A Holographic Model of Impurity in Strongly-Coupled Theory: Kondo Effect and Quantum Quench

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Introduction

Motivation:

- A simple and generic holographic model for the Kondo effect.
- Symmetries close to the actual Kondo systems.
- Starting point for studying open questions of multi-impurity Kondo problem.

The Kondo effect: Screening of magnetic impurity by conduction electrons at low temperatures.

Features of the Kondo system:

- Kondo coupling anti-ferromagnetic.
- Electron hybridise with the impurity, forming a **Kondo singlet**.
- **Asymptotic freedom** & **dynamically generated scale** T_K.
- **Phase shift** due to spectral flow.

CFT Description

Symmetry: $SU(N) \times SU(k) \times U(1)$

EFT: 1D chiral fermions interacting with impurity at origin

$$H = \frac{1}{2\pi(N+k)} J^a J^a + \frac{1}{2\pi(k+N)} J^A J^A + \frac{1}{4\pi N k} J^2 + \lambda_K \, \delta(x) \, \vec{S} \cdot \vec{J}$$

- J^a spin $SU(N)$, J^A channel $SU(k)$, J charge $U(1)$ currents.
- λ_K classically marginal.

UV fixed point: $\lambda_K \to 0$, free $(1+1)$-D CFT

- **Kac-Moody** current algebra $SU(N)_k \times SU(k)_N \times U(1)$.
- Spectrum determined by current algebra and BC's.

IR Fixed Point

Impurity spin "absorbed" by conduction electron:

$$H = \frac{1}{2\pi(N+k)} \mathcal{J}^a \mathcal{J}^a + \frac{1}{2\pi(k+N)} J^A J^A + \frac{1}{4\pi N k} J^2$$

$$\mathcal{J}^a \equiv J^a + \pi(N+k)\lambda_K \delta(x) S^a \,, \quad \lambda_K = \frac{2}{N+k}$$

- Same $SU(N)_k \times SU(k)_N \times U(1)$ symmetry as **UV FP**.

Kondo problem: How representations rearrange in going from UV to IR along the RG flow triggered by the marginally relevant λ_K.

Affleck & Ludwig: Highest weight states of $SU(N)_k$ "fuse" with the impurity spin $\Rightarrow$ spins rearrange relative to charges.

- **Phase shift**: $\pi/2$ for $N = 2$, $k = 1$.
- $k = 2s_{imp}$ **critical screening**, $k < 2s_{imp}$ **under-screening**.
- $k > 2s_{imp}$ **over-screening**, IR FP $\neq$ free fermion theory.

Large N Approach

$SU(N)$ spin: standard large N limit

Slave fermions: $S^a = \chi^\dagger T^a \chi\,,\, a = 1, \ldots, N^2 - 1.$

- Extra $U(1)$ symmetry $\Rightarrow$ **constraint** $\chi^\dagger \chi = Q$.
- Extra $U(N_f)$ if N_f "flavours" of χ.

$\mathcal{O}(t) \equiv \psi_L^\dagger \chi$ $SU(N)$ singlet, $SU(k) \times U(N_f)$ bi-fundamental

$$\lambda_K\, \delta(x)\, J^a S^a = \frac{1}{2} \lambda_K\, \delta(x) \left[\mathcal{O}\mathcal{O}^\dagger - \frac{Q}{N} \left(\psi_L^\dagger \psi_L \right) \right],$$

- $\mathcal{O}\mathcal{O}^\dagger$ classically marginal "double trace" deformation.
- $\langle \mathcal{O} \rangle \neq 0$ when $T \leq T_c \leftrightarrow$ formation of **Kondo singlet**.

Essential Ingredients

Chiral fermions (ψ_L):

FT Currents obeying Kac-Moody algebra.

Dual Chern-Simons gauge field (A_μ) in AdS_3 (D_7).

Impurity:

FT Slave fermion (χ) Wilson line.

Dual Yang-Mills gauge field (a_μ) in AdS_2 (D_5).

Kondo interaction:

FT Bilinear charged scalar operator ($\mathcal{O} = \psi_L^\dagger \chi$).

Dual Complex bi-fundamental scalar (ϕ) with special BC.

A Bottom-Up Model

The action:

$$S = S_{CS} + S_{AdS_2}$$

$$S_{CS} = -\frac{N}{4\pi}\int tr\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right)$$

$$S_{AdS_2} = -N\int d^3x\,\delta(x)\sqrt{-g}\left[\frac{1}{4}tr\,f^2 + |D\Phi|^2 + V(\Phi^\dagger\Phi)\right]$$

$$f = da\,,\quad D\Phi = \partial\Phi + i\,A\,\Phi - i\,a\,\Phi\,,\quad \Phi = \phi e^{i\psi}$$

Bottom-up: Choose $V(\Phi^\dagger\Phi) = M^2\Phi^\dagger\Phi$

Finite temperature: BTZ black hole

$$ds^2 = \frac{1}{z^2}\left(\frac{dz^2}{h(z)} - h(z)\,dt^2 + dx^2\right),\quad h(z) = 1 - \frac{z^2}{z_H^2}$$

The Kondo Coupling

Near the boundary: $\phi(z) = \alpha\sqrt{z}\log(\Lambda z) + \beta\sqrt{z}$

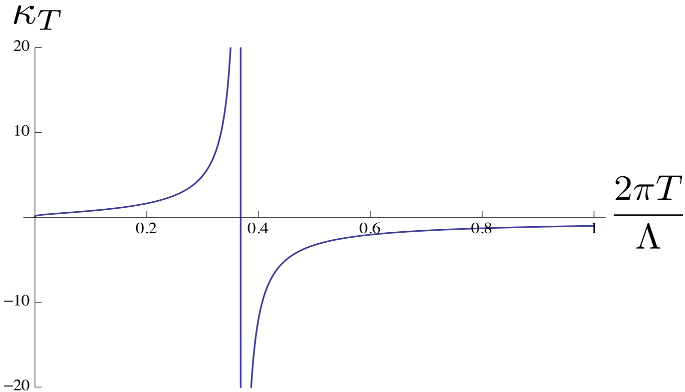
- **BC**: boundary flux $\sqrt{-g}f^{zt}|_{z=0} = Q$
- **Double trace coupling**: $\alpha = \kappa\beta \propto \langle\mathcal{O}\rangle$

Running of coupling: $\phi(z)$ Λ-independent

$$\kappa_T\beta_T = \frac{\kappa\beta}{2\pi T}\,, \quad \kappa_T = \frac{\kappa}{1+\kappa\log\frac{\Lambda}{2\pi T}}$$

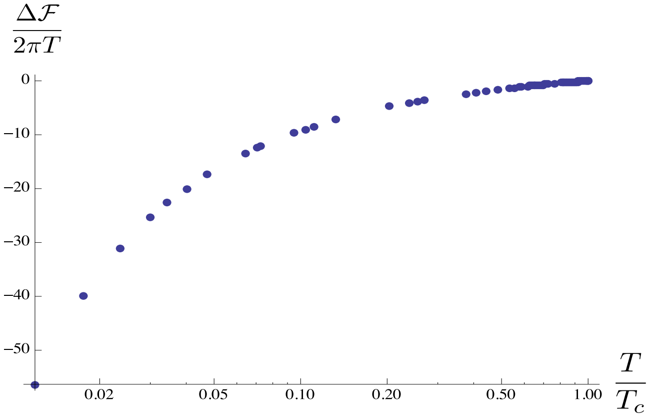
- **Asymptotic freedom**: $\kappa \to 0$ as $\Lambda \to \infty$

Dynamical Scale Generation



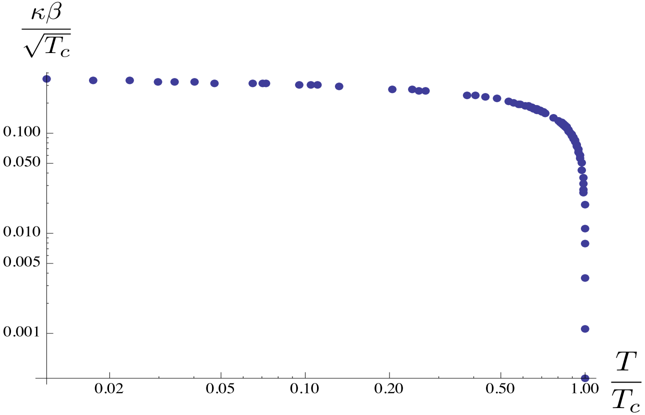
Divergence of κ_T determines $T_K = \frac{1}{2\pi}\Lambda e^{1/\kappa}$

Phase Transition



Free energy difference: $\Delta\mathcal{F} = \mathcal{F}_{\phi(z)\neq 0} - \mathcal{F}_{\phi(z)=0}$, $T_c/T_K \approx 0.90$

The Condensate



Mean-field transition: $\langle \mathcal{O} \rangle \propto \left(1 - \frac{T}{T_c}\right)^{1/2}, \ T \lesssim T_c$

Screening of Impurity

Maxwell's equation: $\partial_z \left(\sqrt{-g} f^{zt} \right) = -J^t$

Electric charge density: $J^t = -2\sqrt{-g} g^{tt} a_t \phi^2$

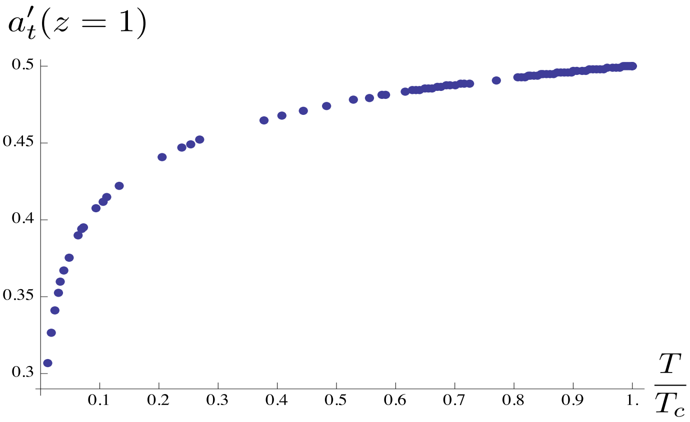
Electric flux $\sqrt{-g} f^{tz} = z^2 a_t'(z) \leftrightarrow$ impurity representation

$T > T_c$: $\phi = 0$, $J^t = 0$, $\sqrt{-g} f^{zt}(z) = Q$ constant

$T < T_c$: $\phi \neq 0$ draws charge away, reducing flux at horizon

- **Under-screening**: Nonzero flux remain at $T = 0$.
- **Critical/Over-screening**: No flux remain at $T = 0$.

Horizon Flux



Impurity screened: $R^{IR}_{imp} < R^{UV}_{imp} = Q$

Phase Shift

Magnetic flux: $\varepsilon^{tzx} F_{zx} = 2\partial_z A_x = -4\pi\delta(x) J^t$

Wilson loop:

$$W(z) \equiv \oint dx\, A_x(z) = -2\pi \int_0^z ds\, J^t(s)$$

$T > T_c$: $\phi = 0,\ J^t = 0,\ W(z) = 0$

$T < T_c$: $\phi \neq 0,\ J^t \neq 0,\ W(z) \neq 0 \Rightarrow$ phase shift $e^{iW(z)}$

Friedel sum rule: IR phase shift $\leftrightarrow$ impurity representation

- Total charge $\lim_{z\to\infty} \int_0^z ds\, J^t(s) = -Q$
- Phase shift at IR FP $= 2\pi Q$

Time-dependent Problem

Scalar equation:

$$\frac{1}{\sqrt{-g}}\left[\partial_t\left(\sqrt{-g}g^{tt}\partial_t\phi\right)+\partial_z\left(\sqrt{-g}g^{zz}\partial_z\phi\right)\right]=\Delta^m\Delta_m\phi+\frac{1}{2}\frac{\partial V}{\partial\phi}$$

Maxwell's equations:

$$\frac{1}{\sqrt{-g}}\partial_m\left(g^{mp}g^{nq}\partial_{[p}\Delta_{q]}\right)=2g^{nm}\Delta_m\phi^2$$

Convenient variable: $\Delta_m=\partial_m\psi-a_m$

Coupling Quench

Quench protocal: $\kappa(u) = \kappa_c e^{-su^2}$, $s \ll 1$

Adiabatic expansion:

$$\phi(\rho, u; \kappa) = \phi_0(\rho; \kappa(u)) + \epsilon\phi_1(\rho, u) + \cdots$$

- Expansion breaks down when $\phi_0 \sim \phi_1$
- Breakdown time scale: $u_{ad} \sim s^{-1/3}$

Critical scaling: $\langle\mathcal{O}\rangle(t; s) = s^{1/6}\langle\mathcal{O}(s^{1/3}t)\rangle$

Scalar Quench