Wave Packets in AdS/CFT Correspondence

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4 Aug. 2023

“String and Fields” at YITP

based on the following papers:

2304.08478 [hep-th] ;

PTEP,2104.11743 [hep-th] ;PRD104 (2021) 8, 2005.05962 [hep-th];

with Sotaro Sugishita JHEP11(2022)041, 2207.06455 [hep-th] and to appear;

Introduction

One way to study quantum gravity is AdS/CFT duality

Maldacena

Quantum gravity on AdS

= conformal field theory (CFT)

Highly non-trivial and important!

Of course, CFT is not quantum gravity in general.

Special class of CFT, called Holographic CFT, is dual to quantum gravity on AdS

Typical holographic CFT:
SU(N) gauge theory with conformal symmetry

$$\text{Newton constant } G_N \sim \frac{1}{N^2}$$

Large N is needed for the bulk spacetime picture

Important property:

(d+1)-dim gravity = d-dim CFT

Gravity theory on AdS_{d+1}
(bulk theory)

$\Updownarrow$

CFT_d on $\mathbf{R} \times S^{d-1}$
(boundary theory)

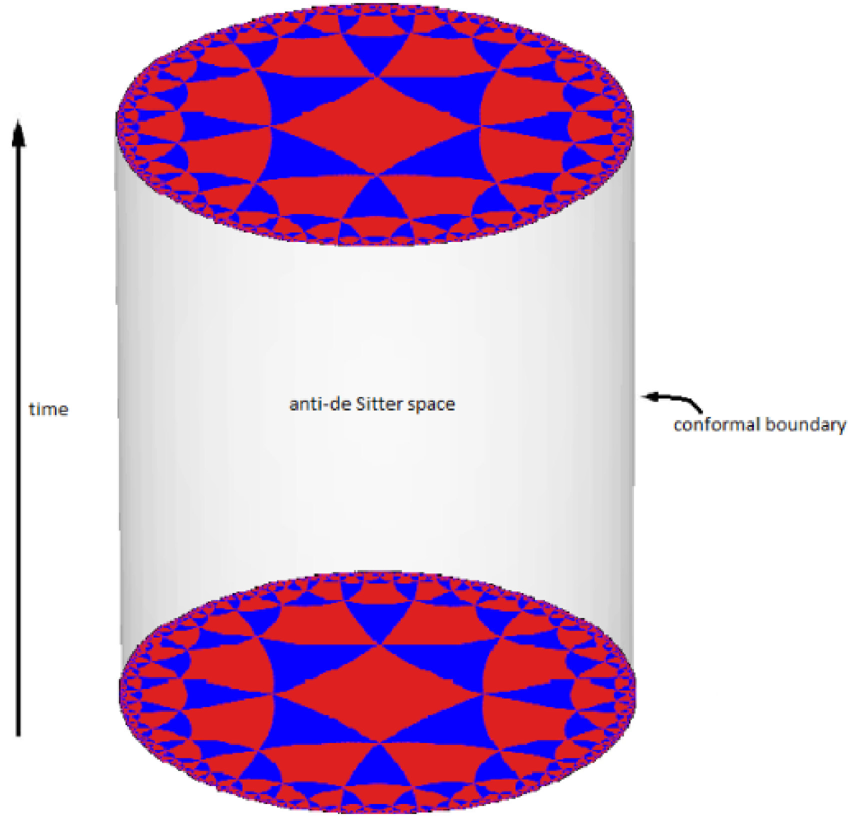


figure from Wikipedia

Usual formulation of AdS/CFT

equivalences of partition function with source

$$Z_{bulk}(J) = Z_{CFT}(J)$$

J as boundary condition in AdS

⇕

J as source terms in CFT

This relation, called GKPW relation, is assumed

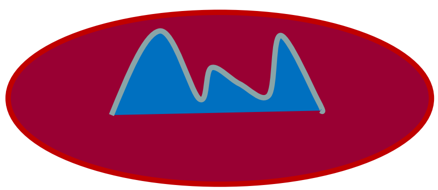
Another formulation of AdS/CFT

In operator formalism,

equivalence between
Hilbert spaces and Hamiltonians
of gravity on AdS and CFT

Low energy states of bulk theory and CFT

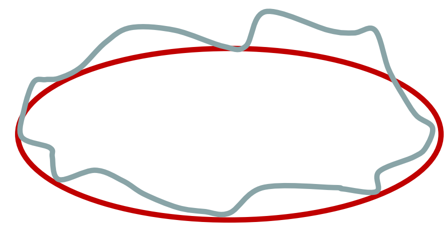
Bulk theory



states on disk

1 to 1

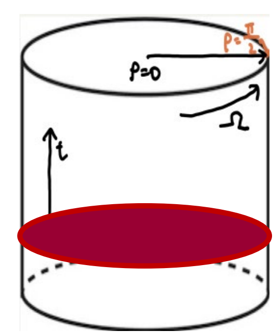
CFT



states on S^1

(Not on boundary, but in bulk.
CFT does not live in boundary.)

They are on a fixed time slice of AdS or cylinder



In this talk, we will study
AdS/CFT in operator formalism

This has not been studied so much,

and,

important to understand
how bulk space-time emerges from CFT

We will focus on Large N limit, which is essential for AdS/CFT duality

We can show that

Low energy spectrum of large N CFT_d

$\Updownarrow$ **equivalent!**

Spectrum of free gravity on AdS_{d+1}

We explicitly construct bulk wave packet state from CFT operator and compute time-evolution of energy density in CFT.

We will see this violates the entanglement wedge reconstruction.

We will explains how AdS/CFT for subregion is realized

Plan

1. **Introduction**
2. **Bulk wave packet**
3. **AdS/CFT for subregion**
4. **Difference between bulk semi-classical gravity theory and finite N CFT**
5. **Conclusion**

Bulk wave packet

Wave packets in Minkowski space

Wave packet of a free scalar field $\phi(t,\vec{x})$ in $d+1$ dimension at $t=\vec{x}=0$ with momentum $\vec{p}$:

$$\int d\vec{x}\, e^{-\frac{\vec{x}^2}{2a^2}+i\vec{p}\cdot\vec{x}}\phi(t,\vec{x})|_{t=0}|0\rangle \propto \int d\vec{k}\, e^{-\frac{a^2(\vec{k}-\vec{p})^2}{2}} a^\dagger_{\vec{k}}|0\rangle$$

where a is size of wave packet

Instead of this, we can use

$$\int dt \prod_{i=2,\cdots,d} dx^i\, e^{-\frac{x^i x_i+t^2}{2a^2}+ip_i x^i+i\omega t}\phi(t,\vec{x})|_{x_1=0}|0\rangle$$

$$\propto \int d\vec{k}\, e^{-\frac{a^2}{2}\left((k^i-p^i)(k_i-p_i)+(\sqrt{(k_1)^2+k^i k_i}-\omega)^2\right)} a^\dagger_{\vec{k}}|0\rangle$$

where i runs only for $2,\cdots,d$.

General wave packets in AdS/CFT

Near the boundary of AdS_{d+1} with scaling, metric is

$$ds^2 = \frac{1}{z^2}\left(-dt^2 + dz^2 + \delta_{ij} dx^i dx^j\right),$$

where $z > 0$ and $i, j = 1, 2, \cdots, d-1$.

General wave packet in AdS space is given by

$$|p, \bar{\omega}\rangle = \lim_{z \to 0} \frac{1}{z^\Delta} \int dt\, dx^i\, e^{-\frac{x^i x_i + t^2}{2a^2} + i p_i x^i - i\bar{\omega} t} \phi(t, z, x^i)|0\rangle$$

General wave packet in AdS space is given by

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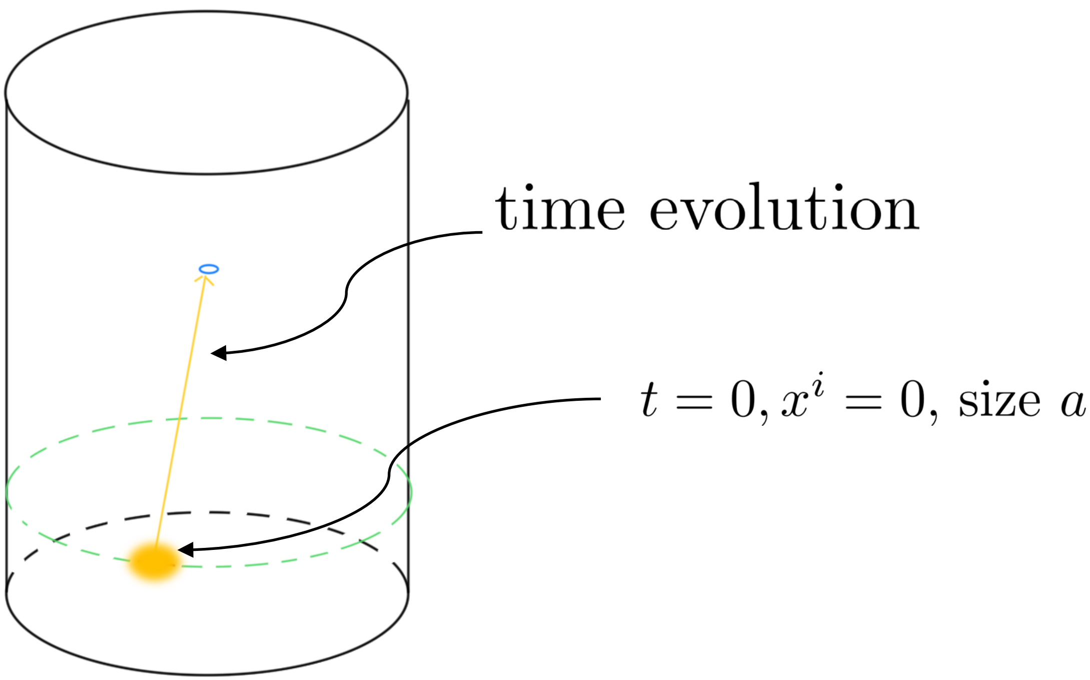


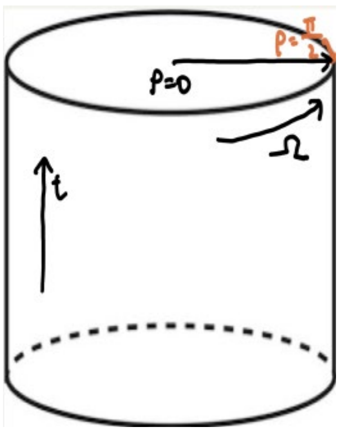
Bulk local field near boundary
(BDHM extrapolation formula):

Bulk operator at boundary is CFT primary field:

$$\lim_{z\to 0} \frac{\phi(t,z,\Omega)}{z^{\Delta}} \sim \mathcal{O}(t,\Omega), \quad \text{where } z = \pi/2 - \rho$$

BDHM

$\mathcal{O}(t,\Omega)$ is CFT primary field



General wave packets in AdS/CFT

General wave packet in AdS/CFT is given by

$$|p, \bar{\omega}\rangle = \lim_{z \to 0} \frac{1}{z^{\Delta}} \int dt\, dx^i\, e^{-\frac{x^i x_i + t^2}{2a^2} + ip_i x^i - i\bar{\omega} t} \phi(t, z, x^i)|0\rangle$$

$$= \int dt\, dx^i\, e^{-\frac{x^i x_i + t^2}{2a^2} + ip_i x^i - i\bar{\omega} t} \mathcal{O}(t, x)|0\rangle$$

in CFT picture

where we have used BDHM relation

Similar construction in Kinoshita-Murata-Takeda

Energy density of the wave packet

$$\mathcal{E}(t,x) \sim \langle p,\bar{\omega}|T_{00}(t=\bar{t},x^i=\bar{x}^i)|p,\bar{\omega}\rangle$$

$$= \int dt_1\, dx_1^i\, e^{-\frac{(x_1^i)^2+t_1^2}{2a^2}-ip_i x_1^i+i\bar{\omega}t_1} \int dt_2\, dx_2^i\, e^{-\frac{(x_2^i)^2+t_2^2}{2a^2}+ip_i x_2^i-i\bar{\omega}t_2}$$

$$\times \langle 0|\mathcal{O}(t_1,x_1)T_{00}(t=\bar{t},x^i=\bar{x}^i)\mathcal{O}(t_2,x_2)|0\rangle$$

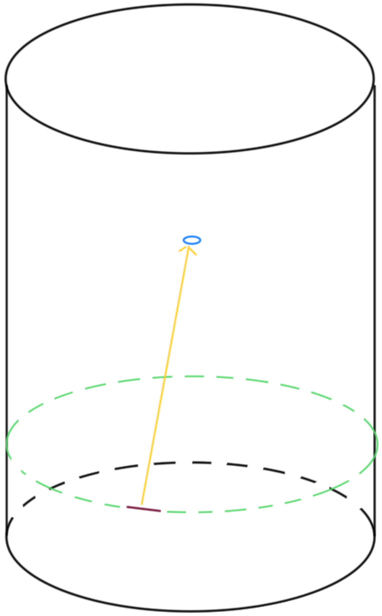
3-point function (TOO) is universal, and then computable

For $d=2$,

$$\mathcal{E}(t,x) \simeq \frac{1}{2\sqrt{2\pi}a}\left(e^{-\frac{(x+t)^2}{2a^2}}(\bar{\omega}-p)+e^{-\frac{(x-t)^2}{2a^2}}(\bar{\omega}+p)\right).$$

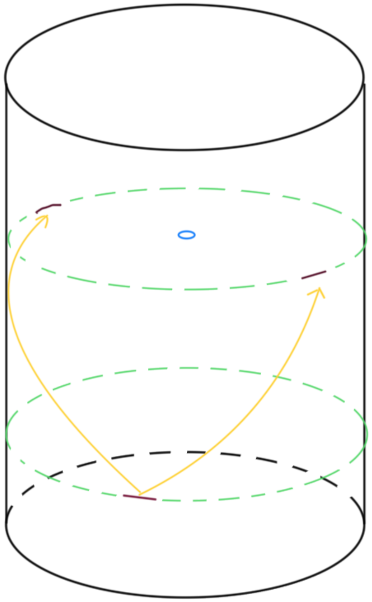
Bulk picture

An example of the bulk wave packet
(moving toward the center as an example).



CFT picture

The corresponding
two ”particles” in the CFT picture



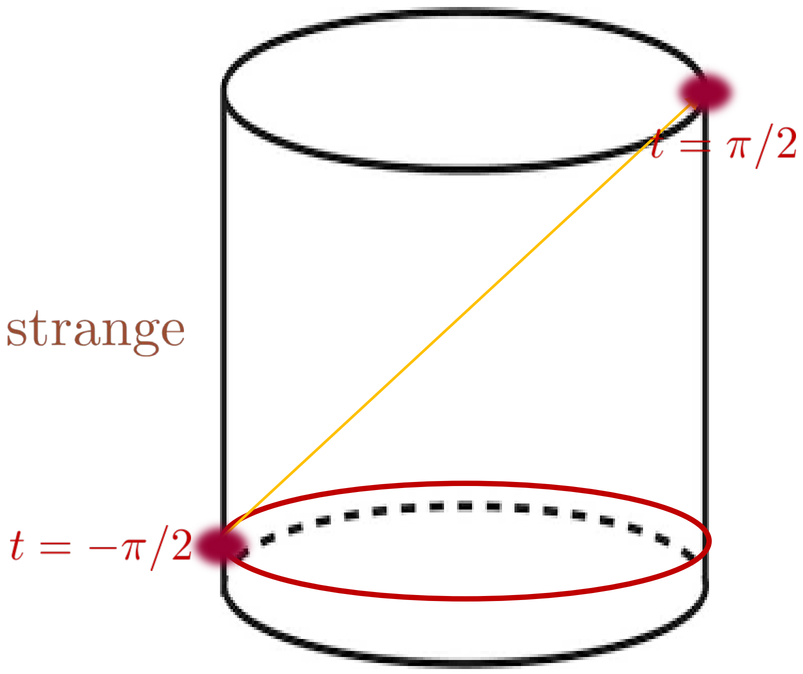
Overlap between the wave packet state and CFT local state

$$\langle 0|\mathcal{O}(\tau,\theta)|p,\bar{\omega}\rangle \simeq a^4 e^{-i\bar{\omega}\tau+ip\theta}\delta(\tau+\pi\mathbf{Z})\delta(\theta-\tau+2\pi\mathbf{Z})$$

This is related to VEV of $\mathcal{O}$ for the coherent state

This is strange

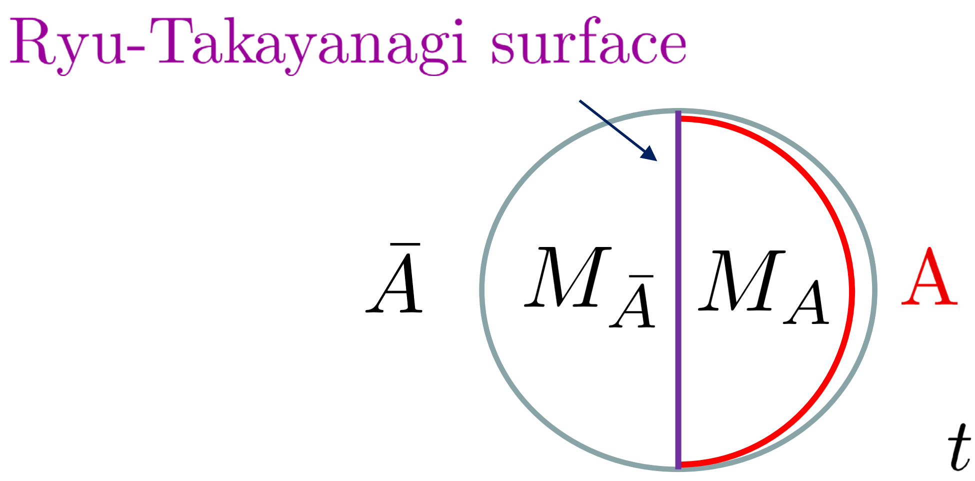
because generalized free field is strange



This violates subregion duality
and entanglement wedge reconstruction

What is subregion duality?

Decompositions for Bulk and CFT

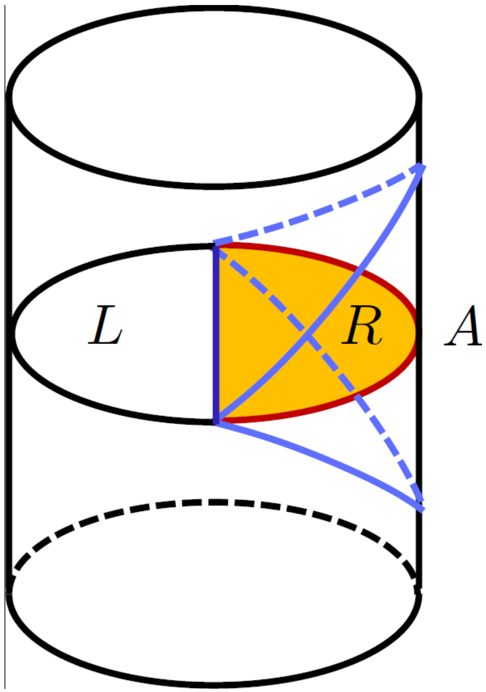


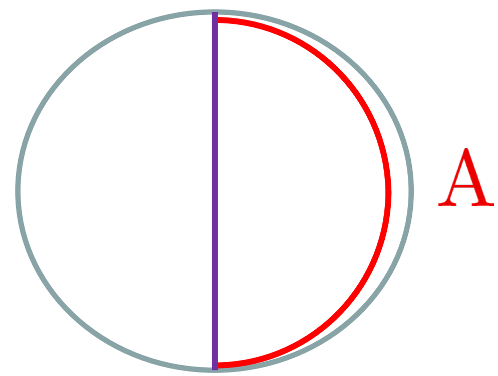
$t = 0$ slice of AdS_3

$$\text{Bulk space} = M_A + M_{\bar{A}}$$

$$\text{CFT space } (= S^1) = A + \bar{A}$$

M_A =Entanglement wedge of A





$t = 0$ slice of AdS_3

Subregion duality:

For density matrices ρ, σ,

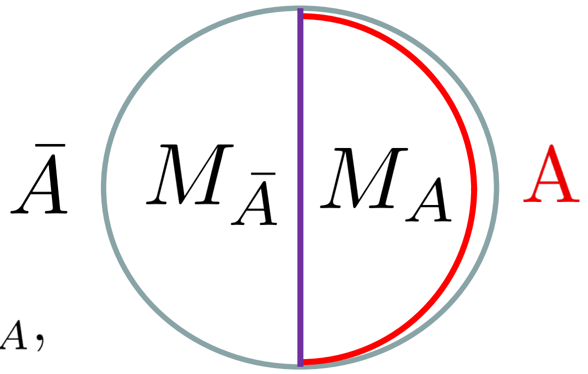
$$\rho_A = \sigma_A \Leftrightarrow \rho_{M_A} = \sigma_{M_A}$$

where $\rho_A = tr_{\bar{A}}(\rho), \rho_{M_A} = tr_{M_{\bar{A}}}(\rho)$

Entanglement wedge reconstruction:

For low enegy state $|\phi\rangle$,

$$^{\forall}\mathcal{O}_{M_A}|\phi\rangle = {}^{\exists}\mathcal{O}_A|\phi\rangle,$$



$\mathcal{O}_{M_A}$ is bulk operator supported in M_A,
$\mathcal{O}_A$ is CFT operator supported in A

"Derivation"

Jafferis-Lewkowycz-Maldacena-Suh show

CFT relative entropy = bulk relative entropy

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Subregion duality

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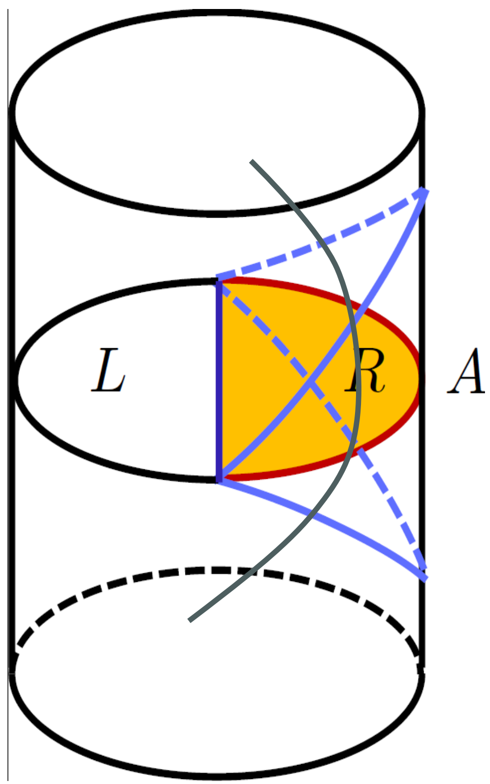
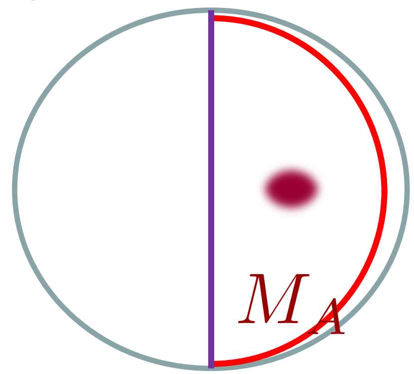
Entanglement wedge reconstruction

Consider "horizon-horizon" wave packet



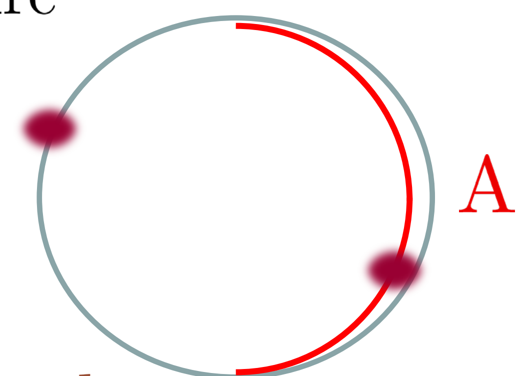
Bulk picture

$t = 0$ slice



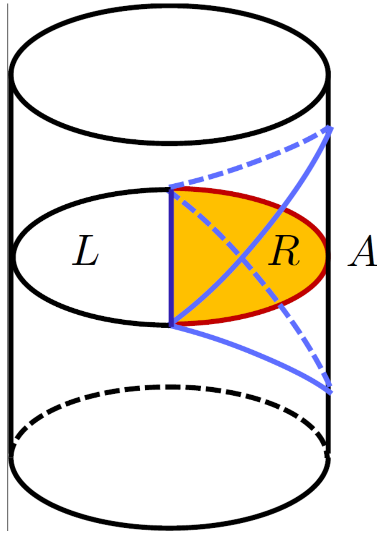
CFT picture

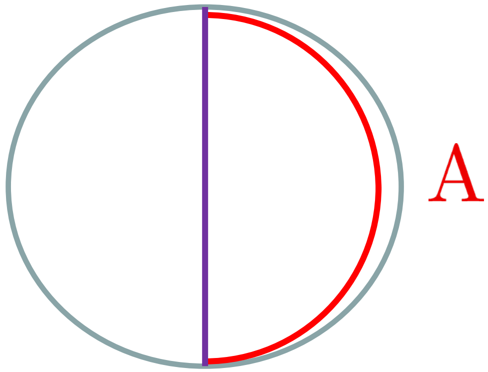
$t = 0$ slice



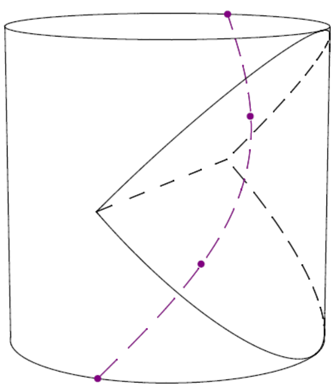
Violation of entanglement wedge reconstruction!

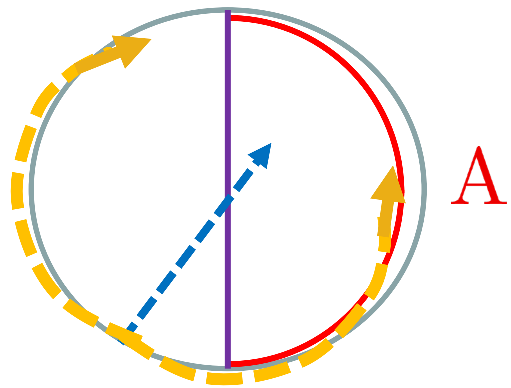
$$\mathcal{O}_{M_A}|\phi\rangle \neq {}^{\exists}\mathcal{O}_A|\phi\rangle,$$





Bulk wave packet on $t = 0$ is in entanglement wedge of A





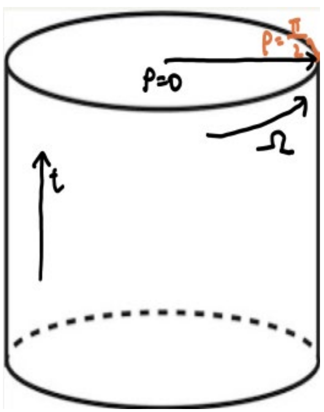
But, CFT wave packet is NOT in A

AdS/CFT for subregion

(Global) AdS_{d+1}

The metric of global AdS_{d+1} ($l_{AdS} = 1$) is

$$ds^2_{AdS} = \frac{1}{\cos^2(\rho)}\left(-dt^2 + d\rho^2 + \sin^2(\rho) d\Omega^2_{d-1}\right)$$



where $0 \leq \rho < \pi/2$

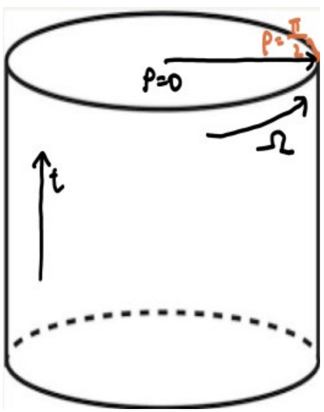
$$z \equiv \pi/2 - \rho$$

Boundary of AdS_{d+1} is located at $\rho = \pi/2$

Boundary of (Global) AdS_3

The boundary of AdS_3 is the cylinder

$$ds^2_{cylinder} = -dt^2 + d\theta^2$$



(Global) HKLL bulk reconstruction

Bulk local fied $\phi(t_0, z_0, \theta_0)$
is related to fields at boundary $\phi(t, z=0, \theta)$
using free e.o.m.

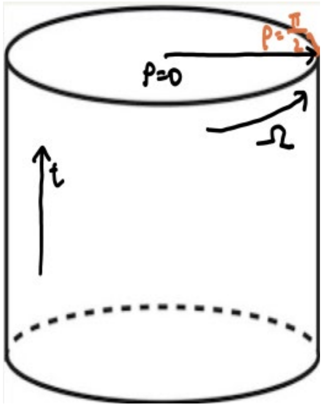
Then, using BDHM relation $\lim_{z \to 0} \frac{\phi(t,z,\theta)}{z^{\Delta}} \sim \mathcal{O}(t,\theta)$

$\Downarrow$

Bulk local fied is given by CFT field:

$z \equiv \pi/2 - \rho$

$$\phi(t,z,\theta) \leftrightarrow \int dt' d\theta' K(\theta', t') \mathcal{O}(\theta', t')$$



(Global) HKLL bulk reconstruction

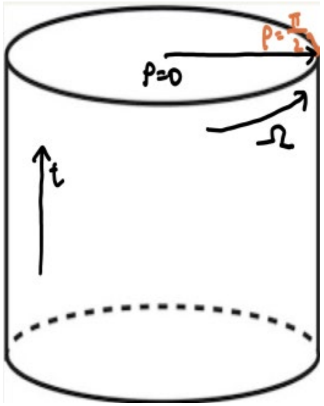
We will denote the CFT operator as

$$\phi^G(t,z,\theta) \equiv \int dt' d\theta' K(\theta',t')\mathcal{O}(\theta',t')$$

Then, we can show

$$\langle 0|\phi(t,z,\theta)\phi(t',z',\theta')|0\rangle = \langle 0|\phi^G(t,z,\theta)\phi^G(t',z',\theta')|0\rangle$$

i.e. bulk correlation function
is reproduced by CFT oeprator



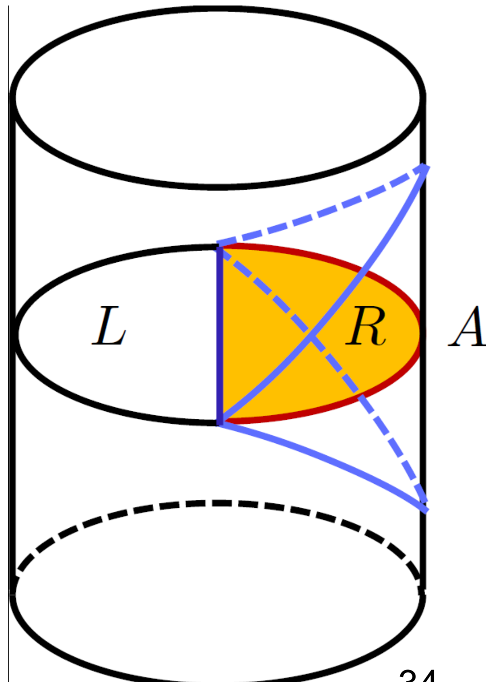
Rindler AdS_3

The metric of Rindler patch of AdS_3 $(l_{AdS} = 1)$ is

$$ds^2 = -\xi^2 dt_R^2 + \frac{d\xi^2}{1+\xi^2} + (1+\xi^2)d\chi^2$$

where $-\infty < t_R < \infty$, $-\infty \leq \chi < \infty$

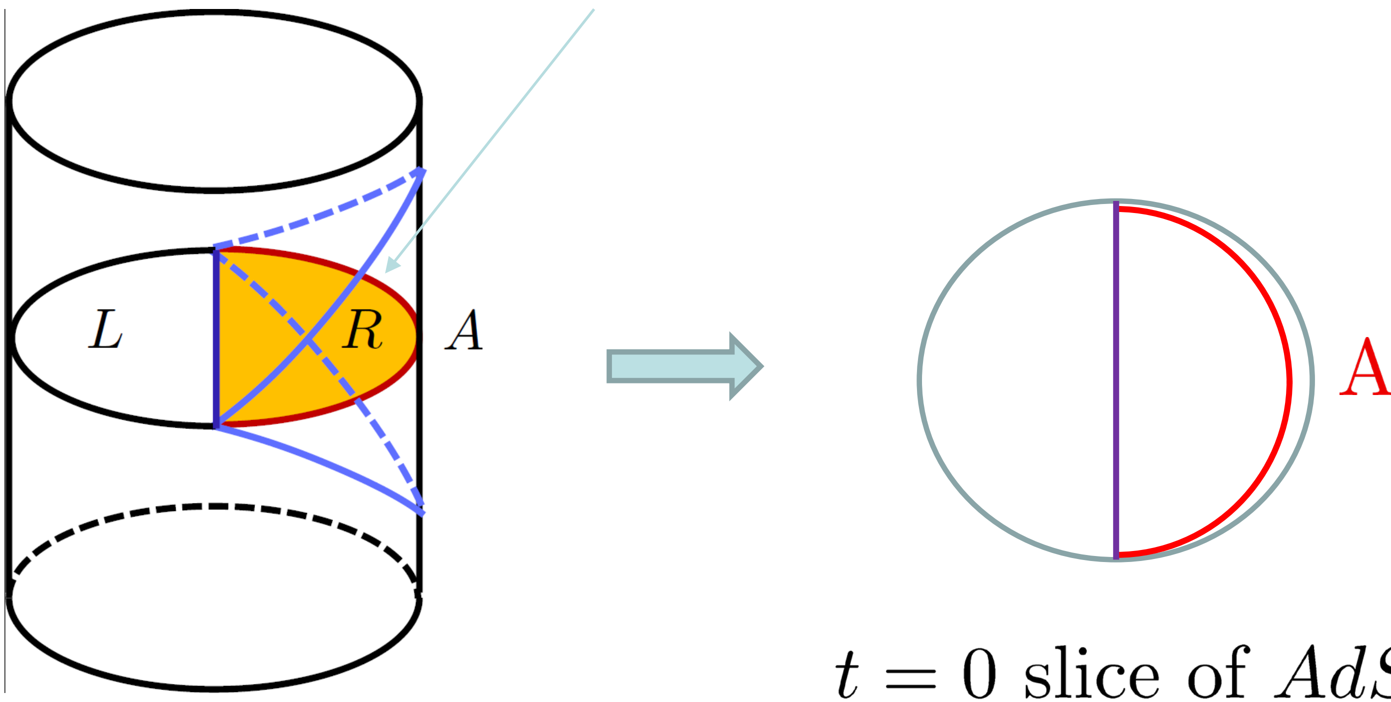
$$0 \leq \xi < \infty$$



Boundary of AdS_3 is located at $\xi = \infty$

Rindler horizon is at $\xi = 0$

Entanglement wedge of A



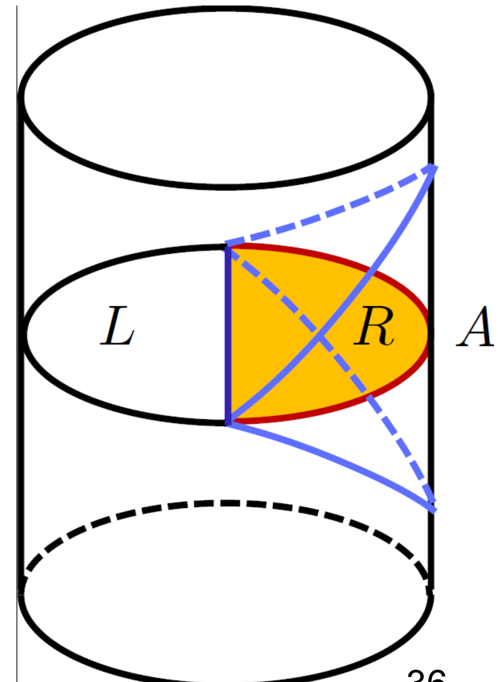
$t = 0$ slice of AdS_3

Boundary of Rindler AdS_3

The boundary of Rindler patch of AdS_3
is (conformally) Minkowski space

$$ds^2 = e^{2\Phi}(-dt_R^2 + d\chi^2)$$

where $-\infty < t_R < \infty, -\infty \leq \chi < \infty$



AdS-Rindler HKLL bulk reconstruction

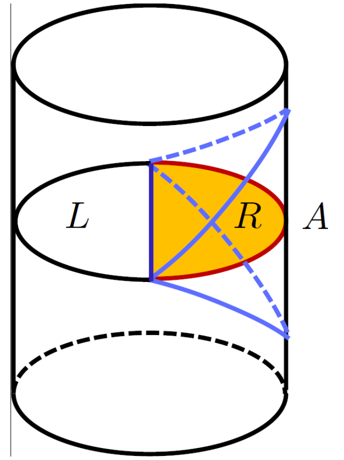
Bulk local fied $\phi(t_{R0}, \xi_0, \chi_0)$
is related to fields at boundary $\phi(t_R, \xi = 0, \chi)$
using free e.o.m.

Then, using BDHM relation $\lim_{\xi \to 0} \frac{\phi(t_R, \xi, \chi)}{z^\Delta} \sim \mathcal{O}(t_R, \chi)$

⇓

Bulk local fied is given by CFT field:

$$\phi(t_R, \xi, \chi) \leftrightarrow \int dt'_R d\chi' K(\chi', t'_R)\mathcal{O}(\chi', t'_R)$$



AdS-Rindler HKLL bulk reconstruction

We will denote the CFT operator as

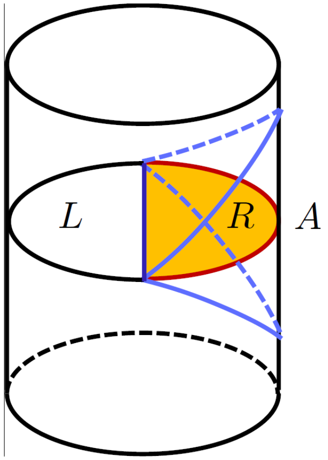
$$\phi^R(t_R,\xi,\chi) \equiv \int dt'_R d\chi' K(\chi',t'_R)\mathcal{O}(\chi',t'_R)$$

If $\{t_R,\xi,\chi\}$ are in AdS-Rindler patch,

$$\langle 0|\phi(t_R,\xi,\chi)\phi(t'_R,\xi',\chi')|0\rangle = \langle 0|\phi^R(t_R,\xi,\chi)\phi^R(t'_R,\xi',\chi')|0\rangle$$

i.e. bulk correlation function

is reproduced by CFT oeprator



Subregion complementarity

(similar(?) to Black hole complementarity)

Although $\phi^R(t_R, \xi, \chi) \neq \phi^G(t'_R, \xi', \chi')$, we find

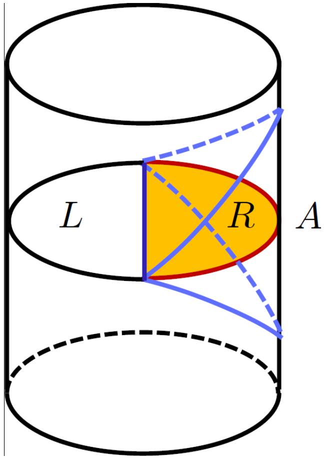
$$\langle 0|\phi(t_R,\xi,\chi)\phi(t'_R,\xi',\chi')|0\rangle = \langle 0|\phi^R(t_R,\xi,\chi)\phi^R(t'_R,\xi',\chi')|0\rangle$$
$$= \langle 0|\phi^G(t_R,\xi,\chi)\phi^G(t'_R,\xi',\chi')|0\rangle$$

Furhtremore, $\phi^R(t_R,\xi,\chi)|0\rangle \neq \phi^G(t'_R,\xi',\chi')|0\rangle$

Same bulk operator in different coordinate patches are realized in CFT differently!

⇓

Complementarity



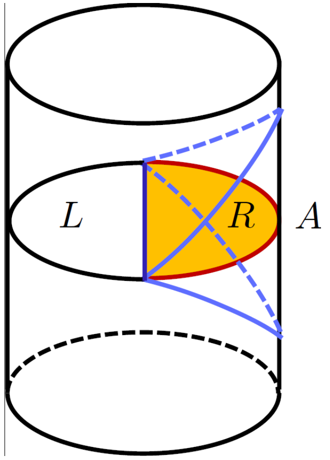
Difference between bulk semi-classical gravity theory and finite N CFT

CFT operator in Rindler patch

By the conformal transformation,

CFT primary operator in Rindler patch is same as

CFT primary operator in Minkowski space



Free scalar in Bulk Rindler patch

We expand ϕ by the modes $v_{\omega,\lambda,\mu}(t_R,\xi,\chi)$,

$$\phi(t_R,\xi,\chi) = \int_0^\infty d\omega \int_{-\infty}^\infty d\lambda \frac{1}{\sqrt{2\pi}} \tilde{\psi}_{\omega,\lambda}(\xi) \left[a_{\omega,\lambda} e^{-i\omega t_R + i\lambda\chi} + a^\dagger_{\omega,\lambda} e^{i\omega t_R - i\lambda\chi} \right].$$

Modes are given as

$$\tilde{\psi}_{\omega,\lambda}(\xi) = \frac{N_{\omega,\lambda}}{\Gamma(\nu+1)} \xi^{i\omega} (1+\xi^2)^{-\frac{i\omega}{2}-\frac{\Delta}{2}} {}_2F_1\left(\frac{i\omega - i\lambda + \nu + 1}{2}, \frac{i\omega + i\lambda + \nu + 1}{2}; \nu+1; \frac{1}{1+\xi^2} \right)$$

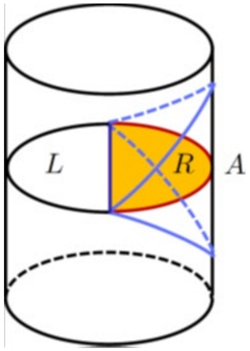
$$N_{\omega,\lambda} = \frac{\left|\Gamma\left(\frac{i\omega - i\lambda + \nu + 1}{2}\right)\right| \left|\Gamma\left(\frac{i\omega + i\lambda + \nu + 1}{2}\right)\right|}{\sqrt{4\pi\omega} |\Gamma(i\omega)|}$$

Here, energy ω is any real number

BDHM relates Bulk and CFT pictures

In global and Rindler, fields are identical.

$$\lim_{\xi \to \infty} \xi^{\Delta} \phi(t_R, \xi, \chi) = O_{\Delta}(t_R, \chi).$$



⇓

$$O_{\Delta}(t_R, \chi) = \int_0^{\infty} d\omega \int_{-\infty}^{\infty} d\lambda \frac{N_{\omega,\lambda}}{\sqrt{2\pi}\Gamma(\nu+1)} \left[a_{\omega,\lambda} e^{-i\omega t_R + i\lambda\chi} + a^{\dagger}_{\omega,\lambda} e^{i\omega t_R - i\lambda\chi} \right]$$

Modes with $\omega < |\lambda|$ are tachyonic!

Thus, different from CFT primary field

What is wrong?

Bulk free theory is only the low energy and large N limit of the (finite N) CFT.

Free theory on the bulk Rindler patch M_A is incorrect as an approximation of the CFT, i.e. the quantum gravity,

Failure of low-energy effective theory(=bulk gravity)!
Asymptotic $1/N$ expansion vs Unitarity

Bulk gravity theory is invalid if we consider a subregion of spacetime, which implies that there are "horizons".

This is because of the UV cut-off, typically the Planck mass, of this effective theory.

We stress that this can be seen by considering finite N because 1/N expansion (i.e. semi-classical expansion) is based on the leading order spectrum. In this sense, this is the non-perturbative quantum gravity effect.

Jafferis-Lewkowycz-Maldacena-Suh used the bulk gravity theory even for the subregion.

But, this is not justified.

(In particular, the entanglement entropy depends on the boundary of the subregion, i.e. “horizon”.)

Such a violation is an essential property of (black hole) horizon, which is universal to general black hole horizons.

(related to “Brick wall”)

Conclusion

- **We reconstruct the wave packets in bulk theory from CFT primary operators.**
- **AdS/CFT for subregion works even though the subregion duality does not work.**
- **Black hole complementarity like properties are important.**

Future directions

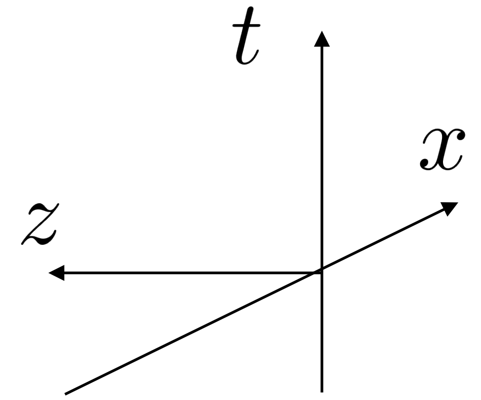
- **There are lots of important things to investigate and understand, at least for me.**

Fin.

backups

Poincare patch for AdS_3 (as an example)

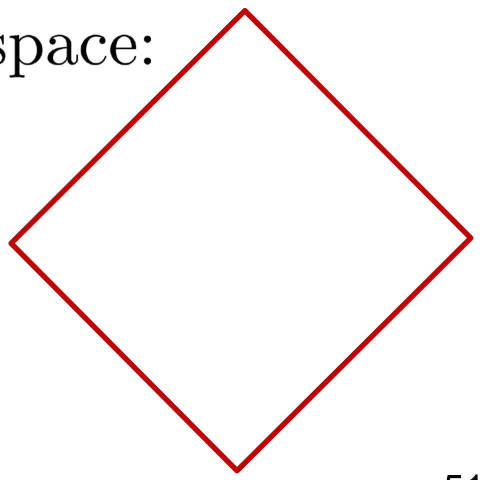
$$ds^2_{AdS} = \frac{1}{z^2}\left(-dt^2 + dz^2 + dx^2\right)$$



where $z = 0$ is boundary

Corresponding CFT_2 is on Minkowski space:

$$ds^2 = -dt^2 + dx^2$$



(Penrose diagram of Minkowski space)

HKLL bulk reconstruction

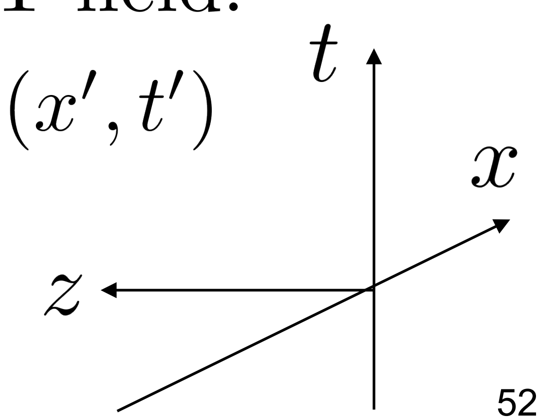
Bulk local fied $\phi(t_0, z_0, x_0)$
is related to fields at boundary $\phi(t, z=0, x)$
using free e.o.m.

Then, using BDHM relation $\lim_{z \to 0} \frac{\phi(t,z,x)}{z^{\Delta}} \sim \mathcal{O}(t,x)$

⇩

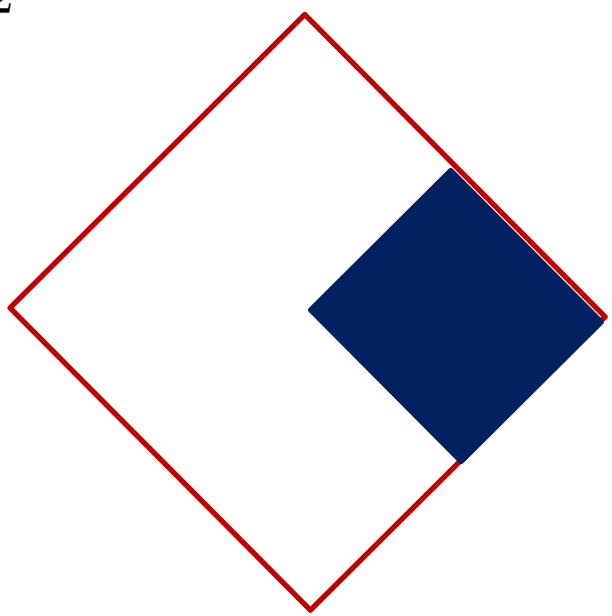
Bulk local fied is given by CFT field:

$$\phi(t, z, x) = \int dt' dx' K(x', t') \mathcal{O}(x', t')$$



Rindler patch of CFT on Minkowski space

$$t_R = \tanh^{-1}(t/x),\ \chi = \ln\sqrt{x^2 - t^2}$$



(conformally equivalent to)
Minkowski space $ds^2 \simeq -dt_R^2 + d\chi^2$

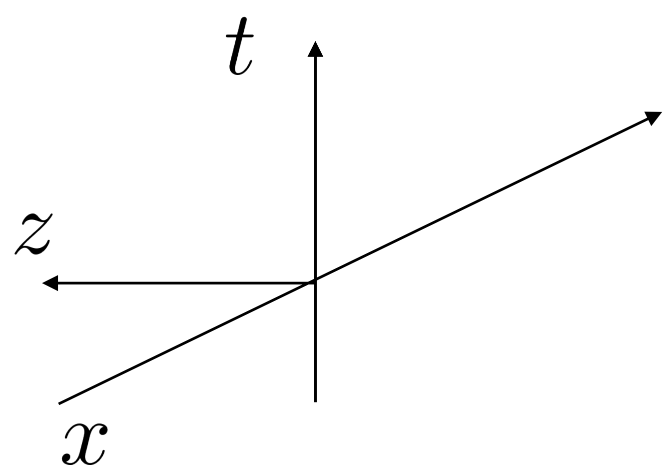
⇓

CFT on Rindler wedge is same CFT with $\{t_R, \chi\}$
by conformal transformation

Entanglement wedge of Rindler space in AdS_3 is AdS-Rindler space:

$$ds^2 = -\xi^2 dt_R^2 + \frac{d\xi^2}{1+\xi^2} + (1+\xi^2)d\chi^2$$

where $\xi = 0$ is boundary and $\xi = \infty$ is horizon



AdS-Rindler HKLL reconstruction

$\phi(t_R, \xi, \chi)$ can be written by $O_\Delta(t_R, \chi)$

i.e. bulk local operator can be reconsructed

⇩

This leads the subregion duality,

entanglement wedge reconstruction,

quatum error correction code proposal,,,

But, this is not consistent!

As we have shown,

CFT is on Rindler patch of boundary, which is (conformally) Minkowski space.

⇓

$O_\Delta(t_R, \chi)$ should be operator on Minkowski space.

But, there are tachyonic modes $(\omega^2 < \lambda^2)$

$$O_\Delta(t_R, \chi) = \int_0^\infty d\omega \int_{-\infty}^\infty d\lambda \frac{N_{\omega,\lambda}}{\sqrt{2\pi}\Gamma(\nu+1)} \left[a_{\omega,\lambda} e^{-i\omega t_R + i\lambda\chi} + a^\dagger_{\omega,\lambda} e^{i\omega t_R - i\lambda\chi} \right]$$

BDHM map is NOT correct for AdS-Rindler!

Remarks:

1. $O_\Delta(t_R, \chi)$ gives correct CFT 2-point function.

   But, CFT 2-point function is universal.

   Different theories can give same 2-point function.

2. $O_\Delta(t_R, \chi)$ is obtained by conformal transformation of generalized free approx. of original CFT operator.

But,

generalized free theory is large N approximation and such a spectrum is only the low energy approximation not realized for the high energy states.

Remarks:

3. Tachyonic modes correspond to
   horizon to horizon modes

Large N approx.

$O_\Delta^{CFT}(\tau, \theta)$ → $O_\Delta^{\mathrm{GF}}(\tau, \theta)$=bulk field

↓ Rindler patch ↓ Rindler patch

$O_\Delta^{CFT}(t_R, \chi)$ ✗→ $O_\Delta(t_R, \chi)$=bulk field

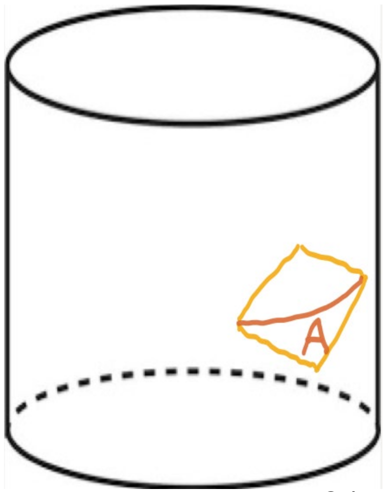
Large N approx.

Generalized free field (bulk field) is not good by finite N effect=nonperturbative QG effect

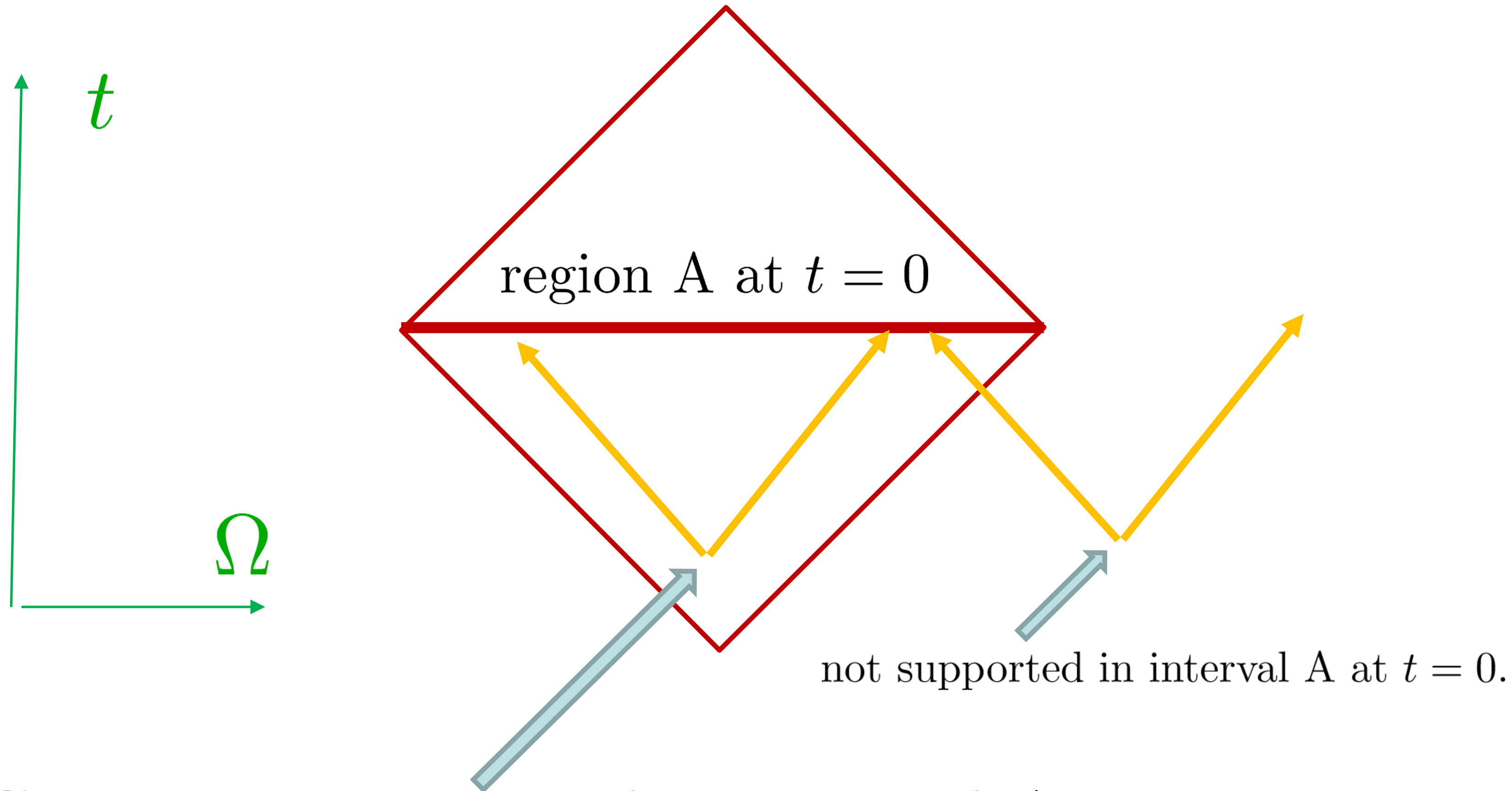
CFT states supported in a region

CFT states supported in a region

Let us consider bulk states correspond to CFT states supported in interval A at $t = 0$.



Causal diamond in CFT

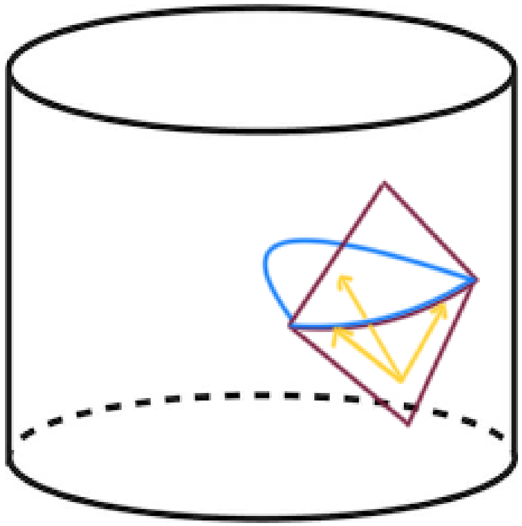


CFT state supported in interval A at $t = 0$.

CFT states supported in a region

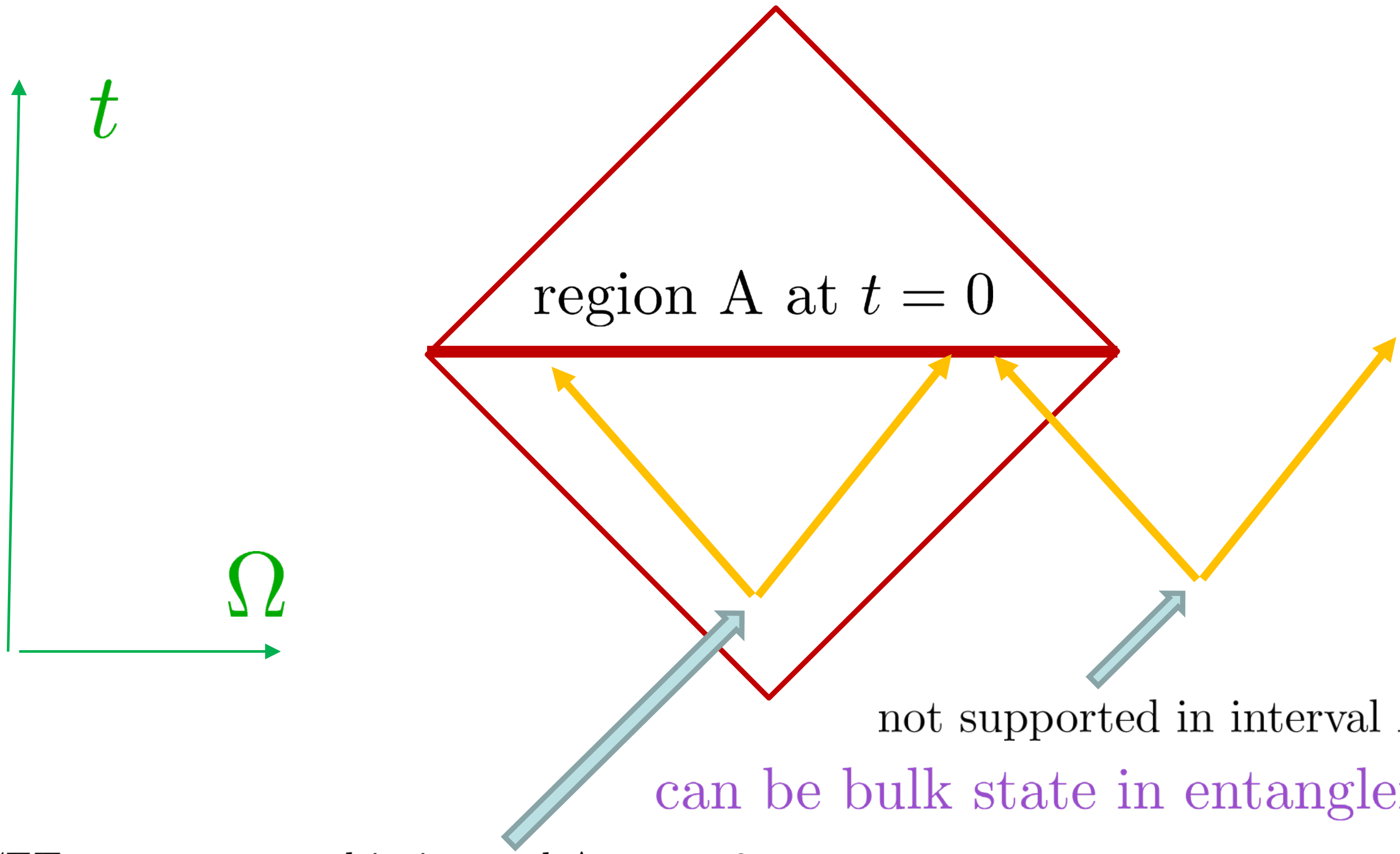
CFT states supported in region A are given by bulk states supported in the causal wedge of A.

The causal wedge of A on $t = 0$ is bulk region inside of blue curve.



Ryu-Takayanagi surface appears!

Causal diamond in CFT



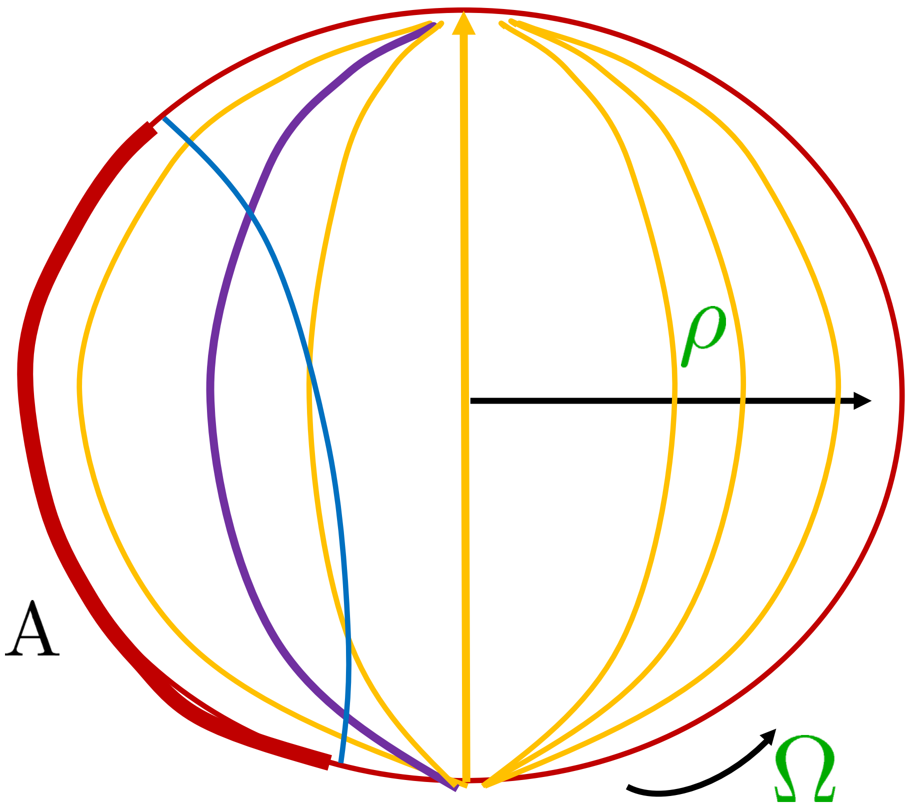
not supported in interval A at $t = 0$.

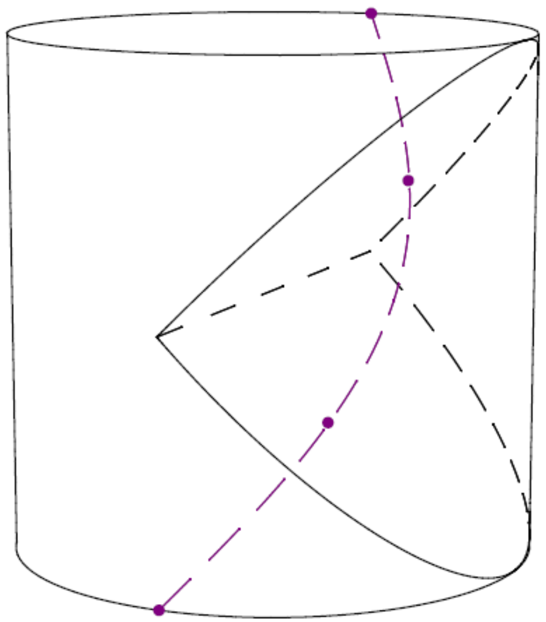
can be bulk state in entanglement wedge

CFT state supported in interval A at $t = 0$.

Null-geodesics connecting horizons

However, some bulk state supported in causal wedge of A can not be CFT state supported in region A !



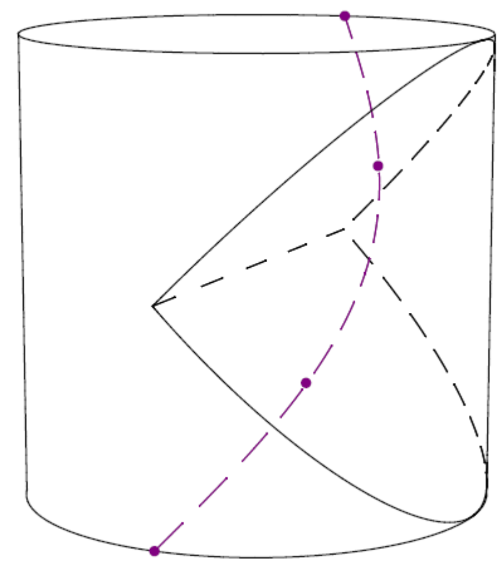


(strong) subregion duality is NOT valid.

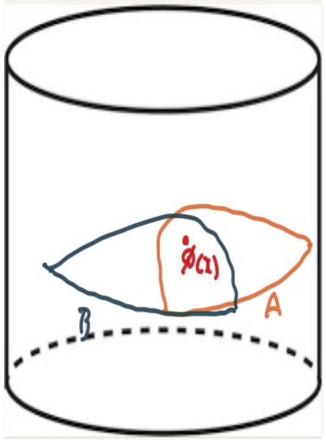
(strong) subregion duality is NOT valid

This problem associated with the null-geodesics was already raised by Bousso-Freivogel-Leichenauer-Rosenhaus-Zukowski in arXiv:1209.4641

Note that
entanglement wedge reconstruction is based on this subregion duality.



Bulk local states at a same bulk point constructed from CFT states supported in different regions are different even in the low energy (gravity) theory

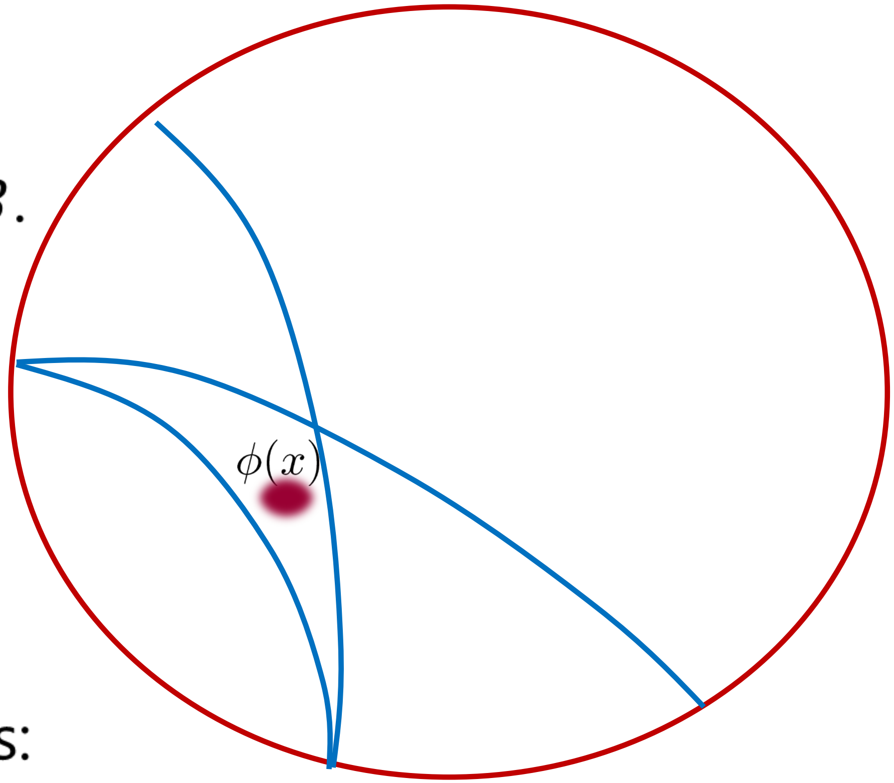


Quantum error correction (QEC) code

If bulk local operators $\phi(x)$ constructed from CFT operators supported in regions A and B are same, It should be constructed from the ones supported in regions $A \cap B$.

However, $\phi(x)$ is outside causal wedge of $A \cap B$.

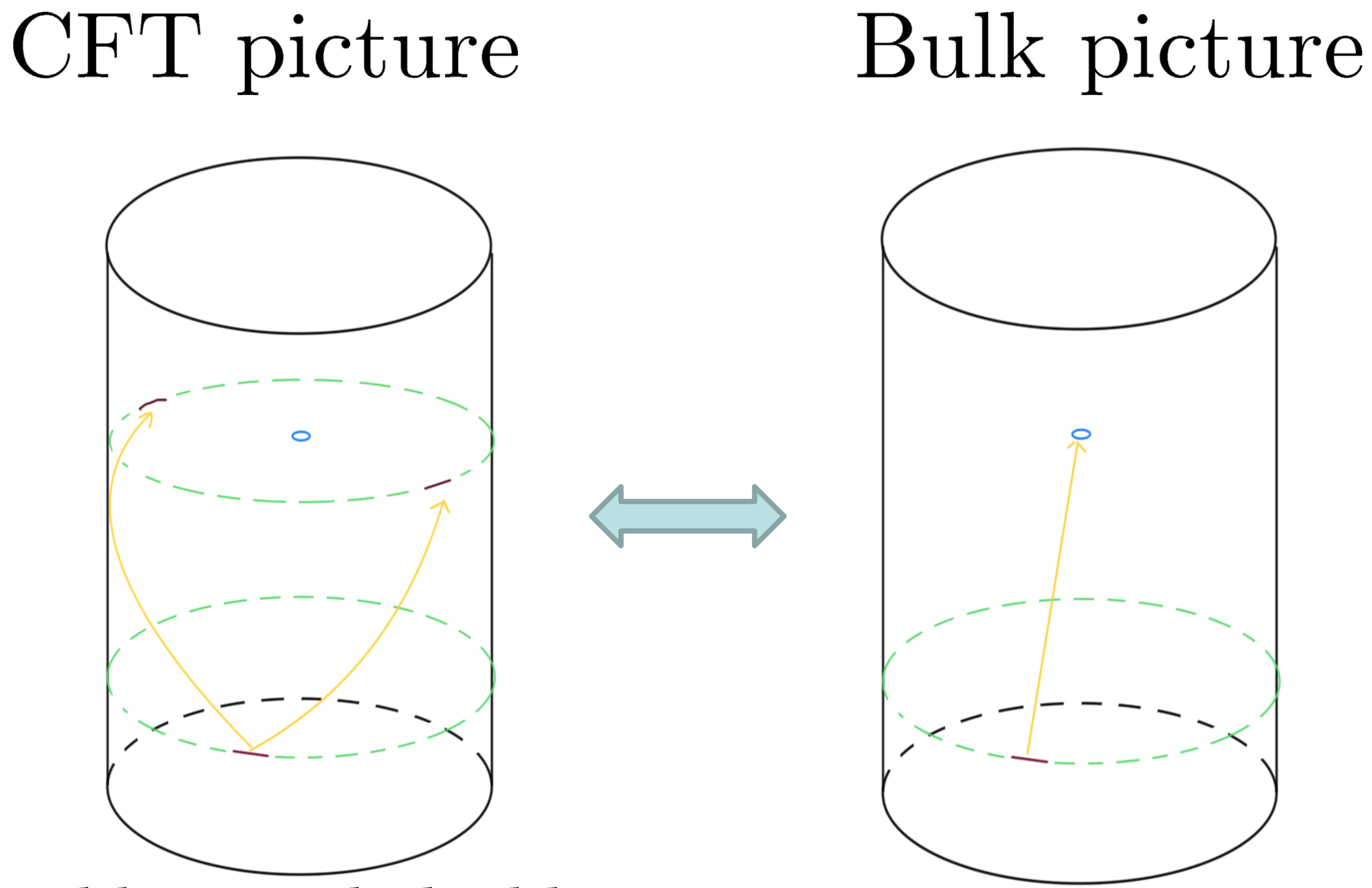
$\phi(x)$ are same only in low energy theory (called code subspace) in QEC proposal.



Our picture is opposite to this:
$\phi(x)$ are different even in low energy theory

Generalization to asymptotic AdS

For asymptotic AdS, assuming BDHM, we have same picture:



Time-like, not light-like Null-geodesics in curved space

Always time-delay by Gao-Wald theorem

Null geodesics in the AdS-Rindler patch

We will regard

tachyonic modes as the bulk local field

Null geodesics in AdS-Rindler patch

There are two types:

(1) horizon ($\xi = 0$) to boundary ($\xi = \infty$), $|b| < 1$

$$\xi(t_R) = \frac{1}{\sqrt{1-b^2}\,|\sinh(t_R - t_0)|}, \quad \chi(t_R) = \chi_0 + \frac{1}{2}\log\frac{1 + b\tanh(t_R - t_0)}{1 - b\tanh(t_R - t_0)}$$

(2) horizon to horizon, $|b| > 1$

$$\xi(t_R) = \frac{1}{\sqrt{b^2-1}\,\cosh(t_R - t_0)}, \quad \chi(t_R) = \chi_0 + \frac{1}{2}\log\frac{b + \tanh(t_R - t_0)}{b - \tanh(t_R - t_0)}$$

Null geodesics in AdS-Rindler patch

For well-localized wave packet

along null-geodesics with b,

modes $a_{\omega,\lambda}$ with $\lambda/\omega = b$ are dominantly contribute

$\Downarrow$

(1) horizon ($\xi = 0$) to boundary ($\xi = \infty$), $|b| < 1$

non-tachyonic modes $\omega^2 > \lambda^2$

(2) horizon to horizon, $|b| > 1$

tachyonic modes $\omega^2 < \lambda^2$

Thus, from the CFT on Rindler patch,

wave packet along horizon to horizon

null-geodesics can not reconstructed