Muon g-2 in Standard Model

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RIKEN BNL Research Center

2023-08-08 YITP String and Field , Kyoto

References

- g-2 Hadronic Vacuum Polarization (HVP)
  RBC/UKQCD 2023 https://arxiv.org/pdf/2301.08696.pdf
  Phys. Rev. Lett. 121 (2018) 022003

- g-2 Hadronic Light-by-Light (HLbL)
  RBC/UKQCD 2023 https://arxiv.org/pdf/2304.04423.pdf
  Phys. Rev. Lett 124 (2020) 13, 132002
  Phys. Rev. D96 (2017) 034515
  Phys. Rev. Lett. 118 (2017) 022005

- talks at 5th Plenary Workshop Muon g-2 Theory Initiative
  https://indico.ph.ed.ac.uk/event/112/timetable/#20220905
  Tom Blum, Christoph Lehner, Mattia Bruno,
  BMWc (Laurent Lellouch) FNAL/HPQCD (Ethan Neil)
  ETMc (Giuseppe Gagliardi) chiQCD (Gen Wang)

- talks at Radiative correction and MC tools for low-energy hadronic cross sections in e+e- collisions,
  https://indico.psi.ch/event/13708/timetable/#20230607.detailed
  Martin Hoferichter , Fedor Ignatov

- talks at Lattice 2023 https://indico.fnal.gov/event/57249/
  A. Keshavarzi, S. Kuberski, C. Lehner,

Collaborators / Machines

g-2 DWF
HVP & HLbL

- Group 1: Regensburg, Christoph Lehner
- Group 2: UKQCD: Vera Gulpers et al.
- Group 3: Millan: Mattia Bruno + Davide Giusti (Regensburg)
- Group 4: Connecticut: Tom Blum
- Group 5: BNL: Taku Izubuchi
  Peter Boyle , Chulwoo Jung, Chris Kelly,
  Aaron Meyer, Nobuyuki Matsumoto

DWFQCD
Global fit

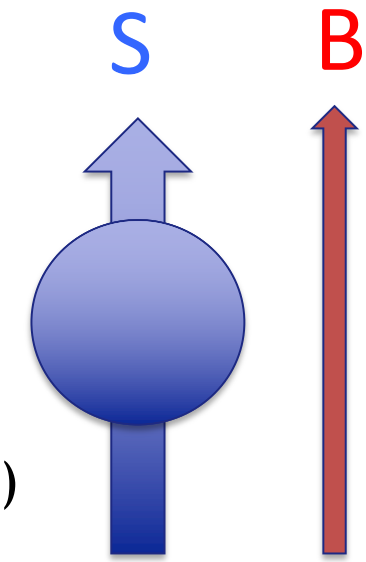
- Group 1 Yong-Chull Jung, Norman Christ, Bob Mawhinney (CU), Chris Kelly (BNL)
- Group2 Christoph Lehner

tau input for
g-2 HVP &
HVP GEVP

Mattia Bruno (Milano)
Aaron Meyer (BNL)
Christoph Lehner (Regensburg)
Taku Izubuchi (BNL & RBRC)

Part of related calculation are done by resources from
USQCD (DOE), HPCI(Japan), RIKEN, XSEDE, ANL BG/Q Mira (DOE, ALCC), Edinburgh BG/Q, Crasher (DOE)
BNL BG/Q, RIKEN BG/Q and Cluster (RICC, HOKUSAI)
Support from RIKEN, JSPS, US DOE, and BNL

Anomalous magnetic moment

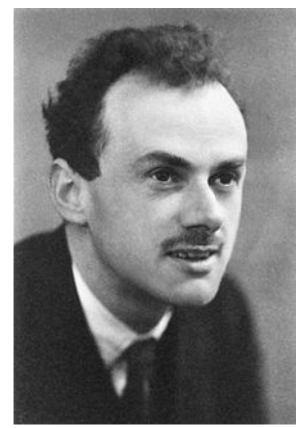


- Fermion's energy in the external magnetic field:

$$V(x) = -\vec{\mu}_l \cdot \vec{B} \qquad \vec{\mu}_l = g_l \frac{e}{2m_l} \vec{S}_l$$

- Magnetic moment Lande g-factor tree level value 2
- 1928 P.A.M. Dirac "Quantum Theory of Electron"
  Dirac equation (relativity, minimal gauge interaction)

$$i[\partial_\mu - ieA_\mu(x)]\gamma^\mu \psi(x) = m\psi(x)$$



- Non-relativistic and weak constant magnetic field
  limits of the Dirac equation :

$$-i\hbar \frac{\partial \psi}{\partial t} = \left[\frac{\nabla^2}{2m} + \frac{e}{2m} \left(\vec{L} + 2\vec{S} \right) \cdot \vec{B} \right] \psi$$

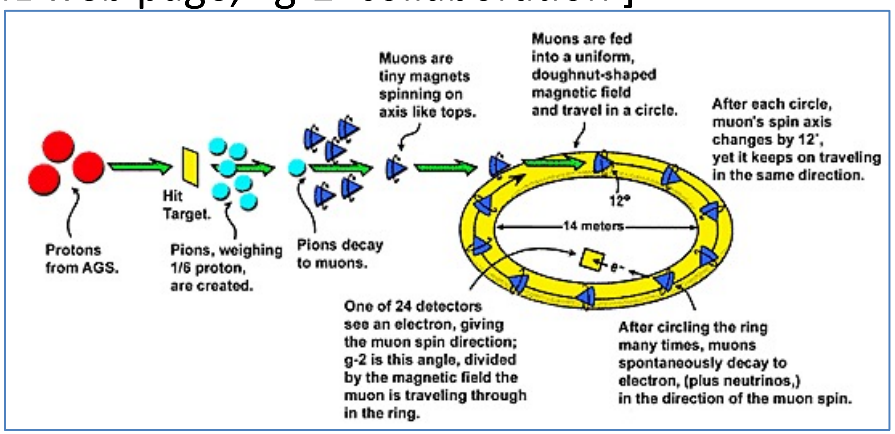
$$g_l = 2$$ (for Dirac Fermion l = e, μ, τ, ...)

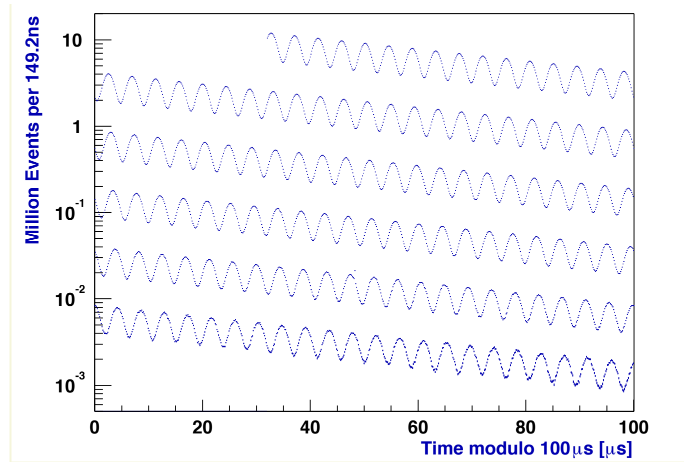
The Muon g-2 experiments BNL E821 (-2004)

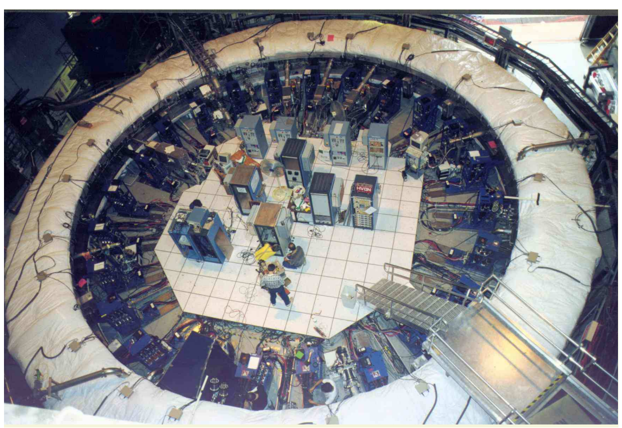
- measure precession of muon spin very accurately

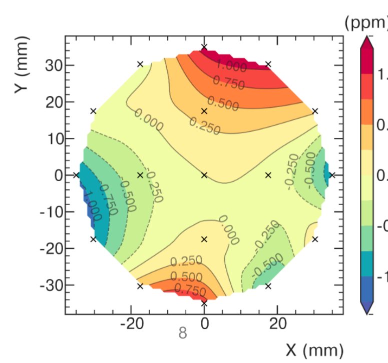
$$N(t) = N_0(E)\ \exp\left(-t/\gamma\tau_\mu\right)\ [1 + A(E)\ \sin(\omega_a t + \phi(E))]$$

[BNL web page, g-2 collaboration]







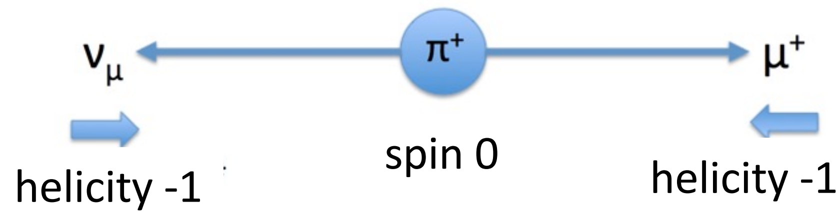


$$a_\mu \propto \frac{\omega_a}{\langle \omega'_p \rangle \times M_\mu \rangle}$$

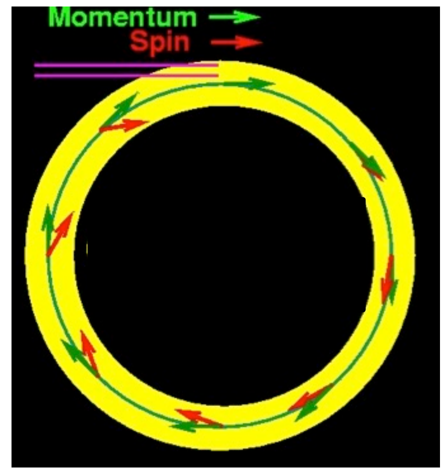
[Aoyama, LAT15] [A.Keshavarzi, LAT23]

Recipe of a g-2 measurement

1. Prepare a polarized muon beam from P-violating pion decay



1. Store in a magnetic field (let muon spin precessed)



$$\vec{\omega} = -\frac{e}{m}\left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} + \frac{\eta}{2}\left(\vec{\beta} \times \vec{B} + \frac{\vec{E}}{c} \right) \right]$$

Magic momentum, γ=30 (p= 3 GeV/c),

2. Measure positron from P-violating muon decay

[Slide from T. Mibe, L. Roberts]

Various corrections, error budget [A. Keshavarzi, LAT23]

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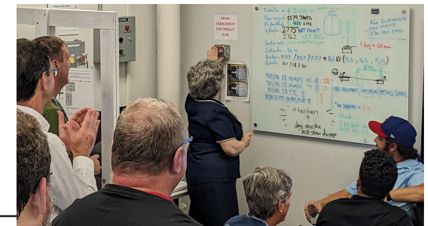
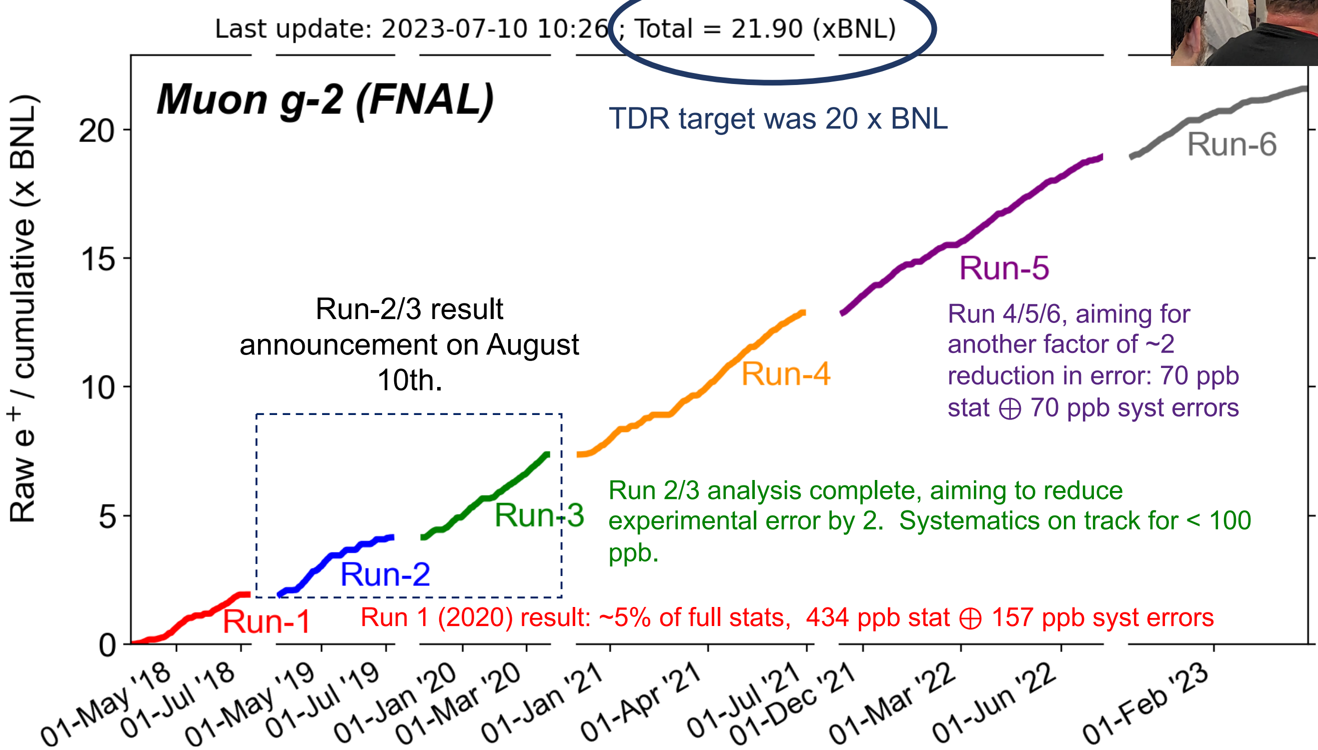
The full data-set

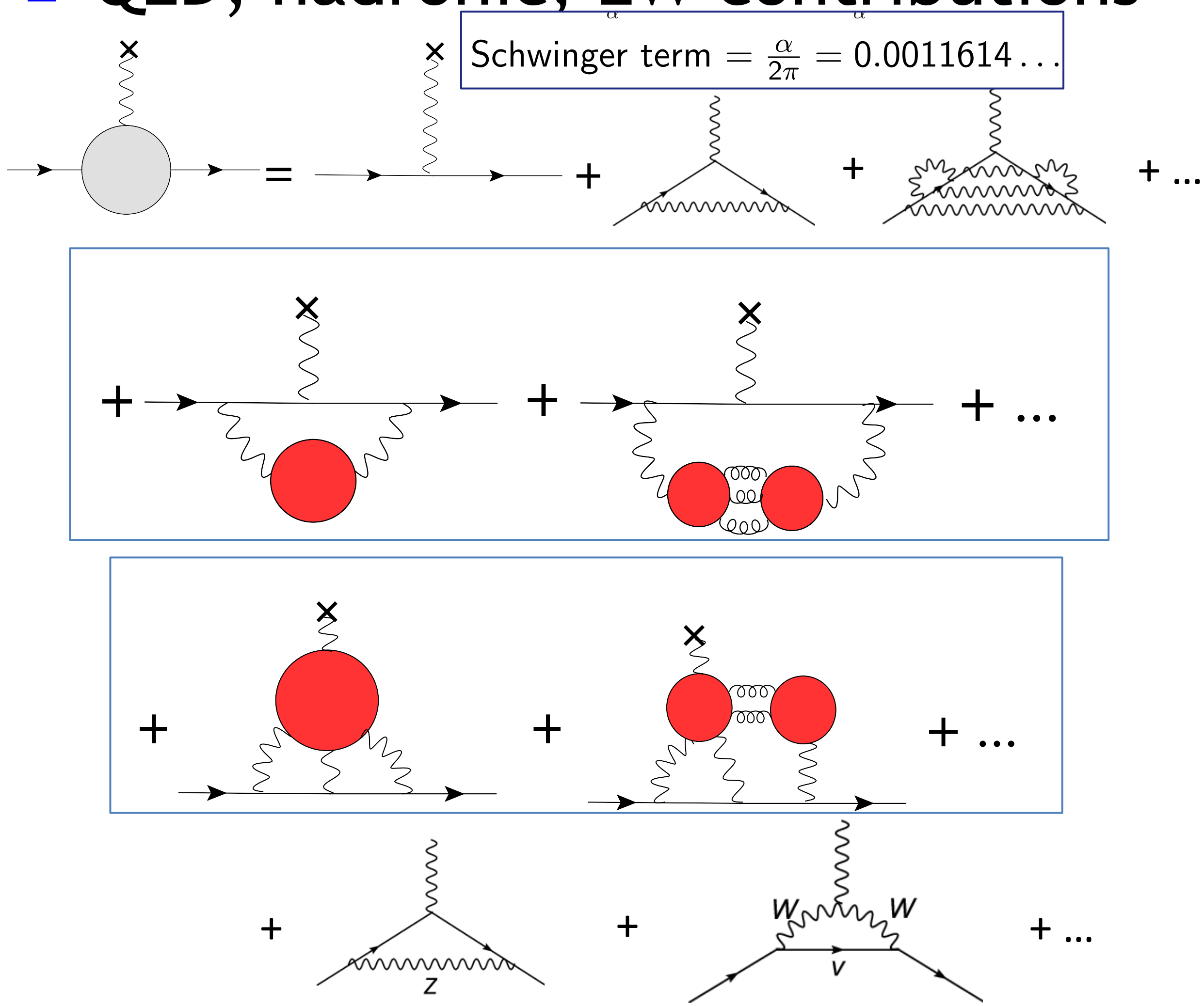




SM Theory

$$\gamma^\mu \rightarrow \Gamma^\mu(q) = \left(\gamma^\mu F_1(q^2) + \frac{i\,\sigma^{\mu\nu} q_\nu}{2m} F_2(q^2)\right)$$

- QED, hadronic, EW contributions



QED (5-loop)
Aoyama Hayakawa, Kinoshita, Nio PRL109,111808 (2012)

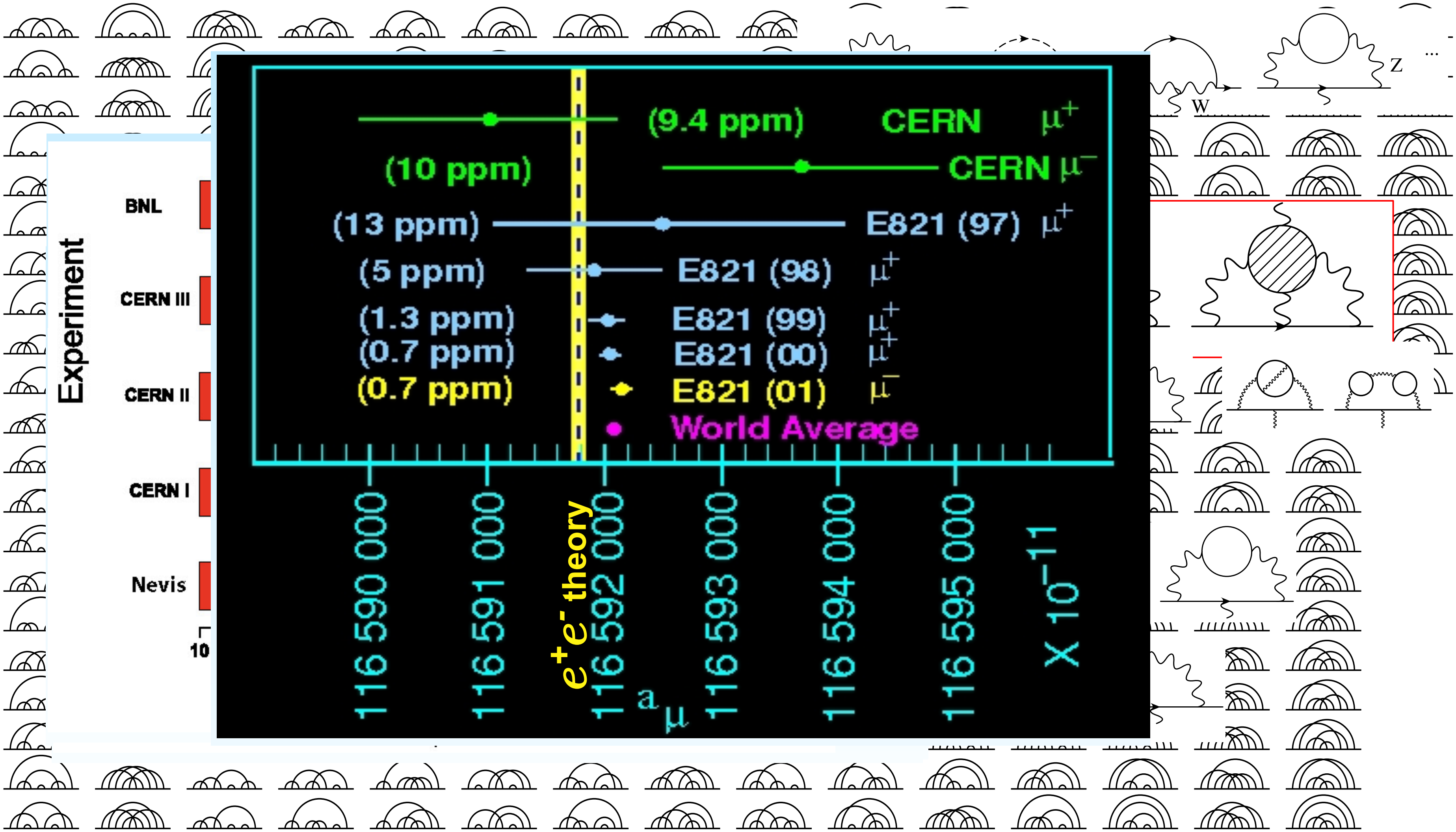
Hadronic vacuum polarization (HVP)

Hadronic light-by-light (Hlbl)

Electroweak (EW)
Knecht et al 02
Czarnecki et al. 02
……

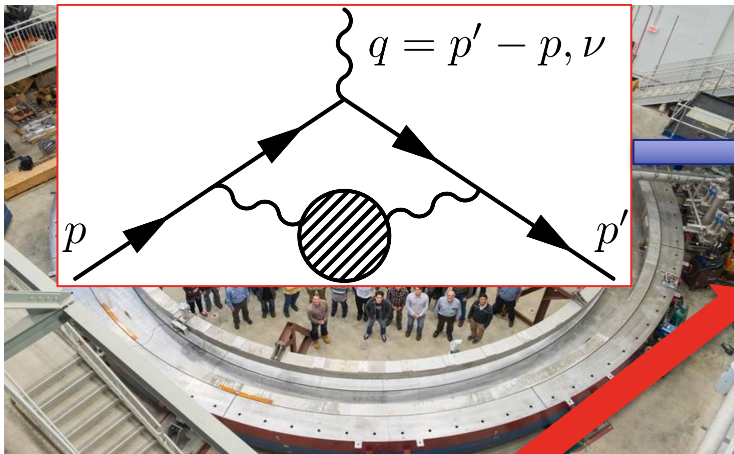
Experiments history

Lee Roberts



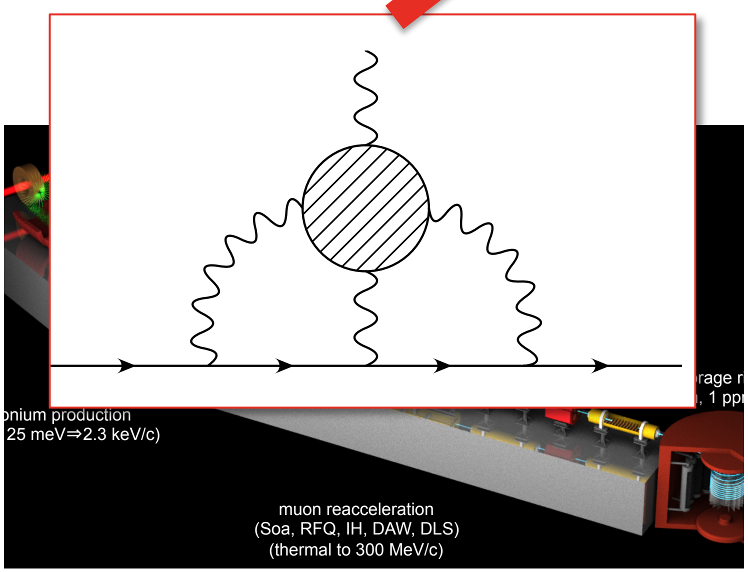
muon anomalous magnetic moment

BNL g-2 till 2004 : ~ 3.7 σ larger than SM prediction



Contribution	Value $\times 10^{10}$	Uncertainty $\times 10^{10}$
QED (5 loops)	11 658 471.895	0.008
EW	15.4	0.1
HVP LO	692.3	**4.2**
HVP NLO	-9.84	0.06
HVP NNLO	1.24	0.01
Hadronic light-by-light	10.5	**2.6**
Total SM prediction	11 659 181.5	4.9
BNL E821 result	11 659 209.1	6.3
FNAL E989/J-PARC E34 goal		$\approx$ **1.6**

$$a_\mu^{\mathrm{EXP}} - a_\mu^{\mathrm{SM}} = 27.4 \underbrace{(2.7)}_{\mathrm{HVP}} \underbrace{(2.6)}_{\mathrm{HLbL}} \underbrace{(0.1)}_{\mathrm{other}} \underbrace{(6.3)}_{\mathrm{EXP}} \times 10^{-10}$$



FNAL E989 (2017-)

2021-04: announces ~BNL level error, 4.6×10^{-10}

2023-08-10: Run-2, Run-3 results

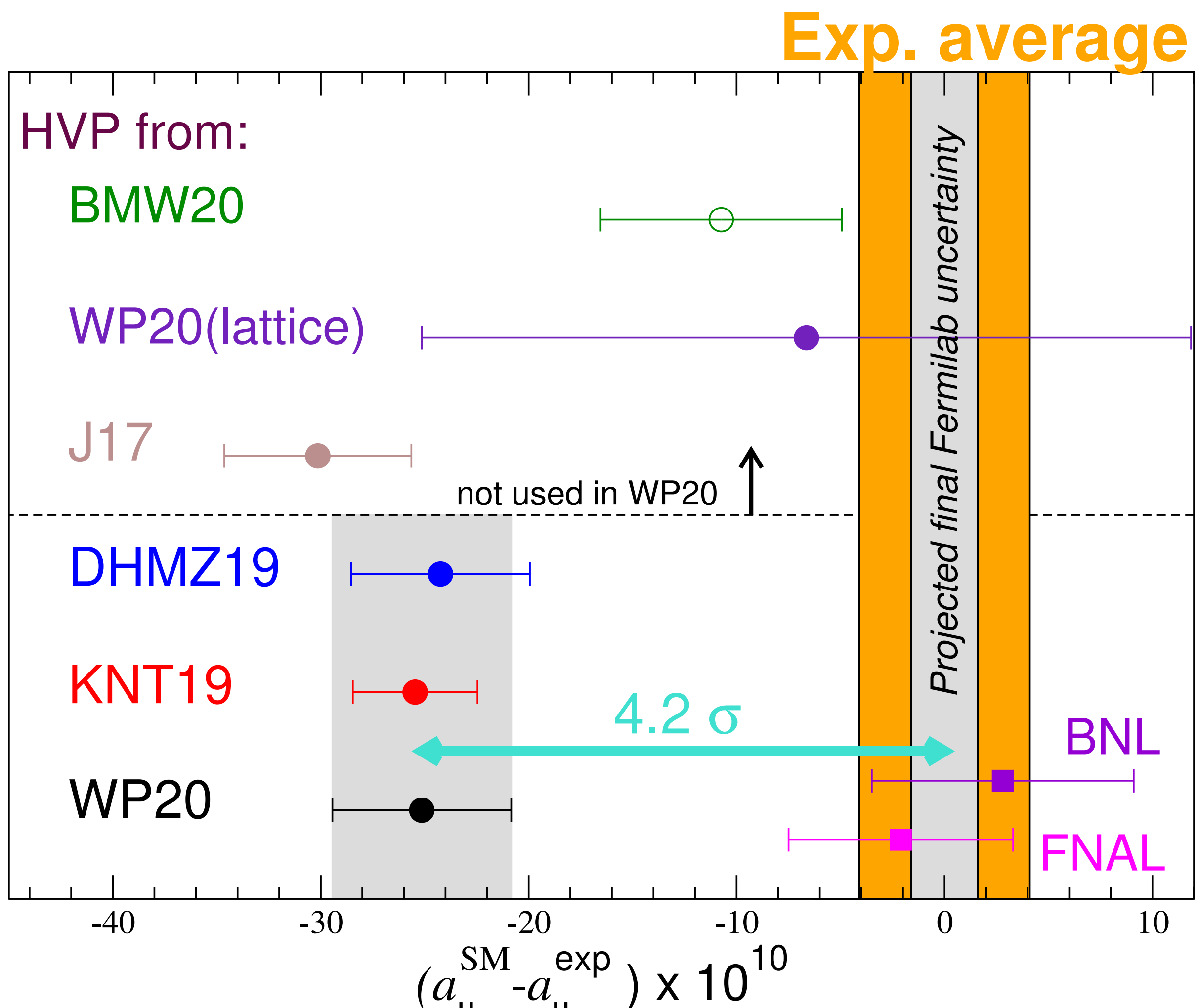
All 6 Run completed, x22 more statistics than BNL aiming for error $1.6\ 10^{-10}$ 0.14ppm

J-PARC E34 (IMPORTANT different systematics !)

ultra-cold muon beam

0.37 ppm then 0.1 ppm, also EDM

April 2020 status muon g-2 HVP



[M. Hoferichter]

G-2 from BSM sources

- Typical new particle contribute g-2

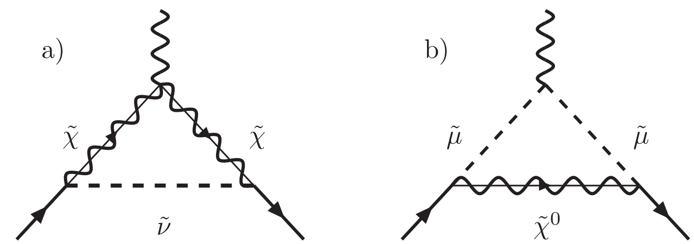
  $g\text{-}2 \sim C \ (m_\mu / m_{NP})^2$

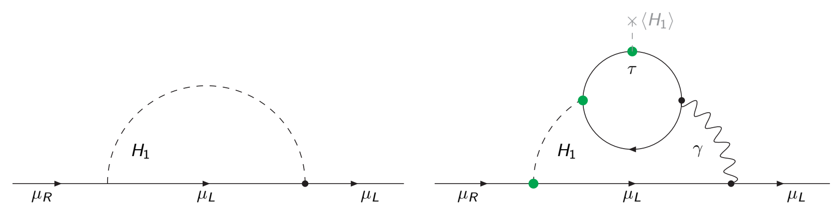
- To explain current discrepancy

$\mathcal{C}$	1	$\frac{\alpha}{\pi}$	$(\frac{\alpha}{\pi})^2$
M_{NP}	$2.0^{+0.4}_{-0.3}$ TeV	100^{+21}_{-13} GeV	5^{+1}_{-1} GeV

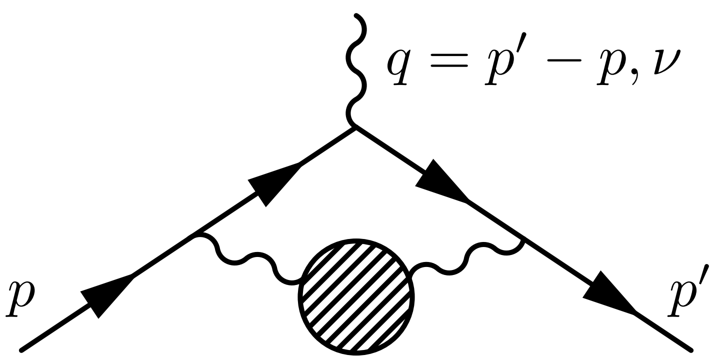
[A. Nyfler]

- SUSY (scalar-lepton)
- 2 Higgs doublet models
  Type-X, ...
- Dark photons
  from kinematical mixings
  $\varepsilon F_{\mu\nu} F'_{\mu\nu}$

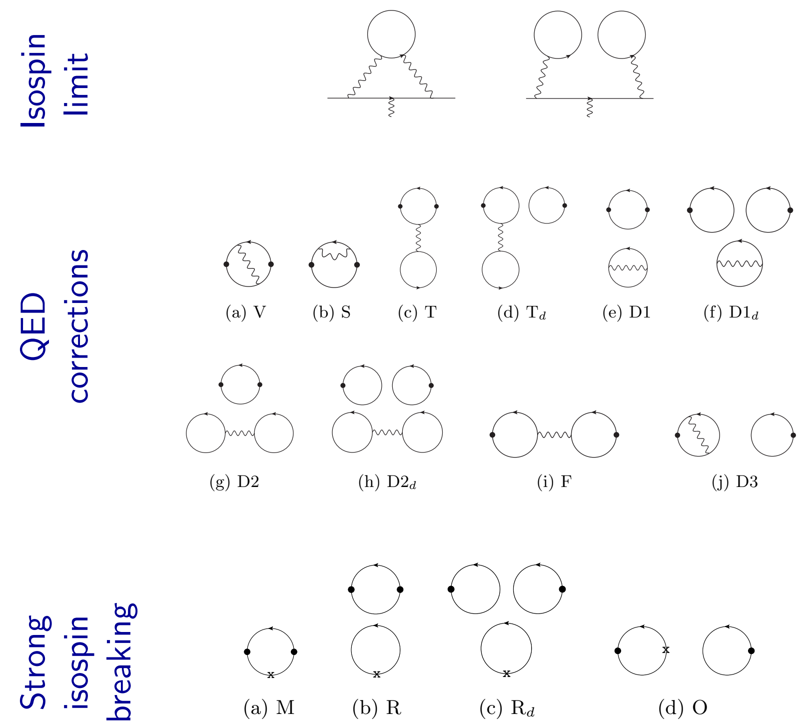




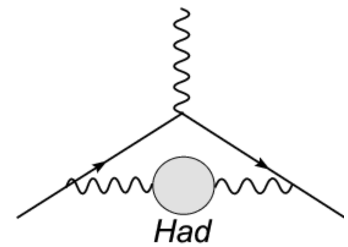
Hadronic Vacuum Polarization (HVP) contribution to g-2



Quark & anti-quark contribution



Leading order of hadronic contribution (HVP)



- Hadronic vacuum polarization (HVP)

V_μ  V_ν $= (q^2 g_{\mu\nu} - q_\mu q_\nu)\Pi_V(q^2)$

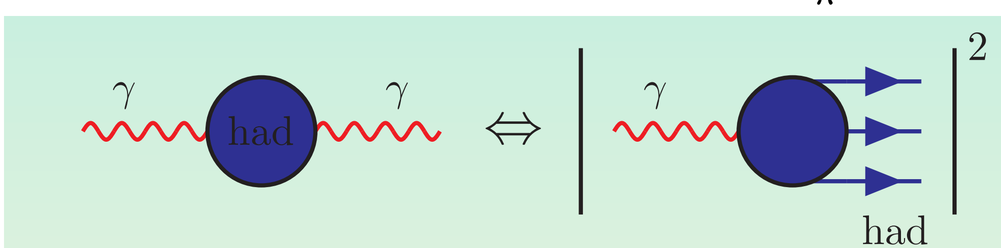
quark's EM current : $V_\mu = \sum_f Q_f \bar{f}\gamma_\mu f$

- Unitarity, Optical Theorem

$$\mathrm{Im}\Pi_V(s) = \frac{s}{4\pi\alpha}\sigma_{\mathrm{tot}}(e^+e^- \to X)$$

- Analycity

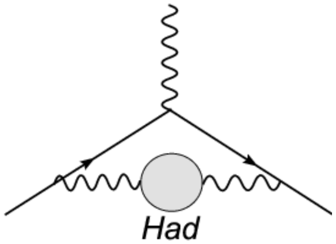
$$\Pi_V(s) - \Pi_V(0) = \frac{k^2}{\pi}\int_{4m_\pi^2}^{\infty} ds \frac{\mathrm{Im}\Pi_V(s)}{s(s-k^2-i\epsilon)}$$



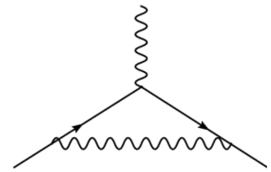
F. Jegerlehner's lecture

Leading order of hadronic contribution (HVP)

- Hadronic vacuum polarization (HVP)



$$a_\mu^{\rm had} = \int ds$$

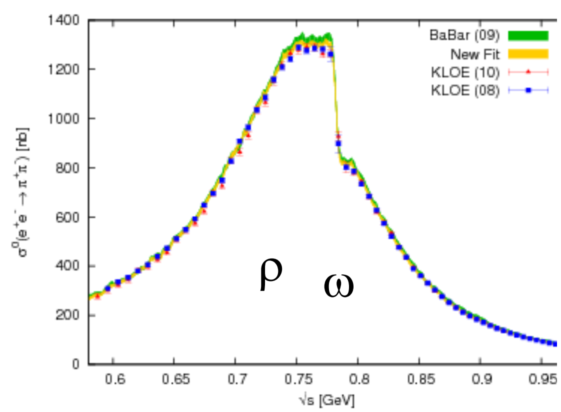


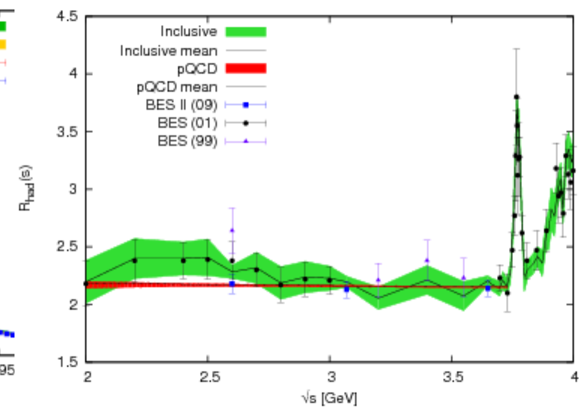
$*$

$$= \frac{\alpha}{\pi^2}\int_{m_\pi^2}^{\infty}\frac{ds}{s}{\rm Im}\Pi(s)K(s) \qquad K(s) = \int_0^1 dx\frac{x^2(1-x)}{x^2+(s/m_\mu^2)(1-x)}$$

$$= \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2\left[\int_{m_\pi^2}^{s_{\rm cut}} ds\frac{K(s)}{s}R_{\rm had}^{\rm data}(s) + \int_{s_{\rm cut}}^{\infty} ds\frac{K(s)}{s}R_{\rm had}^{\rm pQCD}(s)\right]$$

$$R(s) = \sigma_{had}^0(s)/(\frac{4\pi\alpha^2}{3s})$$

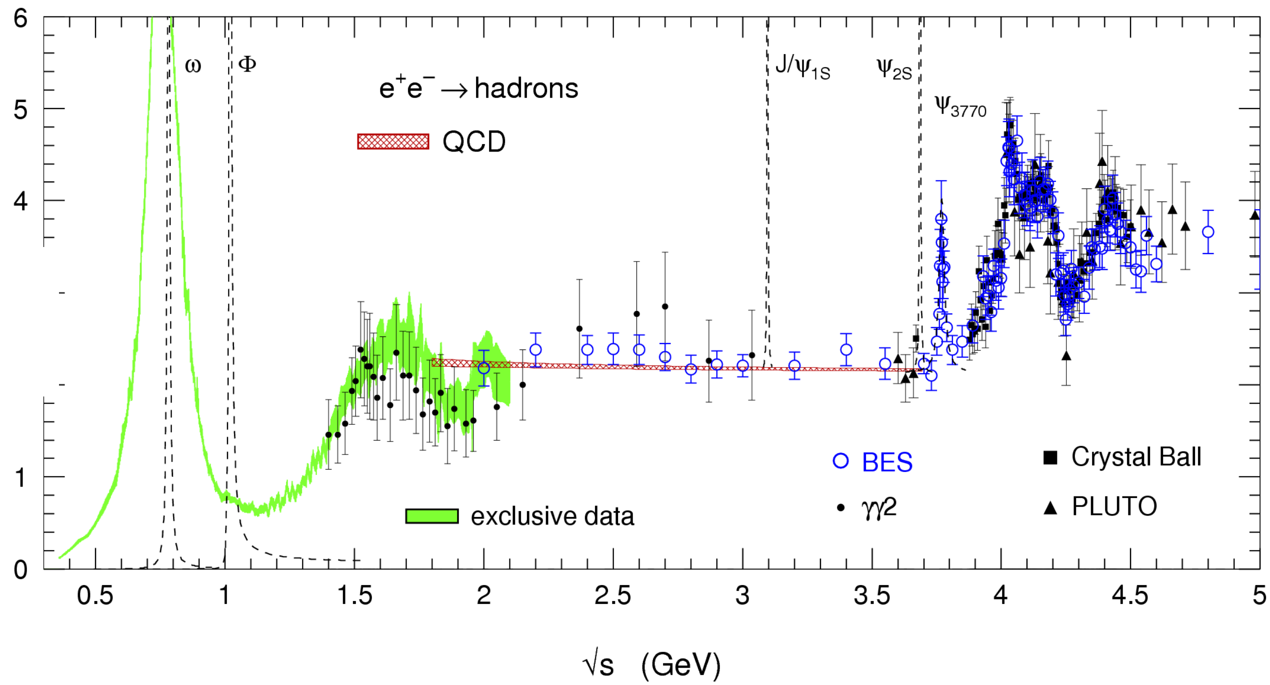


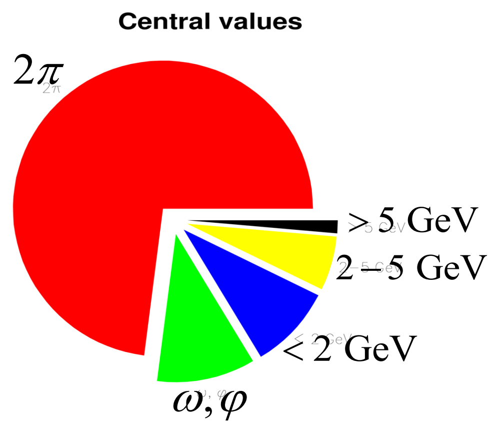


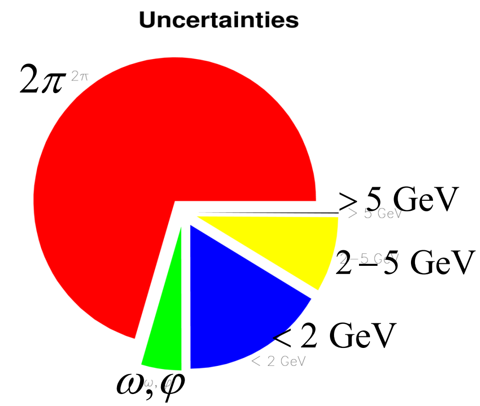
Hagiwara, et al. J.Phys. G38,085003 (2011)

g-2 from R-ratio experiments

$$R(s) = \sigma^0_{had}(s)/(\frac{4\pi\alpha^2}{3s})$$







[A. Keshavarzi, LAT23]

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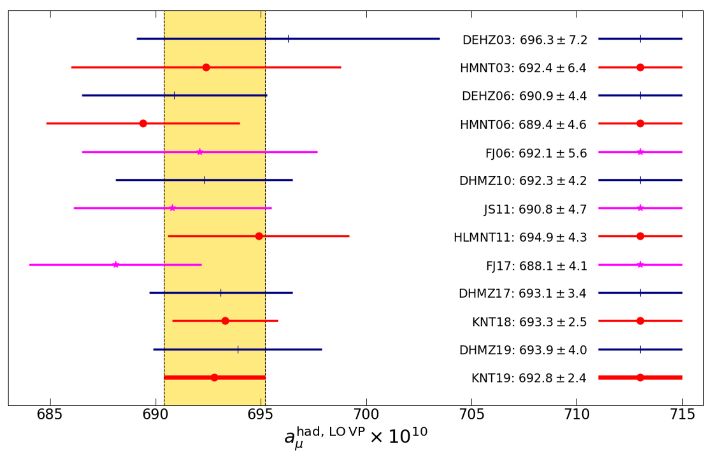
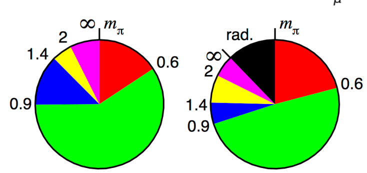
Comparisons and the 2021 WP result

KNT19, *Phys.Rev.D* 97 (2018) 114025, *Phys.Rev.D* 101 (2020) 014029.

$$a_\mu^{\text{had, LOVP}} = 693.84 \pm 1.19_{stat} \pm 1.96_{sys} \pm 0.22_{vp} \pm 0.71_{fsr}$$
$$= 692.78 \pm 2.42_{tot}$$





- Clear $\pi^+\pi^-$ dominance

- Precision better than 0.4% (uncertainties include all available correlations and χ^2 inflation)

Phys.Rept. 887 (2020) 1-166.

Detailed comparisons by-channel and energy range between direct integration results:

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
[1.8, 3.7] GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
[3.7, ∞) GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	694.0(1.0)(3.5)(1.6)(0.1)$_\psi$(0.7)$_{\text{DV+QCD}}$	692.8(2.4)	1.2

+ evaluations using unitarity & analyticity constraints for $\pi\pi$ and $\pi\pi\pi$ channels
[CHS 2018, HHKS 2019]

Conservative merging to obtain a realistic assessment of the underlying uncertainties:

- Account for differences in results from the same experimental inputs.
- Include correlations between systematic errors

$$\Rightarrow a_\mu^{\text{HVP,LO}} = 693.1\,(4.0) \times 10^{-10}$$

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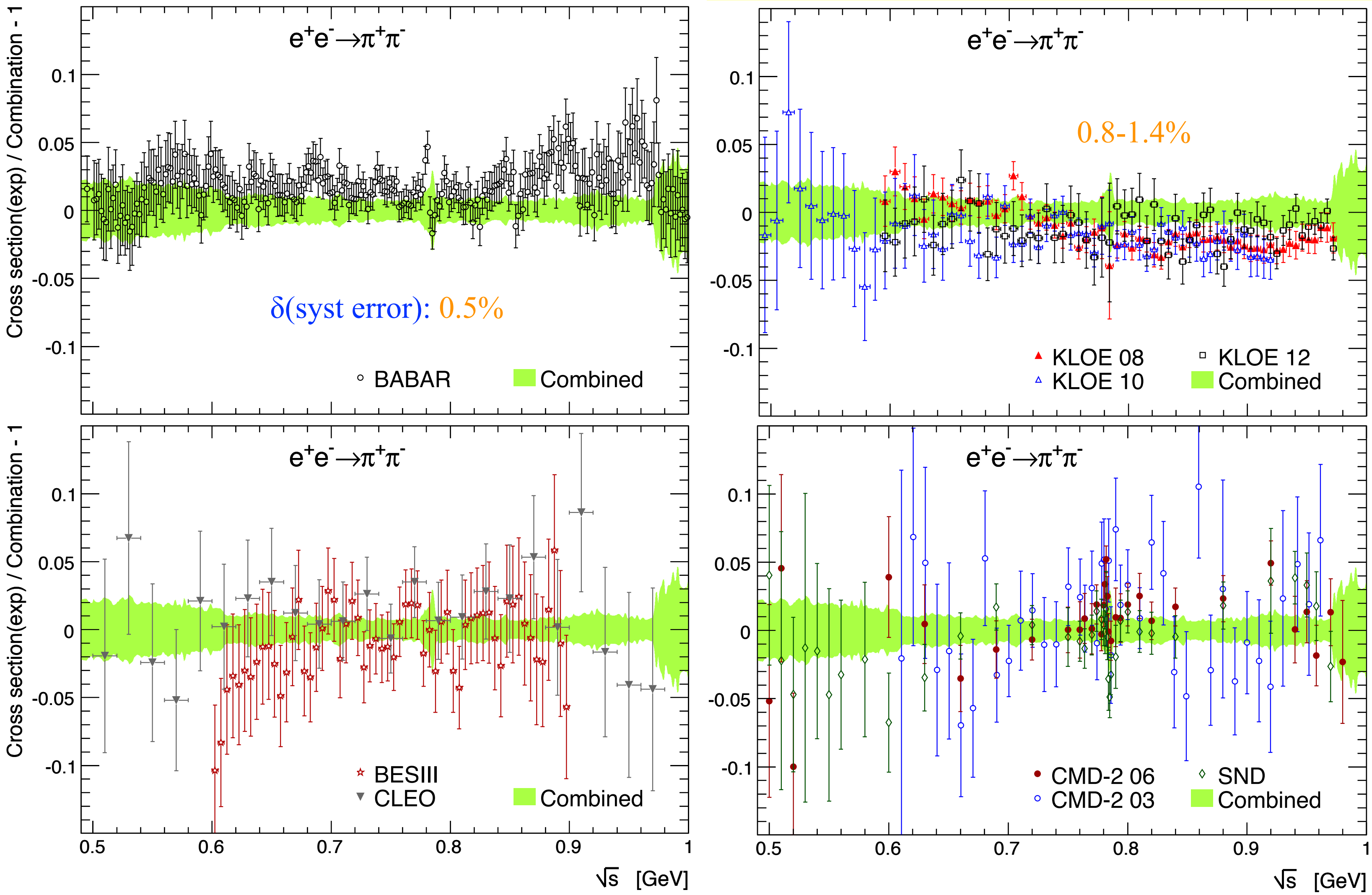
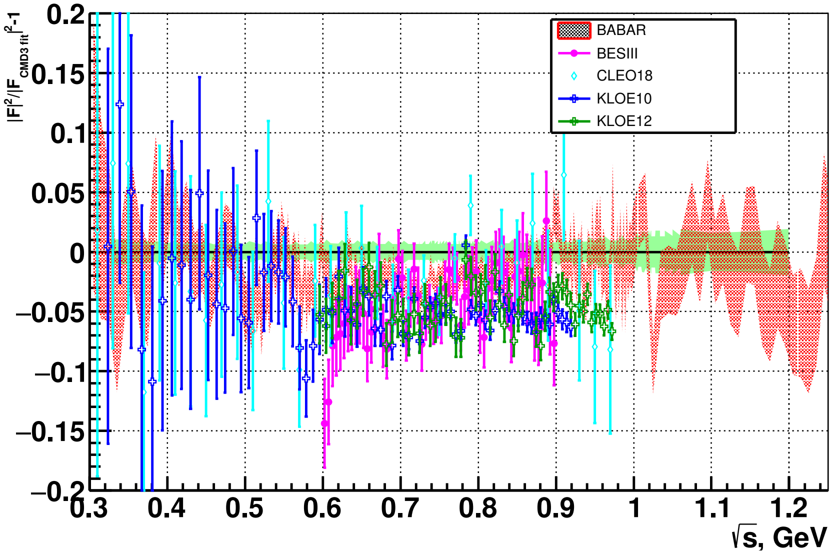
The Dominant $\pi^+\pi^-$ Channel (2)

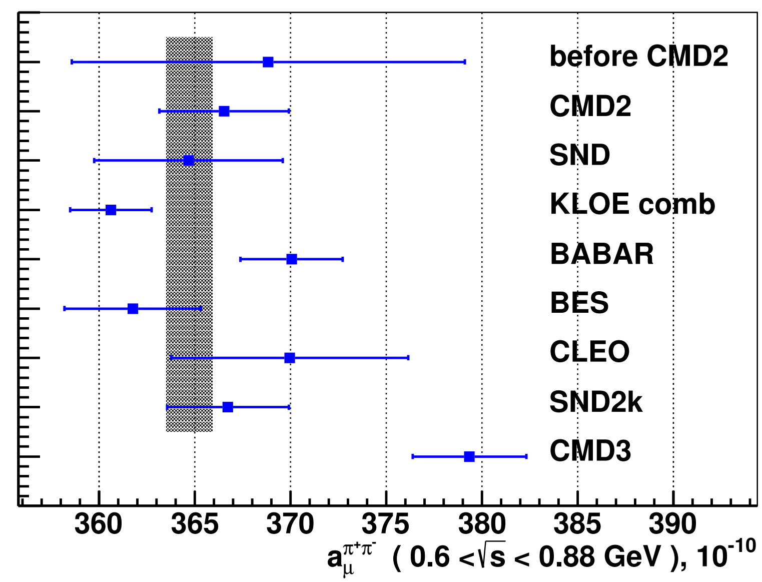


Zhang et al. EPS2019

BABAR & KLOE dominates 0.6-0.9 GeV ππ data,
Has a large discrepancy between BABAR & KLOE -> inflate error (dominant)

2023 CMD-3 e+ e- to π+ π-





CMD-3, 2302.08834

https://indico.psi.ch/event/13708/contributions/43296/attachments/25270/46331/pipiFinal_7June2023_ZurichRadcorMC.pdf

[A. Keshavarzi, LAT23]

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CMD-3 compared to KNT19

In collaboration with Genessa Benton, Diogo Boito, Maarten Golterman, Kim Maltman & Santi Peris.

To be able to compare CMD-3 with KNT19 data combination:

- Data published as pion form factor, $|F_\pi|^2$.
- Must subtract vacuum polarisation effects using Fedor Ignatov's VP correction update.
- Must include final-state-radiation effects.
- Put data on fine, common binning.

In the full 2π data combination range, the KNT19 analysis found:

$$a_\mu^{\pi^+\pi^-}(0.305 \rightarrow 1.937 \text{ GeV}) = (503.46 \pm 1.91) \times 10^{-10}.$$

Replacing KNT19 2pi data in the region $0.33 \rightarrow 1.20$ GeV with CMD-3 data:

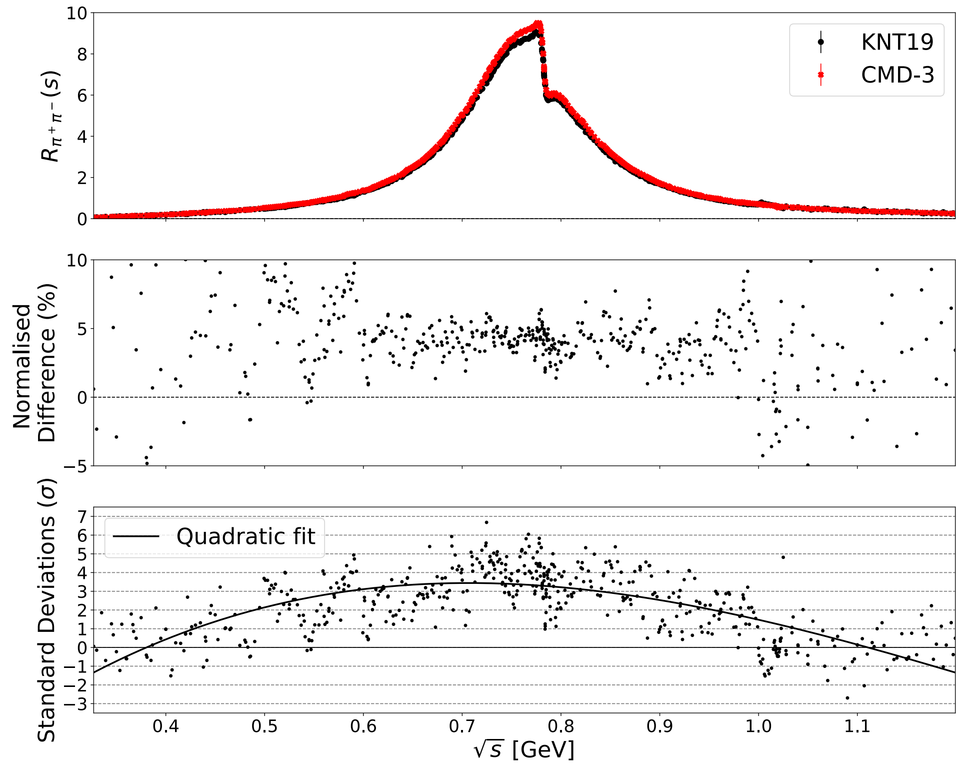
$$a_\mu^{\pi^+\pi^-}(0.305 \rightarrow 1.937 \text{ GeV}) = (525.17 \pm 4.18) \times 10^{-10}.$$

Neglecting possible correlations between e.g. CMD-3 and CMD-2, this results in a difference of:

$$\Delta a_\mu^{\pi^+\pi^-} = (21.71 \pm 4.96) \times 10^{-10} \rightarrow 4.4\sigma\,,$$

This removes the experiment vs. SM Muon g-2 discrepancy.

CMD-3 [F. Ignatov et al, arXiv:2302.08834]



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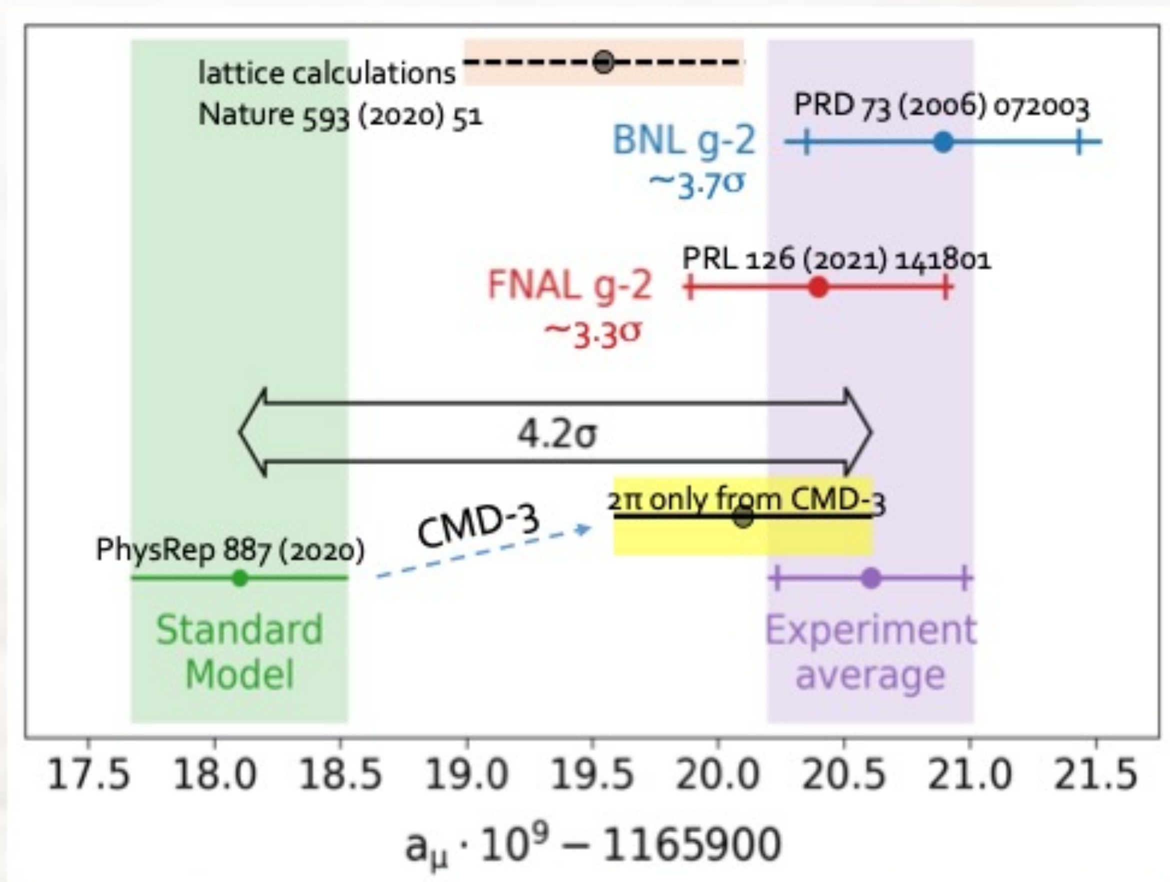


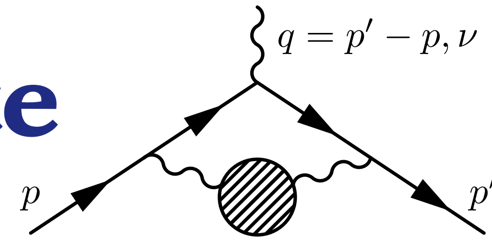
If it will be only CMD-3
than SM will be solved.
But CMD-3 is only one now over
many other experiments
(BaBar, KLOE, BES, CMD-2,
SND, ...)

Unfortunately at the moment, we don't know the reasons of the disagreement between different experiments.

https://indico.psi.ch/event/13708/contributions/43296/attachments/25270/46331/pipiFinal_7June2023_ZurichRadcorMC.pdf

g-2 HVP from Lattice

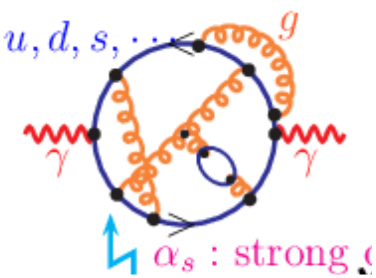
[T. Blum, 2003]]



[Bernecker Meyer 2011, Feng et al. 2013]

In Euclidean space-time, project verctor 2 pt to zero spacial momentum, $\vec{p} = 0$:

$$C(t) = \frac{1}{3} \sum_{x,i} \langle j_i(x) j_i(0) \rangle$$



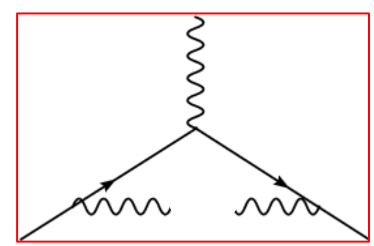
g-2 HVP contribution is

w(t) ~ t^4

$$a_\mu^{HVP} = \sum_t w(t) C(t)$$

$$w(t) = 2 \int_0^\infty \frac{d\omega}{\omega} f_{\text{QED}}(\omega^2) \left[\frac{\cos \omega t - 1}{\omega^2} + \frac{t^2}{2} \right]$$

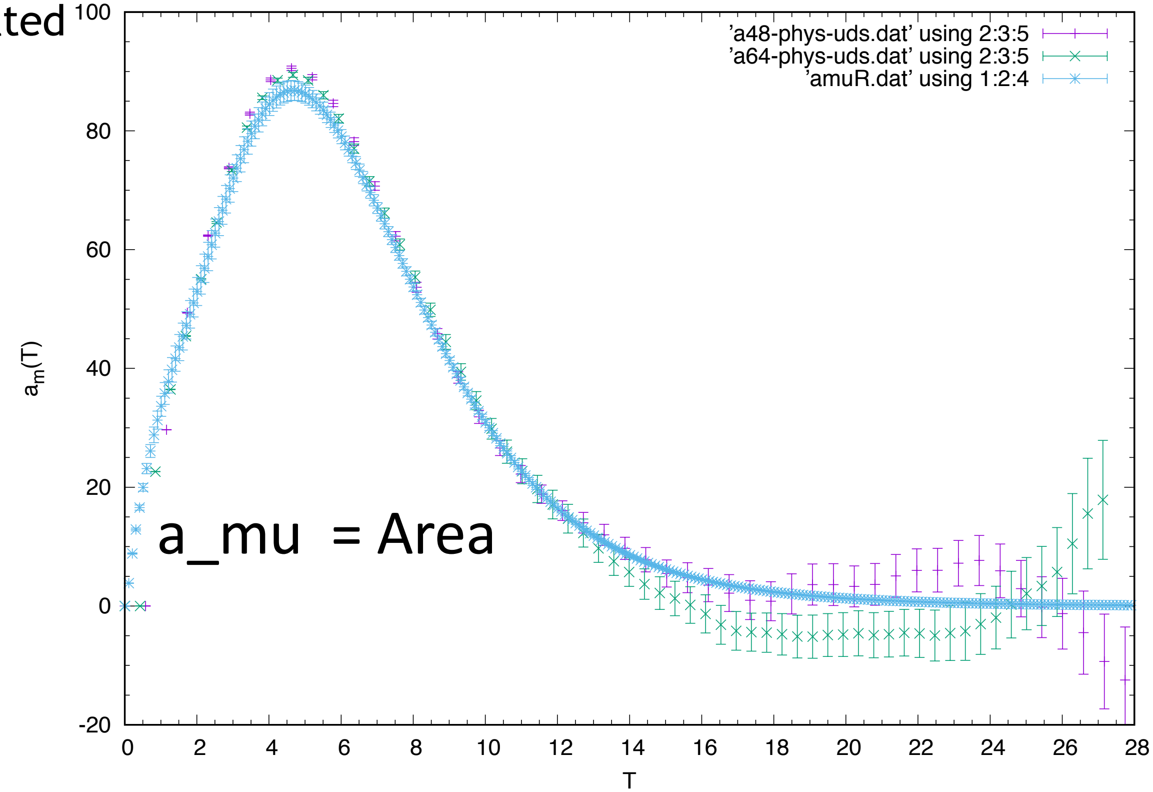
$f_{QED}(\omega^2)$



- Subtraction $\Pi(0)$ is performed.
  Noise/Signal $\sim e^{(E_{\pi\pi} - m_\pi)t}$, is improved [Lehner et al. 2015] .

Comparison of R-ratio and Lattice
[F. Jegerlehner alphaQED 2016]

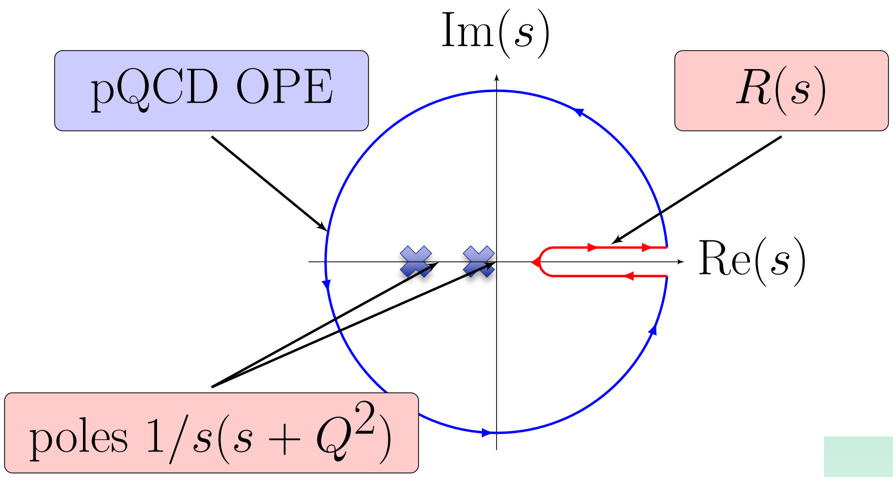
- Covariance matrix among energy bin in R-ratio is not available, assumes 100% correlated

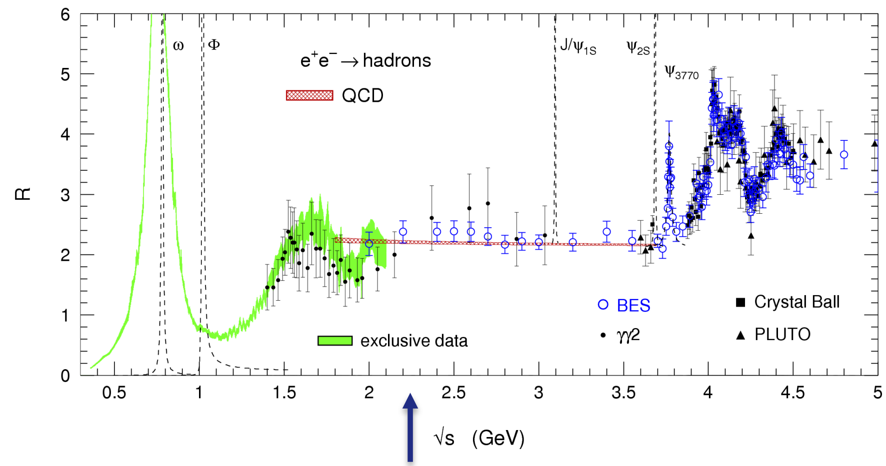


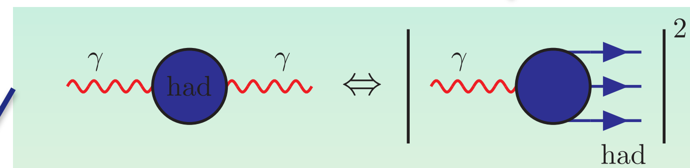
$$a_\mu^{HVP} = \sum_t w(t) C(t)$$

$$w(t) = 2\int_0^\infty \frac{d\omega}{\omega} f_{\text{QED}}(\omega^2)\left[\frac{\cos\omega t - 1}{\omega^2} + \frac{t^2}{2}\right]$$

$$C(t) = \frac{1}{3}\sum_{x,i}\langle j_i(x) j_i(0)\rangle$$



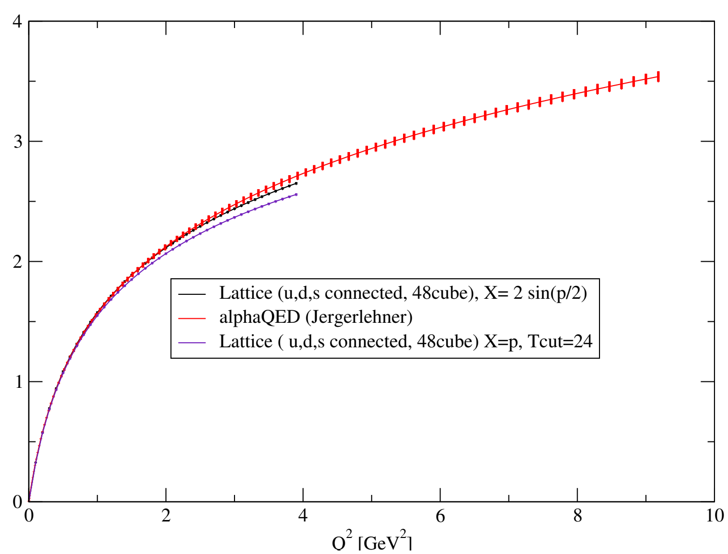




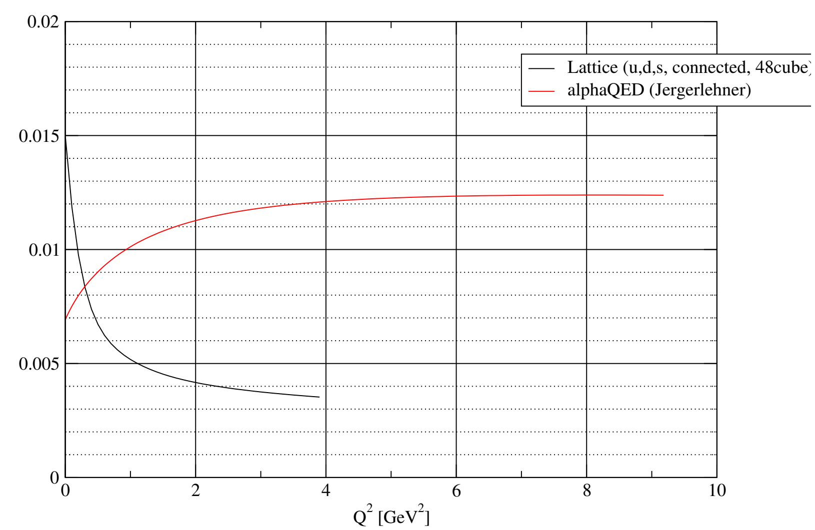
$$\hat{\Pi}(Q^2) = Q^2 \int_0^\infty ds \frac{R(s)}{s(s+Q^2)}$$

($1/a = 1.78$ **GeV, Relative statistical error)**

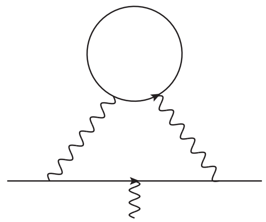
Pihat(Q^2)

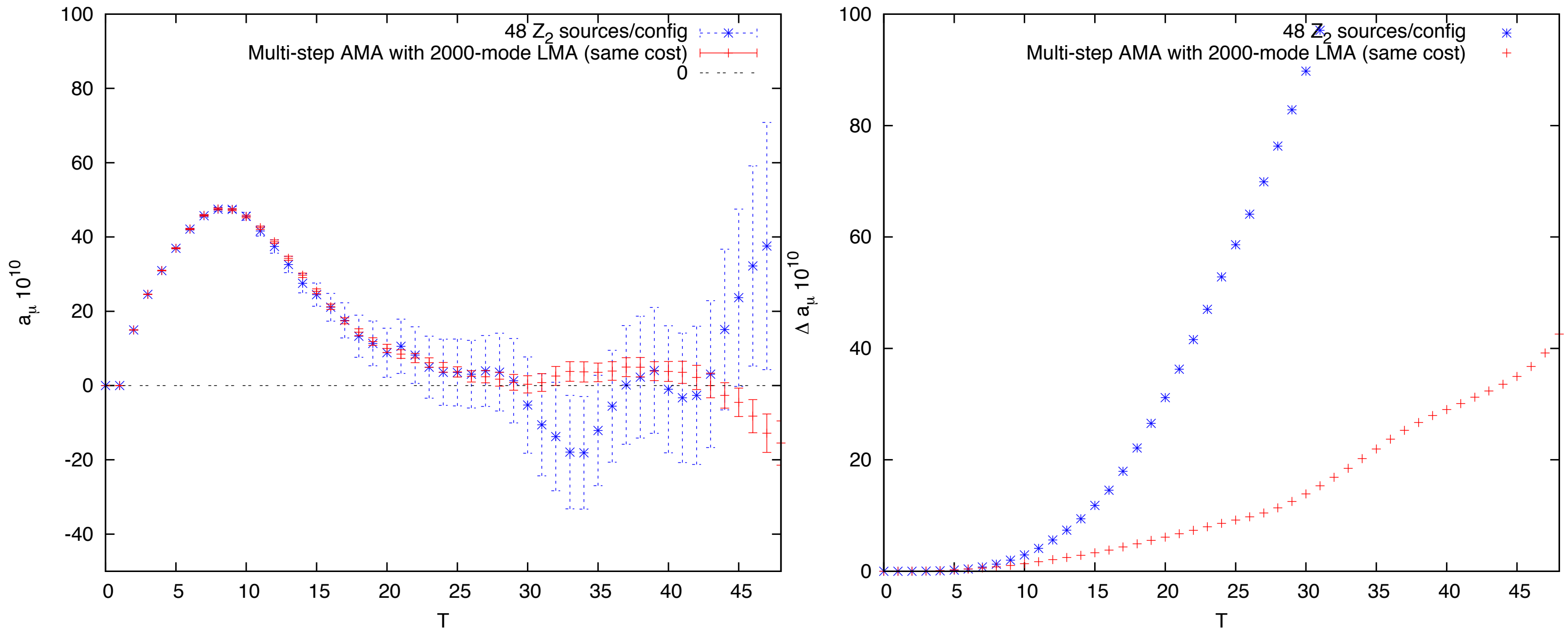


Relative Err of Pihat(Q^2)



up/down quark HVP

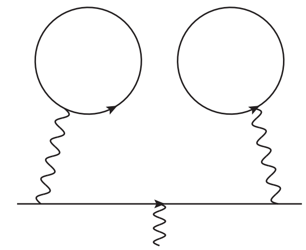




120 conf (a=0.11fm), 80 conf (a=0.086fm) physical point Nf=2+1 Mobius DWF
4D full volume LMA with 2,000 eigen vector (of e/o preconditioned zMobius D^+D)
EV compression (1/10 memory) using local coherence [C. Lehner Lat2017 Poster]
In addition, 50 sloppy / conf via multi-level AMA
more than x 1,000 speed up compared to simple CG

disconnected quark loop contribution

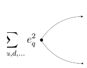
- [C. Lehner et al. (RBC/UKQCD 2015, arXiv:1512.09054, PRL)]
- Very challenging calculation due to statistical noise
- Small contribution, vanishes in SU(3) limit,
  Qu+Qd+Qs = 0

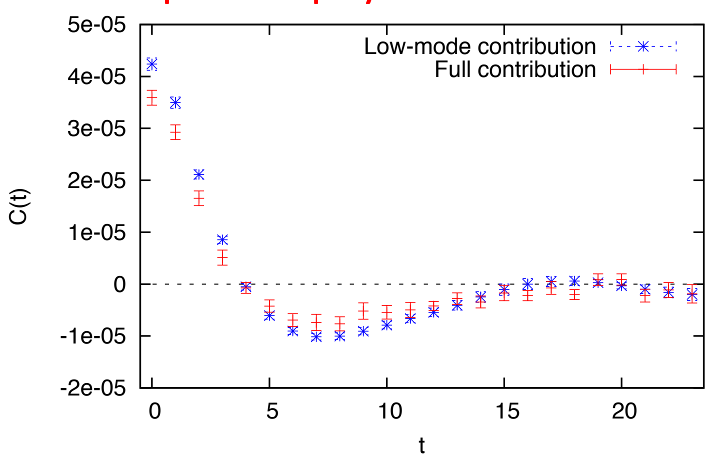


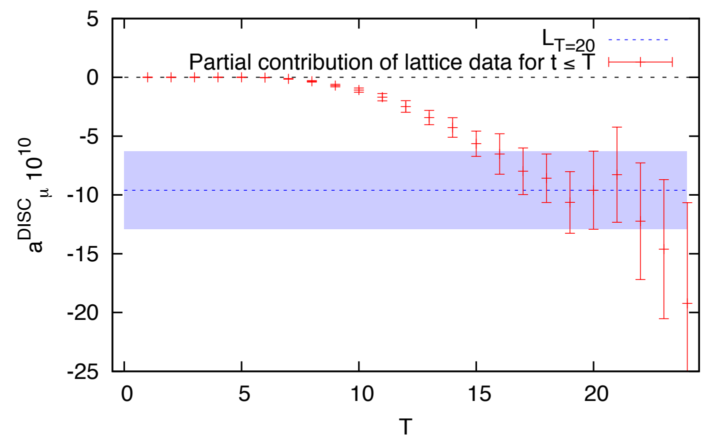
- Use low mode of quark propagator, treat it exactly
  (all-to-all propagator with sparse random source)
- First non-zero signal

$$a_\mu^{\rm HVP\ (LO)\ DISC} = -9.6(3.3)_{\rm stat}(2.3)_{\rm sys} \times 10^{-10}$$

Sensitive to m_π
crucial to compute at physical mass

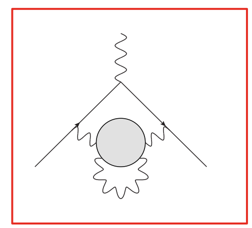


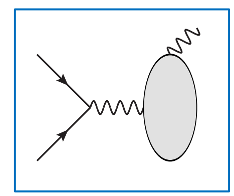


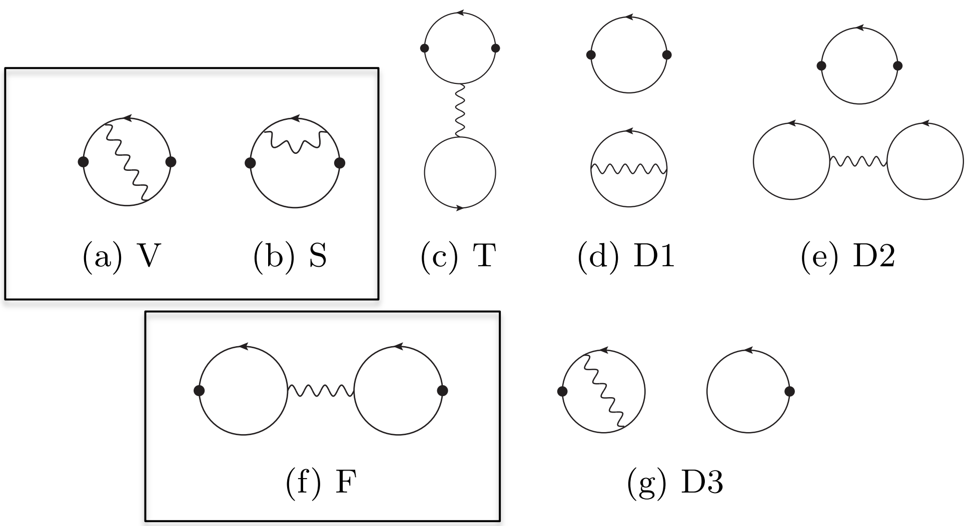


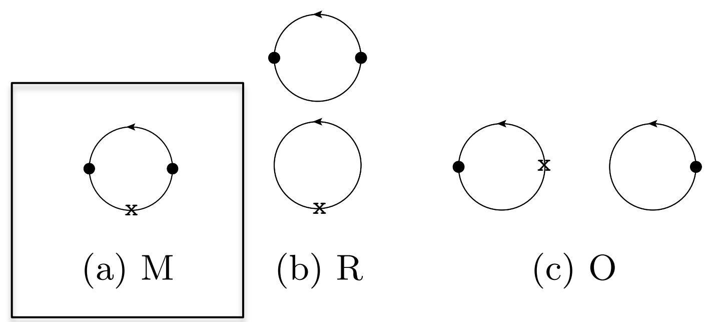
HVP QED+ strong IB corrections

- HVP is computed so far at Iso-symmetric quark mass, needs to compute isospin breaking corrections : Qu, Qd, mu-md ≠0
- u,d,s quark mass and lattice spacing are re-tuned using {charge,neutral} x{pion,kaon} and (Omega baryon masses)
- For now, V, S, F, M are computed : assumes EM and IB of sea quark and also shift to lattice spacing is small (correction to disconnected diagram)
- Point-source method : stochastically sample pair of 2 EM vertices a la important sampling with exact photon









Window values
Combine R-ratio and Lattice
[Christoph Lehner et al PRL18]

- Divide total a_μ into { short, mid, long } distance contribtions
- Useful to crosscheck among different lattice group and R-ratios

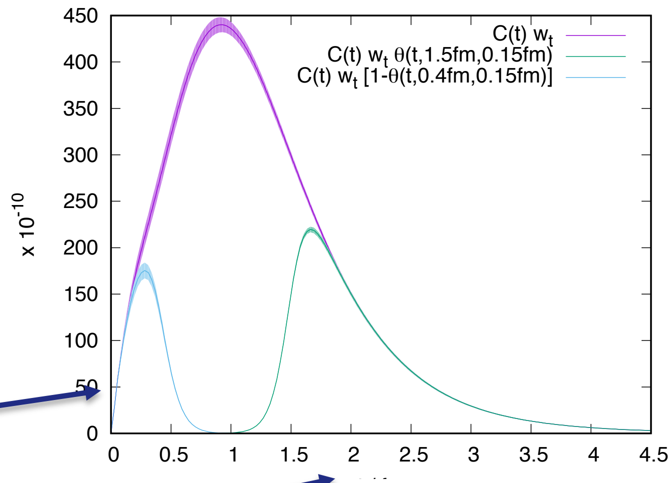
$$\Theta(t,\mu,\sigma) \equiv [1+\tanh[(t-\mu)/\sigma]]$$

$$a_\mu = \sum_t w_t C(t) \equiv a_\mu^{\rm SD} + a_\mu^{\rm W} + a_\mu^{\rm LD}$$

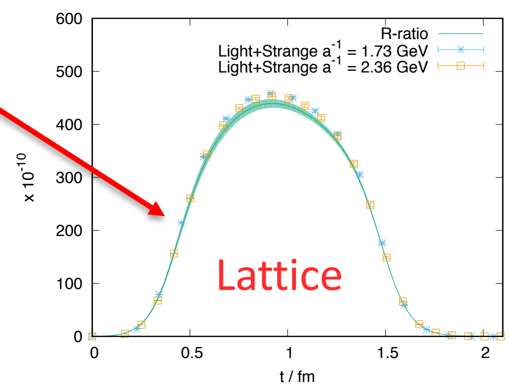
$$a_\mu^{\rm SD} = \sum_t C(t) w_t [1-\Theta(t,t_0,\Delta)]\,,$$

$$a_\mu^{\rm W} = \sum_t C(t) w_t [\Theta(t,t_0,\Delta)-\Theta(t,t_1,\Delta)]\,,$$

$$a_\mu^{\rm LD} = \sum_t C(t) w_t \Theta(t,t_1,\Delta)$$



+



Euclidean time correlation from e^+e^- $R(s)$ data

From e^+e^- $R(s)$ ratio, using disparsive relation, zero-spacial momentum projected Euclidean correlation function $C(t)$ is obtained

$$\hat{\Pi}(Q^2) = Q^2 \int_0^\infty ds \frac{R(s)}{s(s+Q^2)}$$

$$C^{\text{R-ratio}}(t) = \frac{1}{12\pi^2} \int_0^\infty \frac{d\omega}{2\pi} \hat{\Pi}(\omega^2) e^{i\omega t} = \frac{1}{12\pi^2} \int_0^\infty ds\, \sqrt{s} R(s) e^{-\sqrt{s}t}$$

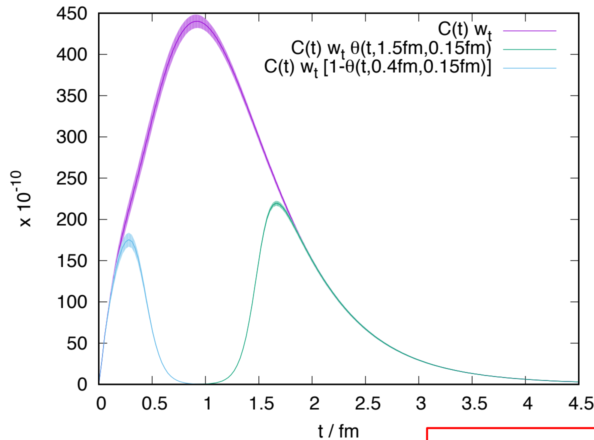
Lattice can compute Integral of Inclusive cross sections accurately

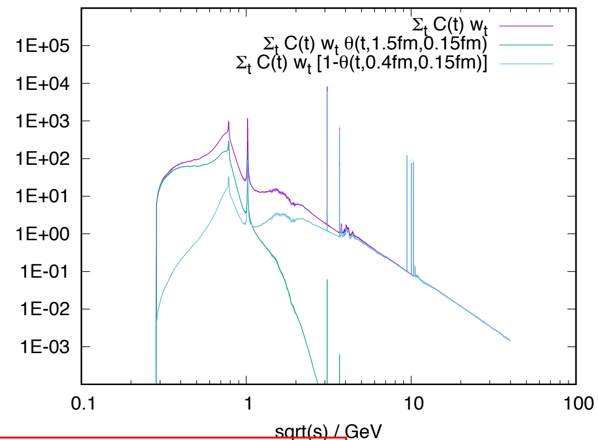
- $C(t)$ or $w(t)C(t)$ are directly comparable to Lattice results with the proper limits ($m_q \to m_q^{\text{phys}}, a \to 0, V \to \infty$, QED ...)
- Lattice: long distance has large statistical noise, (short distance: discretization error, removed by $a \to 0$ and/or pQCD)
- R-ratio : short distance has larger error

How does this translate to the time-like region?

Euclidean from Lattice

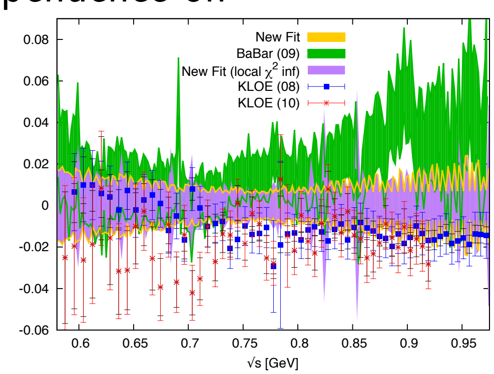
Time-like from e+e-





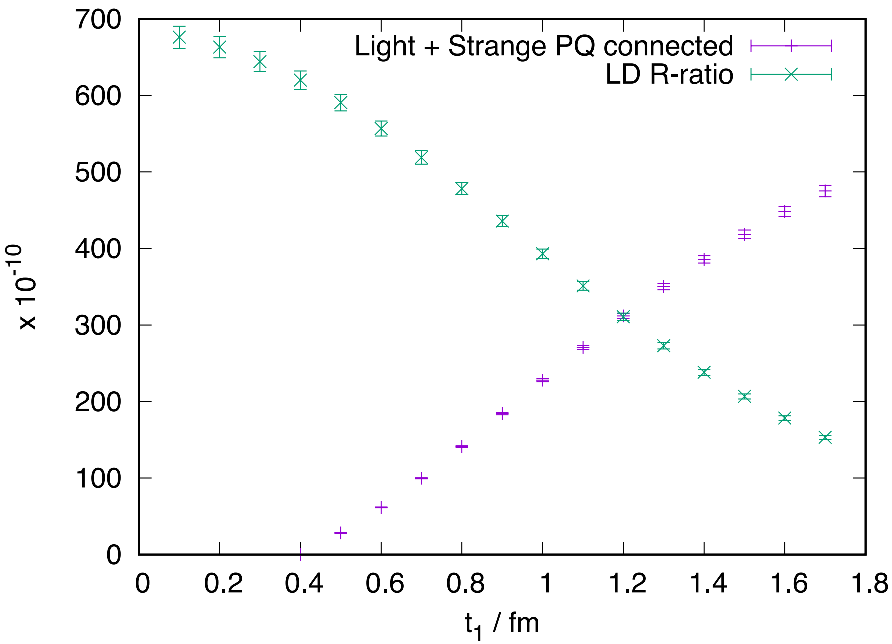
Blue : low-pass window
Green: high-pass window
Purple : total

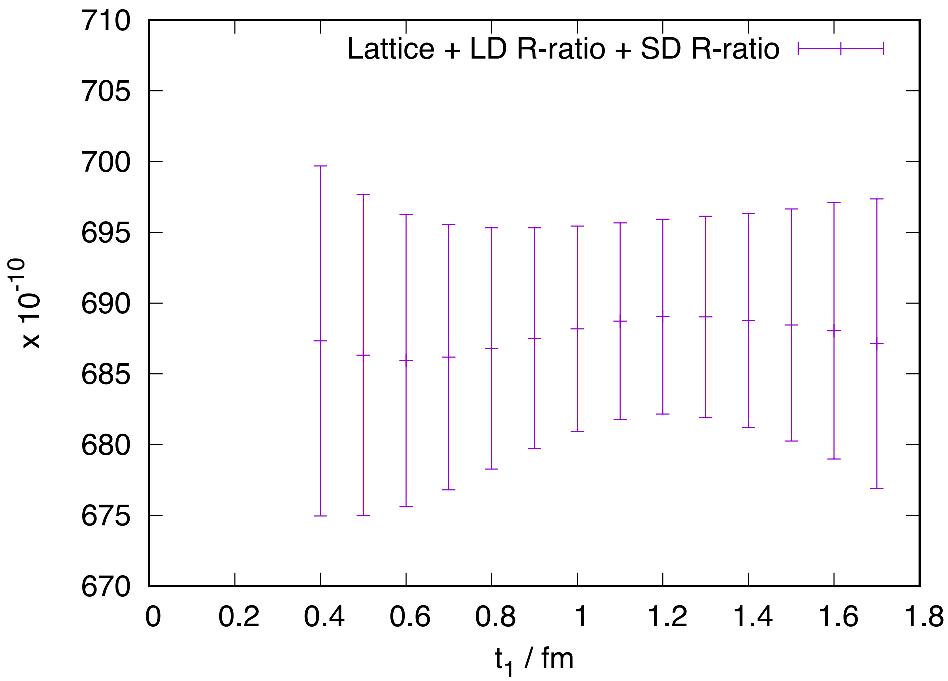
Most of $\pi\pi$ peak is captured by window from $t_0 = 0.4$ fm to $t_1 = 1.5$ fm, so replacing this region with lattice data reduces the dependence on BaBar versus KLOE data sets.



R-ratio + Lattice

t0 = 0.4 fm





t1 dependence is flat => a consistency between R-ratio and Lattice
t1 = 1.2 fm, R-ratio : Lattice = 50:50
t1=1.2 fm current error (note 100% correlation in R-ratio) is minimum

2022/2023 HVP update

- New fine ensemble 96I to check discretization error
- 6 accompanying smaller / heavy pion QCD samples to correct and check various small mistuning and systematic errors, and Nf=2+1+1 ensembles to check sea charm quark effects

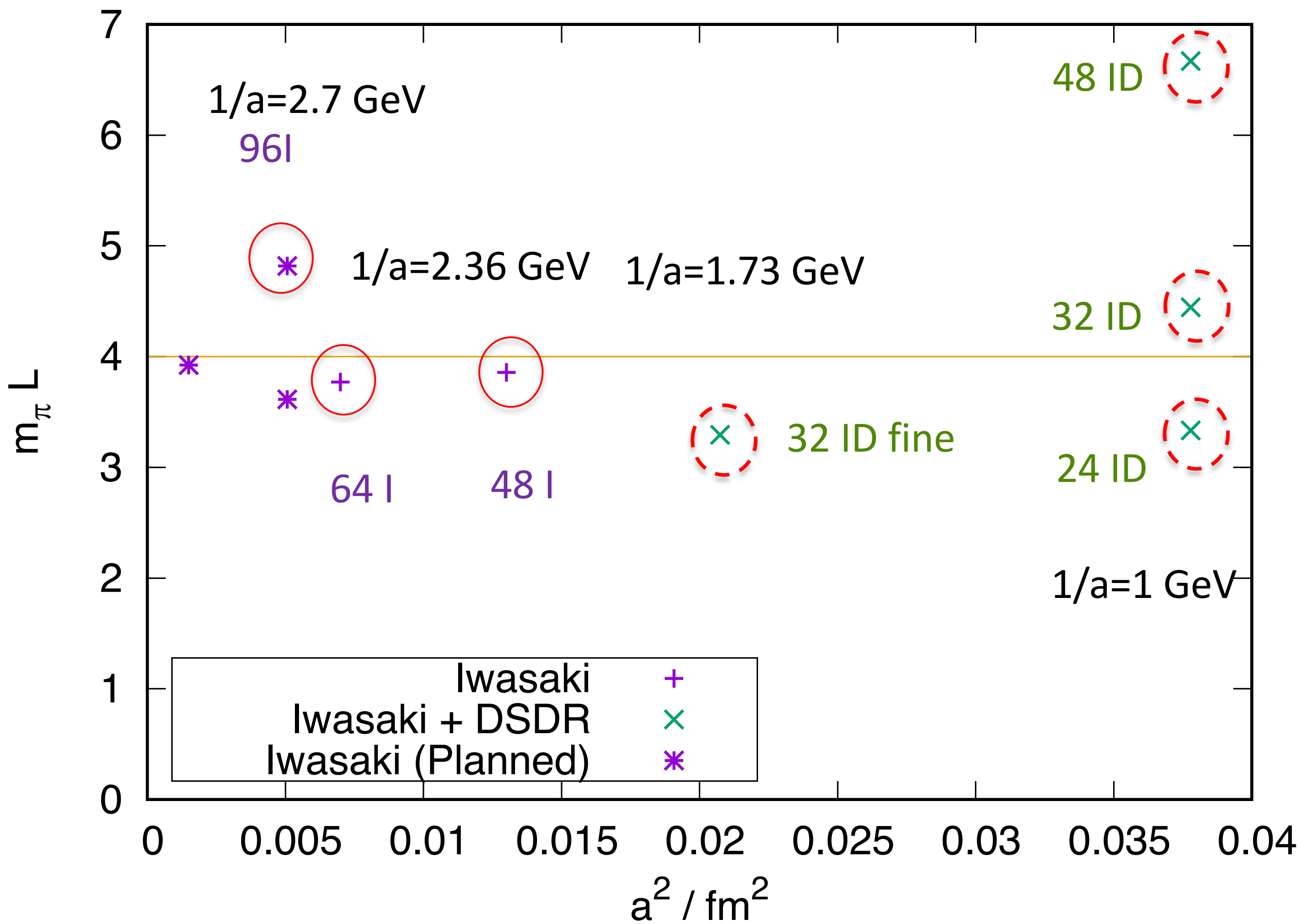
Name	m_l	m_s	L_s	m_π /GeV	m_K /GeV	a^{-1} /GeV	V	$m_\pi L$	β
48I	0.00078	0.0362	24	141	500	1.730	$48^3 \times 96$	3.92	2.13
48I_ml	0.0025	0.0362	24	210	512	1.730	$32^3 \times 64$	3.89	2.13
48I_ms	0.0025	0.05	24	208(2)	600(3)	1.730	$32^3 \times 64$	3.85	2.13
48I_ml2_Ls	0.0055	0.0368	32	288	530	1.730	$24^3 \times 48$	4.00	2.13
48I_ml2_Ls2	0.002356	0.03366	8	280	530	1.730	$24^3 \times 48$	3.88	2.12
48I_ml2	0.0049	0.0362	24	280	530	1.730	$24^3 \times 48$	3.88	2.13
64I_ml2	0.00372	0.0257	12	280	530	2.359	$32^3 \times 64$	3.80	2.25

- Blind analysis by 5 groups (HVP) and 2 groups (lattice scale and quark mass)

$$C_b(t) = (b_0 + b_1 a^2 + b_2 a^4) C_0(t)$$

- Among many other continuum extrapolation and Finite Volume correction were significant and scrutinized

New Nf=2+1 DWF QCD ensemble at physical quark mass



Lattice EM currents: Two Operators and Three normalization schemess

- $C(t) = \sum_{x,i} \langle V_i\ (x,t) V_i(0) \rangle$

- Two variants of lattice EM vector current $V_{i(x,t)}$ are used to check discretization error

  $V_i\ (x)\ = \bar{q}\ (x) \gamma_i q$(x) (local current)

  $V_i(x) = \overline{\psi(x)} (1+\gamma_i) U_i(x) \psi(x+\hat{\mu}) + c.c.$ (conserved current)

- EM vector current $V_i(x,t)$ on lattice is matched to continuum current multiplicatively

$$V_i(x; cont) = Z_V V\_i(x; lattice)$$

by matching matrix element of operator to a state

Three variants of states :

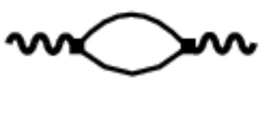
Z_V : 0 momentum single pion state,

Z_K : 0 momentum single Kaon state

Z_r : a 0 momentum state state specified by the Euclidean distance from another vector operator

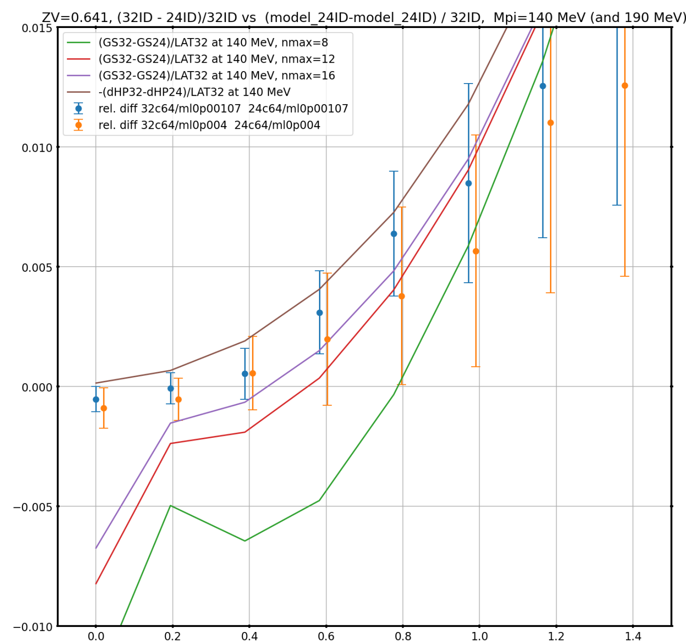
Finite Volume correction estimates

- FV correction by long-distance two pion contributions
  - scalar QED
  - Using pion form factor (Gounaris-Sakurai parametrization) & Lellouche Luscher's FV formula
  - Hansen-Patella FV correction

$$a_\mu^{\rm HVP,LO}(T,L) =$$ 

$$a_\mu^{HVP}(L = 6.22\ \mathrm{fm}) - a_\mu^{HVP}(L = 4.66\ \mathrm{fm})$$

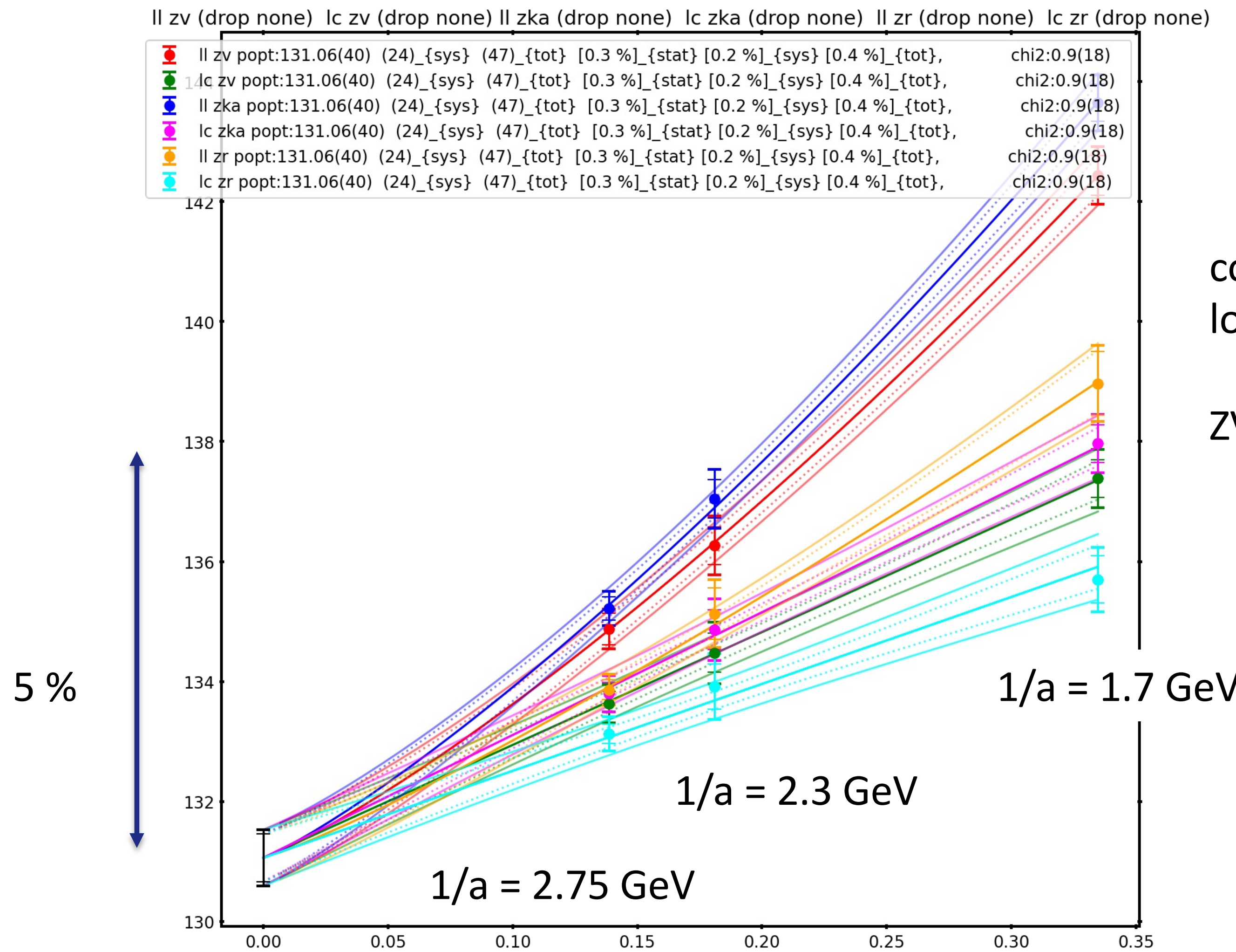
$$= \begin{cases} 12.2 \times 10^{-10} & \text{sQED} \\ 21.6(6.3) \times 10^{-10} & \text{LQCD} \\ 20(3) \times 10^{-10} & \text{GSL} \end{cases}$$



- Revised FV estimation :

$$a_\mu^{HVP}(L = \infty) - a_\mu^{HVP}(L = 5.47\ \mathrm{fm}) = 22(1) \times 10^{-10}$$

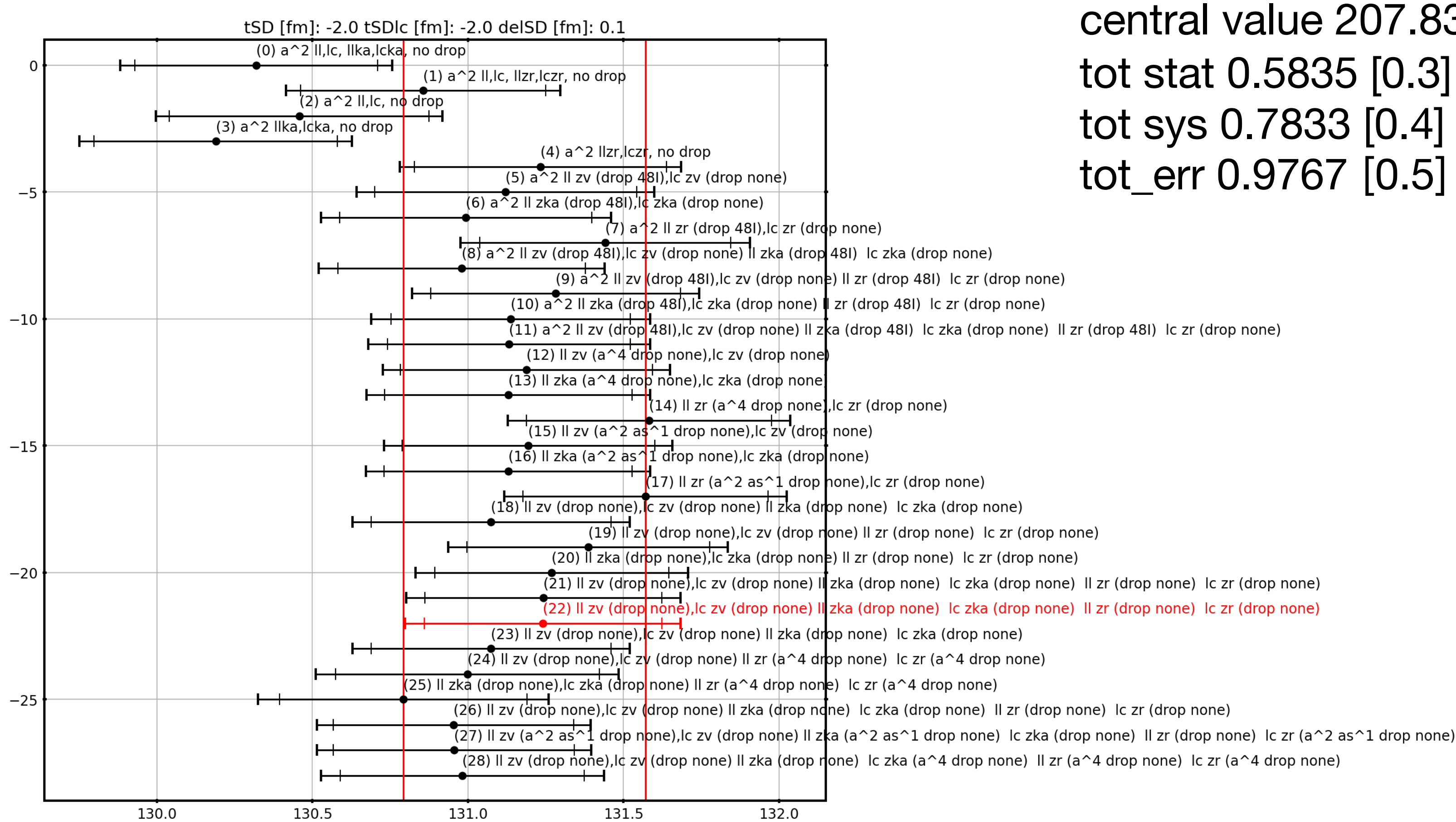
Continuum limit extrapolation window value [0.4 fm, 1.0 fm]



conserved-local
local-local

ZV,ZK, ZR

- Fit forms, a^2, a^2 + a^4, a^2 + a^2 log(a)
- Two currents x Three ZV



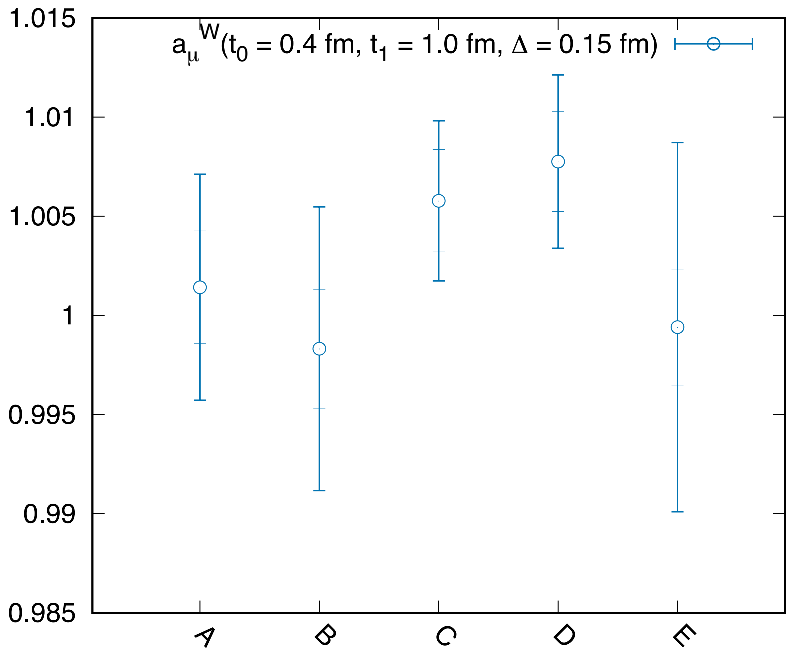
central value 207.8380
tot stat 0.5835 [0.3] %
tot sys 0.7833 [0.4] %
tot_err 0.9767 [0.5] %

Blind analysis

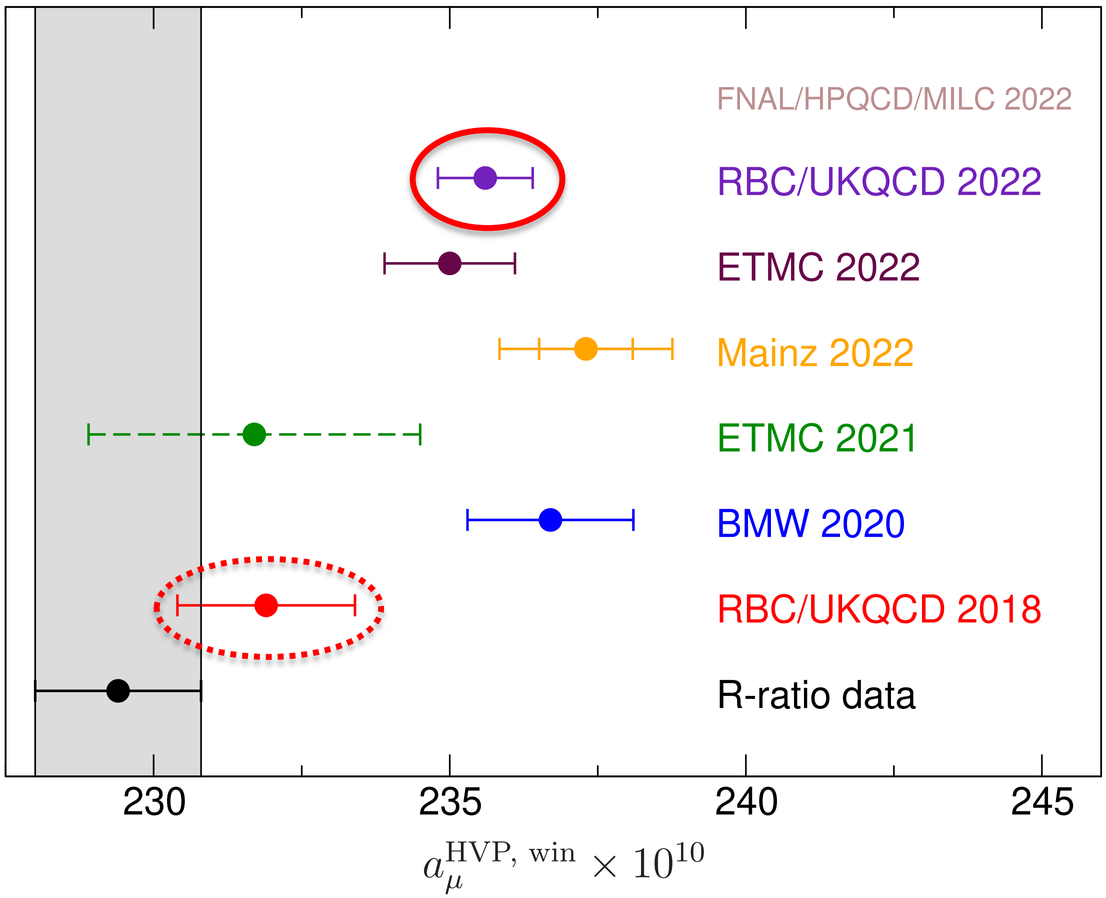
- Blinding procedure :
  lattice spacing dependent blind factor

$$C_b(t) = (b_0 + b_1 a^2 + b_2 a^4) C_0(t)$$

- 5 groups



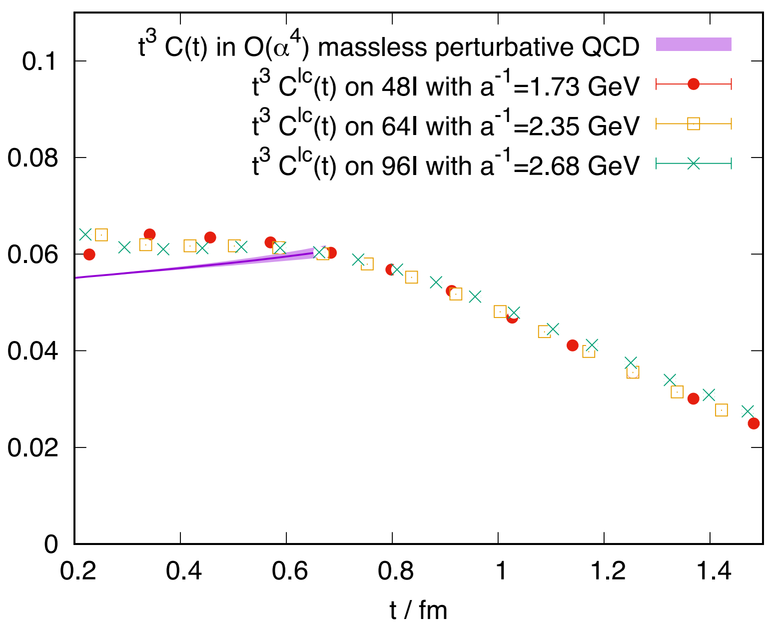
Mid-Window value of g-2
Intermediate energy region

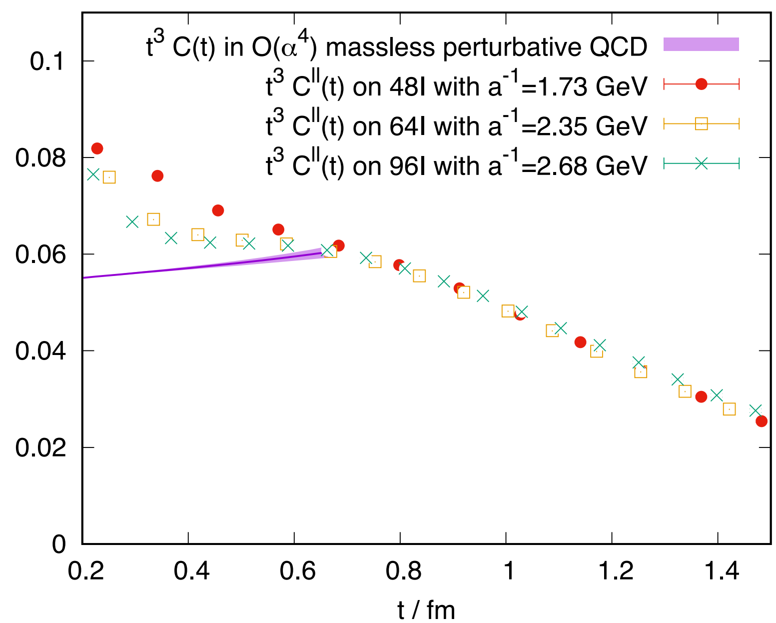


total amu ~ 700, exp vs theory tension ~ 25
mid-window value [0.4 fm, 1.0 fm] ~ rho meson peak

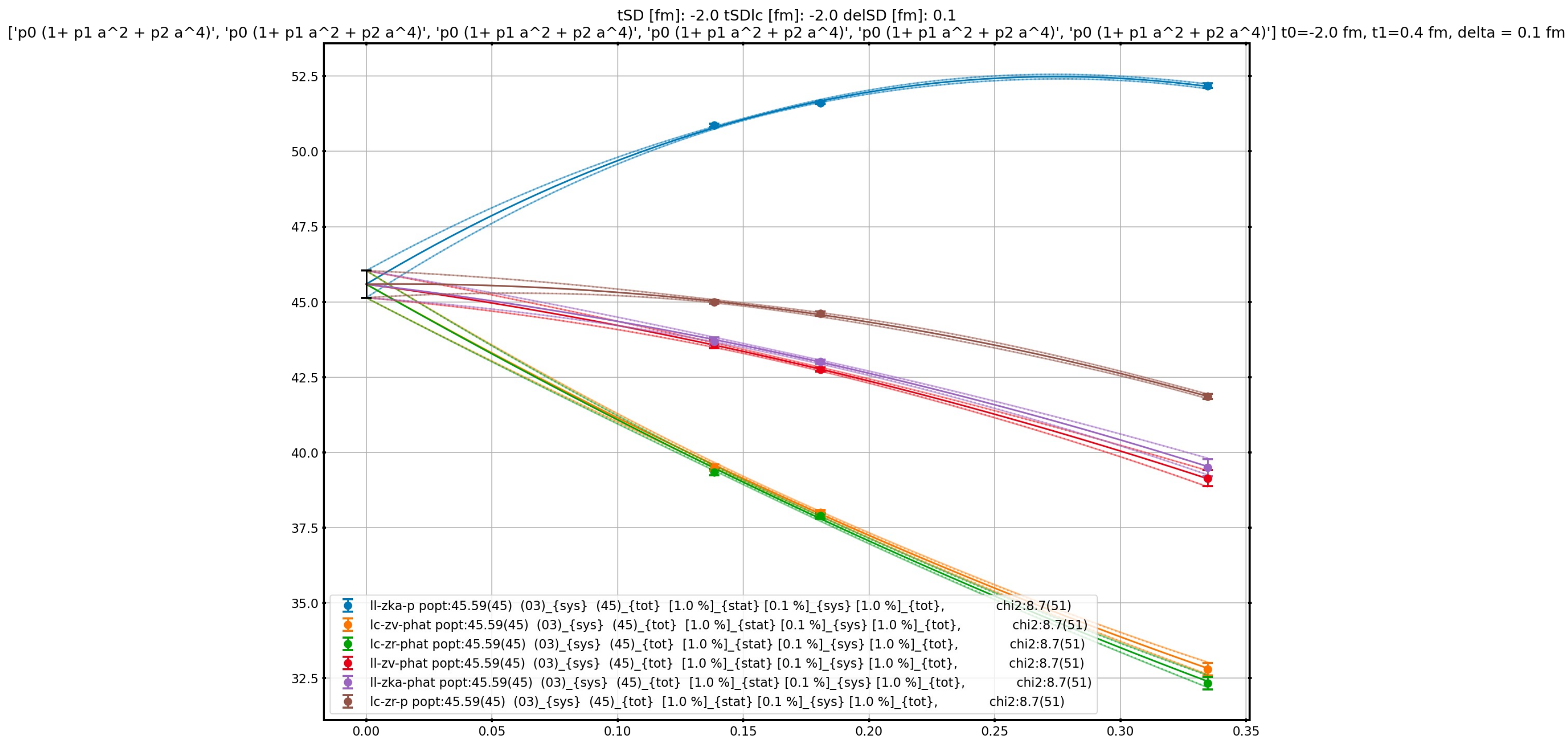
Short distance window discovered a large discretization error

- $t^3\,C(t)$ local-conserved vs local-local
- pQCD, 3 lattice spacings
- Large discretization error at short distance





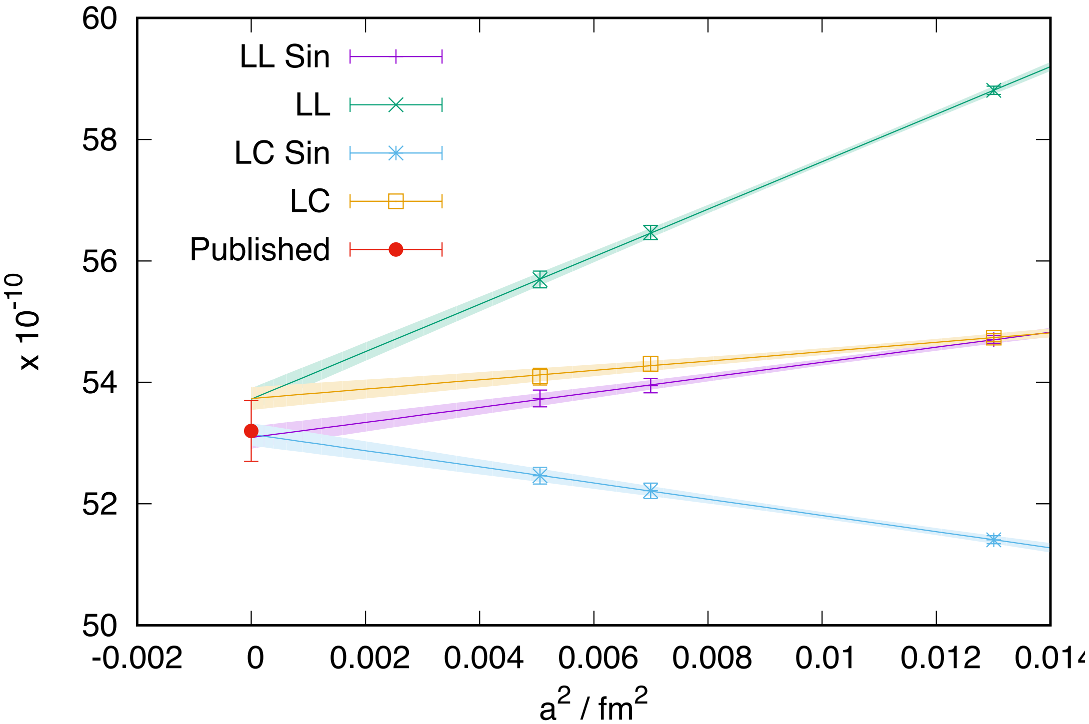
Short distance Window (after MF improved tree-level correction)



Strange quark contribution

Add $a^{-1} = 2.77$ GeV lattice spacing

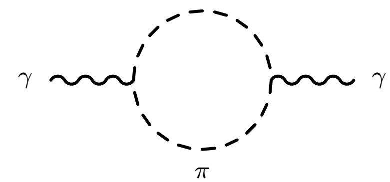
- Third lattice spacing for strange data ($a^{-1} = 2.77$ GeV with $m_\pi = 234$ MeV with sea light-quark mass corrected from global fit):



- For light quark need new ensemble at physical pion mass. Started run on Summit Machine at Oak Ridge this year ($a^{-1} = 2.77$ GeV with $m_\pi = 139$ MeV).

Reconstruction of HVP for LD from multi-channel Greens function

- Correlation function among N operators O_n, n=0,1,…, N-1



- Point (or smeared) vector $\mathcal{O}_0 = \sum_x \bar\psi(x)\gamma_\mu\psi(x),\ \mu \in \{1,2,3\}$
- 2 π operator $\mathcal{O}_n = \left|\sum_{xyz} \bar\psi(x) f(x-z) e^{-i\vec p_\pi \cdot \vec z}\gamma_5 f(z-y)\psi(y)\right|^2$
- (4 π operator) $\mathcal{O}_{4\pi} = \left|\sum_{xyz} \bar\psi(x) f(x-z) e^{-i\vec p_\pi \cdot \vec z}\gamma_5 f(z-y)\psi(y)\right|^2 \left|\sum_{xy} \bar\psi(x) f(x-y)\gamma_5\psi(y)\right|^2$

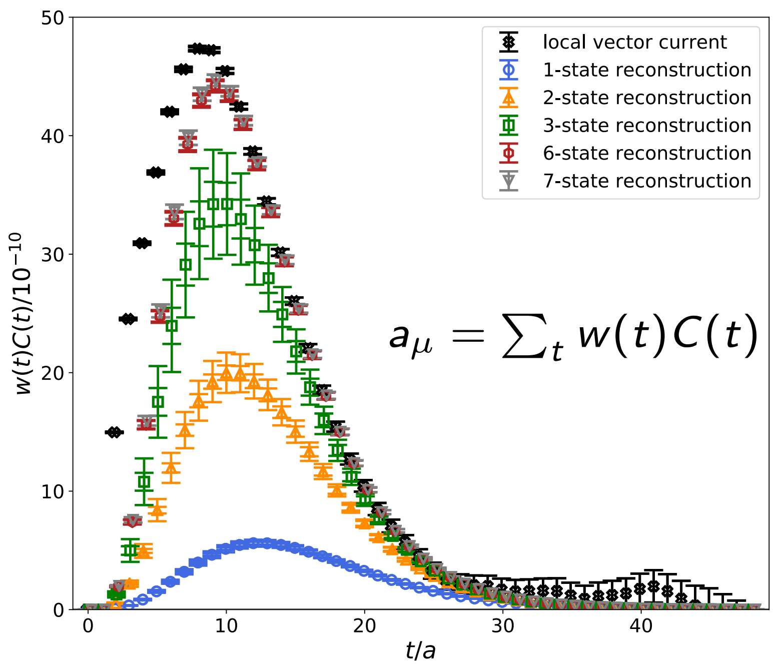
two pion
rho-resonance

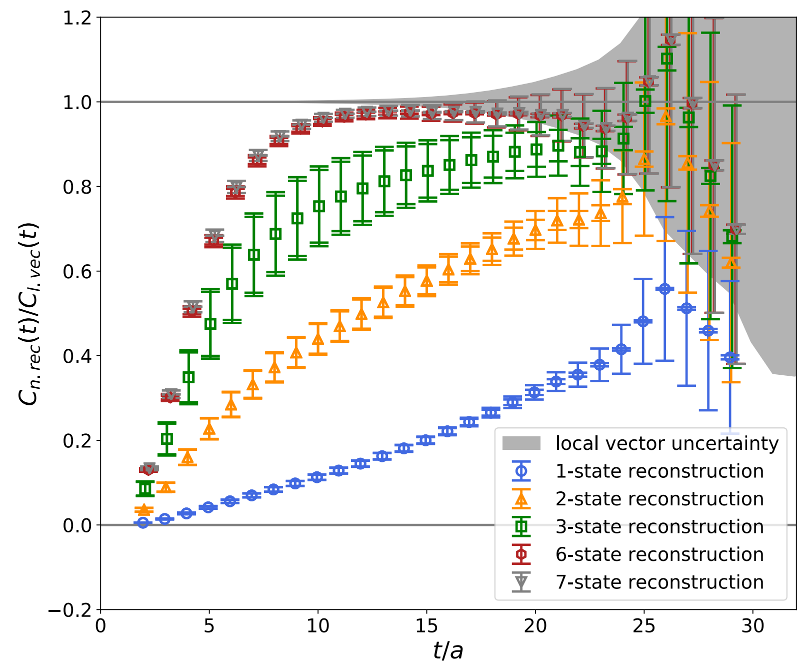
- NxN correlation function < O_i(t) O_j(0) > (using distillation)
- Solve NxN spectrum E_n of eigenstates |E_n> and Overwrap factors <E_n|O_0|0> (GEVP)
- Reconstruct V-V correlator, and bound contribution from the (N+1)-th states and above

$$\langle O_0(t) O_0^\dagger(0)\rangle = \sum_{n=0}^{N-1} |\langle 0|O_0|n\rangle|^2 e^{-E_n t}$$
$$+\ (\text{contributions from} \quad n \geq N \quad \text{states})$$

GEVP & Reconstruction of I=1 VV

[Aaron Meyer]





Left: a_μ integrand, Right: ratio reconstruction/local vector

- More states $\implies$ better reconstruction
- 6 state $\implies$ 1σ consistent at $t \geq 16a \sim 1.7$ fm

Bounds for a_μ

- Upper & lower bounds from unitarity

$$\widetilde{C}(t; t_{\max}, E) = \begin{cases} C(t) & t < t_{\max} \\ C(t_{\max}) e^{-E(t-t_{\max})} & t \geq t_{\max} \end{cases}$$

Upper bound: $E = E_0$, lowest state in spectrum

Lower bound: $E = \log[\frac{C(t_{\max})}{C(t_{\max}+1)}]$

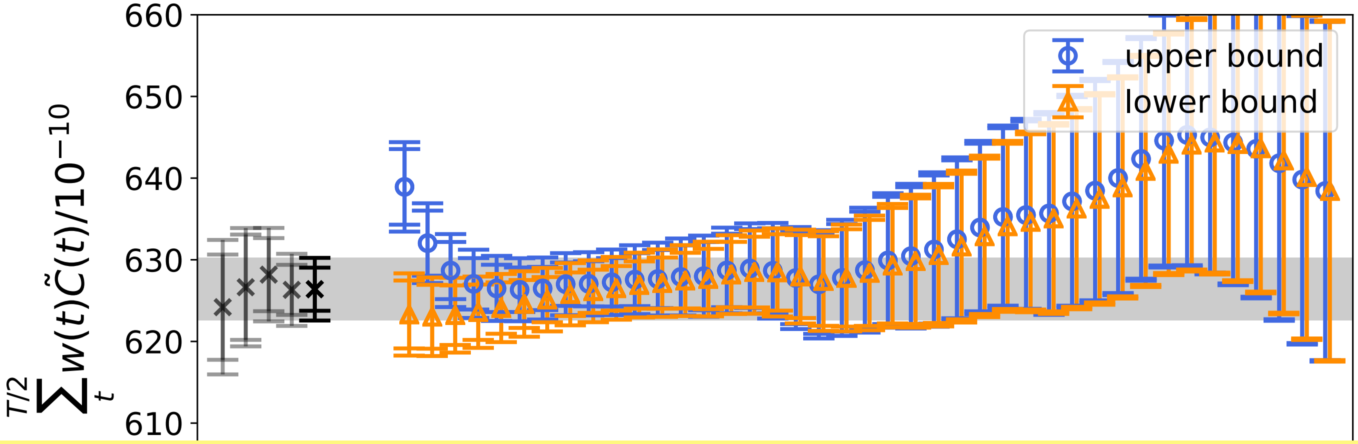
- Also bounds for the n in [N+1, ∞] states contribution

Replace $C(t) \to C(t) - \sum_n^N |c_n|^2 e^{-E_n t}$

$\Longrightarrow$ Long distance convergence now $\propto e^{-E_{N+1} t}$

$\Longrightarrow$ Smaller overall contribution from neglected states

GEVP + Bounding Method [A. Meyer]



a factor of 2.5 smaller statistical error by bounding method
a factor of 5 smaller statistical error by bounding method + 5 state reconstruction

continuum limit and other systematic studies are on-going (again blinding)

Bounding method $t_{\max} = 3.0$ fm, 1 state reconstruction: $a_\mu^{HVP} = 626.6(7.2)$
Bounding method $t_{\max} = 2.5$ fm, 2 state reconstruction: $a_\mu^{HVP} = 628.2(5.7)$
Bounding method $t_{\max} = 2.1$ fm, 5 state reconstruction: $a_\mu^{HVP} = 626.3(4.4)$
Bounding method $t_{\max} = 1.7$ fm, 6 state reconstruction:

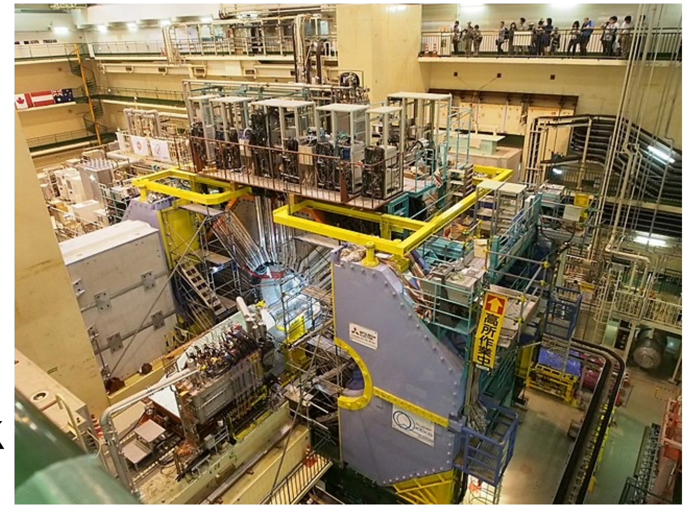
$$a_\mu^{HVP\text{,conn, iso, 48I}} = 626.6(2.7)_{\text{stat}}(0.4)_{Z_V\text{,48I}}(2.6)_{a^{-1}\text{,48I}}(0.5)_{\text{bound}}(0.5)_{\text{exc}}$$

Bounding method gives factor of 2.5 improvement over no bounding method

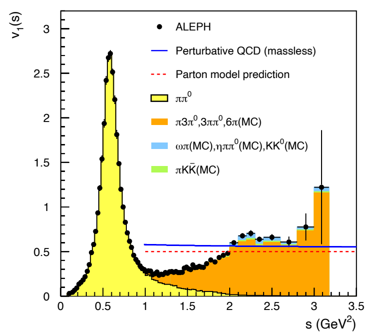
Improving the bounding method increases gain to factor of 5, including systematics

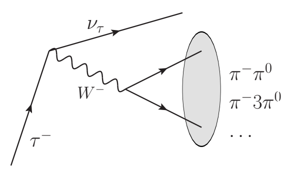
Use of tau decay data
[M. Bruno]

- Belle II : tau factory
- Isospin corrections from Lattice QCD
- Different systematic errors to cross-check



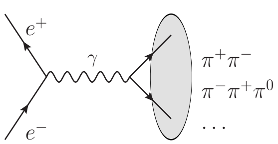
Motivations for τ





$V-A$ current

Final states $I=1$ charged



EM current

Final states $I=0,1$ neutral

τ data can improve $a_\mu[\pi\pi]$

$\to$ 72% of total Hadronic LO

$\to$ competitive precision on a_μ^W

Neutral vs Charged

$$\tfrac{i}{2}(\bar u\gamma_\mu u - \bar d\gamma_\mu d)\,,\begin{bmatrix} I=1 \\ I_3=0\end{bmatrix} \to j_\mu^{(1,-)} = \tfrac{i}{\sqrt{2}}(\bar u\gamma_\mu d)\,,\begin{bmatrix} I=1 \\ I_3=-1\end{bmatrix}$$

Isospin 1 charged correlator $G_{11}^W = \frac{1}{3}\sum_k \int d\boldsymbol{x}\,\langle j_k^{(1,+)}(x) j_k^{(1,-)}(0)\rangle$

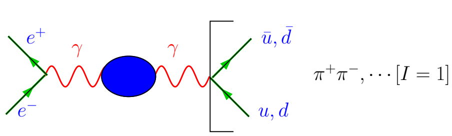
$\delta G_{11} \equiv G_{11}^\gamma - G_{11}^W$ [MB et al.' Latt18]

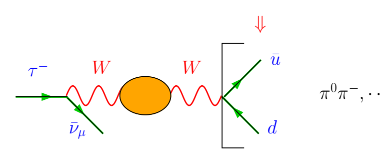
$$= Z_V^4 (4\pi\alpha)\frac{(Q_u - Q_d)^4}{4}\Big[\ \ +\ \ \Big]$$

$$G_{01}^\gamma = Z_V^4 \frac{(Q_u^2-Q_d^2)^2}{2}(4\pi\alpha)\Big[\ \ + 2\times\ \ +\ \ + \ldots\Big]$$

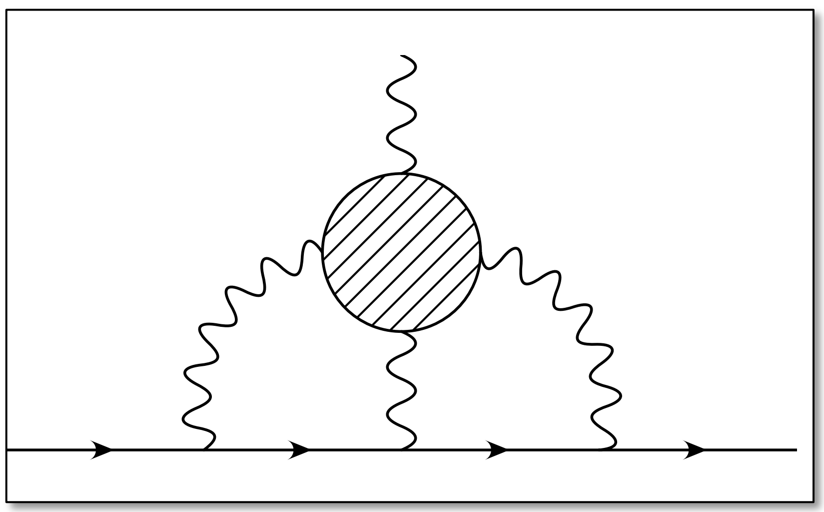
$$+ Z_V^2 \frac{Q_u^2 - Q_d^2}{2}(m_u - m_d)\Big[2\times\ \ + \ldots\Big]$$

$\ldots$ = subleading diagrams





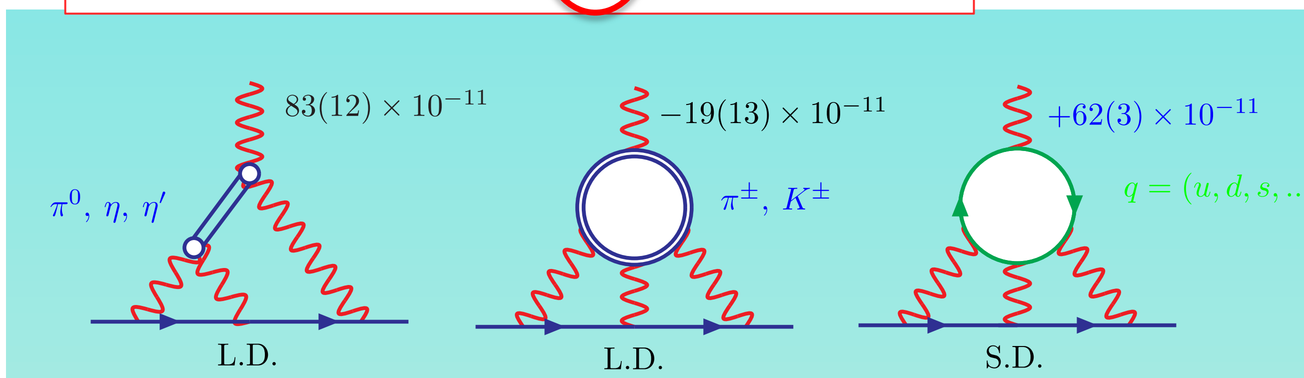
Hadronic Light-by-Light (HLbL) contributions



HLbL from Models

- Model estimate with non-perturbative constraints at the chiral / low energy limits using anomaly : (9—12) x 10^{-10} with 25-40% uncertainty

$$a_\mu^{\rm EXP} - a_\mu^{\rm SM} = 27.4\,\underbrace{(2.7)}_{\rm HVP}\underbrace{(2.6)}_{\rm HLbL}\underbrace{(0.1)}_{\rm other}\underbrace{(6.3)}_{\rm EXP}\times 10^{-10}$$



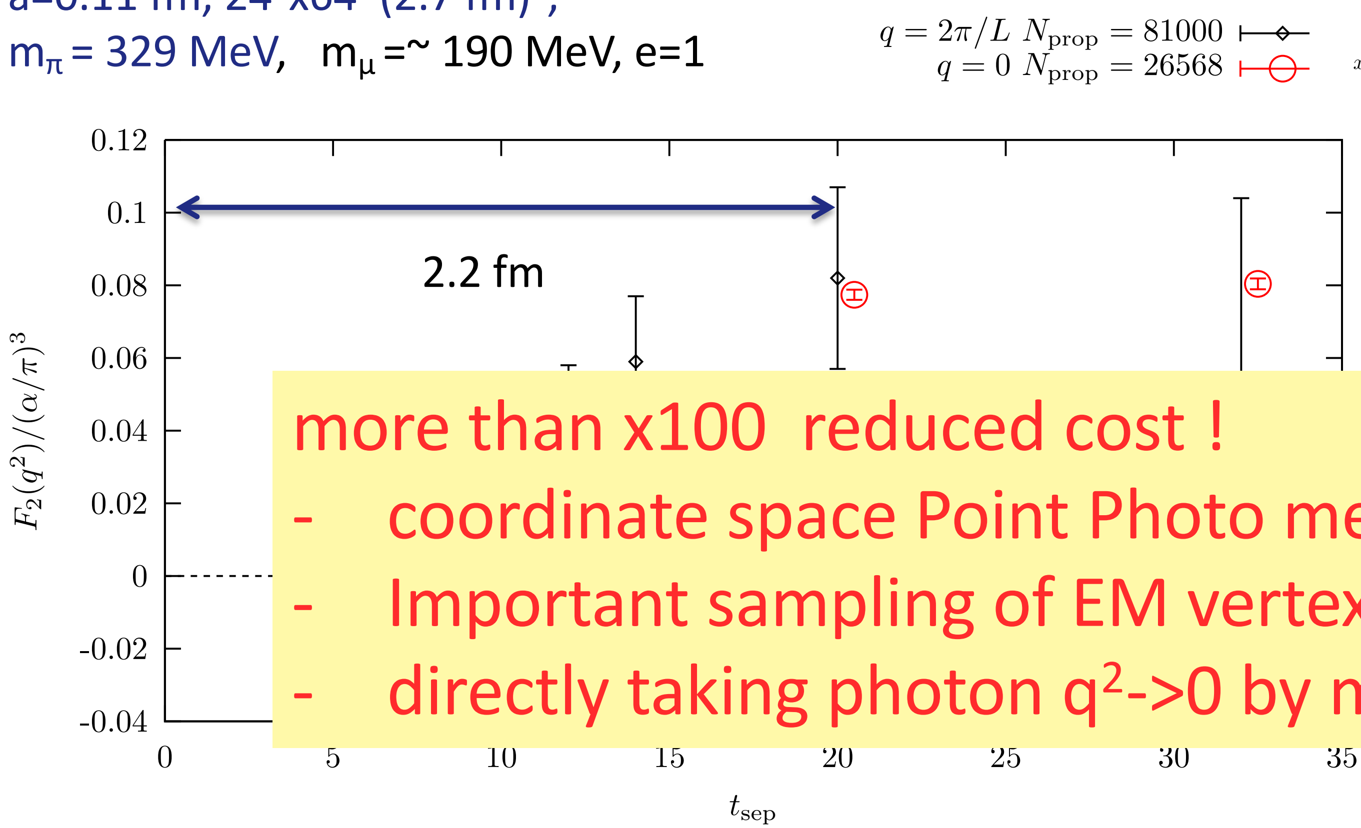
F. Jegerlehner , x 10^{11}

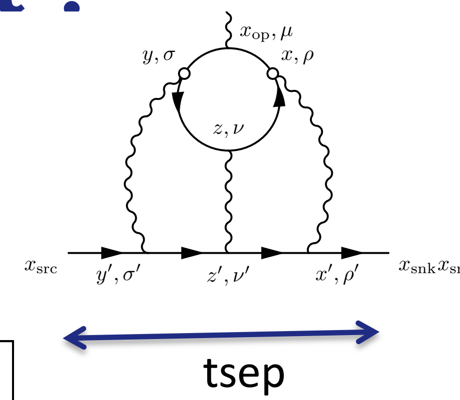
Contribution	BPP	HKS	KN	MV	PdRV	N/JN
π^0,η,η'	85±13	82.7±6.4	83±12	114±10	114±13	99±16
π, K loops	−19±13	−4.5±8.1	−	0±10	−19±19	−19±13
axial vectors	2.5±1.0	1.7±1.7	−	22± 5	15±10	22± 5
scalars	−6.8±2.0	−	−	−	−7± 7	−7± 2
quark loops	21± 3	9.7±11.1	−	−	2.3	21± 3
total	83±32	89.6±15.4	80±40	136±25	105±26	116±39

Dramatic Improvement !

Luchang Jin

a=0.11 fm, 24^3x64 $(2.7\ \text{fm})^3$,
m_π = 329 MeV, m_μ =~ 190 MeV, e=1



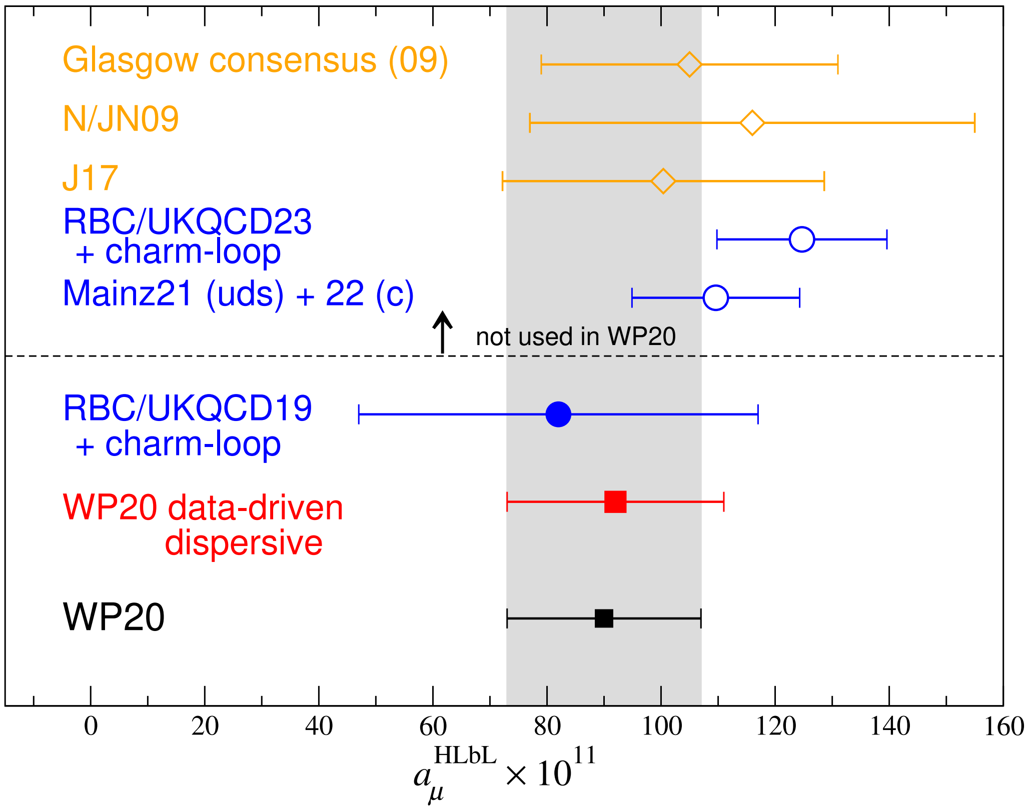


more than x100 reduced cost !

- coordinate space Point Photo method
- Important sampling of EM vertex
- directly taking photon q^2->0 by moment

Method	$F_2/(\alpha/\pi)^3$	$N_{\rm conf}$	$N_{\rm prop}$	$\sqrt{\rm Var}$
Conserved	0.0825(32)	12	$(118+128)\times 2 \times 7$	0.65
Mom.	0.0804(15)	18	$(118+128)\times 2 \times 3$	0.24

2023 HLbL status



[M. Hoferichter]

- Lattice QCD Mainz 2021, 2022:

$$a_\mu^{\mathrm{HLbL}}[uds] = 107(15) \times 10^{-11}$$

$$a_\mu^{\mathrm{HLbL}}[c] = 2.8(5) \times 10^{-11}$$

- New result RBC/UKQCD 2023:

$$a_\mu^{\mathrm{HLbL}}[uds] = 122(15) \times 10^{-11}$$

Summary

- FNAL muon g-2 Run-2, Run-3 results will be announced

  https://indico.fnal.gov/event/60738/

  August 10, 10AM central == August 11, 0:00 Japan

  expect a factor of 2 smaller error

- R-ratio data driven approach and muon g-2 has 4.2 σ tension,
- New CMD-3 two pion data is significantly larger, may make muon g-2 value consistent with SM, if difference between other R-ratio results will be understood.

- Lattice QCD
- For short distance and mid distance, lattice calculations are now mostly agreeing to each other, a tension with R-ratio may
- For window value [0.4 fm, 1.0 fm], adding new finer ensemble analysis, our 2018 analysis seems to underestimate a^4 discretization error

  (a Λ)^4 ~ 0.5 % for Λ = 0.4 fm
- BMW results so far only precise SD, MD, LD, close to g-2

- New full g-2 HVP results including Long Distance contribution soon

- Tau decay experiment input + Lattice IB correction may shed a light
- For HLbL, good agreement between Lattice and phenomenology, but needs another factor of 2 improvement for 10% relative error goal.