A Holographic Realization of Ferromagnets

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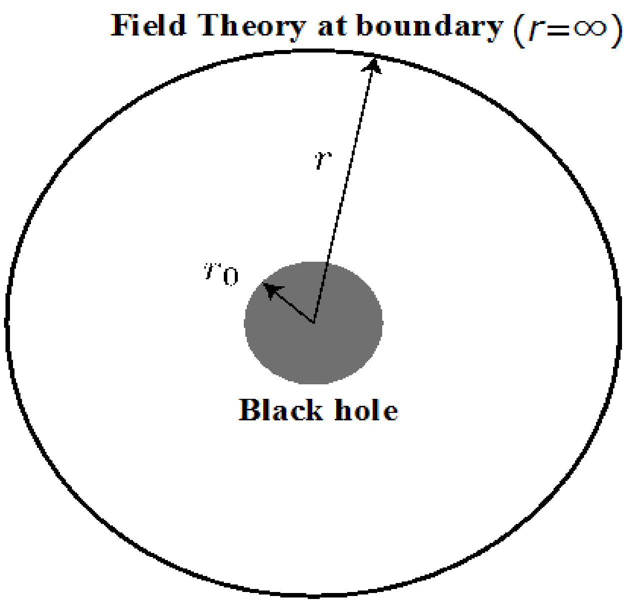
Holographic duality

New Duality from string theory: Holographic Duality (Holography)

J.M. Maldacena 1998,



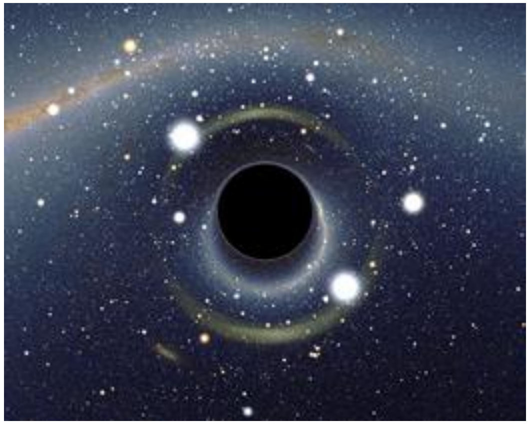
QFT in d-dimension

Holography

gravity in (d+1)-dimension

Holographic Ferromagnet

http://ja.wikipedia.org/wiki/%E7%A3%81%E7%9F%B3



http://en.wikipedia.org/wiki/Black_hole

We construct the dual gravity model of ferromagnet by holography

Rotational SU(2) sym. → **U(1)**

Contents

✓ Introduction

Ferromagnet (condensed matter theory)

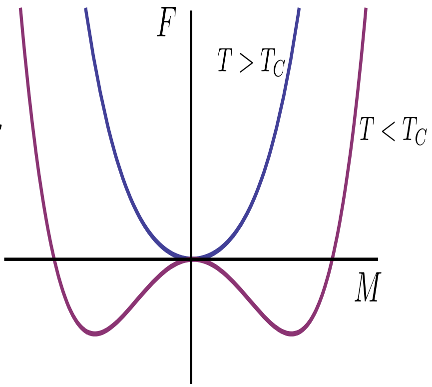
Ferromagnet (Holographic duality)

Numerical Result

Summary

Ginzburg-Landau Theory

Ferromagnet : SU(2) is broken to U(1)



GL theory: useful near critical temperature $T \sim T_c$

***F:* Free energy** ***M*: Magnetization** ***H*: Magnetic field**

$$F = F_0 + \frac{1}{2}a(T - T_c)M^2 + \frac{1}{4}bM^4 - MH$$

For ***H=0***, $\frac{\partial F}{\partial M} = a(T - T_c)M + bM^3 = 0$

$$\leftrightarrow \begin{cases} M \propto (T - T_c)^{\frac{1}{2}} & \text{for } T < T_c \\ M = 0 & \text{for } T > T_c \end{cases}$$

For $H \neq 0$, $\frac{\partial F}{\partial M} = a(T - T_c)M + bM^3 - H = 0$

$\leftrightarrow$ $M \propto H^{\frac{1}{3}}$ at $T \sim T_c$,

Curie-Weiss Law

Susceptibility $\chi \equiv \frac{\partial M}{\partial H}$

$$\frac{\partial F}{\partial M} = a(T - T_c)M + bM^3 - H = 0$$

$$\leftrightarrow \qquad a(T - T_c)\chi + 3bM^2\chi - 1 = 0$$

Curie-Weiss Law

$$\chi\Big|_{H=0} = \begin{cases} \dfrac{2C}{T - T_c} & (T > T_c) \\ \dfrac{C}{T_c - T} & (T < T_c) \end{cases}$$

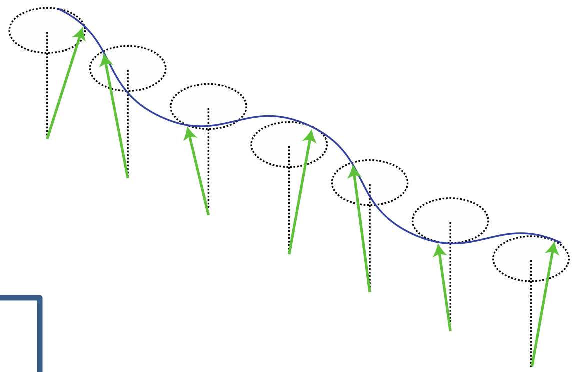
C: constant

Low Temperature and magnons

At low temperatures, magnetization is mostly aligned.
elementary excitations: magnons (quantized spin wave)

Reduction of magnetization is proportional to magnon density ***n*** :

$$\boldsymbol{M = M_0 - \Delta M \qquad \Delta M \propto n}$$



Dispersion of magnons : $\boldsymbol{\epsilon_k = Dk^2 + \alpha H}$

Magnon density : $\boldsymbol{n = T^{\frac{3}{2}} e^{-\alpha H / k_B T}}$

Bloch $T^{3/2}$ law : $\boldsymbol{\Delta M|_{H \to 0} \propto T^{\frac{3}{2}}}$

Prescription of Holography

Find the 1-dimensional higher gravity action with the same symmetry (breaking) as the Ferromagnetic system.

Solve the equation of motion from the gravitational action.

Extract the physical quantities from the solution by using "holographic dictionary".

Gravity action dual to Ferromagnet

(3+1)D Ferromagnetic system :

SU(2) symmetry which is spontaneously broken to U(1)

⬍

(4+1)D Gravitational system with SU(2) fields which is spontaneously broken to U(1)

$$S_g = \int d^5x \sqrt{-g}\left(\frac{1}{2\kappa^2}(R-2\Lambda) - \frac{1}{4e^2}G_{MN}G^{MN} - \frac{1}{4g^2}F^a_{MN}F^{aMN} - \frac{1}{2}(D_M\phi^a)^2 + V(|\phi|)\right)$$

$F^a_{MN} = \partial_M A^a_N - \partial_N A^a_M + \epsilon^{abc}A^b_M A^c_N$ **SU(2) gauge field**

$a = 1,2,3$

$x^M = (t, x, y, z, r)$

$G_{MN} = \partial_M B_N - \partial_N B_M$ **U(1) gauge field**

$\phi^a = (0, 0, \phi(r))$ **: triplet scalar**

$$V = \frac{\lambda}{4}\left(|\phi|^2 - \frac{m^2}{\lambda}\right)^2$$

: potential for scalar

Dictionary

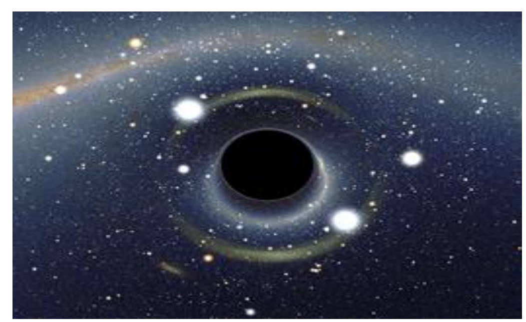
GKP-Witten relation $Z_{QFT}[J] = e^{-S_{gravity}[J]}$

J: source

S.S. Gubser, I.R. Klebanov and A.M. Polyakov 1998 , E.Witten 1998

(3+1)D Ferromagnet		(4+1)D gravity
Magnetization M		Scalar field ϕ
External Magnetic Field H	Holography	
Temperature T	⟷	Black Hole temperature T
Charge current J_μ		U(1) gauge field B_M
Spin Current J^a_μ		SU(2) gauge field A^a_M

http://ja.wikipedia.org/wiki/%E7%A3%81%E7%9F%B3



http://en.wikipedia.org/wiki/Black_hole

Black Hole solution

$$S_g = \int d^5x \sqrt{-g}\left(\frac{1}{2\kappa^2}(R-2\Lambda) - \frac{1}{4e^2}G_{MN}G^{MN} - \frac{1}{4g^2}F^a_{MN}F^{aMN} - \frac{1}{2}(D_M\phi^a)^2 + V(|\phi|)\right)$$

Solution of EOM for $\phi = 0$

(4+1)-Dim AdS charged Black Hole metric

$$ds^2_{AdS-cBH} = \frac{r^2}{l^2}\left(-f(r)dt^2 + dx^2 + dy^2 + dz^2\right) + \frac{l^2}{f(r)}\frac{dr^2}{r^2}$$

$$f(r) = 1 + Q^2\left(\frac{r_H}{r}\right)^6 - \left(1+Q^2\right)\left(\frac{r_H}{r}\right)^4$$

$$A_0^3 = \mu_s\left(\frac{r_H}{l}\right)\left(1 - \frac{r_H^2}{r^2}\right)$$

$$Q^2 = \frac{2\kappa^2}{3}\left(\frac{\mu^2}{e^2} + \frac{\mu_s^2}{g^2}\right)$$

$$B_0 = \mu\left(\frac{r_H}{l}\right)\left(1 - \frac{r_H^2}{r^2}\right)$$

Black Hole Temperature : $T = \frac{2-Q^2}{2\pi}$

$\Lambda = -\frac{6}{l^2}$ $\quad r_H = l = 1$

C. P. Herzog and S. S. Pufu (2009) N. Iqbal, H. Liu, M. Mezei, and Q. Si 2010

Equation of motion for ϕ

Action for ϕ

$$S_\phi = \int dr\sqrt{-g_{AdS-cBH}}\left(-\frac{1}{2}\left(\partial\phi(r)\right)^2 - V\left(\phi(r)\right)\right)$$

Equation of motion

$$\lambda\phi^3(r) - m^2\phi(r) - \left(5f(r)r + f'(r)r^2\right)\phi'(r) - f(r)r^2\phi''(r) = 0$$

$$f(r) = 1 + \frac{Q^2}{r^6} - \frac{1+Q^2}{r^4} \qquad T = \frac{2-Q^2}{2\pi}$$

Asymptotic solution:

$$\phi(r) = \frac{H}{r^{2-\Delta}} + \frac{M}{r^{2+\Delta}} + \cdots \qquad \Delta \equiv \sqrt{4-m^2} \qquad 3.5 \leq m^2 \leq 4$$

→ *H*: External magnetic field

M: Magnetization

from GKP-Witten relations ($Z_{QFT}[J] = e^{-S_{gravity}[J]}$)

Numerical method

We solve the EOM numerically. $(m^2 = 3.89,\ \lambda = 1)$

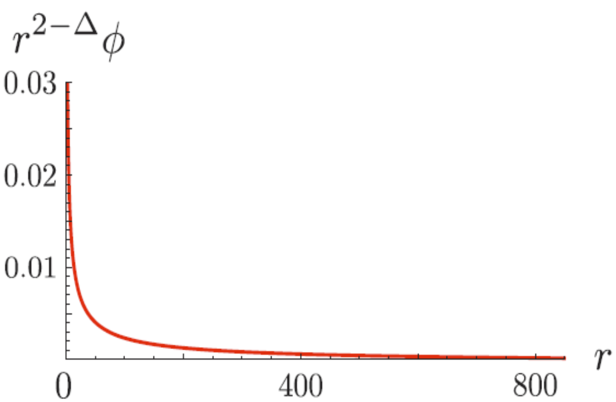
$$\lambda\phi(r)^3 - m^2\phi(r) - \left(5f(r)r + f'(r)r^2\right)\phi'(r) - f(r)r^2\phi''(r) = 0$$

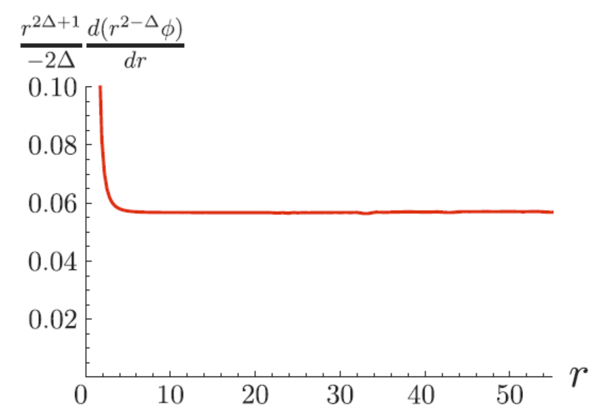
$$f(r) = 1 + \frac{Q^2}{r^6} - \frac{1+Q^2}{r^4} \qquad T = \frac{2-Q^2}{2\pi}$$

We will focus on **Spontaneous magnetization** **(M when $H = 0$)**

$$\phi(r) = \frac{H}{r^{2-\Delta}} + \frac{M}{r^{2+\Delta}} + \cdots \qquad \Delta \equiv \sqrt{4 - m^2}$$

$$H = r^{2-\Delta}\phi(r)|_{r\to\infty} \qquad\qquad M = \frac{r^{2\Delta+1}}{-2\Delta}\frac{d\left(r^{2-\Delta}\phi(r)\right)}{dr}|_{r\to\infty}$$



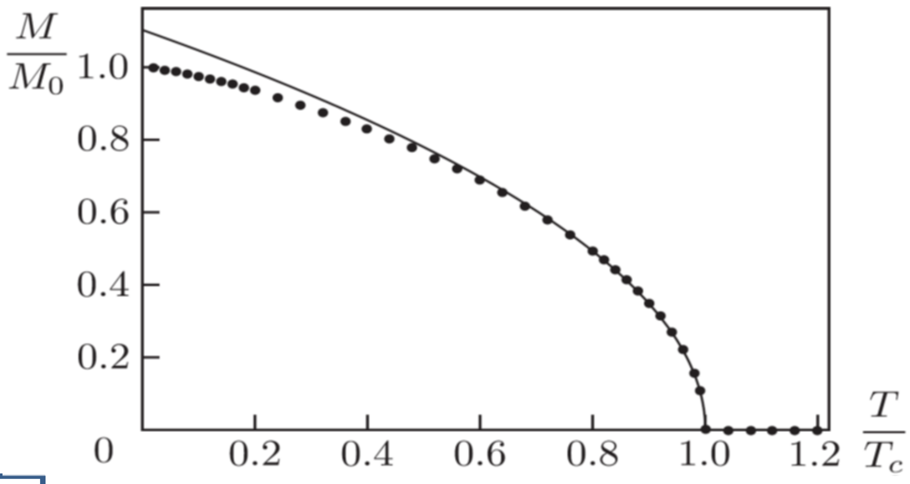


Result: $T \sim T_c$

Spontanious Magnetization M

$$M \propto \left(1 - \frac{T}{T_c}\right)^{\frac{1}{2}}$$

●: result by holographic duality

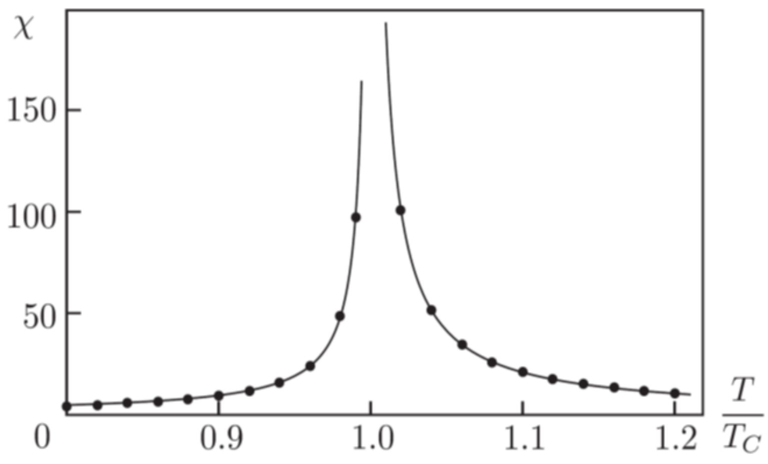


Magnetic susceptibility $\chi \equiv \frac{dM}{dH}|_{H=0}$

we can get **Curie–Weiss law**

$$\chi = \begin{cases} \dfrac{c_+}{T/T_c - 1} & (T > T_c) \\ \dfrac{c_-}{1 - T/T_c} & (T < T_c) \end{cases}$$

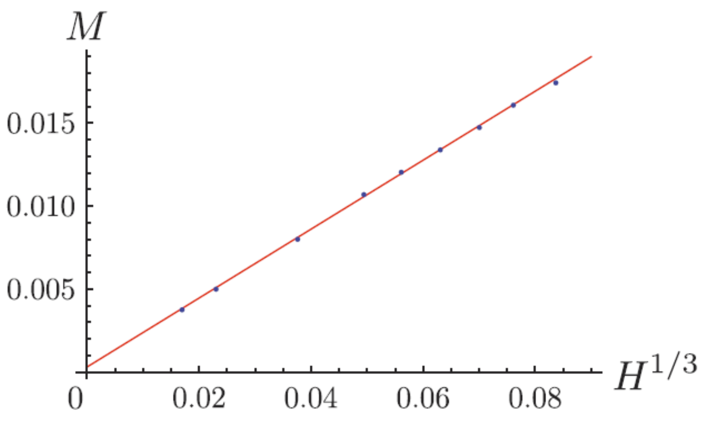
$$c_+/c_- \sim 2.22$$



Result : T ~ T_c

H: **External magnetic field**
M: **Magnetization**

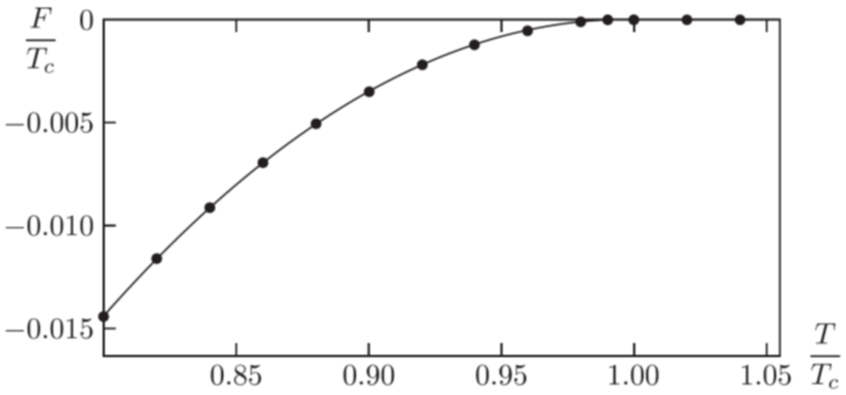
$$M \propto H^{\frac{1}{3}}$$



F: **Free energy**

The scalar part of the on-shell action

$$F \propto (T - T_c)^2$$



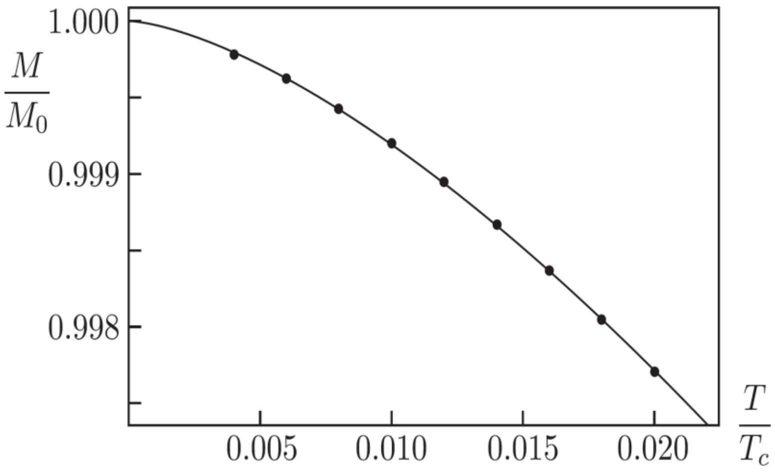
Results near T_c are consistent with Ginzburg-Landau **Theory**

Result: low temperature ($T \sim 0$)

Magnetization M

we can reproduce **the Bloch $T^{\frac{3}{2}}$ law**

$$M \propto 1 - C\left(\frac{T}{T_c}\right)^{\frac{3}{2}}$$

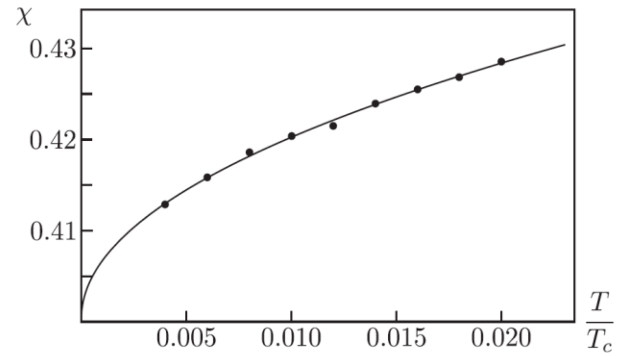


At low *T*, Results are consistent with magnons.

Result: low temperature ($T \sim 0$)

Magnetic susceptibility: χ

$$\chi \sim \chi_0 + D\left(\frac{T}{T_c}\right)^{\frac{1}{2}}$$

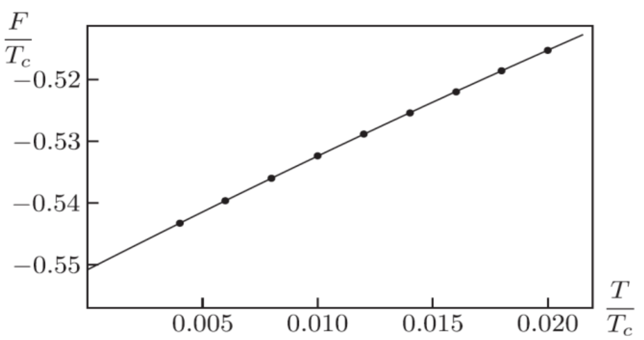


First term χ_0: Pauli paramagnetic susceptibility from conduction electrons

Second term: susceptibility from magnons

***F*: Free energy**

$$F \sim -F_0 + E\left(\frac{T}{T_c}\right) - \gamma\left(\frac{T}{T_c}\right)^2$$



γ: linear in *T* of the specific heat from conduction eletctrons

Summary

We have constructed a holographic dual model of ferromagnet and found the holographic dictionary between ferromagnet and gravity.

Using the dictionary, we analyzed the temperature dependence of Magnetization ***M***, Susceptibility χ , Free energy F

Our results are consistent with

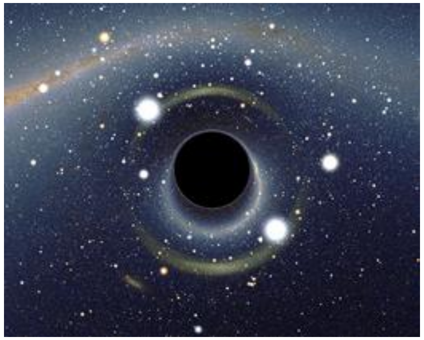
$\mathrm{T} \sim 0$: Magnon + Conduction electron
$\mathrm{T} \sim T_c$: GL theory

Black Hole captures the ferromagnetic system both near T_c and low temperatures

Outlook

1 Magnon dynamics

2 Correlation functions



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