A_∞ structure from the Berkovits formulation of open superstring field theory

Tomoyuki Takezaki, The University of Tokyo, Komaba

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1. Introduction

For the Neveu-Schwarz sector of open superstring field theory, we now have two formulations: the Berkovits formulation [1] and the A_∞ formulation [2].

	Berkovits	A_∞
Hilbert space	large HS	small HS
gauge fixing	difficult	straightforward
closed form expression	yes	no

We find the same structure in these two formulations.

2. The Berkovits formulation

The Berkovits formulation [1] is based on the large Hilbert space of the superconformal ghosts. The equation of motion and the gauge transformation in the free theory are

$$Q\eta\Phi_B = 0, \quad \delta\Phi_B = Q\Lambda + \eta\Omega$$

where Q is the BRST operator of the superstring, and η is the zero mode of the superconformal ghost $\eta(z)$. The full action takes the Wess-Zumino-Witten-like form

$$S_{\rm WZW}[\Phi_B] = -\int_0^1 dt\,\langle\, B_t(t), QB_\eta(t)\,\rangle$$

with

$$B_\eta(t) = (\eta e^{\Phi_B(t)})e^{-\Phi_B(t)}, \quad B_t(t) = (\partial_t e^{\Phi_B(t)})e^{-\Phi_B(t)}$$

where $\Phi_B(0) = 0$ and $\Phi_B(1) = \Phi_B$. The t dependence of the action is topological, and the action is a functional of Φ_B. The gauge invariance and the topological t dependence follow from

$$\eta B_\eta(t) = B_\eta(t)B_\eta(t)\,,$$
$$\partial_t B_\eta(t) = \eta B_t(t) - B_\eta(t)B_t(t) + B_t(t)B_\eta(t)\,.$$

A regular formulation in small Hilbert space (reduced Berkovits formulation) can be obtained by fixing gauge freedom generated by η. The point is that we can use ξ; a line integral of superconformal ghost $\xi(z)$ [3].

3. The A_∞ formulation

The A_∞ formulation [2] is based on the small Hilbert space of the superconformal ghosts. The action and the gauge transformation are given by the same set of multi-string products $\{M_n\}_{n\in\mathbb{N}}$:

$$S_{A_\infty}[\Psi_A] = \frac{1}{2}\,\omega(\Psi_A, Q\Psi_A) + \frac{1}{3}\,\omega(\Psi_A, M_2(\Psi_A,\Psi_A)) + \frac{1}{4}\,\omega(\Psi_A, M_3(\Psi_A,\Psi_A,\Psi_A)) + \cdots$$

$$\delta_\Lambda\Psi_A = Q\Lambda + M_2(\Lambda,\Psi_A) + M_2(\Psi_A,\Lambda) + M_3(\Lambda,\Psi_A,\Psi_A) + M_3(\Psi_A,\Lambda,\Psi_A) + M_3(\Psi_A,\Psi_A,\Lambda) + \cdots$$

where $M_1 \equiv Q$. The gauge invariance of the action follows from A_∞ relations

$$0 = Q^2 a$$
$$0 = QM_2(a,b) + M_2(Qa,b) + M_2(a,Qb)$$
$$0 = QM_3(a,b,c) + M_2(M_2(a,b),c) + M_2(a,M_2(b,c)) + M_3(Qa,b,c) + M_3(a,Qb,c) + M_3(a,b,Qc)$$
$$0 = QM_4(a,b,c,d) + \cdots$$

where $a, b, c, d, \ldots$ are arbitrary string fields. If M_2 is associative, we can set $M_{n\geq3} = 0$. However, such a choice leads to divergent results (Witten's original theory).

In the A_∞ formulation, the singularity in Witten's original theory is regularized. $\{M_n\}_{n\in\mathbb{N}}$ are constructed from Q, star product $*$ and ξ [2].

4. Main results of our paper

In our paper, we transform the action of the A_∞ formulation into the same form as the Berkovits formulation.

$$S_{A_\infty}[\Psi_A] = -\int_0^1 dt\,\langle\, A_t(t), QA_\eta(t)\,\rangle$$

where $A_\eta(t)$ and $A_t(t)$ satisfies

$$\eta A_\eta(t) = A_\eta(t)A_\eta(t)\,,$$
$$\partial_t A_\eta(t) = \eta A_t(t) - A_\eta(t)A_t(t) + A_t(t)A_\eta(t)\,.$$

The expression of $A_\eta(t)$ and $A_t(t)$ to the second order is

$$A_\eta(t) = \Psi_A(t) + \frac{1}{3}\xi(\Psi_A(t)\Psi_A(t)) - \frac{1}{3}(\xi\Psi_A(t))\Psi_A(t) + \frac{1}{3}\Psi_A(t)(\xi\Psi_A(t)) + \cdots$$

$$A_t(t) = \xi\partial_t\Psi_A(t) + \frac{1}{3}\xi\Big(\Psi_A(t)(\xi\partial_t\Psi_A(t)) - (\xi\partial_t\Psi_A(t))\Psi_A(t)\Big) - \frac{1}{3}(\xi\partial_t\Psi_A(t))(\xi\Psi_A(t)) - \frac{1}{3}(\xi\Psi_A(t))(\xi\partial_t\Psi_A(t)) + \cdots$$

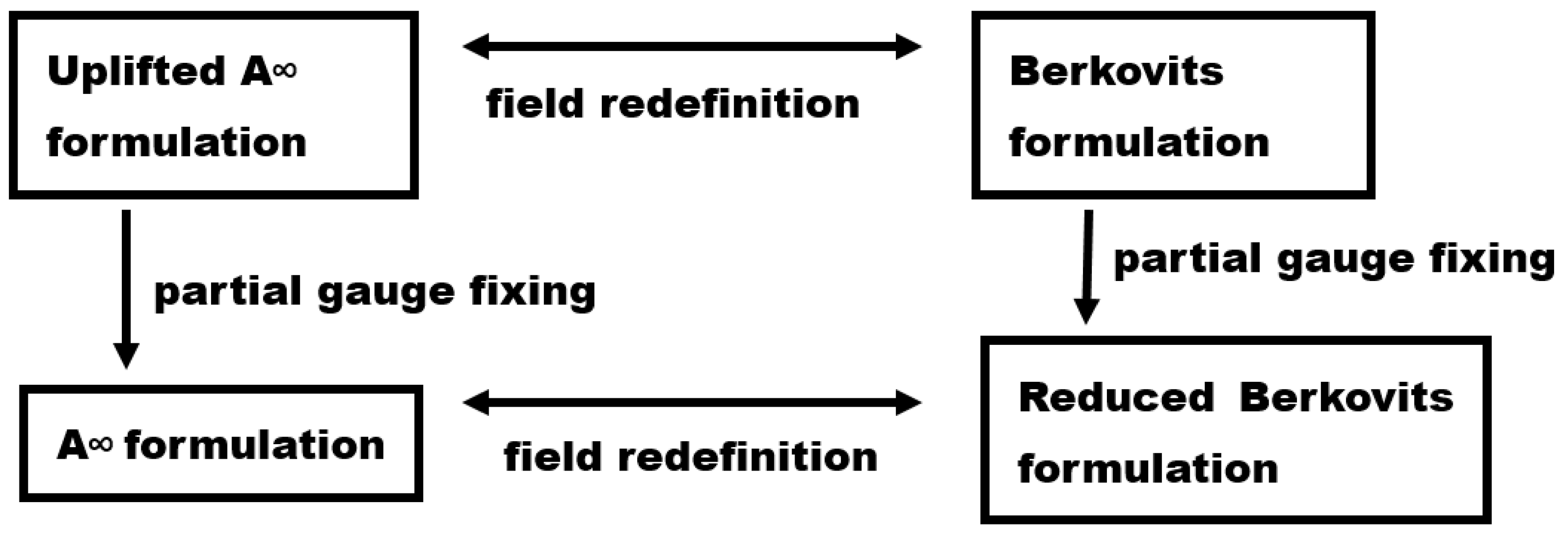
where $\Psi_A(0) = 0$ and $\Psi_A(1) = \Psi_A$.

Equating A_η and B_η, we obtain a field redefinition between the reduced Berkovits formulation and the A_∞ formulation.

We also uplift the A_∞ formulation to the large Hilbert space, and find its relation to the Berkovits formulation.

5. Conclusion

We can understand the Berkovits formulation and the A_∞ formulation as different parametrizations of A_η, A_t.



6. Future directions

Recentry, an action of open superstring field theory including the Ramond sector is constructed [4]. They started with the Berkovits action, and coupled it to the Ramond string field. In our next paper [5], we construct an A_∞ action with Ramond sector, and show its equivalence with [4].

[1] N. Berkovits, "SuperPoincare invariant superstring field theory," Nucl. Phys. B **450**, 90 (1995) [Erratum-ibid. B **459**, 439 (1996)] [arXiv:9503.099 [hep-th]].

[2] T. Erler, S. Konopka and I. Sachs, "Resolving Witten's superstring field theory," JHEP **1404**, 150 (2014) [arXiv:1312.2948 [hep-th]].

[3] Y. Iimori, T. Noumi, Y. Okawa and S. Torii, "From the Berkovits formulation to the Witten formulation in open superstring field theory," JHEP **1403**, 044 (2014) [arXiv:1312.1677 [hep-th]].

[4] H. Kunitomo and Y. Okawa, "Complete action of open superstring field theory," [arXiv:1508.00366 [hep-th]].

[5] T. Erler, Y. Okawa and T. Takezaki, *To appear*.