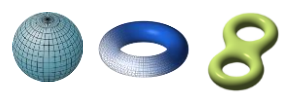
Three-generation solutions of equations of motion in heterotic supergravity



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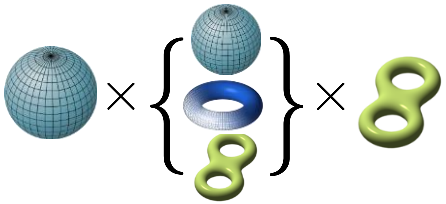
Introduction

超弦理論の低エネルギー極限として、
$E_8 \times E_8$超重力理論のコンパクト化を考える

近年新しいコンパクト化解が見つかった(Tsuyuki, 2021)



目標:
このコンパクト化でフェルミオン3世代が
存在しうることを示す

(pictures from Wikipedia)

Lagrangian

10D Heterotic SUGRA with R^2 term [84 Bergshoeff, de Roo]

$$L = \sqrt{-g}e^{-2\phi}\left[R + 4(\nabla\phi)^2 - \frac{1}{12}H_{MNP}H^{MNP} + \frac{\alpha'}{8}R_{MNPQ}R^{MNPQ} - \frac{\alpha'}{8}\mathrm{tr}(F_{MN}F^{MN})\right]$$

EoM for *H*:

$$\partial^M(e^{-2\phi}H_{MNP}) = 0$$

Assumption: $\partial_M\phi = 0, \quad H_{MNP} = 0$

We do NOT assume SUSY after compactification

Equations of motion

$$R + \frac{\alpha'}{8} R_{MNPQ} R^{MNPQ} - \frac{\alpha'}{8} \mathrm{tr}(F_{MN} F^{MN}) = 0, \quad (6)$$

$$R_{MN} + \frac{\alpha'}{4} R_{MPQR} R_N{}^{PQR} - \frac{\alpha'}{4} \mathrm{tr}(F_{MP} F_N{}^P) = 0, \quad (7)$$

$$\nabla_M F^{MN} + [A_M, F^{MN}] = 0. \quad (8)$$

Multiplying (7) by g^{MN}, (6) reduces to

$$R = 0.$$

Bianchi identity

Condition for anomaly cancellation [84 Green, Schwarz]

$$0 = dH = \frac{\alpha'}{4}(\mathrm{tr}R \wedge R - \mathrm{tr}F \wedge F)$$

‘Standard embedding’ $R = F$ がよく仮定される

新たな解では**Standard embeddingを仮定しない**

Product space assumption

Assume 10D spacetime is

$$M^{10} = M_0 \times M_1 \times M_2 \times M_3,$$

M_0: 4D Minkowski spacetime

$M_{1,2,3}$: 2D spaces with constant curvature

Advantages of this assumption:

1. Bianchi id. is satisfied without 'standard embedding'
2. F_{MN} can be in Freund-Rubin configuration

Curvatures

$$(M^{10} = M_0 \times \prod_{i=1}^{3} M_i)$$

Riemann tensor for M_i (i=1,2,3):

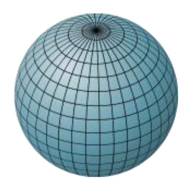
$$R^{(i)}_{mnpq} = \lambda_i (g^{(i)}_{mp} g^{(i)}_{nq} - g^{(i)}_{mq} g^{(i)}_{np}),$$

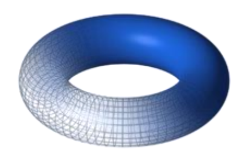
$$(m, n, p, q \in \{4,5\}(i=1), \{6,7\}(i=2), \{8,9\}(i=3))$$

λ_i >0: Sphere S_2

=0: Torus T_2

<0: genus $\geq$ 2 Riemann surface H_2/Γ





First term of Bianchi id. vanishes

$$\mathrm{tr} R \wedge R \propto R_{[MN}{}^{RS} R_{PQ]RS} = 0$$

Gauge fields

$$(M^{10} = M_0 \times \prod_{i=1}^{3} M_i)$$

Freund-Rubin configuration [80 Freund Rubin]

$$F_{mn}^{A(i)} = \sqrt{g_i} f_i^A \epsilon_{mn}^{(i)},$$

f_i^A: constant, g_i: determinant of the metric of M_i

A: index of U(1) in $E^8 \times E^8$

$\varepsilon_{mn}^{(i)}$: Levi-Civita symbol

EoM for A_M is satisfied:

$$\nabla_M F^{Mn} = \frac{1}{\sqrt{g_i}} \partial_m \left(\sqrt{g_i} \frac{f_i^A}{\sqrt{g_i}} \epsilon^{mn} \right) T^A = 0.$$

Field equations

$$(M^{10} = M_0 \times \prod_{i=1}^{3} M_i)$$

Flux quantization:

(χ_i: Euler characteristic)

$$f_i^A = \frac{n_i^A}{\chi_i}\lambda_i. \qquad (n_i^A \in \mathbb{Z})$$

を運動方程式、Bianchi恒等式に代入すると

EoM:

$$\lambda_1 + \lambda_2 + \lambda_3 = 0,$$

$$\lambda_i = \frac{\chi_i^2}{\sum_{A=1}^{11} n_{Ai}^2 - \chi_i^2}$$

Bianchi id.:

$$\sum_{A=1}^{11} n_{Ai} n_{Aj} = 0 \quad (i \neq j).$$

これらに世代数の条件を加える

フェルミオンの世代数

Standard embeddingを仮定しない場合の世代数

(Bars, Visser 1985)

$$N_{gen} = \frac{1}{(2\pi)^3}\frac{1}{6}\left|\int \mathrm{tr}(F^3)\right|.$$

今の解では次のようにフラックスの整数で決まる

$$N_{gen} = \left|\prod_{i=1}^{3}\sum_{A=1}^{3} n_{Ai} - \sum_{A=1}^{3}\prod_{i=1}^{3} n_{Ai}\right|.$$

運動方程式、Bianchi恒等式、世代数の4式を連立

主な解析結果

運動方程式・Bianchi恒等式で世代数が制限

①トーラスがあると世代数0

世代数が非0なコンパクト化は

$S^2 \times H^2/\Gamma \times H^2/\Gamma$ 又は $S^2 \times S^2 \times H^2/\Gamma$

②特にgenusが(0,2,2)の時、世代数は3以下

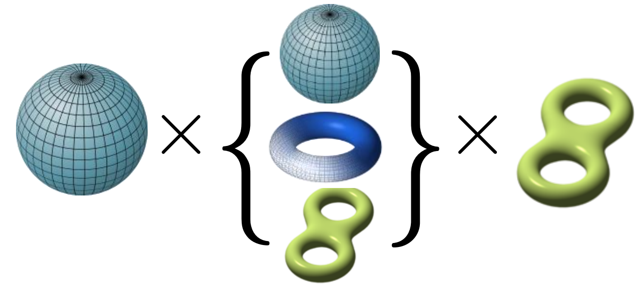
③H_2/Γのgenusがいくつでも、3世代の解は常に存在

3世代の解の具体例

	solution 1	solution 2	solution 3	solution 4	solution 5	solution 6
χ_2	-2	-2	-4	2	2	2
χ_3	-2	-4	-4	-2	-4	-6
(n_{Ai})	$\begin{pmatrix} 2 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 2 & 0 & -1 \\ 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 2 & 0 & -1 \\ 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Table 1: Samples of three-generation solutions. These Euler characteristics χ_i and fluxes (n_{Ai}) satisfy the equations of motion (18), (19), the Bianchi identity (20), and $N_{gen} = 3$ [N_{gen} is given in Eq. (27)]. Solutions 1-3 are for $S^2 \times H^2/\Gamma \times H^2/\Gamma$ cases, and solutions 4-6 are for $S^2 \times S^2 \times H^2/\Gamma$ cases.

Summary



- 10次元超重力理論の新たなコンパクト化

  $S^2\times S^2\times H^2/\Gamma$, $S^2\times T^2\times H^2/\Gamma$, $S^2\times H^2/\Gamma\times H^2/\Gamma$

  について、フェルミオンの世代数を求めた

- 世代数は運動方程式・Bianchi恒等式で制限

- トーラスを使わなければ3世代の解は存在

- 2次元定曲率空間の直積を用いているため、湯川結合の計算ができるかもしれない（将来）

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