Open-closed string field theory using Kaku vertex

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場の理論と弦理論 2023

弦の場の理論

弦の場の理論 = 第 2 量子化した弦理論

$$
\begin{aligned}
S = \frac{1}{2}\omega(\Psi, Q_o\Psi) + \sum_{k\geq 2}\frac{1}{k+1}\omega(\Psi, m_k(\Psi^k)) \\
+ \sum_{p\geq 1, q\geq 0}\frac{1}{p!(q+1)}\omega(\Psi, n_{p,q}(\Phi^p, \Psi^q)) \\
+ \frac{1}{2!}\omega(\Phi, Q_c\Phi) + \sum_{k\geq 2}\frac{1}{(k+1)!}\omega(\Phi, L_k(\Phi^k))
\end{aligned}
$$

1. 第 1 量子化の結果と矛盾しないように相互作用を入れる.
2. 運動方程式の古典解を見つける.
3. その解周りで振幅や物理的状態を調べる.

今日注目したいもの

D ブレーン 0 枚の古典解
(タキオン真空解)

D ブレーンが一枚も無く開弦が励起出来ない = 閉弦しかいない

$$Q^{\mathrm{tv}}\Psi = 0, \quad \Psi \simeq 0 + Q^{\mathrm{tv}}\Lambda$$

タキオン真空解周りで閉弦の振幅を確認したい.

開弦の場の理論

Witten 型 (E.Witten '86) $\Longleftrightarrow$ 光円錐型 (T.Kugo-B.Zweibach '92)

$$\frac{1}{2}\omega(\Psi, Q_o\Psi) + \frac{1}{3}\omega(\Psi, m_2^{\mathrm{W}}(\Psi,\Psi))$$

$$\frac{1}{2}\omega(\Psi, Q_o\Psi) + \frac{1}{3}\omega(\Psi, m_2^{\mathrm{lc}}(\Psi,\Psi)) + \frac{1}{4}\omega(\Psi, m_3^{\mathrm{lc}}(\Psi,\Psi,\Psi))$$

Witten 型:

○ タキオン真空解

× 閉弦

光円錐型:

× タキオン真空解

○ 閉弦

- タキオン真空解が作れて閉弦の振幅を計算できる理論はまだ無い.

$\Downarrow$

- 両者の欠点を克服した理論を見つけ
  タキオン真空解周りで閉弦振幅を見たい.

開弦の場の理論

Witten 型 (E.Witten '86) ⟺ Kaku 型 ⟺ 光円錐型 (T.Kugo-B.Zweibach '92)

Witten 型:

$$\frac{1}{2}\omega(\Psi, Q_o\Psi) + \frac{1}{3}\omega(\Psi, m_2^{\mathrm{W}}(\Psi,\Psi))$$

- ○ タキオン真空解
- × 閉弦

光円錐型:

$$\frac{1}{2}\omega(\Psi, Q_o\Psi) + \frac{1}{3}\omega(\Psi, m_2^{\mathrm{lc}}(\Psi,\Psi)) + \frac{1}{4}\omega(\Psi, m_3^{\mathrm{lc}}(\Psi,\Psi,\Psi))$$

- × タキオン真空解
- ○ 閉弦

- タキオン真空解が作れて閉弦の振幅を計算できる理論はまだ無い.

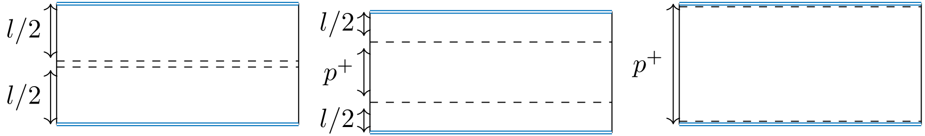
$\Downarrow$

- 両者の欠点を克服した理論を見つけ
タキオン真空解周りで閉弦振幅を見たい.

今回は Witten 型と光円錐型の中間の理論である Kaku 型の理論に注目する.

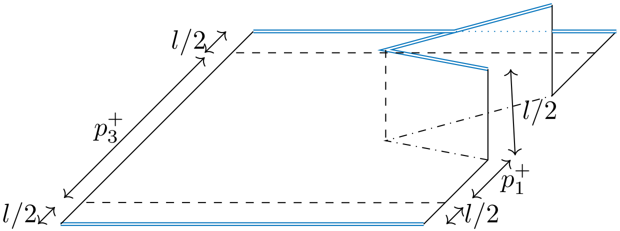
Kaku vertex

(M.Kaku '88)



Witten 型
$l \to \infty$

光円錐型
$l = 0$

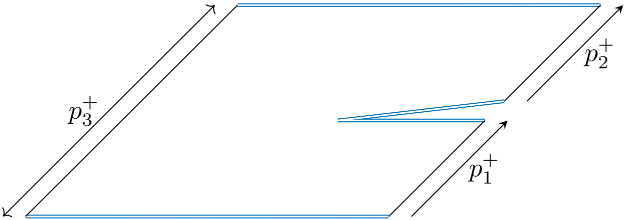


Kaku vertex

(M.Kaku '88)

Witten 型 $l \to \infty$

光円錐型 $l = 0$



Kaku vertex

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Witten 型
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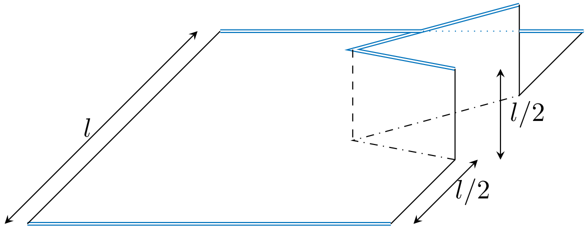

Kaku 理論

Witten 型 ($l = \infty$) $\longleftarrow$ l $\longrightarrow$ 光円錐型 ($l = 0$)

Witten 型 ($l = \infty$)

$$\frac{1}{2}\omega(\Psi, Q_o\Psi) + \frac{1}{3}\omega(\Psi, m_2^{\mathrm{W}}(\Psi,\Psi)) + \underline{\hspace{4cm}}$$

+ (開弦-閉弦相互作用)

◯ タキオン真空解

× 閉弦

l

$$\frac{1}{2}\omega(\Psi, Q_o\Psi) + \frac{1}{3}\omega(\Psi, m_2^{l}(\Psi,\Psi)) + \frac{1}{4}\omega(\Psi, m_3^{l}(\Psi,\Psi,\Psi))$$

$$+ \sum_{p,q}\frac{1}{p!(q+1)}\omega(\Psi, n_{p,q}(\Phi^p;\Psi^q)) + \sum_{r}\frac{1}{(r+1)!}\omega(\Phi, L_r(\Phi^r))$$

◯ タキオン真空解

? 閉弦

光円錐型 ($l = 0$)

$$\frac{1}{2}\omega(\Psi, Q_o\Psi) + \frac{1}{3}\omega(\Psi, m_2^{\mathrm{lc}}(\Psi,\Psi)) + \frac{1}{4}\omega(\Psi, m_3^{\mathrm{lc}}(\Psi,\Psi,\Psi))$$

+ (開弦-閉弦相互作用)

+ (閉弦の場の理論)

× タキオン真空解

◯ 閉弦

開弦-閉弦場の理論を考える.

OCHA

以下の関係式を満たす (n, L) を Open-Closed-Homotopy-Algebra と呼ぶ.

(H.Kajiura-J.Stasheff '04)

$$n_{p,q} : (p \text{ 個の開弦}) \times (q \text{ 個の開弦}) \to (\text{開弦}), \quad L_r : (r \text{ 個の閉弦}) \to (\text{閉弦})$$

$$\begin{aligned}
0 = &\sum_{\sigma} \sum_{m=1}^{k} (-1)^{\epsilon(\sigma)} \frac{1}{m!(k-m)!} n_{k-m+1,t}\Big(L_m\big(y_{\sigma(1)}, \ldots, y_{\sigma(m)}\big), y_{\sigma(m+1)}, \ldots, y_{\sigma(k)}; x_1, \ldots, x_t\Big) \\
&+ \sum_{\sigma} \sum_{m=0}^{k} \sum_{j=0}^{t} \sum_{i=0}^{t-j} (-1)^{\mu_{m,i}(\sigma)} \frac{1}{m!(k-m)!} \\
&\times n_{m,t-j+1}\Big(y_{\sigma(1)}, \ldots, y_{\sigma(m)}; x_1, \ldots, x_i, \\
&\qquad n_{k-m,j}\big(y_{\sigma(m+1)}, \ldots, y_{\sigma(k)}; x_{i+1}, \ldots, x_{i+j}\big), x_{i+j+1}, \ldots, x_t\Big)
\end{aligned}$$

$n_{0,q} = m_q^l$ として関係式を満たすように閉弦の理論を作る.

OCHA

以下の関係式を満たす (n, L) を Open-Closed-Homotopy-Algebra と呼ぶ.

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$$n_{p,q} : (p\text{ 個の開弦}) \times (q\text{ 個の閉弦}) \to (\text{開弦}), \quad L_r : (r\text{ 個の閉弦}) \to (\text{閉弦})$$

$$\begin{aligned}0 = &\sum_{\sigma}\sum_{m=1}^{k}(-1)^{\epsilon(\sigma)}\frac{1}{m!(k-m)!}n_{k-m+1,t}\Big(L_m\big(y_{\sigma(1)},\dots,y_{\sigma(m)}\big),y_{\sigma(m+1)},\dots,y_{\sigma(k)};x_1,\dots,x_t\Big)\\ &+\sum_{\sigma}\sum_{m=0}^{k}\sum_{j=0}^{t}\sum_{i=0}^{t-j}(-1)^{\mu_{m,i}(\sigma)}\frac{1}{m!(k-m)!}\\ &\times n_{m,t-j+1}\Big(y_{\sigma(1)},\dots,y_{\sigma(m)};x_1,\dots,x_i,\\ &\qquad n_{k-m,j}\big(y_{\sigma(m+1)},\dots,y_{\sigma(k)};x_{i+1},\dots,x_{i+j}\big),x_{i+j+1},\dots,x_t\Big)\end{aligned}$$

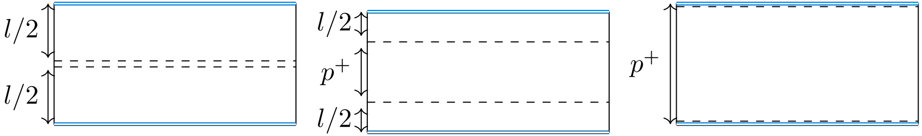
$n_{0,q} = m_q^l$ として関係式を満たすように閉弦の理論を作る.

ex) $k = 1, t = 0$

$$\underbrace{Q_o}_{\text{開弦の BRS 演算子}}(n_{1,0}(\Phi)) = n_{1,0}(\underbrace{Q_c}_{\text{閉弦の BRS 演算子}}(\Phi))$$

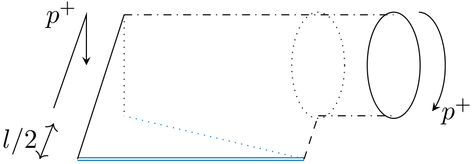
$$\frac{1}{2}\omega(\Psi, Q_o\Psi) + \omega(\Psi, n_{1,0}(\Phi)) + \frac{1}{2}\omega(\Phi, Q_c\Phi) + \dots$$

Kaku 理論の開弦-開弦 vertex



Witten 型
$l \to \infty$

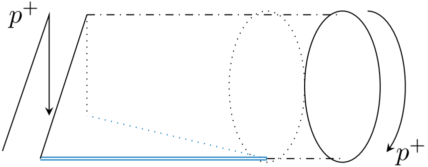
光円錐型
$l = 0$



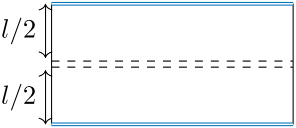
Kaku 理論の開弦-開弦 vertex

Witten 型
$l \to \infty$

光円錐型
$l = 0$

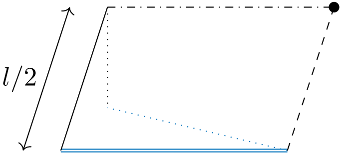


Kaku 理論の開弦-開弦 vertex

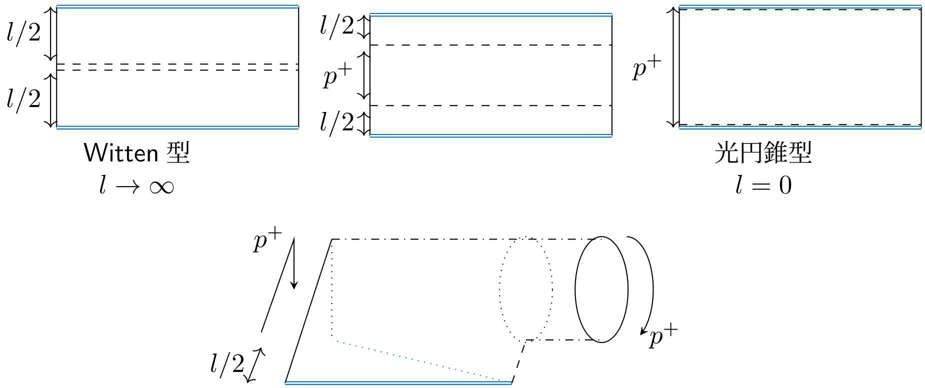


Witten 型
$l \to \infty$

光円錐型
$l = 0$



Kaku 理論の開弦-開弦 vertex



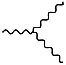
Kaku 型の開弦-閉弦の場の理論

Witten 型 ($l = \infty$)	$\longleftarrow \quad l \quad \longrightarrow$	光円錐型 ($l = 0$)
$S^{\mathrm{W}}_{\text{open}}$	$S^{l}_{\text{open}} + \omega(\Psi, n^{l}_{1,0}(\Phi)) + \frac{1}{2}\omega(\Psi, n^{l}_{1,1}(\Phi;\Psi))$	$S^{\mathrm{lc}}_{\text{open}} + \omega(\Psi, n^{\mathrm{lc}}_{1,0}(\Phi)) + \frac{1}{2}\omega(\Psi, n^{\mathrm{lc}}_{1,1}(\Phi;\Psi))$
	$+ \frac{1}{2}\omega(\Phi, Q_c\Phi) + \frac{1}{3!}\omega(\Phi, L^{\mathrm{lc}}_2(\Phi,\Phi))$	$+ \frac{1}{2}\omega(\Phi, Q_c\Phi) + \frac{1}{3!}\omega(\Phi, L^{\mathrm{lc}}_2(\Phi,\Phi))$

開弦:Kaku vertex　閉弦: 光円錐 vertex
タキオン真空解周りの振幅を調べる.

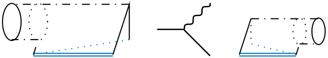
閉弦 3 点振幅

$\mathcal{A}_{3-\mathsf{closed}} =$



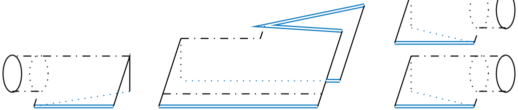
$\omega(\Phi, L_2^{\mathrm{lc}}(\Phi, \Phi))$

$+$



$\omega(n_{1,0}^l(\Phi), n_{1,1}^l(\Phi; n_{1,0}^l(\Phi)))$

$+$

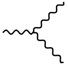


$\omega(n_{1,0}^l(\Phi), m_2^l(n_{1,0}^l(\Phi), n_{1,0}^l(\Phi)))$

外線は on-shell : $Q_c\Phi = 0$

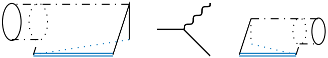
閉弦 3 点振幅

$\mathcal{A}_{3-\text{closed}} =$



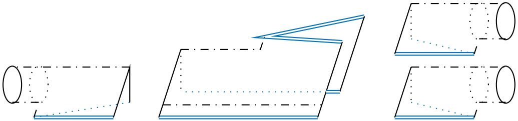
$\omega(\Phi, L_2^{\text{lc}}(\Phi, \Phi))$

$+$



$\omega(n^l_{1,0}(\Phi), n^l_{1,1}(\Phi; n^l_{1,0}(\Phi)))$

$+$



$\omega(n^l_{1,0}(\Phi), m^l_2(n^l_{1,0}(\Phi), n^l_{1,0}(\Phi)))$

外線は on-shell : $Q_c\Phi = 0$

$$\Downarrow \quad n^l_{1,0}(Q_c(\Phi)) = Q^{\text{tv},l}(n^l_{1,0}(\Phi))$$

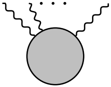
$n^l_{1,0}(\Phi)$ は $Q^{\text{tv},l}$-exact

$$n^l_{1,0}(\Phi) \simeq 0$$

純粋な閉弦の振幅だけが残る.

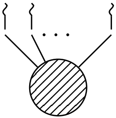
一般の閉弦振幅

$\mathcal{A}_{n-\mathsf{closed}} =$



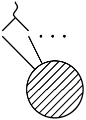
純粋な閉弦振幅

$+$



開弦-閉弦 vertex を含む

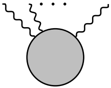
$+$



開弦-閉弦 vertex を含まない

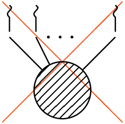
一般の閉弦振幅

$\mathcal{A}_{n-\text{closed}} =$



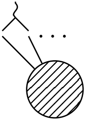
純粋な閉弦振幅

$+$



開弦-閉弦 vertex を含む

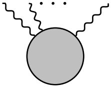
$+$



開弦-閉弦 vertex を含まない

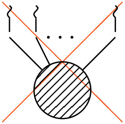
一般の閉弦振幅

$\mathcal{A}_{n-\text{closed}} =$



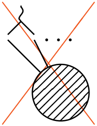
純粋な閉弦振幅

$+$



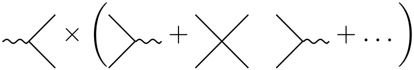
開弦-閉弦 vertex を含む

$+$



開弦-閉弦 vertex を含まない

必ず loop になるので tree 振幅には効いてこない



まとめと課題

まとめ

- 既に知られている開弦の Kaku vertex を開弦-閉弦 vertex に拡張し OCHA を満たす作用を考えた.
- タキオン真空解周りの tree レベルの閉弦振幅を計算すると，開弦を含まない純粋な閉弦振幅が得られることがわかった.

課題

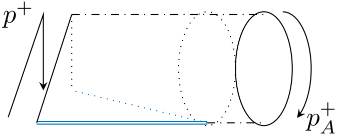
- パラメータを $l \to \infty$ に極限をとれば Kaku vertex は Witten vertex になる．そのため今回の結果を使えば Witten 理論で閉弦の振幅を計算する処方箋を与えるかもしれない.

目次

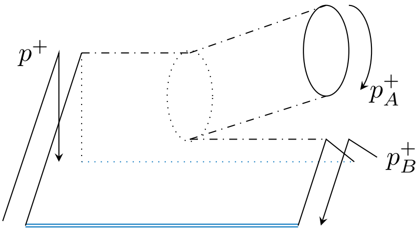
光円錐型開弦-閉弦場の理論

(Y.Saitoh-Y.Tanii '89)
(T.Kugo-T.Takahashi '98)
(T.Asakawa-T.Kugo-T.Takahashi '98)

$$
\begin{aligned}
S^{\mathrm{lc}}_{\text{open-closed}} &= \frac{1}{2}\omega(\Psi, Q\Psi) + \frac{1}{3}\omega(\Psi, m_2^{\mathrm{lc}}(\Psi,\Psi)) + \frac{1}{4}\omega(\Psi, m_3^{\mathrm{lc}}(\Psi,\Psi,\Psi)) \\
&\quad + \omega(\Psi, n^{\mathrm{lc}}_{1,0}(\Phi)) + \frac{1}{2}\omega(\Psi, n^{\mathrm{lc}}_{1,1}(\Phi;\Psi)) \\
&\quad + \frac{1}{2!}\omega(\Phi, Q_c\Phi) + \frac{1}{3!}\omega(\Phi, L_2^{\mathrm{lc}}(\Phi,\Phi))
\end{aligned}
$$



$n^{\mathrm{lc}}_{1,0}(A)$

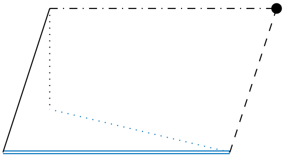


$n^{\mathrm{lc}}_{1,1}(A;B)$

Witten 型開弦-閉弦場の理論

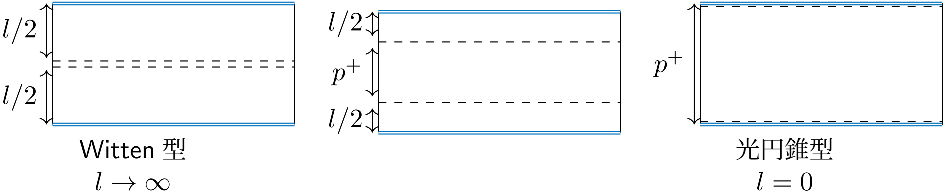
閉弦を背景に固定した理論 (B.Zweibach '92)

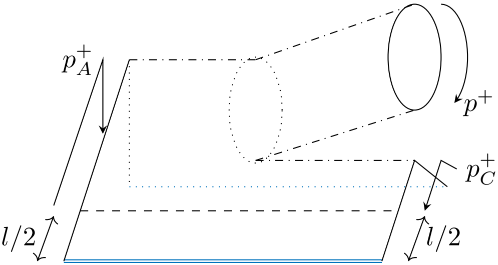
$$S^{\mathrm{W}}_{\text{open-closed}} = \frac{1}{2}\omega(\Psi, Q\Psi) + \frac{1}{3}\omega(\Psi, m_2^{\mathrm{W}}(\Psi, \Psi)) + \omega(\Psi, n^{\mathrm{W}}_{1,0}(\Phi))$$



$n^{\mathrm{W}}_{1,0}(\Phi)$

開弦-開弦-開弦 Kaku vertex





相互作用点が $l/2$ から $p^+ + l/2$ まで動く.

$l \to \infty$ 極限で moduli の積分領域が消えるので Witten 型の理論と無矛盾

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