Quantum Entanglement of Excited States by Heavy Local Operators in Large-c 2d CFT at Finite Temperature

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Motivations

- Entanglement Entropy (EE) for Excited States by local operators

$$S_A = -\mathrm{Tr}\rho_A \log \rho_A \qquad \rho = \mathcal{O}(x)|\Psi\rangle\langle\Psi|\mathcal{O}^\dagger(x) \qquad \rho_A = \mathrm{Tr}_{\bar{A}}\,\rho$$

→ "Can we characterize the local operators from entanglement measures ?"

- Free scalar [Nozaki-Numasawa-Takayanagi, Nozaki 14]
- RCFT [He-Numasawa-Takayanagi-KW 14]
- Large-N [Caputa-Nozaki-Takayanagi 14]
- Large-c [Asplund-Bernamonti-Galli-Hartman 14]
- Finite T [Caputa-Simon-Stikonas-Takayanagi 14]

For some case, good ! For other case, looks difficult···

etc···

→ **Next! Large-c & Finite T !** (Heavy local operator)

- Mutual Information (MI)

$$I_{A:B} = S_A + S_B - S_{A\cup B}$$

$$\geq \frac{(\langle\mathcal{O}_A\mathcal{O}_B\rangle - \langle\mathcal{O}_A\rangle\langle\mathcal{O}_B\rangle)^2}{2||\mathcal{O}_A||^2||\mathcal{O}_B||^2}$$

[Wolf-Verstraere-Hastings-Cirac 08] (for finite dimensional Hilbert space)

Upper bound for the connected 2-pt functions !

→ **When does MI vanish ?** $I_{A:B} = 0$

No entanglement (and correlation) between A and B !

- Scrambling Time t_ω^* [Hayden-Preskill 07]

→ "The minimum time required for the information about the initial state to be lost"

- Fast Scrambling [Sekino-Susskind 08]···

Black holes (BHs) are the fastest scramblers in nature.

Diffusion time $t_D \sim \beta$

$$t_\omega^* \sim \beta \log S$$

- Holographic model (BTZ BH + local perturbation)

$$I_{A:B}(t_\omega^*) = 0$$ [Shenker-Stanford 13 14]···

Small perturbations at the boundary will get exponentially blue-shifted energy at BH horizon.

Very heavy back-reaction → Shock wave

In the boundary system, → "Chaotic behavior" "Butterfly effect"

$$t_\omega^* \sim \frac{\beta}{2\pi}\log S$$

→ **"How about the perturbations by heavy local operators ?"**

"How scrambled ?"

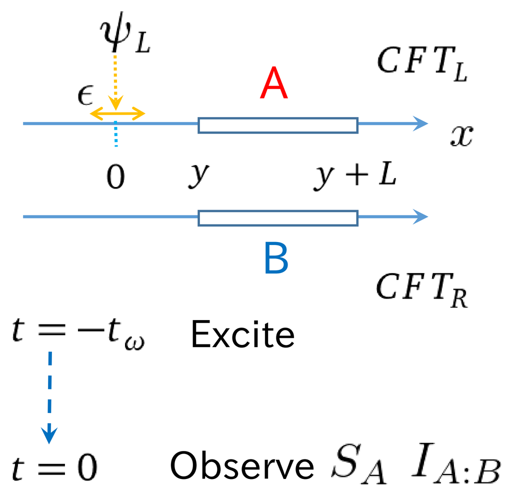
Set-up

- Large-c 2d CFT $c \to \infty$
- Finite T Thermo-field double (TFD) state

$$|TFD\rangle_\beta = \sum_n e^{-\frac{\beta}{2}E_n}|n\rangle_L|n\rangle_R \qquad |n\rangle_{L,R} \in \mathcal{H}_{L,R} \text{ in } CFT_{L,R}$$

$$\mathrm{Tr}_R|TFD\rangle\langle TFD| \to \sum_n e^{-\beta E_n}|n\rangle\langle n|_L$$

[Morrison-Roberts 12] [Hartman-Maldacena 13]



- Heavy local operator

$$\psi_L(x=0, t=-t_\omega) \qquad h_\psi \sim O(c) \text{ : Heavy} \qquad h_\psi/c \text{ : fixed}$$

Twist operators σ_n, (Anti-) $\tilde{\sigma}_n$: $H_\sigma/c = \frac{1}{24}\left(n-\frac{1}{n}\right) \xrightarrow{n\to 1} 0$: Light

→ $$\rho = \mathcal{N}\cdot\psi_L(-t_\omega)|TFD\rangle_\beta\langle TFD|_\beta\,\psi_L^\dagger(-t_\omega)$$

$\mathrm{Tr}\rho$ → Correlators on the cylinder C_1

- Time evolution

Consider the time evolution by $e^{-it(H_L-H_R)}$

$$\psi_L \to e^{-itH_L}\psi_L e^{+itH_L} \qquad |TFD\rangle_\beta \to |TFD\rangle_\beta$$

Invariant ! (isometry in the dual geometry)

- Replica trick

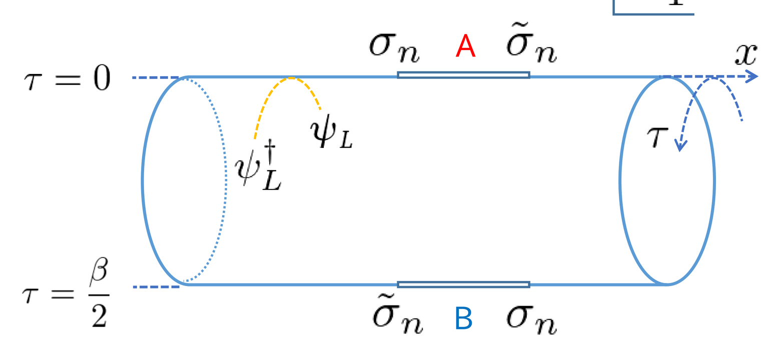
First compute this!

$$S_A^{(n)} = \frac{1}{1-n}\log \mathrm{Tr}\rho_A^n \xrightarrow{n\to 1} S_A$$

n replica fields $\Psi_L = \psi_L^{(1)}\cdots\psi_L^{(n)}$

$$\mathrm{Tr}\rho_A^n = \frac{\langle\Psi_L\sigma_n^A\tilde{\sigma}_n^A\Psi_L^\dagger\rangle_{C_1}}{(\langle\psi_L\psi_L^\dagger\rangle_{C_1})^n} \qquad \mathrm{Tr}\rho_B^n = \frac{\langle\Psi_L\sigma_n^B\tilde{\sigma}_n^B\Psi_L^\dagger\rangle_{C_1}}{(\langle\psi_L\psi_L^\dagger\rangle_{C_1})^n}$$

$$\mathrm{Tr}\rho_{A\cup B}^n = \frac{\langle\Psi_L\sigma_n^A\tilde{\sigma}_n^A\sigma_n^B\tilde{\sigma}_n^B\Psi_L^\dagger\rangle_{C_1}}{(\langle\psi_L\psi_L^\dagger\rangle_{C_1})^n}$$



→ Task :

Compute the correlators on the cylinder in the limit $c\to\infty$, $n \to 1$

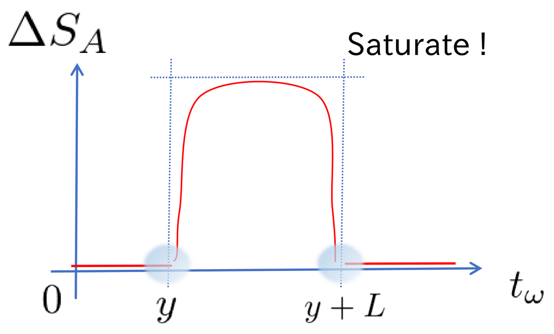
2 Heavy-2 Light (or -4 Light) — 4-pt (or 6-pt) functions

- Large-c computations → Detail

Results

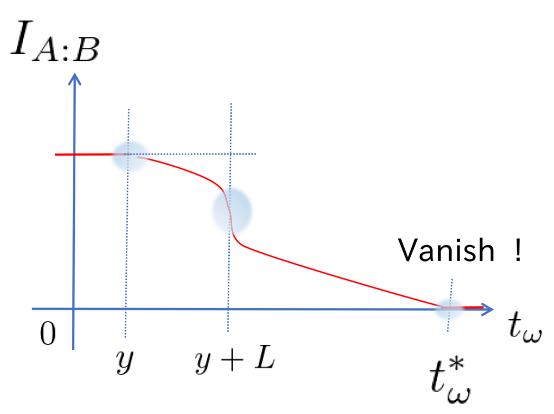
- ΔS_A

$$\Delta S_A \xrightarrow{\epsilon/\beta \ll 1} \begin{cases} 0 & (t_\omega < y,\ y+L<t_\omega) \quad (z,\bar{z}) \to (1,1) \\ \frac{c}{6}\log\left[\frac{\beta}{\pi\epsilon}\frac{\sin\pi\alpha_\psi}{\alpha_\psi}\frac{\sinh\frac{\pi(L-t_\omega)}{\beta}\sinh\frac{\pi t_\omega}{\beta}}{\sinh\frac{\pi L}{\beta}}\right] & (y<t_\omega<y+L) \quad (z,\bar{z}) \to (e^{2\pi i},1)\end{cases}$$



- $I_{A:B}$

$$I_{A:B} \xrightarrow{\epsilon/\beta \ll 1} \begin{cases} \frac{2c}{3}\log\left(\sinh\frac{\pi L}{\beta}\right) & (t_\omega < y) \\ \frac{c}{6}\log\left(\frac{\sinh^3\frac{\pi L}{\beta}\sinh\frac{2\pi(y+L-t_\omega)}{\beta}}{\cosh\frac{\pi(t_\omega-y)}{\beta}}\right) & (y<t_\omega<y+L) \\ \frac{c}{6}\log\left(\frac{4\sinh^4\frac{\pi L}{\beta}}{\sinh\frac{2\pi(t_\omega-y-L)}{\beta}\sinh\frac{2\pi(t_\omega-y)}{\beta}}\right) - \frac{c}{3}\log\left(\frac{\beta}{\pi\epsilon}\frac{\sin\pi\alpha_\psi}{\alpha_\psi}\right) & (y+L<t_\omega)\end{cases}$$



- Scrambling time t_ω^* (for large t_ω)

$$I_{A:B}(t_\omega^*) = 0$$

→ $$t_\omega^* \sim y + \frac{L}{2} - \frac{\beta}{2\pi}\log\frac{\sin\pi\alpha_\psi}{\alpha_\psi} + \frac{\beta}{\pi}\log\left(2\sinh\frac{\pi L}{\beta}\right)$$

cf. [Roberts-Stanford 14] (Read from 2-pt function)

$$\alpha_\psi \ll 1 \to +\frac{\beta}{2\pi}\log\left(\frac{\pi S_{density}}{4E_\psi}\right) \qquad S_{density} = \frac{\pi c}{3\beta} \qquad E_\psi = \frac{\pi h_\psi}{\epsilon}$$

Holographic computations

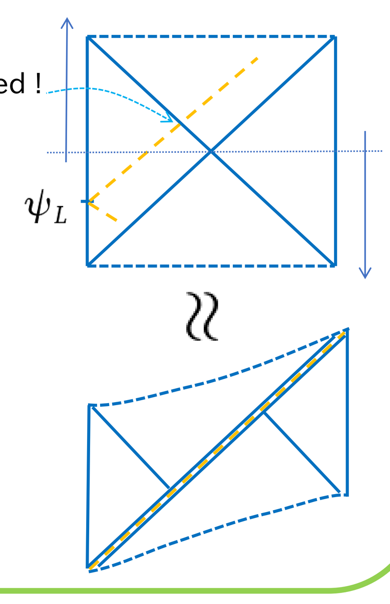
By using Ryu-Takayanagi formula, we can also compute in the holographic model

→ **Perfect matching to the Large-c 2d CFT (leading) results !**

The dual geometry:

Free falling particle in eternal BTZ BH (including the back-reaction)

[Nozaki-Numasawa-Takayanagi 13] [Caputa-Simon-Stikonas-Takayanagi 14],···



Especially, for large t_ω

→ Almost null with energy localized at the horizon

→ ≈ Shock wave geometry

mathching to [Shenker-Stanford,Susskind, Roberts-Stanford··· 13 14]

Some more physics ?

1/c corrections, Recurrence···

Complexity of states [Susskind, Susskind-Stanford,Roberts-Stanford··· 13 14]

Bounds on chaos [Maldacena-Shenker-Stanford 15]

Other approaches based on quantum information??

e.g.Quantum information metric [Miyaji-Numasawa-Shiba-Takayanagi-KW15] [Lashkari-Raamdonk15]

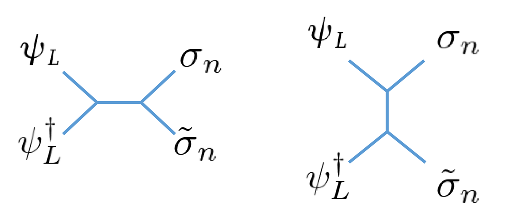
Detail : Large-c computations

By using Conformal map $w = e^{\frac{2\pi}{\beta}x}$ from cylinder to plane,

$$S_A^{(n)} = \frac{c}{6}(n+1)\log\left(\frac{\beta}{\pi\varepsilon_{UV}}\sinh\frac{\pi L}{\beta}\right) - \underbrace{\frac{1}{n-1}\log(|1-z|^{4H_\sigma}G_n(z,\bar{z}))}_{=\Delta S_A^{(n)}}$$

$$G_n(z,\bar{z}) = \langle\psi_L|\sigma_n(z,\bar{z})\tilde{\sigma}_n(1,1)|\psi_L\rangle$$

$(z,\bar{z})$: cross ratio



$c\to\infty$ & Sparse spectrum of low-dimension operators

$$G_n(z,\bar{z}) \simeq \exp\left[-\frac{nc}{6}\cdot f\left(\frac{h_\psi}{c}, \frac{H_\sigma}{nc}, 1-z\right) + c.c.\right]$$ [Zamoldchikov 87]

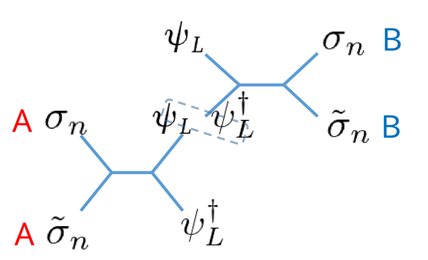
2 Heavy- 2 Light [Fitzpatrick-Kaplan-Walters 14 15]

Expand f around $z\sim 1$ & $n \to 1$ [Asplund-Bernamonti-Galli-Hartman 14] (at T = 0)

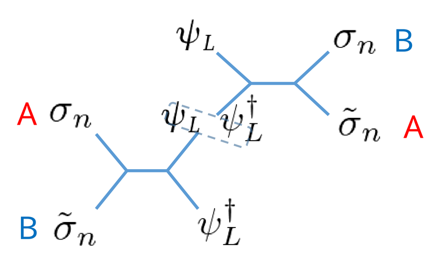
$$\Delta S_A = \frac{c}{6}\log\left(\frac{z^{\frac{1}{2}(1-\alpha_\psi)}(1-z^{\alpha_\psi})\bar{z}^{\frac{1}{2}(1-\bar{\alpha}_\psi)}(1-\bar{z}^{\bar{\alpha}_\psi})}{\alpha_\psi(1-z)\bar{\alpha}_\psi(1-\bar{z})}\right) \qquad \alpha_\psi = \sqrt{1-\frac{24h_\psi}{c}}$$

Similarly, $S_B = \frac{c}{3}\log\left(\frac{\beta}{\pi\varepsilon_{UV}}\sinh\frac{\pi L}{\beta}\right)$ $\quad \Delta S_B = 0$ (for any t_ω)

The 6-pt function can be approximated by 2 dominant contributions : (choose bigger one)



$$S_{A\cup B} = S_A + S_B \quad \to \quad I_{A:B} = 0$$



$$S_{A\cup B} \neq S_A + S_B \quad \to \quad I_{A:B} \neq 0$$