Duality of Adjoint SQCD and Supersymmetry Enhancement

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Emily Nardoni, Jaewon Song, arXiv: 2306.08867
Emily Nardoni, Jaewon Song, in progress

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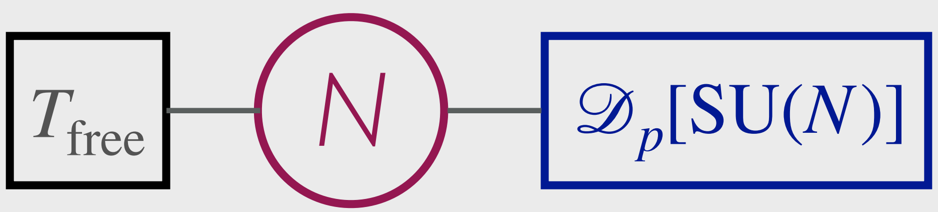
Duality proposal

N=1 SU(*N*) SQCD
+
$\mathscr{D}_p(\mathrm{SU}(N))$ **with superpotential**
$W = u_0$



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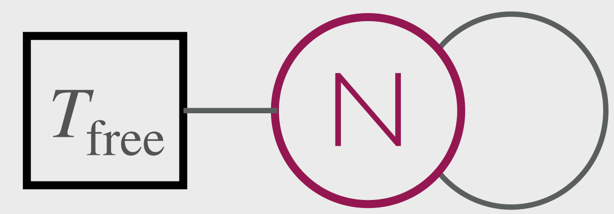
N=1 SU(*N*) SQCD with adjoint chiral mulitplet X with superpotential
$W = \mathrm{Tr} X^{p+1}$



1. $\mathscr{D}_p[\mathrm{SU}(N)]$ **theory**
2. **N=1 gauge theory with** $\mathscr{D}_p(\mathrm{SU}(N))$
3. **Duality**
4. **Supersymmetry enhancement**
5. **Discussion**

I. $\mathscr{D}_p[SU(N)]$ theory

[Cecotti-Del Zotto]

[Cecotti-Del Zotto-Giacomelli]

is an N=2 SCFT of Argyres-Douglas type

- Global symmetry:

  $SU(2)_R \times U(1)_r$ R-symmetry; SU(N) for $(N,p)=1$

- **The Coulomb branch** operators $u_{s,j}$ whose dimensions are given by

$$\Delta(s,j) = \left[j - \frac{N}{p} s \right]_+ + 1 \qquad (j = 1,2,\ldots,N-1 \text{ and } s = 1,2,\ldots,p-1)$$

- **Higgs branch** is identified with the principal nilpotent orbit of A_{N-1} algebra.

  The associated VOA is $\widehat{\mathfrak{su}}(N)_{-\frac{N(p-1)}{p}}$ (we assume $(N,p)=1$ and p>N here)

  [Xie-Yan-Yau, Arakawa]

  The moment map operator μ which is the lowest component of the conserved current multiplet satisfies $\left.\mu^p\right|_{\text{adj}} = 0, \quad \mathrm{Tr}\mu^k = 0$

  [Agarwal-Lee-Song]

Schur index of $\mathscr{D}_p[\mathrm{SU}(N)]$

The index of $\mathscr{D}_p[\mathrm{SU}(N)]$ turns out to be **[Song-Xie-Yan]**

$$I_{\mathrm{Schur}}(\mathfrak{q}; a_i) = \mathrm{PE}\left[\frac{\mathfrak{q} - \mathfrak{q}^p}{(1-\mathfrak{q})(1-\mathfrak{q}^p)}\chi_{\mathrm{adj}}(a_i)\right],$$

where χ_{adj} is the character of the adjoint representation of SU(N) and $\mathrm{PE}[x] = \exp(\sum_{n=1}^{\infty} x^n/n)$.

Actually this index is recovered from the index of the N=1 chiral multiplet with N=1 R-charge r:

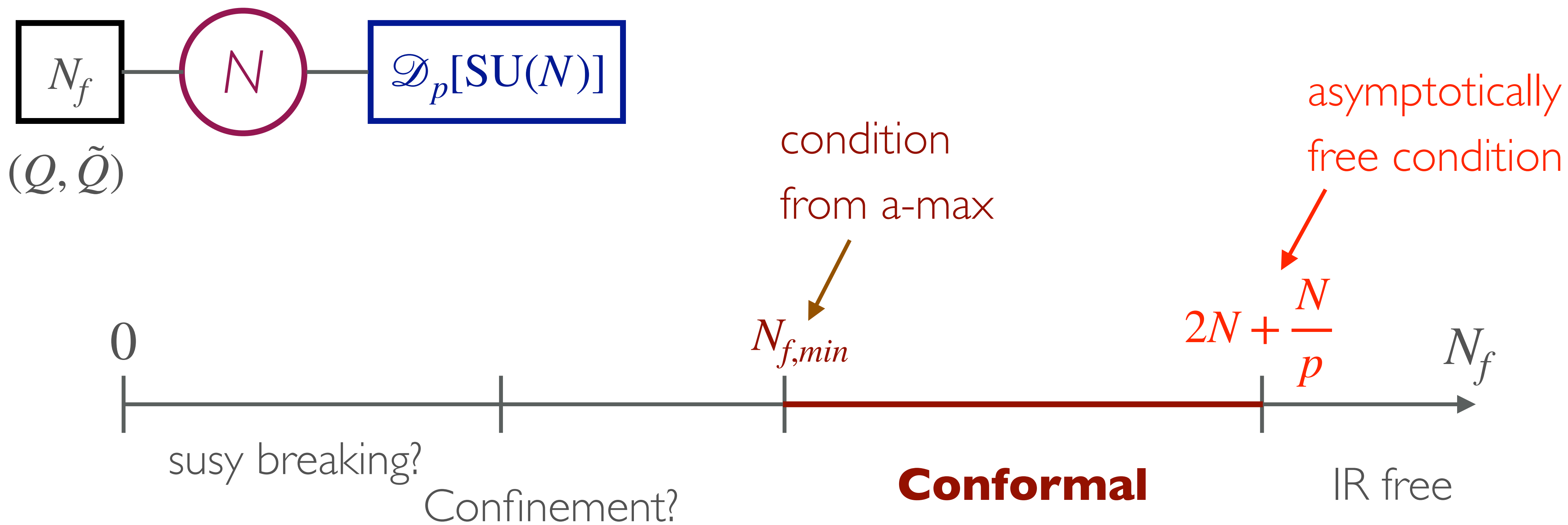
$$I(\mathfrak{p}, \mathfrak{q}; a_i) = \mathrm{PE}\left[\frac{(\mathfrak{p}\mathfrak{q})^{r/2} - (\mathfrak{p}\mathfrak{q})^{1-r/2}}{(1-\mathfrak{p})(1-\mathfrak{q})}\chi_{\mathrm{adj}}(a_i)\right],$$

by taking the limit $\mathfrak{p} \to \mathfrak{q}^p,\ r \to \dfrac{2}{p+1}$.

ϕ^{p+1} **has R-charge 2**

2. N=1 gauge theory with $\mathscr{D}_p[\mathrm{SU}(N)]$

We couple the N=1 vector multiplet with the $\mathscr{D}_p[\mathrm{SU}(N)]$ theory and N_f pairs of fundamental and anti-fundamantal chiral multiplets



(The one-loop beta function is calculated as

$$b_{1-\mathrm{loop}} = 3C(\mathrm{adj}) - 2N_f C(\square) - \frac{k_{\mathrm{SU}(N)}}{2} = 3N - N_f - \frac{p-1}{p}N$$

The upper bound comes from the asymptotically free condition $b_{1-\mathrm{loop}} > 0$.)

Adding the superpotential $W = u_0$, where u_0 is the lowest dimension $\Delta(u_0) = \frac{p+1}{p}$ Coulomb branch operator.

In general, the IR R symmetry of the IR N=1 SCFT is given by

$$R_{\mathrm{IR}} = R_0 + \epsilon F$$

where R_0 is the N=1 R-symmetry and F is the global symmetry:

$$R_0 = \frac{1}{3}r + \frac{4}{3}R, \qquad F = -r + 2R$$

where R is the Cartan part of $SU(2)_R$.

- **The superpotential term is charge 2 under the IR R-symmetry**. **This determines** ϵ **to be** $\epsilon = \frac{1-2p}{3(p+1)}$.
- **The condition of vanishing NSVZ beta-function gives**

$$R(q) = R(\tilde{q}) = 1 - \frac{2N}{N_f(p+1)}$$

- **The dimensions of the operators are given by**

$$\Delta(\mu) = \frac{3}{p+1}, \quad \Delta(u_0) = 3, \quad \Delta(v_0) = \frac{6}{p+1}$$

- **The central charges are given by**

$$a = \frac{3\left(-12N^4 + N^2N_f^2\left(5p^2+p+2\right) + N_f^2\left(-4p^2+p-1\right)\right)}{8N_f^2(p+1)^3}$$

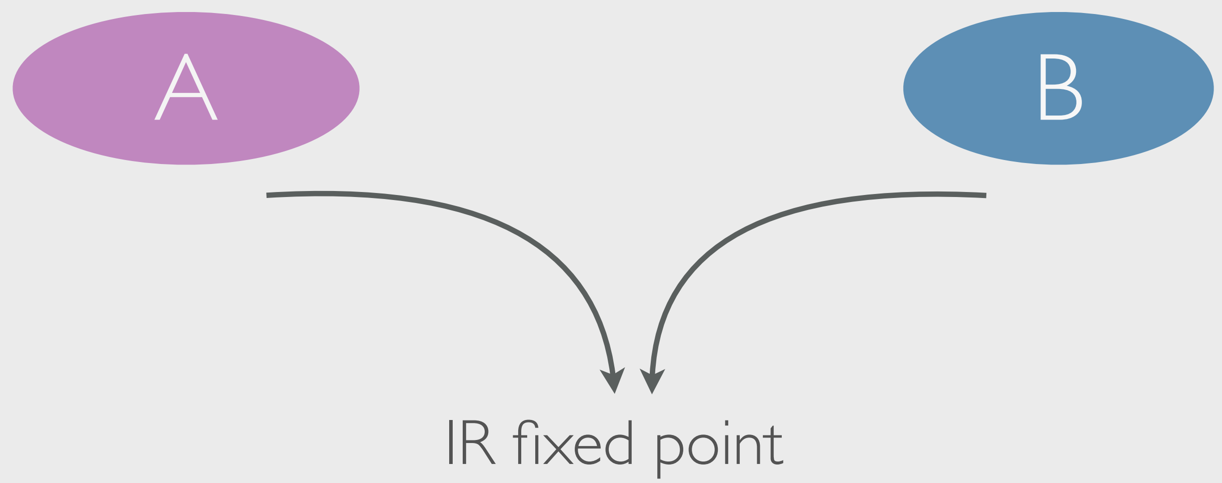
$$c = \frac{-36N^4 + N^2N_f^2\left(16p^2+5p+7\right) + N_f^2\left(-11p^2+5p-2\right)}{8N_f^2(p+1)^3}$$

$\mathcal{O}$	$R(\mathcal{O})$
$u_i,\ i=0,\ldots,p-2$	$\frac{2(p+1+i)}{p+1}$
$v_i,\ i=0,\ldots,p-2$	$\frac{2(2+i)}{p+1}$
$M_j = Q\mu^{j-1}\widetilde{Q},\ j=1,\ldots,p$	$2\left(1-\frac{N}{N_f}\frac{2}{1+p}\right)+(j-1)\frac{2}{1+p}$
$B^{(n_i,\ldots,n_p)}$	$N\left(1-\frac{N}{N_f}\frac{2}{1+p}\right)+\sum_{\ell=1}^{p} n_\ell(\ell-1)\left(\frac{2}{1+p}\right)$

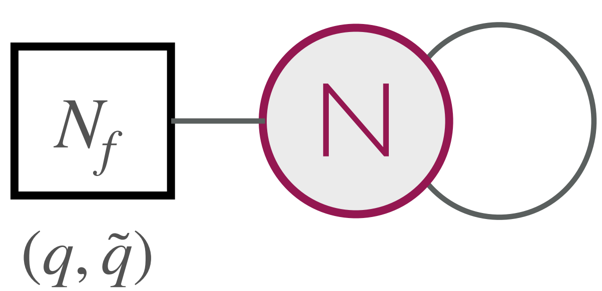
The chiral ring relation of the $\mathscr{D}_p[\mathrm{SU}(N)]$ theory prohibits $\mathrm{Tr}\mu^k$ type operators and $Q\mu^{j-1}\tilde{Q}$ operators with j>p.

3. Duality

duality (or IR duality): two different theories flows to the same IR fixed point or shows the same IR physics **[Seiberg]**.



As the theory B, consider N=1 SU(N) gauge theory with adjoint chiral multiplet X and N_f flavors.



plus $W = \mathrm{Tr} X^{p+1}$

duality checks:

- Central charges and 't Hooft anomalies

- Operator matching:

$Q\mu^j\tilde{Q}$ $(j = 0,1,\ldots,p-1)$,
v_{j-2} $(i = 2,3,\ldots,p)$

$\longleftrightarrow$

$qX^j\tilde{q}$ $(j = 0,1,\ldots,p-1)$
$\mathrm{Tr}X^k$ $(k = 2,3,\ldots p)$,

- superconformal index (Schur index)

- ….

4. Supersymmetry enhancement

Consider a special case where p=2 and Nf=2N:

If we add the superpotential $W = u_0 + Q\mu\tilde{Q}$,
then we expect the supersymmetry is enhanced in the IR.

Indeed, the central charges of this case

$$a = \frac{7N^2 - 5}{24}, \quad c = \frac{2N^2 - 1}{6}$$

and, also

$$\Delta(u_{j,1}) = 3,5,\ldots,N, \;\; \Delta(v_{j,1}) = 2,4,\ldots,N-1,$$

The enhancement is natural from the dual side:
The matter content is those of N=2 SQCD where $Q\mu\tilde{Q}$ is mapped to $QX\tilde{Q}$. At this fixed point the $\mathrm{Tr}X^3$ operator is marginally irrelevant. Therefore the fixed point has N=2 supersymmetry.

5. Discussion

- Non-conformal case (low N_f)
- $N_f = 0$ case: dual to Dijkgraaf-Vafa superpotential ?
- Hint from string/M theory
- Duality for D, or E type SCFT of two adjoints-SQCD **[Intriligator-Wecht]**
- D or E type gauge group **[Leigh-Strassler, Brodie-Strassler]**