Distribution of Number of Generations in Flux Compactification

July 23 (wed), 2014 at YITP workshop

Taizan Watari (Kavli IPMU)

arXiv:1408.xxxx w/ A. Braun (King's)

cf. arXiv:1401.???? w/ A. Braun Y. Kimura (YITP)

flux compactification of IIB/F-theory

$W_{GVW} \propto \int_X G \wedge \Omega_X$ ⟺ cpx str moduli stabilized (isolated minimum)

$H^4(X;"\mathbb{Z}") \supset \{ \; G \; \}$ ⟺ (sub)-ensemble of low-energy eff. theories

string landscape: theoretical foundation for "naturalness"

Flux				Low-energy eff. theories
		algebra		gauge group, matter repr. …
?	⟸	topology	⟹	matter multiplicity
		moduli		eff. coupling constants

Specify $(B_3, [S], R)$.

R: A4, D5, … unif. symmetry of your interest

[S] : divisor class of B_3

$\mathcal{M}_*$:moduli space of $\pi_X : X \to B_3$ with S = “7-brane of sym. R”

X : smooth (resolved) 4-fold

$$h^{1,1}(X) = 1 + h^{1,1}(B_3) + \mathrm{rank}(R).$$

$$\begin{array}{ccccccc} & & & \mathbf{1} & & & \\ & & & h^{1,1} & & & \\ & & h^{2,1} & & h^{2,1} & & \\ \mathbf{1} & h^{3,1} & & h^{2,2} & & h^{3,1} & \mathbf{1} \\ & & h^{2,1} & & h^{2,1} & & \\ & & & h^{1,1} & & & \\ & & & \mathbf{1} & & & \end{array}$$

Hodge diamond of X

Decomposition

cf. Greene Morrison Plesser

$$H^4(X) = H^{2,2}_V(X) \oplus H^{2,2}_{RM}(X) \oplus H^4_H(X);$$

$$H^4_H(X;\mathbb{C}) = Span_{\mathbb{C}}\{\Omega_X, D\Omega_X, D^2\Omega_X, \cdots\}$$

$$= H^{4,0} + H^{3,1} + H^{2,2}_H + H^{1,3} + H^{4,0}.$$

cf: IIB orientifold 3-forms = $H^4_H(X;\mathbb{R})$

(Denef Douglas ’04)

A.Braun, Kimura, TW '14
A.Braun, TW '14

- Observations
  - Generally $H^{2,2}_{RM}(X) \neq \phi.$ (be aware)
    - K3 x K3 $h^{2,2}_{RM} = \rho_1(22-\rho_2)+(22-\rho_1)\rho_2.$
    - toric hypersurface CY4: many examples
  - Flux in $H^{2,2}_{RM}(X)$ often breaks the unif. symm. R.
  - Net chirality is generated by a flux in $H^{2,2}_{V}(X)$
    - because the matter surface for R=SU(5) is vertical.
- We are led to a proposal of flux ensembles

$$\{G_{fix}+G_{scan} \mid G_{scan} \in H^4_H(X)\} \subset H^4(X)$$

$G_{fix} \in H^{2,2}_V(X)$ controls N_gen
constructed in Marsano et.al. '11 (dual to Het)

- Ashok-Denef-Douglas' theory (contin. approx)

'03, '04

vacuum index density distribution $$d\mu_I \approx \frac{(2\pi L_*)^{K/2}}{(K/2)!}\rho_I; \qquad K \ll L_*.$$

  - K = dim[flux scanning space], L*= D3-tadpole.
  - if $K \gg L_*$, the prefactor becomes $\exp[\sqrt{2\pi K L_*}]$.
  - the distribution on $\mathcal{M}_*$

$$\rho_I = \det\left[-\frac{R}{2\pi i} + \frac{\omega}{2\pi}1_{m\times m}\right], \qquad m = h^{3,1},$$

    - if the scanning space covers all of non-verticals (Denef '08)
    - whenever the scanning space contains $H^4_H(X)$ (Braun Kimura TW '14)

- #vac from the prefactor, copling distrib from ρ_I

- computation in examples

A.Braun TW '14

$B_3 = \mathbb{P}[\mathcal{O}_{\mathbb{P}^2} \oplus \mathcal{O}_{\mathbb{P}^2}(n)]$, S is the zero of $\mathcal{O}_{\mathbb{P}^2}$

n	-3	-2	-1	0	1	2	3
$L_*^{\max}$	237	297	387	507	657	837	[1047]
K	7557	8603	10403	12953	16253	20303	25104

prelim. result. containing error

$$K = \dim[H_H^4(X)].$$

$$L_* = \frac{\chi(X)}{24} - \frac{1}{2}(G_{fix})^2 = \frac{2163}{4} + \frac{125}{8} n(n+7) - \frac{5 N_{\text{gen}}^2}{2(18-n)(3-n)}.$$

- more generally, whenever $h^{3,1} \gg h^{1,1}, h_H^{2,2} \gg h_V^{2,2}, h_{RM}^{2,2}$.

$$\chi(X) \approx K, \quad (24 L_*^{\max}) \approx 8\pi L_* \approx K.$$

Gaussian distribution

$$\#(vac) \approx \exp\left[\left[\sqrt{2\pi K L_*}\right]\right] \approx e^{K/2} \exp[-(4\pi) c N_{gen}^2].$$

algebraic topological

- $K_{A4} - K_{D5} \approx \mathcal{O}(10)$?

(based on K3 x K3 or the examples above)