Semiclassical understanding of bifundamental QCD phase diagrams with anomaly-preserving compactification

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場の理論と弦理論 2023

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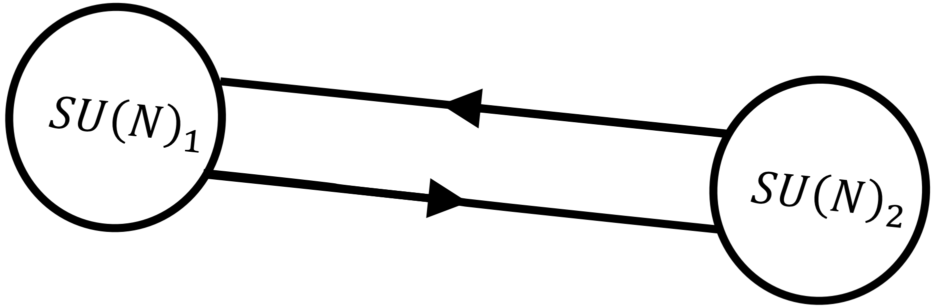


arXiv:2307.13954 [hep-th]

What is bifundamental QCD?

can be obtained by gauging $SU(N_f)$ of flavor-symmetric $N_f = N_c$ QCD

Bifundamental QCD =
$SU(N)_1 \times SU(N)_2$ gauge theory + one Dirac fermion in the bi-fundamental representation $(\square, \overline{\square})$

Field contents:
a_1 : $SU(N)_1$ gauge field
a_2 : $SU(N)_2$ gauge field
Ψ : $N \times N$ matrix-valued Dirac field (fundamental in $SU(N)_1$, anti-fundamental in $SU(N)_2$)

Action:

$$S = \int \frac{1}{2g_1^2}|f_1|^2 + \frac{1}{2g_2^2}|f_2|^2 + \mathrm{tr}\,\overline{\Psi}\,(\not{D} + m)\Psi + \frac{\mathrm{i}\,\theta_1}{8\,\pi^2}\mathrm{tr}\,f_1 \wedge f_1 + \frac{\mathrm{i}\,\theta_2}{8\,\pi^2}\mathrm{tr}\,f_2 \wedge f_2$$

Motivations: why bifundamental QCD?

1. Large N orbifold equivalence [Kachru-Silverstein ‘98][Bershadsky-Johansen ‘98] etc.etc……

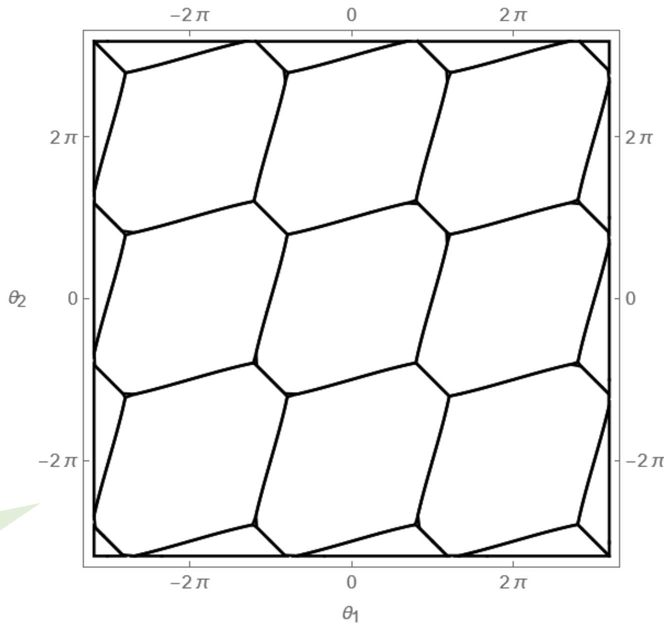
$SU(2N)$ $\mathcal{N}=1$ **SYM** $=$ (at large N) $SU(N)_1 \times SU(N)_2$ **bifundamental QCD**

- SUSY-like structure in non-SUSY theory?
- Nonperturbative validity of equivalence?

unbroken/broken of the exchange symmetry [Kovtun-Ünsal-Yaffe ’03 ‘04]

2. Rich phase diagram:

- Good playground for understanding confining vacuum



(expected) typical phase diagram

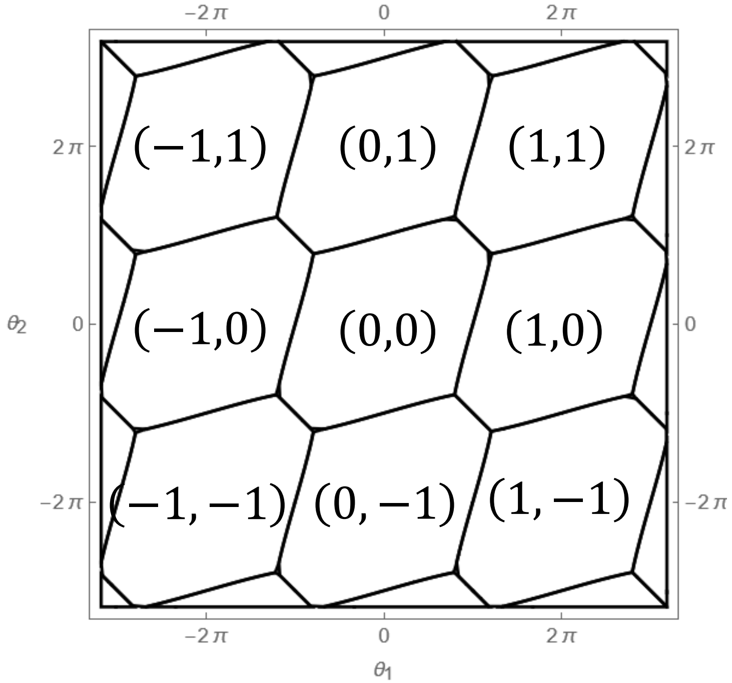
Summary

describing a confining vacuum by semiclassics of **center vortices** [Tanizaki-Ünsal '22]

We study the bifundamental QCD through semiclassics on $\mathbb{R}^2 \times T^2$ with 't Hooft flux.

2d effective theory = N^2 semiclassical vacua $(k_1, k_2) \in \mathbb{Z}_N \times \mathbb{Z}_N$
+ a dynamical 2d fermion on each vacuum

→ phase diagram on (θ_1, θ_2)-plane



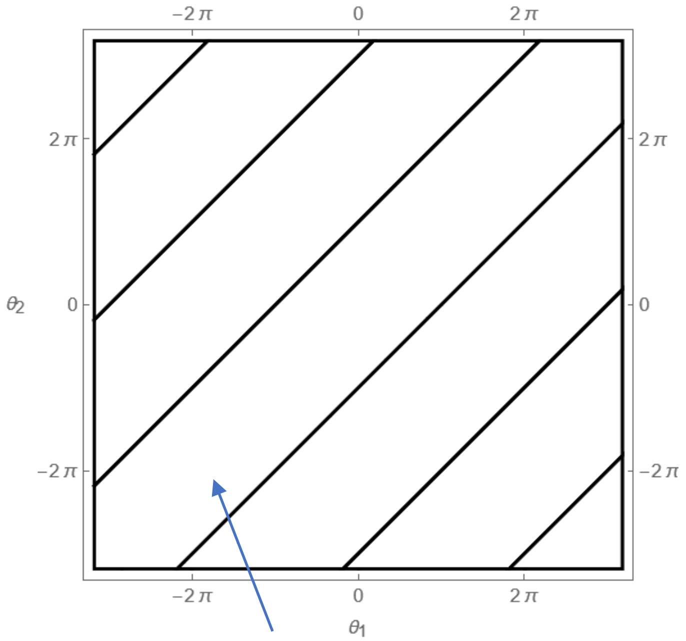
- ☑ phase diagrams from explicit calculations (new!)
- ☑ consistent with (one of) the conjectured phase diagrams
- ☑ on exchange-symmetric line ($\theta_1 = \theta_2$), the exchange symmetry is unbroken

→ evidence for nonperturbative orbifold equivalence.

Conjectured phase diagram

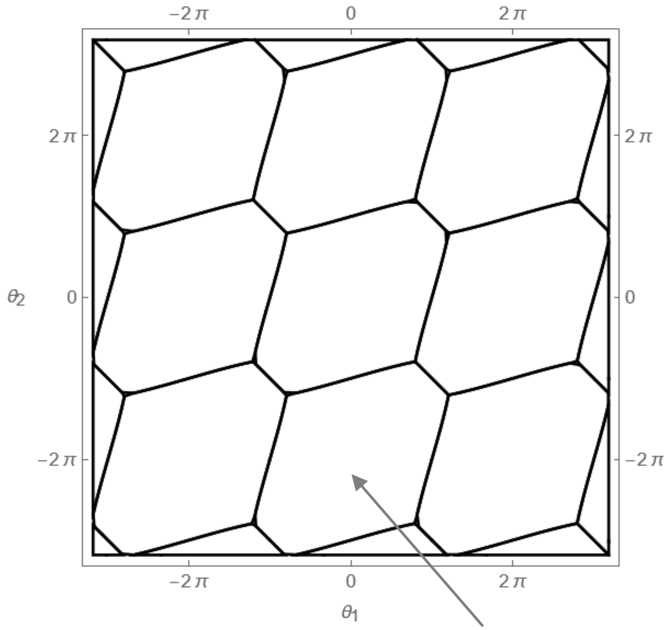
A minimal plausible scenario from anomaly-matching/global inconsistency and calculable limits [Tanizaki-Kikuchi '17][Karasik-Komargodski '19]:

massless case

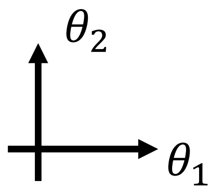


N-fold degenerate vacua on each phase
($(\mathbb{Z}_{2N})_{axial} \to \mathbb{Z}_2$ SSB)

massive case



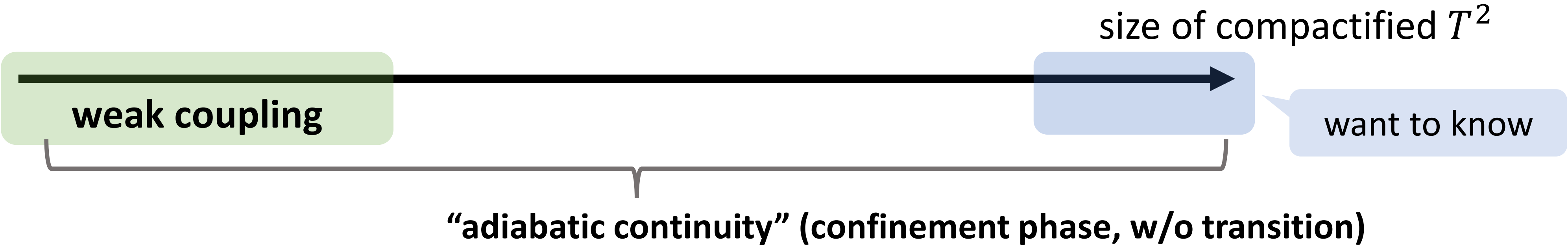
single vacuum on each phase



Methodology: Semiclassics on $\mathbb{R}^2 \times T^2$ with 't Hooft flux

Main ansatz: adiabatic continuity conjecture

theory on $\mathbb{R}^2 \times T^2$ with 't Hooft flux (small T^2) $\simeq$ **theory on $\mathbb{R}^4$**



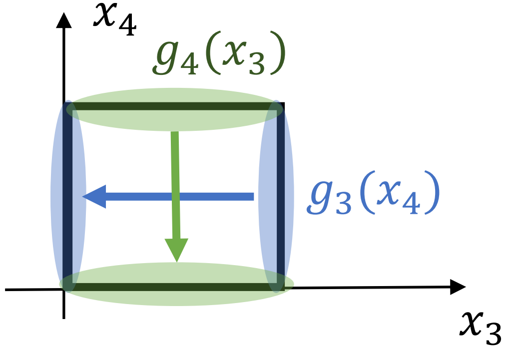
⇒ Semiclassical calculations at small T^2 can predict qualitative features

☑ this method successfully gives a reasonable picture for confining vacuum in SU(N) YM, SU(N) N=1 SYM, QCD(F), QCD(Sym), QCD(AS) [Tanizaki-Ünsal '22 '23]. (cf. [Yamazaki-Yonekura '17])

Example: Semiclassics on $\mathbb{R}^2 \times T^2$ in $SU(N)$ YM [Tanizaki-Ünsal '22]

- **'t Hooft flux on T^2** ($\leftrightarrow \mathbb{Z}_N^{[1]}$ background)

In gauge theory on T^2, the field is periodic up to a gauge transformation $g_3(x_4), g_4(x_3)$ (transition function).



$$\begin{cases} a(\vec{x}, x_3 + L, x_4) = g_3^\dagger a g_3 - \mathrm{i}\, g_3^\dagger \mathrm{d} g_3 \\ a(\vec{x}, x_3, x_4 + L) = g_4^\dagger a g_4 - \mathrm{i}\, g_4^\dagger \mathrm{d} g_4 \end{cases}$$

Without fundamental matter, it is possible to choose

$$g_3(0) g_4(L) g_3^\dagger(L) g_4^\dagger(0) = e^{\frac{2\pi \mathrm{i}}{N}}$$

We can take $g_3 = S,\ g_4 = C$ (shift and clock matrices of $SU(N)$)

e.g.) $N = 3$

$$S = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix},\ C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{2\pi i}{3}} & 0 \\ 0 & 0 & e^{\frac{4\pi i}{3}} \end{pmatrix}$$

- **Center vortex**

If we further compactify $\mathbb{R}^2$ with 't Hooft flux, the minimal topological charge is $Q_{\mathrm{top}} = \frac{1}{N}$. $\to Q_{\mathrm{top}} = \frac{1}{N},\ S_{YM} = \frac{8\pi^2}{N g^2}$ (self-dual solution).

It is numerically confirmed that such a self-dual solution (**fractional instanton/center vortex**) exists and will survive in the decompactifying limit $L_{12} \to \infty$ [Gonzalez-Arroyo–Montero '98, Montero '99].

Example: Semiclassics on $\mathbb{R}^2 \times T^2$ in $SU(N)$ YM [Tanizaki-Ünsal '22]

- Perturbative excitation: gapped!

No gluon constant modes under the twisted boundary condition
$$\begin{cases} a(\vec{x}, x_3 + L, x_4) = S^\dagger a(\vec{x}, x_3, x_4) S \\ a(\vec{x}, x_3, x_4 + L) = C^\dagger a(\vec{x}, x_3, x_4) C \end{cases}$$

→ gluon acquires O(1/NL) KK mass and is **ignored in 2d effective description**.

- Semiclassics by fractional instantons/center vortices

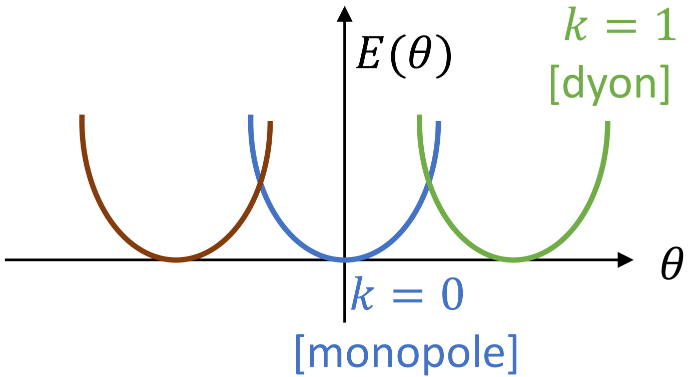
We write (one fractional instanton amplitude) = $K e^{-\frac{8\pi^2}{Ng^2} + i\,\theta/N}$ with a dimensionful constant K.

Then, the dilute gas approximation yields, (note that only configurations with $Q_{top} \in \mathbb{Z}$ are admitted)

$$Z_{YM} = \sum_{n,\bar{n} \geq 0} \frac{1}{n!\,\bar{n}!} \delta_{n-\bar{n} \in \mathbb{Z}} \left(VKe^{-\frac{8\pi^2}{Ng^2} + i\frac{\theta}{N}} \right)^n \left(VKe^{-\frac{8\pi^2}{Ng^2} - i\frac{\theta}{N}} \right)^{\bar{n}}$$
$$= \sum_{k \in \mathbb{Z}_N} \exp\left[-V \left(-2Ke^{-\frac{8\pi^2}{Ng^2}} \cos\left(\frac{\theta - 2\pi k}{N} \right) \right) \right]$$

N semiclassical vacua

Energy density of k-th vacuum
→multibranch structure!



Semiclassics in bifundamental QCD (1)

Apply the previous method to bifundamental QCD:

- ‘t Hooft flux for both $SU(N)_1$ and $SU(N)_2$
- **Two types of center vortices**: $\left(Q_{\text{top}}^{(1)}, Q_{\text{top}}^{(2)}\right) = \left(\frac{1}{N}, 0\right)$ and $\left(0, \frac{1}{N}\right)$
- The boundary condition for the bifundamental fermion,

$$\Psi(\vec{x}, x_3 + L, x_4) = S^\dagger \Psi(\vec{x}, x_3, x_4) S, \qquad \Psi(\vec{x}, x_3, x_4 + L) = C^\dagger \Psi(\vec{x}, x_3, x_4) C$$

allows one constant mode $\Psi \propto 1_{N\times N}$: $^\exists$**dynamical fermion ψ in 2d effective theory**.

- Effect of center vortex on the fermion: **a center vortex changes the chirality**

$$2\,\mathrm{Index}(D) = 2N\left(Q_{\text{top}}^{(1)} + Q_{\text{top}}^{(2)}\right) = 2$$

Thus, the vertex operator of a center vortex carries $\bar{\psi}_L \psi_R$.

e.g.) the (1/N,0) center vortex gives $-K^{(1)} e^{-\frac{8\pi^2}{N g_1^2} + i\,\theta_1/N} \bar{\psi}_L \psi_R$.

Semiclassics in bifundamental QCD (2)

- Dilute gas approximation

$$Z_{YM} = \sum_{k_1,k_2\in\mathbb{Z}_N} \int \mathcal{D}\psi\mathcal{D}\bar{\psi} \exp\left[-S_{2d}[\psi,\bar{\psi}] - \int d^2x \ (M_{k_1,k_2}\bar{\psi}_L\psi_R + c.c.)\right]$$

$$S_{2d}[\psi,\bar{\psi}] = \int d^2x\, \bar{\psi}(\not{\partial}_{2d} + m)\psi, \qquad M_{k_1,k_2} = K^{(1)} e^{-\frac{8\pi^2}{N g_1^2} + i\frac{\theta_1 - 2\pi k_1}{N}} + K^{(2)} e^{-\frac{8\pi^2}{N g_2^2} + i\frac{\theta_2 - 2\pi k_2}{N}}$$

fermion “constant” mode

Summing up center vortices

- 2d low-energy effective description:

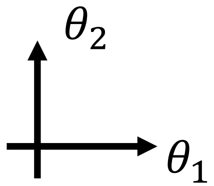
N^2 semiclassical vacua $(k_1, k_2) \in \mathbb{Z}_N \times \mathbb{Z}_N$ + a dynamical 2d fermion on each vacuum

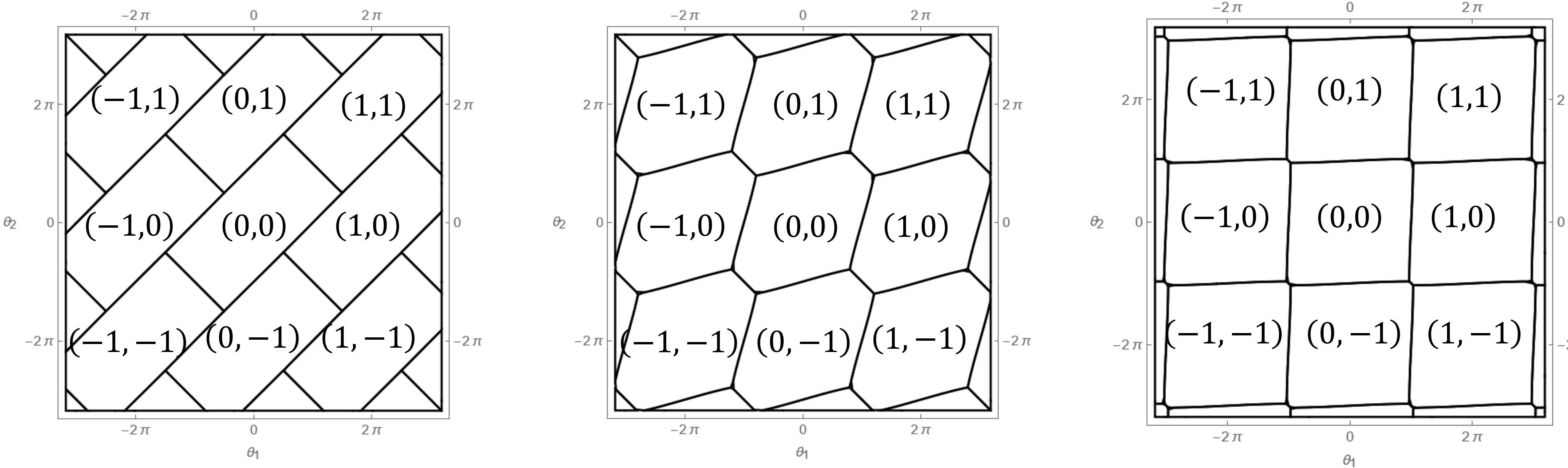
Fermion vacuum energy splits the N^2 degenerate vacua; **the semiclassical vacuum $(\boldsymbol{k_1}, \boldsymbol{k_2})$ with the largest $|\boldsymbol{m} + \boldsymbol{M_{k_1,k_2}}|$ is the ground state.**

Semiclassics in bifundamental QCD (3)

For simplicity, let us draw phase diagrams at $g_1 = g_2$ ($\to K^{(1)} = K^{(2)}$)

massless → fermion mass m





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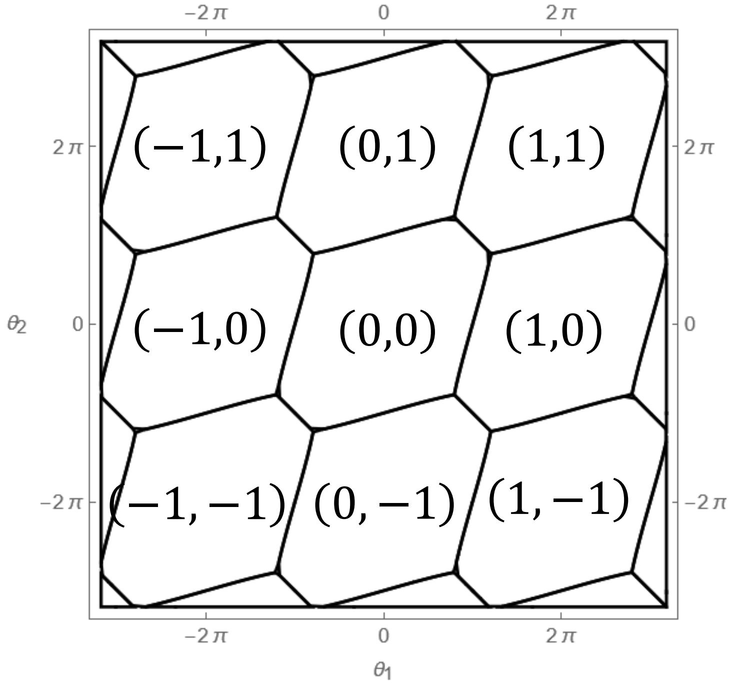

- ☑ phase diagrams from explicit calculations (new for small/intermediate fermion mass)
- ☑ consistent with the conjectured phase diagrams
- ☑ on exchange-symmetric line $(\theta_1 = \theta_2)$, the exchange symmetry is unbroken

Adiabatic continuity works well!

→ evidence for nonperturbative orbifold equivalence.