EPR = ER and Scattering

Shigenori Seki

Research Institute for Natural Science, Hanyang University

This talk is based on the work:

“EPR = ER, Scattering Amplitude and Entanglement Entropy Change,”
Shigenori Seki and Sang-Jin Sin, Phys. Lett. B735 (2014) 272.

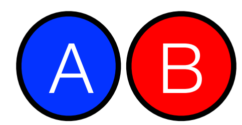
at YITP, Kyoto on 9 May 2014

EPR = ER conjecture

EPR pair

<u>Einstein-Podolsky-Rosen pair</u> [Einstein-Podolsky-Rosen, Phys.Rev. 47 (1935) 777]

entangled two particles



e.g. a spin-0 particle decays to two spin-1/2 particles.

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_A \otimes |\downarrow\rangle_B - |\downarrow\rangle_A \otimes |\uparrow\rangle_B\right)$$

Separate them from each other at long distance

A and B are still entangled.

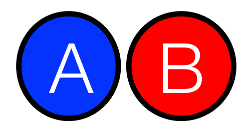
(If the state of A is observed, then the one of B is determined.)

$$S_A = -\mathrm{tr}(\rho_A \log \rho_A) = \log 2$$

EPR = ER conjecture

[Maldacena-Susskind, Fortsch.Phys. 61 (2013) 781]

entangled two particles



Separate them from each other at long distance

ER bridge (or wormhole)

Quantum mechanics

entanglement

geometric interpretation

Gravity

two entangled systems are connected by ER bridge

From the viewpoint of AdS/CFT correspondence,
let us see two examples supporting the EPR = ER conjecture.

- Accelerating quark and anti-quark

  [Jensen-Karch, Phys.Rev.Lett. 111 (2013) 211602]

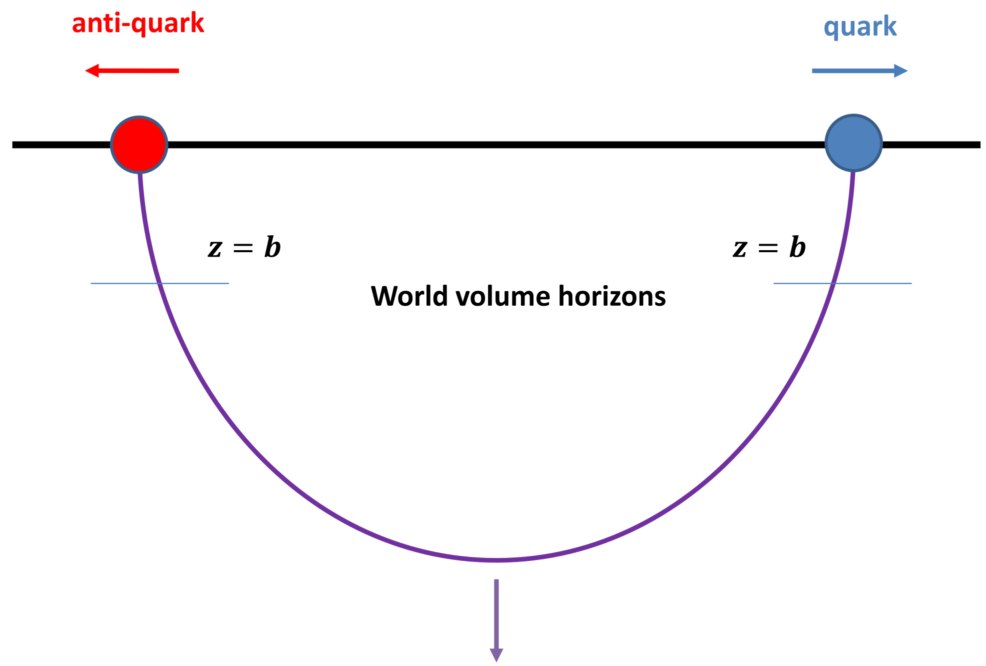
- Scattering gluons

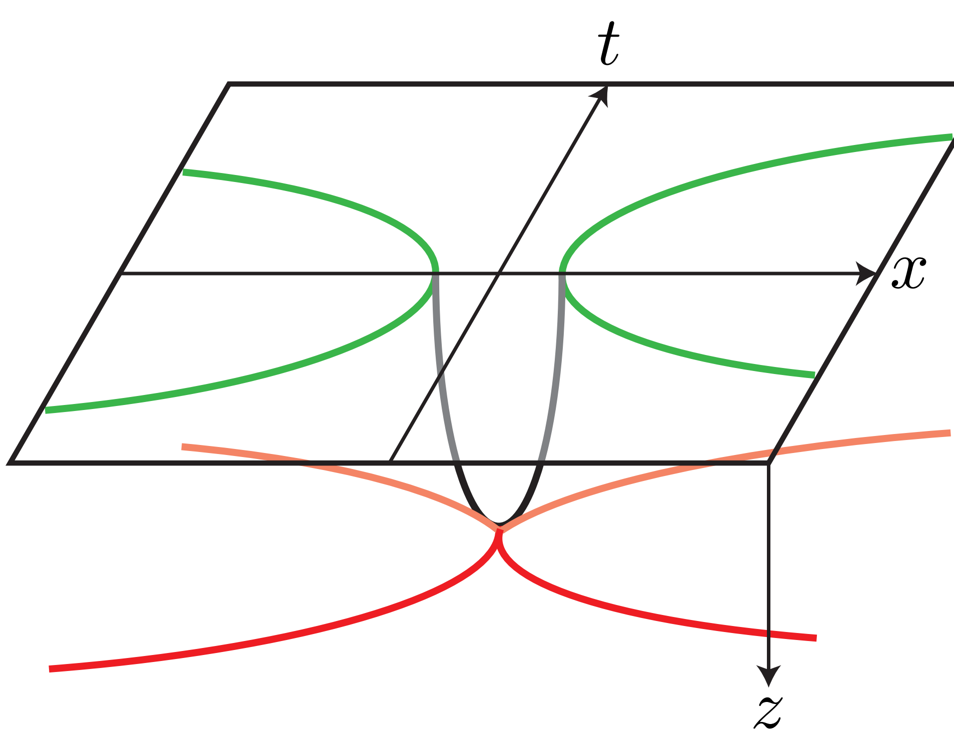
  [SS-Sin, Phys.Lett. B735 (2014) 272]

Entanglement $\Longleftrightarrow$ Wormhole on world-sheet

Accelerating quark and anti-quark

The holographic surface of accelerating quark and anti-quark





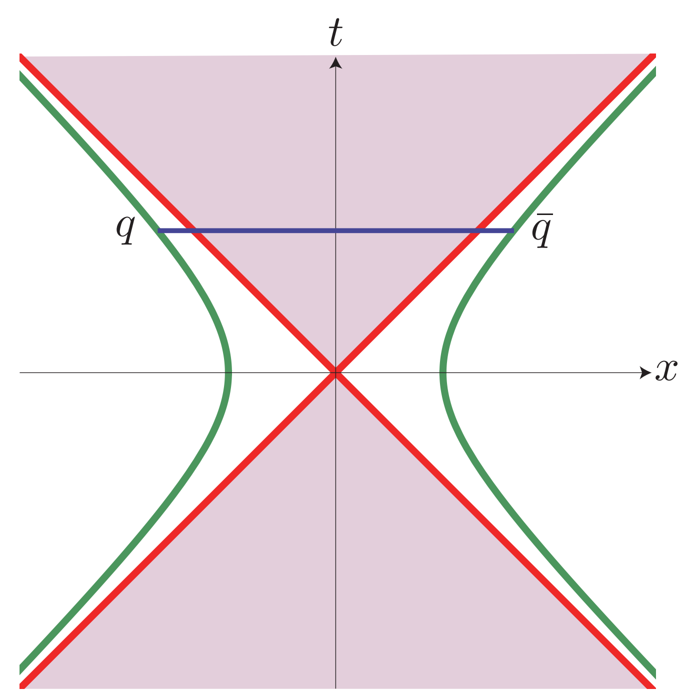
[Xiao, Phys.Lett. B 665 (2008) 173]

AdS bulk metric

$$ds^2 = \frac{1}{z^2}(-dt^2 + dx^2 + dz^2)$$

Minimal surface

$$x^2 = t^2 + b^2 - z^2$$



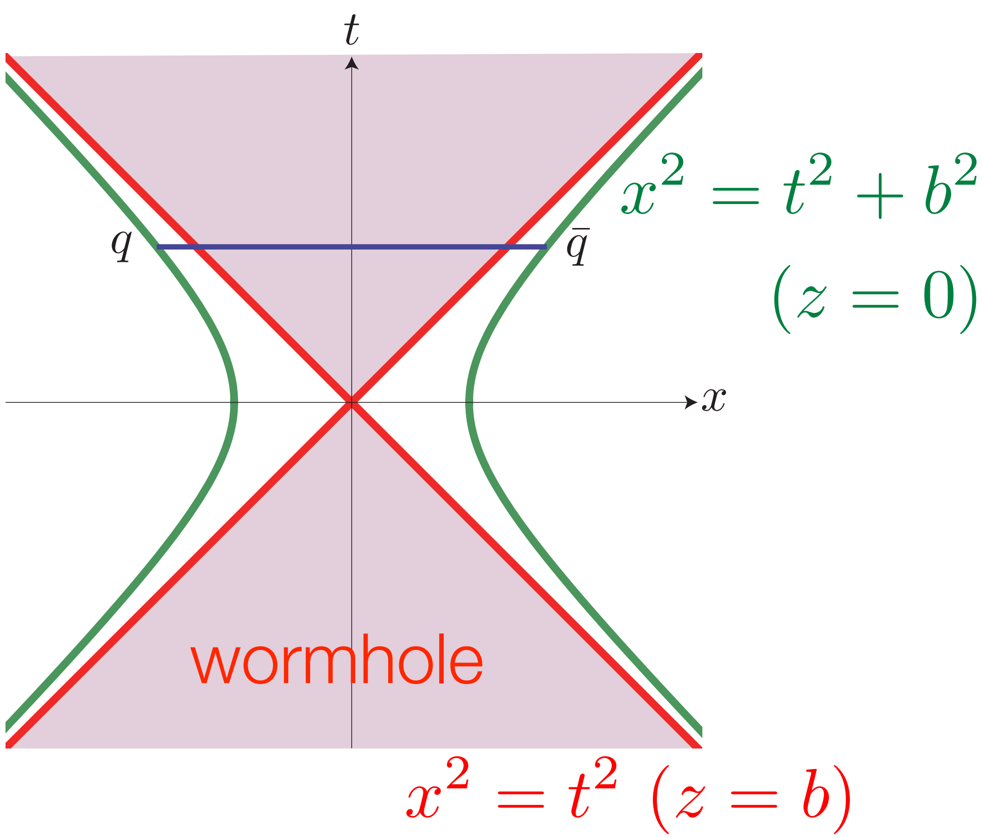
[Jensen-Karch, Phys.Rev.Lett. 111 (2013) 211602]

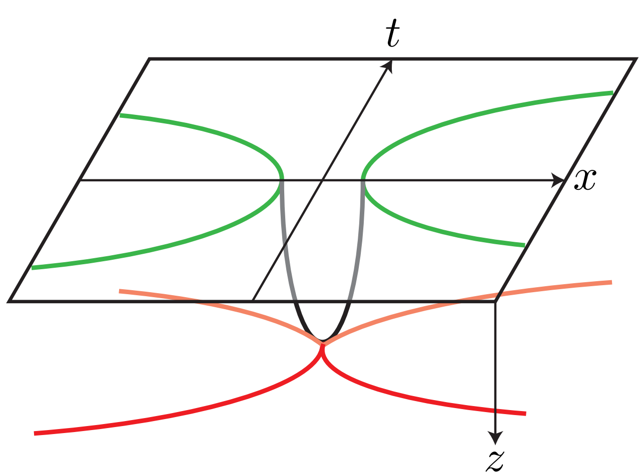
Static gauge

$$t = \tau, \quad z = \sigma$$

World-sheet induced metric

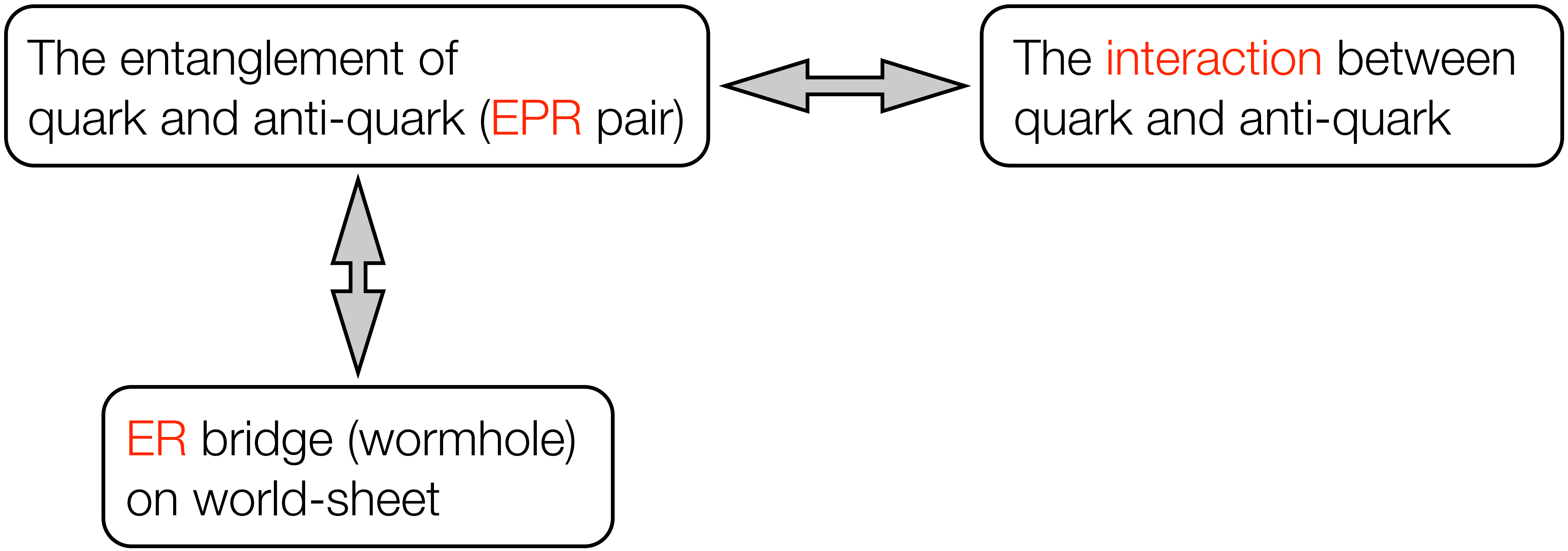
$$ds^2_{\rm ws} = \frac{1}{\sigma^2(\tau^2 + b^2 - \sigma^2)}\left[-(b^2 - \sigma^2)d\tau^2 + (\tau^2 + b^2)d\sigma^2 - 2\tau\sigma d\tau d\sigma\right]$$





The trajectories of quark and anti-quark are causally disconnected on the world-sheet.

The quark and anti-quark are entangled by the wormhole that the open string goes through.



We can naturally guess that the entanglement of final states is different from that of initial states due to interaction. Therefore the scattering process induces the entanglement entropy change.

Are other interacting particles also related to a wormhole on world-sheet?

Fortunately, we know the minimal surface in AdS that describes a gluon-gluon scattering.

Scattering gluons

Minimal surface solution for gluon scattering

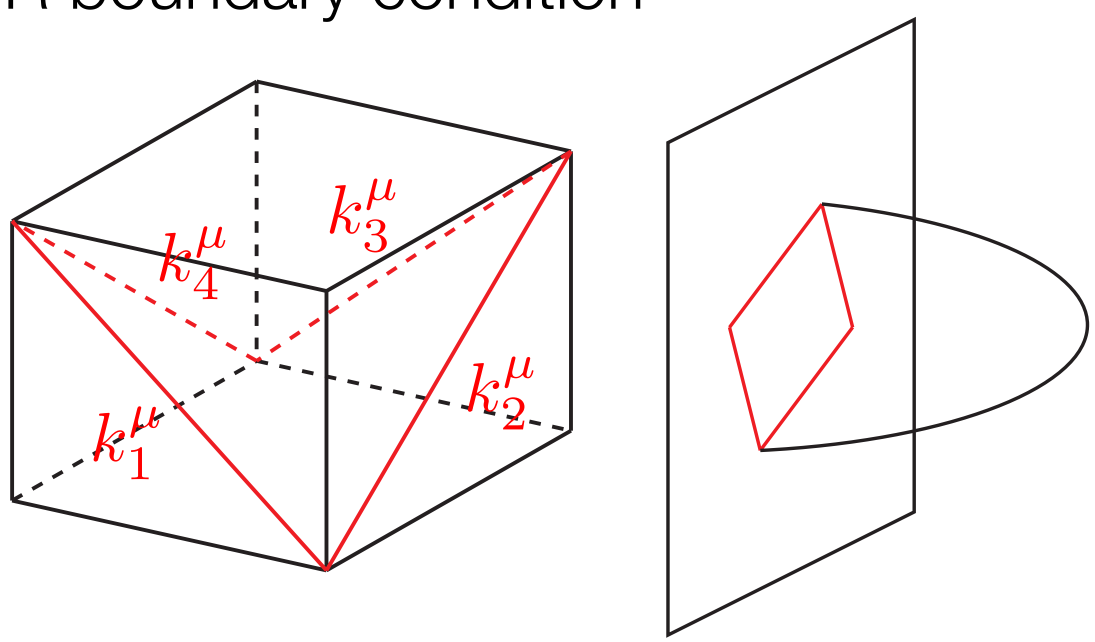
AdS_5 (momentum space)

[Alday-Maldacena, JHEP 0706 (2007) 064]

$$ds^2 = \frac{R^2}{r^2}(\eta_{\mu\nu}dy^\mu dy^\nu + dr^2)$$

$$\Delta y^\mu = 2\pi k^\mu$$

$r = 0$ IR boundary condition



The solution of Nambu-Goto action

$$y_0 = \frac{\alpha\sqrt{1+\beta^2}\sinh u_1 \sinh u_2}{\cosh u_1 \cosh u_2 + \beta \sinh u_1 \sinh u_2},$$

$$y_1 = \frac{\alpha \sinh u_1 \cosh u_2}{\cosh u_1 \cosh u_2 + \beta \sinh u_1 \sinh u_2},$$

$$y_2 = \frac{\alpha \cosh u_1 \sinh u_2}{\cosh u_1 \cosh u_2 + \beta \sinh u_1 \sinh u_2},$$

$$y_3 = 0,$$

$$r = \frac{\alpha}{\cosh u_1 \cosh u_2 + \beta \sinh u_1 \sinh u_2},$$

Mandelstam variables:

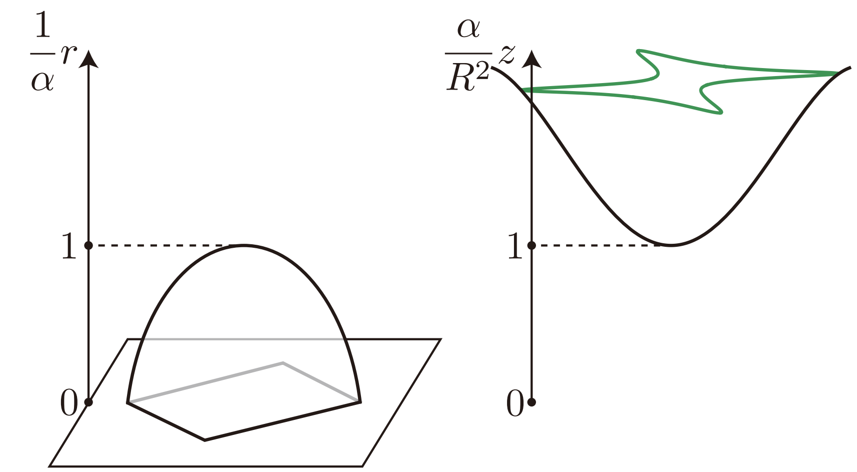
$$-s(2\pi)^2 = \frac{8\alpha^2}{(1-\beta)^2},$$

$$-t(2\pi)^2 = \frac{8\alpha^2}{(1+\beta)^2}.$$

AdS_5 (momentum space) [Kallosh-Tseytlin, JHEP 9810 (1098) 016]

$\big\downarrow$ "T-dual" transformation: $\partial_m y^\mu = \dfrac{R^2}{z^2}\epsilon_{mn}\partial_n x^\mu \,, \quad z = \dfrac{R^2}{r}$

AdS_5 (position space)



$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2)$$

The Alday-Maldacena solution is mapped to

$$x_0 = -\frac{R^2}{2\alpha}\sqrt{1+\beta^2}\sinh u_+ \sinh u_- \,,$$

$$x_+ := \frac{x_1 + x_2}{\sqrt{2}} = -\frac{R^2}{2\sqrt{2}\alpha}\left[(1+\beta)u_- + (1-\beta)\cosh u_+ \sinh u_-\right] \,,$$

$$x_- := \frac{x_1 - x_2}{\sqrt{2}} = \frac{R^2}{2\sqrt{2}\alpha}\left[(1-\beta)u_+ + (1+\beta)\sinh u_+ \cosh u_-\right] \,,$$

$$x_3 = 0 \,,$$

$$z = \frac{R^2}{2\alpha}\left[(1+\beta)\cosh u_+ + (1-\beta)\cosh u_-\right]$$

where $u_\pm := u_1 \pm u_2$. For later convenience, we introduce

$$X_\mu := \frac{\alpha}{R^2}x_\mu \quad (\mu = 0, +, -, 3) \,, \quad Z := \frac{\alpha}{R^2}z \, (\geq 1)$$

Causal structure on world-sheet

The induced metric on world-sheet [SS-Sin, Phys.Lett. B735 (2014) 272]

$$ds^2_{\rm ws} = R^2\left(g_{++}du_+^2 + 2g_{+-}du_+du_- + g_{--}du_-^2\right)$$

$$g_{++} = \frac{4(1+\beta)^2\sinh^2 u_+ + 4(1+\beta^2) - \left[(1+\beta)\cosh u_+ - (1-\beta)\cosh u_-\right]^2}{2\left[(1+\beta)\cosh u_+ + (1-\beta)\cosh u_-\right]^2}\,,$$

$$g_{+-} = \frac{2(1-\beta^2)\sinh u_+ \sinh u_-}{\left[(1+\beta)\cosh u_+ + (1-\beta)\cosh u_-\right]^2}\,,$$

$$g_{--} = \frac{4(1-\beta)^2\sinh^2 u_- + 4(1+\beta^2) - \left[(1+\beta)\cosh u_+ - (1-\beta)\cosh u_-\right]^2}{2\left[(1+\beta)\cosh u_+ + (1-\beta)\cosh u_-\right]^2}\,.$$

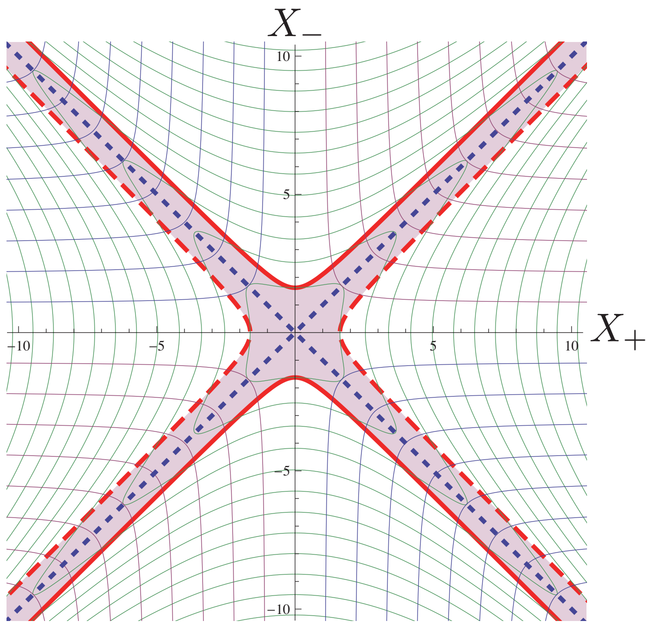
"Horizons"

$$g_{++} = 0: \quad (1-\beta)\cosh u_- = (1+\beta)\cosh u_+ + 2\sqrt{(1+\beta)^2\sinh^2 u_+ + 1 + \beta^2}$$

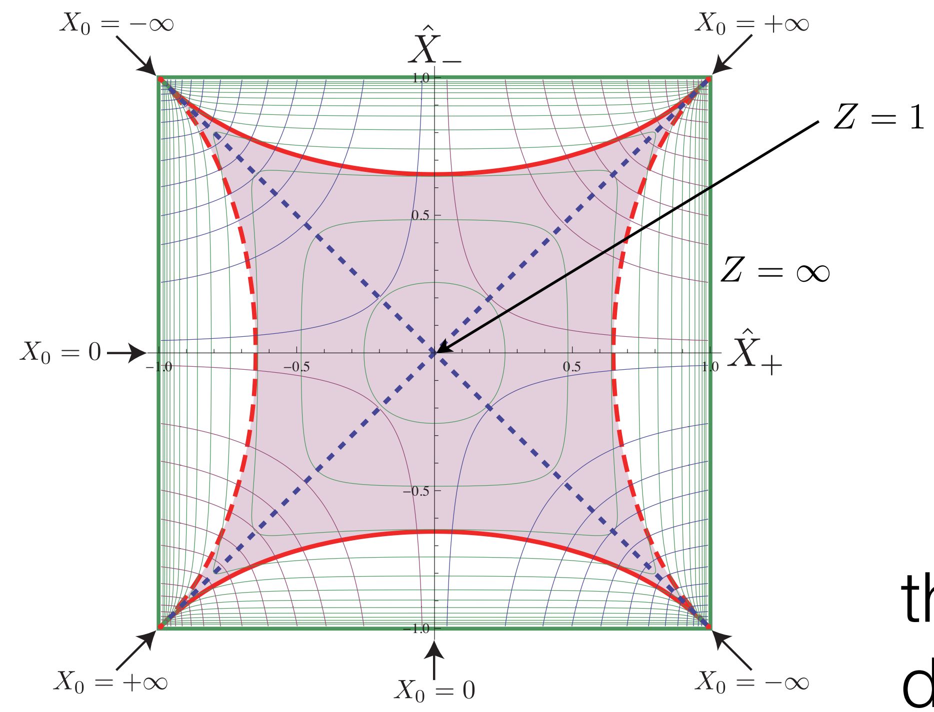
$$g_{--} = 0: \quad (1+\beta)\cosh u_+ = (1-\beta)\cosh u_- + 2\sqrt{(1-\beta)^2\sinh^2 u_- + 1 + \beta^2}$$

$$\boxed{0 \leq \beta < 1} \qquad X_{\pm} \in (-\infty, +\infty) \qquad \hat{X}_{\pm} := \frac{2}{\pi} \arctan X_{\pm} \in [-1, 1]$$

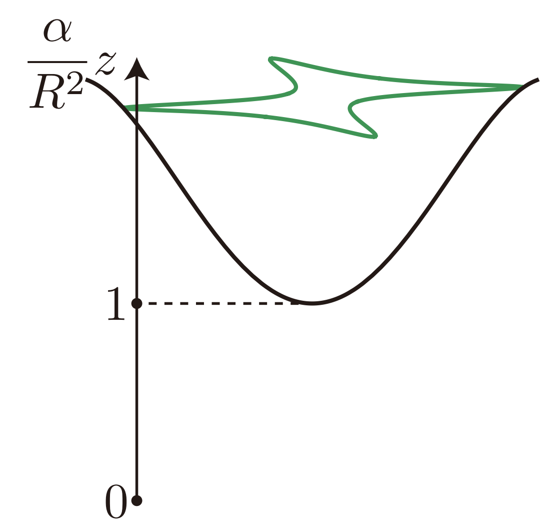
$\beta = 0$



(a)

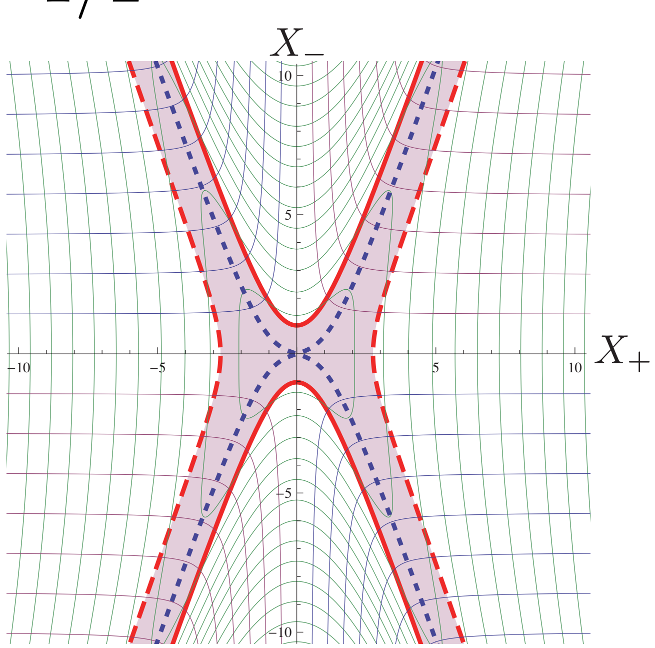


(b)

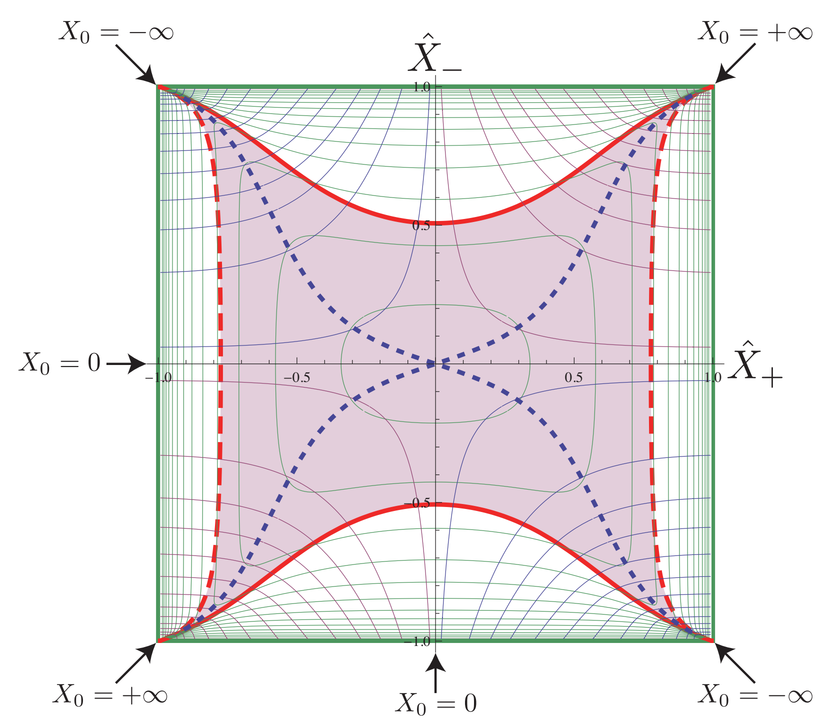


thick red: $g_{--} = 0$,
dashed red: $g_{++} = 0$,
dotted blue:
$g_{++} = g_{--}$

$\beta = 1/2$



(a)

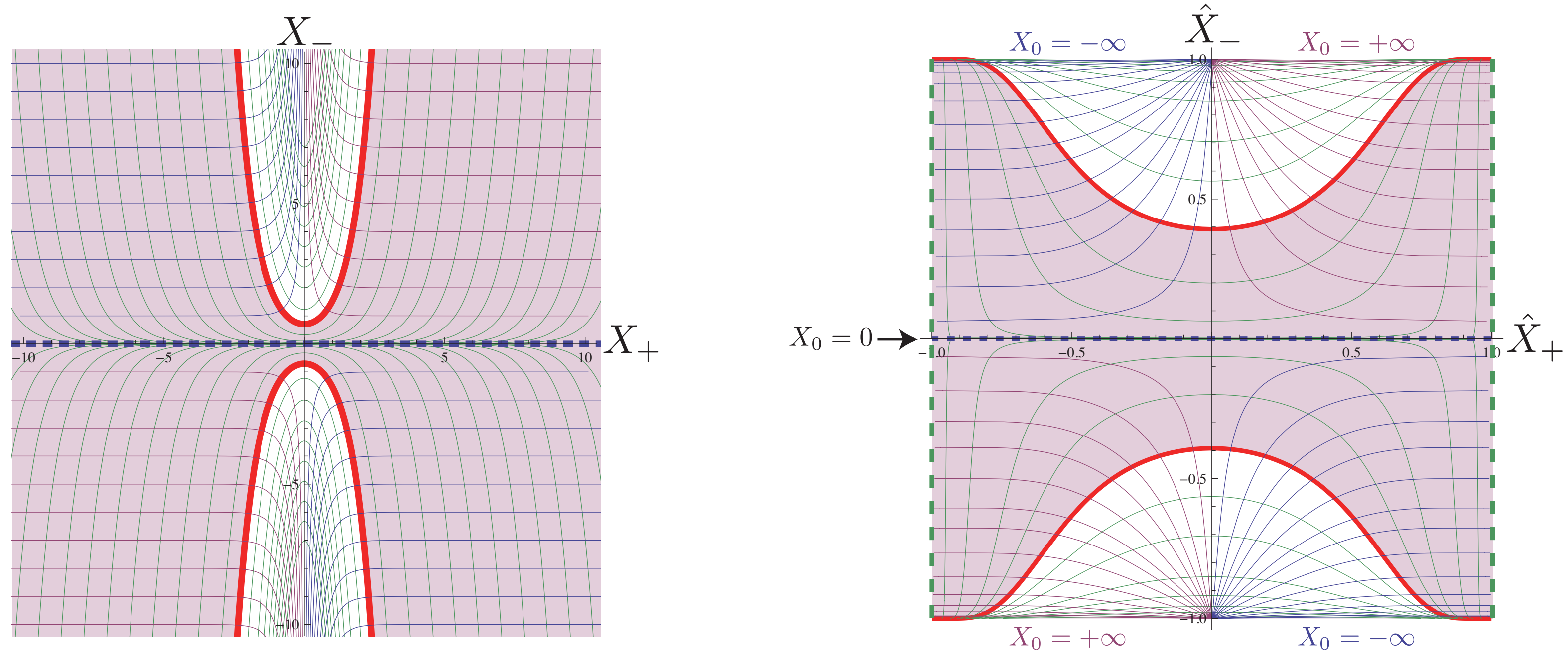


(b)

blue & red: constant X_0
green: constant Z

$\boxed{\beta = 1}$ Regge limit: $-s \to \infty$ with $-t$ fixed.

$$X_0 = -\frac{1}{\sqrt{2}} \sinh u_+ \sinh u_- \,, \quad X_+ = -\frac{1}{\sqrt{2}} u_- \,, \quad X_- = \frac{1}{\sqrt{2}} \sinh u_+ \cosh u_- \,,$$
$$X_3 = 0 \,, \quad Z = \cosh u_+ \,.$$

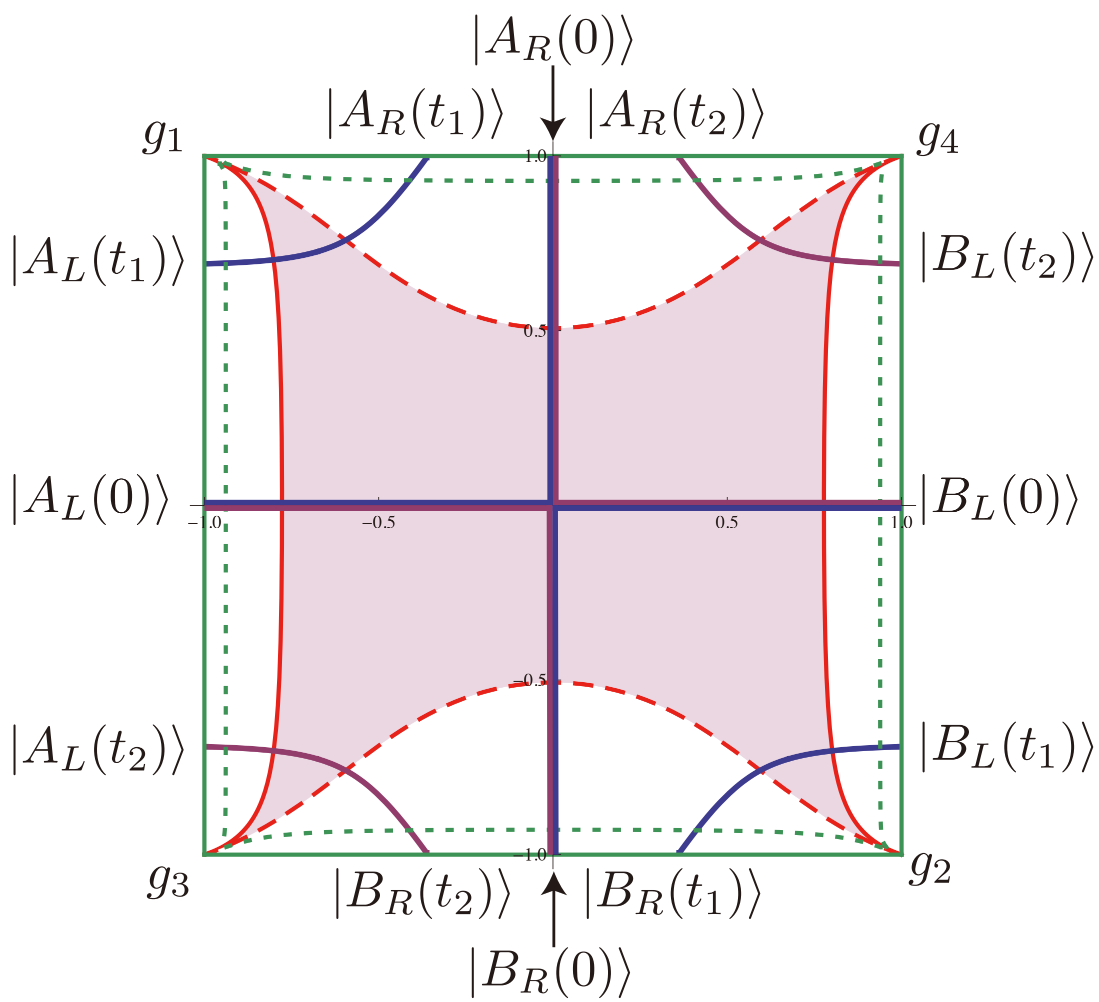


(a) (b)

While g_{++} is positive definite, g_{--} vanishes on $\cosh u_+ = \sqrt{2}$.

EPR = ER in gluon scattering?

$$\{g_1, g_2\} \to \{g_3, g_4\}$$



Incoming gluons:

$$|g_1(t_1)\rangle\rangle = \sum_{i,j} c_{ij}^{(1)} |A_{Li}(t_1)\rangle \otimes |A_{Rj}(t_1)\rangle$$

$$|g_2(t_1)\rangle\rangle = \sum_{i,j} c_{ij}^{(2)} |B_{Li}(t_1)\rangle \otimes |B_{Rj}(t_1)\rangle$$

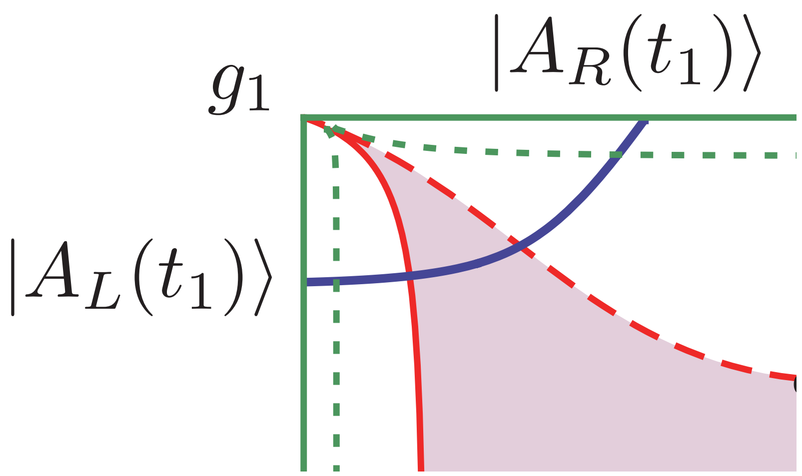
Outgoing gluons:

$$|g_3(t_2)\rangle\rangle = \sum_{i,j} c_{ij}^{(3)} |A_{Li}(t_2)\rangle \otimes |B_{Rj}(t_2)\rangle$$

$$|g_4(t_2)\rangle\rangle = \sum_{i,j} c_{ij}^{(4)} |B_{Li}(t_2)\rangle \otimes |A_{Rj}(t_2)\rangle$$

We can see two types of entanglement which are interpreted to wormholes.

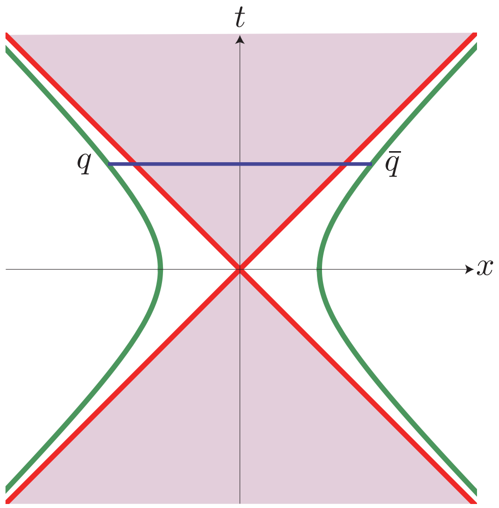
1. Internal entanglement



$$|g_1(t_1)\rangle\rangle = \sum_{i,j} c_{ij}^{(1)} |A_{Li}(t_1)\rangle \otimes |A_{Rj}(t_1)\rangle$$

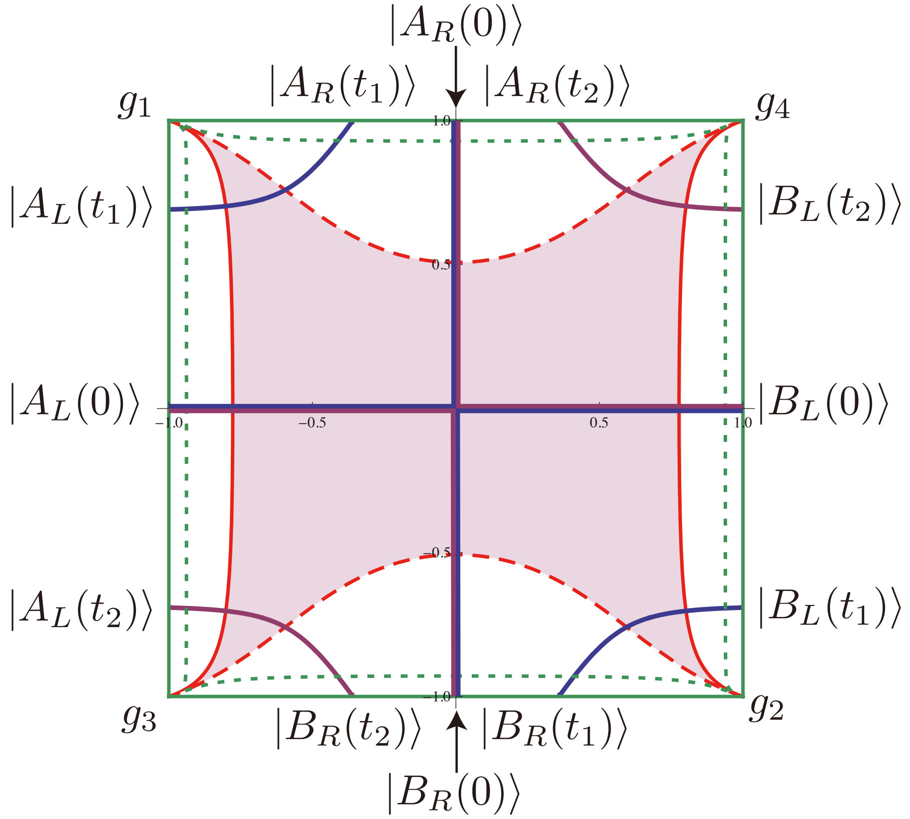
The open string endpoints in each gluon are entangled by the open string going through the wormhole.

This is in the same way as the entanglement of quark and anti-quark.

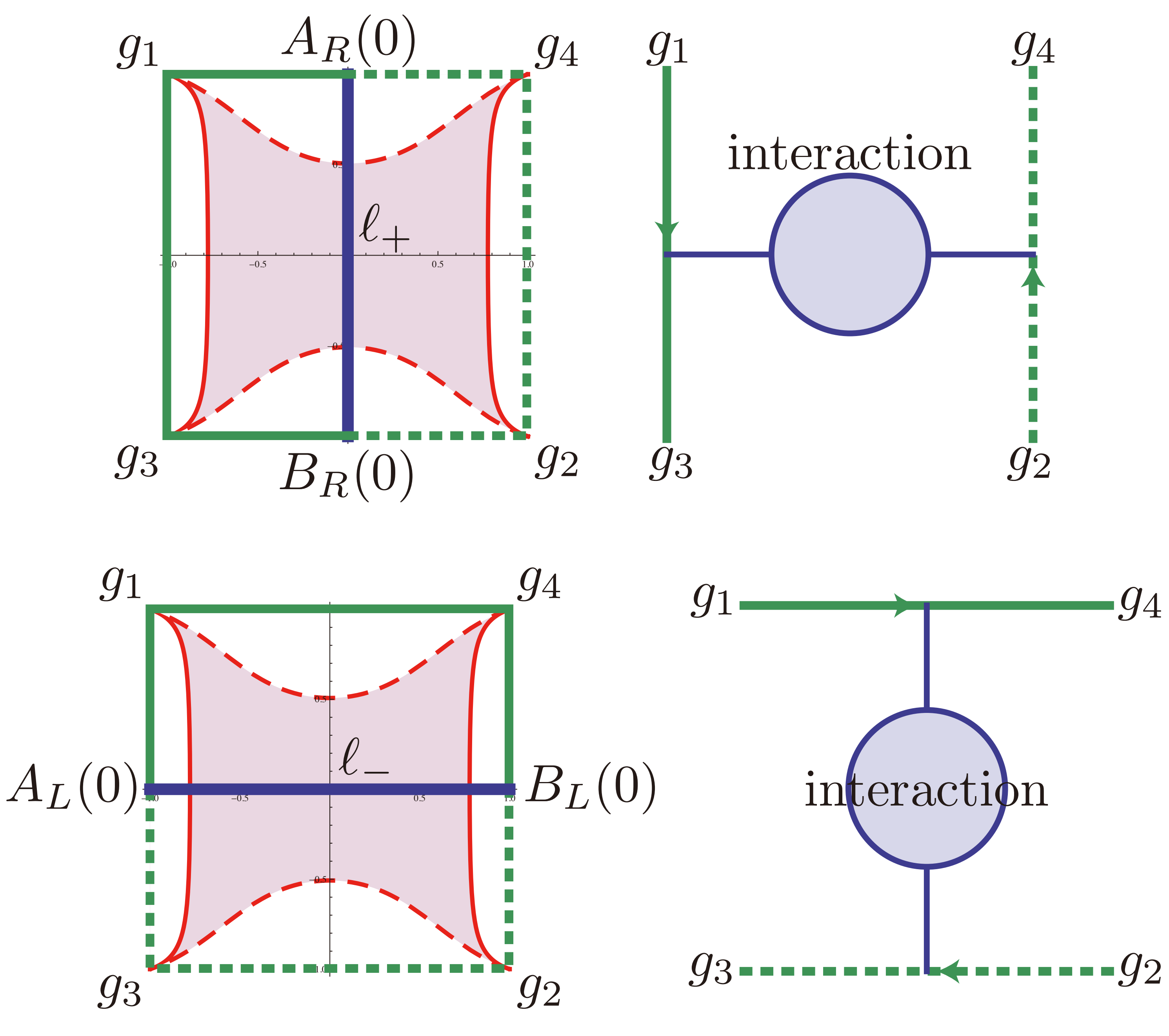


2. Entanglement of gluons

Any paths connecting the gluons must go through the wormhole region.



There are two channels.



How can we measure the change of entanglement in gluon scattering process?

i) (naively) log of scattering amplitude

[Lewkowycz-Maldacena, JHEP 1405 (2014) 025]

The scattering amplitude corresponds to the Wilson loop which is given by the area of minimal surface. And naively $S = (1 - n\partial_n)\log\langle W\rangle|_{n\to 1}$.

$$\mathcal{A} \sim e^{-\mathrm{Area}}, \quad \Delta S \sim \log\mathcal{A} = \frac{\sqrt{\lambda}}{2\pi}\left(\log\frac{1+\beta}{1-\beta}\right)^2 \qquad \frac{s}{t} = \left(\frac{1+\beta}{1-\beta}\right)^2$$

ii) the length between boundaries at the contacting points

$$\ell_+(\beta) = R\int_{-u_{+\infty}}^{+u_{+\infty}} du_+\sqrt{g_{++}}\Big|_{u_-=0}, \quad \ell_-(\beta) = R\int_{-u_{-\infty}}^{+u_{-\infty}} du_-\sqrt{g_{--}}\Big|_{u_+=0}$$

where we introduced the cutoff, $z_\infty\ (\to\infty)$.

$$2\frac{\alpha z_\infty}{R^2} = (1+\beta)\cosh u_{+\infty} + 1 - \beta = (1-\beta)\cosh u_{-\infty} + 1 + \beta$$

$$\ell_\pm(\beta) = R\left[\sqrt{6}\log\frac{2\alpha z_\infty}{R^2} + \sqrt{6}\log\frac{1}{1\pm\beta} + \mathcal{O}\left(\frac{1}{z_\infty}\right)\right]$$

$$\Delta S \sim (\ell_+(\beta) - \ell_-(\beta))^2$$

ΔS diverges at the Regge limit, $\beta = 1$, and vanishes at $\beta = 0$.

Scattering vs Entanglement

Scattering process $\Longleftarrow$ S-matrix

$$\{p_1, p_2\} \to \{k_1, k_2\}$$

$$|\mathrm{ini}\rangle = |p_1, p_2\rangle$$

$$|\mathrm{fin}\rangle = \sum_{k_i} |k_1, k_2\rangle\langle k_1, k_2|S|\mathrm{ini}\rangle$$

The entanglement of particles is changed from the initial state to the final one.

$$e^{i(H_0 + H_{\mathrm{int}})t}$$

Problem: we need to understand the relation between S-matrix theory and entanglement entropy both in the quantum field theory directly and in holography.