Mixed boundary conditions and Celestial holography

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Flat space holographies

- A holography between
  Conformal field theory, $(d-2)$-dim. $\leftrightarrow$
  gravitational theory on asymptotically flat space, d-dim
- types of flat space holography:
  - Celestial holography
  - Codimension-2 Wedge holography

Flat space holographies

- A holography between
  Conformal field theory, $(d-2)$-dim. $\leftrightarrow$
  gravitational theory on asymptotically flat space, d-dim

- types of flat space holography:
  - Celestial holography $\leftarrow$ we will focus on this
  - Codimension-2 Wedge holography

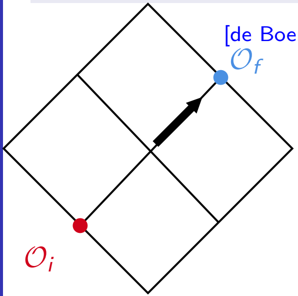
What is Celestial Holography?

Bottom up statement of Celestial holography

(scattering amplitude in boost basis)=(correlation function)

$$\left(\prod_{l=1,\ldots,n}\int d\omega_l\ \omega_l^{\Delta_l-1}\right)\mathcal{A}_{1\ldots n}(p_1,\ldots)=\langle\mathcal{O}_f^{\Delta_1}(z_1,\bar{z}_1)\ldots\mathcal{O}_i^{\Delta_n}\rangle,$$
$$p=\omega(1+z\bar{z},1-z\bar{z},z+\bar{z},i(z-\bar{z})).$$

[de Boer,Solodukhin(2003)],[Pasterski,Shao,Strominger(2017)]



- Celestial CFT operators $O_f(O_i)$ live on the future(past) null infinity of the lightcone
- Δ_l: boost eigenvalue in the bulk
  $\Rightarrow$ conformal dim. of Celestial CFT

Asymptotic symmetries in Celestial holography

Why is Celestial holography important?

- Because it extracts physical **"Asymptotic symmetries"** in a 4D asymptotic flat space.
- What is "asymptotic symmetry"?

$$\cdots \frac{\text{physical gauge symmetry}}{\text{trivial symmetry at infinity}} \quad i.e. \quad Q \neq 0$$

(equivalent to Weinberg's soft theorem)

e.g.1 BMS symmetry [Bondi,van der Burg,Metzner,Sachs(1962)]

e.g.2 large U(1) gauge symmetry [He et al.(2014)]

Large U(1) gauge symmetry

e.g.2 large U(1) gauge symmetry

$\Leftrightarrow$ in Celestial CFT: $\exists \mathcal{J}_{w(\bar{w})}$:current s.t.

$$\langle \mathcal{J}_z(z,\bar{z}) O_{\Delta_1}(w_1,\bar{w}_1)\dots\rangle = \sum_i \frac{Q_i}{\bar{z}-\bar{w}_i}\langle O_{\Delta_i}(w_1,\bar{w}_1)\dots\rangle$$

- $\mathcal{S}$ corresponds to the Goldstone boson of the large U(1) gauge symmetry,

$$[\mathcal{J}_{\bar{w}}(w,\bar{w}), \partial_z \mathcal{S}(z,\bar{z})] = 2\pi i \delta^{(2)}(w-z).$$

They are called **"soft operators"**.

What to do:

1. **Review double-trace deformation in AdS/CFT**

2. **See double-trace-like deformation in Celestial holography**

   2.1 scalar field

   2.2 soft operators of gauge field

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AdS/CFT correspondence

Statement of AdS_{d+1}/CFT_d correspondence (GKPW dictionary) [Gubser,Klebanov,Polyakov(1998)]

(partition function with bdy cond.)=(correlation function)

$$Z_{\text{grav}}[\phi(x)|_{\partial M} = \phi_0(x)] = \left\langle \exp - \int d^d x \phi_0(x) \mathcal{O}(x) \right\rangle \quad (1)$$

($\mathcal{O}$: Single-trace operator)

- (scalar field's mass in the bulk)↔ (conformal dim.)

$$m^2 = \Delta(\Delta - d) \Rightarrow \Delta_{\pm} = \frac{d}{2} \pm \sqrt{\frac{d}{2}^2 + m^2}$$

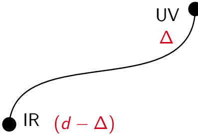
Double-trace deformation in AdS/CFT

- CFT side

  -Relevant deformation

  $\delta S = \frac{f}{2} \int d^d z \, \mathcal{O}_{\Delta_-}^2$ parametrized by f

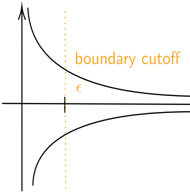
  $\rightarrow$ *RG flow from UV to IR*



$\Updownarrow$ equivalent [Berkooza,Severb,Shomerb(2003)]

- Bulk AdS

  -Mixing Neumann/Dirichlet boundary conditions



$$\gamma \phi(x, \epsilon) + \hat{n} \cdot \partial \phi(x, \epsilon) = 0, \qquad (2)$$

$$\gamma = -\Delta_- - f \epsilon^{2\nu} \left(2\pi^{d/2} \frac{\Gamma(\Delta_- - \frac{d}{2} + 1)}{\Gamma(\Delta_-)} \right)$$

[Hartman, Rastelli(2007)]

δS works as a IR regulator

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Reformulation of Celestial CFT

Reformulate Celestial CFT in the path integral way

- Setup: Massless scalar field on 4D Minkowski

- Coherent quantization is useful
  -coherent quantization
  : Take annihilation(creation) eigenstates as a basis

Relation

For asymptotic operators, [Donnay,Pasterski,Puhm(2020)]

$$\int d\omega\ \omega^{\Delta-1} a^{(\dagger)}(p) = O_{\Delta}^{f(i)}(w, \bar{w}) \tag{3}$$

- Bulk free scalar lagrangian $\leftrightarrow$ Celestial CFT lagrangian:

$$S_{\text{bulk}} = \int d^4x \; \partial_\mu \phi(x) \partial^\mu \phi(x) \tag{4}$$

$$\leftrightarrow \;\; S_{\text{CCFT}} = -\int d^2w \int_{1-i\infty}^{1+i\infty} \frac{d\Delta}{2\pi} \alpha_f^{\Delta^*} \bar{\alpha}_i^{\Delta}, \tag{5}$$

$\alpha_f, \bar{\alpha}_i$ are the eigenvalues of $O_{f(i)}$ respectively.

¡¡Unlike AdS/CFT, conformal dim. can take any $1 + i\mathbb{R}$!!

Celestial CFT side

- we define **double-trace-like deformation** as adding $\frac{f}{2}\int |O_\Delta|^2$ to the Celestial CFT correlation function
- lagrangian:

$$iL_{\text{CCFT}} = \int d^2w \int \frac{d\Delta}{2\pi i}\left[-\bar{\alpha}_i^{\Delta}\alpha_f^{\Delta^*} + \frac{f_i}{2}|\bar{\alpha}_i|^2 + \frac{f_f}{2}|\alpha_f|^2\right] \quad (6)$$

- two-point function

$$\langle \mathcal{O}_f^{\Delta_f}\mathcal{O}_i^{\Delta_i}\rangle = 2\pi i\frac{1}{1-f_i f_f}\delta(\Delta_i-\Delta_f^*)\delta^{(2)}(w_i-w_f) \quad (7)$$

$\rightarrow$ *works as a regulator*

(The role of double-trace-like deformation as an RG-flow is more obvious in Wedge holography.)

Double-trace-like deformation of Celestial CFT

The double-trace-like deformation equivalents to...

Bulk side

- boundary condition

$$z_i = f_i \bar{z}_i, \quad \bar{z}_f = f_f z_f \tag{8}$$

($z, \bar{z}$ are the eigenvalues of annihilation/creation operators)

...It means that we expand the def. of asymptotic states beyond Fock space(squeezed state).

$$|in\rangle = \exp \frac{f}{2} \int \frac{d^3\mathbf{k}}{2k^0} (a^\dagger(\mathbf{k}))^2 |\mathbf{k}_1, \ldots\rangle$$

Large U(1) gauge theory

- The soft current $J_{w(\bar{w})} = \partial_{w(\bar{w})}\mathcal{J}$ are conformal dim. $\Delta = 1$ Celestial operators.

bulk side

Adding

$$|in\rangle = \exp \frac{f_i}{2} \int \frac{d^3\mathbf{k}}{2k^0} |\partial S(\mathbf{k})|^2 |\mathbf{k}_1, \ldots \rangle$$

for the past,

$$\langle out| = \langle \mathbf{k}_1, \ldots | \exp \frac{f_f}{2} \int \frac{d^3\mathbf{k}}{2k^0} |\mathcal{J}_w(\mathbf{k})|^2$$

for the future

Double-trace like deformation in gauge theory

- For U(1) gauge theory,

Double-trace-like deformed Lagrangian:

$$L_{\text{soft}} = \int \frac{d\Delta}{2\pi i} d^2 w \left[-\partial\mathcal{S}\bar{\partial}\mathcal{J} - \partial\mathcal{J}\bar{\partial}\mathcal{S} + f_J \partial\mathcal{J}\bar{\partial}\mathcal{J} + f_S \partial\mathcal{S}\bar{\partial}\mathcal{S} \right] \tag{9}$$

This yields

$$\langle \partial\mathcal{S}\bar{\partial}\mathcal{S} \rangle = 0$$
$$\langle \partial\mathcal{J}\bar{\partial}\mathcal{S} \rangle = -\pi i \delta^{(2)}(w - w')$$
$$\langle \partial\mathcal{J}\bar{\partial}\mathcal{J} \rangle = -\pi i f_S \delta^{(2)}(w - w').$$

$$\langle \partial\mathcal{S}\bar{\partial}\mathcal{S} \rangle = -\pi i f_J \delta^{(2)}(w - w')$$
$$\langle \partial\mathcal{J}\bar{\partial}\mathcal{S} \rangle = -\pi i \delta^{(2)}(w - w')$$
$$\langle \partial\mathcal{J}\bar{\partial}\mathcal{J} \rangle = 0.$$

$\rightarrow$ anomalous dimension?

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Summary

- We have seen double-trace-like deformation in Celestial holography.
  1. scalar field
     ...double-trace-like deformation in CFT corresponds to mixing boundary condition in the bulk.
     It also works as a regulator.
  2. soft operators of gauge field

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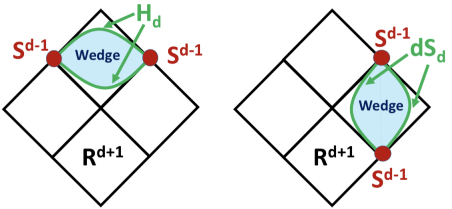

Penrose diagram of codim.-2 Wedge holography [Ogawa et al.(2022)]

- codimension-2 Wedge holography:
  4-dim. classical gravity
  in the "wedge" region surrounded by EoW
  =3-dim. quantum gravity on the EoW[Akal et al.(2020)]
  =2-dim. CFT on the edge of the EoW

In the Minkowski space, we have two kinds of EoWs(end of the world) branes : (H_d, dS_d).

Reformulation of wedge holography

How to define "wedge operator" O_W^{Δ}?
(From now on we restrict on AdS region)

POSTULATIONS

1. shifting $\Delta \to \Delta + \epsilon$(=dimensional regularization)
2. extrapolate dictionary;

$$\lim_{\rho\to\infty} \Psi(X) = \int \frac{d\Delta}{2\pi i} e^{(\Delta-2)\rho} O_W^{\Delta}$$

¡ We cannnot use naive GKPW dictionary except of IR fixed point!
We derive two-point function by just substituting the extrapolate dictionary for the action.
(according with AdS/CFT case:[Terashima(2018)].)

double-trace-like deformation of Wedge holography

Bulk side

Mixing Neumann/Dirichlet bdy cond;

$$[\gamma\Psi + \partial_\rho|_{\rho=R}\Psi](\eta, \rho = R \gg 1, \Omega) = 0 \quad (10)$$

CFT side

- two-point function

$$\langle O_W^{\Delta} O_W^{\Delta'} \rangle^{-1} \sim \quad (11)$$
$$(\gamma - 1)\delta(\Omega - \Omega')e^{2\epsilon R} + (\gamma - \Delta)e^{-2(\Delta-1)R}\frac{\Delta - 1}{4\pi}\left(\frac{2}{1 - \cos\Theta}\right)^{\Delta}$$

Taking $\gamma = 1$ ⇔holographic renormalization, we have

$$\langle O_W^{\Delta} O_W^{\Delta'} \rangle^{-1} \sim e^{-2(\Delta-1)R}\frac{\Delta - 1}{4\pi}\left(\frac{2}{1 - \cos\Theta}\right)^{\Delta} \quad (12)$$

Celestial:UV fixed point ↔ Wedge:IR fixed point?