Exact Computations in Confining Phase using SUSY Localization

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based on JHEP 1503 (2015) 097 (arXiv:1410.3630) and arXiv:1509.02916

Summary

Theory : **4d N=1** supersymmetric gauge theory

Method: **localization** with new choice of SUSY generator

We exactly compute
gaugino condensation $\langle \mathrm{Tr}\,(\lambda\lambda) \rangle$ in confining phase

Apply to : nonperturbative proof of Dijkgraaf-Vafa conjecture

Motivation

Analytic computations in QFT are important, and **difficult**

But, for **SUSY field theory,** we can !
two major techniques:

1. Holomorphy
Simple, But, Indirect

2. Localization
Direct and Systematic,
But, had **not** been applied to theory in confining phase

Apply localization technique to
"dynamical object"(gaugino condensation)
in confining phase !

What is done in this work

	Holomorphy	Localization
N=2 SUSY	Seiberg-Witten curve	Nekrasov partition function
N=1 SUSY	Seiberg, ADS	**This work!**

Localization

For SUSY theory,
take
$$\begin{cases} \delta = \text{a SUSY generator} \\ I = \int (\delta\lambda)^\dagger \lambda \end{cases}$$
(λ is a fermion)

Then, we found
$$\begin{cases} \delta I|_{bosonic} = \int (\delta\lambda)^\dagger \delta\lambda \geq 0 \\ \delta^2 I(\phi) = 0 \end{cases}$$

Let us consider a correlator:
$$Z(t) = \int D\phi\, e^{-S(\phi) - t\,\delta I(\phi)} \mathcal{O}_1(\phi) \cdots \mathcal{O}_n(\phi)$$
where we assume
$$\begin{cases} \delta S(\phi) = 0 \\ \delta \mathcal{O}_n(\phi) = 0 \end{cases}$$

This is independet with t!
$$\frac{dZ(t)}{dt} = \int D\phi\, \delta \left(I e^{-S(\phi) - t\,\delta I(\phi)} \mathcal{O}_1(\phi) \cdots \mathcal{O}_n(\phi) \right)$$
$$= 0$$

Taking $t \to \infty, Z(0) = Z(t \to \infty)$
In this limit,
path-integral localized on $\delta I(\phi_0) = 0$
$$Z(0) = \int_{\text{saddle pt}} D\phi_0 \left(e^{-S(\phi_0)} \times \text{ (1-loop determinant)} \right)$$

Exactly computable!

4D $N = 1$SUSY gauge theory on $\mathbf{R}^3 \times S^1_R$

Fields (in Vector multiplet)

adjoint representations of gauge group G

- A_m : vector
- λ : chiral spinor
- $\bar{\lambda}$: anti − chiral spinor
- D : auxilary field

$SU(2) \times SU(2) (= Spin(4))$

SUSY

$$\begin{aligned} \delta A_m &= \frac{i}{2}(\epsilon\sigma_m\bar{\lambda} - \bar{\epsilon}\bar{\sigma}_m\lambda), \\ \delta\lambda &= \frac{1}{2}\sigma^{mn}\epsilon F_{mn} - \epsilon D, \\ \delta\bar{\lambda} &= \frac{1}{2}\bar{\sigma}^{mn}\bar{\epsilon}F_{mn} - \bar{\epsilon}D, \\ \delta D &= -\frac{i}{2}\epsilon\sigma^m D_m\bar{\lambda} - \frac{i}{2}\bar{\epsilon}\bar{\sigma}^m D_m\lambda \end{aligned}$$

In Euclidian space, ϵ and $\bar{\epsilon}$ are independent

SUSY invariant Lagrangian: $L_g = \mathrm{Tr}\left[\frac{1}{g^2}\left(\frac{1}{2}F_{mn}F^{mn} + D^2 + i\bar{\lambda}\bar{\sigma}^m D_m\lambda\right) + i\frac{\theta}{16\pi^2}F\tilde{F}\right]$, $\tau \equiv \frac{\theta}{2\pi} + \frac{4\pi i}{g^2}$

For Localization, adding $\delta I = \int d^4x \delta V$ where $V = (\delta\lambda)^\dagger\lambda + (\delta\bar{\lambda})^\dagger\bar{\lambda}$ with a choice of $\epsilon, \bar{\epsilon}$

Localization with $\epsilon \neq 0$, $\bar{\epsilon} \neq 0$ (known choice)

$$(\delta\lambda)^\dagger\delta\lambda \sim (F^+_{mn}F^{+mn} + D^2)(\epsilon^\dagger\epsilon)$$
$$(\delta\bar{\lambda})^\dagger\delta\bar{\lambda} \sim (F^-_{mn}F^{-mn} + D^2)(\bar{\epsilon}^\dagger\bar{\epsilon})$$

→

$V|_{bosnic} = F_{mn}F^{mn} + D^2$

saddle point is trivial: $F_{mn} = D = 0$

$$L_g + t\,\delta V \sim (\frac{1}{2g^2} + t)(F^2 + D^2) + i\frac{\theta}{16\pi^2}F\tilde{F}$$

Weak coupling in $t \to \infty$ and No gauge couling dependence. This gives superconformal index

Localization with $\epsilon = 0$, $(\bar{\epsilon} \neq 0)$ (NEW choice)

$V| = F^-_{mn}F^{-mn} + D^2 = \frac{1}{2}F_{mn}F^{mn} + \frac{1}{2}F_{mn}\tilde{F}^{mn} + D^2$, saddle point is **instanton:** $F^-_{mn} = D = 0$

$$L_g + t\,\delta V \sim (\frac{1}{2g^2} + t)(F^2 + D^2) + (i\frac{\theta}{16\pi^2} + t)F\tilde{F} \quad \to \quad i\frac{\tau}{8\pi}F\tilde{F} \text{ at saddle point}$$

Weak coupling in $t \to \infty$, **gauge couling dependent.** Usual instanton analysis becomes exact!

Moreover, $\mathrm{Tr}\,(f(\lambda))$ is an "observable" in localization, because $\delta\lambda = 0$

Gaugino condensation $\langle\mathrm{Tr}\,(\lambda\lambda)\rangle$ computation

We need precisly 2 fermion zero modes for nonzero $\langle\mathrm{Tr}\,(\lambda\lambda)\rangle$

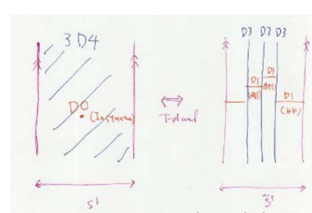
⇓

$1/N$ fractional instantons (=T-dual of BPS monopoles)

Usual 1-loop comutations around these give

the correct exact result ! $\left\langle\frac{\mathrm{Tr}\lambda^2}{16\pi^2}\right\rangle = \Lambda^3\omega$ where $\omega^N = 1$ for $G = SU(N)$, $\Lambda^3 = \mu^3\frac{1}{g^2(\mu)}\exp\frac{2\pi i\tau(\mu)}{N}$

This is R independent



Brane picture (SU(3) case)

Localization for theory on $\mathbf{R}^4$?	Localization for theory on $\mathbf{S}^4$?
Theory DOESN'T become weak in $t \to \infty$	We can NOT take $t \to \infty$

General 4d $N = 1$ gauge theories (including chiral multiplets)

Kinetic terms for chiral multiplets can be arbitrary large. Now gauge coupling can be weak also.
Thus, we can integrate out them perturbatively! → semi-classical monopole computations
Therefore, we can compute the correlators of observables(chiral ring) always in weak coupling
→ a Nonperturbative proof of Dijkgraff-Vafa conjecture (Marvelous Proof Which This Margin Is Too Narrow To Contain)