Entanglement Entropy of de Sitter Space α-Vacua

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Based on arXiv:1404.7487 (& work in progress)

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Aspects of Vacua on de Sitter space

by

Quantum Entanglement

Plan to Talk

1. dS Space & α-Vacua

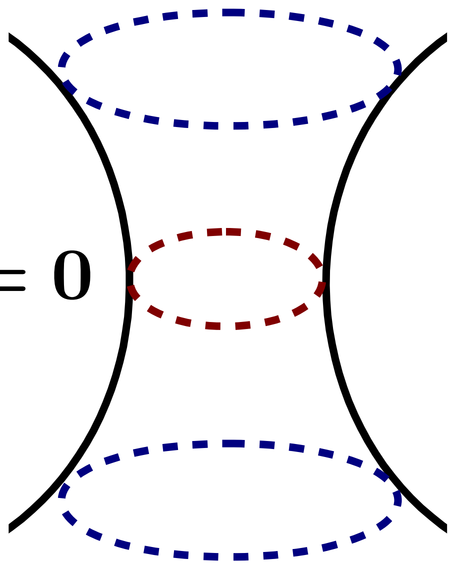
2. Entanglement Entropy of α-Vacua

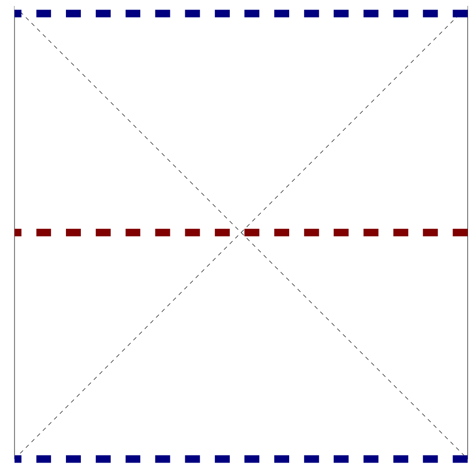
3. Summary

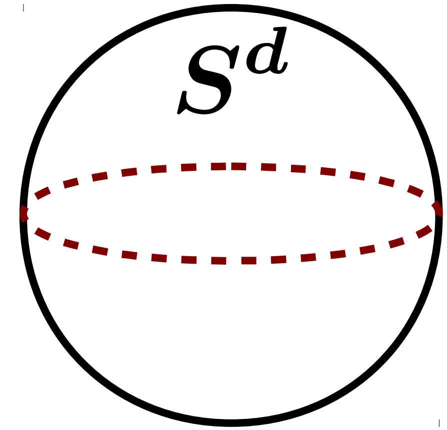
de Sitter Space

$$-X_0^2 + X_i^2 = 1$$ (embedding)

$t = 0$







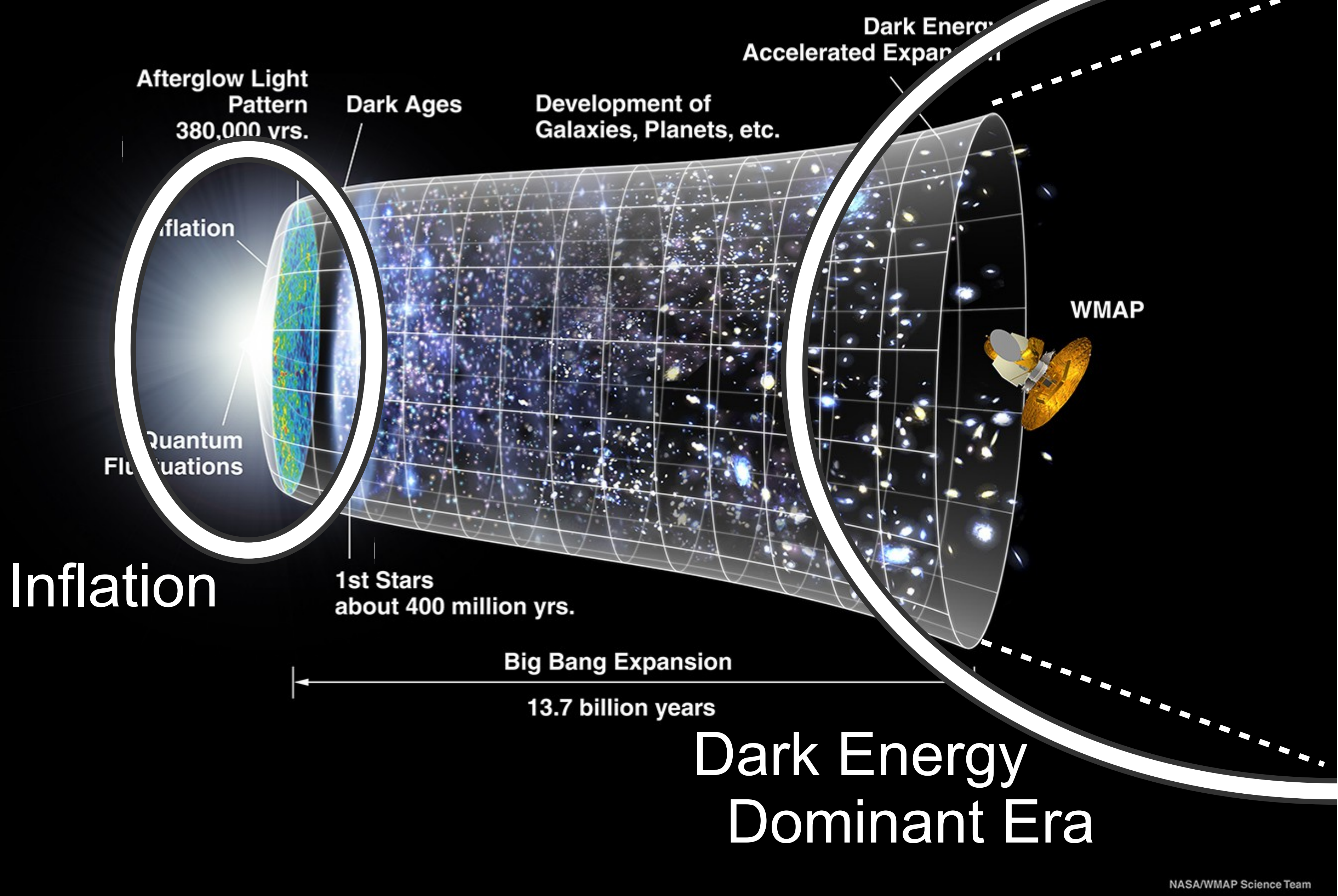
Euclidean

$(t \to i\theta)$

$$ds^2 = -dt^2 + \cosh^2 t \, d\Omega_{d-1}^2$$

(global coordinate)

dS Eras of the Universe



α-Vacua in dS

Mottola('85), Allen('85)

Massive free scalar on dS

$$S = -\frac{1}{2}\int d^4x\,\sqrt{-g}(D_\mu\Phi D^\mu\Phi + m^2\Phi^2)$$

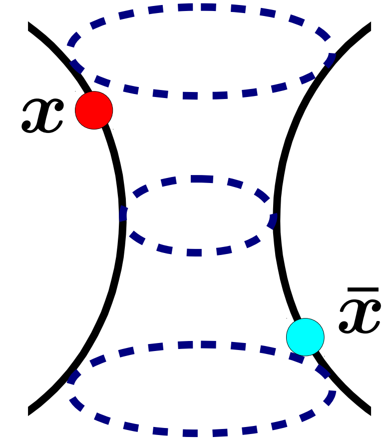
SO(1,d) preserving vacua, with parameters (α, β)

standard (Bunch-Davies) vacuum

$$\tilde{G}(x,y) = \cosh 2\alpha\, G_0(x,y) + \sinh 2\alpha(\cos\beta\, G_0(\bar{x},y) - \sin\beta\, D(\bar{x},y))$$

$$G(x,y) = <\{\Phi(x),\Phi(y)\}>$$

$$D(x,y) = <[\Phi(x),\Phi(y)]>$$



α-Vacua in dS (2)

$$\Phi(x) = \sum_n \left(\phi_n(x) a_n + \phi_n^*(x) a_n^\dagger\right)$$
$$= \sum_n \left(\tilde{\phi}_n(x) \tilde{a}_n + \tilde{\phi}_n^*(x) \tilde{a}_n^\dagger\right)$$

$$\tilde{\phi}_n(x) = (\cosh\alpha)\phi_n(x) + e^{i\beta}(\sinh\alpha)\phi_n^*(x)$$

$$\tilde{a}_n = (\cosh\alpha) a_n - e^{-i\beta}(\sinh\alpha) a_n^\dagger$$

$$\tilde{a}_n|\tilde{0}> = 0$$

"α-vacuum"

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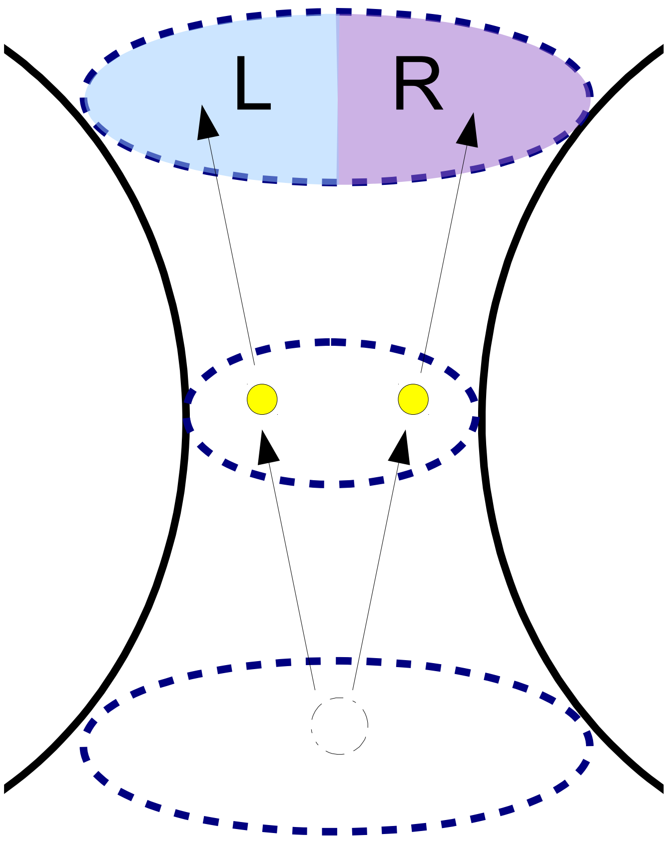
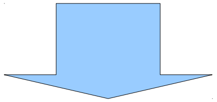
Particle Creation and Entanglement



Particle creations in dS:

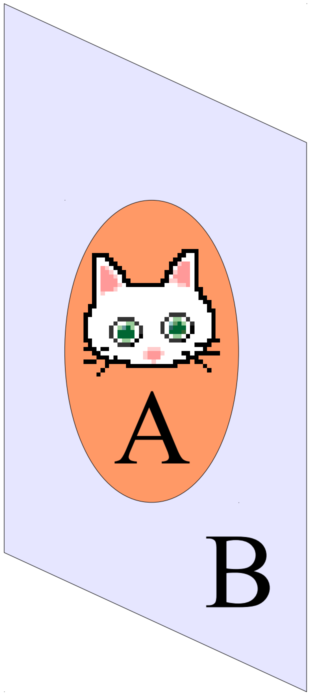
- Not exist in Minkowski
- Depend on vacua
- Generate Entanglement



Entanglement Entropy
characterizes dS & different vacua.

Bunch-Davies vacuum: by Maldacena-Pimentel(2012)

Entanglement Entropy



$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

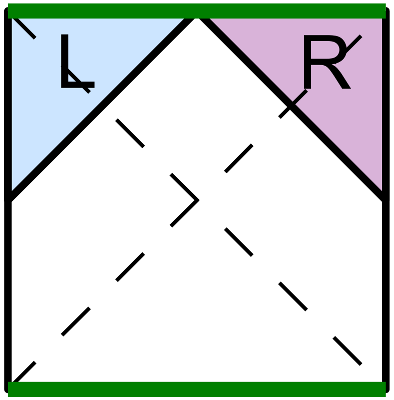
$$\rho_{total} = |\Psi >< \Psi|$$

$$\rho_A = \mathrm{Tr}_B[\rho_{total}]$$

$$S_A = -\mathrm{Tr}\rho_A \log \rho_A$$

Vacuum Condition by {L,R}-Oscillators

Total & LR mode functions are related as:



$$\chi_\sigma(x) = \sum_{q=L,R} \left(\alpha_{q\sigma}\psi_q(x_q) + \beta_{q\sigma}\psi_q^*(x_q)\right)$$

$(\sigma = \pm 1)$

Sasaki-Tanaka-Yamamoto ('94)

$$a_\sigma = \sum_{q=L,R} (\gamma_{q\sigma} b_q + \delta_{q\sigma}^* b_q^\dagger)$$

$\alpha_{q\sigma}(\nu, p)$ $\beta_{q\sigma}(\nu, p)$
$\gamma_{q\sigma}(\nu, p)$ $\delta_{q\sigma}(\nu, p)$

Known functions

α-Vacua: $\tilde{a}_\sigma |\tilde{0}> = 0$

$$\tilde{a}_\sigma = \sum_{q=L,R} (\tilde{\gamma}_{q\sigma} b_q + \tilde{\delta}_{q\sigma}^* b_q^\dagger)$$

$$\tilde{\gamma} = (\cosh\alpha)\gamma - e^{-i\beta}(\sinh\alpha)\delta$$

$$\tilde{\delta} = (\cosh\alpha)\delta - e^{-i\beta}(\sinh\alpha)\gamma$$

α-Vacuum Wavefunction

α-Vacua: $\tilde{a}_\sigma|\tilde{0}>=0$ $\quad$ $\tilde{a}_\sigma = \sum_{q=L,R} (\tilde{\gamma}_{q\sigma} b_q + \tilde{\delta}^*_{q\sigma} b^\dagger_q)$

$$|\tilde{0}>=\exp\left(\frac{1}{2}\tilde{m}_{ij}b^\dagger_i b^\dagger_j\right)|0>_L\,|0>_R$$

$$\tilde{m}_{ij} = -\tilde{\delta}^*_{i\sigma}(\tilde{\gamma}^{-1})_{\sigma j}$$

$$= \sum_{n\geq 0}\tilde{\kappa}^n|\tilde{n}'>_L\,|\tilde{n}'>_R$$

Diagonalize

$$= \begin{pmatrix} \rho & \zeta \\ \zeta & \rho \end{pmatrix}$$

$$|\tilde{\kappa}|^2 = \tilde{\Lambda} - \sqrt{\tilde{\Lambda}^2 - 1}$$

$$\tilde{\Lambda} = \frac{|\zeta|^4 + (|\rho|^2-1)^2 - (\rho^2\zeta^{*2} + \rho^{*2}\zeta^2)}{2|\zeta|^2}$$

Entanglement Entropy

$$|\tilde{0}> = \sum_{n\geq 0} \tilde{\kappa}^n |\tilde{n}'>_L |\tilde{n}'>_R$$

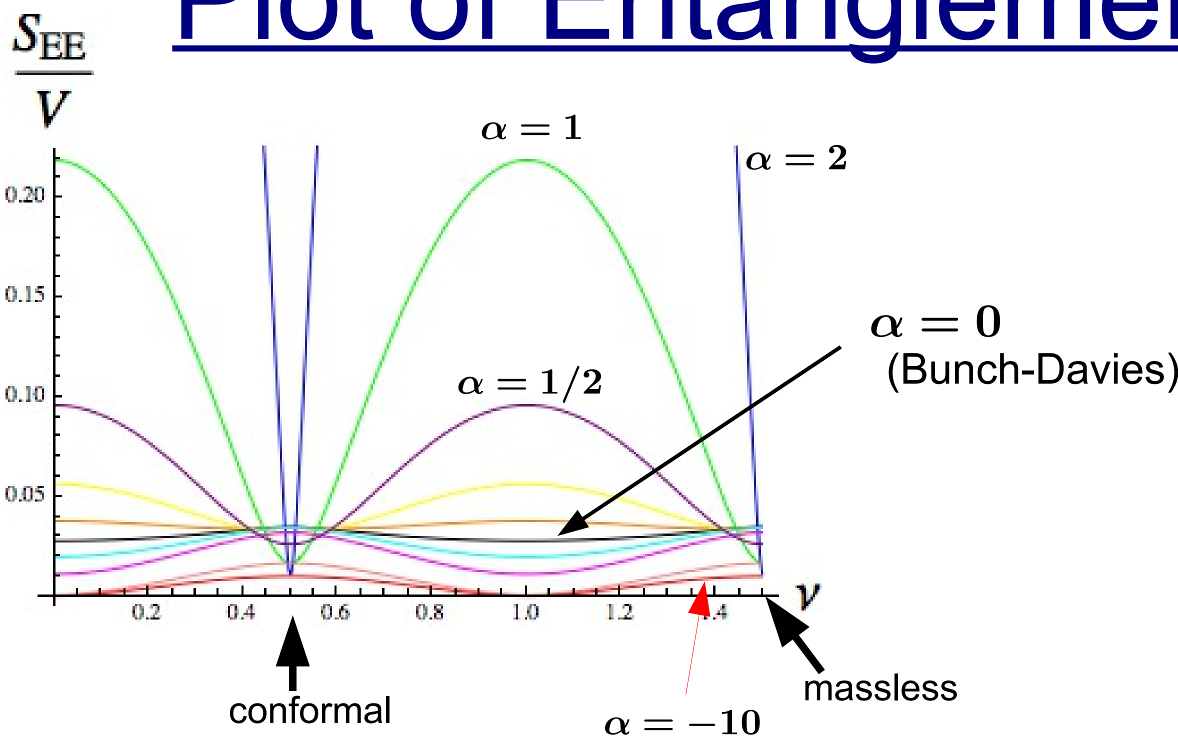
$$\Rightarrow \quad \rho_L = \frac{1}{1-|\tilde{\kappa}|^2} \sum_{n\geq 0} |\tilde{\kappa}|^{2n} |\tilde{n}'>_L < \tilde{n}'|_L$$

$$\Rightarrow \quad S_{EE}(p) = -\log(1-|\tilde{\kappa}|^2) - \frac{|\tilde{\kappa}|^2}{1-|\tilde{\kappa}|^2}\log|\tilde{\kappa}|^2$$

$$\Rightarrow \quad S_{EE}/V = \int_0^\infty dp\, \mathcal{D}(p) S_{EE}(p)$$

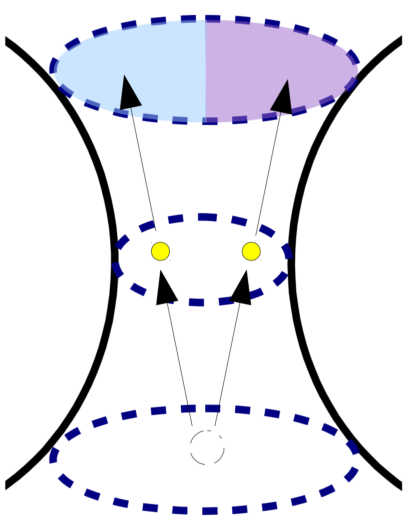
$$\mathcal{D}(p) \propto p^2 \quad (d=4)$$

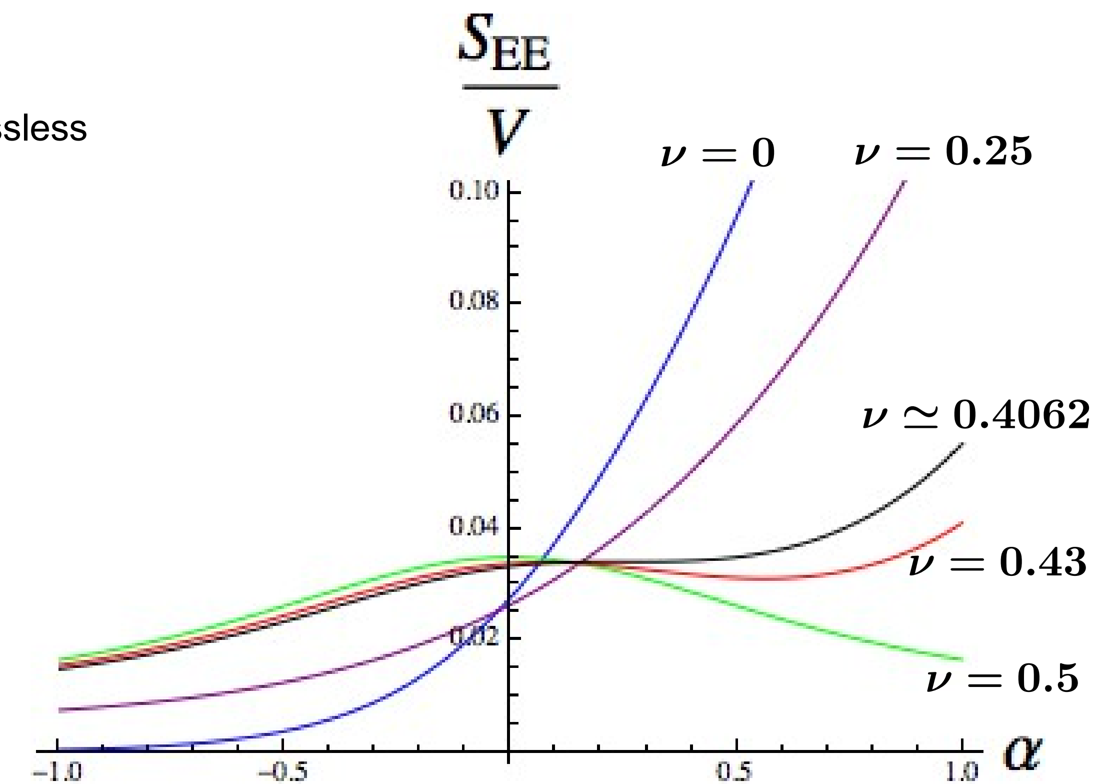
Plot of Entanglement Entropy



(d=4, β=0, future infinity)

$$\nu = \sqrt{\frac{9}{4} - m^2}$$





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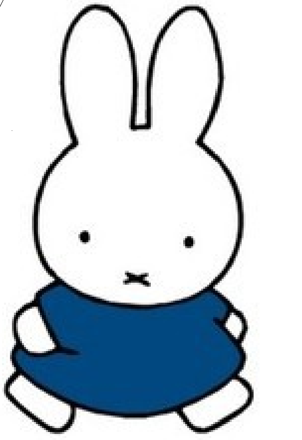
Summary

- dS has nontrivial α-Vacua
  → generate entanglement in different ways.

- We computed EE in α-Vacua by direct evaluation of the wavefunction.

Thank You
for
This Fantastic
Workshop !!!!



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