $\mathbf{C}P^1$ model

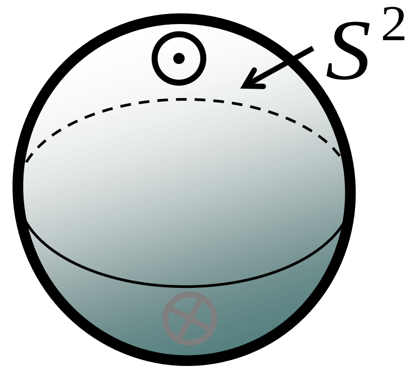
$$SO(3)/SO(2) = S^2 \cong \mathbf{C}P^1$$

$$\mathcal{L}_{\mathrm{nrel}} = \frac{i(u^*\dot{u} - \dot{u}^* u)}{2(1+|u|^2)} - \frac{|\nabla u|^2}{(1+|u|^2)^2}$$

Berry phase

Relativistic version

$$\mathcal{L}_{\mathrm{rel}} = \frac{|\dot{u}|^2 - |\nabla u|^2}{(1+|u|^2)^2}$$



momentum

$$v \overset{\text{nrel}}{=} \frac{\partial \mathcal{L}_{\text{nrel}}}{\partial \dot{u}} = \frac{i u^*}{2(1+|u|^2)}$$

u and u^* are **momentum conjugate**!

→ **Only 1 NG mode** is independent

Cf: relativistic $v \overset{\text{rel}}{=} \frac{\partial \mathcal{L}_{\text{rel}}}{\partial \dot{u}} = \frac{\dot{u}^*}{(1+|u|^2)^2}$

More generally $\omega = g(I*,*) = d\alpha$ Kahler 2-form

$\alpha = pdq$ (p, q) symplectic pair

Classification of NG modes completed for internal symmetry

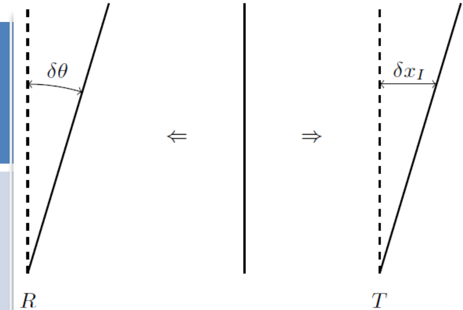
NG modes localized around vortices,solitons

but not yet for space-time symmetry

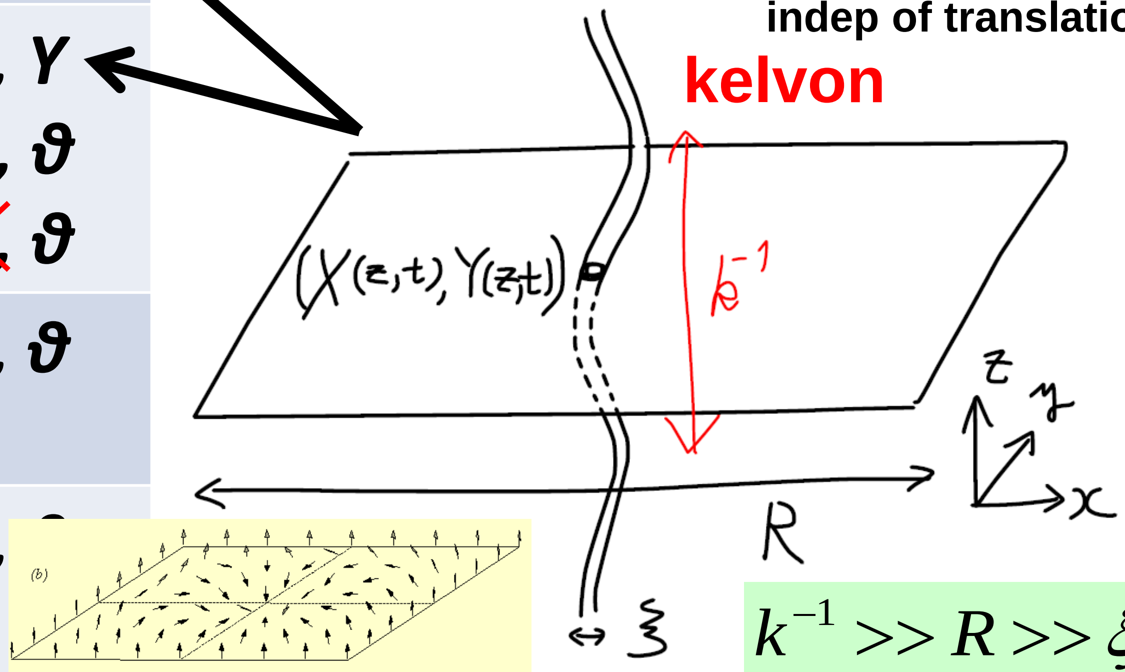
objects	systems
Quantized vortex	Superfluid He, BEC
Domain wall	Anisotropic ferromagnets 2 component BEC
Skyrmion lines	Isotropic ferromagnets
Non-Abelian vortices	Multicomponent BEC

(1)Finite *R*	Broken sym	NG type	Dispersion for finite *R*	
Vortex line in superfluid	*X*, *Y*	II	$e \sim \log R\ \ k^2$ **kelvon**	e.g. Kobayashi & MN ('13.07)
Skyrmion line	*X*, *Y*	II	$e \sim k^2$ **kelvon**	Well-known
(scale inv)	*D*, ϑ	II	$e \sim k^2$ **dilaton-magnon**	Kobayashi & MN ('14.03)
(scale violated)	~~*D*~~, ϑ	I	$e \sim k$	
Domain wall in ferromagnet	*X*, ϑ	II	$e \sim k^2$ **ripplon-magnon**	Kobayashi & MN ('14.02)
Domain wall in 2comp BEC	*X*, ϑ	II	$e \sim R^{1/2} k^2$ **ripplon-magnon**	Takeuchi & Kasamatsu('13.09) well-known for ripplon

(1)Finite *R*	Broken sym	NG type	Dispersion for finite *R*
Vortex line in superfluid	*X, Y*	II	$e \sim \log R \ \ k^2$
Skyrmion line (scale inv) (scale violated)	*X, Y* *D*, ϑ ~~*D*~~, ϑ		
Domain wall in ferromagnet	*X*, ϑ		
Domain wall in 2comp BEC	*X*,		



Rotation is not indep of translation



$$k^{-1} >> R >> \xi$$

(1)Finite *R*	Broken sym	NG type	Dispersion for finite *R*
Vortex line in superfluid	*X, Y*	II	$e \sim \log R \;\; k^2$
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Domain wall in ferromagnet	*X*, ϑ		
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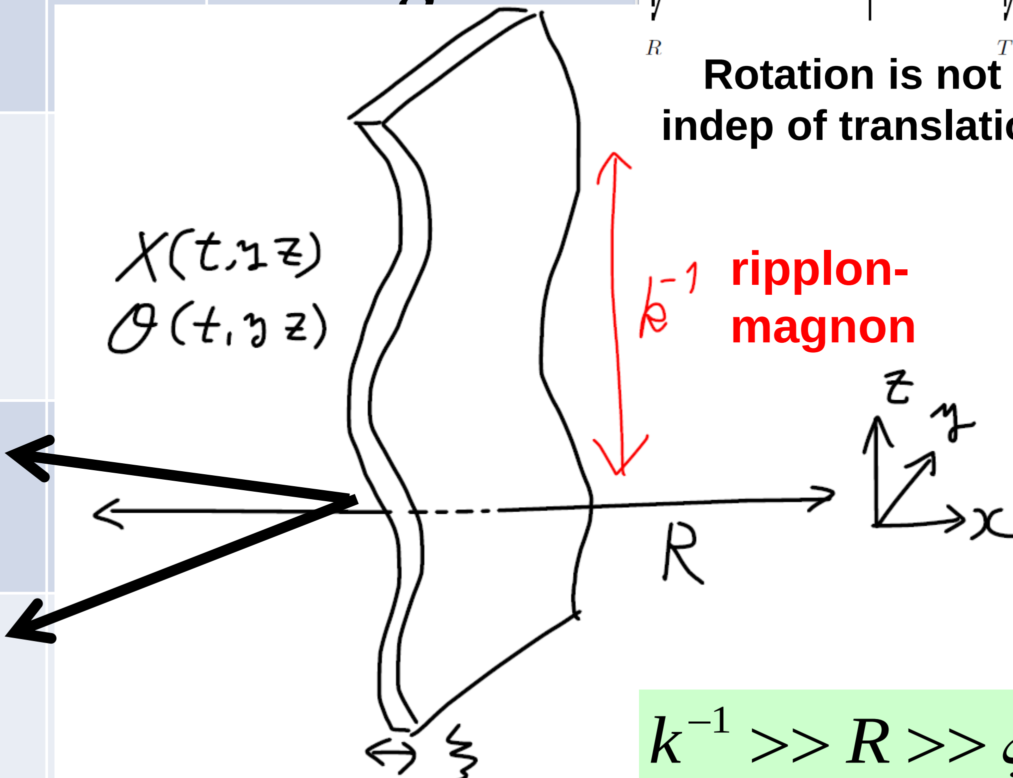

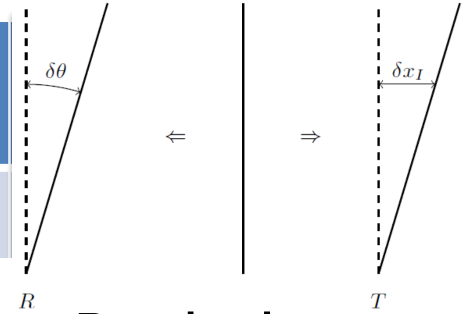

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$$k^{-1} >> R >> \xi$$

(1)Finite *R*	Broken sym	NG type	Dispersion for finite *R*	
Vortex line in superfluid	X, Y	II	$e \sim \log R \;\; k^2$ kelvon	e.g. Kobayashi & MN ('13.07)
Skyrmion line	X, Y	II	$e \sim k^2$ kelvon	Well-known
(scale inv)	D, ϑ	II	$e \sim k^2$ dilaton-magnon	Kobayashi & MN ('14.03)
(scale violated)	~~D~~, ϑ	I	$e \sim k$	
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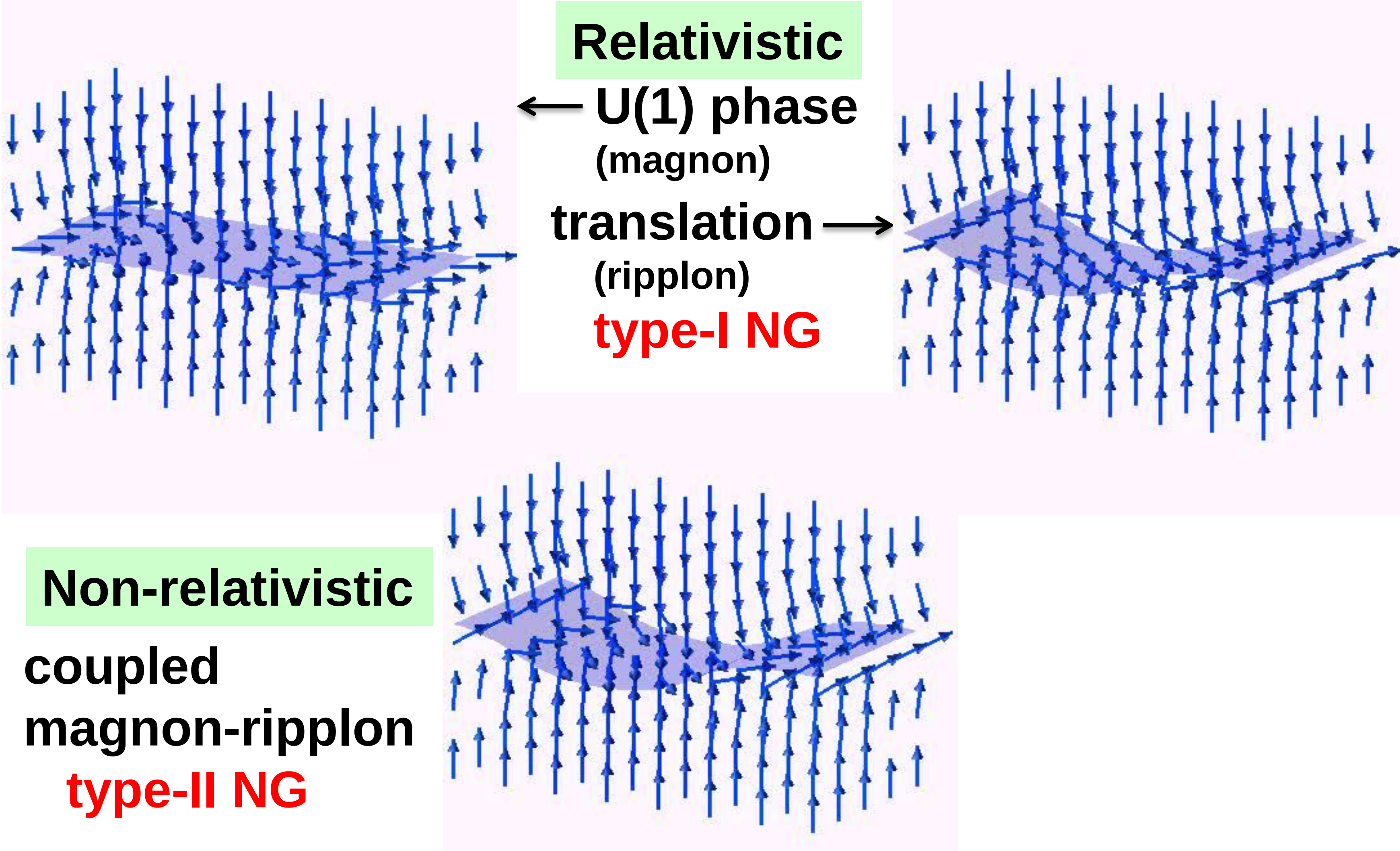
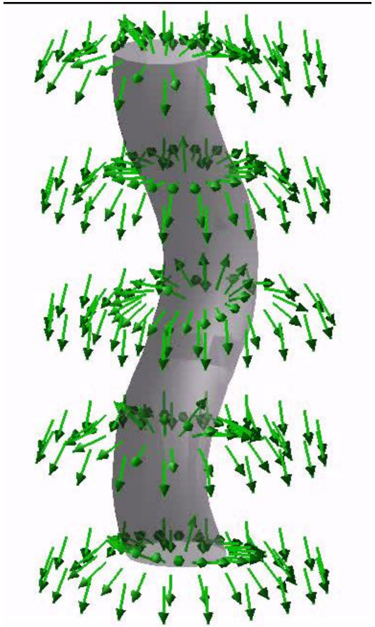
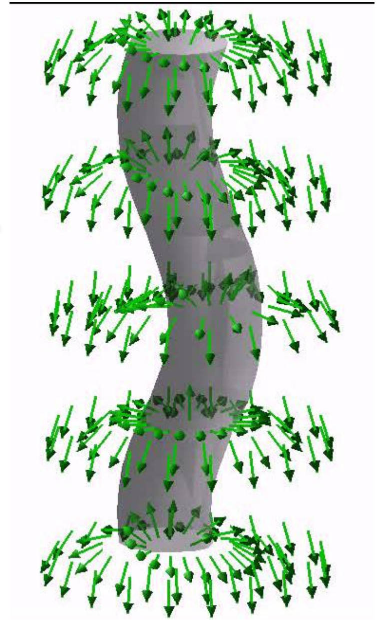

Relativistic
U(1) phase
(magnon)
translation
(ripplon)
type-I NG
Non-relativistic
coupled
magnon-ripplon
type-II NG

Relativistic



← translation X

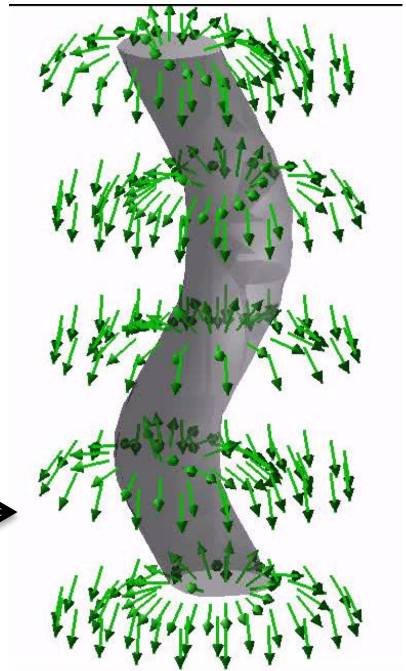
translation Y →



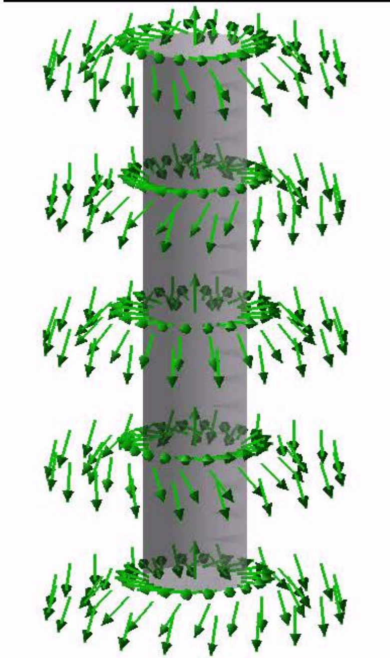
type-I NG

Non-relativistic

coupled translation (Kelvon) (X, Y) →



type-II NG

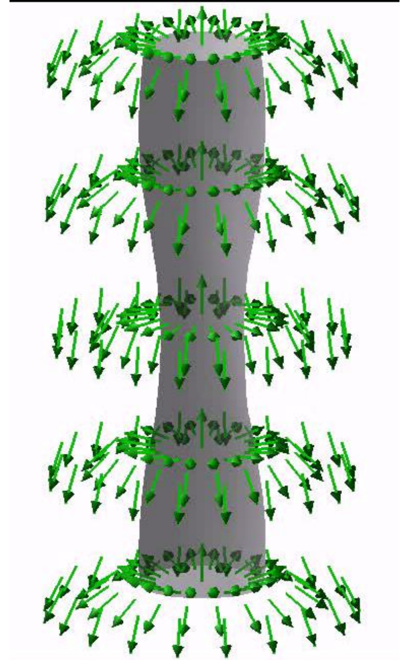


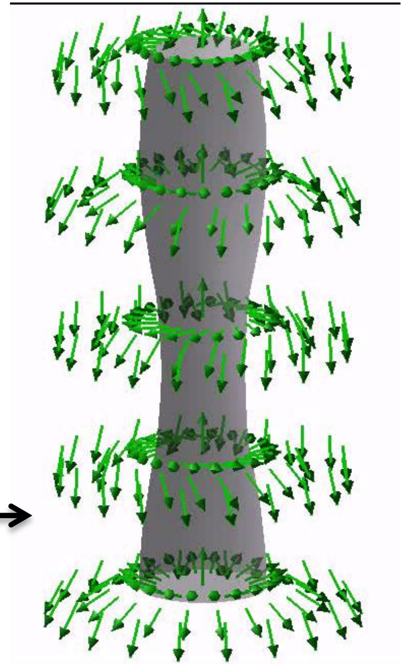
Relativistic

← **U(1) phase (magnon)** θ

size (dilaton) R →

type-I NG





Non-relativistic

coupled magnon-dilaton →

(R, θ) **type-II NG**

(2)Symmetry	Broken sym
Vortex line in superfluid	X, Y
Skyrmion line (scale inv) (scale violated)	X, Y D, ϑ ~~D~~, ϑ
Domain wall in ferromagnet	X, ϑ
Domain wall in 2comp BEC	X, ϑ

Watanabe-Brauner relation

<[broken,broken]>
=/=0
for type-II

(2)Symmetry	Broken sym	Watanabe-Brauner relation
Vortex line in superfluid	*X, Y*	$[P_x, P_y] \sim \text{vortex charge}$ Watanabe&Murayama('14.01)
Skyrmion line	*X, Y*	$[P_x, P_y] \sim \text{skyrmion charge}$ Watanabe&Murayama('14.01)
(scale inv)	*D, ϑ*	$[D, \Theta] \sim r^2\ (\text{skyrmion charge}$ Kobayashi&MN('14.03)
(scale violated)	~~*D*~~, *ϑ*	
Domain wall in ferromagnet	*X, ϑ*	$[P_x, \Theta] \sim \text{wall charge}$ Kobayashi&MN('14.02)
Domain wall in 2comp BEC	*X, ϑ*	$[P_x, \Theta] \sim \text{wall charge}$ Kobayashi&MN('14.02) Watanabe&Murayama('14.03)

(2)Symmetry	Broken sym	Watanabe-Brauner relation
Vortex line in superfluid	*X, Y*	$[P_x, P_y] \sim \text{vortex charge}$ Watanabe&Murayama('14.01)
Skyrmion line (scale inv)	*X, Y* *D, ϑ*	$[P_x, P_y] \sim \text{skyrmion charge}$ Watanabe&Murayama('14.01) $[D, \Theta] \sim r^2\ (\text{skyrmion charge}$ Kobayashi&MN('14.03)
[illegible] in ferromagnet	[illegible]	$[P_x, \Theta] \sim \text{wall charge}$ Kobayashi&MN('14.02)
Domain wall in 2comp BEC	*X, ϑ*	$[P_x, \Theta] \sim \text{wall charge}$ Kobayashi&MN('14.02) Watanabe&Murayama('14.03)

Central extension of algebra

(2)Symmetry	Broken sym
Vortex line	X, Y
Skyrmion line	X, Y
(scale inv)	D, ϑ
(scale violated)	~~D~~, ϑ
Domain wall	X, ϑ

well-known (Magnus force)

[space-time,internal] =/=0

Cf) Coleman&Mandula('67) for relativistic case

Watanabe-Brauner relation

$[P_x, P_y] \sim \text{vortex charge}$

Watanabe&Murayama('14.01)

$[P_x, P_y] \sim \text{skyrmion charge}$

Watanabe&Murayama('14.01)

$[D, \Theta] \sim r^2 \ (\text{skyrmion charge}$

Kobayashi&MN('14.03)

$[P_x, \Theta] \sim \text{wall charge}$

Kobayashi&MN('14.02)

$[P_x, \Theta] \sim \text{wall charge}$

Kobayashi&MN('14.02)

Watanabe&Murayama('14.03)

Central extension of algebra due to topological charge

$$[P, \Theta] = \left[\frac{|u|^2}{1+|u|^2} \right]_{z=-\infty}^{z=+\infty} = \frac{1}{2} [1 - n_z]_{z=-\infty}^{z=+\infty}$$

translation U(1)

topological charge

Somehow similar to supersymmetry algebra

$$\{Q_\alpha, \overline{Q}_{\dot\beta}\} \sim \sigma^\mu{}_{\alpha\dot\beta} P_\mu$$

Dvali & Shifman('96)

$$\{Q_\alpha, Q_\beta\} \sim W$$

Tensorial central charge

topological charge

(3)Infinite R	Broken sym	NG type	Dispersion for finite R	Dispersion for infinite R
Vortex line	X, Y	II	$e \sim \log R\, k^2$	$e \sim - k^2 \log k$
Skyrmion line	X, Y	II	$e \sim k^2$	$e \sim k^2$
(scale inv)	D, ϑ	II	$e \sim k^2$	$e \sim k^2$
(scale violated)	~~D~~, ϑ	I	$e \sim k$	$e \sim k$
Domain wall in ferromagnet	X, ϑ	II	$e \sim k^2$	$e \sim k^2$
Domain wall in 2comp BEC	X, ϑ	II	$e \sim R^{1/2} k^2$	$e \sim k^{3/2}$

$$R >> k^{-1} >> \xi$$

Vortex line: non-normalizable; $R \to k^{-1}$; well-known

Skyrmion line and domain wall in ferromagnet: normalizable

Takahashi&MN('14.04)

Domain wall in 2comp BEC: non-normalizable; $R \to k^{-1}$

Takeuchi & Kasamatsu ('13.09) well-known for ripplon

Summary

(1) Dispersion relations in finite systems

type-I NG: $e\sim k$, type-II NG: $e\sim k^2$

(2) Symmetry (commutation relation)

Watanabe-Brauner relation

<[X,Y]>=/=0: 1 type-II **<[space-time,internal]>=/=0**

<[X,Y]> = 0: 2 type-I

(3) Dispersion relations in infinite systems

normalizable : the same with finite system

non-normalizable $e\sim f(\mathrm{R})k^n$: $R\to k^{-1}$,non-integer power

(Kelvon&ripplon were *not* recognized as NG thus far)

What I didn't talk about:

1) So far, mean field,
   Beyond mean field: Coleman-Mermin-Wargner
   type-II NG seem to be stable at quantum level
   ….can be proved by Bethe ansatz (for some case)
2) Bogoliubov theory approach: Gram matrix

Disscussion:

1) Proof for general cases (finite & infinite *R*)
2) Jacobi id. do not hold. Consistent algebra?
3) Nonrelativistic supersymmetry
4) NG fermions (SSB of fermionic symmetry)

3-comp BEC with U(2) x U(1) symmetry

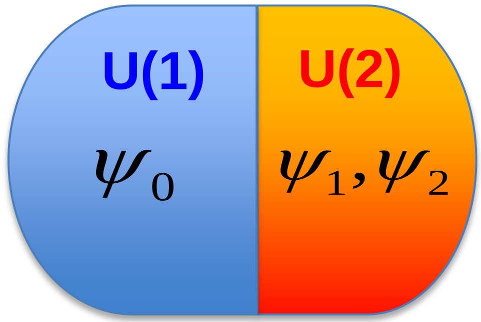
MN, S.Uchino & W.Vinci,
arXiv:1311.5408 [hep-th]

Lagrangian for GP

$$\mathcal{L} = i\hbar\psi_0^\dagger\dot{\psi}_0 + i\hbar\Psi^\dagger\dot{\Psi} - \frac{\hbar^2}{2m_0}\nabla_i\psi_0^\dagger\nabla_i\psi_0 - \frac{\hbar^2}{2m}\nabla_i\Psi^\dagger\nabla_i\Psi$$

$$+\mu_0|\psi_0|^2 + \mu|\Psi|^2 - \frac{1}{2}\lambda_0|\psi_0|^4 - \frac{1}{2}\lambda|\Psi|^4 - \kappa|\psi_0|^2|\Psi|^2$$

ψ_0 — U(1) $\qquad \Psi = (\psi_1, \psi_2)^T$ — U(2)



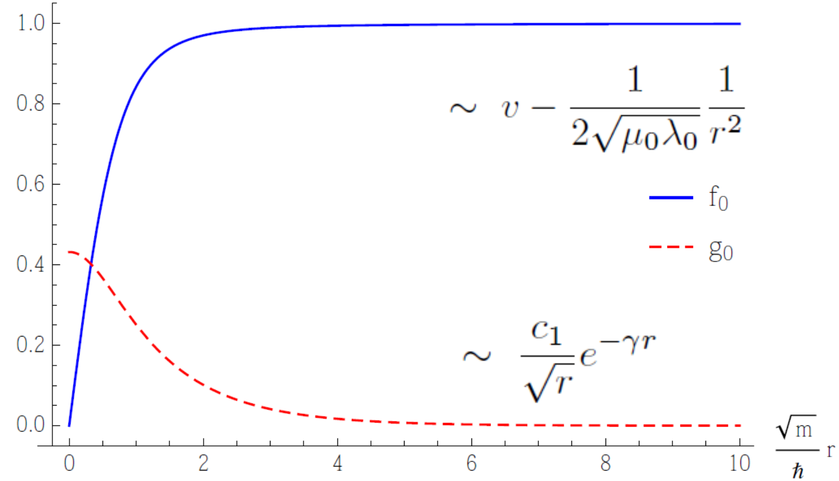
Stability $\lambda_0\lambda - \kappa^2 > 0$

$\mu_0\lambda - \mu\kappa > 0,\ \mu\lambda_0 - \mu_0\kappa < 0 \quad \mu^2\lambda_0 - \mu_0^2\lambda < 0$

ground state $(\psi_0, \Psi) = (v_0, 0, 0)^T, (0, v, 0)^T$

Non-Abelian vortex

$$\begin{pmatrix} \psi_0 \\ \Psi \end{pmatrix} = \begin{pmatrix} f_0(r,\theta)e^{i\theta} \\ g_0(r,\theta)\eta \end{pmatrix}$$



$$\eta^\dagger \eta = 1$$

$S^3 \cong U(2)/U(1)$

NG modes

Effective theory

$$\mathcal{L} = i\hbar g_0^2 \eta^\dagger \dot{\eta} + K\hbar^2(\eta^\dagger\dot{\eta})^2 - \frac{\hbar^2}{2m} g_0^2 |\nabla_z \eta|^2$$

$$\alpha = \int_0^\infty 2\pi r dr g_0^2 \quad \beta = \int_0^\infty 2\pi r dr K$$

$$K \equiv -(\mu_0 f_1^2 + \mu g_1^2 - 3\lambda_0 f_0 f_1^2 - 3\lambda g_0 g_1^2 - 4\kappa f_0 g_0 f_1 g_1 - \kappa g_0^2 f_1^2 - \kappa f_0^2 g_1^2).$$

$$\eta = e^{i\varphi} n$$

S^3

$S^1 \ltimes S^2 \sim \mathbf{C}P^1$

type-I NG | **type-II NG**

$$\mathcal{L}_{\text{eff.}} = K\hbar^2\left(\dot{\varphi} - \frac{i}{2}(n^\dagger\dot{n} - \dot{n}^\dagger n)\right)^2 - \frac{\hbar^2}{2m}\left(\partial_z\varphi - \frac{i}{2}(n^\dagger\partial_z n - \partial_z n^\dagger n)\right)^2 + 2i\hbar(n^\dagger\dot{n} - \dot{n}^\dagger n) - \frac{\hbar^2}{2m}\left((\partial_z n)^2 - (n^\dagger\partial_z n)^2\right)$$

Quantum level

MN, S.Uchino & W.Vinci, arXiv:1311.5408 [hep-th]

ψ_0 vortex plays as a trapping pot

~Two components in 1d trap

$$\eta = e^{i\varphi} n$$

S^3

$S^1 \ltimes S^2 \sim CP^1$

type-I NG

type-II NG → SU(2)/U(1) remains broken

NG remains gapless

Quantum analysis: *Bethe ansatz*

→ U(1) recovers: Lieb-Liniger gas,

(*c*=1 CFT, Tomonaga-Luttinger liquid)

Quantum Exact Non-Abelian vortex

1st example in physics

Can be generalized to $\frac{SU(N)}{SU(N-1) \times U(1)} = CP^{N-1}$