A case study of 2HDM in the Higgs basis

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Main Ref.: 2204.05728 [hep-ph] Y.H., Dong-Won Jung, Jae Sik Lee

1. Introduction – ρ parameter

- SMのヒッグス場の自発的対称性の破れの結果、
  W, Zボソンの質量比は、電弱ゲージ結合定数で決まる

$$\frac{m_W^2}{m_Z^2} = \cos^2\theta_w = \frac{g^2}{g^2 + g'^2}\ .$$

- $\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_w}$ とすると、**標準模型では** $\rho_0 = 1$ となる。$\left(\rho \equiv \frac{1}{1-\Delta\rho}, \rho_0 = 1, \Delta\rho = 0\right)$

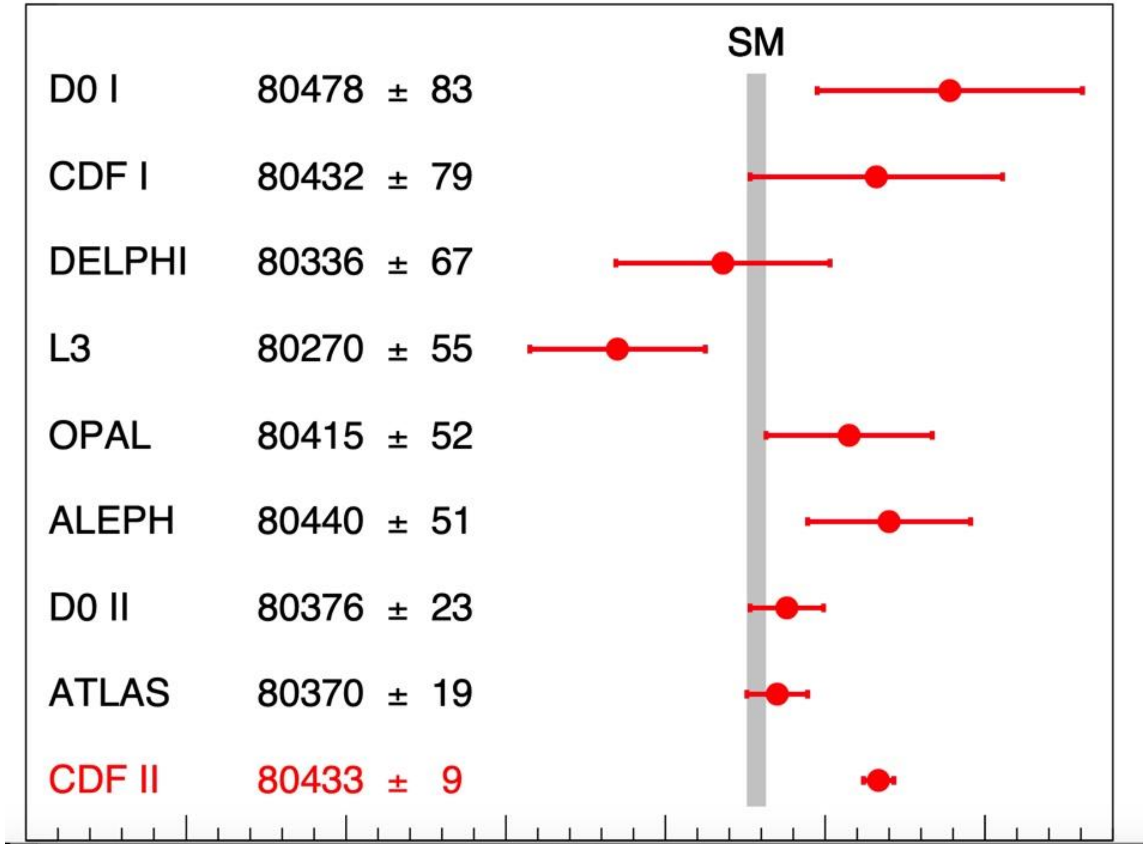
- **New Physics**があると、その影響で ρ値の補正が生じる (またはボソンの質量)

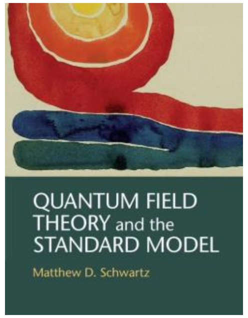
$$m_W^2\left(1 - \frac{m_W^2}{m_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_\mu}(1 + \Delta r)\ .$$

1. Introduction – Peskin-Takeuchi parameter

[Peskin-Takeuchi, 1992]

- <u>ρ値の補正</u>を見るために、Peskin-竹内 parameter **S, T, U** (通称 Oblique parameter)がよく使われている。新模型構築の際、精密な実験で測定された **S, T, U** 値を新模型は満たさなければならない。
  - 標準模型では **S=T=U=0.**
    新模型の影響で
    S, Tが大きくなる。
    (U≪1より無視)
- そして 2022年4月7日…
  理論値と **7 σ** 離れた
  <u>**Wボソン質量値が報告される**</u>。





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1. Introduction – Peskin-Takeuchi parameter

[Peskin-Takeuchi, 1992]

- ρ値の補正を見るために、Peskin-竹内 parameter **S, T, U** (通称 Oblique parameter)がよく使われている。新模型構築の際、精密な実験で測定された **S, T, U** 値を新模型は満たさなければならない。

$$m_W^{\mathrm{CDF}} = 80.4335 \pm 0.0094 \text{ GeV}$$

[CDF Collaboration, T. Aaltonen et al., Science 376 (2022) 6589

Table III. Same as Tab. II, but for S and T with $\Delta U = 0$.

$U = 0$	PDG 2021			CDF 2022		
	Result	Correlation		Result	Correlation	
14 dof	$\chi^2_{\min} = 15.48$	S	T	$\chi^2_{\min} = 17.82$	S	T
S	0.05 ± 0.08	1.00	0.92	0.15 ± 0.08	1.00	0.93
T	0.09 ± 0.07		1.00	0.27 ± 0.06		1.00

[C.-T. Lu, L. Wu, Y. Wu, and B. Zhu, arXiv:2204.0379]

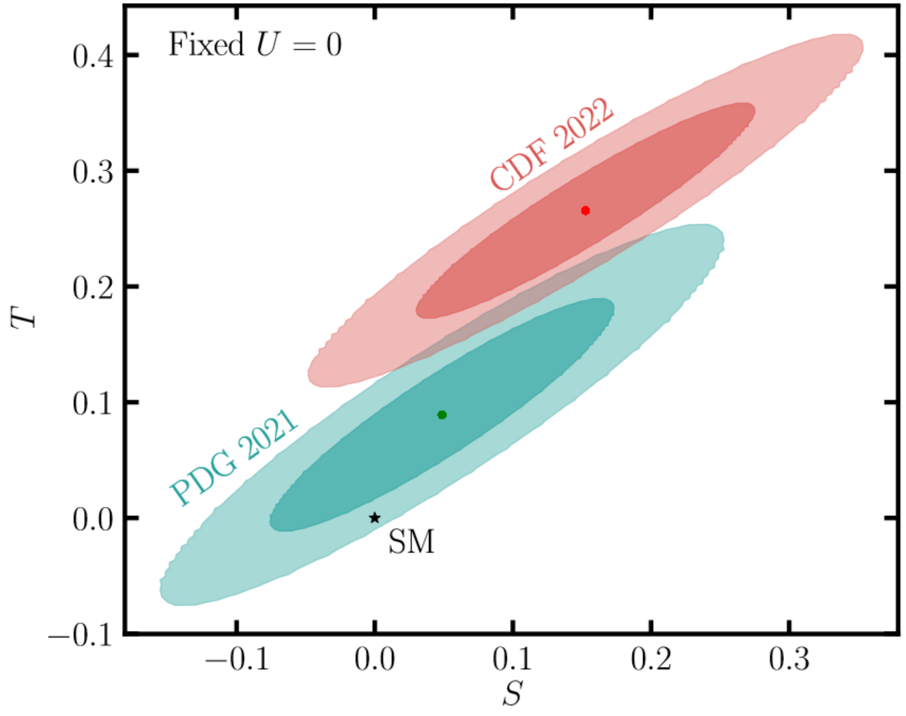


Figure 1. The 1- and 2-σ allowed regions in S-T plane from the electroweak fits using the PDG 2021 data set with the old value of m_W (green region) and the new CDF value of m_W (red region).

1. Introduction – New Physics

- Models with extended Higgs sectors:
  - ヒッグス場が１つでなければならない理由はないので、スカラーセクターを拡張する。
  - Minimal Supersymmetry Standard Model (MSSM)などでは自然にヒッグス場２つが現れる。
  - 模型の例

1. SM with additional Higgs singlet
2. **Two Higgs Doublet Model (2HDM)**: type I, **II**, III, IV, …
3. 2HDM with singlet extensions: N2HDM, S2HDM, 2HDMS, . . .
4. Minimal Supersymmetric Standard Model (MSSM)
5. MSSM with one extra singlet (NMSSM)
6. MSSM with more extra singlets (μνSSM)
7. SM/MSSM with Higgs triplets

1. Introduction

- **今まで最も正しい測定であるCDF実験で W-mass anomalyが観測された**。

  この場合、2HDMの予言はどうなるか？

- **Higgs-basis** 2HDMに基づいて、その様相をみて、
  実験でその検証の可能性を考える。

- 今後の流れ
  - Higgs-basis 模型の説明
  - CP-Conservingにおける模型のConstraints
  - 模型の予言

2. Higgs basis 2HDM

2. Φ basis 2HDM

- 通常の Φ basis 2HDM (Type II)

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + \phi_1 + i a_1) \end{pmatrix}, \quad \Phi_2 = e^{i\xi} \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \phi_2 + i a_2) \end{pmatrix}$$

  - $v = \sqrt{v_1^2 + v_2^2}$, $\tan\beta = \frac{v_2}{v_1}$, $s_\beta = \sin\beta$, $c_\beta = \cos\beta$

- Potential $V_\Phi = \mu_1^2\left(\Phi_1^\dagger\Phi_1\right) + \mu_2^2\left(\Phi_2^\dagger\Phi_2\right) + m_{12}^2\left(\Phi_1^\dagger\Phi_2\right) + m_{12}^{*2}\left(\Phi_2^\dagger\Phi_1\right)$

$$+\lambda_1\left(\Phi_1^\dagger\Phi_1\right)^2 + \lambda_2\left(\Phi_2^\dagger\Phi_2\right)^2 + \lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$
$$+\lambda_5\left(\Phi_1^\dagger\Phi_2\right)^2 + \lambda_5^*\left(\Phi_2^\dagger\Phi_1\right)^2 + \lambda_6\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_1^\dagger\Phi_2\right) + \lambda_6^*\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_1\right)$$
$$+\lambda_7\left(\Phi_2^\dagger\Phi_2\right)\left(\Phi_1^\dagger\Phi_2\right) + \lambda_7^*\left(\Phi_2^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right)$$

2. Higgs basis 2HDM potential

- Higgs basis

$$\mathcal{H}_1 = c_\beta \Phi_1 + e^{-i\xi} s_\beta \Phi_2 = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(v + \varphi_1 + iG^0) \end{pmatrix}, \quad \mathcal{H}_2 = -s_\beta \Phi_1 + e^{-i\xi} c_\beta \Phi_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(\varphi_2 + ia) \end{pmatrix}$$

- $\begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix}, \quad \begin{pmatrix} G^0 \\ a \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}, \quad \begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix},$
- $v = \sqrt{v_1^2 + v_2^2}\ , \ t_\beta = \tan(\beta) = v_2/v_1$

- Potential $V_{\mathcal{H}} = Y_1(\mathcal{H}_1^\dagger \mathcal{H}_1) + Y_2(\mathcal{H}_2^\dagger \mathcal{H}_2) + Y_3(\mathcal{H}_1^\dagger \mathcal{H}_2) + Y_3^*(\mathcal{H}_2^\dagger \mathcal{H}_1)$

$$+Z_1(\mathcal{H}_1^\dagger \mathcal{H}_1)^2 + Z_2(\mathcal{H}_2^\dagger \mathcal{H}_2)^2 + Z_3(\mathcal{H}_1^\dagger \mathcal{H}_1)(\mathcal{H}_2^\dagger \mathcal{H}_2) + Z_4(\mathcal{H}_1^\dagger \mathcal{H}_2)(\mathcal{H}_2^\dagger \mathcal{H}_1)$$
$$+Z_5(\mathcal{H}_1^\dagger \mathcal{H}_2)^2 + Z_5^*(\mathcal{H}_2^\dagger \mathcal{H}_1)^2 + Z_6(\mathcal{H}_1^\dagger \mathcal{H}_1)(\mathcal{H}_1^\dagger \mathcal{H}_2) + Z_6^*(\mathcal{H}_1^\dagger \mathcal{H}_1)(\mathcal{H}_2^\dagger \mathcal{H}_1)$$
$$+Z_7(\mathcal{H}_2^\dagger \mathcal{H}_2)(\mathcal{H}_1^\dagger \mathcal{H}_2) + Z_7^*(\mathcal{H}_2^\dagger \mathcal{H}_2)(\mathcal{H}_2^\dagger \mathcal{H}_1)$$

2. 質量項 in the Higgs Basis

- Mass terms

$$V_{\mathcal{H}\,\mathrm{mass}} = M_{H^\pm}^2 H^+ H^- + \frac{1}{2}(\varphi_1 \ \ \varphi_2 \ \ a)\, \mathcal{M}_0^2 \begin{pmatrix} \varphi_1 \\ \varphi_2 \\ a \end{pmatrix}$$

  - The charged Higgs boson mass: $M_{H^\pm}^2 = Y_2 + \frac{1}{2} Z_3 v^2$
  - The neutral Higgs bosons: $\mathcal{M}_0^2 = M_A^2 \mathrm{diag}(0,1,1) + \mathcal{M}_Z^2$
    - $M_A^2 = M_{H^\pm}^2 + \left[\frac{1}{2} Z_4 - \Re\mathrm{e}(Z_5)\right] v^2$,
    - $\mathcal{M}_Z^2 = v^2 \begin{pmatrix} 2Z_1 & \Re\mathrm{e}(Z_6) & -\Im\mathrm{m}(Z_6) \\ \Re\mathrm{e}(Z_6) & 2\Re\mathrm{e}(Z_5) & -\Im\mathrm{m}(Z_5) \\ -\Im\mathrm{m}(Z_6) & -\Im\mathrm{m}(Z_5) & 0 \end{pmatrix}$
    - 3×3 real and symmetric mass-squared matrix $\mathcal{M}_0^2$:
      - $(\varphi_1 \ \ \varphi_2 \ \ a)_\alpha^T = O_{\alpha i} (H_1 \ \ H_2 \ \ H_3)_i^T$,
      - $O^T \mathcal{M}_0^2 O = \mathrm{diag}\left(M_{H_1}^2, M_{H_2}^2, M_{H_3}^2\right)$

2. Input parameters

- The tadpole conditions relate the quadratic parameters $Y_{1,3}$ to $Z_{1,6}$

$$Y_1 + Z_1 v^2 = 0; \quad Y_3 + \frac{1}{2} Z_6 v^2 = 0.$$

- In this work we consider the **CP-conserving** case assuming $\mathfrak{Im}(Y_3) = \mathfrak{Im}\big(Z_{5,6,7}\big) = 0.$
  - $(\varphi_1,\ \varphi_2)^T_\alpha = O_{\alpha i}(h, H)^T_i,\ \ O = \begin{pmatrix} c_\gamma & s_\gamma \\ -s_\gamma & c_\gamma \end{pmatrix}$
  - The quartic couplings

$$Z_1 = \frac{1}{2v^2}\left(c_\gamma^2 M_h^2 + s_\gamma^2 M_H^2\right), \qquad Z_4 = \frac{1}{v^2}\left(s_\gamma^2 M_h^2 + c_\gamma^2 M_H^2 + M_A^2 - 2M_{H^\pm}^2\right)$$

$$Z_5 = \frac{1}{2v^2}\left(s_\gamma^2 M_h^2 + c_\gamma^2 M_H^2 - M_A^2\right), \quad Z_6 = \frac{1}{v^2}\left(-M_h^2 + M_H^2\right) c_\gamma s_\gamma$$

  - In the decoupling limit of $Z_6 \to 0$, $M_h^2 = 2 Z_1 v^2$

- Using these conditions and Higgs masses

$$\mathcal{J} = \{Y_1, Y_2, Y_3; Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7\} \ \rightarrow\ \mathcal{J}' = \left\{v; M_{H^\pm}, M_{H_1}, M_{H_2}, M_{H_3}, \gamma; Z_3; Z_2, Z_7\right\}$$

3. Constraints

3. Conditions for the potential (1/3); UNIT

D. J. and L. L., JHEP 12 (2018)
S. K. and K. Y., Phys. Lett. B 751 (2015), 289-296

- Constraints 1: Perturbative Unitarity (UNIT)
  - The three scattering matrices in CP-conserving

$$\mathcal{M}_1^S = \begin{pmatrix} \eta_{00} - I & \eta^T \\ \eta & E + I \times \mathbf{1}_{3\times3} \end{pmatrix}_{4\times4}, \quad \mathcal{M}_2^S = \begin{pmatrix} 3\eta_{00} - I & 3\eta^T \\ 3\eta & 3E + I \times \mathbf{1}_{3\times3} \end{pmatrix}_{4\times4}$$

$$\mathcal{M}_3^S = \begin{pmatrix} 2Z_1 & 2Z_5 & \sqrt{2}Z_6 \\ 2Z_5 & 2Z_2 & \sqrt{2}Z_7 \\ \sqrt{2}Z_6 & \sqrt{2}Z_7 & Z_3 + Z_4 \end{pmatrix}_{3\times3}$$

- $\eta_{00} = Z_1 + Z_2 + Z_3, \quad I = Z_3 - Z_4, \quad \eta = \begin{pmatrix} \Re\mathrm{e}(Z_6 + Z_7) \\ 0 \\ Z_1 - Z_2 \end{pmatrix}, \quad E = \begin{pmatrix} Z_4 + 2\Re\mathrm{e}(Z_5) & 0 & -\Re\mathrm{e}(Z_7) \\ 0 & Z_4 & 0 \\ -\Re\mathrm{e}(Z_7) & 0 & Z_1 + Z_2 - Z_3 \end{pmatrix}$

3. Simplified UNIT constraints

- The 3 scattering matrices $\mathcal{M}^S_{1,2,3}$ and I should have their moduli smaller than 4π
- When $= Z_{\{6,7\}} = 0$ or $Z_{\{1,2,3,4,5\}} = 0$

$Z_{\{6,7\}} = 0$	$Z_{\{1,2,3,4,5\}} = 0$
$$\begin{aligned} \lvert Z_3 \pm Z_4 \rvert &< 4\pi, \\ \left\lvert Z_3 \pm 2\lvert Z_5\rvert \right\rvert &< 4\pi, \\ \left\lvert Z_3 + 2Z_4 \pm 6\lvert Z_5\rvert \right\rvert &< 4\pi, \\ \left\lvert Z_1 + Z_2 \pm \sqrt{(Z_1 - Z_2)^2 + 4\lvert Z_5\rvert^2} \right\rvert &< 4\pi, \\ \left\lvert Z_1 + Z_2 \pm \sqrt{(Z_1 - Z_2)^2 + Z_4^2} 2\lvert Z_5\rvert \right\rvert &< 4\pi, \\ \left\lvert 3Z_1 + 3Z_2 \pm \sqrt{9(Z_1 - Z_2)^2 + (2Z_3 + Z_4)^2} \right\rvert &< 4\pi. \end{aligned}$$	$$\begin{aligned} \sqrt{\lvert Z_6\rvert^2 + \lvert Z_7\rvert^2} &< 2\sqrt{2}\pi, \\ \sqrt{\lvert Z_6\rvert^2 + \lvert Z_7\rvert^2 + \lvert Z_6^2 + Z_7\rvert^2} &< \frac{4\pi}{3}. \end{aligned}$$

- Combining them,

$$\left\lvert Z_{1,2,5} \right\rvert < \frac{2\pi}{3}, \quad \left\lvert Z_{6,7} \right\rvert < \frac{2\sqrt{2}\pi}{3}, \quad \lvert Z_3 - Z_4 \rvert < 4\pi \ \cup\ \lvert 2Z_3 + Z_4 \rvert < 4\pi \cup \lvert Z_3 + 2Z_4 \rvert < 4\pi$$

3. Conditions for the potential (2/3); BFB

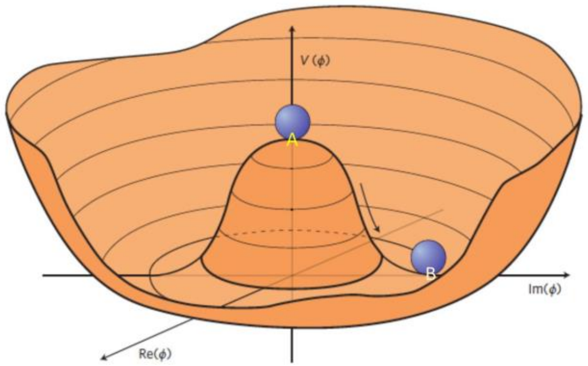
- Constraints 2: Bounded from below (BFB)

G. C. B., P. M. F., L. L., M. N. R., M. S. and J. P. S., Phys. Rept. 516 (2012)
D. J. and L. L., JHEP 12 (2018)

$$Z_1 \geq 0, \quad Z_2 \geq 0;$$

$$2\sqrt{Z_1 Z_2} + Z_3 \geq 0, \quad 2\sqrt{Z_1 Z_2} + Z_3 + Z_4 - 2|Z_5| \geq 0;$$

$$Z_1 + Z_2 + Z_3 + Z_4 + 2|Z_5| - 2|Z_6 + Z_7| \geq 0.$$



Mexican hat $\lambda > 0, \mu < 0$の2HDM ver.

- The couplings Z_2 and Z_7 have no direct relations to the masses and mixing of Higgs bosons

- But they are interrelated with the other five quartic couplings of $Z_{1,3-6}$ through the UNIT and BFB conditions.

3. Conditions for the potential (3/3); EWPO

- Constraints 3: Electroweak Precision Observables, ***S*** and ***T*** parameters.
- The electroweak oblique corrections to the so-called ***S***, ***T*** and *U*.
  - Fixing *U* = 0, by M_Z^2/M_{BSM}^2

M. E. P. and T. T., Phys. Rev. Lett. 65 (1990), 964-967
M. E. P. and T. T. , Phys. Rev. D 46 (1992), 381-409

$$\frac{\left(S-\widehat{S_0}\right)^2}{\sigma_S^2}+\frac{\left(T-\widehat{T_0}\right)^2}{\sigma_T^2}-2\rho_{ST}\frac{\left(S-\widehat{S_0}\right)\left(T-\widehat{T_0}\right)}{\sigma_S\sigma_T}\leq R^2(1-\rho_{ST}^2)$$

$$(R^2=9.21 \text{ at } 95\% \text{ CLs})$$

- Performing a global fit of electroweak data with the high-precision CDF measurement while fixing *U* = 0, one may find the large central values of the oblique parameters S and T together with the standard deviations such as

arXiv:2204.03796

$$\left(\widehat{S_0},\sigma_S\right)=(0.15,0.08),\qquad \left(\widehat{T_0},\sigma_T\right)=(0.27,0.06),\qquad \rho_{ST}=0.93$$

3. Expressions of S and T in 2HDM

[Particle Data Group], PTEP 2020 (2020) no.8
D. T., Phys. Rev. D 18 (1978), 1626

- Then, the S and T parameters take the following forms:

$$S = -\frac{1}{4\pi}\left[F'_\Delta\left(M_{H^\pm}, M_{H^\pm}\right) - c_\gamma^2 F'_\Delta(M_A, M_H) - s_\gamma^2 F'_\Delta(M_A, M_h)\right]$$

$$T = -\frac{\sqrt{2}G_F}{16\pi^2\alpha_{\rm EM}}\left[F_\Delta\left(M_A, M_{H^\pm}\right) + c_\gamma^2 F_\Delta\left(M_H, M_{H^\pm}\right) + s_\gamma^2 F_\Delta\left(M_h, M_{H^\pm}\right) - c_\gamma^2 F_\Delta(M_A, M_H) - s_\gamma^2 F_\Delta(M_A, M_h)\right]$$

  - One-loop functions

$$F_\Delta(m_0, m_1) = F_\Delta(m_1, m_0) = \frac{m_0^2 + m_1^2}{2} - \frac{m_0^2 m_1^2}{m_0^2 - m_1^2}\ln\frac{m_0^2}{m_1^2},$$

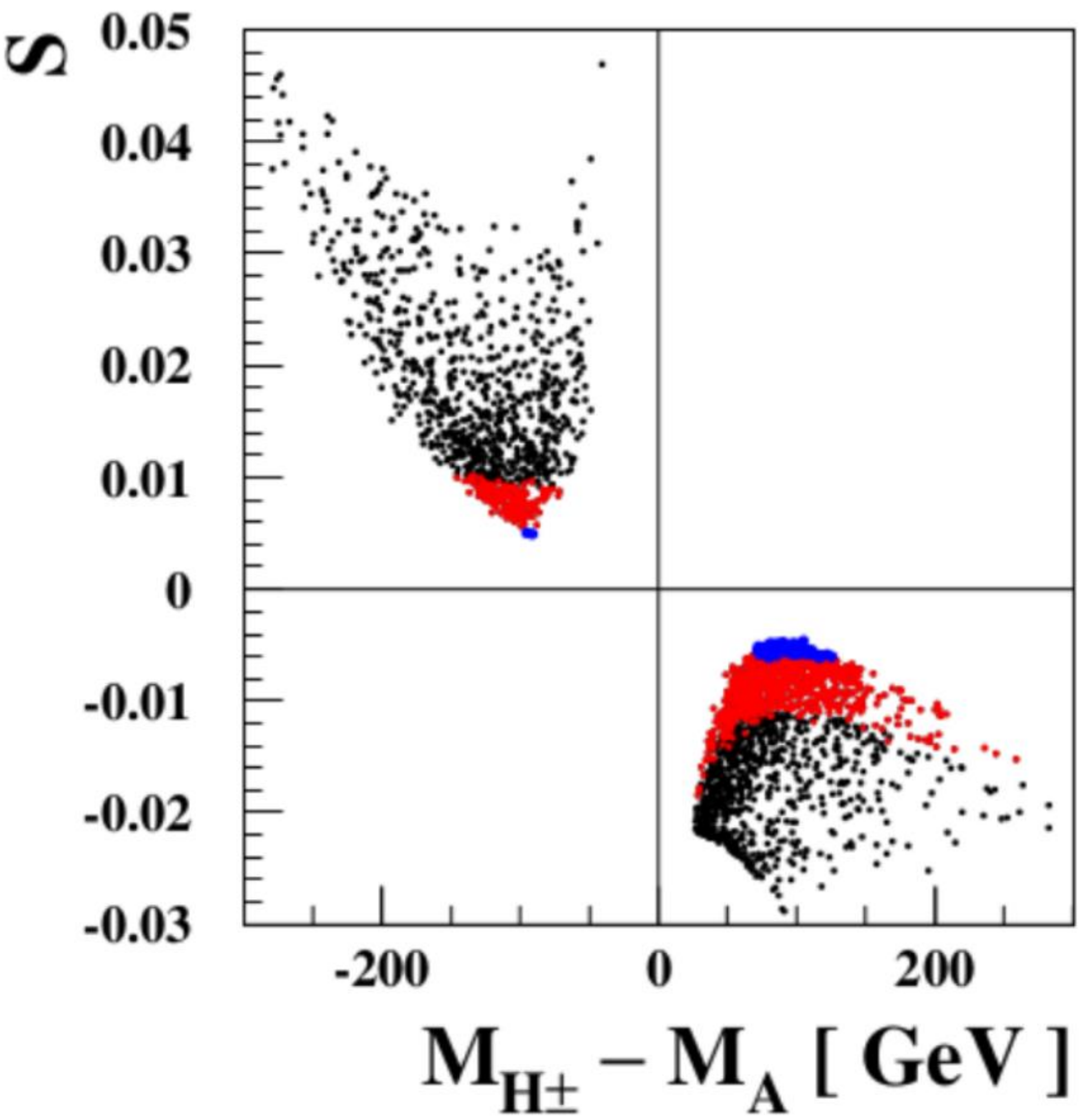
$$F'_\Delta(m_0, m_1) = F'_\Delta(m_1, m_0) = -\frac{1}{3}\left[\frac{4}{3} - \frac{m_0^2\ln m_0^2 - m_1^2\ln m_1^2}{m_0^2 - m_1^2} - \frac{m_0^2 + m_1^2}{(m_0^2 - m_1^2)^2}F_\Delta(m_0, m_1)\right]$$

$$F_\Delta(m, m) = 0, \quad F'_\Delta(m, m) = \frac{1}{3}\ln m^2$$

4. Analysis

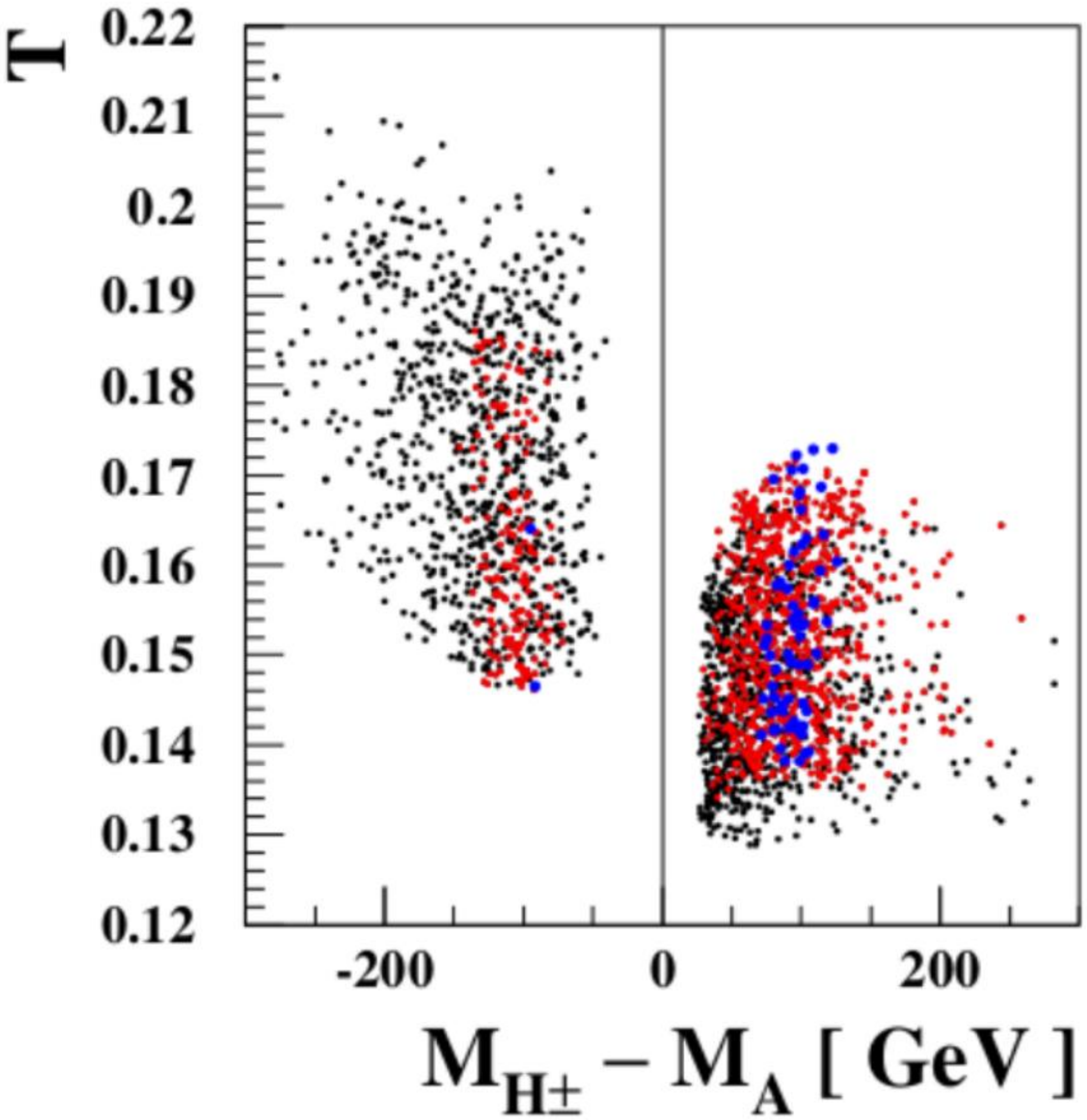
4. Scatter plots of S versus $M_{H^\pm} - M_A$

- Combining UNIT, BFB, and EWP constraints ($\text{abbr. UNIT} \oplus \text{BFB} \oplus \text{ELW}_{95\%}$)
- The heavy Higgs masses squared are scanned up to $(1.5\ \text{TeV})^2$
  - The red points are for $M_{H^\pm} > 500$ GeV, the blue points are for $M_{H^\pm} > 900$ GeV.
- Parameter range;
  - $S = -0.03 \sim 0.05$, S is negative (positive) when $M_{H^\pm} > (<) M_A$.
  - The narrow region around 0 about 0.004 is not allowed



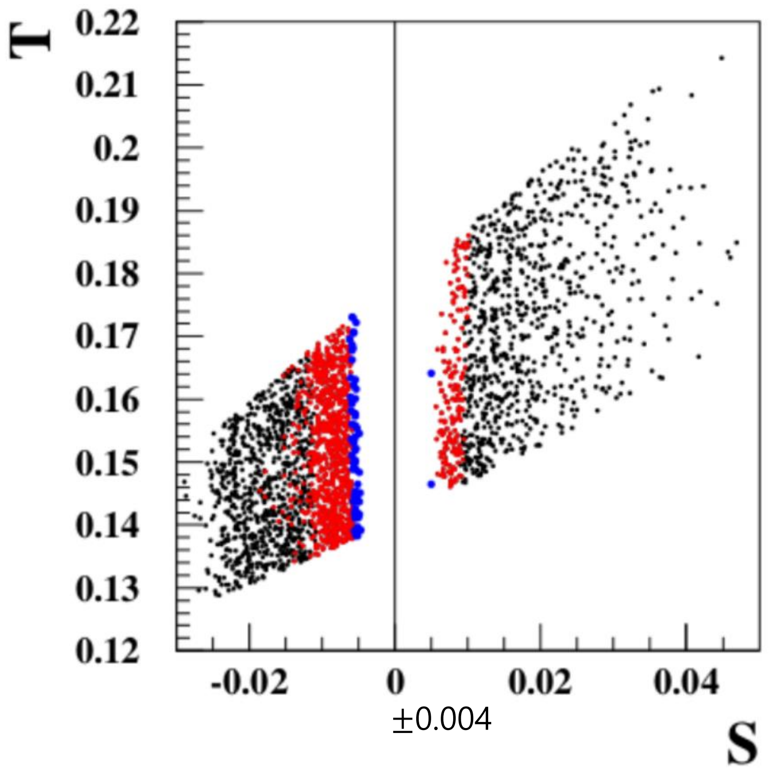
4. Scatter plots of T versus $M_{H^\pm} - M_A$

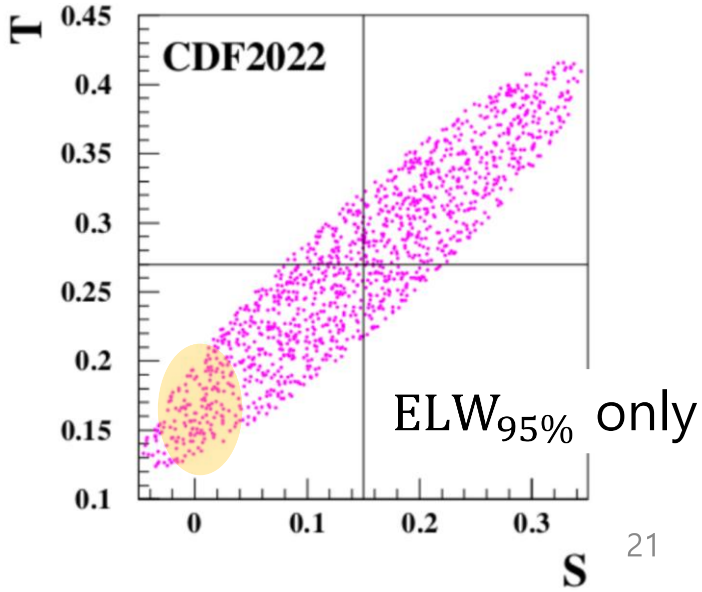
- Combining UNIT, BFB, and EWP constraints ($\text{abbr. UNIT} \oplus \text{BFB} \oplus \text{ELW}_{95\%}$)
- The heavy Higgs masses squared are scanned up to $(1.5\ \text{TeV})^2$
  - The red points are for $M_{H^\pm} > 500$ GeV, the blue points are for $M_{H^\pm} > 900$ GeV.
- Parameter range;
  - $T = 0.13 \sim 0.22$, T is positive and sizable, thus, $M_{H^\pm} = M_A$ is forbidden.
  - The region $-40 \lesssim M_{H^\pm} - M_A \lesssim 20\ \text{GeV}$ is ruled out



4. Scatter plots of S versus T

- Combining UNIT, BFB, and EWP constraints (abbr. $\mathrm{UNIT} \oplus \mathrm{BFB} \oplus \mathrm{ELW}_{95\%}$)
- The heavy Higgs masses squared are scanned up to $(1.5\ \mathrm{TeV})^2$
  - The red points are for $M_{H^\pm} > 500$ GeV, the blue points are for $M_{H^\pm} > 900$ GeV.
- Parameter range;
  - $S = -0.03 \sim 0.05$,
  - $T = 0.13 \sim 0.22$.
- $\frac{(S-\widehat{S_0})^2}{\sigma_S^2} + \frac{(T-\widehat{T_0})^2}{\sigma_T^2} - 2\rho_{ST}\frac{(S-\widehat{S_0})(T-\widehat{T_0})}{\sigma_S \sigma_T} \leq 9.21^2(1-\rho_{ST}^2)$
  - $(\widehat{S_0}, \sigma_S) = (0.15, 0.08)$, $(\widehat{T_0}, \sigma_T) = (0.27, 0.06)$, $\rho_{ST} = 0.93$

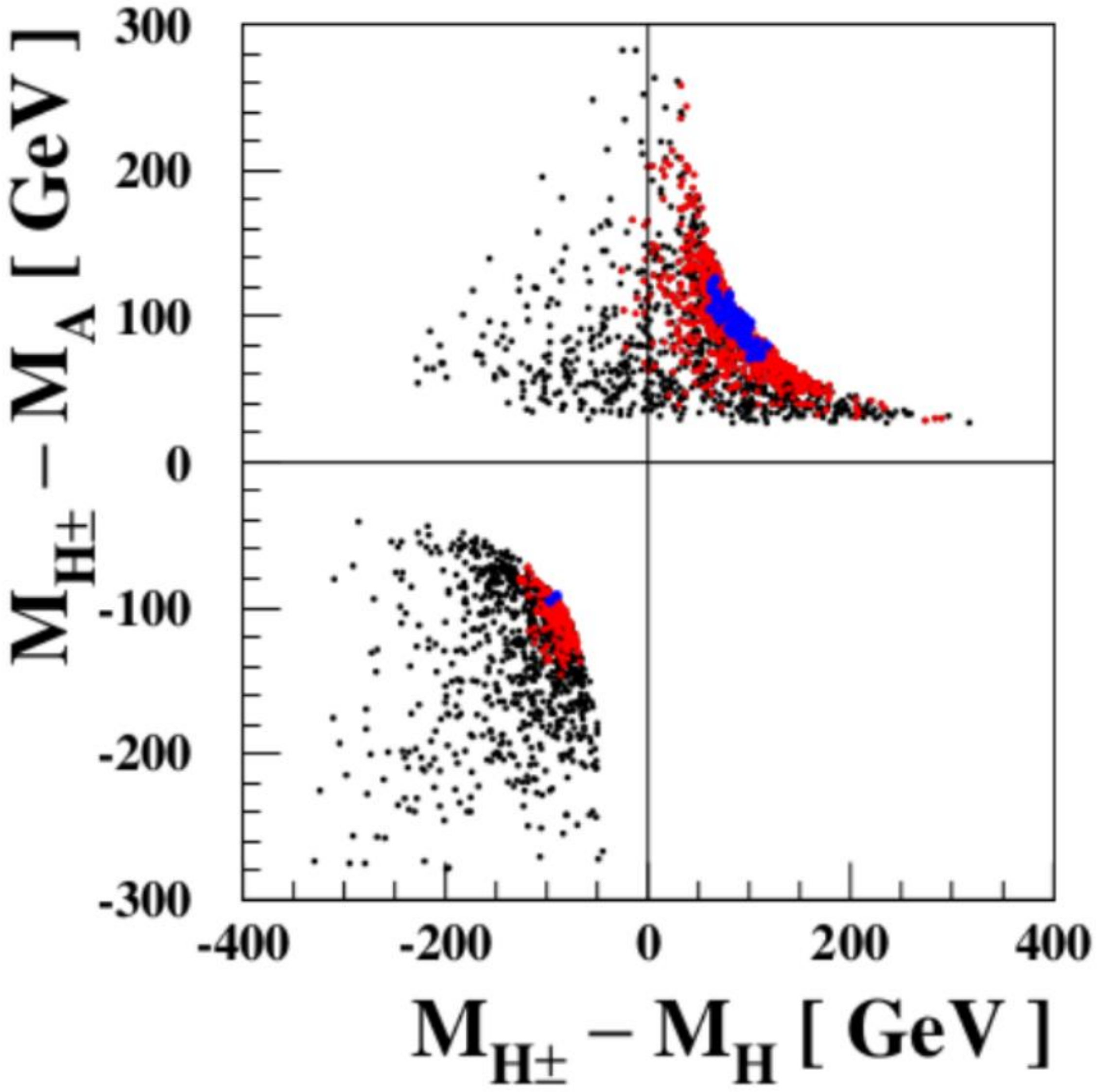




4. Scatter plots of $M_{H^\pm} - M_A$ versus $M_{H^\pm} - M_H$

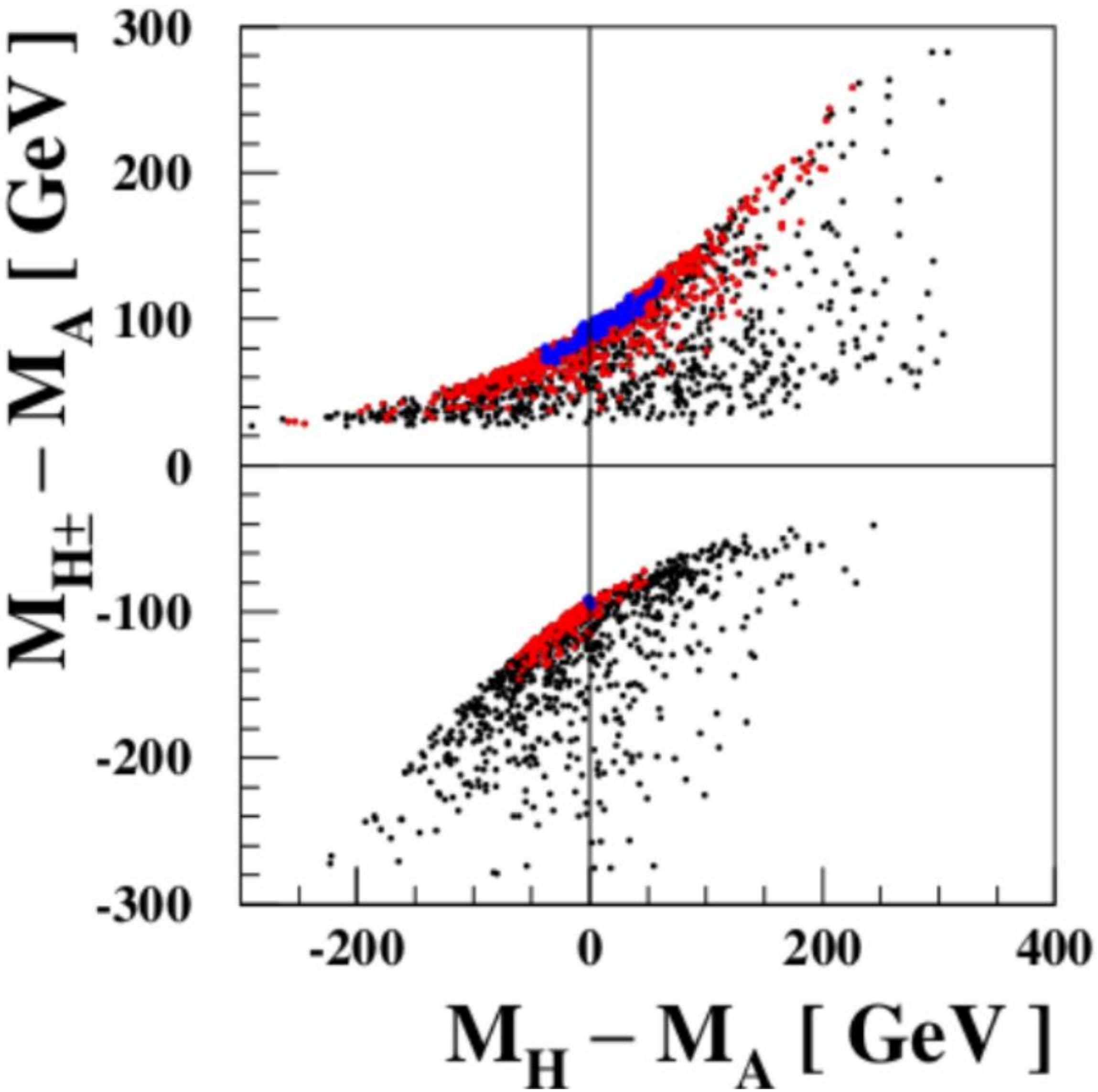
- The correlations among the mass differences
- The heavy Higgs masses squared are scanned up to $(1.5\ \mathrm{TeV})^2$
  - The red points are for $M_{H^\pm} > 500$ GeV, the blue points are for $M_{H^\pm} > 900$ GeV.

- As $M_{H^\pm}$ increases,
  the mass difference between
  the charged and neutral Higgs bosons
  $\left|M_{H^\pm} - M_{A,H}\right|$ converges to the value of
  about 100 GeV



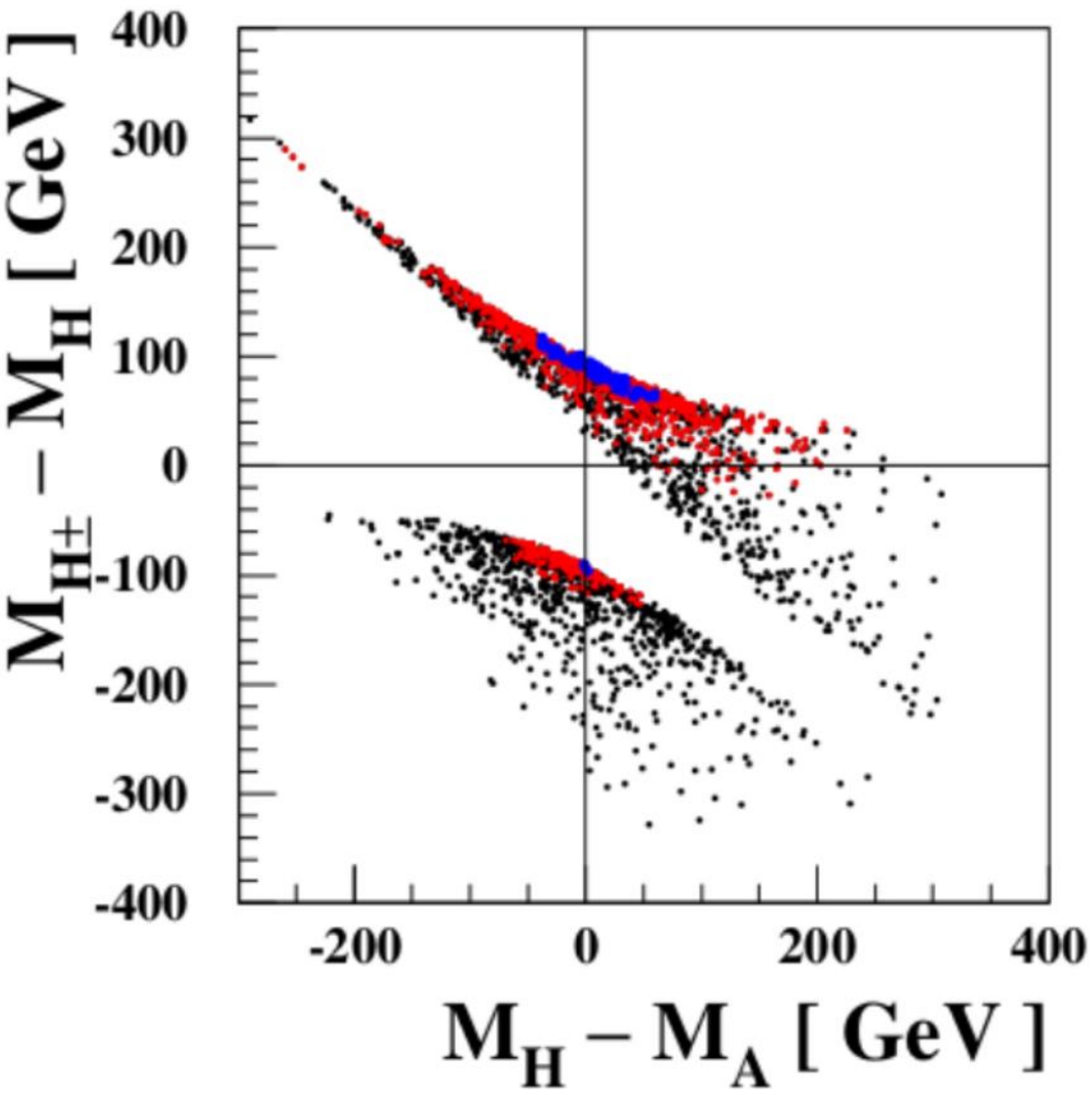
4. Scatter plots of $M_{H^\pm} - M_A$ versus $M_H - M_A$

- The correlations among the mass differences
- The heavy Higgs masses squared are scanned up to $(1.5\ \mathrm{TeV})^2$
  - The red points are for $M_{H^\pm} > 500$ GeV, the blue points are for $M_{H^\pm} > 900$ GeV.
- As $M_{H^\pm}$ increases, the mass difference between the charged and neutral Higgs bosons $\left|M_{H^\pm} - M_{A,H}\right|$ converges to the value of about 100 GeV
- $|M_H - M_A| \lesssim 250\ (50)$ GeV when $M_{H^\pm} > 500\ (900)$ GeV



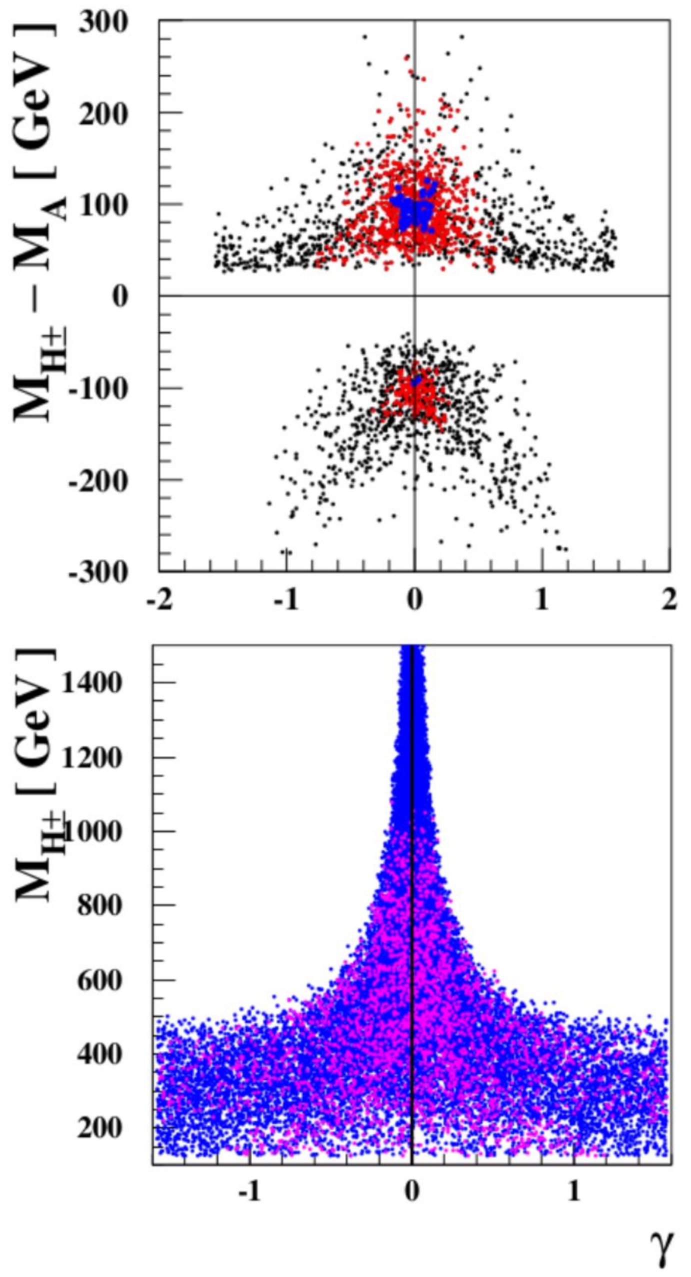
4. Scatter plots of $M_{H^\pm} - M_H$ versus $M_H - M_A$

- The correlations among the mass differences
- The heavy Higgs masses squared are scanned up to $(1.5\ \mathrm{TeV})^2$
  - The red points are for $M_{H^\pm} > 500$ GeV, the blue points are for $M_{H^\pm} > 900$ GeV.
- As $M_{H^\pm}$ increases, the mass difference between the charged and neutral Higgs bosons $|M_{H^\pm} - M_{A,H}|$ converges to the value of about 100 GeV
- $|M_H - M_A| \lesssim 250\ (50)$ GeV when $M_{H^\pm} > 500\ (900)$ GeV

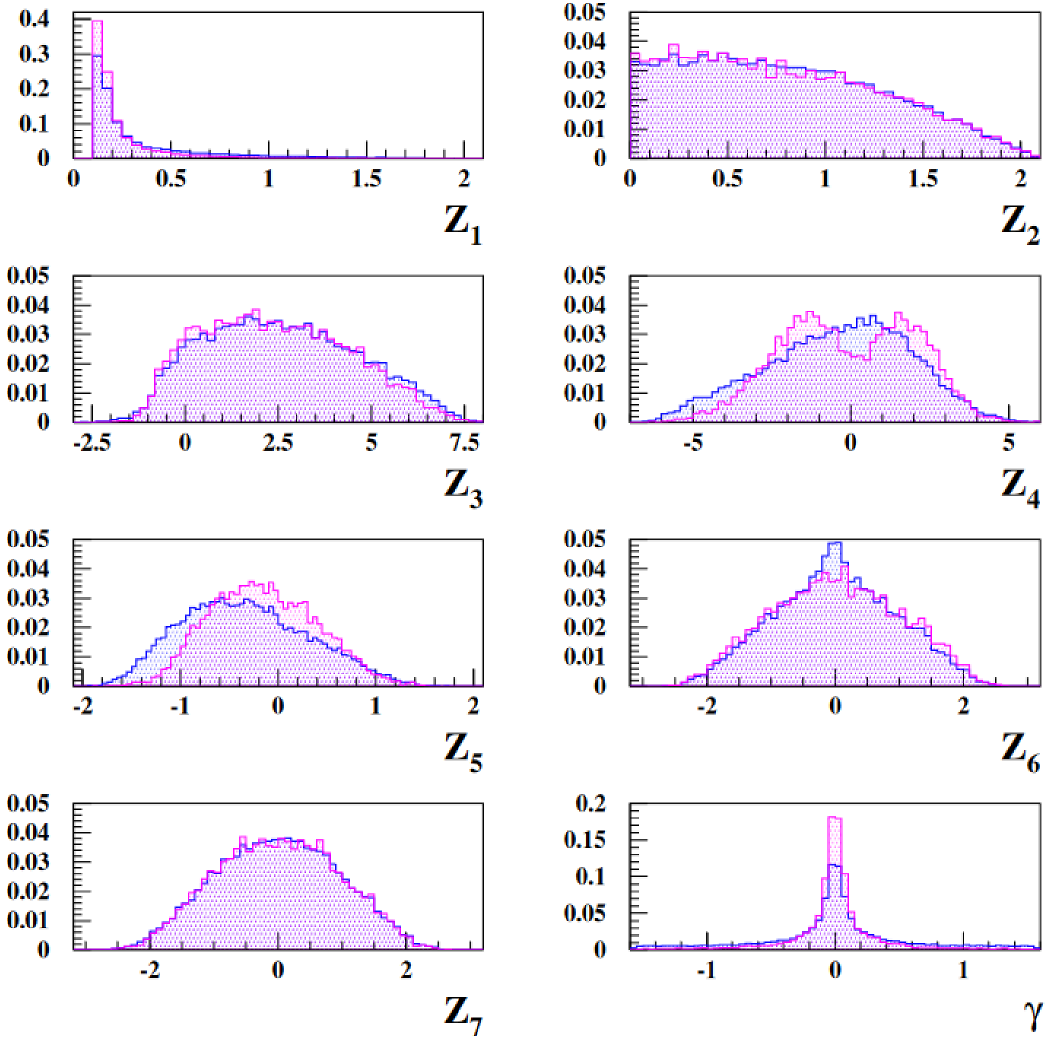


4. Scatter plots of $M_{H^\pm}$ versus γ

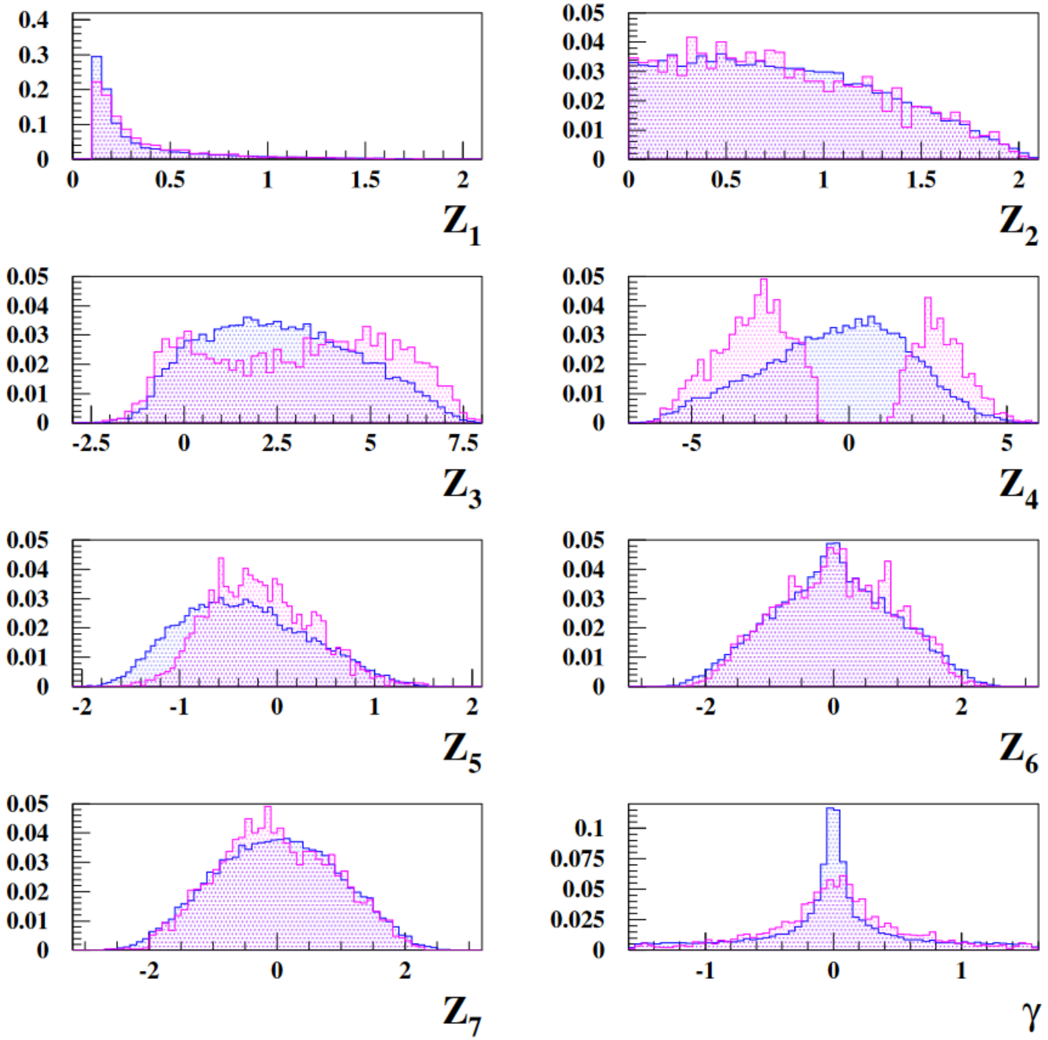
- The correlations among the heavy Higgs-boson masses and the mixing angle γ.
- The heavy Higgs masses squared are scanned up to $(1.5\ \mathrm{TeV})^2$
  - The red points are for $M_{H^\pm} > 500$ GeV, the blue points are for $M_{H^\pm} > 900$ GeV.
  - The blue points are for UNIT $\oplus$ BFB constraints, the magenta points are for UNIT $\oplus$ BFB $\oplus$ ELW$_{95\%}$ constraints.



4. The normalized distributions of couplings



UNIT ⊕ BFB ⊕ ELW$_{95\%}$ **[PDG]**



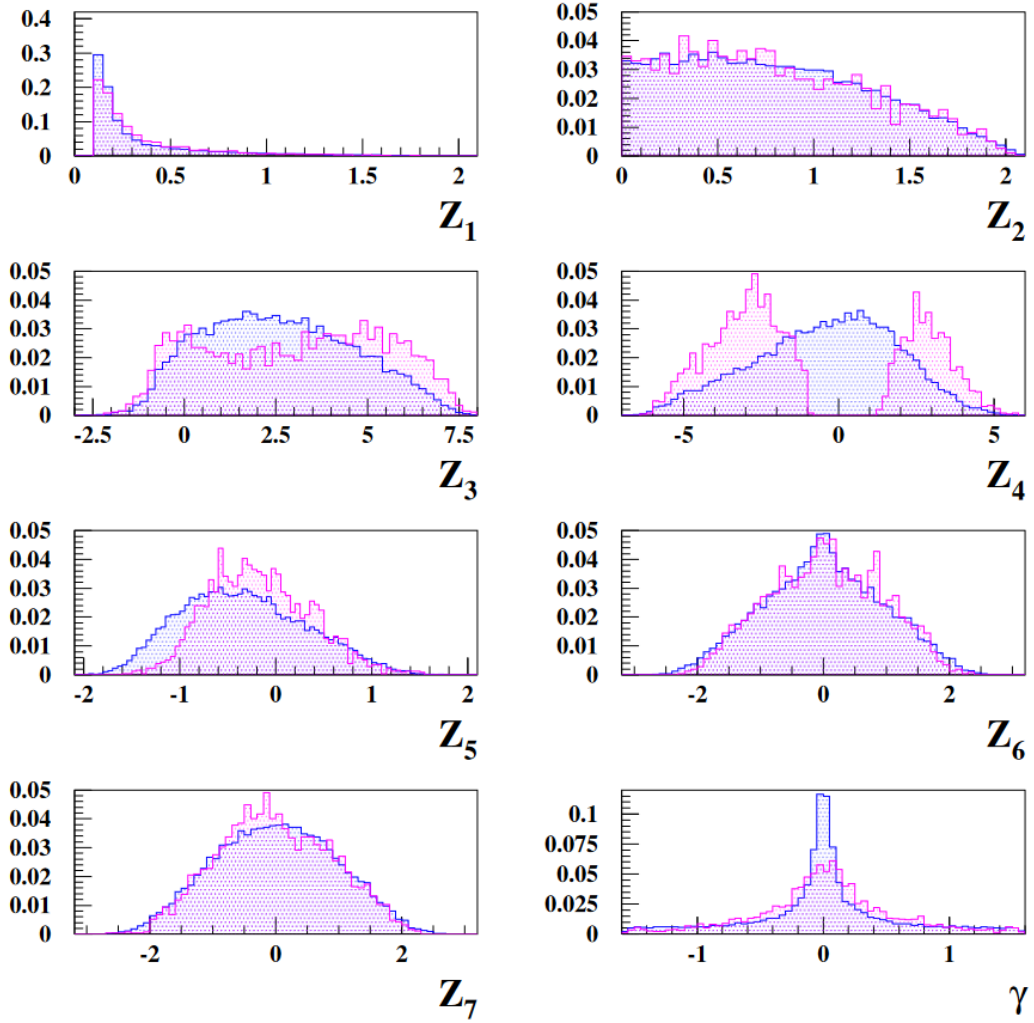
UNIT ⊕ BFB ⊕ ELW$_{95\%}$ **[CDF]**

4. The normalized distributions of couplings

- In $\mathrm{UNIT} \oplus \mathrm{BFB} \oplus \mathrm{ELW}_{95\%}$ constraints,
  - Z_1 and γ take values near to 0 less likely,
  - while Z_5 positive ones more likely,
  - Z_2 and Z_7 distributions remain almost the same since they are irrelevant to the masses of Higgs bosons and the mixing angle γ,
  - Z_3 and Z_6 distributions undergo some change,
  - **Z_4 distribution changes most drastically excluding the region $|Z_4| \lesssim 1$**
    - Reminder: $Z_4 = \frac{1}{v^2}\left[s_\gamma^2 M_h^2 + c_\gamma^2 M_H^2 + M_A^2 - 2M_{H^\pm}^2\right]$
    - Taking $\gamma = 0$ and $M_H = M_A$ for the simplicity,

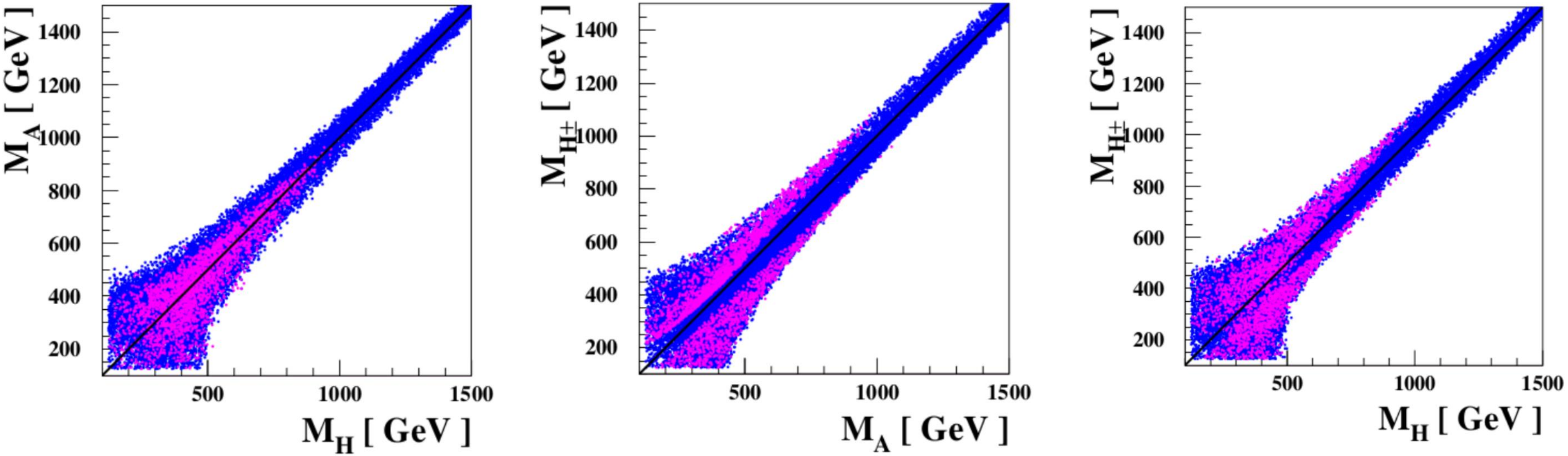
$$Z_4 = v^{-2} 4\Delta_M \bar{M}$$

$$\Delta_M \equiv M_A - M_{H^\pm}, \qquad \bar{M} \equiv (M_A + M_{H^\pm})/2$$



∴ **Z_4 can not vanish in the presence of the misalignment between $M_{H^\pm}$ and M_A,** which is required to achieve the sizable central value of the T parameter.

4. Scatter plots of M_A versus M_H



- The mass difference $|\Delta_M|$ approaches to 100 GeV as $M_{H^\pm}$ grows.
- $\bar{M} \equiv \frac{\left(M_A + M_{H^\pm}\right)}{2} = \frac{Z_4 v^2}{4\Delta_M} \le \frac{v^2}{4}\left(\frac{Z_4}{\Delta_M}\right)_{\max} \le \frac{v^2}{4}\frac{|Z_4|_{\max}}{|\Delta_M|_{\min}}.$
- **We find that $\left(\frac{Z_4}{\Delta_M}\right)_{\max} \sim \frac{6}{(100\ \mathrm{GeV})}$ leading to the upper limit of about 1 TeV.**

5. Conclusion

- We consider the implication of the recent CDF W-mass anomaly in the general framework of 2HDM.
- We find that the large deviation of the $\boldsymbol{S}$ and $\boldsymbol{T}$ parameters from their SM values of zero leads to **the upper limit of about 1 TeV on the masses of the heavy charged and neutral Higgs bosons** when it is combined with the theoretical constraints from the perturbative unitarity (UNIT) and for the Higgs potential to be bounded from below (BFB).

Appendix - Higgs basis 2HDM potential

- The potential parameters $Y_{1,2,3}$ and Z_{1-7} in the Higgs basis
  - $Y_1 = \mu_1^2 c_\beta^2 + \mu_2^2 s_\beta^2 + \Re\mathrm{e}\left(m_{12}^2 e^{i\xi}\right) s_{2\beta}$,
  - $Y_2 = \mu_1^2 s_\beta^2 + \mu_2^2 c_\beta^2 - \Re\mathrm{e}\left(m_{12}^2 e^{i\xi}\right) s^{2\beta}$,
  - $Y_3 = -(\mu_1^2 - \mu_2^2) c_\beta s_\beta + \Re\mathrm{e}\left(m_{12}^2 e^{i\xi}\right) c_{2\beta} + i\Im\mathrm{m}\left(m_{12}^2 e^{i\xi}\right)$,

$\lambda_{345} = \frac{(\lambda_3+\lambda_4)}{2} + \Re\mathrm{e}\left(\lambda_5 e^{2i\xi}\right)$

  - $Z_1 = \lambda_1 c_\beta^4 + \lambda\_2 s_\beta^4 + 2\lambda_{345} c_\beta^2 s_\beta^2 + \left[\Re\mathrm{e}\left(\lambda_6 e^{i\xi}\right) c_\beta^2 + \Re\mathrm{e}\left(\lambda_7 e^{i\xi}\right) s_\beta^2\right] s_{2\beta}$,
  - $Z_3 = \lambda_3 + 2(\lambda_1 + \lambda_2 - 2\lambda_{345}) c_\beta^2 s_\beta^2 - \left[\Re\mathrm{e}\left(\lambda_6 e^{i\xi}\right) - \Re\mathrm{e}\left(\lambda_7 e^{i\xi}\right)\right] c_{2\beta} s_{2\beta}$,
  - $Z_6 = \left(-\lambda_1 c_\beta^2 + \lambda_2 s_\beta^2\right) s_{2\beta} + 2\lambda_{345} c_{2\beta} c_\beta s_\beta + \Re\mathrm{e}\left(\lambda_6 e^{i\xi}\right)\left(c_\beta^2 - 3 s_\beta^2\right) c_\beta^2 + \Re\mathrm{e}\left(\lambda_7 e^{i\xi}\right)\left(3 c_\beta^2 - s_\beta^2\right) s_\beta^2$
    $+i\ |\Im\mathrm{m}\left(\lambda_5 e^{2i\xi}\right) s_{2\beta} + \Im\mathrm{m}\left(\lambda_6 e^{i\xi}\right) c_\beta^2 + \Im\mathrm{m}\left(\lambda_7 e^{i\xi}\right) s_\beta^2]$,
    - $Z_1 \leftrightarrow Z_2,\ Z_3 \leftrightarrow Z_4,\ Z_6 \leftrightarrow Z_7;\ c_\beta \leftrightarrow s_\beta, \lambda_3 \leftrightarrow \lambda_4, \left(\lambda_5 e^{2i\xi}\right) \leftrightarrow \left(\lambda_5 e^{2i\xi}\right)^*, \left(\lambda_{6,7} e^{i\xi}\right) \leftrightarrow -\left(\lambda_{6,7} e^{i\xi}\right)^*$
  - $Z_5 = (\lambda_1 + \lambda_2 - 2\lambda_{345}) c_\beta^2 s_\beta^2 + \Re\mathrm{e}\left(\lambda_5 e^{2i\xi}\right) - \left[\Re\mathrm{e}\left(\lambda_6 e^{i\xi}\right) - \Re\mathrm{e}\left(\lambda_7 e^{i\xi}\right)\right] c_{2\beta} c_\beta s_\beta$
    $+i\left[\Im\mathrm{m}\left(\lambda_5 e^{2i\xi}\right) c_{2\beta} - \Im\mathrm{m}\left(\lambda_6 e^{i\xi}\right) c_\beta s_\beta + \Im\mathrm{m}\left(\lambda_7 e^{i\xi}\right) c_{\beta s_\beta}\right]$

Appendix – Φ basis 2HDM potential

- Potential $V_{\Phi} = \mu_1^2(\Phi_1^\dagger\Phi_1) + \mu_2^2(\Phi_2^\dagger\Phi_2) + m_{12}^2(\Phi_1^\dagger\Phi_2) + m_{12}^{*2}(\Phi_2^\dagger\Phi_1)$

$$+\lambda_1(\Phi_1^\dagger\Phi_1)^2 + \lambda_2(\Phi_2^\dagger\Phi_2)^2 + \lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$

$$+\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \lambda_5^*(\Phi_2^\dagger\Phi_1)^2 + \lambda_6(\Phi_1^\dagger\Phi_1)(\Phi_1^\dagger\Phi_2) + \lambda_6^*(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_1)$$

$$+\lambda_7(\Phi_2^\dagger\Phi_2)(\Phi_1^\dagger\Phi_2) + \lambda_7^*(\Phi_2^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)$$

- Tadpole conditions: the square of the charged Higgs-boson mass
  - $\mu_1^2 = -v^2\left[\lambda_1 c_\beta^2 + \frac{1}{2}\lambda_3 s_\beta^2 + c_\beta s_\beta \Re\mathrm{e}\left(\lambda_6 e^{i\xi}\right)\right] + s_\beta^2 M_{H^\pm}^2$,
  - $\mu_1^2 = -v^2\left[\lambda_1 c_\beta^2 + \frac{1}{2}\lambda_3 s_\beta^2 + c_\beta s_\beta \Re\mathrm{e}\left(\lambda_6 e^{i\xi}\right)\right] + s_\beta^2 M_{H^\pm}^2$,
  - $\Im\mathrm{m}\left(m_{12}^2 e^{i\xi}\right) = -\frac{v^2}{2}\left[\lambda_1 c_\beta^2 + \frac{1}{2}\lambda_3 s_\beta^2 + c_\beta s_\beta \Re\mathrm{e}\left(\lambda_6 e^{i\xi}\right)\right] + s_\beta^2 M_{H^\pm}^2$.
- The square of the charged Higgs-boson mass
  - $M_{H^\pm}^2 = -\frac{\Re\mathrm{e}\left(m_{12}^2 e^{i\xi}\right)}{c_\beta s_\beta} = -\frac{v^2}{2c_\beta s_\beta}\left[\lambda_4 c_\beta s_\beta + 2c_\beta s_\beta \Re\mathrm{e}\left(\lambda_5 e^{2i\xi}\right) + c_\beta^2\, \Re\mathrm{e}\left(\lambda_6 e^{i\xi}\right) + s_\beta^2 \Re\mathrm{e}\left(\lambda_7 e^{i\xi}\right)\right]$

Appx. - Interactions with massive vector bosons

- The cubic interactions of the neutral and charged Higgs bosons with the massive gauge bosons Z and $W^{\pm}$ are described by the three interaction lagrangians:

$$\mathcal{L}_{HVV} = gM_W \left(W_\mu^+ W^{-\mu} + \frac{1}{2c_W^2} Z_\mu Z^\mu \right) \sum_i g_{H_iVV} H_i \,,$$

$$\mathcal{L}_{HHZ} = \frac{g}{2c_W} \sum_{i>j} g_{H_iH_jZ} Z^\mu \left(H_i \overleftrightarrow{\partial}_\mu H_j \right) ,$$

$$\mathcal{L}_{HH^\pm W^\mp} = -\frac{g}{2} \sum_i g_{H_iH^+W^-} W^{-\mu} \left(H_i i \overleftrightarrow{\partial} H^+ \right) + \text{h.c.}$$

  - The normalized couplings
    - $g_{H_iVV} = O_{\varphi_1 i}, \quad g_{H_iH_jZ} = \text{sign}[\det(O)] \epsilon_{ijk} g_{H_kVV} = \text{sign}[\det(O)] \epsilon_{ijk} O_{\varphi_1 k}, \quad g_{H_iH^+W^-} = -O_{\varphi_2 i} + i O_{ai}$
    - Sum rules

$$\sum_{i=1}^{3} g_{H_iVV}^2 = 1 \quad \text{and} \quad g_{H_iVV}^2 + \left| g_{H_iH^+W^-} \right|^2 = 1 \quad \text{for each} \quad i = 1,2,3$$

Appx. - Input parameters in CPV

- Using the tadpole conditions and Higgs masses

$$\{Y_1, Y_2, Y_3; Z_1, Z_2, Z_3, Z_4, Z_5, Z_6, Z_7\} \to \mathcal{I}' = \left\{v; M_{H^\pm}, M_{H_1}, M_{H_2}, M_{H_3}, \{O_{3\times3}\}; Z_3; Z_2, Z_7\right\}$$

  - $O = O_\gamma O_\eta O_\omega \equiv \begin{pmatrix} c_\gamma & s_\gamma & 0 \\ -s_\gamma & c_\gamma & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_\eta & 0 & s_\eta \\ 0 & 1 & 0 \\ -s_\eta & 0 & c_\eta \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\omega & s_\omega \\ 0 & -s_\omega & c_\omega \end{pmatrix} = \begin{pmatrix} c_\gamma c_\eta & s_\gamma c_\omega - c_\gamma s_\eta s_\omega & s_\gamma s_\omega + c_\gamma s_\eta c_\omega \\ -s_\gamma c_\eta & c_\gamma c_\omega + s_\gamma s_\eta s_\omega & c_\gamma s_\omega - s_\gamma s_\eta c_\omega \\ -s_\eta & -c_\eta s_\omega & c_\eta c_\omega \end{pmatrix}$

- $Z_1 = \frac{1}{2v^2}\left(M_{H_1}^2 O_{\varphi_1 1}^2 + M_{H_2}^2 O_{\varphi_1 2}^2 + M_{H_3}^2 O_{\varphi_1 3}^2\right)$
- $Z_4 = \frac{1}{v^2}\left[M_{H_1}^2\left(O_{\varphi_2 1}^2 + O_{a1}^2\right) + M_{H_2}^2\left(O_{\varphi_2 2}^2 + O_{a2}^2\right) + M_{H_3}^2\left(O_{\varphi_2 3}^2 + O_{a3}^2\right) - 2M_{H^\pm}^2\right]$
- $Z_5 = \frac{1}{2v^2}\left[M_{H_1}^2\left(O_{\varphi_2 1}^2 - O_{a1}^2\right) + M_{H_2}^2\left(O_{\varphi_2 2}^2 - O_{a2}^2\right) + M_{H_3}^2\left(O_{\varphi_2 3}^2 - O_{a3}^2\right)\right]$
  $\quad - \frac{i}{v^2}\left(M_{H_1}^2 O_{\varphi_2 1} O_{a1} + M_{H_2}^2 O_{\varphi_2 2} O_{a2} + M_{H_3}^2 O_{\varphi_2 3} O_{a3}\right)$
- $Z_6 = \frac{1}{v^2}\left(M_{H_1}^2 O_{\varphi_1 1} O_{\varphi_2 1} + M_{H_2}^2 O_{\varphi_1 2} O_{\varphi_2 1} + M_{H_3}^2 O_{\varphi_1 3} O_{\varphi_2 3}\right)$
  $\quad - \frac{i}{v^2}\left(M_{H_1}^2 O_{\varphi_1 1} O_{a1} + M_{H_1}^2 O_{\varphi_1 2} O_{a2} + M_{H_3}^2 O_{\varphi_1 3} O_{a3}\right)$

Yukawa couplings in the Higgs basis

- The Yukawa couplings

$$-\mathcal{L}_Y = \sum_{k=1,2} \overline{Q_L^0} \boldsymbol{y}_k^u \widetilde{\mathcal{H}_k} u_R^0 + \overline{Q_L^0} \boldsymbol{y}_k^d \mathcal{H}_k d_R^0 + \overline{L_L^0} \boldsymbol{y}_k^e \mathcal{H} e_R^0 + \text{h.c.}$$

$$\widetilde{\mathcal{H}_i} = i\tau_2 \mathcal{H}_i^*$$

- The mass terms in the Yukawa interactions

$$-\mathcal{L}_{Y\,\text{mass}} = \frac{v}{\sqrt{2}} \left(\overline{u_L^0} \boldsymbol{y}_1^u u_R^0 + \overline{d_L^0} \boldsymbol{y}_1^d d_R^0 + \overline{e_L^0} \boldsymbol{y}_1^e e_R^0 + \text{h.c.} \right)$$

$$u_L^0 = \mathcal{U}_{u_L} u_L, \quad d_L^0 = \mathcal{U}_{d_L} d_L, \quad e_L^0 = \mathcal{U}_{e_L} e_L, \quad u_R^0 = \mathcal{U}_{u_R} u_R, \quad d_R^0 = \mathcal{U}_{d_R} d_R, \quad e_R^0 = \mathcal{U}_{e_R} e_R$$

$$\mathbf{M}_u = \frac{v}{\sqrt{2}} \mathcal{U}_{u_L}^\dagger \boldsymbol{y}_1^u \mathcal{U}_{u_R} = \text{diag}\,(m_u, m_c, m_t),$$

$$\mathbf{M}_d = \frac{v}{\sqrt{2}} \mathcal{U}_{d_L}^\dagger \boldsymbol{y}_1^d \mathcal{U}_d = \text{diag}\,(m_d, m_s, m_b),$$

$$\mathbf{M}_e = \frac{v}{\sqrt{2}} \mathcal{U}_{e_L}^\dagger \boldsymbol{y}_1^e \mathcal{U}_{e_R} = \text{diag}\left(m_e, m_\mu, m_\tau\right)$$

- Finally $-\mathcal{L}_{Y\,\text{mass}} = \overline{u_L}\, \mathbf{M}_u u_R + \overline{d_L}\, \mathbf{M}_d d_R + \overline{e_L} \mathbf{M}_e e_R + \text{h.c.}$

Couplings of Higgs bosons

- The couplings of the neutral Higgs bosons to two fermions

$$-\mathcal{L}_{H\bar{f}f} = \frac{1}{v}[\bar{u}\mathbf{M}_u u]\varphi_1 + [\bar{u}(\mathbf{h}_u^H + \mathbf{h}_u^A\gamma_5)u]\varphi_2 + [\bar{u}(-i\mathbf{h}_u^A - i\mathbf{h}_u^H\gamma_5)u]a$$
$$+\frac{1}{v}\left[\bar{d}\mathbf{M}_d d\right]\varphi_1 + \left[\bar{d}\left(\mathbf{h}_d^H + \mathbf{h}_d^A\gamma_5\right)d\right]\varphi_2 + \left[\bar{d}\left(i\mathbf{h}_d^A + i\mathbf{h}_d^H\gamma_5\right)d\right]a$$
$$+\frac{1}{v}[\bar{e}\mathbf{M}_e e]\varphi_1 + [\bar{e}(\mathbf{h}_e^H + \mathbf{h}_e^A\gamma_5)e]\varphi_2 + [\bar{e}(i\mathbf{h}_e^A + i\mathbf{h}_e^H\gamma_5)e]a$$

  - Hermitian and three anti-Hermitian Yukawa coupling matrices

$$\mathbf{h}_f^H \equiv \frac{\mathbf{h}_f + \mathbf{h}_f^\dagger}{2}, \qquad \mathbf{h}_f^A \equiv \frac{\mathbf{h}_f - \mathbf{h}_f^\dagger}{2}, \qquad \mathbf{h}_f \equiv \frac{1}{\sqrt{2}}\mathcal{U}_{f_L}^\dagger \boldsymbol{y}_2^f\, \mathcal{U}_{f_R}, \quad (f = u, d, e)$$

- The couplings of the charged Higgs bosons to two fermions

$$-\mathcal{L}_{H^\pm \overline{f_\uparrow} f_\downarrow} = -\sqrt{2}\left[\overline{u_R}\left(\boldsymbol{h}_u^\dagger V\right)d_L\right]H^+ + \sqrt{2}[\overline{u_L}(V\boldsymbol{h}_d)d_R]H^+ + \sqrt{2}\,[\overline{\nu_L}\boldsymbol{h}_e e_R]H^+ + \mathrm{h.c.}.$$

  - $\mathcal{U}_{u_L}^\dagger \mathcal{U}_{d_L} = V_{\mathrm{CKM}} \equiv V$ and $\boldsymbol{h}_u = t_\beta^{-1}\frac{\boldsymbol{M}_u}{v},\ \boldsymbol{h}_d = -t_\beta\frac{\boldsymbol{M}_d}{v},\ \boldsymbol{h}_e = -t_\beta\frac{\boldsymbol{M}_e}{v}$