Band spectrum is D-brane

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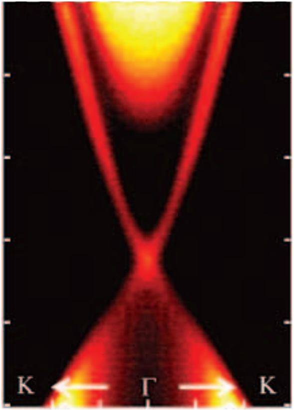
collaboration with K. Hashimoto (Osaka) [arXiv:1509.04676]

What's this?



D-brane

Topological phase

- characterized by **topological number**

- ex.) (integer) quantum Hall effect

$$\sigma_{\mathsf{H}} = \nu \frac{e^2}{h} \qquad \nu \in \mathbb{Z}$$

Wavefunction topology

- ex.) Massive Dirac fermion in $d = 2$ ($d = 2$ IQHE)

$$\mathcal{H}(p) = \begin{pmatrix} m & p_x - i p_y \\ p_x + i p_y & -m \end{pmatrix}$$

- Topological $\#$: the base (p-space) to the Hilbert space

$$\nu = \frac{1}{2\pi} \int_{\mathbb{R}^2} dp\, \mathcal{F} = \frac{1}{2} \operatorname{sgn}(m) \qquad \text{w/} \quad \vec{\mathcal{A}} = \psi^\dagger i \frac{\partial}{\partial \vec{p}} \psi$$

- Topology change: $\Delta\nu = \pm 1$ at $m = 0$

Wavefunction topology

- ex.) Massive Dirac fermion in $d = 4$ ($d = 4$ IQHE)

$$\mathcal{H}(p) = \begin{pmatrix} m & p \cdot \bar{\sigma} \\ p \cdot \sigma & -m \end{pmatrix}$$

- Topological #: the base (p-space) to the Hilbert space

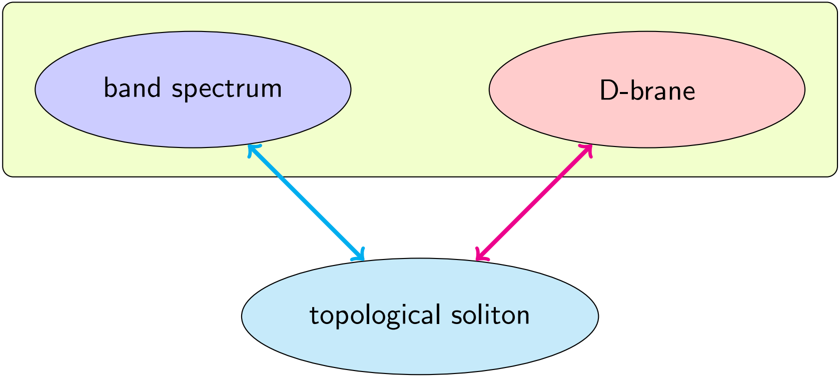
$$\nu = \frac{1}{8\pi^2} \int_{\mathbb{R}^4} dp \, \mathrm{tr}\, \mathcal{F}^2 = \frac{1}{2} \mathrm{sgn}(m) \qquad \text{w/} \quad \vec{\mathcal{A}} = \psi^\dagger i \frac{\partial}{\partial \vec{p}} \psi$$

- Topology change: $\Delta\nu = \pm 1$ at $m = 0$

- **Topological solitons** in **momentum space**

$$\nu = \frac{1}{2\pi} \int_{\mathbb{R}^2} \mathcal{F} \qquad \text{or} \qquad \frac{1}{8\pi^2} \int_{\mathbb{R}^4} \mathrm{tr}\, \mathcal{F}^2$$

Nahm & ADHM construction


band spectrum
D-brane
topological soliton

Nahm construction of monopole

- "Dirac eq": $\nabla^\dagger v(\xi) = 0$ w/ $\nabla^\dagger = i\frac{d}{d\xi} + i\sigma_i \left(x^i - T^i(\xi) \right)$
- Nahm eq: $\frac{d}{d\xi} T_i = i\epsilon_{ijk} T^j T^k$
- BPS monopole: $\partial_i \Phi(x) = \frac{1}{2}\epsilon_{ijk} F^{jk}(x)$

  where

$$\Phi(x) = \int d\xi \, v^\dagger \xi v \qquad A_i(x) = \int d\xi \, v^\dagger i \frac{d}{dx_i} v$$

Schrödinger eq = "Dirac eq"

- Schrödinger eq in $d = 2$

$$\left[i\frac{\partial}{\partial t} - \mathcal{H}(p) \right] v(t) = 0 \qquad \text{(Wick rotation: } it = \xi\text{)}$$

  - Correspondence: $(x_1, x_2, x_3) \longleftrightarrow (p_1, p_2, m)$

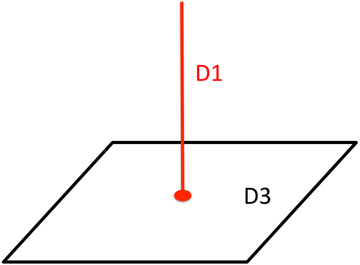
- Magnetic flux

$$1 = \frac{1}{2\pi} \int d\mathbf{S} \cdot \mathbf{B} = \frac{1}{2\pi} \int d^2p\, F_{12} \bigg|_{m>0} - \frac{1}{2\pi} \int d^2p\, F_{12} \bigg|_{m<0}$$

The role of Φ

1. $\nabla^\dagger v = 0$ & $\nabla^\dagger \sim \dfrac{\partial}{\partial \xi} \pm \epsilon$ (eigen eq: $\mathcal{H}(p) v = \epsilon(p)\, v$)
2. $\Phi = \displaystyle\int d\xi\, v^\dagger \xi v \sim \pm \frac{1}{\epsilon}$ $\left(\epsilon(p) = \sqrt{\vec{p}^2 + m^2}\right)$

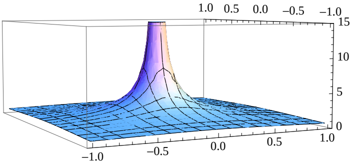
- D1-D3 brane system



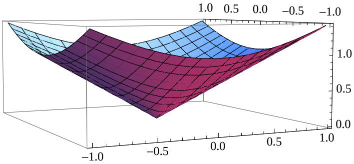
Band spectrum = D-brane

Summary

- Topological number from electron wavefunction
- Topological charge in momentum space
- Topological soliton in momentum space
- Nahm's Φ: **D-brane shape** & **band spectrum**

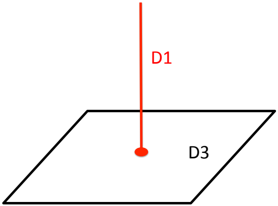


Scalar: Φ

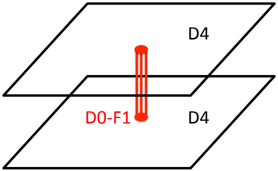


Energy: $X = 1/\Phi$

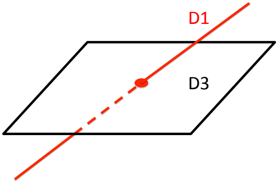
Generalizations



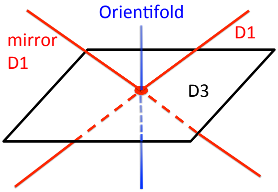
$d = 2$ QHE (monopole)



$d = 4$ QHE (instanton)



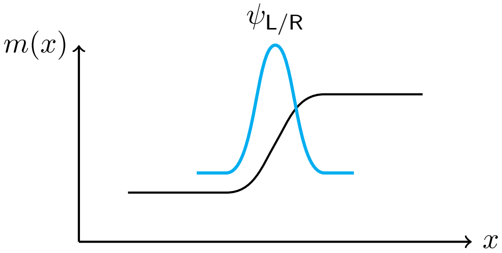
chiral edge state (QHE)



helical edge state (QSHE)

Edge state

1. Topology changes at the boundary
2. Mass flips at the boundary
3. Domain-wall mass configuration



Edge state

- Mass profile: $m(x_1) = \theta x_1$

- Hamiltonian: $\mathcal{H} = p_1 \sigma_2 + p_2 \sigma_3 + m(x_1) \sigma_1$

$$\longrightarrow \quad \mathcal{H} = \begin{pmatrix} p_2 & \sqrt{2\theta}\, \hat{a}^\dagger \\ \sqrt{2\theta}\, \hat{a} & -p_2 \end{pmatrix} \qquad \hat{a} = \frac{1}{2\theta} (\theta x_1 + i p_1)$$

θ is...

- noncommutativity: $[x_1, x_3] = -i\theta \qquad (x_1, x_3) \leftrightarrow (p_1, m)$
- magnetic field: $\theta = B$

Nahm construction in **noncommutative space**

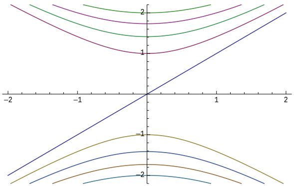
- Nahm eq in noncomm sp:

$$\frac{d}{d\xi}T_i = i\epsilon_{ijk}T^jT^k - \theta\,\delta_{i2}$$

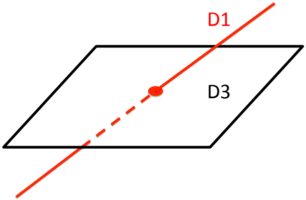
- Dirac eq:

$$\left[\frac{d}{d\xi} + \theta\xi\sigma_2 + \sigma_i x^i\right] v = 0$$

- Nahm's Φ and band spectrum as D1-brane



chiral edge state



tilted D1-brane

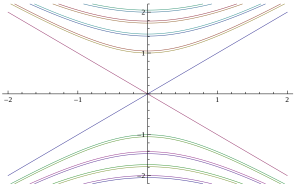
Generalization: additional symmetry

- Time-reversal symmetry: $\Theta = -i\sigma_2 \otimes K$

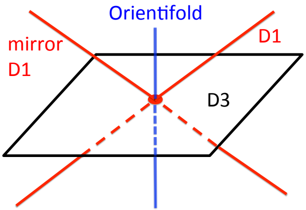
$$\Theta \mathcal{H}(p) \Theta = \mathcal{H}(-p)$$

  - Topological classification: QHE $(\mathbb{Z})$ $\longrightarrow$ QSHE $(\mathbb{Z}_2)$

- Band spectrum and D1-brane



helical edge state



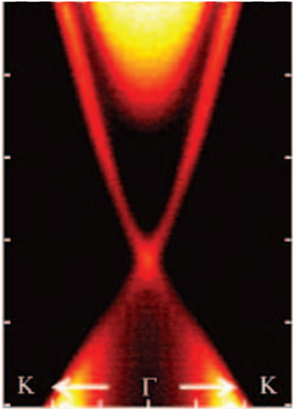
D1-brane w/ orientifold

Summary & discussion

- Topology of electron & soliton in momentum space
- Nahm's Φ: **D-brane shape** & **band spectrum**

- Another construction of topological system [Ryu–Takayanagi]
- Non-perturbative study of D-brane to cond-mat physics

Band spectrum is D-brane!



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