Free energy from toric diagram in $\mathrm{AdS}_4/\mathrm{CFT}_3$

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Collaboration with Sangmin Lee (SNU)
Strings and Fields @ YITP2014

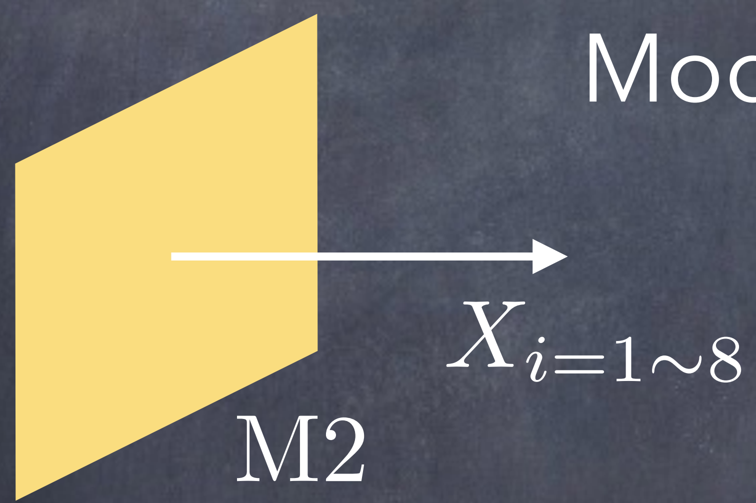
M2-branes in 11 dim

$\mathrm{AdS}_4/\mathrm{CFT}_3$

$3d\ \mathcal{N}=2$ Chern-Simons theory

Free energy
in large *N* limit

Moduli space



4d (toric) Calabi-Yau

$$ds^2_X = dr^2 + r^2 ds^2_Y$$

Toric diagram

SUGRA on $AdS_4 \times Y_7$

Volume

Global sym = Isometry

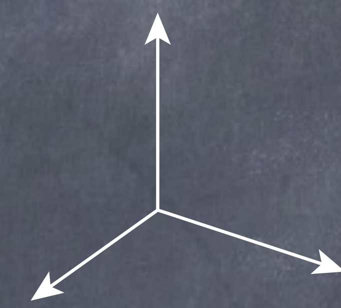
$$F = N^{3/2}\sqrt{\frac{2\pi^6}{27\mathrm{Vol}(Y_7)}}$$

Toric diagram

CY_n

$n = 3$

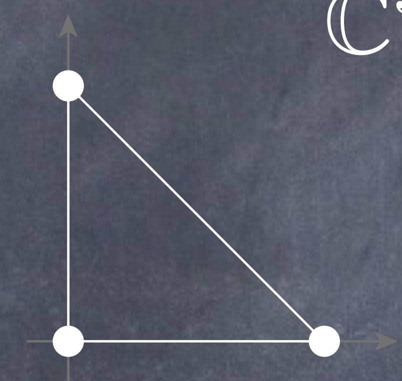
$$v^I = \begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}0\\1\\0\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}$$



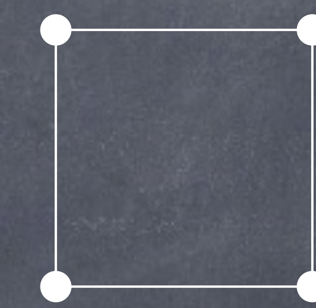
$SL(3, \mathbb{Z})$

$$v^I = \begin{pmatrix}1\\0\\1\end{pmatrix}, \begin{pmatrix}0\\1\\1\end{pmatrix}, \begin{pmatrix}0\\0\\1\end{pmatrix}$$

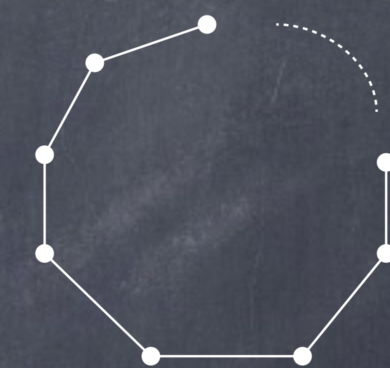
$\mathbb{C}^3$



Conifold

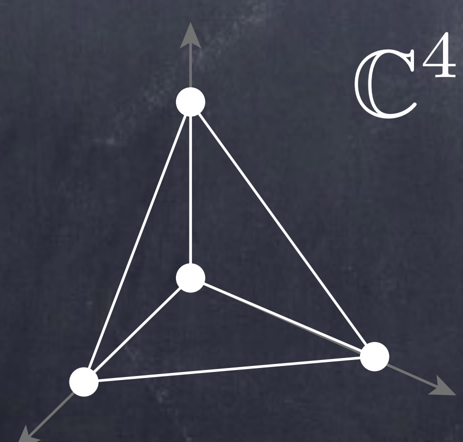


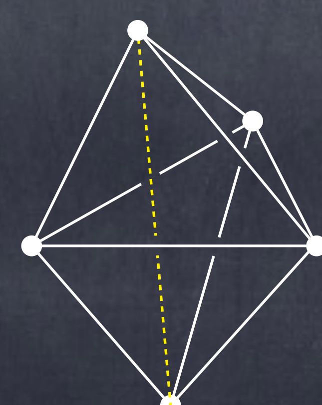
General

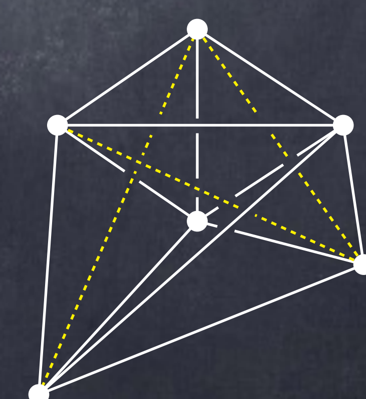


$n = 4$

$\mathbb{C}^4$







Free energy & Volume

Equivalent ?

$$F^2 \overset{?}{=} \frac{1}{\mathrm{Vol}(Y_7)}$$

F-max

[Jafferis '10][Closset '12]

$$F = -\log |Z(R)|$$

$$R = \sum_{I=1}^{d} a_I \Delta^I$$

R-sym mixes with global syms in RG flow

$$\sum_I \Delta^I = 2$$

Marginality of superpotential

Vol-min

[Martelli, Sparks, Yau '05]

$$\mathrm{Vol}(Y_7) = V_{\mathrm{MSY}}(b)$$

$$K_R = \sum_{i=1}^{4} b^i \frac{\partial}{\partial \phi^i}$$

Reeb vector

$$b^4 = 4$$

CY condition

?

Toric diagram

a-max = Vol-min

a-max

[Intriligator, Wecht '03]

$$a = \frac{1}{32}\left(9\mathrm{tr}R^3 - 3\mathrm{tr}R\right)$$

$$R = \sum_{I=1}^{d} a_I \Delta^I$$

Equivalent

$$a = \frac{1}{\mathrm{Vol}(Y_5)}$$

Vol-min

[Martelli, Sparks, Yau '05]

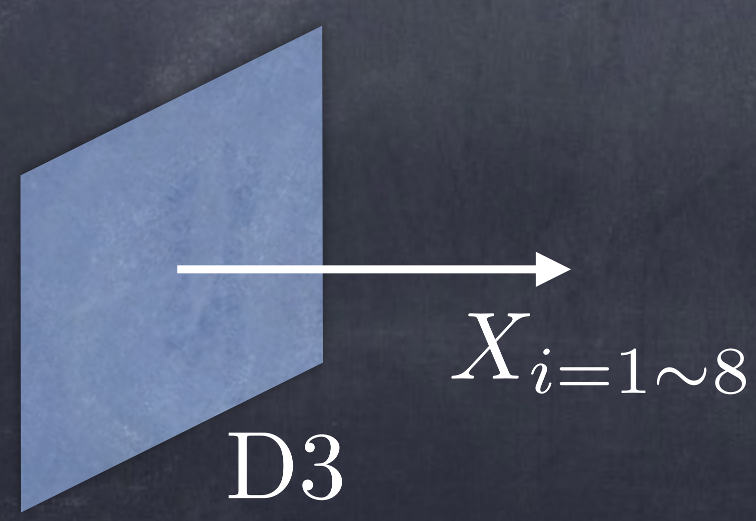
$$\mathrm{Vol}(Y_5) = V_{\mathrm{MSY}}(b)$$

$$\mathrm{AdS}_5 \times Y_5$$

$$K_R = \sum_{i=1}^{3} b^i \frac{\partial}{\partial \phi^i}$$

Toric diagram

$$\mathrm{CFT}_4/\mathrm{AdS}_5$$

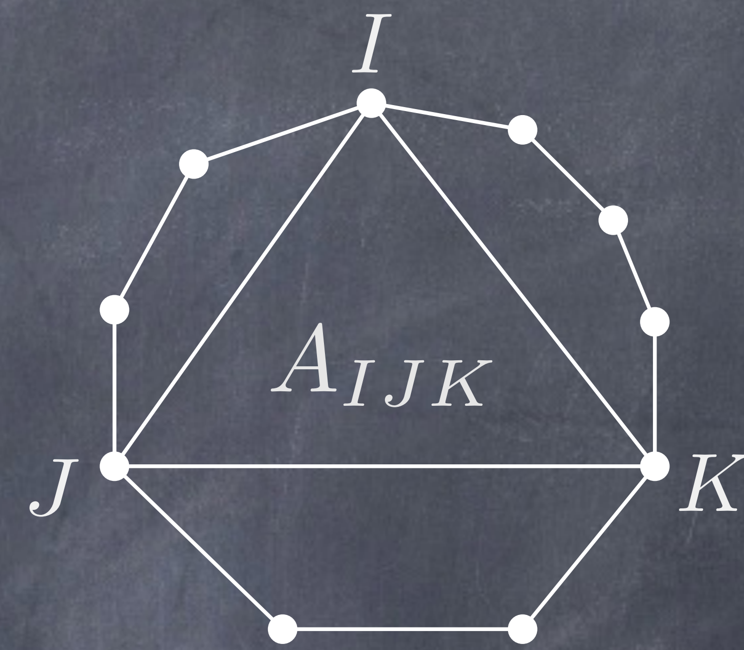


a-max = Vol-min

[Butti, Zaffaroni '05][Soo-Jong Rey, Sangmin Lee '06]

$$a = \sum_{I<J<K} C_{IJK} \Delta^I \Delta^J \Delta^K$$

$$C_{IJK} = A_{IJK}$$



$$\Delta^I = t^a Q^I_a + s^i F^I_i$$

Baryon (homology) — Meson (isometry)

$$I = 1 \sim d$$
$$i = 1 \sim n = 3$$
$$a = 1 \sim d - n$$

$$a \sim \cancel{t^3} + t^2 s + t s^2 + s^3$$

Extremize along t

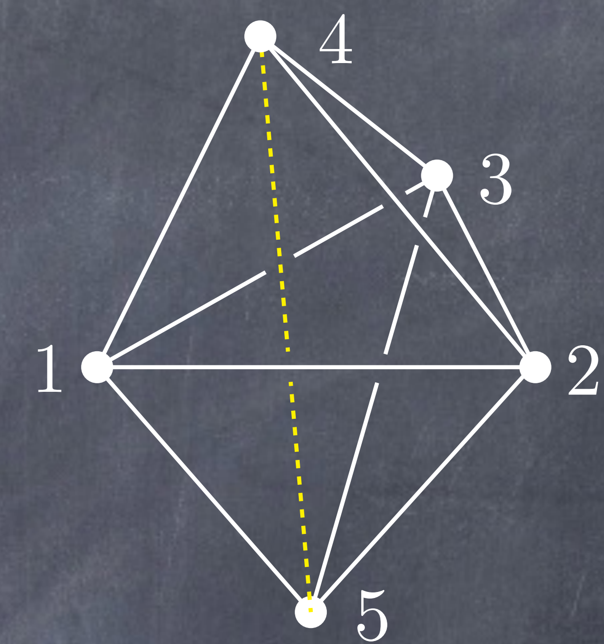
$$a(s) = \frac{1}{\mathrm{Vol}(b = s)}$$

$$\mathrm{Vol}(Y_5) = \frac{\pi^3}{3} \sum_I \frac{\langle v_{I-1}, v_I, v_{I+1} \rangle}{\langle b, v_{I-1}, v_I \rangle \langle b, v_I, v_{I+1} \rangle}$$

t'Hooft anomaly calculation

Free energy from toric diagram

[Amariti, Franco '12]

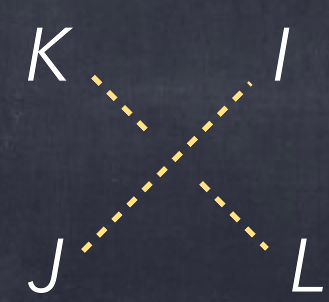


$$F^2(\Delta) = \sum_{I<J<K<L} C_{IJKL} \Delta^I \Delta^J \Delta^K \Delta^L$$

$$= \sum_{I<J<K<L} V_{IJKL} \Delta^I \Delta^J \Delta^K \Delta^L$$

$$+(''\text{corrections}'')$$

$$(\text{factor})(\Delta^I \Delta^J)^2$$



$$(\text{factor})(\Delta^I \Delta^J)(\Delta^K \Delta^L)$$

Strategy

$$F^2(\Delta) = \sum_{I<J<K<L} V_{IJKL}\Delta^I\Delta^J\Delta^K\Delta^L + \delta(''\text{corrections}'')$$

$$\Delta^I = t^a Q_a^I + s^i F_i^I$$

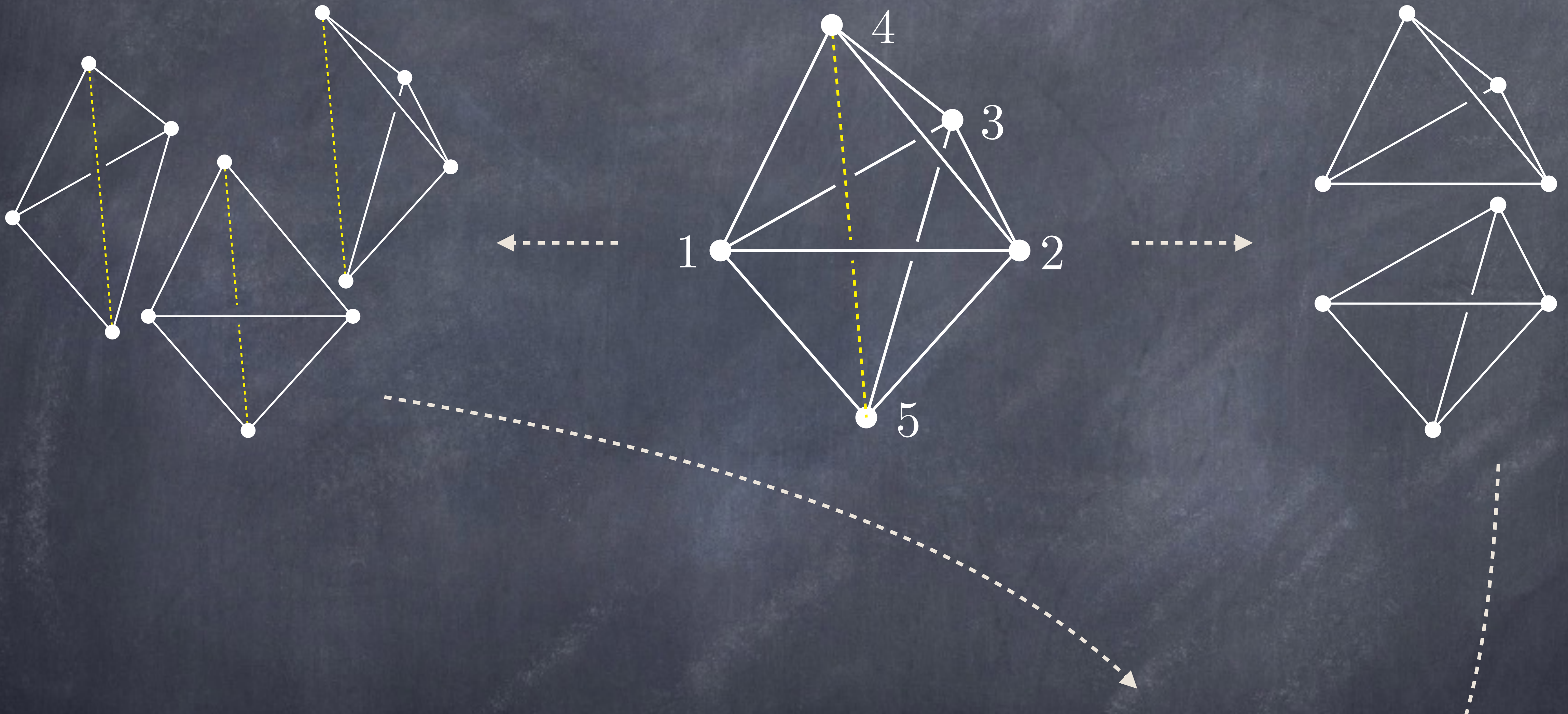
Baryon (homology) Meson (isometry)

$$F^2 \sim \cancel{t^4} + \cancel{t^3}s + t^2s^2 + ts^3 + s^4$$

Extremize along t

$$F^2(s) = \frac{1}{\text{Vol}(s)}$$

5-vertex model

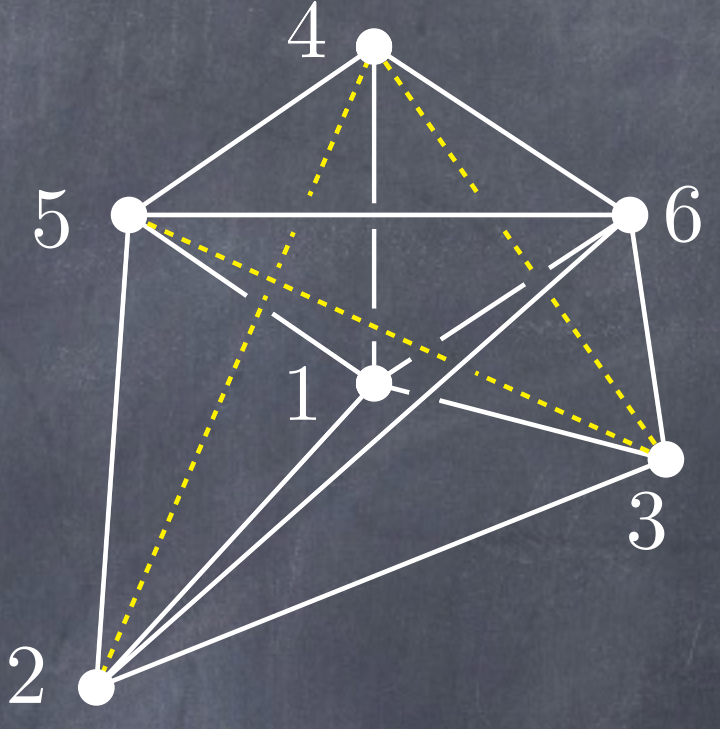


$$F^2(\Delta) = \sum_{I<J<K<L} V_{IJKL}\Delta^I\Delta^J\Delta^K\Delta^L - \frac{V_{1245}V_{2345}V_{3145}}{V_{1234}V_{1235}}(\Delta^4\Delta^5)^2$$

"Correction" term

6-vertex model (1)

$$R = \frac{V_{1245} V_{2356} V_{3164}}{V_{3145} V_{1256} V_{2364}}$$

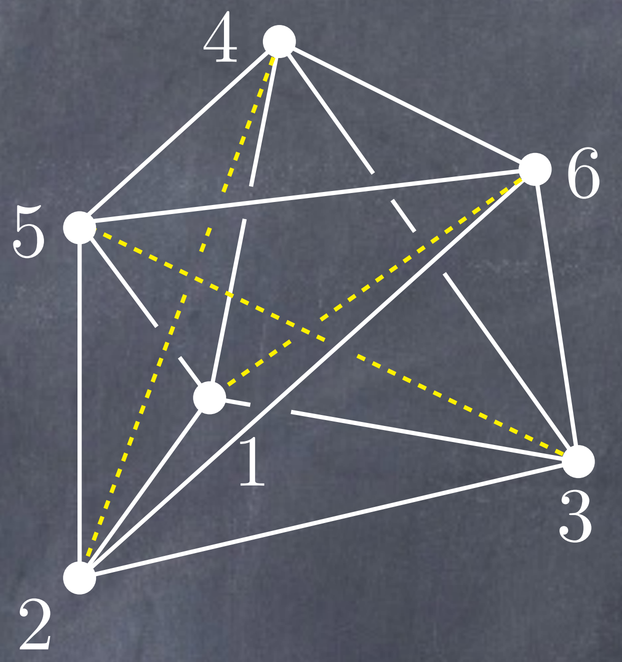


$$\delta_1 = -\frac{V_{2456} V_{2461} V_{2415}}{V_{2561} V_{4561}} (\Delta^2 \Delta^4)^2 - \frac{V_{5321} V_{5316} V_{5362}}{V_{2561} V_{2361}} (\Delta^3 \Delta^5)^2 - (1 - R) \frac{V_{2346} V_{1345} V_{3461}}{V_{1236} V_{1456}} (\Delta^3 \Delta^4)^2$$

$$\delta_2 = -2 \frac{V_{2415} V_{3461} V_{4256}}{V_{1256} V_{4561}} (\Delta^2 \Delta^4)(\Delta^3 \Delta^4) - 2 \frac{V_{2356} V_{3461} V_{3125}}{V_{1236} V_{6125}} (\Delta^3 \Delta^4)(\Delta^3 \Delta^5) + 2 \frac{V_{1245} V_{2356}}{V_{1256}} (\Delta^2 \Delta^4)(\Delta^3 \Delta^5)$$

6-vertex model (2)

$$R = \frac{V_{1245}V_{2356}V_{3164}}{V_{3145}V_{1256}V_{2364}}$$



$$\delta_1 = -\frac{1}{1+R}\left(\frac{V_{4256}V_{2134}V_{2415}}{V_{1256}V_{5134}}(\Delta^2\Delta^4)^2 + (\text{cyclic})\right)$$

$$\delta_{2A} = \frac{2}{1+R}\left(\frac{V_{1245}V_{3164}}{V_{3145}}(\Delta^2\Delta^4)(\Delta^1\Delta^6) + (\text{cyclic})\right)$$

$$\delta_{2B} = -\frac{2R}{1+R}\left(V_{2416}(\Delta^2\Delta^4)(\Delta^1\Delta^6) + (\text{cyclic})\right)$$

Conclusions

- We found the way to derive a free energy from a toric diagram for general 5,6-vertex models
- ‘tHooft anomaly ??
- Generalization to general toric diagrams ?
- Can be used for determining a prepotential of 4d gauged SUGRA ?