Lee-Wick's Complex Ghost violates Unitarity in Quadratic Gravity Theory

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1 Introduction

Lee-Wick: Finite QED '69 $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}\left(1 + \frac{\Box}{m^2}\right)F^{\mu\nu}$

Quadratic Gravity Theory $\mathcal{L} = m^2 R + \alpha R^2 + \beta R_{\mu\nu}R^{\mu\nu}$

Propagator

$$\frac{1}{k^2 + k^4/m^2} = \underbrace{\frac{1}{k^2}}_{\substack{\text{massless}\\ \text{positive metric}}} - \underbrace{\frac{1}{k^2 + m^2}}_{\substack{\text{massive (ghost)}\\ \text{negative metric}}}$$

Finite QED becomes finite. Quadratic gravity becomes renormalizable

But, Masssive Ghost $\Longrightarrow$ Unitarity is violated

Lee-Wick noted:

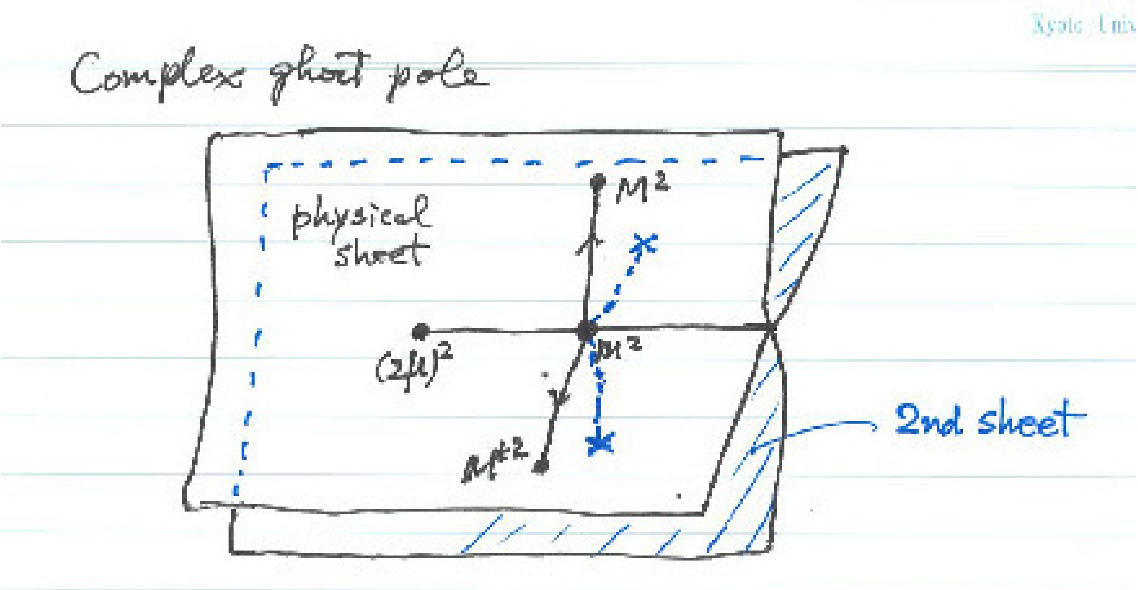
massive ghost can "decay" into lepton pair

Pole at one-loop : lepton-loop graph $\Sigma(q)$

$$q^2 + q^4/m^2 - \Sigma(q) = 0 \quad \rightarrow \quad q^2 = M^2,\ M^{*2}$$

$$\boxed{M^2 = m^2 + i\gamma^2}$$

Complex poles on the physical sheet! for ghost



Complex energy $E_{\boldsymbol{q}} = \sqrt{\boldsymbol{q}^2 + M^2} \quad \Rightarrow \quad$ Complex Ghost

$$\langle\phi\phi\rangle = \underset{\langle AA\rangle}{\frac{1}{q^2}} - \frac{1}{2}\Big(\underset{\langle\varphi\varphi\rangle}{\frac{1}{q^2+M^2}} + \underset{\langle\varphi^\dagger\varphi^\dagger\rangle}{\frac{1}{q^2+M^{*2}}} \Big)$$

$$\longrightarrow \quad \boxed{\phi = A + \frac{1}{\sqrt{2}}(\varphi + \varphi^\dagger)}$$

They argue:

Energy Conservation $\rightarrow$ Complex ghosts never produced from scattering of physical particles alone $\rightarrow$ physical unitarity

Even ghost-anti-ghost pair have complex energy except measure-zero points

$$E = \sqrt{\left(\boldsymbol{P}/2 + \boldsymbol{k}\right)^2 + M^2} + \sqrt{\left(\boldsymbol{P}/2 - \boldsymbol{k}\right)^2 + M^{*2}}$$
$$= \sum_{\pm} \sqrt{\boldsymbol{P}^2/4 \pm \boldsymbol{P}\cdot\boldsymbol{k} + \boldsymbol{k}^2 + m^2 \pm i\gamma^2}$$

is real only when $\boldsymbol{P} \perp \boldsymbol{k}$; measure zero (2d) points in 3d $\boldsymbol{k}$ space.

このトークでは

$$\boxed{\text{これは全くの誤解}} \tag{1}$$

で、Complex ghost はノンゼロノルムで現れることを示す。

不思議な事に

$$\begin{aligned} \text{Nakanishi} &\quad\rightarrow\quad \text{Lorentz inv. broken} \\ \text{Coleman} &\quad\rightarrow\quad \text{Causality, broken} \end{aligned} \tag{2}$$

と批判しながら、Energy 保存による unitarity の議論は認めている!

最近、Quadratic Gravity 理論で、この Lee-Wick complex ghost の議論が revival

Anselmi(2017,2018), Donoghue(2019, 2021)

これがOK なら、gravity は、Quadratic Gravity の理論だとすれば、摂動論的にくり込み可能で、UV complete となる！

2 Lee-Wick の間違いI: Energy保存と複素デルタ関数

”Energy 保存則とは？”

$$\int_{-\infty}^{\infty} e^{-iEt} dt = 2\pi\delta(E), \qquad E = \sum_i E_i$$

where E_i: energy of i-th particle coming into a interaction vertex.
E_i にcomplex energyが入っていると、we need regularization

$$\mathcal{L}_{\rm int}(t) \quad \to \quad e^{-a^2t^2}\mathcal{L}_{\rm int}(t)$$

and take $\lim_{a\to 0}$ finally.
Then, $\delta(E)$ replaced by

$$\Delta_a(z) := \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\, e^{-a^2t^2} e^{-izt} = \frac{1}{2\sqrt{\pi}\, a}\, e^{-z^2/a^2}\, . \tag{3}$$

Its $a \to 0$ limit defines Complex Delta function (distribution)

$$\lim_{a\to 0} \Delta_a(z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dt\, e^{-izt} =: \delta_{\rm c}(z)\ , \tag{4}$$

このdistributionのsupportは、$z = 0$ に局在しているわけではない!

Consider the single ghost production in Lee's scalar model[1]

$$\psi(\boldsymbol{p}_1) + \psi_(\boldsymbol{p}_2) \to \phi(\boldsymbol{q}), \qquad \text{by} \quad \mathcal{L}_{\rm int} = \frac{f}{2}\psi^2\phi$$

. Then, the invariant amplitude square (Ghost production probabolity) is proportional to

$$|\mathcal{M}|^2 \sim f^2 \left(\frac{1}{\omega_{\boldsymbol{q}}}\delta_c(E - \omega_{\boldsymbol{q}}) + \frac{1}{\omega^*_{\boldsymbol{q}}}\delta_c(E - \omega^*_{\boldsymbol{q}}) \right) \qquad \omega_{\boldsymbol{q}} = \sqrt{\boldsymbol{q}^2 + M^2}$$

and $E = \sqrt{\boldsymbol{p}_1^2 + \mu^2} + \sqrt{\boldsymbol{p}_2^2 + \mu^2}$

So let us examine

$$\Delta_a(E - \omega) \propto \frac{1}{a} e^{-(E-\omega)^2/a^2} = \frac{1}{a} \exp\left[-\frac{(E - {\rm Re}\,\omega)^2 - ({\rm Im}\,\omega)^2}{a^2} \right] \cdot e^{i\Theta}$$

$$\text{with} \quad \Theta = \frac{2}{a^2}(E - {\rm Re}\,\omega){\rm Im}\,\omega.$$

Since

$$(E - {\rm Re}\,\omega)^2 - ({\rm Im}\,\omega)^2 = (E - {\rm Re}\,\omega + {\rm Im}\,\omega)(E - {\rm Re}\,\omega - {\rm Im}\,\omega)$$

[1] next section

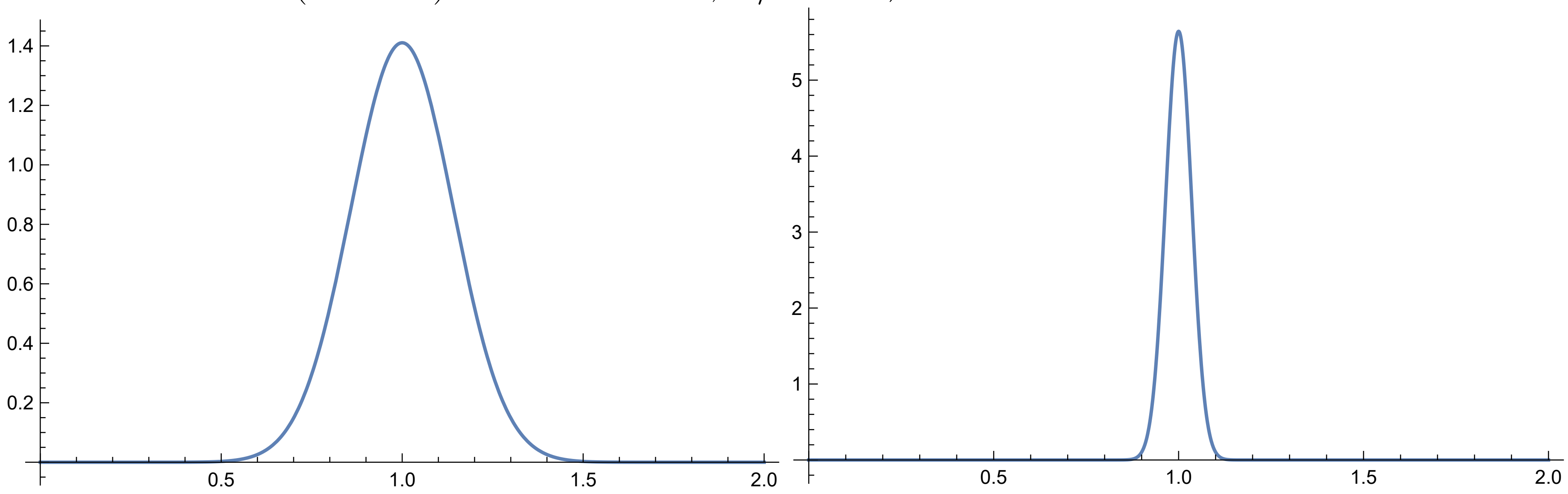
we have

$$\delta_{\mathrm{c}}(E-\omega)=\lim_{a\to 0}\Delta_a(E-\omega)=0 \quad \text{for} \quad \begin{cases} E<\mathrm{Re}\,\omega-\mathrm{Im}\,\omega \\ \mathrm{Re}\,\omega+\mathrm{Im}\,\omega<E \end{cases} . \tag{5}$$

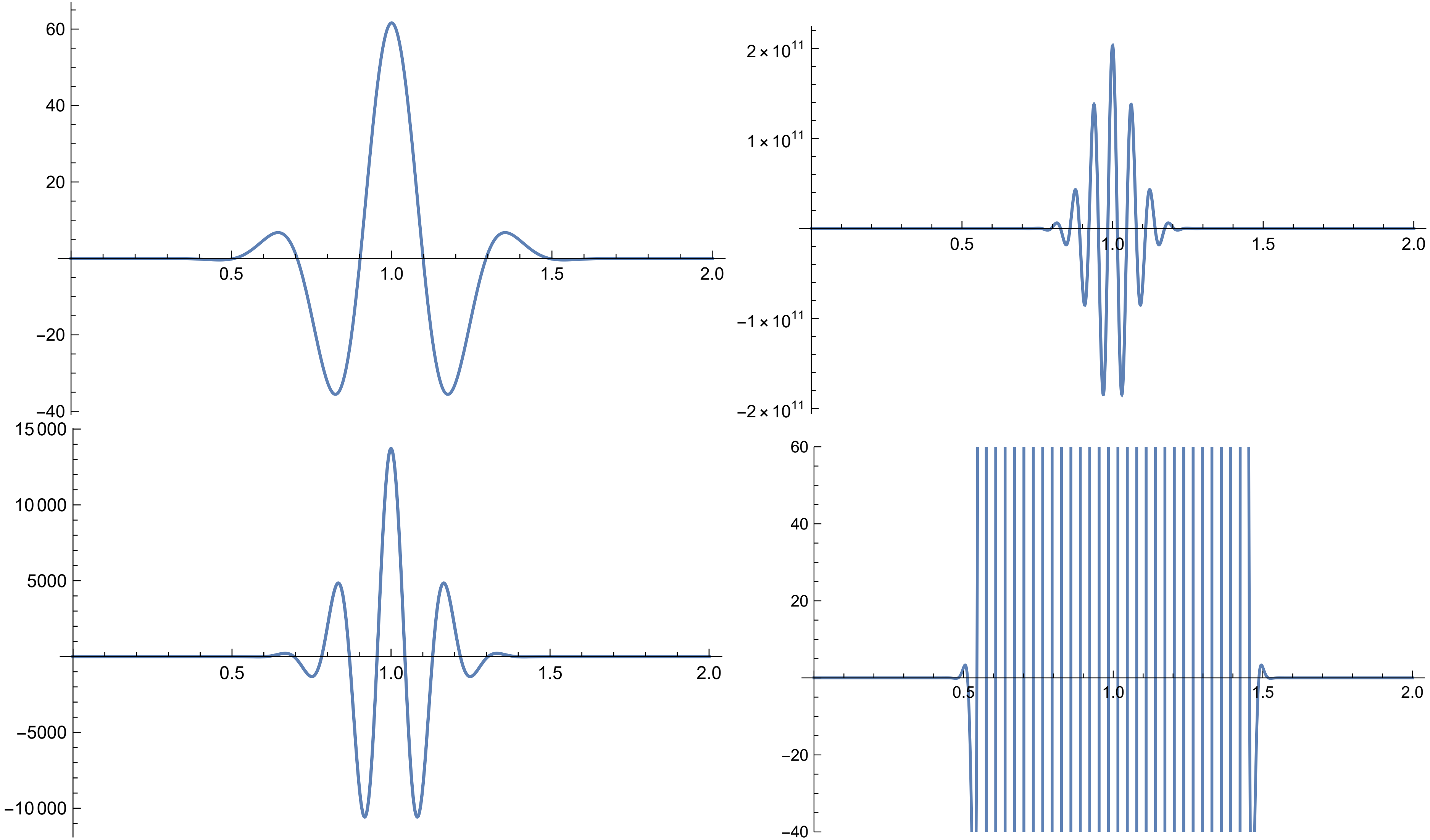
So, in the limit $a \to 0$, it has the support only in $|E-\mathrm{Re}\,\omega| \leq \mathrm{Im}\,\omega$. On the support, it has no definite limit; since it is divergent and rapidly oscillating. It gives a well-defined distribution. It is as usual for the distribution.

$\delta_{\mathrm{c}}(E-\omega)=\lim_{a\to 0}\Delta_a(E-\omega)$ の図

- Dirac delta $\delta(E-\omega)$: $\omega=1+0i,\ 1/a=5,20$



- Complex delta $\delta_c(E-\omega)$: $\omega = 1+0.5i$, $1/a = 4, 10$



左下:$1/a = 6$, $E_{\mathrm{Lth}} = \mathrm{Re}\,\omega - \mathrm{Im}\,\omega = 0.5$, $E_{\mathrm{Uth}} = \mathrm{Re}\,\omega + \mathrm{Im}\,\omega = 1.5$

Property of δ_{c}

For test fn $\forall f(E)$: analytic in a rectangular strip D,

$$\boxed{\int_R dE\, \delta_{\mathrm{c}}(E-\omega)\, f(E) = f(\omega)} \tag{6}$$

($\because$)

$$\int_{-\infty}^{\infty} dE\, \delta_{\mathrm{c}}(E-\omega)\, f(E) = \lim_{a\to 0} \int_{-\infty}^{\infty} dE\, \frac{1}{2a\sqrt{\pi}} \exp\left[-\frac{(E-\omega)^2}{a^2}\right] f(E)$$

is evaluated by deforming the contour

$$\begin{aligned} R[-\infty \to +\infty] \;\Rightarrow\; & \quad C_1[-\infty + i0 \to -\infty + i\mathrm{Im}\,\omega] \\ & + R'[-\infty + i\mathrm{Im}\,\omega \to +\infty + i\mathrm{Im}\,\omega] \\ & + C_2[+\infty + i\mathrm{Im}\,\omega \to +\infty + i0] \end{aligned} \tag{7}$$

If $f(E)$ が D $(= [C_1 + R' + C_2 - R]$で囲まれた短冊領域）で正則

$$\begin{aligned} \int_{R'} dE\, \delta_{\mathrm{c}}(E-\omega) f(E) &= \int_{-\infty}^{\infty} dE'\, \delta(E' - \mathrm{Re}\,\omega) f(E' + i\mathrm{Im}\,\omega) \\ &= f(\mathrm{Re}\,\omega + i\mathrm{Im}\,\omega) = f(\omega)\,.\ \text{q.e.d.} \end{aligned}$$

しかし、Feynman graphの計算では、$f(E)$ はmeromorphic.

Well-defined distribution?
実際の実験では、入射エネルギーに幅σがある：

$$f_{P^0}(E) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2}\left(\frac{E-P^0}{\sigma}\right)^2\right]$$

Then, the ghost production probability becomes

$$\begin{aligned} P_\varphi &= -\mathrm{Re}\left[f^2 2\pi \int_{-\infty}^{\infty} dE\, f_{P^0}(E)\delta_c(E-\omega)\frac{1}{2\omega}\right] \\ &= -\mathrm{Re}\left[f^2\frac{\pi}{\omega}f_{P^0}(\omega)\right] = -\frac{f^2}{\sqrt{2\pi\sigma^2}}\mathrm{Re}\left[\frac{\pi}{\omega}\exp\left[-\frac{1}{2}\left(\frac{\omega-P^0}{\sigma}\right)^2\right]\right]. \end{aligned}$$

This is finite.

Complex Ghost は、$\mathrm{Re}\,\omega - \mathrm{Im}\,\omega < E < \mathrm{Re}\,\omega + \mathrm{Im}\,\omega$ のエネルギー範囲で有限確率で生成される。

3 Lee's Model in "Quanta"

漸近場: ghost 場 $\phi = \underset{\text{'photon'}}{A} + \underbrace{\frac{1}{\sqrt{2}}(\varphi + \varphi^\dagger)}_{=B}$

matter 場 ψ

$$\mathcal{L} = \mathcal{L}_\phi + \mathcal{L}_\psi + \mathcal{L}_{\text{int}}$$

$$\mathcal{L}_\phi = -\frac{1}{2}\left[(\partial_\mu A)^2 + \delta^2 A^2\right] + \frac{1}{2}\left[(\partial_\mu B)^2 + m^2 B^2\right] - \frac{1}{2}\left[(\partial_\mu C)^2 + m^2 C^2\right]$$
$$\underline{-\gamma^2 BC}$$
$$= -\frac{1}{2}\left[(\partial_\mu A)^2 + \delta^2 A^2\right] + \frac{1}{2}\left[\partial_\mu\varphi\,\partial^\mu\varphi + M^2\varphi^2 + \partial_\mu\varphi^\dagger\,\partial^\mu\varphi^\dagger + M^{*2}\varphi^{\dagger 2}\right],$$
$$\mathcal{L}_{\text{matter}} = -\frac{1}{2}(\partial_\mu\psi)^2 - \frac{1}{2}\mu^2\psi^2$$

where $M^2 = m^2 + i\gamma^2$ and

$$\varphi = \frac{1}{\sqrt{2}}(B - iC) \qquad \text{or} \qquad \begin{cases} B = (\varphi + \varphi^\dagger)/\sqrt{2} \\ C = i(\varphi - \varphi^\dagger)/\sqrt{2} \end{cases}.$$

$$\mathcal{L}_{\text{int}}(\phi,\psi) = f\psi^2\phi, \quad f\psi^2\phi^2, \cdots \tag{8}$$

'Photon' A and ghost φ , anti-ghost$\varphi^\dagger$ interact only through $\mathcal{L}_{\text{int}}(\phi,\psi)$

Free fields:

$$A(x) = \int \frac{d^3\boldsymbol{q}}{\sqrt{(2\pi)^3 2\nu_{\boldsymbol{q}}}} \left(a(\boldsymbol{q}) e^{i\boldsymbol{q}\boldsymbol{x} - i\nu_{\boldsymbol{q}} x^0} + a^\dagger(\boldsymbol{q}) e^{-i\boldsymbol{q}\boldsymbol{x} + i\nu_{\boldsymbol{q}} x^0} \right), \quad \nu_{\boldsymbol{q}} = \sqrt{\boldsymbol{q}^2 + \delta^2}$$

$$\psi(x) = \int \frac{d^3\boldsymbol{p}}{\sqrt{(2\pi)^3 2E_{\boldsymbol{p}}}} \left(d(\boldsymbol{p}) e^{i\boldsymbol{p}\boldsymbol{x} - iE_{\boldsymbol{p}} x^0} + d^\dagger(\boldsymbol{p}) e^{-i\boldsymbol{p}\boldsymbol{x} + i\nu_{\boldsymbol{p}} x^0} \right), \quad E_{\boldsymbol{p}} = \sqrt{\boldsymbol{p}^2 + \mu^2} \ .$$

$$[a(\boldsymbol{p}), a^\dagger(\boldsymbol{q})] = [d(\boldsymbol{p}), d^\dagger(\boldsymbol{q})] = -\delta^3(\boldsymbol{p} - \boldsymbol{q}),$$

Ghost

$$\varphi(x) = \int \frac{d^3\boldsymbol{q}}{\sqrt{(2\pi)^3 2\omega_{\boldsymbol{q}}}} \left(\alpha(\boldsymbol{q}) e^{i\boldsymbol{q}\boldsymbol{x} - i\omega_{\boldsymbol{q}} x^0} + \beta^\dagger(\boldsymbol{q}) e^{-i\boldsymbol{q}\boldsymbol{x} + i\omega_{\boldsymbol{q}} x^0} \right)$$

where $\omega_{\boldsymbol{q}}$ is the complex energy

$$\omega_{\boldsymbol{q}} = \sqrt{\boldsymbol{q}^2 + M^2} = \sqrt{\boldsymbol{q}^2 + m^2 + i\gamma^2} \tag{9}$$

CCR

$$\begin{aligned} [\alpha(\boldsymbol{p}), \beta^\dagger(\boldsymbol{q})] &= [\beta(\boldsymbol{p}), \alpha^\dagger(\boldsymbol{q})] = -\delta^3(\boldsymbol{p} - \boldsymbol{q}), \\ [\alpha(\boldsymbol{p}), \alpha^\dagger(\boldsymbol{q})] &= [\beta(\boldsymbol{p}), \beta^\dagger(\boldsymbol{q})] = 0. \end{aligned}$$

1-ghost states

$$|\alpha(\boldsymbol{p})\rangle := \alpha^\dagger(\boldsymbol{p})\,|0\rangle\,, \quad |\beta(\boldsymbol{p})\rangle := \beta^\dagger(\boldsymbol{p})\,|0\rangle$$

have off-diagonal innerproduct structure

$$\begin{aligned} \langle\alpha(\boldsymbol{p})\,|\alpha(\boldsymbol{q})\rangle &= 0, \qquad \langle\beta(\boldsymbol{p})\,|\alpha(\boldsymbol{q})\rangle = \delta^3(\boldsymbol{p} - \boldsymbol{q}), \\ \langle\beta(\boldsymbol{p})\,|\beta(\boldsymbol{q})\rangle &= 0, \qquad \langle\alpha(\boldsymbol{p})\,|\beta(\boldsymbol{q})\rangle = \delta^3(\boldsymbol{p} - \boldsymbol{q}). \end{aligned}$$

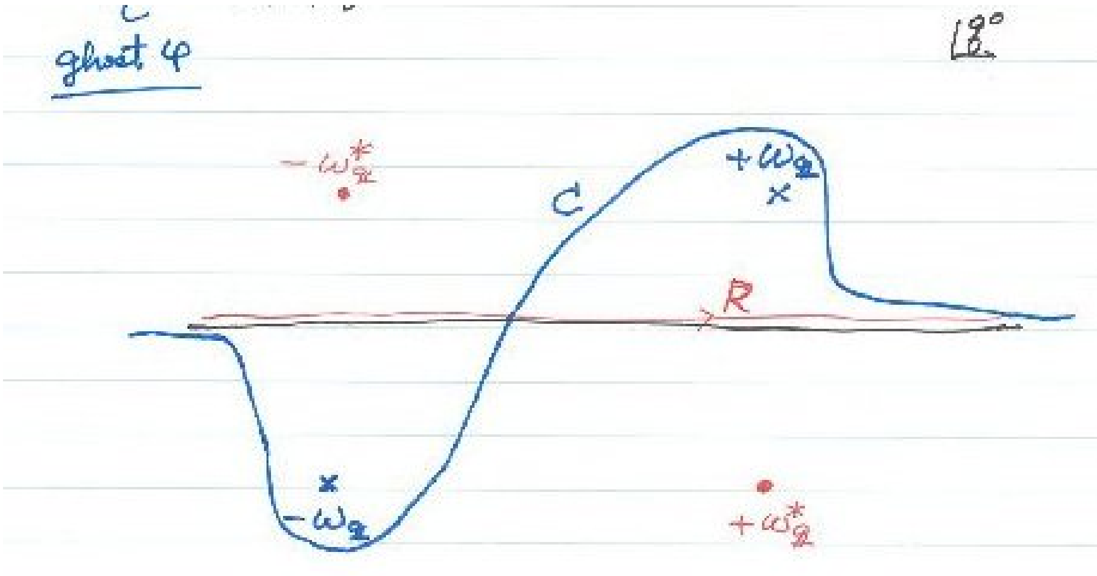
Propagator:

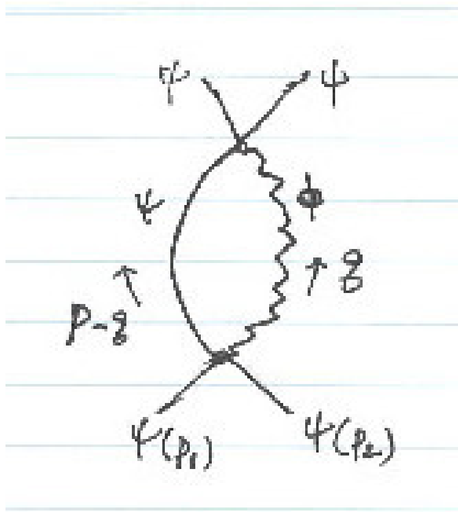
$$\begin{aligned}
&\langle 0|\, \mathrm{T}\varphi(x)\, \varphi(y)\, |0\rangle \\
&= \int \frac{d^3\boldsymbol{q} d^3\boldsymbol{p}}{(2\pi)^3\sqrt{2\omega_{\boldsymbol{q}} 2\omega_{\boldsymbol{p}}}} \Big\{ \theta(x^0 - y^0) e^{i(\boldsymbol{q}\boldsymbol{x} - \omega_{\boldsymbol{q}} x^0) - i(\boldsymbol{p}\boldsymbol{y} - \omega_{\boldsymbol{p}} y^0)} \langle 0|\, \alpha(\boldsymbol{q}) \beta^\dagger(\boldsymbol{p})\, |0\rangle \\
&\qquad + \theta(y^0 - x^0) e^{i(\boldsymbol{p}\boldsymbol{y} - \omega_{\boldsymbol{p}} y^0) - i(\boldsymbol{q}\boldsymbol{x} - \omega_{\boldsymbol{q}} x^0)} \langle 0|\, \alpha(\boldsymbol{p}) \beta^\dagger(\boldsymbol{q})\, |0\rangle \Big\} \\
&= - \int \frac{d^3\boldsymbol{q}}{(2\pi)^3 2\omega_{\boldsymbol{q}}} \Big\{ \theta(x^0 - y^0) e^{i\boldsymbol{q}(\boldsymbol{x} - \boldsymbol{y}) - i\omega_{\boldsymbol{q}}(x^0 - y^0)} + \theta(y^0 - x^0) e^{-i\boldsymbol{q}(\boldsymbol{x} - \boldsymbol{y}) + i\omega_{\boldsymbol{q}}(x^0 - y^0)} \Big\}
\end{aligned}$$

Note that the over-all minus sign implying negative norm.

This 3d expression over $d^3\boldsymbol{q}$ can be rewritten into the usual 4d form over $d^4q = d^3\boldsymbol{q}dq^0$

$$= -\int \frac{d^3q}{(2\pi)^3} e^{i\boldsymbol{q}(\boldsymbol{x}-\boldsymbol{y})} \left[\int_C \frac{dq^0}{2\pi i} \frac{e^{-iq^0(x^0-y^0)}}{q^2+M^2} \right], \qquad (10)$$





4 Lee-Wickの間違いII: 複素デルタ関数の使い方

Single ghsot ϕ + matter ψ production

$$\psi(\boldsymbol{p}_1) + \psi(\boldsymbol{p}_2) \quad \rightarrow \quad \phi(\boldsymbol{q}) + \psi(\boldsymbol{p} - \boldsymbol{q})$$

$(\boldsymbol{p}_1 + \boldsymbol{p}_2 \equiv \boldsymbol{p})$ の cross section を

$$\psi(\boldsymbol{p}_1) + \psi(\boldsymbol{p}_2) \quad \underset{(\phi(\boldsymbol{q})+\psi(\boldsymbol{P}-\boldsymbol{q})\text{ の Loop})}{\longrightarrow} \quad \psi(\boldsymbol{p}_1) + \psi(\boldsymbol{p}_2)$$

の前方散乱振幅の Imaginary part から計算。　Figure

$$\mathcal{L}_{\rm int} = \frac{1}{3!}\psi^3\phi$$

Lee wrote $\quad [\ \phi_i = (A, \varphi, \varphi^\dagger),\ M_i^2 = (\delta^2, M^2, M^{*2})\ ]$

$$\Sigma(p) = -f^2 \sum_j \int_{C_j} \frac{d^4q}{i(2\pi)^4} \overbrace{\frac{1}{q^2 + M_j^2}}^{D_\phi(q)} \overbrace{\frac{1}{(p-q)^2 + \mu^2 - i\varepsilon}}^{D_\psi(p-q)} \tag{11}$$

and computed it by closing the contour adding infinite semi-circle in upper (or lower) half plane: ($\boldsymbol{k} \equiv \boldsymbol{p} - \boldsymbol{q}$)

$$
\begin{aligned}
&= \frac{f^2}{32\pi^2} \int d^3\boldsymbol{q} \\
&\quad \times \Bigg\{ \frac{1}{\nu_{\boldsymbol{q}}\, E_{\boldsymbol{k}}} \left(\frac{1}{p^0 - \nu - E - i\varepsilon} - \frac{1}{p^0 + \nu + E - i\varepsilon} \right) && \leftarrow A\,\psi \\
&\qquad - \frac{1}{2} \Bigg[\frac{1}{\omega_{\boldsymbol{q}}\, E_{\boldsymbol{k}}} \left(\frac{1}{p^0 - \omega - E} - \frac{1}{p^0 + \omega + E} \right) && \leftarrow \varphi\,\psi \\
&\qquad\quad + \frac{1}{\omega^*_{\boldsymbol{q}}\, E_{\boldsymbol{k}}} \left(\frac{1}{p^0 - \omega^* - E} - \frac{1}{p^0 + \omega^* + E} \right) \Bigg] \Bigg\} && \leftarrow \varphi^\dagger\,\psi
\end{aligned}
$$

Sum of the last two terms is real, has no imaginary part! He concluded Ghost is not produced.

What's wrong?

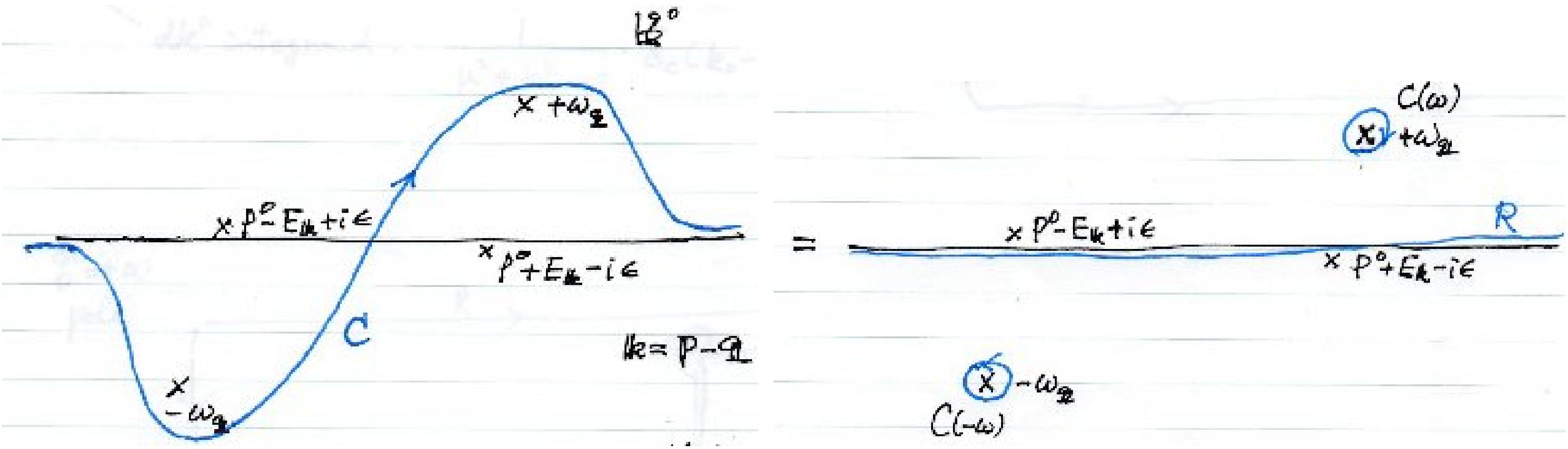
正しくやると、ghost φ の エネルギーの $\delta_{\rm c}$ 関数の積分が非自明！

$$-\Sigma(p) = f^2 \int \frac{d^3\boldsymbol{q}}{i(2\pi)^4}$$

$$\times \int_C dq^0 \int_R dk^0\, \delta_{\rm c}(k^0 + q^0 - p^0) \overset{D_\varphi(q)}{\frac{1}{q^2 + M^2}} \overset{D_\psi(k)}{\frac{1}{k^2 + \mu^2 - i\varepsilon}}\Bigg|_{\boldsymbol{k}=\boldsymbol{p}-\boldsymbol{q}}$$

q^0 積分を先ずやり、$\delta_{\rm c}(k^0 + q^0 - p^0)$ の積分はその後の dk^0 積分でやる。

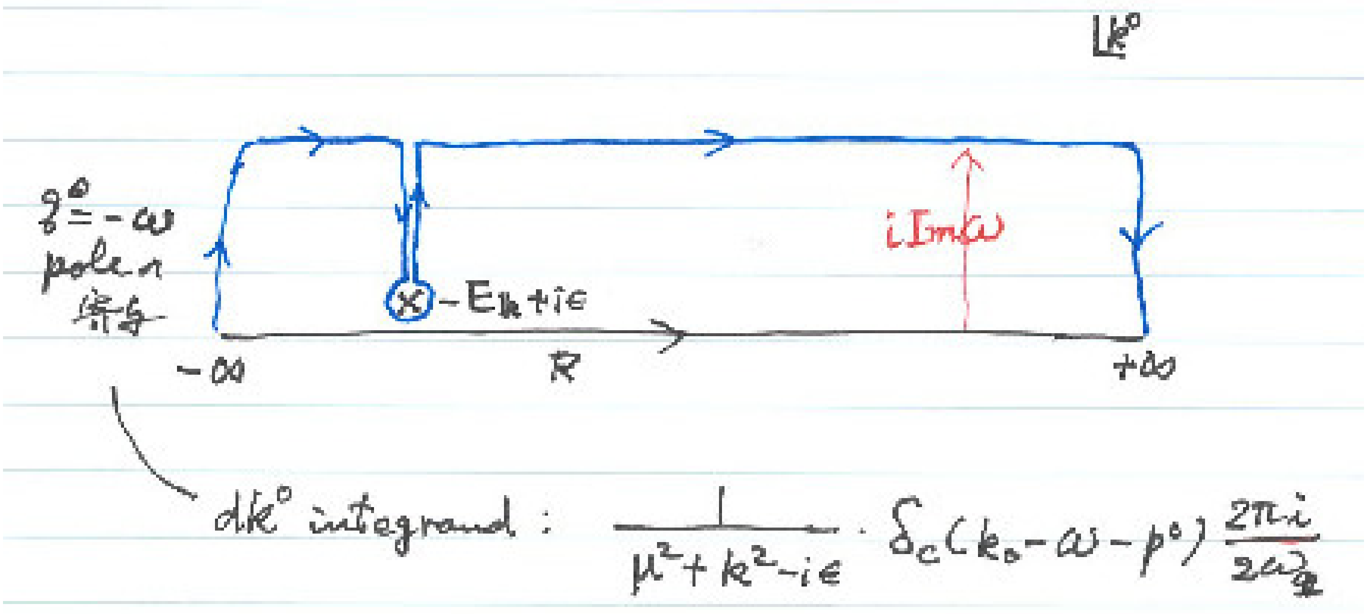
$$\int_C dq^0 = \int_R dq^0 + \int_{C(-\omega)} dq^0 + \int_{C(+\omega)} dq^0 \qquad (12)$$



後者2つの pole contribution は、

$$\int d^3\boldsymbol{q} \left(\frac{2\pi i}{2\omega_{\boldsymbol{q}}}\right) \int_R dk^0 \left[\delta_{\rm c}(p^0 + \omega - k^0) + \delta_{\rm c}(p^0 - \omega - k^0)\right] \frac{1}{E_{\boldsymbol{k}}^2 - k^{0\,2}} \qquad (13)$$

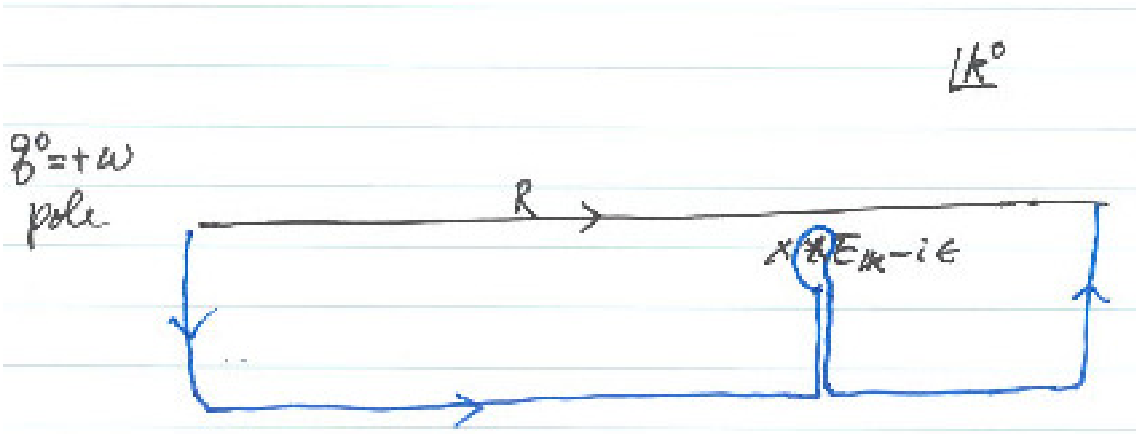
ここで k^0 積分をやるのに、積分経路 R を、（第1項では）$\mathrm{Im}\,\omega$ だけ持ち上げ、（第2項では）$\mathrm{Im}\,\omega$ だけ下げれば、それぞれ実数の普通の Dirac delta になる。その際におつりが出る。



通常の Dirac delta の公式を使える $\mathrm{Im}\,\omega_{\boldsymbol{q}}$ だけ持ち上げた積分路の寄与に加えて、

$$\text{おつり項} = \frac{2\pi i}{2E_{\boldsymbol{k}}}\frac{2\pi i}{2\omega_{\boldsymbol{q}}}\delta_{\mathrm{c}}(E_{\boldsymbol{k}} - \omega_{\boldsymbol{q}} - p^0)$$

が出る。



後者の寄与を加えて、おつりはネットに、

$$\text{おつり} = -f^2 \int \frac{d^3\boldsymbol{q}}{i(2\pi)^4} \left\{ \frac{(2\pi i)^2}{2\omega_{\boldsymbol{q}}\, 2E_{\boldsymbol{k}}} \left(\delta_{\rm c}(p^0 + \omega_{\boldsymbol{q}} + E_{\boldsymbol{k}}) + \delta_{\rm c}(p^0 - \omega_{\boldsymbol{q}} - E_{\boldsymbol{k}}) \right) \right.$$

もとのRに沿ってのdq^0積分の部分も素朴にdelta関数を適用出来、後者二つの素朴にdelta関数公式を使える寄与を加えると、Leeの最初から素朴なデルタ関数公式を使った表式を与える。それはrealだったので、Imaginary partは全ておつりの項から来て

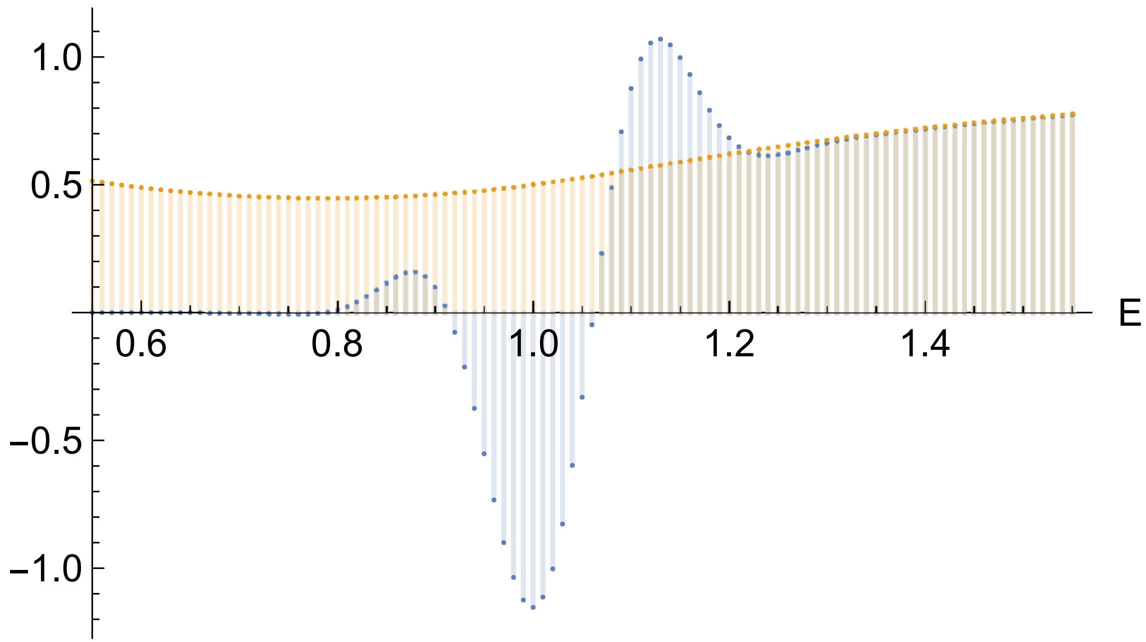
$$\mathrm{Im}\,\Sigma(p) = f^2 \frac{-\pi^2}{(2\pi)^4} \int d^3\boldsymbol{q}\, \mathrm{Re} \left[\frac{1}{\omega_{\boldsymbol{q}}\, E_{\boldsymbol{k}}} \Big(\delta_{\rm c}(p^0 + \omega_{\boldsymbol{q}} + E_{\boldsymbol{k}}) + \delta_{\rm c}(p^0 - \omega_{\boldsymbol{q}} - E_{\boldsymbol{k}}) \Big) \right]$$

これは、直接production cross-sectionを計算した結果と一致した答えを与える。

Ghost production probability の計算例：

$\psi + \psi \to \varphi + \varphi$

$m^2 = 1,\ \gamma^2 = 0.5,\ a = 1/14,\ E_{\rm Lth} = 0.786,\ E_{\rm Uth} = 1.272$



5 Conclusion

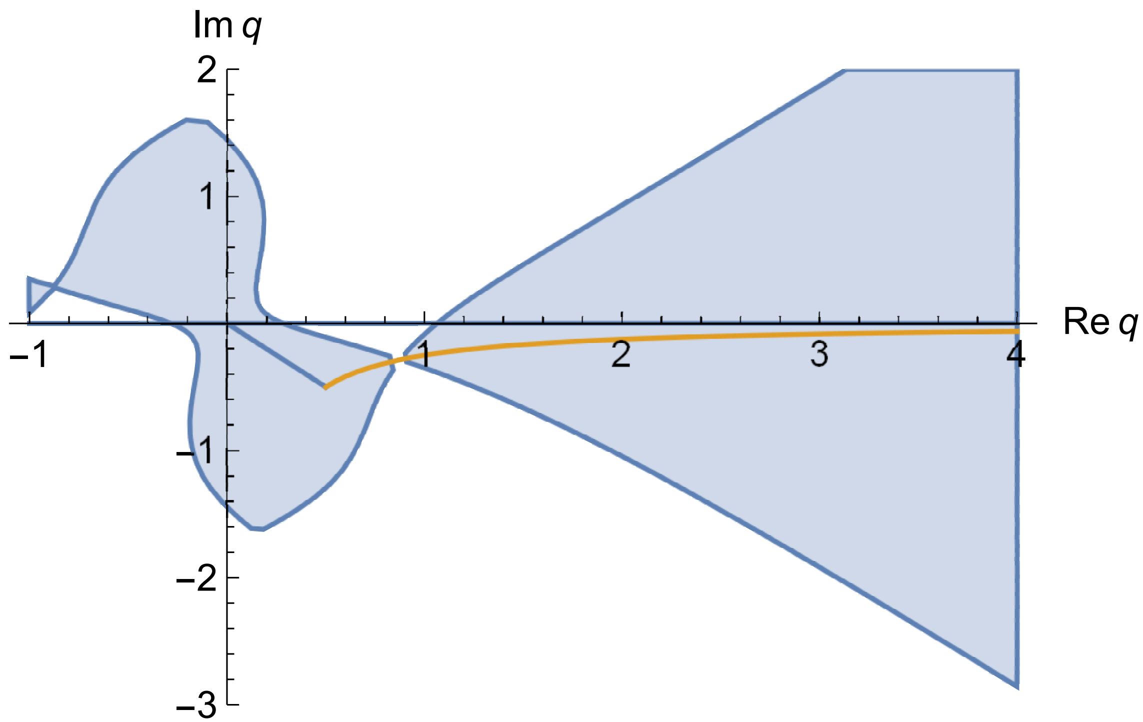
- Complex ghosts は、有限(non-zero)確率で現れ、(physical) Unitarity を壊す。

- Complex ghost は、unstable particle ではない。
むしろ安定 or Anti-stable (Coleman) である。

Hamiltonian はエルミートなので、ノルムは保存する。親粒子が complex ghost だと initial state は負ノルム。それが decay して、全てが安定な正ノルムの physical particle ばかりになることは出来ない。必ず、負ノルムの粒子が残っている必要がある。

- Complex ghost theory は、場の理論として数学的には consistent.
しかし、physical unitarity が壊れるので、物理的理論としては、このままでは inconsistent.

- Lower threshold 以下のエネルギー $E < \mathrm{Re}\sqrt{M^2} - \mathrm{Im}\sqrt{M^2}$ では、ghost は生成されないので、unitarity, renormalizability は OK.



Positive $\mathrm{Re}\,[(E-\omega_{\boldsymbol{q}})^2]$ region and q-contour at E above upper threshold $E_{\mathrm{Uth}} = \mathrm{Re}\,\omega_{\boldsymbol{q}} + \mathrm{Im}\,\omega_{\boldsymbol{q}}$.