Defect branes as Alice strings

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based on **work in progress** with **T. Okada (RIKEN)**

Motivation and Results

7-branes in string theory usually have monodromy in the 2-dimensional transverse space.
Then, as a probe goes around the 7-brane, the charge can be changed by the action of the monodromy group.
We discuss the charge conservation focusing on the similarity between 7-brane and Alice string.
In addition, we construct a new T-fold background with non-locally distributed F1 charge.

Review: Alice string

[A. S. Schwarz, Nucl. Phys. B 208, 141 (1982)]

Example: 4-dimensional gauge theory with G = SO(3).

$$\mathcal{L}=\tfrac{1}{8}\,\mathrm{tr}\,(F_{\mu\nu}F^{\mu\nu})+\tfrac{1}{2}\,(D_\mu\Phi)^\dagger\,(D^\mu\Phi)-V(\Phi)$$

(Φ_{ij} : 5-dim rep., real symmetric traceless matrix)

Vacuum configuration:

$$(\Phi_{ij})=\begin{pmatrix} a & 0 & 0\\ 0 & a & 0\\ 0 & 0 & -2a\end{pmatrix}$$

unbroken subgroup $H=U(1)\rtimes\mathbb{Z}_2$

$$\mathrm{e}^{\mathrm{i}\,\alpha\,Q}=\begin{pmatrix}\cos\alpha & \sin\alpha & 0\\ -\sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{pmatrix}\qquad X=\begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & -1\end{pmatrix}$$

$X\,Q\,X^{-1}=-Q$: charge conjugation is local symmetry.

Alice string (Alice vortex):

In order to make the energy of the vortex finite,
We should impose $D_\mu\Phi=0$ at spatial infinity.
This is realized if we assume

$$\Phi(\theta)=U(\theta)\,\Phi(-\pi)\,U^{-1}(\theta)\,,\quad U(\theta)=\exp\big(e\int_{-\pi}^{\theta}\mathrm{d}\theta\,A_\theta\big).$$

⇒ generator of unbroken $U(1)$ also depends on θ :

$$Q(\theta)=U(\theta)\,Q(-\pi)\,U^{-1}(\theta)\,.$$

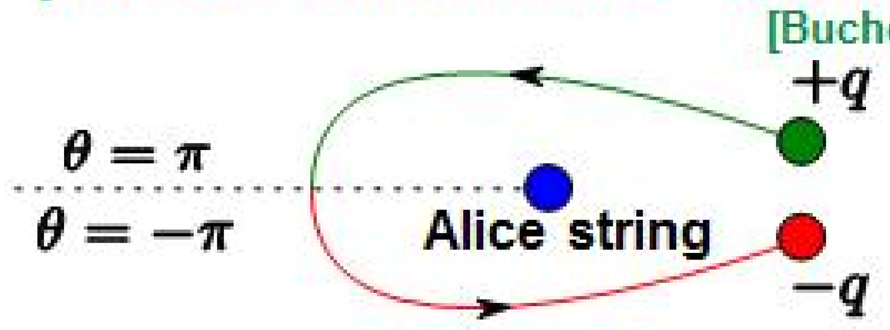
Alice string is obtained by setting $U(\pi)=X$.

⇒ As we go around the Alice string,
the sign of $U(1)$ generator is changed:

$$Q(\pi)=X\,Q(-\pi)\,X^{-1}=-Q(-\pi)\,.$$

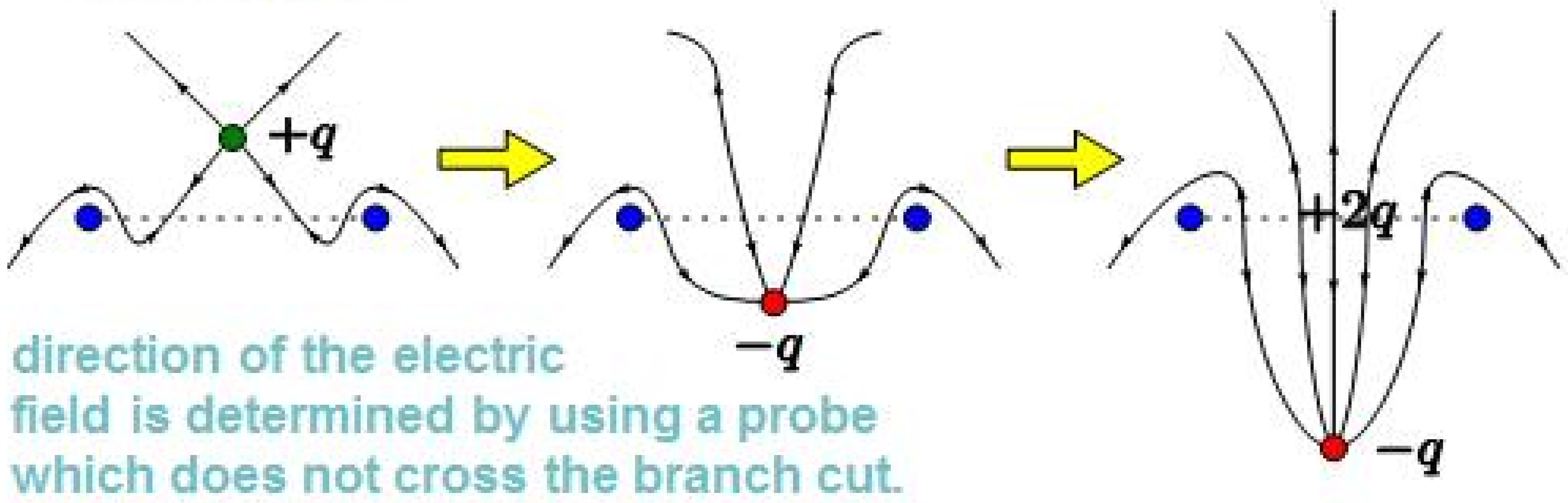
Review: Cheshire charge

[Alford-Benson-Coleman-March-Russell-Wilczek (1990, 1991)]
[Bucher-Lo-Preskill (1992)]



Naively, the total charge is not conserved.

Resolution:



direction of the electric field is determined by using a probe which does not cross the branch cut.

Non-localized charge, called Cheshire charge, appears on the branch cut, and the total charge Is kept invariant under the above process.

Since the position of the branch cut is not physical, local distribution of Cheshire charge is unphysical.

Similar things occur in string theory.

Defect branes in string theory

Example: 5^2_2 -background

type II theory : Kaluza-Klein Monopole (56789,4)

↓ smear along the x^3-direction and take T-duality.

[de Boer-Shigemori (2010. 2012)]

$$\mathrm{d}s^2=H(r)\,(\mathrm{d}r^2+r^2\mathrm{d}\theta^2)+H(r)\,K^{-1}(r,\theta)\,\mathrm{d}x_{34}^2+\mathrm{d}x_{056789}^2\,,$$
$$B^{(2)}=-\sigma\,K^{-1}\,\theta\,\mathrm{d}x^3\wedge\mathrm{d}x^4\,,\quad \mathrm{e}^{2\Phi}=H\,K^{-1}\,,$$
$$H(r)\equiv h+\sigma\,\log(\mu/r)\,,\quad K(r,\theta)\equiv H^2+(\sigma\theta)^2\,.$$

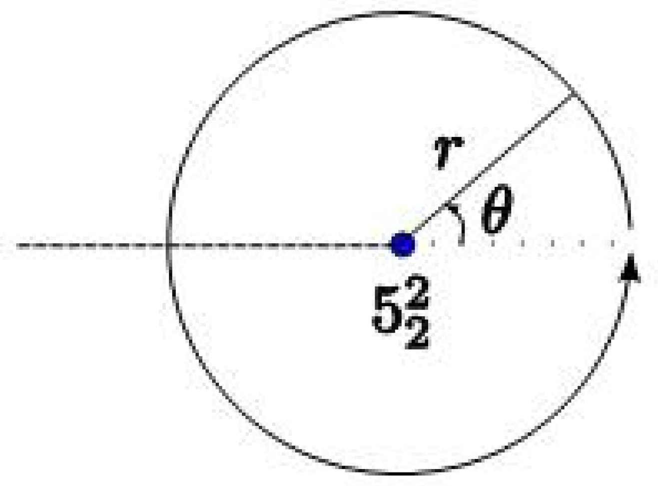
$\epsilon=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$

If we define the generalized metric by

$$(\mathcal{H}_{AB})\equiv\begin{pmatrix}G^{-1} & G^{-1}B\\ -B\,G^{-1} & G-B\,G^{-1}B\end{pmatrix}=\frac{1}{H}\begin{pmatrix}K\,\mathbf{1} & -\sigma\theta\epsilon\\ \sigma\theta\,\epsilon & \mathbf{1}\end{pmatrix},$$

it satisfies the relation:

$$\mathcal{H}(\theta)=U(\theta)\,\mathcal{H}(-\pi)\,U^{-1}(\theta)$$
$$U(\theta)=\begin{pmatrix}\mathbf{1} & 0\\ \sigma(\theta+\pi)\,\epsilon & \mathbf{1}\end{pmatrix}\in \mathrm{O}(2,2,\mathbf{R})$$



Consider a probe with charge P(3),

$$\begin{pmatrix}(1/R_3)\times \mathrm{P}(3)\\ (1/R_4)\times \mathrm{P}(4)\\ (R_3/l_s^2)\times \mathrm{F1}(3)\\ (R_4/l_s^2)\times \mathrm{F1}(4)\end{pmatrix}=\begin{pmatrix}1/R_3\\0\\0\\0\end{pmatrix}\to U^{-1}(\pi)\begin{pmatrix}1/R_3\\0\\0\\0\end{pmatrix}=\begin{pmatrix}1/R_3\\0\\0\\R_4/l_s^2\end{pmatrix}.$$

As the probe goes around the 5^2_2 brane,
the charge becomes P(3) + F1(4) . charge is not conserved?

In order to keep the total charge invariant,
anti-F1(4) "Cheshire charge" should appear on the branch cut !

Results: seven branes with Cheshire charges

Using 4-dimensional electric-magnetic duality,
we found a solution with the following form:

$$\mathrm{d}s^2=-\frac{\mathrm{c}^2+\mathrm{d}^2\,\tilde\tau_2}{\tilde\tau_2}\,\mathrm{d}t^2+\tau_{2,\infty}\,|f|^2\,(\mathrm{c}^2+\mathrm{d}^2\,\tilde\tau_2)\,\mathrm{d}z\,\mathrm{d}\bar z$$
$$+\frac{\tau_{2,\infty}^{-1}\,\tilde\tau_2}{\mathrm{c}^2\,\tilde\tau_2+\mathrm{d}^2\,\tau_{2,\infty}^{-2}\,|\tau|^2}\Big(\mathrm{d}x^3-\frac{\mathrm{c}\,\mathrm{d}\,\tau_1}{\tilde\tau_2}\,\mathrm{d}t\Big)^2+\frac{\tau_{2,\infty}^{-1}\,(\mathrm{c}^2+\mathrm{d}^2\,\tilde\tau_2)}{\mathrm{c}^2\,\tilde\tau_2+\mathrm{d}^2\,\tau_{2,\infty}^{-2}\,|\tau|^2}\,\mathrm{d}x_4^2+\mathrm{d}x_{56789}^2\,,$$
$$\hat B^{(2)}=-\frac{\mathrm{c}\,\tau_{2,\infty}^{-1/2}\,(\mathrm{c}^2+\mathrm{d}^2\,\tilde\tau_2)}{\mathrm{c}^2\,\tilde\tau_2+\mathrm{d}^2\,\tau_{2,\infty}^{-2}\,|\tau|^2}\,\mathrm{d}t\wedge\mathrm{d}x^4-\frac{\mathrm{d}\,\tau_{2,\infty}^{-2}\,\tau_1}{\mathrm{c}^2\,\tilde\tau_2+\mathrm{d}^2\,\tau_{2,\infty}^{-2}\,|\tau|^2}\,\mathrm{d}x^3\wedge\mathrm{d}x^4\,,$$
$$\mathrm{e}^{2\Phi}=\frac{\tau_{2,\infty}^{-3/2}\,(\mathrm{c}^2+\mathrm{d}^2\,\tilde\tau_2)}{\mathrm{c}^2\,\tilde\tau_2+\mathrm{d}^2\,\tau_{2,\infty}^{-2}\,|\tau|^2}\,.$$

Properties:

- This has the same monodromy as the 5^2_2 background (i.e. T-fold).
- F1 charge is non-locally distributed on the branch cut.
  (in addition, the total F1-charge is properly quantized.)
  The configuration (with Cheshire charge)
  can be interpreted as a background
  which is realized after a probe (with P(3) charge)
  goes around the 5^2_2 brane.

⇓ taking dualities

Various 7-brane backgrounds can be obtained:

- KKM background with F1 Cheshire charge
- NS7 background with D1 Cheshire charge
- D7 background with F1 Cheshire charge, etc.

Future direction

- brane configuration corresponding to the new 7-brane background?
- generalization to the supertube solution and its relevance to the black hole microstates.