How baryons appear in low-energy QCD: Domain-wall Skyrmion phase in strong magnetic fields

String and Fields
YITP, Aug 4, 2023

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e-Print: 2304.02940 [hep-ph]

References

Collaborators of the whole project

Minoru Eto(Yamagata U.), Kentaro Nishimura(KEK),
Zebin Qiu (Keio U.), Yuki Amari (Keio U.),

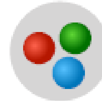
[1] M.Eto, K.Nishimura & MN, e-Print: 2304.02940

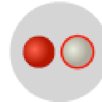
[2] Quasicrystals in QCD,
Z.Qiu & MN, *JHEP* 05 (2023) 170, 2304.05089

[3] Quantum nucleation of topological solitons,
M.Eto & MN, *JHEP* 09 (2022) 077 2207.00211

[4] Non-Abelian chiral solitons under rotation
M.Eto, K. Nishimura & MN, *JHEP* 08 (2022) 305 2112.01381

QCD is a theory of quarks & gluons





In the vacuum, only hadrons (mesons, baryons).

➔ ***Can one calculate the nucleon mass*** $m_N \sim 939\ \mathrm{MeV}$ ***?***
(Lattice QCD, holographic QCD ?…).

➔**Low-energy QCD is described by mesons(pions,** η**) or Nambu-Goldstone modes. Chiral perturbation theory.**

How are baryons described? Cf: The Skyrme model.

In this talk, I show the effective nucleon mass (in a certain situation)

$$"m_N" = \frac{16\pi f_\pi^2}{3m_\pi} \sim 1.03\ \mathrm{GeV}$$

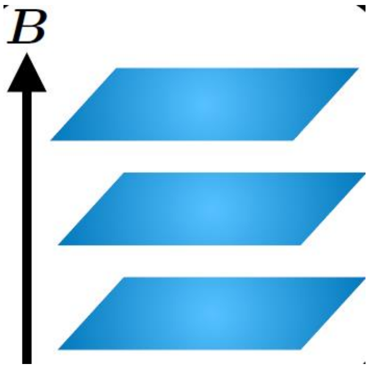
Vacuum values

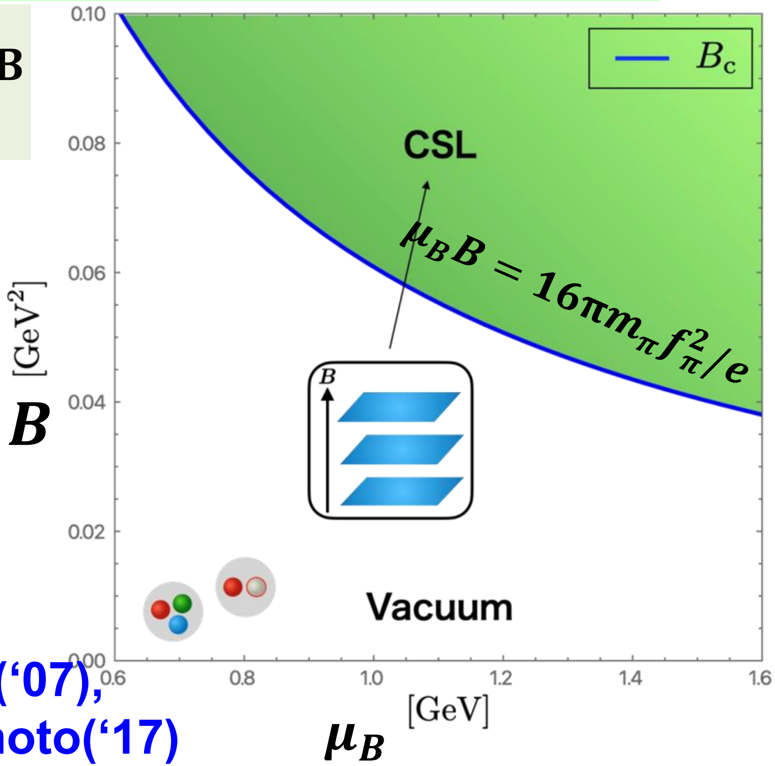
$f_\pi \sim 93\mathrm{MeV}$,
$m_\pi \sim 140\ \mathrm{MeV}$

Summary of my talk

Chiral Soliton Lattice(CSL) phase

chemical pot. μ_B
magnetic field B





Son & Stephanov('07),
Brauner & Yamamoto('17)

Solitons carry baryon #

Summary of my talk

Our results: New phase in QCD

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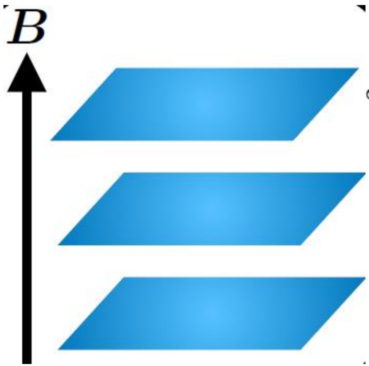

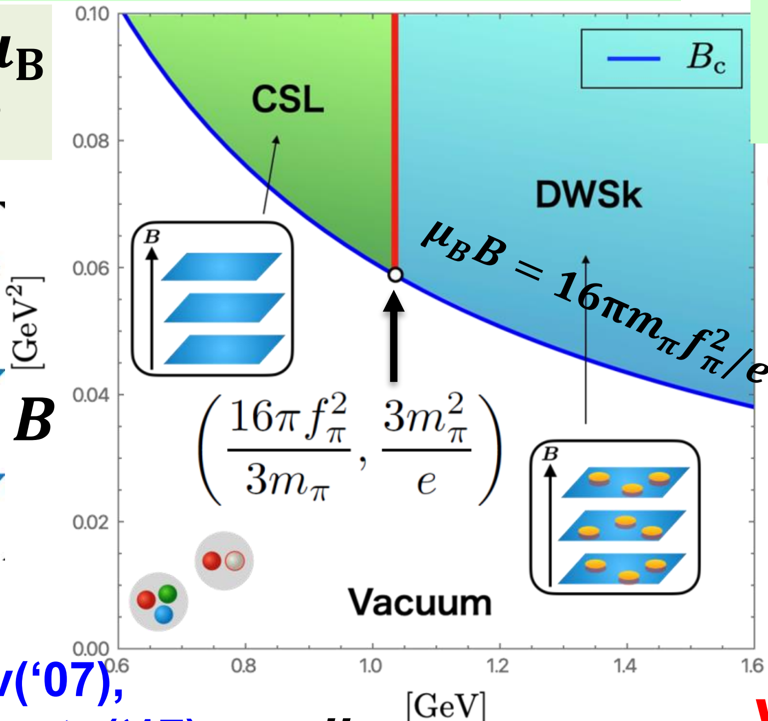

Son & Stephanov('07),
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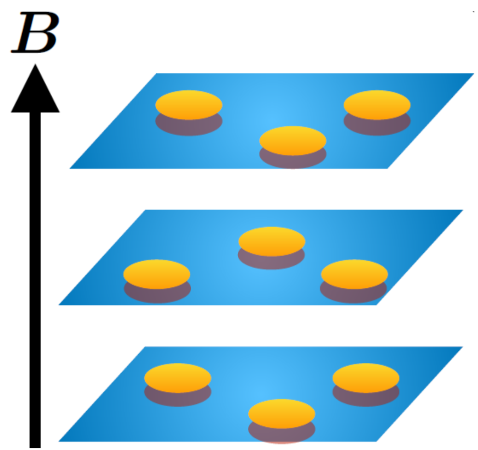
Solitons carry baryon #

Domain-Wall Skyrmion Crystal (DW-SkX) phase

@ $\mu_B \geq 1.03$ GeV



Walls & Skyrmions carry baryon #

Remarkable points of our work

(1) We show this in the chiral perturbation theory @ the leading order $\mathcal{O}(p^2)$ *without higher derivative (Skyrme) term*. Thus, it is model independent.

(Skyrmions are stable without the Skyrme term.)

(2) The critical μ_B coincides with the instability of CSL via charged pion condensation (Brauner-Yamamoto '17).

(3) The μ_B corresponds to the nuclear saturation density

$$\mu_{\mathrm{B}} \geq \mu_{\mathrm{c}} = \frac{16\pi f_\pi^2}{3m_\pi} \sim 1.03 \ \ \mathrm{GeV}$$

remarkably, written in terms of only pion information.

Chiral sine-Gordon model in QCD [Son-Stephanov('07)]

SU(2) Nambu-Goldstone fields $\Sigma = \exp\left(\frac{i\sigma^a\pi^a}{f_\pi}\right) = \exp(i\sigma^a\chi_a)$

Chiral Lagrangian with Wess-Zumino-Witten(WZW) term

$$\mathcal{L}_{\chi\mathrm{PT}} = \frac{f_\pi^2}{4}\mathrm{Tr}\left[D_\mu\Sigma D^\mu\Sigma^\dagger + m_\pi^2(\Sigma+\Sigma^\dagger)\right] + \mathcal{L}_{\mathrm{WZW}}$$

$$\mathcal{L}_{\mathrm{WZW}} = -\left(A_\mu^{\mathrm{B}} + \frac{1}{2}A_\mu^{\mathrm{EM}}\right)j_{\mathrm{B}}^\mu$$

$$D_\mu\Sigma \equiv \partial_\mu\Sigma + \mathrm{i}eA_\mu[Q,\Sigma]\,, \quad Q = \frac{1}{6}\mathbf{1} + \frac{1}{2}\tau_3$$

$$L_\mu = \Sigma\partial_\mu\Sigma^\dagger,\ R_\mu = \partial_\mu\Sigma^\dagger\Sigma$$

$$j_{\mathrm{B}}^\mu = -\frac{1}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}\left\{\mathrm{Tr}L_\nu L_\alpha L_\beta - \frac{3}{2}i\partial_\nu\left(A_\alpha\sigma^3(L_\beta+R_\beta)\right)\right\}$$

A constant magnetic field $\boldsymbol{B}$

A baryon chemical potential $A_\mu^{\mathrm{B}} = (\mu_{\mathrm{B}}, \vec{0})$

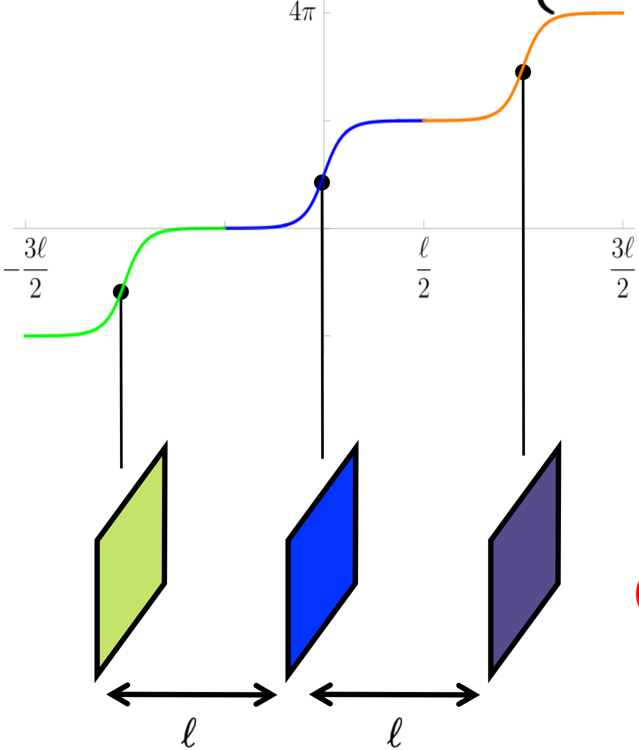
Ignoring charged pions $\chi_{1,2} = 0$

$$A_\mu^{\mathrm{B}} j_{\mathrm{B}}^\mu = \mu_{\mathrm{B}} j_{\mathrm{B}}^{\mu=0}$$

Baryon# density

Previous works assume 1D structure

$$\mu_B\, j_{\mathrm{B}}^{\mu} = -\frac{\mu_{\mathrm{B}}}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}\left\{\mathrm{Tr}L_\nu L_\alpha L_\beta - \underline{\frac{3}{2}i\partial_\nu\left(A_\alpha\sigma^3(L_\beta+R_\beta)\right)}\right\}$$



In 1D, this part gives

$$\sim\mu_{\mathrm{B}}B\partial_z\chi_3 \qquad (\chi_{1,2}=0)$$

magnetic field

chemical potential (finite density)

Chiral soliton lattices

Son & Stephanov('07), Brauner & Yamamoto('17)

Ignoring charged pions **(chiral sine-Gordon model)**

= Sine-Gordon model + topological term

$$\mathcal{L} = \frac{f_\pi^2}{2}(\partial_\mu \chi_3)^2 - f_\pi^2 m_\pi^2 (1 - \cos\chi_3) + \boxed{\frac{e\mu_{\mathrm{B}}}{4\pi^2}\boldsymbol{B}\cdot\boldsymbol{\nabla}\chi_3}$$

topological term

Cf. The same model appears in chiral magnets in which a topological term is the Dzyaloshinskii–Moriya interaction.

The condition that domain walls appear in the g.s.

$$E = 8 m_\pi f_\pi^2 \boxed{- \frac{e\mu_{\mathrm{B}}B}{2\pi}} \leq 0 \text{ !!} \quad \Leftrightarrow \quad \boxed{\mu_B B = 16\pi m_\pi f_\pi^2/e}$$

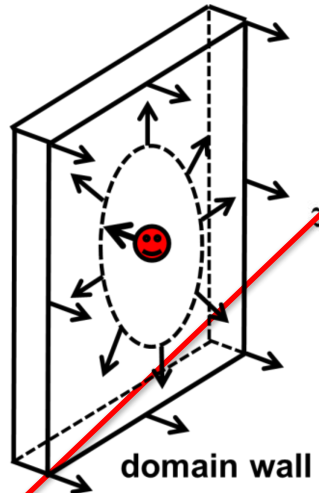
DW tension of a single soliton

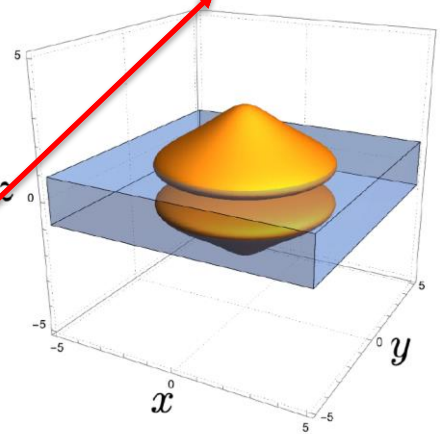
Topological term

$$\chi_3 = 4\tan^{-1} e^{m_\pi(z-Z)}$$

Previous works assume 1D structure

$$\mu_B\, j_{\mathrm{B}}^{\mu} = -\frac{\mu_{\mathrm{B}}}{24\pi^2}\epsilon^{\mu\nu\alpha\beta}\left\{ \mathrm{Tr} L_\nu L_\alpha L_\beta - \underline{\frac{3}{2} i\partial_\nu \left(A_\alpha \sigma^3 (L_\beta + R_\beta)\right)} \right\}$$





In 1D, this part gives

$$\sim \mu_{\mathrm{B}} B \partial_z \chi_3 \qquad (\chi_{1,2} = 0)$$

magnetic field

chemical potential (finite density)

$\chi_{1,2} \neq 0$

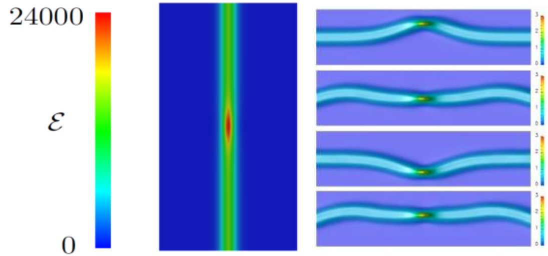
Due to this term ($\neq 0$ with charged pions) in full 3D,

DW Skyrmion lattices are true ground state!!

(in certain parameter region).

A history of domain-wall Skyrmion

MN, Kobayashi, Gudnason, Eto, Ross ….



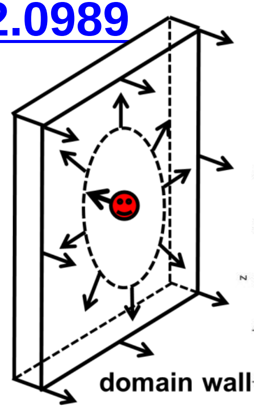
P. Jennings
& P. Sutcliffe ('13)

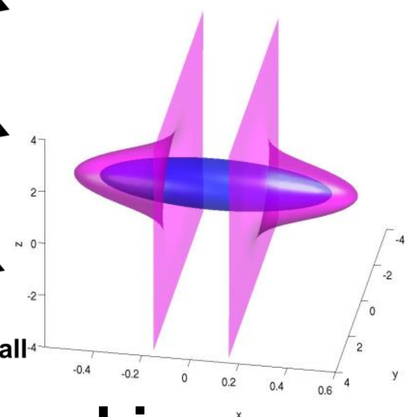
(1) 2+1d version

[1] MN, *PR*D86 ('12) 125004, 1207.6958
[2] M.Kobayashi & MN, *PRD*87 ('13)085003 1302.0989

(2) 3+1d version

[3] MN, *PRD*87 ('13) 025013 1210.2233
[4] S.B.Gudnason & MN,
*PRD*89 ('14) 085022 1403.1245
[5] M.Eto & MN, *PRD*91 ('15) 085044 1501.07038





(3) In condensed matter physics, it's observed in 2+1d chiral magnets. T.Nagase et.al, Nature Comm. ('21)

[6] C.Ross & MN, *PRB*107 ('23) 024422 2205.11417

§ 2 Derivation & Some more details

(1) Considering a single soliton
(2) Constructing DW world-volume effective theory
(3) Constructing lumps (baby Skyrmions)

Technical details

(1) Considering a single soliton

$$\chi_3^{\rm single} = 4\tan^{-1} e^{m_\pi(z-Z)} \qquad \Sigma_0 = e^{i\tau_3\chi_3}$$

More general solution with S^2 moduli (collective coordinates)

$$\Sigma = g\Sigma_0 g^\dagger \qquad g \in SU(2)_{\rm V}$$

$$= [\mathbf{1}_2 + (u^2-1)\phi\phi^\dagger]u^{-1} \qquad u \equiv e^{i\chi_3^{\rm single}}$$

Non-Abelian soliton

$$\phi \in \mathbb{C}^2, \qquad \phi^+\phi = 1$$

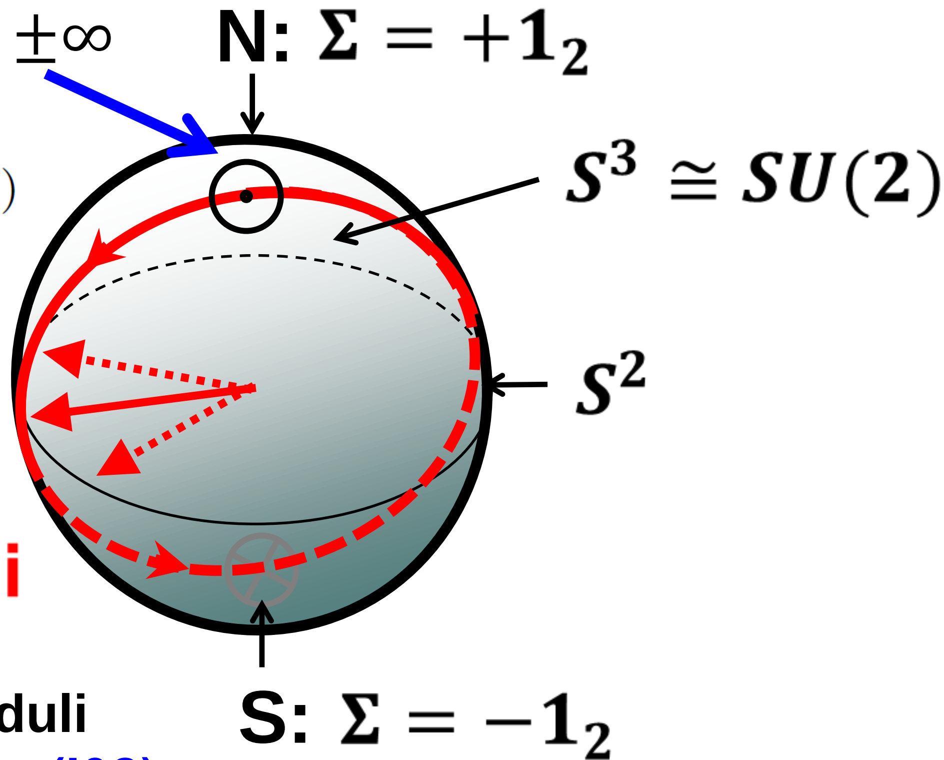
$$g\sigma_3 g^+ = 2\phi\phi^+ - 1_2$$

Cf: η-solitons under rotation are also non-Abelian

Eto, Nishimura & MN, *JHEP* 08 (2022) 305, 2112.01381 [hep-ph]

$$\chi_3^{\text{single}} = 4\tan^{-1} e^{m_\pi (z-Z)}$$

$$\Sigma_0 = \mathrm{e}^{\mathrm{i}\tau_3 \chi_3}$$



Charged pions

***S^2* moduli**

Cf: Domain wall with *S^2* moduli

Losev, Shifman & Vainshtein ('02)

Ritz, Shifman & Vainshtein ('04)

(2) Constructing DW world-volume effective theory via the moduli (Manton) approximation

(a) Promote moduli to fields on D=2+1 worldvolume

$$\phi \to \phi(x^\alpha)\,, \qquad (\alpha = 0, 1, 2)$$

Translational and orientational moduli

(b) Integrate over codimension x^3

$\Rightarrow$ **D=2+1 worldvolume effective theory**

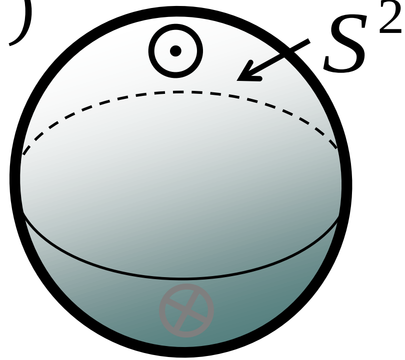
**** A well-defined & established method to construct e.g. monopole or instanton moduli space.**

A review: O(3) nonlinear sigma model

$$\mathcal{L} = \frac{1}{2}\partial^{\mu} n \partial_{\mu} n$$

$$n = (n_1, n_2, n_3)$$

$$n^2 = 1$$



Target space $S^2 = \frac{SO(3)}{SO(2)}$

$$\mathbb{C}P^1 = \frac{SU(2)}{U(1)} \cong S^2$$

$\mathbb{C}P^1$ model

$$\mathcal{L} = \partial^{\mu}\phi^{+}\partial_{\mu}\phi + \phi^{+}\partial^{\mu}\phi\phi^{+}\partial_{\mu}\phi$$

$$n \equiv \phi^{\dagger}\boldsymbol{\sigma}\phi$$

$\phi \sim exp(ia)\phi,\ |\phi|^2 = 1 \quad \phi \in \mathbb{C}^2$

(2) Constructing DW world-volume effective theory via the moduli (Manton) approximation

$$\mathcal{L}_{\mathrm{DW}} = -8m_\pi f_\pi^2 + \frac{e\mu_{\mathrm{B}}B}{2\pi} + \mathcal{L}_{\mathrm{norm}} + \mathcal{L}_{\mathrm{WZW}}$$

DW tension

Topological term for DW

Gauged $\mathbb{C}P^1$ model

$$\mathcal{L}_{\mathrm{norm}} = \frac{16 f_\pi^2}{3m_\pi}[(\phi^\dagger D_\alpha \phi)^2 + D_\alpha \phi^\dagger D^\alpha \phi]$$

$$D_\alpha \phi = (\partial_\alpha + \mathrm{i}\tfrac{e}{2}\tau_3 A_\alpha)\phi$$

Background gauge field at $\mathcal{O}(p^2)$

$$\mathcal{L}_{\mathrm{WZW}} = \boxed{2\mu_{\mathrm{B}} q} + \frac{e\mu_{\mathrm{B}}}{2\pi}\epsilon^{03jk}\partial_j[A_k(1-n_3)]$$

$$n_a = \phi^\dagger \tau_a \phi$$

Topological term for 2D (baby) Skyrmions $\pi_2(S^2) \cong \mathbb{Z}$

$$q \equiv -\frac{\mathrm{i}}{2\pi}\epsilon^{ij}\partial_i\phi^\dagger\partial_j\phi = \frac{1}{8\pi}\epsilon^{ij}\boldsymbol{n}\cdot(\partial_i\boldsymbol{n}\times\partial_j\boldsymbol{n})$$

Remark: chiral perturbation theory

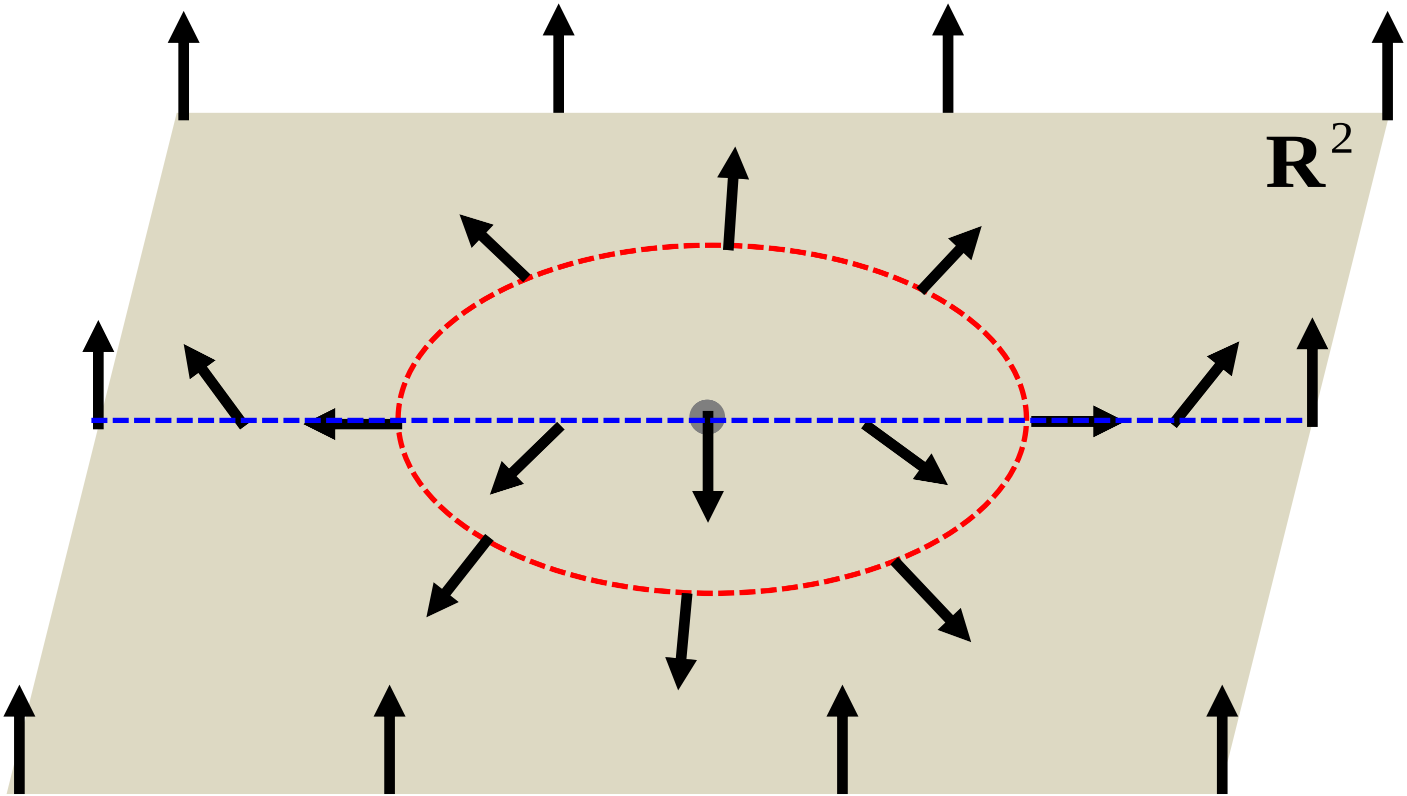
$$\partial_\mu \, , m_\pi \, , A_\mu = \mathcal{O}(p^1) \, , \qquad A_\mu^{\mathrm{B}} = \mathcal{O}(p^{-1})$$

$$\boldsymbol{F_{\mu\nu}^2} \in \boldsymbol{\mathcal{O}(p^4)}$$

Gauge field is nondynamical at the leading $\boldsymbol{\mathcal{O}(p^2)}$

2D (baby) Skyrmion (or lump)

$\pi_2(S^2) \cong \mathbb{Z}$



(3) Constructing lumps (baby Skyrmions).

$$\mathcal{H}_{\mathrm{DW}} = \frac{4f_\pi^2}{3m_\pi}(\partial_i \boldsymbol{n})^2 - 2\mu_{\mathrm{B}} q - \frac{e\mu_{\mathrm{B}}}{2\pi}\epsilon^{03jk}\partial_j[A_k(1-n_3)]$$

$$\partial_i \boldsymbol{n} \cdot \partial_i \boldsymbol{n} = \frac{1}{2}\underbrace{(\partial_i \boldsymbol{n} \pm \epsilon_{ij}\boldsymbol{n} \times \partial_j \boldsymbol{n})^2}_{=\,\mathbf{0}} \pm 8\pi q$$

BPS equation (the same as usual)

Bogomol'nyi bound

$$E_{\mathrm{DW}} \geq \underbrace{\frac{32 f_\pi^2 \pi \boxed{|k|}}{3m_\pi} - 2\mu_{\mathrm{B}} k}_{\text{Can become < 0 ?}} + \frac{e\mu_{\mathrm{B}} B}{4\pi}\oint \mathrm{d}S_i\, x^i (n_3 - 1)$$

➔ **nontrivial constraint**

$$k = \int \mathrm{d}^2 x\, q \in \mathbb{Z}$$ **lump number**

BPS lumps (the same with Belavin &Polyakov)

k **lump solutions**

$$w \equiv x + \mathrm{i}y$$

$$n_3 = \frac{1-|f|^2}{1+|f|^2}, \quad f = \frac{b_{k-1}w^{k-1} + \cdots + b_0}{w^k + a_{k-1}w^{k-1} + \cdots + a_0}$$

Baryons appear pairwise

$$\boxed{N_{\rm B} = \int \mathrm{d}^3x\, \mathcal{B} = 2\int \mathrm{d}^2x\, q = 2k}$$

$\pi_2(S^2) \cong \mathbb{Z}$ **on a wall (in D=2+1)**

2:1 $\updownarrow$

$\pi_3(S^3) \cong \mathbb{Z}$ **in the bulk (in D=3+1)**

However, it's not the end of the story.

There are <u>two nontrivial constraints</u> as follows.

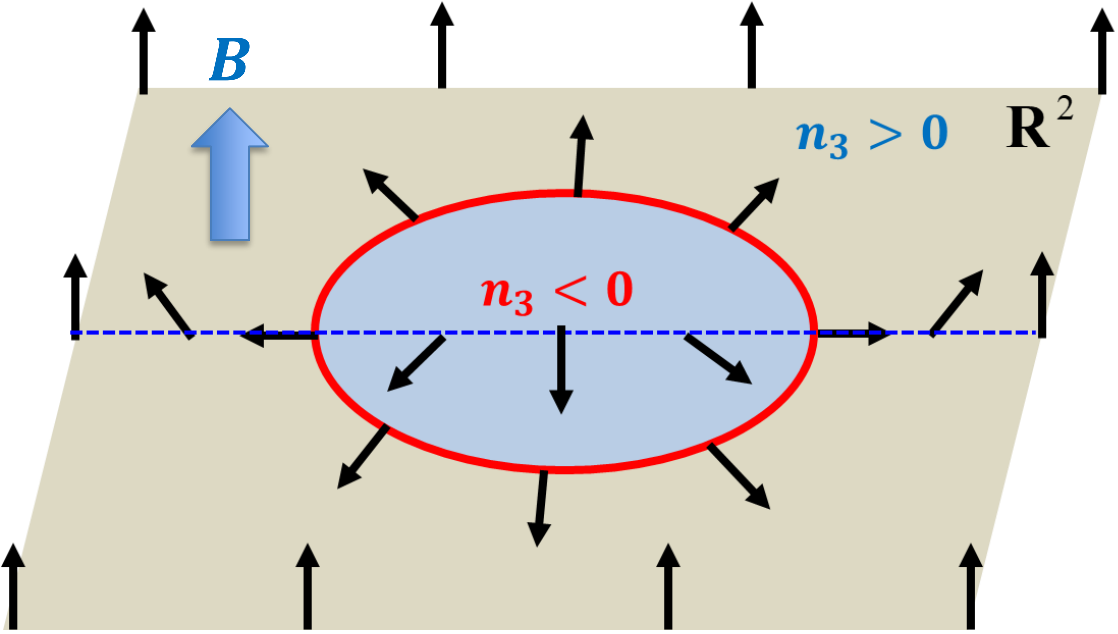
superconducting ring

$$n_1 + \mathrm{i}n_2 \rightarrow \mathrm{e}^{-\mathrm{i}\lambda}(n_1 + \mathrm{i}n_2)$$

n_3 is neutral in $\mathrm{U}(1)_{\mathrm{EM}}$

Area quantization

$$\boxed{BS_D = \int_D \mathrm{d}^2x\, B = \oint_C \mathrm{d}x^i A_i = \frac{1}{e}\oint_C \mathrm{d}x^i \partial_i \psi = \frac{2\pi k}{e}}$$



$$n_1 + \mathrm{i}n_2 = \mathrm{e}^{i\psi}$$

$$|D_\alpha(n_1 + \mathrm{i}n_2)|^2 = 0$$

$$\partial_\alpha \psi = eA_\alpha$$

superconducting ring

$$n_1 + \mathrm{i}n_2 \rightarrow \mathrm{e}^{-\mathrm{i}\lambda}(n_1 + \mathrm{i}n_2)$$

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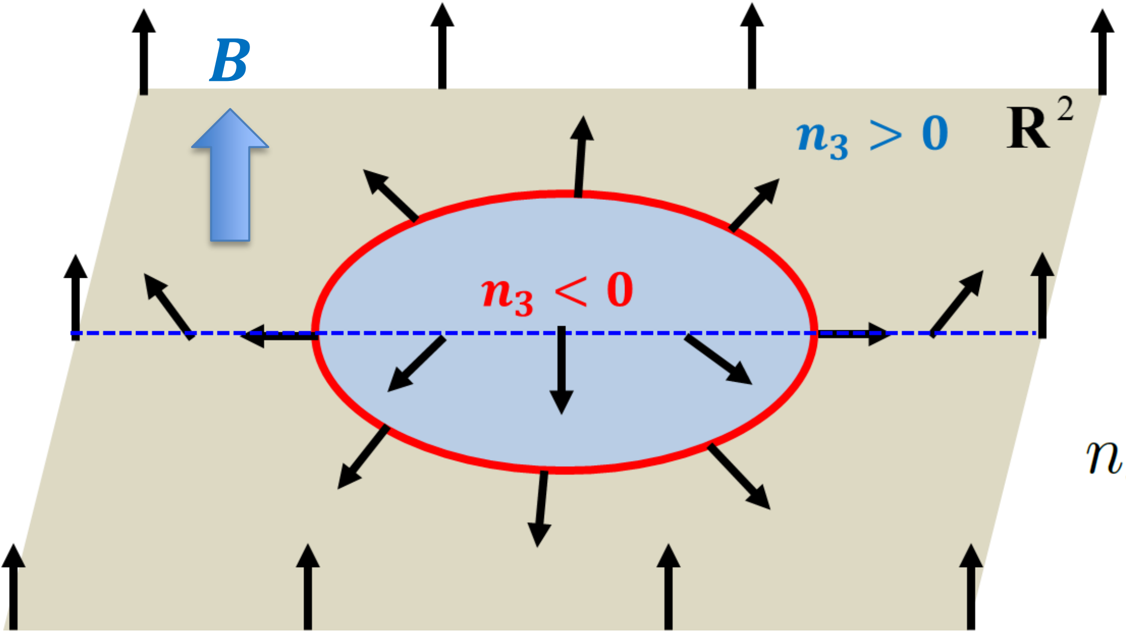


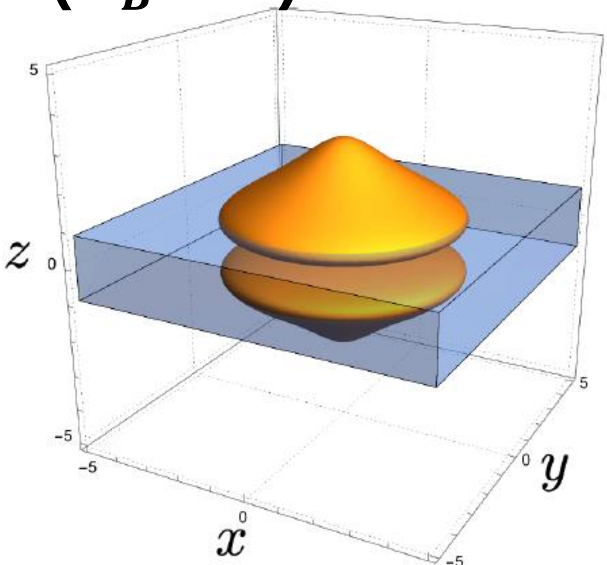
$$k = 1 \qquad f = b_0/w$$

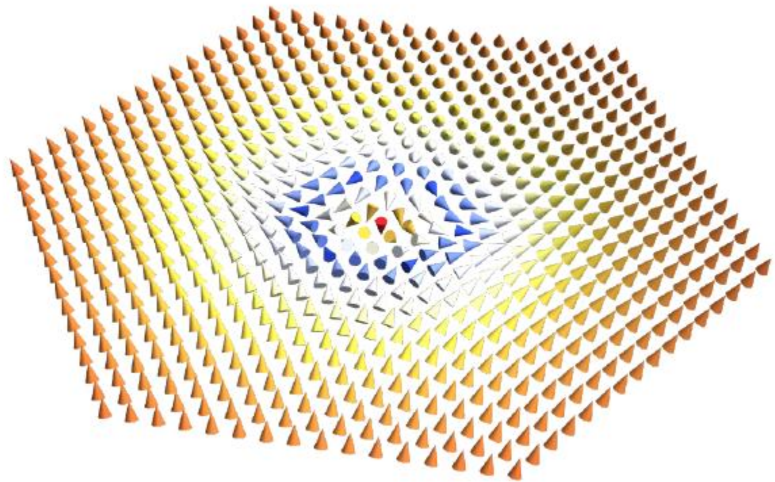
$$n_3 = \frac{|w|^2 - |b_0|^2}{|w|^2 + |b_0|^2}$$

$$n_3 = 0 \;@\; |w| = |b_0|$$

$$b_0 = 4/eB$$

$k = 1$ ($N_B = 2$) domain-wall Skyrmion





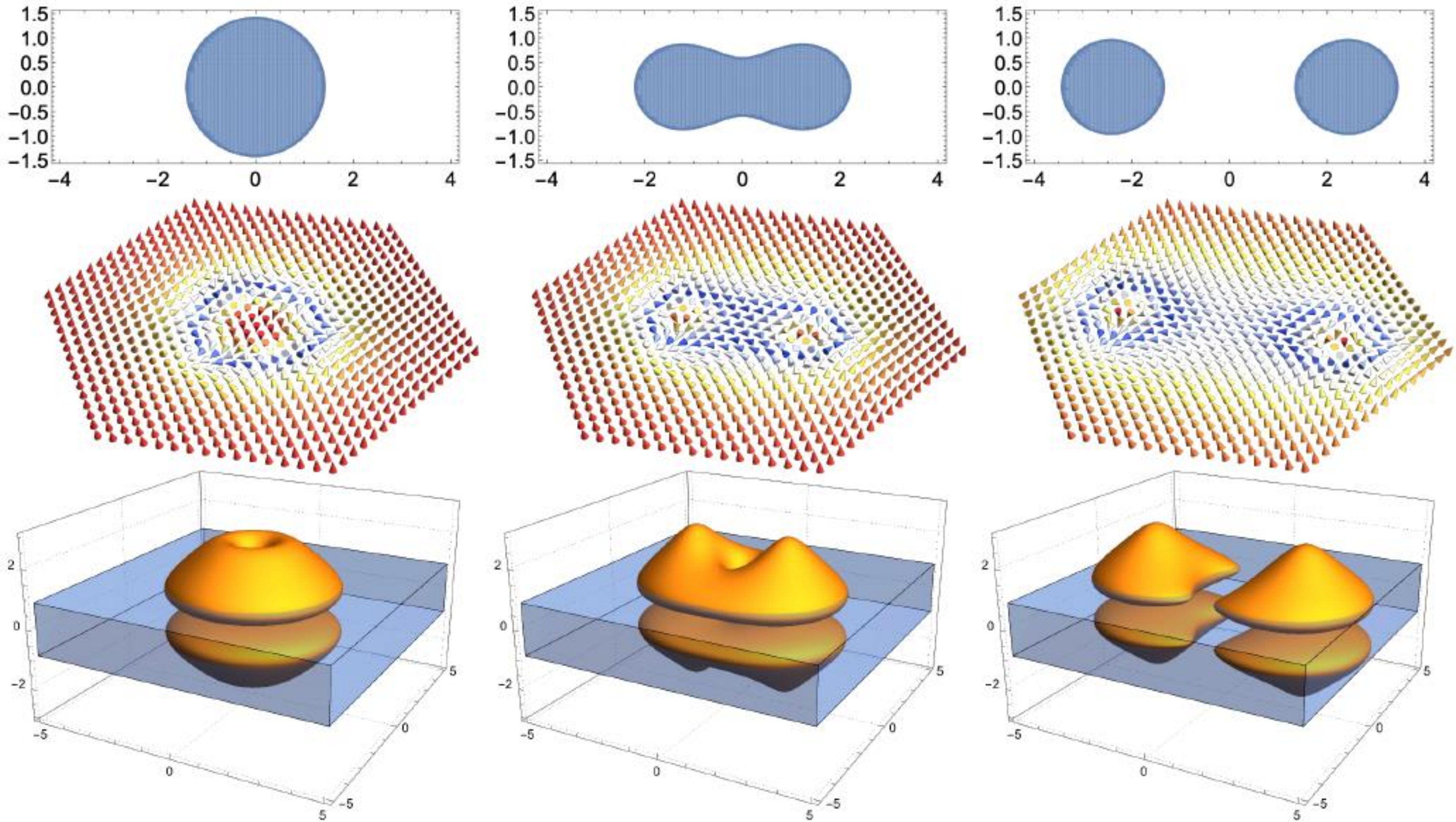
Iso-baryon number density surface

Macaron

Dorayaki
(Japanese sweets)

(kaiundo.co.jp)

$k = 2$ $(N_B = 4)$ DW Skyrmion: Area preserving deformation



DW-Skyrmion energy

Different physics between
$\boldsymbol{k = 1\ (N_B = 2)\ \&\ k \geq 2\ (N_B \geq 4)}$

$$E_{\mathrm{DWSk}} = \frac{32\pi f_\pi^2}{3m_\pi}|k| - 2\mu_{\mathrm{B}}k + e\mu_{\mathrm{B}}B|b_{k-1}|^2$$

DW-Skyrmion energy

Different physics between
$k=1$ $(N_B=2)$ & $k\geq 2$ $(N_B\geq 4)$

$$E_{\rm DWSk} = \frac{32\pi f_\pi^2}{3m_\pi}|k| \underbrace{- 2\mu_{\rm B}k + e\mu_{\rm B}B|b_{k-1}|^2}_{+2\mu_B}$$

<u>$k=1$ $(N_B=2)$</u> $\quad b_0 = 4/eB$

Always positive energy.

DW-Skyrmion energy

Different physics between
$k = 1$ ($N_B = 2$) & $k \geq 2$ ($N_B \geq 4$)

$$E_{\rm DWSk} = \underbrace{\frac{32\pi f_\pi^2}{3m_\pi}|k| - 2\mu_{\rm B}k}_{\text{Can become} < 0} + \cancel{e\mu_{\rm B}B|b_{k-1}|^2}$$

Quantization $|b_0| = \left(\frac{2k}{eB}\right)^{\frac{k}{2}}$

<u>$k \geq 2$ ($N_B \geq 4$)</u> **Further constraint** $b_{k-1} = 0$

The condition that 2D Skyrmions appear on a wall in the ground state

$$E_{\rm DWSk} = \frac{32\pi f_\pi^2}{3m_\pi}|k| - 2\mu_{\rm B} k \leq 0$$

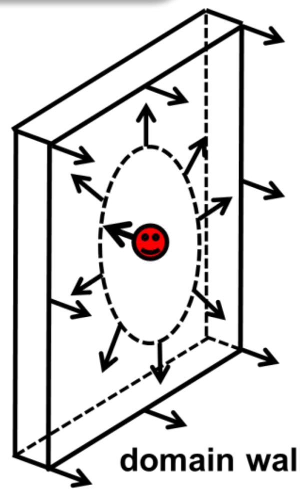
$$\boxed{\mu_B \geq \mu_c = \frac{16\pi f_\pi^2}{3m_\pi} \sim 1.03\ \text{GeV}}$$

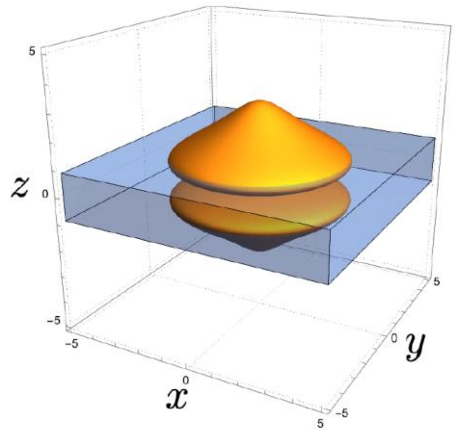
Nucleon mass in terms of pions' constants

$$(\mu_{\rm B}, B) = \left(\frac{16\pi f_\pi^2}{3m_\pi}, \frac{3m_\pi^2}{e}\right)$$

$$0.06\,\text{GeV}^2 \sim 1.0 \times 10^{19}\,\text{G}$$

Future heavy-ion collider





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Our results: New phase in QCD

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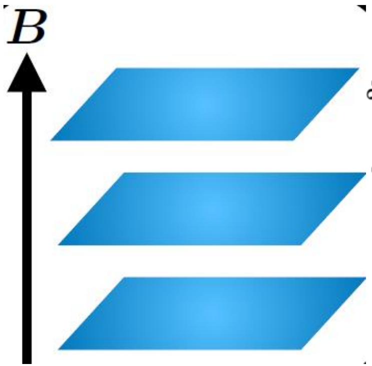

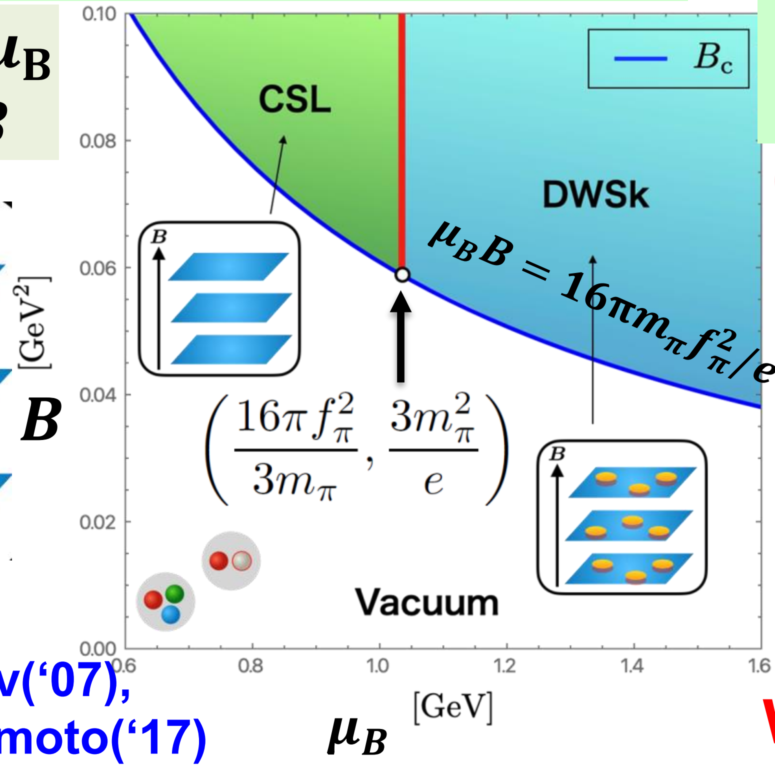

Son & Stephanov('07),
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Solitons carry baryon #

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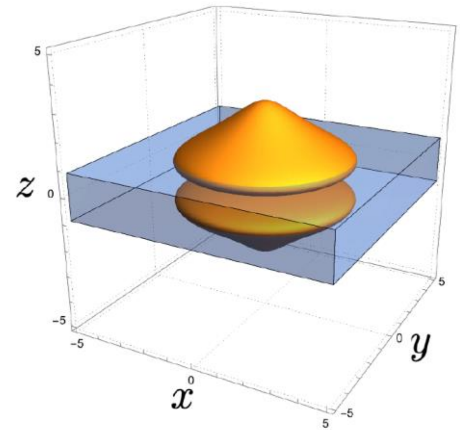
@ $\mu_B \geq 1.03$ GeV



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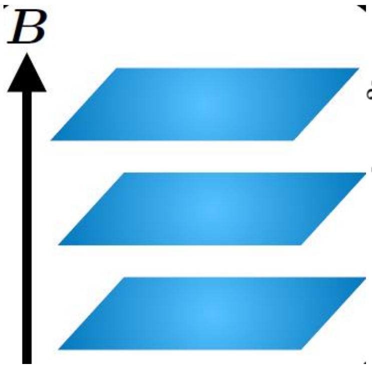
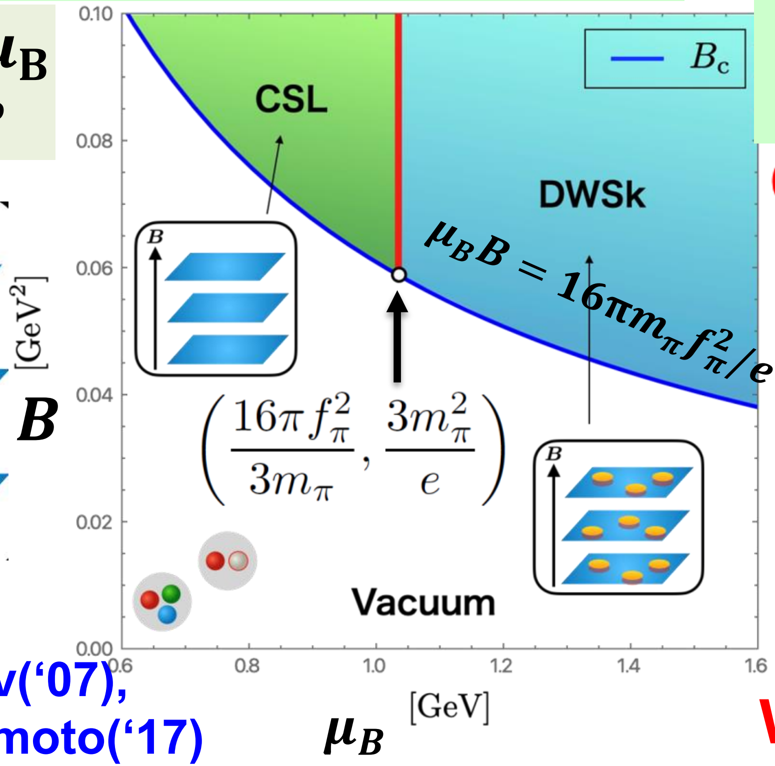
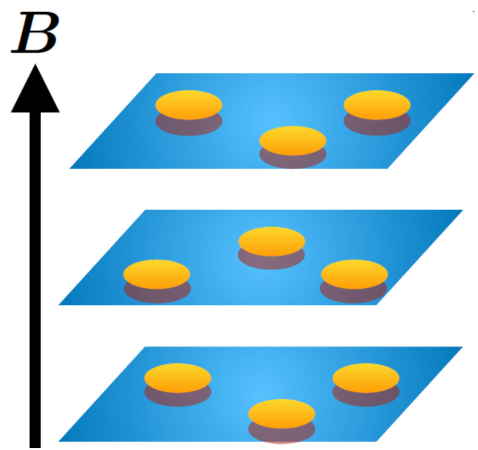
chemical pot. μ_{B}
magnetic field B





Domain-Wall Skyrmion Crystal (DW-SkX) phase

@ $\mu_B \geq 1.03$ GeV



Son & Stephanov('07), Brauner & Yamamoto('17)

Solitons carry baryon #

Walls & Skyrmions carry baryon #

(1) Quasicrystals in QCD

Z.Qiu & MN, *JHEP* 05 (2023) 170, 2304.05089 [hep-ph]

$$\phi_0 \equiv \frac{\eta}{f_\eta}, \quad \phi_3 \equiv \frac{\pi_3}{f_\pi}$$

WZW $\mathcal{L}_B = \frac{\mu}{4\pi^2} \boldsymbol{B} \cdot \left(\nabla \phi_3 + \underline{\frac{1}{3} \nabla \phi_0} \right)$ ➔both η and π modulate

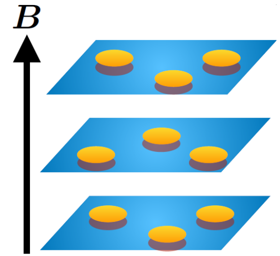
If $\alpha \equiv \frac{f_\pi^2}{f_\eta^2}$ is $\begin{cases} \text{rational} \rightarrow \text{lattice(crystal)} \\ \text{irrational} \rightarrow \text{quasicrystal} \end{cases}$

(2) Rotation (instead of magnetic field)

Eto, Nishimura & MN, *JHEP* 08 (2022) 305 2112.01381 [hep-ph]

WZW $\frac{\Omega \mu_B^2}{2\pi^2 N_c} \partial_z \frac{\eta}{f_\eta}$ ➔ η CSL(Nishimura & Yamamoto), Non-Abelian CSL

A lot of future directions!!



(1) Structure of SkX depending on μ_B & B

Interaction, triangular or square lattice

(2) Multi-solitons⬄ instability curve of Brauner & Yamamoto

(3) CPT @ $\mathcal{O}(p^4)$ ➔ dynamical gauge field

(4) ➔ Bulk SkX (Chen, Fukushima & Qiu $B \neq 0$. Klebanov $B = 0$)

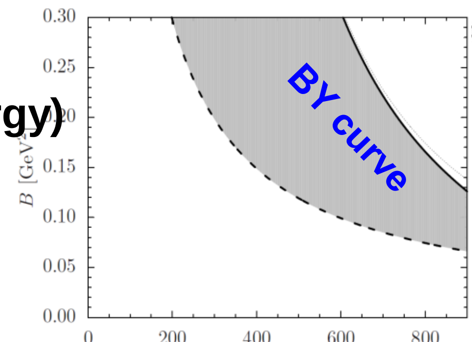
(5) Quantization ➔ proton/neutron (or anyon?)

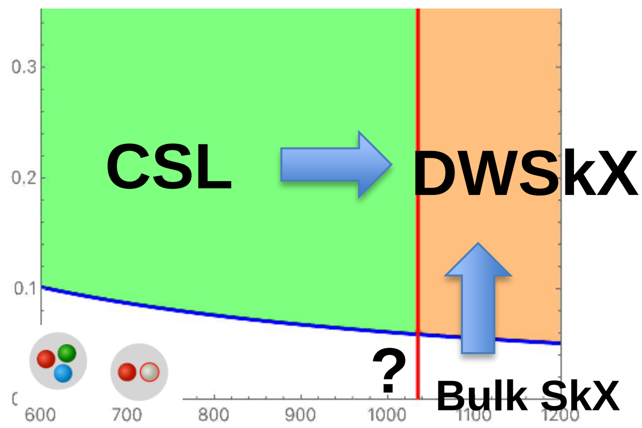
(6) $SU(3)_F$ ➔ $\mathbb{C}P^2$ model

(7) DWSkX under Rotation

(8) Nucleon mass?

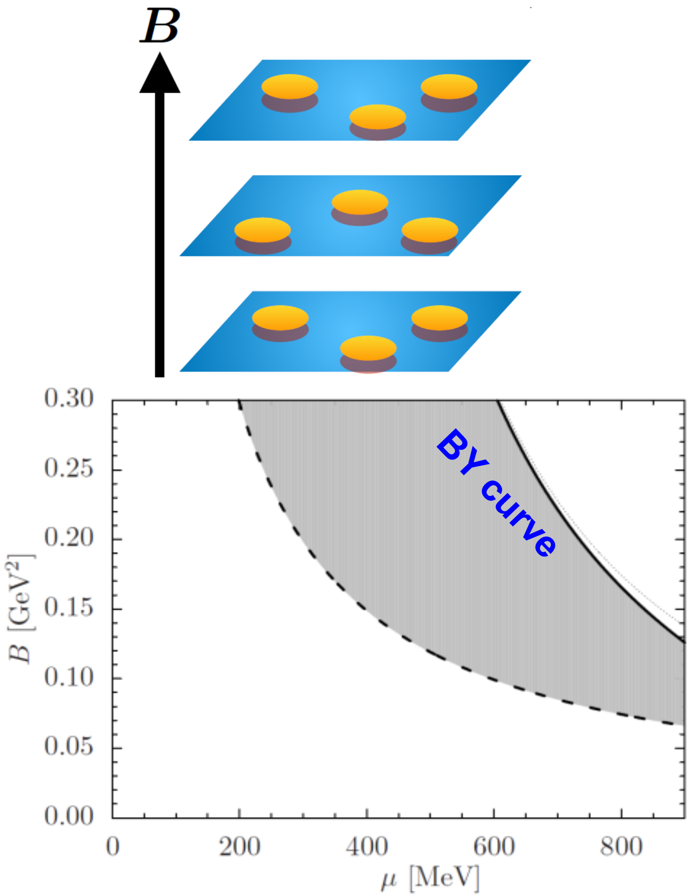
(medium effect, binding energy)

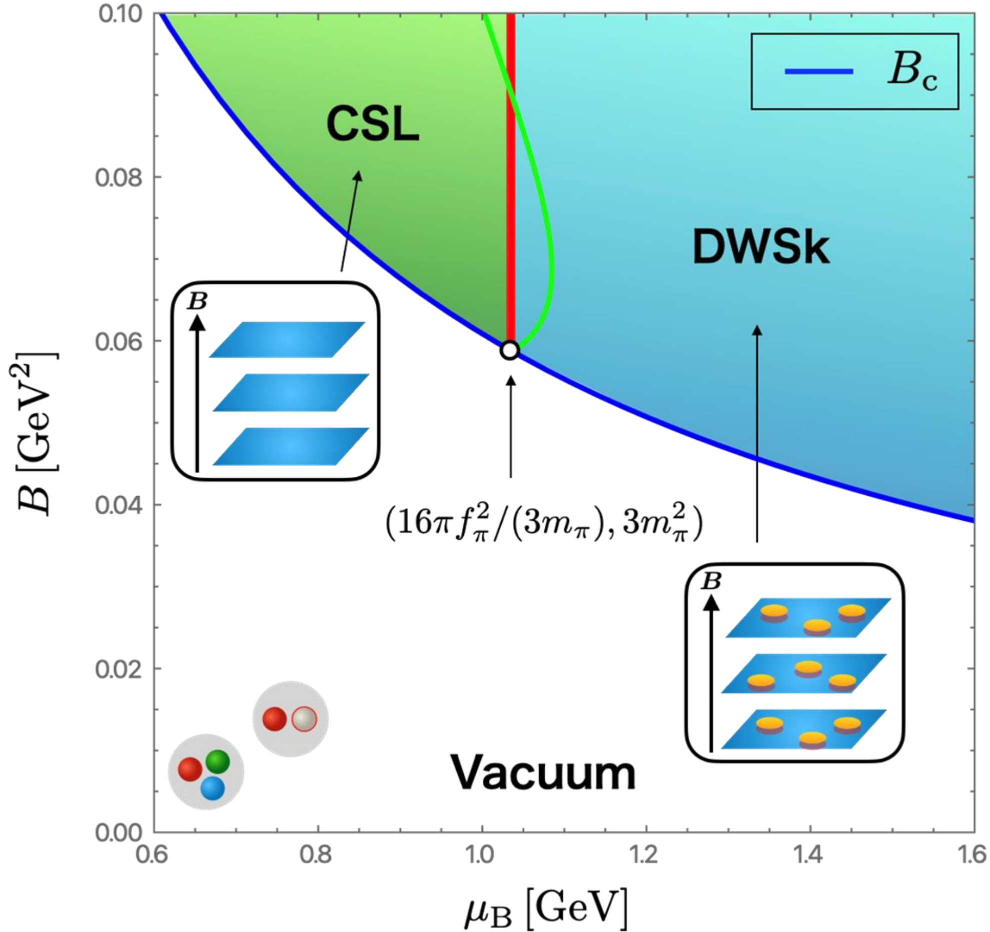




Welcome to join to Collaboration !!

(2) Multi-solitons⇔ instability curve of Brauner & Yamamoto





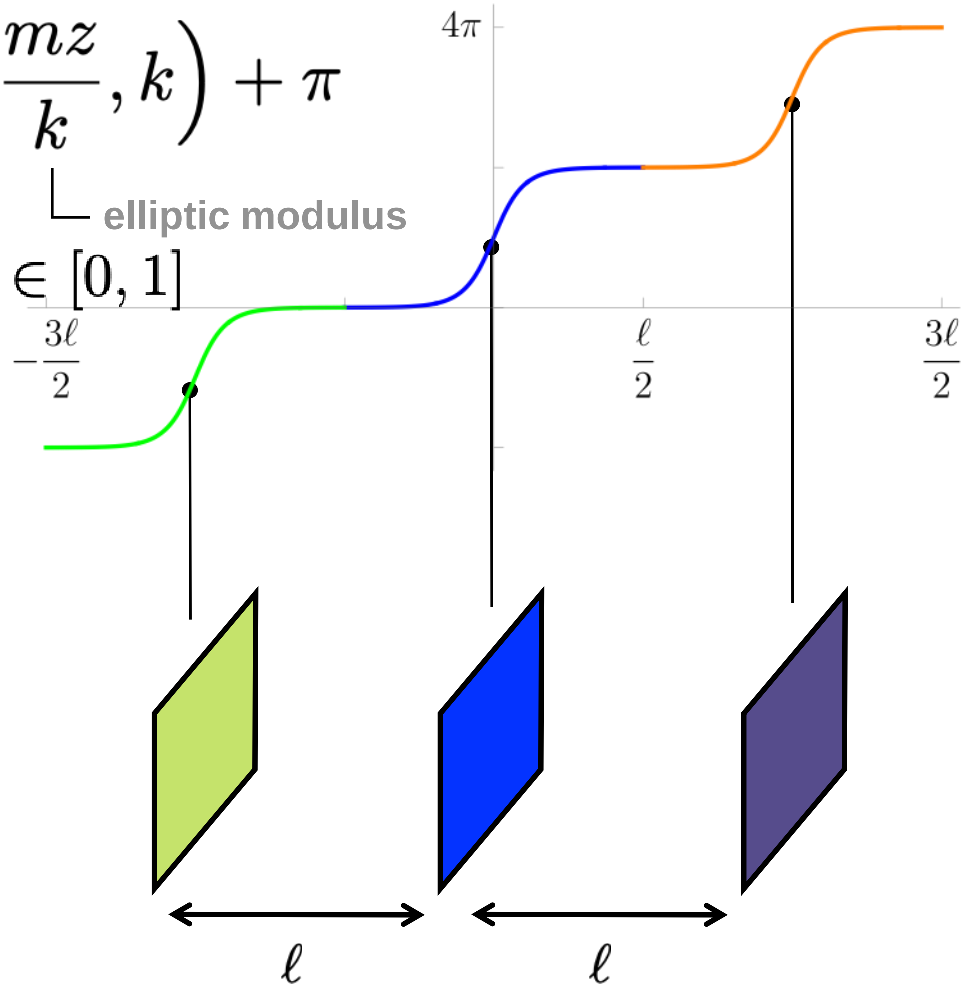
Chiral soliton lattice

$$\chi_3(z) = 2\mathrm{am}\left(\frac{mz}{k}, k\right) + \pi$$

Jacobi amplitude function

elliptic modulus

$k \in [0, 1]$



lattice spacing

$$\ell(k) = 2kK(k)/m$$

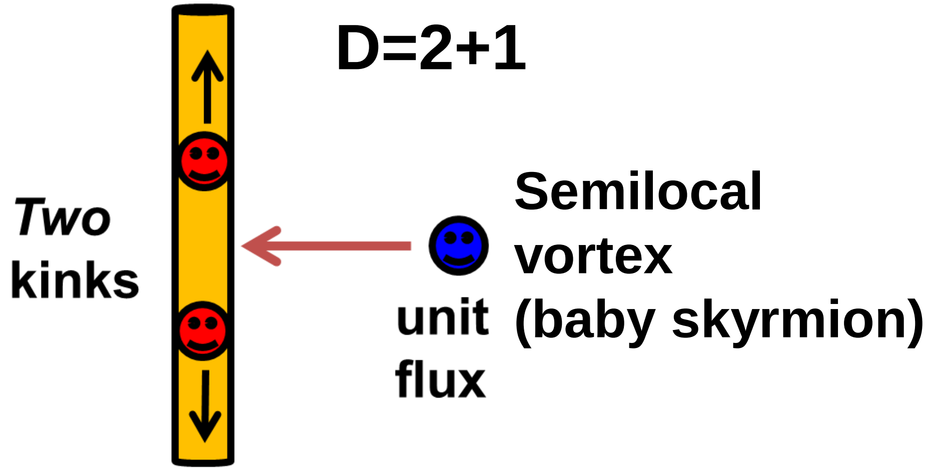
elliptic integral of 1st kind

Comments on supersymmetry

(1) D=2+1 version

N =1 supersymmetry

Auzzi, Shifman & Yung ('06)



(2) Baryons on domain wall in supersymmetric QCD

Armoni & Shifman ('03)