On the superconformal index of Argyres-Douglas theories

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Introduction

Argyres-Douglas (AD) theories

4d N=2 SCFTs (superconformal field theories)
with Coulomb branch ops. of fractional dim.

(e.g. $\mathcal{O}$ such that $[\mathcal{O}] = \frac{3}{2}$)

cf.) SU(N) w/ 2N flavors

$$\mathcal{O} = \mathrm{Tr}(\phi^k) \longrightarrow [\mathcal{O}] = k$$

Coulomb branch op.

Introduction

Argyres-Douglas (AD) theories

asymptotically free gauge theory → (RG-flow) → AD theory

$U(1)_R$ is accidental

The physics of AD theories has been unclear for 20 years!

Introduction

superconformal index (Schur limit)

flavor sym.

$$\mathcal{I}(q;\vec{x}) = \mathrm{Tr}_{\mathcal{H}}\,(-1)^F q^{E-R} \prod_{k=1}^{\mathrm{rank}G_F} (x_k)^{A_k}$$

$\mathcal{H}$: Hilbert sp. of local operators $q,\ x_k \in \mathbb{C}$

E : scaling dim. R : $SU(2)_R$ charge A_k : flavor charge

This index captures the spectrum of BPS local operators.

Q: What is $\mathcal{I}(q;\vec{x})$ of AD theories ???

Our answer

- We conjecture exact expressions for the superconformal indices (in the Schur limit) of two infinite series of AD theories.

(A_1, A_{2n-3}) theory

$$\mathcal{I}_{(A_1,A_{2n-3})}(q;x) = \sum_R d_R\, \tilde{f}_R^{(n)}(x)$$

(A_1, D_{2n}) theory

$$\mathcal{I}_{(A_1,D_{2n})}(q;x_1,x_2) = \sum_R \tilde{f}_R^{(n)}(x_1) f_R^{\text{reg}}(x_2)$$

(n = 1, 2, 3, 4, 5, ...)

(R : irreducible representations of su(2))

Outline

1. Argyres-Douglas theories
2. Superconformal index
3. Our conjecture

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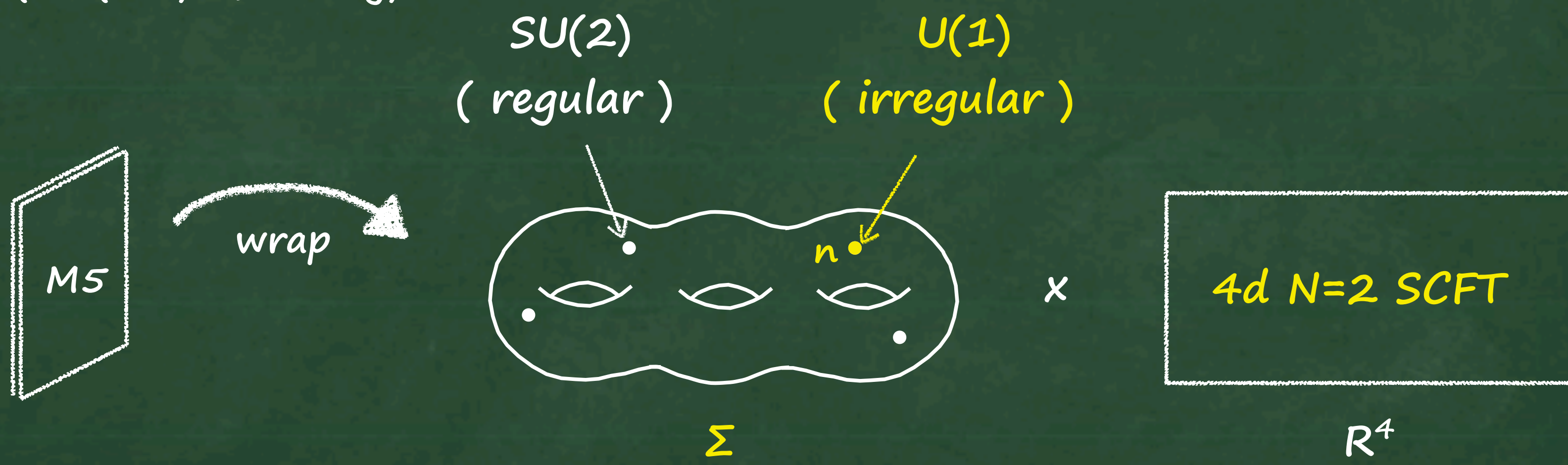
Argyres-Douglas theories

- Class S theories

  2 M5-branes compactified on a Riemann surface Σ w/ punctures

  (6d (2,0) A_1 theory)



regular puncture ⟶ SU(2) flavor sym., only 1 type

irregular puncture ⟶ U(1) flavor sym., various types labeled by $n = 1,2,3,4,\ldots$

Higgs field in the Hitchin system

$$\Phi(z) \sim dz\left[\frac{M_n}{z^{n+1}} + \cdots + \frac{M_1}{z^2} + \frac{M_0}{z} + \mathcal{O}(1)\right]$$

Argyres-Douglas theories

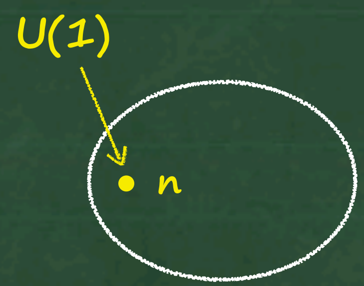
[Bonelli-Maruyoshi-Tanzini]
[Xie]

(A_1, A_{2n-3}) theory

genus 0

1 irregular puncture

(labeled by n)



x

R^4

(A_1, A_{2n-3}) theory

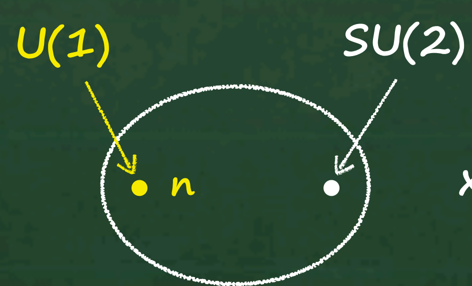
U(1) flavor sym.

(A_1, D_{2n}) theory

genus 0

1 irregular + 1 regular

(labeled by n)



x

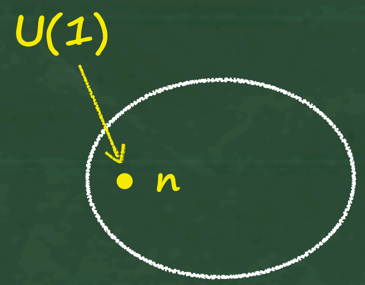
R^4

(A_1, D_{2n}) theory

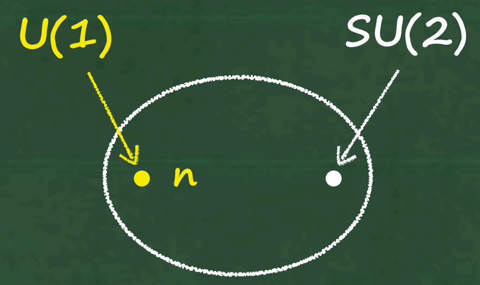
SU(2) x U(1) flavor sym.

Argyres-Douglas theories

[Bonelli-Maruyoshi-Tanzini]
[Xie]



(A_1, A_{2n-3}) theory



(A_1, D_{2n}) theory

- Coulomb branch ops. $\mathcal{O}_k$ such that $[\mathcal{O}_k] = 1 + \dfrac{k}{n}$

$$\begin{cases} k = 1, 2, 3, \ldots, n-2 & \text{for } (A_1, A_{2n-3}) \\ k = 1, 2, 3, \ldots, n-1 & \text{for } (A_1, D_{2n}) \end{cases}$$

(A_1, A_1) = a free hypermultiplet

(A_1, D_2) = 2 free hypermultiplets

Outline

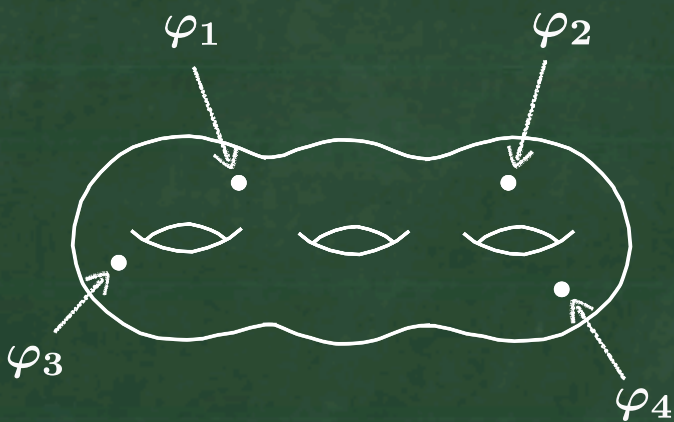
1. Argyres-Douglas theories
2. Superconformal index
3. Our conjecture

Superconformal index

- 2 M5-branes on Σ

φ_k : operator insertion at the k-th puncture
(depending on the 4d flavor fugacity x_k)

Σ



m = # of punctures

2d TQFT on Σ
(q-deformed Yang-Mills)

$$\langle \varphi_1 \, \varphi_2 \cdots \varphi_m \rangle_{q\mathrm{YM}}$$

×

|| '11 [Gadde-Rastelli-Razamat-Yan]

R^4

4d N=2 SCFT

$$\mathcal{I}(q; \vec{x})$$

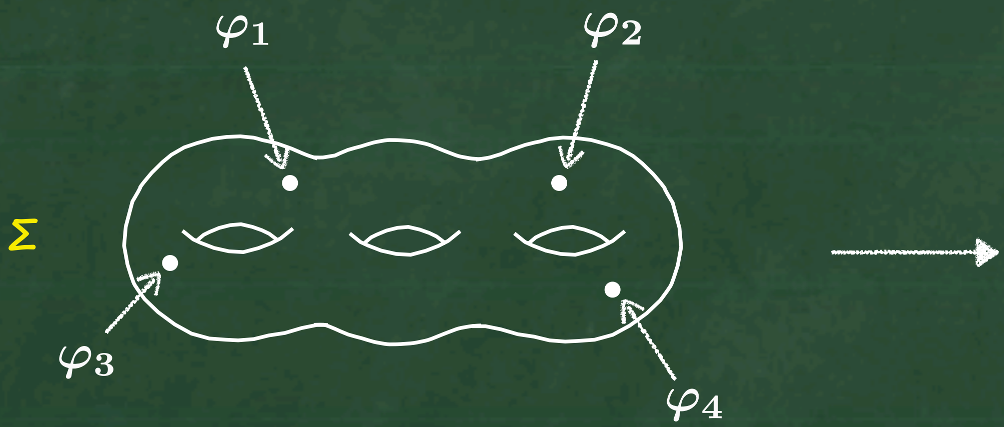
4d Superconformal index

Superconformal index

$$d_R \equiv \frac{[\dim R]_q}{(q^2;q)_\infty} \qquad [k]_q \equiv \frac{q^{k/2}-q^{-k/2}}{q^{1/2}-q^{-1/2}}$$

$$(z;q)_n \equiv \prod_{k=0}^{n-1}(1-q^k z)$$

- 2 M5-branes on Σ



2d TQFT on Σ

(q-deformed Yang-Mills)

$$\langle \varphi_1\, \varphi_2 \cdots \varphi_m \rangle_{q\text{YM}}$$

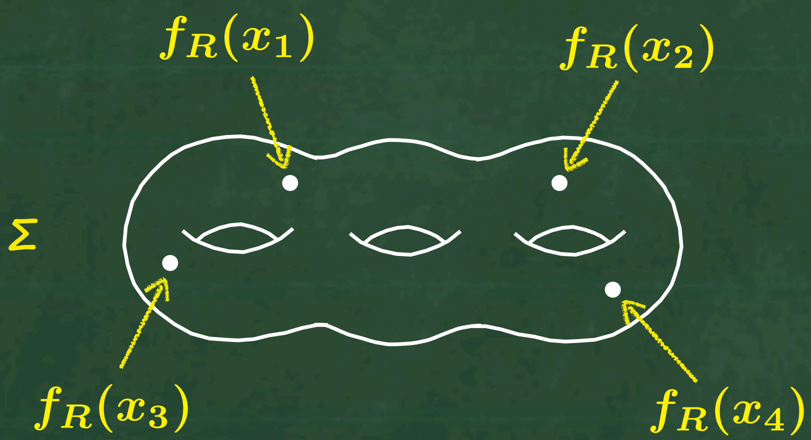
$$= \sum_{R:\ \text{irrep of } su(2)} (d_R)^{2-2g-m} \prod_{k=1}^{m} f_R(x_k)$$

Superconformal index

$$d_R \equiv \frac{[\dim R]_q}{(q^2;q)_\infty} \qquad [k]_q \equiv \frac{q^{k/2}-q^{-k/2}}{q^{1/2}-q^{-1/2}}$$

$$(z;q)_n \equiv \prod_{k=0}^{n-1}(1-q^k z)$$

- 2 M5-branes on Σ



2d TQFT on Σ
(q-deformed Yang-Mills)

$$\langle \varphi_1 \, \varphi_2 \cdots \varphi_m \rangle_{q\text{YM}}$$

$$\shortparallel$$

$$\sum_{R:\ \text{irrep of } su(2)} (d_R)^{2-2g-m} \prod_{k=1}^{m} f_R(x_k)$$

$f_R(x_k)$: wave function for the k-th puncture

regular puncture : $f_R(x) = f_R^{\text{reg}}(x) \equiv \dfrac{\chi_R^{su(2)}(x)}{(q;q)_\infty (qx^2;q)_\infty (qx^{-2};q)_\infty}$ [Gadde-Rastelli-Razamat-Yan]

irregular puncture : $f_R(x) =$??????

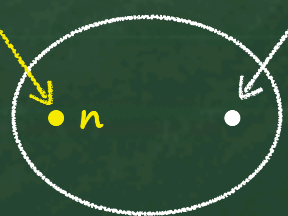
The 4d Index from 2d TQFT

$$\mathcal{I}(q;\vec{x}) = \sum_{R:\ \text{irrep of } su(2)} (d_R)^{2-2g-m} \prod_{k=1}^{m} f_R(x_k)$$

(2-2g-m = 1)

(A_1, A_{2n-3}) $$\mathcal{I}_{(A_1,A_{2n-3})}(q;x) = \sum_R d_R\, \tilde{f}_R^{(n)}(x)$$

wave function for an *irregular puncture* $\tilde{f}_R^{(n)}(x)$ $\qquad f_R^{\text{reg}}(y)$



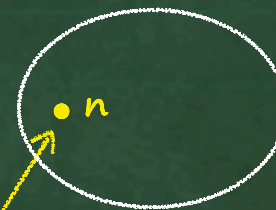
(A_1, D_{2n}) $$\mathcal{I}_{(A_1,D_{2n})}(q;x,y) = \sum_R \tilde{f}_R^{(n)}(x)\, f_R^{\text{reg}}(y)$$

(2-2g-m = 0)

The 4d Index from 2d TQFT

$$\mathcal{I}(q;\vec{x}) = \sum_{R:\ \text{irrep of } su(2)} (d_R)^{2-2g-m} \prod_{k=1}^{m} f_R(x_k)$$

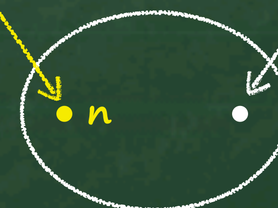
(2-2g-m = 1)



(A_1, A_{2n-3}) $\quad \mathcal{I}_{(A_1,A_{2n-3})}(q;x) = \sum_R d_R \tilde{f}_R^{(n)}(x)$

wave function for an irregular puncture $\tilde{f}_R^{(n)}(x)$ $\quad f_R^{\text{reg}}(y)$

(A_1, D_{2n}) $\quad \mathcal{I}_{(A_1,D_{2n})}(q;x,y) = \sum_R \tilde{f}_R^{(n)}(x) f_R^{\text{reg}}(y)$



(2-2g-m = 0)

Q: What is $\tilde{f}_R^{(n)}(x)$????

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Our conjecture

(A_1, A_{2n-3})

$$\mathcal{I}_{(A_1,A_{2n-3})}(q;x) = \sum_R d_R\, \tilde{f}_R^{(n)}(x)$$

(A_1, D_{2n})

$$\mathcal{I}_{(A_1,D_{2n})}(q;x,y) = \sum_R \tilde{f}_R^{(n)}(x) f_R^{\text{reg}}(y)$$

w/

$$\tilde{f}_R^{(n)}(x) = \frac{q^{nC_2(R)}}{(q;q)_\infty} \text{Tr}_R \left[x^{2J_3} q^{-n(J_3)^2} \right]$$

$C_2(R)$: quadratic Casimir

J_3 : Cartan of su(2)

This is our conjecture! Very simple!!!!

Consistency checks

- (A_1, A_1) = 1 free hypermultiplet

$$\mathcal{I}_{(A_1,A_1)}(q;x) = \sum_R d_R\, \tilde{f}_R^{(2)}(x) \quad \text{our conjecture}$$

$$= \prod_{k=0}^{\infty} \frac{1}{(1-q^{k+1/2}x)(1-q^{k+1/2}x^{-1})} \quad \text{path-integral}$$

- (A_1, D_2) = 2 free hypermultiplets

$$\mathcal{I}_{(A_1,D_2)}(q;x,y) = \sum_R \tilde{f}_R^{(1)}(x)\, f_R(y) \quad \text{our conjecture}$$

$$= \prod_{s_1,s_2=\pm 1} \prod_{k=0}^{\infty} \frac{1}{(1-q^{k+1/2}x^{s_1}y^{s_2})} \quad \text{path-integral}$$

The next simplest examples are (A_1, A_3) and (A_1, D_4).

For these theories,

there is another conjecture for the index.

Consistency checks

- 2d chiral algebra '13 [Beem-Lemos-Liendo-Peelaers-Rastelli-van Rees]

  For any 4d N=2 SCFT $\mathcal{T}$ with flavor symmetry G_F, there exists a 2d chiral algebra (VOA) such that

  1. VOA $\supset$ Virasoro algebra, affine G_F algebra

  2. $$\mathcal{I}_{\mathcal{T}}(q;\vec{x}) = \mathrm{Tr}_{\mathrm{VOA}}(-1)^F q^{L_0} \prod_{k=1}^{\mathrm{rank}\, G_F} (x_k)^{A_k}$$

     (character of VOA)

<u>conjectures</u>

$\mathcal{T}$ = (A_1, A_3) ⟶ VOA = $\widehat{su}(2)_{-4/3}$

$\mathcal{T}$ = (A_1, D_4) ⟶ VOA = $\widehat{su}(3)_{-3/2}$

(Virasoro is given by Sugawara)

($\mathcal{T}$ = SU(2) w/ 4 flavors ⟶ VOA = $\widehat{so}(8)_{-2}$)

($\mathcal{T}$ = Minahan-Nemeshanski E_6 ⟶ VOA = $(\widehat{E_6})_{-3}$)

Consistency checks

- (A_1, A_3)

$$\mathcal{I}_{(A_1,A_3)}(q;x) = \sum_R d_R\, \tilde{f}_R^{(3)}(x)$$ our conjecture

$$= 1 + q\chi_3^{su(2)}(x) + q^2\left[1 + \chi_3^{su(2)}(x) + \chi_5^{su(2)}(x)\right] + \cdots$$

$$= \mathrm{Tr}_{\widehat{su}(2)_{-4/3}}\, q^{L_0} x^J$$ chiral algebra

- (A_1, D_4)

$$\mathcal{I}_{(A_1,D_4)}(q;x,y) = \sum_R \tilde{f}_R^{(2)}(x)\, f_R(y)$$ our conjecture

$$= 1 + q\chi_8^{su(3)}(x) + q^2\left[1 + \chi_8^{su(3)}(x) + \chi_{27}^{su(3)}(x)\right] + \cdots$$

$$= \mathrm{Tr}_{\widehat{su}(3)_{-3/2}}\, q^{L_0} (yx^{1/3})^{J_1} (x^{2/3})^{J_2}$$ chiral algebra

Perfectly consistent !!!!!

Summary

- We conjectured exact expressions for the superconformal indices of (A_1, A_{2n-3}) and (A_1, D_{2n}) theories (in the Schur limit) in terms of TQFT on sphere.

$$\mathcal{I}_{(A_1,A_{2n-3})}(q;x) = \sum_R d_R\, \tilde{f}_R^{(n)}(x)$$

$$\mathcal{I}_{(A_1,D_{2n})}(q;x,y) = \sum_R \tilde{f}_R^{(n)}(x) f_R^{\mathrm{reg}}(y)$$

- Our formula passes a lot of non-trivial consistency checks!
  - reproduces free hypermultiplet indices
  - agrees with the 2d chiral algebra conjecture
  - consistent with S-dualities of AD theories
  - consistent with RG-flows of AD theories
  - consistent with the $q \to 1$ limit

  Talk to me if you're interested in these!

- The Macdonald limit of the index was also conjectured in our latest paper.

(1509.05402)

$q \to 1$ limit

- In the limit $q \to 1$, we checked that our Schur index reduces to the S^3 partition function of the 3d reduction of the 4d theory.

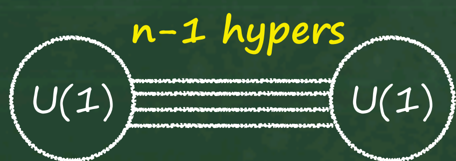
[Dolan-Spiridonov-Vartanov]
[Gadde-Yan]
[Imamura]

$$\mathcal{I}(q; \vec{x}) \xrightarrow{q \to 1} \mathcal{Z}_{S^3}(\vec{x})$$

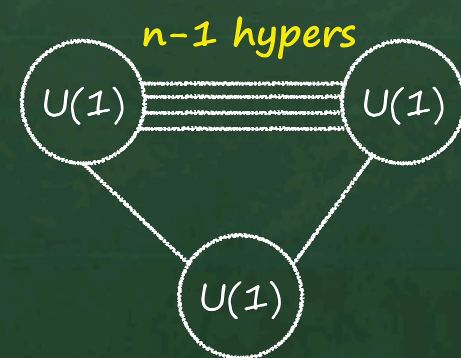
The 3d mirror is a 3d N=4 gauge theory.

[Boalch]
[Xie]

(A_1, A_{2n-3})



(A_1, D_{2n})



4d $SU(2)_R \times U(1)_R$ $\longrightarrow$ a subgroup of 3d $SU(2)_R \times SU(2)_L \times$ topological $U(1)$