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Large order behavior and instanton action in supersymmetric matrix model

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+ recent progress

Backgrounds

Sugino-K.
Endres-K.-Sugino-Suzuki

0-dim. SUSY matrix model:

$$S = N\mathrm{tr}\left[\frac{1}{2}B^2 + iB(\phi^2 - \mu^2) + \bar{\psi}(\phi\psi + \psi\phi)\right]$$

nilpotent SUSY:
$$Q\phi = \psi, \quad Q\psi = 0, \quad Q\bar{\psi} = -iB, \quad QB = 0,$$
$$\bar{Q}\phi = -\bar{\psi}, \quad \bar{Q}\bar{\psi} = 0, \quad \bar{Q}\psi = -iB, \quad \bar{Q}B = 0,$$

- (predicted to be) nonperturbative formulation of a two-dim. superstring theory in RR background
- two SUSY's preserved in all order in $1/N$-expansion
- SUSY's are broken nonperturbatively and spontaneously

(if established) the first example of nonpert. superstring theory realizing spont. SUSY breaking!!

Evidence

- perturbative: reproduce several kinds of and infinitely many two-pt. functions at the tree level in the two-dim. superstring theory (agreement of functional form)
- nonperturbative: prove spont. SUSY breaking in MM side by computing the VEV of SUSY inv. operator (order parameter)

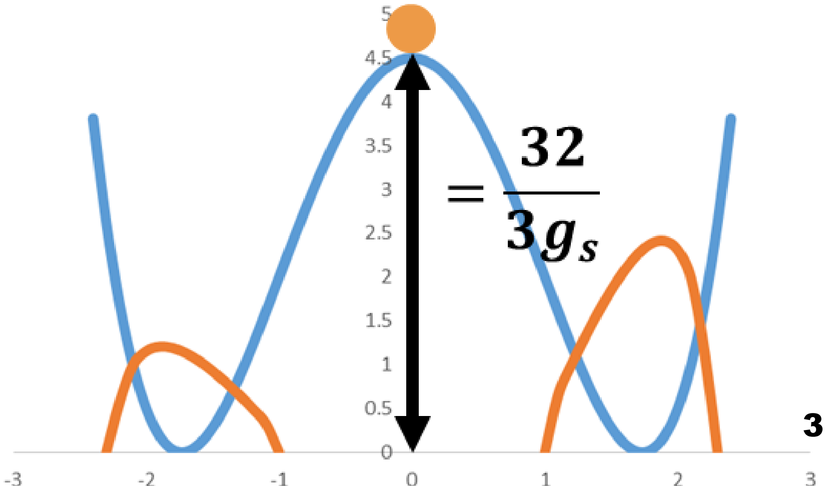
In particular, $F = 0$ in all order in $1/N$-expansion reflecting perturbative SUSY, but nonzero nonperturbatively

origin: instanton in MM

∼isolated eigenvalue of ϕ: $F \sim e^{-\frac{32}{3g_s}}$

counterpart in superstring side?

(D-brane?)



Evidence

- perturbative: reproduce several kinds of and infinitely many two-pt. fu ctions at the tree level in the two-dim. superstring theory (ag nt of functional form)
- nonpert breaking in MM side by c (order parameter)

In on reflecting pertu batively

more evidence necessary!!
→ higher order , multi-pt.
this talk

origin: instanton in MM
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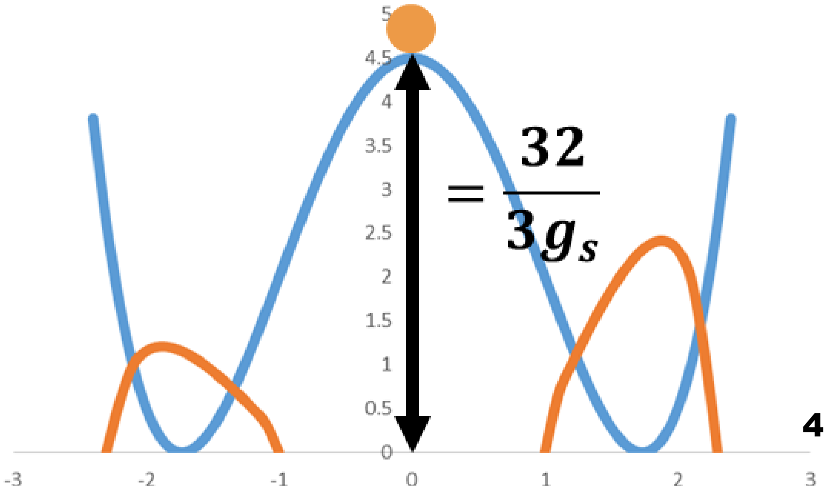


Perturbation series & nonperturbative effect (bosonic string)

$c=0$ noncritical string (2D pure gravity)

Brezin-Kazakov '90
Gross-Migdal '90
Douglas-Shenker '90

$u=\partial_t^2 F$: $\boxed{u^2+\frac{1}{6}\partial_t^2 u=t}$ ($t^{-\frac{5}{4}}\sim g_s$: C.C. or string coupling)

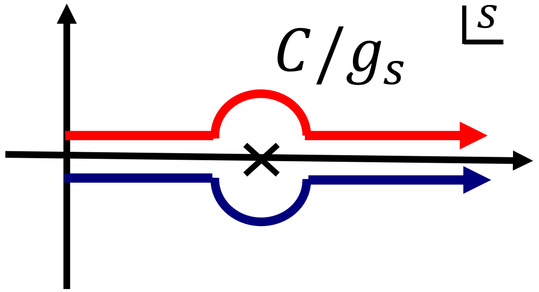
perturbative soln.: $u=\sum_{h\geq 0} u_h\, t^{\frac{1}{2}-\frac{5}{2}h}=\sum_{h\geq 0} u_h\, g_s^{-2+2h} \rightarrow u_h\sim C^{-2h}(2h)!$:

large order behavior

- characteristic of string (vol. of moduli sp. of Riemann surface) cf. particle: $n!$
- nonperturbative ambiguity:

Borel transf.: $u\rightarrow \tilde{u}(s)=\sum_{h\geq 0}\frac{u_h}{(2h)!} g_s^{-2+2h} s^{2h} \sim \frac{1}{1-\left(\frac{s}{C/g_s}\right)^2}$

inverse: $u=\int_0^\infty \tilde{u}(s)e^{-s}ds\sim \int_0^\infty \frac{1}{1-\left(\frac{s}{C/g_s}\right)^2}e^{-s}ds$



$\rightarrow$ ambiguity $\sim e^{-\frac{C}{g_s}}$, where C: instanton action in MM = tension of D-brane

Perturbation series & nonperturbative effect (SUSY DWMM)

$$S = N\mathrm{tr}\left[\frac{1}{2}B^2 + iB(\phi^2 - \mu^2) + \bar{\psi}(\phi\psi + \psi\phi)\right]$$

perturbative SUSY $\rightarrow F = 0$ in all order in $1/N^2 \sim g_s^2$-expansion, but $^\exists$nonpert. effect

$F \sim e^{-\frac{32}{3g_s}} \neq 0$ & ~~SUSY~~ $\rightarrow$ in SUSY theory, no relation between LOB & nonpert. effect??

genus h quantities: $\mathcal{O}(1/N^{2h}) \sim \mathcal{O}(g_s^{2h-2})$:

$\partial_{\mu^2} F_h = -i\left\langle\frac{1}{N}\mathrm{tr}\,B\right\rangle_h \sim \left\langle\frac{1}{N}\mathrm{tr}(\phi^2 - \mu^2)\right\rangle_h$: SUSY protected (Note: $iQ(\mathrm{tr}\bar{\psi}) = \mathrm{tr}B$)

perturbatively zero $F \sim e^{-\frac{32}{3g_s}}$ $\rightarrow$ no prediction

$\rightarrow$ due to huge cancellation

$\rightarrow$ non-SUSY inv. operator: $\left\langle\frac{1}{N}\mathrm{tr}\,\phi^{2k+1}\right\rangle_h$ $\rightarrow$ stringy LOB? instanton action?

All order result for non-SUSY operator

$B, \psi, \bar{\psi}$: Gaussian $\rightarrow$ MM only for ϕ $\rightarrow$ eigenvalue density

$$\rho_h(z) = \left\langle \frac{1}{N} \mathrm{tr}\, \delta(z-\phi) \right\rangle_h = \frac{1}{\pi} \mathrm{Im} \left\langle \frac{1}{N} \mathrm{tr} \frac{1}{z-i\epsilon-\phi} \right\rangle_h$$

$$\rightarrow \left\langle \frac{1}{N} \mathrm{tr}\, \phi^{2k+1} \right\rangle_h = \int dx \rho_h(x) x^{2k+1}$$

difficult to compute!!

trick

$$\rho_h(z) = \frac{2\sqrt{z}}{\pi} \mathrm{Im} \left\langle \frac{1}{N} \mathrm{tr} \frac{1}{z-i\epsilon-\phi^2} \right\rangle_h$$: SUSY protected,

Nicolai mapping

[Haagerup-Thorbjørnsen '10 [math.PR]]

$\rightarrow$ GUE: all order result in $1/N$-expansion known!

$$\rho_h(x) = \frac{8}{3\pi}\left(-\frac{1}{12}\right)^h \frac{(6h-3)!}{h!\,(3h-2)!}\left(4-(x-\mu^2)^2\right)^{\frac{1}{2}}$$

:alternating series (Borel summable, no ambiguity) reflecting SUSY protected

$$\left\langle \frac{1}{N}\mathrm{tr}\phi^{2k+1}\right\rangle_h = \int dx\, \rho_h(x) x^{2k+1}$$

$$= N^{-\frac{2}{3}(k+2)}(-1)^{k+1}\frac{\Gamma\left(k+\frac{3}{2}\right)}{2\pi^{\frac{3}{2}}}\left(\frac{1}{12}\right)^h \frac{(3h-k-3)!}{h!}\left(\frac{g_s^2}{64}\right)^{h-\frac{k+2}{3}}$$

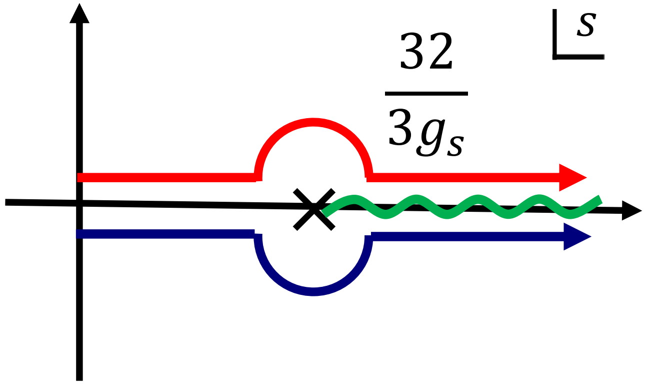
non-SUSY op. → positive term series

→ non-Borel summable (obtained from ρ_h)

standard expansion in terms of g_s^2,

LOB: $(2h)!$ → stringy!

inverse Borel: $\int_0^\infty \left(1-\frac{s^2}{\frac{16\cdot 64}{9g_s^2}}\right)^{k+\frac{5}{2}} e^{-s}ds \sim e^{-\frac{32}{3g_s}}$



:exact agreement with instanton action!

Conclusions

1-pt. function of SUSY op.

Nicolai mapping

resolvent of GUE: all order in $1/N$-exp.

Im

Borel summable

Im

eigenvalue density : all order

integration

1-pt. function of non-SUSY op. :all order

- MM is a superstring theory
- spontaneous ~~SUSY~~ proved

→ FIRST EX. established

- non-Borel summable
- stringy LOB
- reproduce instanton action exactly

Discussions

- Superstring counterpart of instanton which triggers SUSY breaking

  likely D-brane → D-brane generation triggers spont. SUSY breaking!

- 1-pt. function of ϕ^{2k+1} in higher order in $1/N$-expansion

  =1-pt. function of RR field at higher genus: problem of supermoduli

  MM side: completely well-defined

  → further perturbative confirmation of our claim with solving the problem of supermoduli

  K-Sugino, work in progress

- Application to SUSY gauge theories?

  existence of $\rho(x)$ is essential

  localization works?

  ABJM: different story?