Thermodynamics of regular black holes in effective loop quantum gravity

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Introduction

- Though debates on quantum nature of black holes are crucial and long standing, there is no systematic study by quantum gravity so far to lay a solid theoretical foundation for arguments.

- Some different models of quantum gravity are effective ways to understand gravity behaviors at a sufficiently small scale: *String theory*, *Loop quantum gravity*, *Asymptotically safe gravity*, *Noncommutative gravity*, *Quadratic gravity*, *Rainbow gravity*, …

- Several aspects of quantum black holes have been discussed: *Thermodynamics*, *Hawking radiation*, *Quasinormal modes*, *Stabilities*, *Accretion disk*, *Periapsis shift*, *Gravitational time delay*, *Light deflection*, *Gravitational lensing*, *Black hole shadow*, …

Introduction

- Thermodynamics of quantum corrected polymer black hole and asymptotically safe gravity black hole have been discussed [Mele *et al.*, Mandal, Gangopadhyay].

- These spacetime admit extremal minimal-sized configurations of quantum gravitational nature characterized by vanishing temperature and entropy.

- If black holes would not evaporate completely but turn into remnants at the end of evaporation process, such remnants might be a constituent of dark matter.

✓ We study thermodynamics and evaporation of regular black holes in loop quantum Oppenheimer-Snyder model as one of quantum gravity effects in Hawking radiation.

Loop quantum cosmology

- Ashtekar-Pawlowski-Singh (APS) model

$$ds^2_{\rm APS} = -d\tau^2 + a(\tau)^2(d\tilde{r}^2 + \tilde{r}^2 d\Omega^2)$$: pressureless homogeneous star

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_c}\right)$$: deformed Friedmann equation

$$\rho = \frac{M}{\frac{4}{3}\pi\tilde{r}_0^3 a^3}$$: uniform density, $\rho_c = \sqrt{3}/(32\pi^2\gamma^3 G^2\hbar)$: critical density

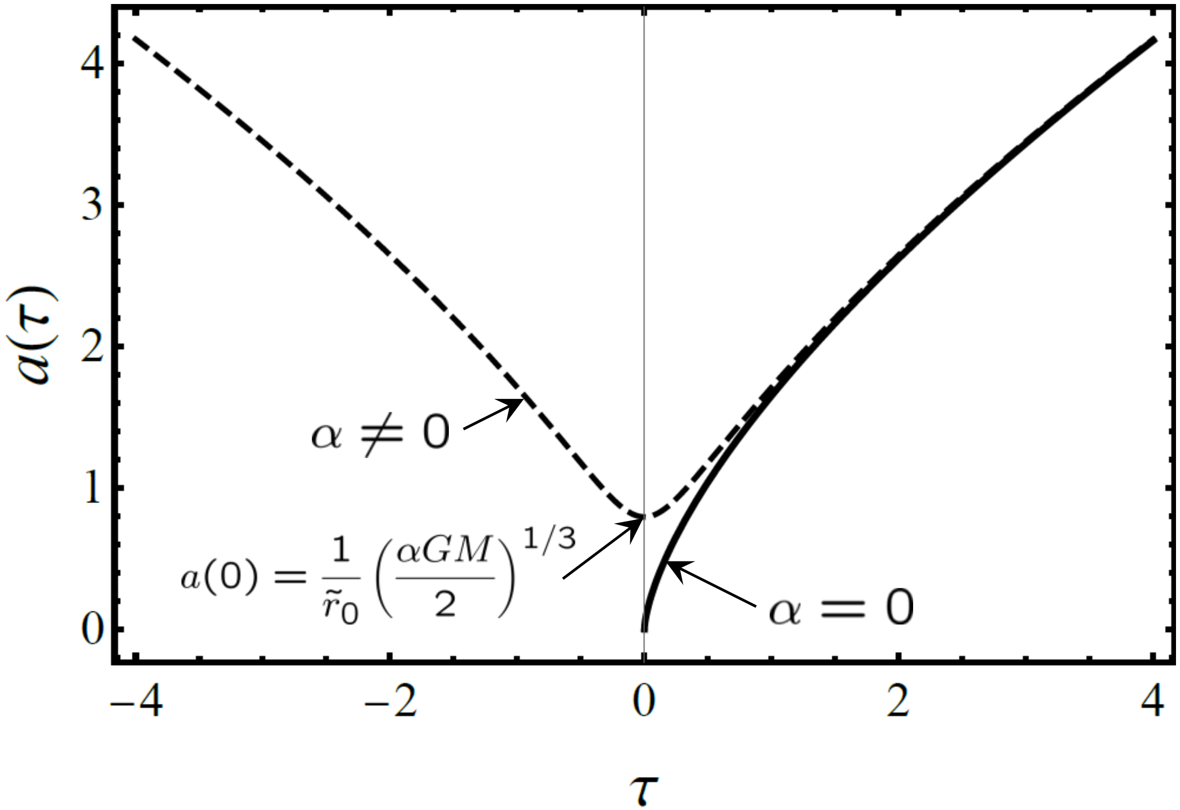
M: mass of dust ball with radius $a(\tau)\tilde{r}_0$, γ : Barbero-Immirzi parameter

- $\rho \ll \rho_c$: classical regime ⇒ usual Friedmann equation
- $\rho \sim \rho_c$: quantum regime ⇒ deformed Friedmann equation:

$$(\tilde{r}_0\dot{a}(\tau))^2 = \frac{2GM}{\tilde{r}_0 a(\tau)} - \frac{\alpha G^2 M^2}{(\tilde{r}_0 a(\tau))^4}, \quad \alpha = 16\sqrt{3}\pi\gamma^3\ell_p^2$$

Loop quantum cosmology bounce

$$\begin{cases} ds^2_{\mathrm{APS}} = -d\tau^2 + a(\tau)^2(d\tilde{r}^2 + \tilde{r}^2 d\Omega^2) \\ a(\tau) = \frac{1}{\tilde{r}_0}\left(\frac{GM\left(9\tau^2+\alpha\right)}{2}\right)^{1/3} \end{cases}$$



- Metric is nowhere and never singular.
- $a(\tau)$ can be extend to whole interval $(-\infty,\infty)$.

Quantum Oppenheimer-Snyder model [Lewandowski *et al.*]

- Gluing interior APS geometry with exterior spherically symmetric geometry:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

along radial free fall geodesics by identification $(\tau, \tilde{r}_0, \theta, \phi) \sim (t(\tau), r(\tau), \theta, \phi)$ such that induced metric and extrinsic curvature are equal on gluing surfaces that become a single C^1 surface of dusty part of spacetime.

$$\Rightarrow \quad f(r) = 1 - \frac{2GM}{r} + \frac{\alpha G^2 M^2}{r^4}$$

A quantum deformation of Schwarzschild spacetime
as derived in effective loop quantum gravity [Kelly *et al.*]

- A Killing observer perceives energy density:

$$T^q_{\mu\nu} = \frac{G_{\mu\nu}}{8\pi G}, \qquad \rho^q = \frac{3\alpha G M^2}{8\pi r^6}$$: dark matter candidate

Loop quantum regular black holes

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \quad f(r) = 1 - \frac{2GM}{r} + \frac{\alpha G^2 M^2}{r^4}$$

- Metric is determined for $r \geq r_b = \left(\frac{\alpha GM}{2}\right)^{\frac{1}{3}}$

⇔ Dust surface radius $a(\tau)\tilde{r}_0$ runs over $[r_b, \infty)$

- Introducing parameter $0 < \beta < 1$ by $G^2 M^2 = \frac{4\beta^4}{(1-\beta^2)^3}\alpha$

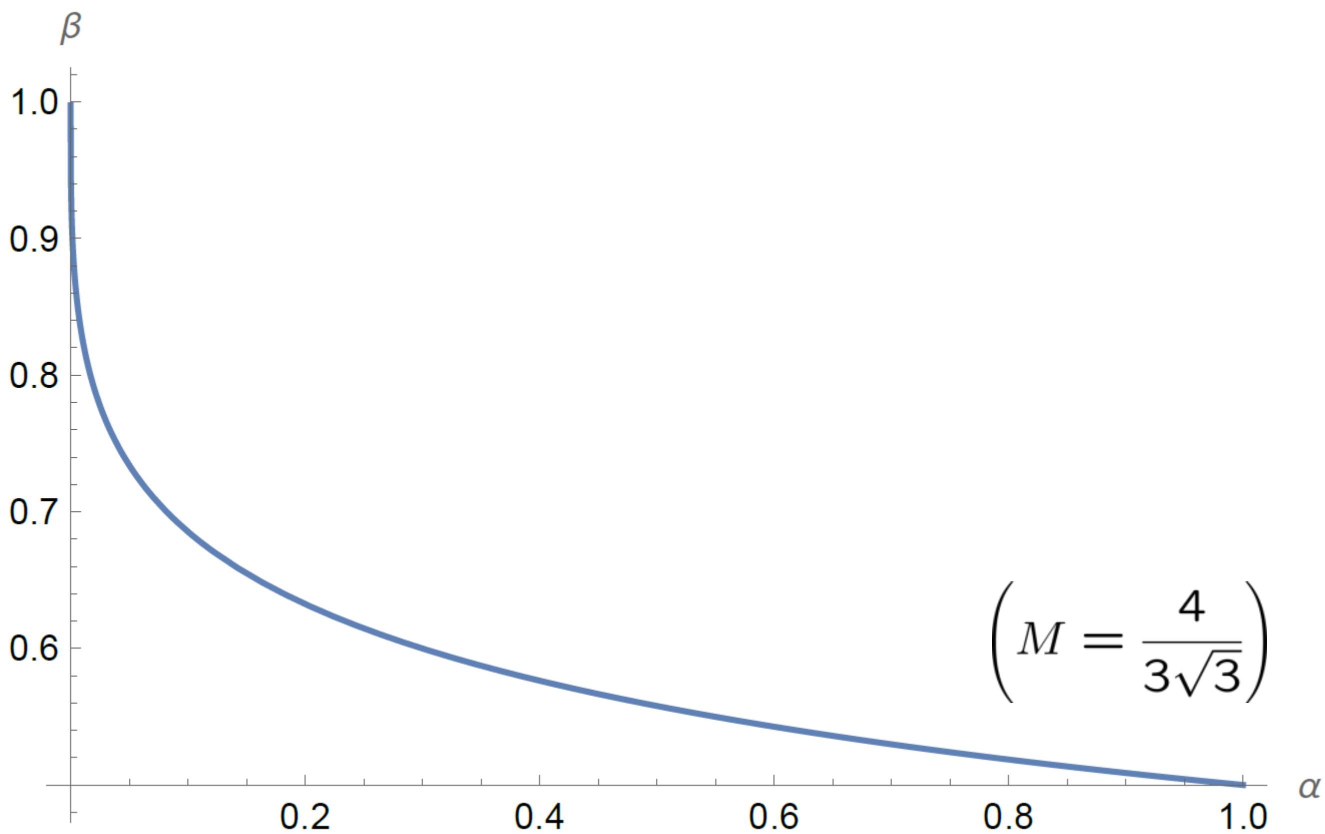
⇨
- $0 < \beta < 1/2$: $M < M_{\min} := \frac{4}{3\sqrt{3}G}\sqrt{\alpha}$: No horizon
- $1/2 < \beta < 1$: $M > M_{\min}$: two Killing horizons

$$r_\pm = \frac{M(1+\beta)\left(1 \pm \sqrt{2\beta - 1}\right)}{2\beta}$$

➢ White hole horizon = Cauchy horizon: most likely unstable with respect to perturbations of spacetime by analogy with Reissner-Nordström spacetime

Loop quantum regular black holes

$$\beta^2 = 1 - \frac{4\alpha}{3M^2} + \frac{\sqrt[3]{2}}{3M^2}\sqrt[3]{\alpha\left(-32\alpha^2 - 27M^4 + 72\alpha M^2 + 3M^3\sqrt{81M^2 - 48\alpha}\right)} + \frac{4 \times 2^{2/3}\alpha\left(2\alpha - 3M^2\right)}{3M^2\sqrt[3]{\alpha\left(-32\alpha^2 - 27M^4 + 72\alpha M^2 + 3M^3\sqrt{81M^2 - 48\alpha}\right)}}$$



Loop quantum regular black holes

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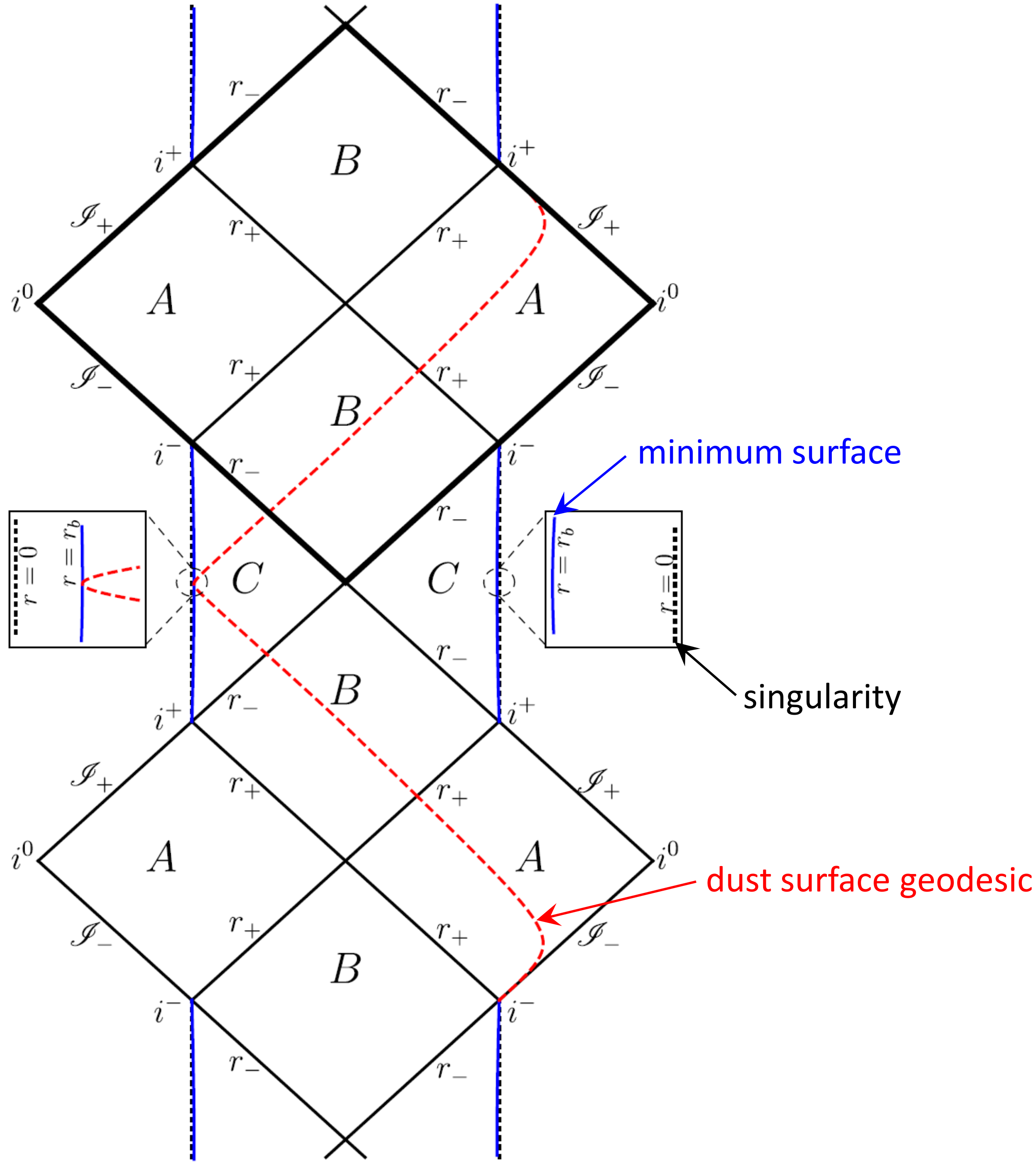
Maximal extension of BH spacetime: $1/2 < \beta < 1$



Equation of motion for scalar particles

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2, \quad f(r) = 1 - \frac{2GM}{r} + \frac{\alpha G^2 M^2}{r^4}$$

- Klein-Gordon equation for uncharged massive scalar particles:

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}g^{\mu\nu}\partial_\nu\Psi\right) - \frac{m^2}{\hbar^2}\Psi = 0$$

✓ Wave function of Klein-Gordon equation: $\Psi = \exp\left(\frac{i}{\hbar}I\left(x^\mu\right)\right)$

- WKB approximation to leading order in $\hbar$ in black hole geometry:

$$-\frac{1}{f}\left(\partial_t I\right)^2 + f\left(\partial_r I\right)^2 + \frac{1}{r^2}\left(\partial_\theta I\right)^2 + \frac{1}{r^2\sin^2\theta}\left(\partial_\phi I\right)^2 + m^2 = 0$$

✓ According to Killing vector fields, the action ansatz: $I = -\omega t + W(r,\theta) + J\phi$

(*ω, J* : scalar particle's energy and angular momentum)

- Equation of motion for scalar particles:

$$fr^2\left(\frac{\partial W}{\partial r}\right)^2 - \frac{\omega^2 r^2}{f} + m^2 r^2 + \left(\frac{\partial W}{\partial \theta}\right)^2 + \frac{J^2}{\sin^2\theta} = 0$$

- Function *W* can be written as $W(r,\theta) = R(r) + \Theta(\theta)$

Tunneling of scalar particles

- Action for outgoing and ingoing modes for classically forbidden trajectory:

$$\mathrm{Im}R_{\mathrm{out}} = -\mathrm{Im}R_{\mathrm{in}} = \frac{\pi\omega r_+^4}{\left(r_+ - r_-\right)\left(r_+^2 + \left(r_+ + r_- - 2M\right)r_+ + \alpha M^2 / \left(r_+ r_-\right)\right)}$$

- Tunneling probability amplitude of scalar particles:

$$\Gamma \simeq \frac{\exp\left(-2\mathrm{Im}R_{\mathrm{out}}\right)}{\exp\left(-2\mathrm{Im}R_{\mathrm{in}}\right)} \simeq \exp\left(-\frac{4\pi r_+^4}{\left(r_+ - r_-\right)\left(r_+^2 + \left(r_+ + r_- - 2M\right)r_+ + \alpha M^2 / \left(r_+ r_-\right)\right)}\omega\right)$$

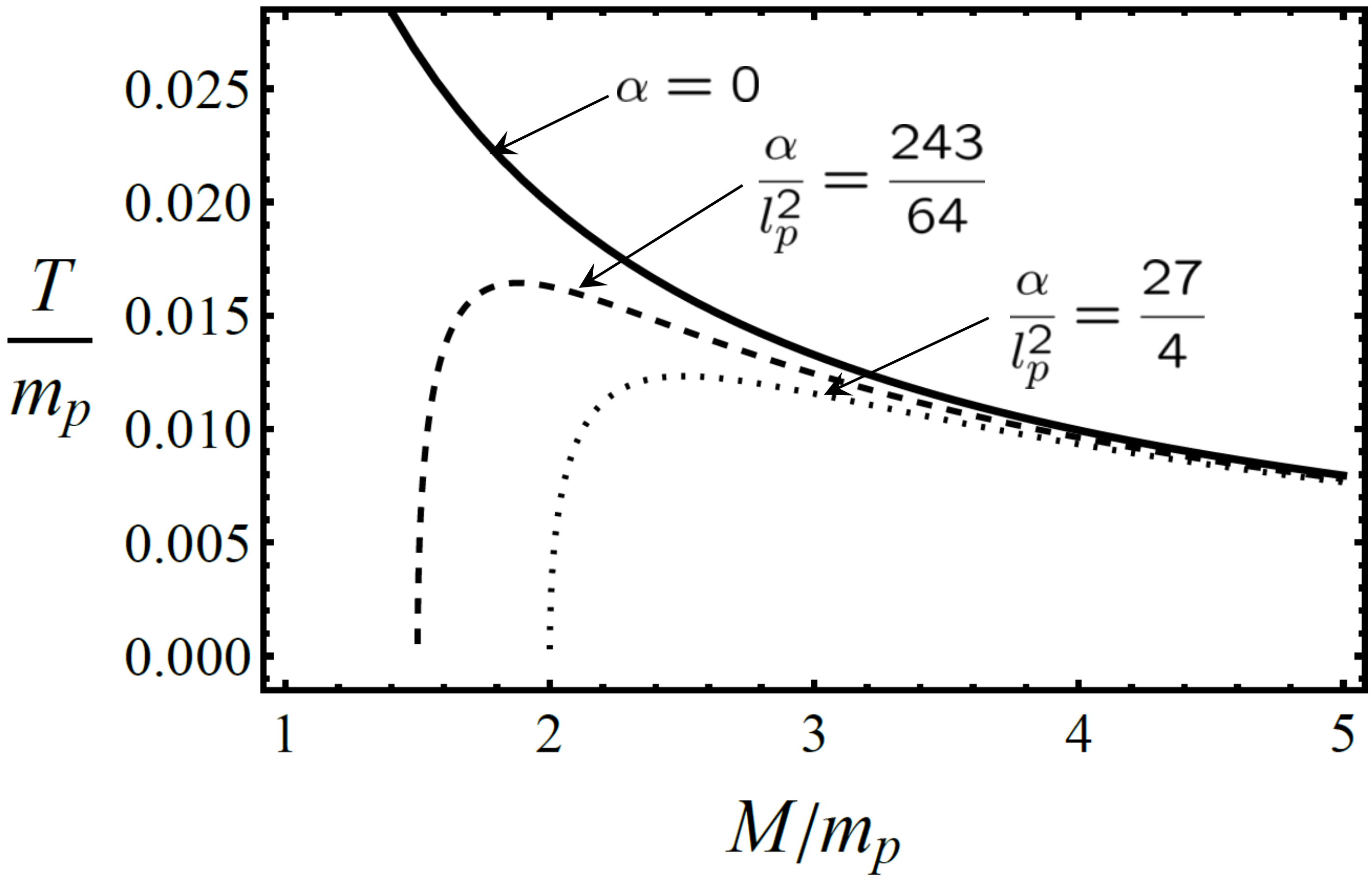
- Comparing tunneling probability amplitude with Boltzmann factor $\Gamma = \exp\left(-\omega / T\right)$ at temperature *T*, we obtain **modified Hawking temperature of loop quantum black hole**:

$$T = \frac{\left(r_+ - r_-\right)\left(r_+^2 + \left(r_+ + r_- - 2M\right)r_+ + \alpha M^2 / \left(r_+ r_-\right)\right)}{4\pi r_+^4}$$

$$= \frac{\beta\left(1 - 4\beta^2 - \left(1 - 3\beta - \beta^2\right)\sqrt{2\beta - 1}\right)}{\pi M\left(1 - \beta^2\right)^2}, \quad \beta = \beta(M, \alpha)$$

Modified thermodynamics

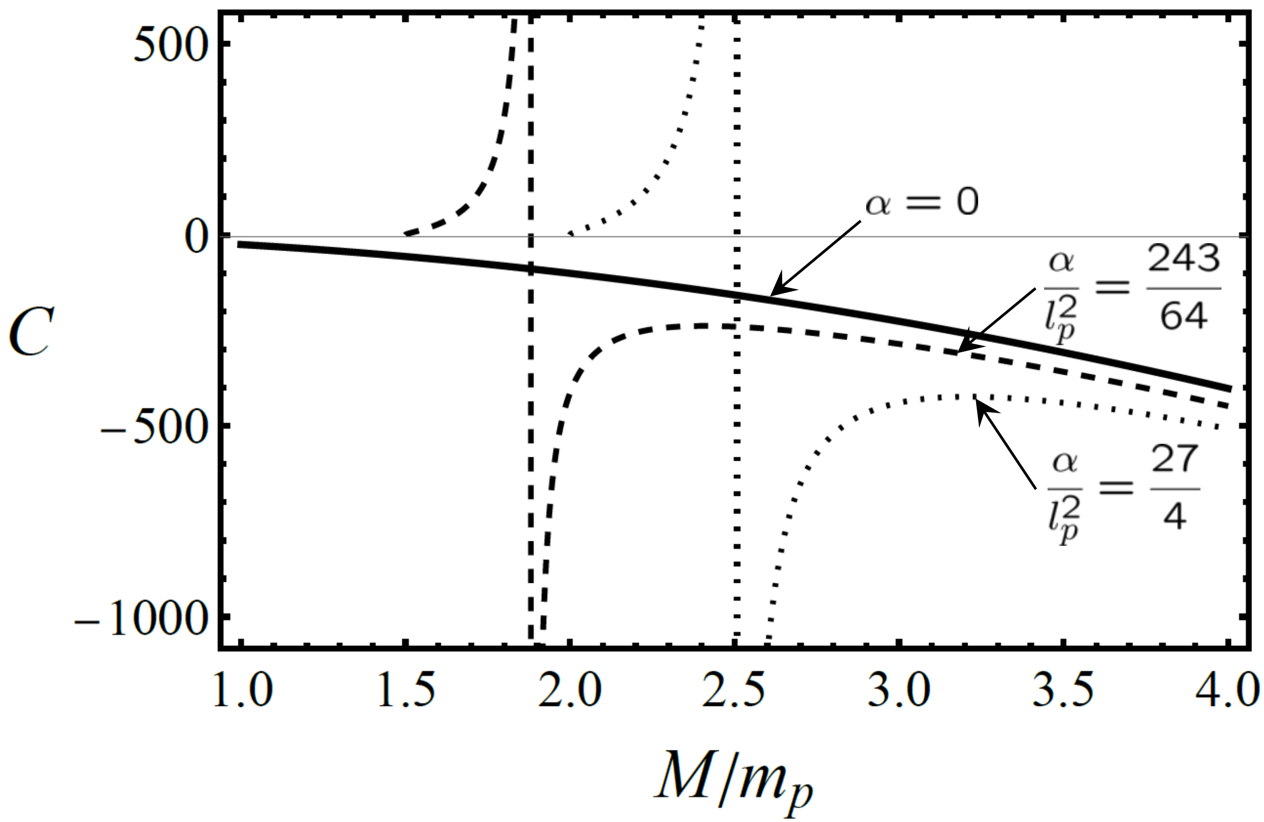
- Hawking temperature:



✓ As black hole mass decreases, temperature with $\alpha \neq 0$ reaches local maximum value and then decreases to zero at minimum value of mass $M_{\min} = \frac{4 m_p \sqrt{\alpha}}{3\sqrt{3} l_p}$.

Modified thermodynamics

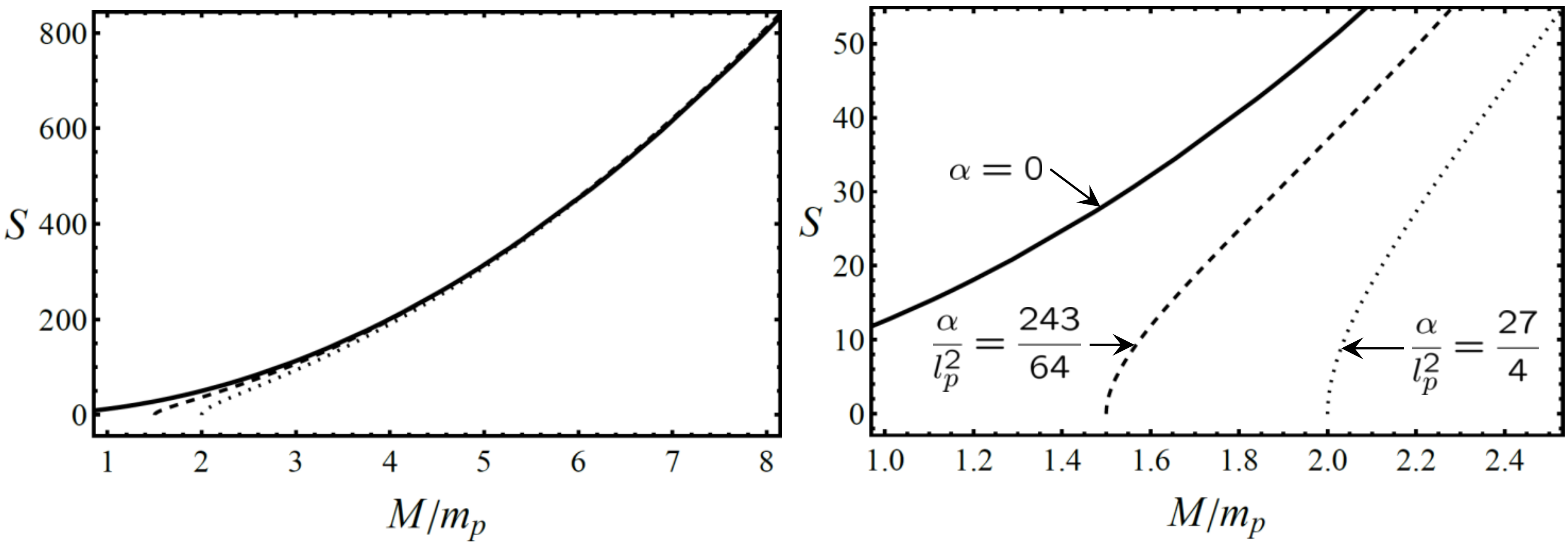
- Heat capacity: $C = \dfrac{\partial M}{\partial T} = \left(\dfrac{\partial T}{\partial M}\right)^{-1}$



✓ At local maximum temperature, system with $\alpha \neq 0$ undergoes transition from unstable negative heat capacity phase to stable positive heat capacity cooling down towards cold extremal configuration with mass M_{min}.

Modified thermodynamics

- Entropy: $S = \int_{M_{\text{min}}}^{M} \frac{dM'}{T(M')}$



✓ At minimum mass M_{min}, since Hawking temperature, heat capacity and entropy vanish, black hole may not exchange its energy with surrounding environment.

➢ **Loop quantum gravity prevents black hole to completely evaporate and results in thermodynamic stable remnant.**

Sparsity of Hawking radiation during BH evaporation

[Gray *et al.*]

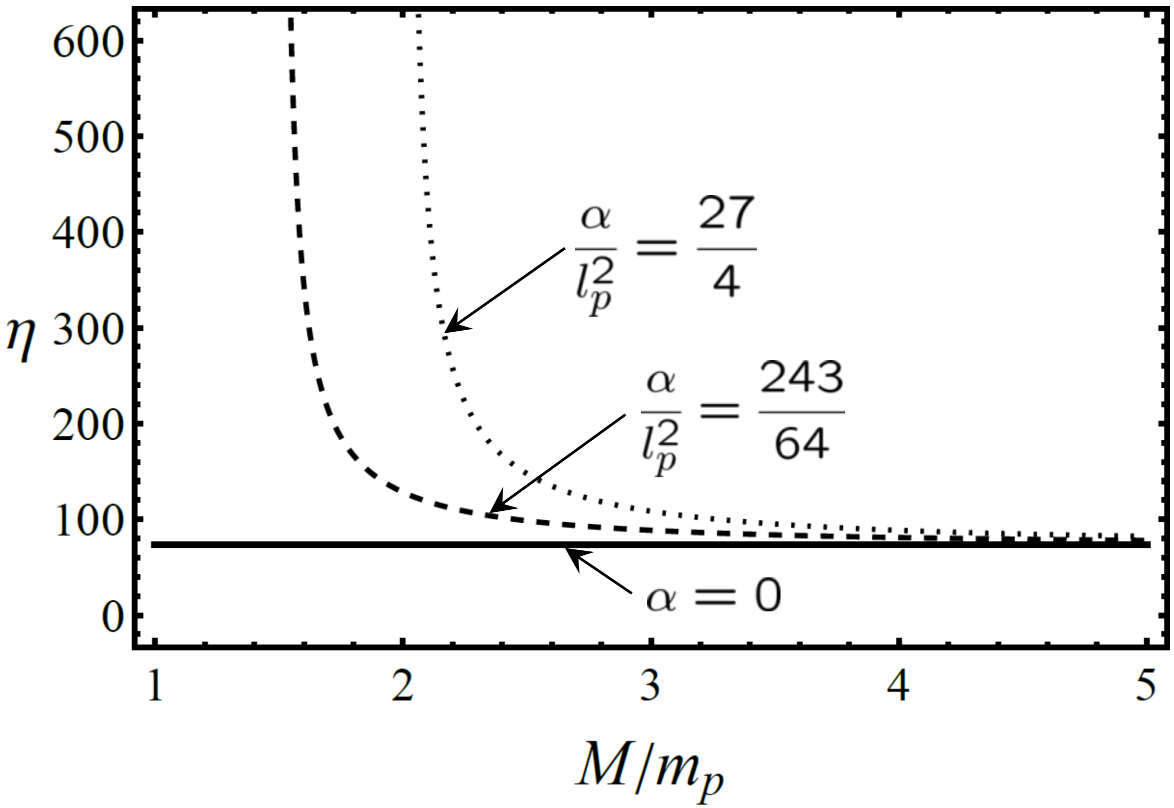
- Sparsity: average time gap between two successive particle emissions over characteristic timescale of individual particle emission

$$\eta = \frac{\lambda^2}{A_{\rm eff}} \begin{cases} \lambda = \dfrac{2\pi}{T} & \text{: thermal wavelength of Hawking particle} \\ A_{\rm eff} = \dfrac{27}{4} A_{\rm BH} & \text{: universal cross section at high frequencies} \end{cases}$$

- $\eta \ll 1$: Hawking radiation is a typical blackbody radiation where its thermal wavelength is much shorter than emitting body size.
- $\eta \gg 1$: Hawking radiation is not a continuous emission of particles but a sparse radiation, i.e., most particles are randomly emitted in a discrete manner with pauses in between.

  ex) $\eta_{\rm 4D\ Sch.} = 64\pi^3/27$

Modified sparsity of Hawking radiation

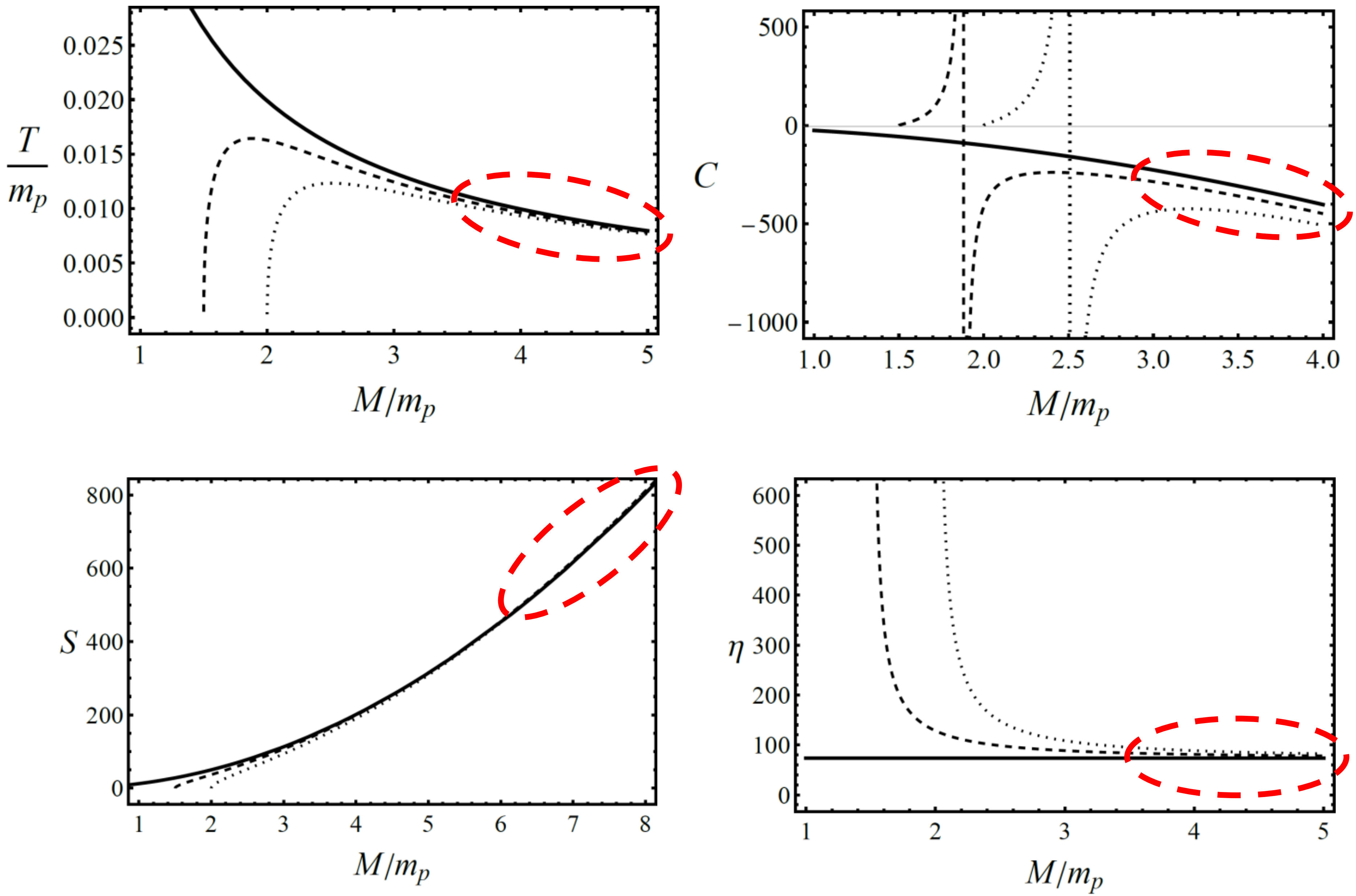


✓ η with $\alpha \neq 0$ diverges at $M = M_{\text{min}}$.

➢ Sparsity is enhanced by quantum gravity effect.

✓ Similar to black hole evaporations in noncommutative model and asymptotically safe gravity, loop quantum black hole would take an infinite amount of time to radiate a particle at final stage of evaporation, and then turn into remnant when black hole mass M approaches mass M_{min}.

Large black hole mass limit



Large black hole mass limit

$$\begin{cases} T = \frac{1}{8\pi M}\left(1 - \frac{\alpha}{8M^2}\right) + O\left(\frac{\alpha^2}{M^5}\right) & \text{: Temperature} \\ C = -8\pi M^2\left(1 + \frac{3\alpha}{8M^2}\right) + O\left(\frac{\alpha^2}{M^2}\right) & \text{: Heat capacity} \\ \eta = \frac{64\pi^3}{27}\left(1 + \frac{3\alpha}{8M^2}\right) + O\left(\frac{\alpha^2}{M^4}\right) & \text{: Sparsity} \end{cases}$$

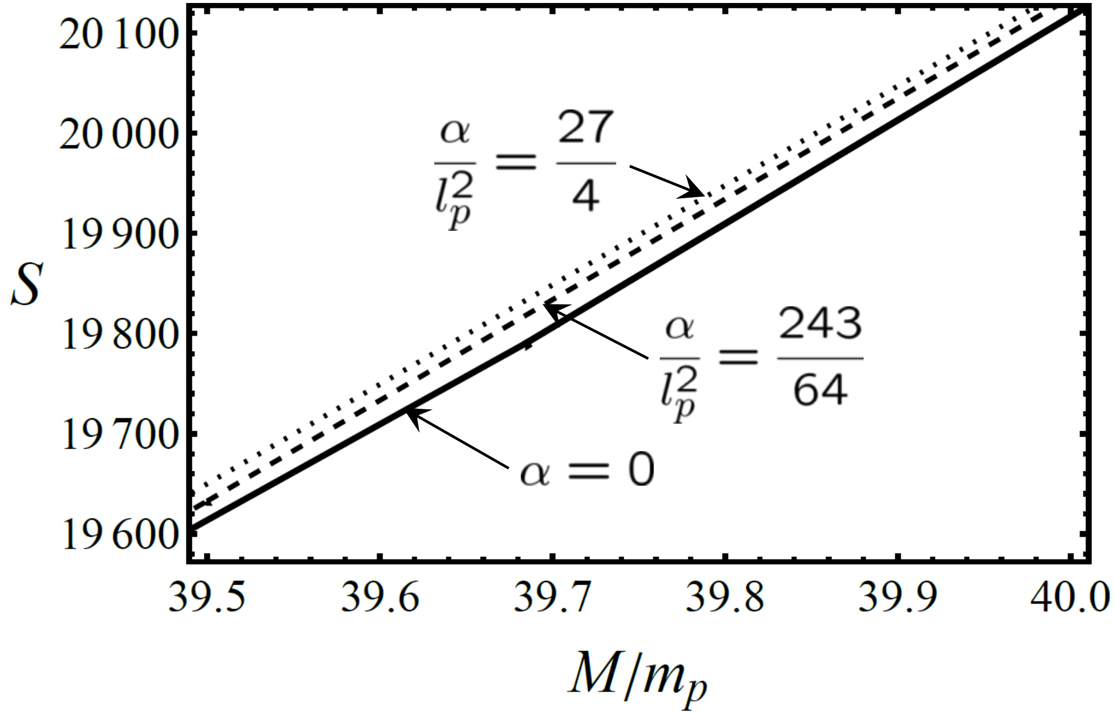
✓Entropy :

$$S = 4\pi\left(M^2 - M_{\text{min}}^2\right) + \pi\alpha \log\left(\frac{M}{M_{\text{min}}}\right) + O\left(\frac{\alpha^2}{M^2}\right)$$

$$\simeq \frac{1}{4}(A_{\text{BH}} - A_{\text{min}}) + \frac{\pi\alpha}{2}\log\left(\frac{A_{\text{BH}}}{A_{\text{min}}}\right)$$

$\left(A_{\text{BH/min}} \simeq 16\pi M_{\text{BH/min}}^2\right)$ Logarithmic term

Large black hole mass limit



✓Entropy :

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$\left(A_{\text{BH/min}} \simeq 16\pi M_{\text{BH/min}}^2\right)$ Logarithmic term

Summary

We consider thermodynamics of four-dimensional static spherically symmetric black holes with minimal area gap in loop quantum gravity inspired by effective field theory.

- We derive modified Hawking temperature, heat capacity and entropy of loop quantum black hole based on scalar particle tunneling mechanism.

✓ Loop quantum gravity may slow down increase of Hawking temperature due to radiation and result in thermodynamic stable remnant, similar to black hole evaporations in noncommutative model and asymptotically safe gravity.

✓ Modified sparsity of Hawking radiation may become infinite when mass of loop quantum black hole approaches its remnant mass.

Discussion

- If Barbero-Immirzi parameter is $\gamma \doteqdot 0.24$, mass of black hole remnant is $M_{\min} \doteqdot 10^{-8}$ kg, which is of order Planck mass like black hole remnants in minimally geometric deformation model, quadratic gravity and asymptotically safe gravity.

- Since such a black hole remnant would not radiate and its gravitational interaction would be very weak, it would be difficult to observe remnants in our Universe directly.

- It would be expected that one possible indirect signature of black hole remnant might be associated with cosmic gravitational wave background [Chen, Adler].