Witten型超弦の場の理論から Kugo-Zwiebach型超弦の場の理論への場の再定義

吉中　譲次郎　（京大）

場の理論と弦理論 2023 (ポスター)

work in progress

with Y. Ando, R. Fujii and H. Kunitomo.

背景

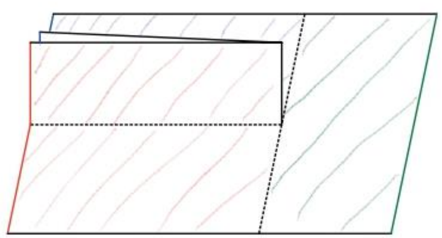
3つのSFT

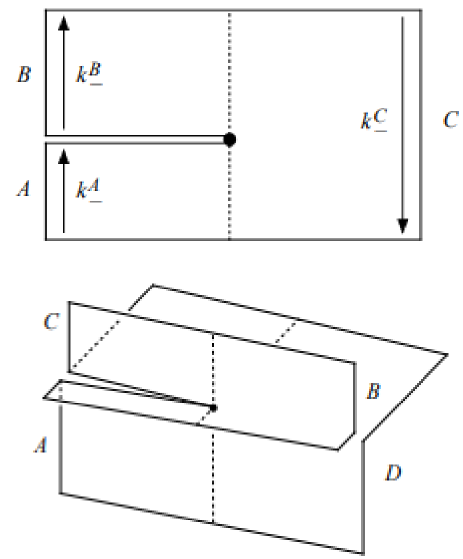
ボソン弦

[Erler-Matsunaga (2021)]

	Witten型 SFT	Kugo-Zwiebach型 SFT	光円錐 SFT
第一量子化：	BRST形式	BRST形式	光円錐ゲージ

相互作用：





Eerler-Matsunaga, JHEP11(2021)208

SFT間のmapping

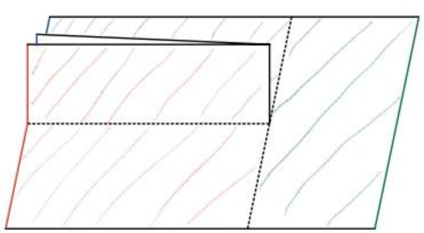
ボソン弦

[Erler-Matsunaga (2021)]

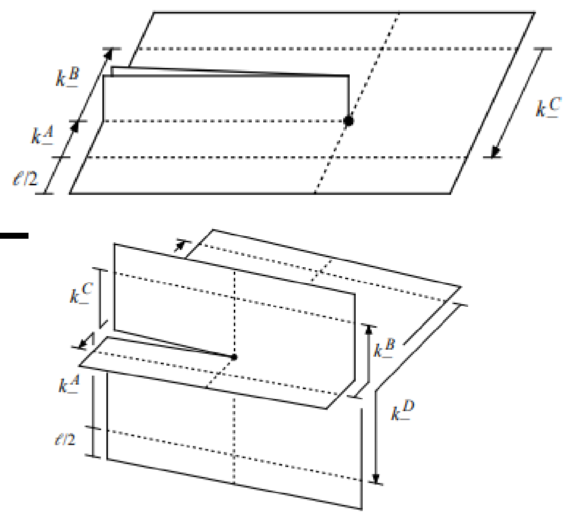
Witten型 SFT ⟷ Kugo-Zwiebach型 SFT ⟷ 光円錐 SFT

8/7 藤井僚太氏のトーク

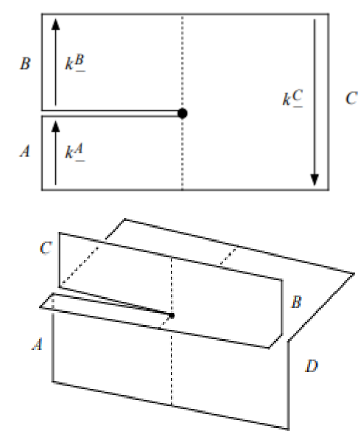
Kaku vertex



$l \to \infty$ ←

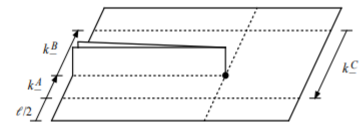


→ $l \to 0$

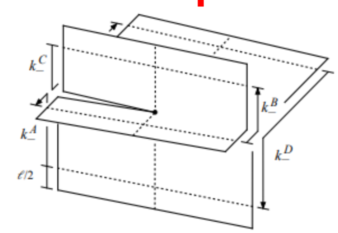


Kaku 理論 [Kaku (1988)]

長さ l の Kaku 理論の弦場：Ψ_l



$$\text{作用：}S[\Psi_l] = \frac{1}{2}\omega(\Psi_l, Q_B\Psi_l) + \frac{1}{3}\omega(\Psi_l, m_2^l(\Psi_l, \Psi_l)) + \frac{1}{4}\omega(\Psi_l, m_3^l(\Psi_l, \Psi_l, \Psi_l))$$



$\boxed{A_\infty \text{ 関係式}}$

$$\{\boldsymbol{m}^l, \boldsymbol{m}^l\} = 0, \qquad \boldsymbol{m}^l = \boldsymbol{Q_B} + \boldsymbol{m}_2^l + \boldsymbol{m}_3^l$$

※**太字**はテンソル代数上の余微分

場の再定義

長さ $l+\epsilon$ の Kaku 理論の弦場を再定義：

$$\Psi_{l+\epsilon} = \Psi_l - \epsilon\big[\mu_2^l\big(\Psi_l, \Psi_l\big) + \mu_3^l\big(\Psi_l, \Psi_l, \Psi_l\big) + \ldots\big]$$

$S[\Psi_l] = S[\Psi_{l+\epsilon}]$ である条件

$$\frac{d}{dl}\boldsymbol{m}^{\boldsymbol{l}} = [\boldsymbol{m}^{\boldsymbol{l}}, \boldsymbol{\mu}^{\boldsymbol{l}}], \qquad \boldsymbol{\mu}^{\boldsymbol{l}} = \boldsymbol{\mu}_{\boldsymbol{2}}^{\boldsymbol{l}} + \boldsymbol{\mu}_{\boldsymbol{3}}^{\boldsymbol{l}} + \ldots$$

これを満たす μ^l は実際に構成可能 [Erler-Matsunaga (2021)]

Q. 超弦の場合は？

超 Kaku 理論

弦場：$\Psi_l = \Psi_l^{(\mathrm{NS})} + \Psi_l^{(\mathrm{R})}$

picture=-1 　 picture=-1/2

作用：$$S[\Psi_l] = \frac{1}{2}\Omega(\Psi_l, Q_B\Psi_l) + \frac{1}{3}\Omega(\Psi_l, M_2^l(\Psi_l, \Psi_l)) + \frac{1}{4}\Omega(\Psi_l, M_3^l(\Psi_l, \Psi_l, \Psi_l))$$

$\vdots$ 　 高次の項

$\boxed{A_\infty \text{ 関係式}}$

$$\{\boldsymbol{M}^{\boldsymbol{l}}, \boldsymbol{M}^{\boldsymbol{l}}\} = 0, \qquad \boldsymbol{M}^{\boldsymbol{l}} = \boldsymbol{Q_B} + \boldsymbol{M_2^l} + \boldsymbol{M_3^l} + \dots$$

picture 数をもつ

超 Kaku vertex

$\boldsymbol{M^l}$ は $\boldsymbol{m^l}$ とPCOを組み合わせて構成可能

[Erler-Konopka-Sachs (2013)]
[Kunitomo (2021)]
⋮

例）NSだけの場合

$$\boxed{\xi \circ m_n = \frac{1}{n+1}[\xi m_n - m_n(\xi \otimes \mathbb{I}^{\otimes n-1}) - \cdots - m_n(\mathbb{I}^{\otimes n-1} \otimes \xi)]}$$

picture=+1

$$\lambda_2^{l(1)} = \xi \circ m_2^l$$

$$\lambda_3^{l(1)} = 2\xi \circ m_3^l$$

$$\boldsymbol{m_3^{l(1)} = [Q_B, \lambda_3^{l(1)}] + [m_2^l, \lambda_2^{l(1)}]}$$

$$\lambda_3^{l(2)} = \xi \circ m_3^{l(1)}$$

$$\boldsymbol{M_2^l = [Q_B, \lambda_2^{l(1)}]} \longrightarrow \boldsymbol{M_3^l = \frac{1}{2}([Q_B, \lambda_3^{l(2)}] + [M_2^l, \lambda_2^{l(1)}])}$$

Re: 場の再定義

長さ $l+\epsilon$ の超 Kaku 理論の弦場を再定義：

$$\Psi_{l+\epsilon} = \Psi_l - \epsilon\big[\nu_2^l\big(\Psi_l, \Psi_l\big) + \nu_3^l\big(\Psi_l, \Psi_l, \Psi_l\big) + \dots\big]$$

picture 数をもつ

$\boldsymbol{\nu}^{\boldsymbol{l}}$ が満たすべき条件

$$\frac{d}{dl}\boldsymbol{M}^{\boldsymbol{l}} = [\boldsymbol{M}^{\boldsymbol{l}}, \boldsymbol{\nu}^{\boldsymbol{l}}], \qquad \boldsymbol{\nu}^{\boldsymbol{l}} = \boldsymbol{\nu}_{\boldsymbol{2}}^{\boldsymbol{l}} + \boldsymbol{\nu}_{\boldsymbol{3}}^{\boldsymbol{l}} + \dots$$

Q. このような $\boldsymbol{\nu}^{\boldsymbol{l}}$ を構成できるか？

Re: 場の再定義

A. $\boldsymbol{\mu}^{\boldsymbol{l}}$から構成可能

例）NSだけの場合

$$\frac{d}{dl}\boldsymbol{M}_{\boldsymbol{2}}^{\boldsymbol{l}} = [\boldsymbol{Q}_{\boldsymbol{B}}, \boldsymbol{\nu}_{\boldsymbol{2}}^{\boldsymbol{l}}]$$

picture=+1

$$\frac{d}{dl}\boldsymbol{M}_{\boldsymbol{2}}^{\boldsymbol{l}} = \frac{d}{dl}[\boldsymbol{Q}_{\boldsymbol{B}}, \boldsymbol{\lambda}_{\boldsymbol{2}}^{\boldsymbol{l(1)}}] = [\boldsymbol{Q}_{\boldsymbol{B}}, \frac{d}{dl}\boldsymbol{\lambda}_{\boldsymbol{2}}^{\boldsymbol{l(1)}}]$$

$$= [\boldsymbol{Q}_{\boldsymbol{B}}, \xi \circ \frac{d}{dl}\boldsymbol{m}_{\boldsymbol{2}}^{\boldsymbol{l}}] = [\boldsymbol{Q}_{\boldsymbol{B}}, \xi \circ [\boldsymbol{Q}_{\boldsymbol{B}}, \boldsymbol{\mu}_{\boldsymbol{2}}^{\boldsymbol{l}}]]$$

➡ $$\boldsymbol{\nu}_{\boldsymbol{2}}^{\boldsymbol{l}} = \xi \circ [\boldsymbol{Q}_{\boldsymbol{B}}, \boldsymbol{\mu}_{\boldsymbol{2}}^{\boldsymbol{l}}]$$

Re: 場の再定義

$$\frac{d}{dl}\boldsymbol{M}_3^l = [\boldsymbol{Q_B}, \boldsymbol{\nu}_3^l] + [\boldsymbol{M}_2^l, \boldsymbol{\nu}_2^l]$$

picture=+2

$$\begin{aligned}
\frac{d}{dl}\boldsymbol{M}_3^l &= \frac{1}{2}\frac{d}{dl}([\boldsymbol{Q_B}, \boldsymbol{\lambda}_3^{l(2)}] + [\boldsymbol{M}_2^l, \boldsymbol{\lambda}_2^{l(1)}]) \\
&= \frac{1}{2}([\boldsymbol{Q_B}, \frac{d}{dl}\boldsymbol{\lambda}_3^{l(2)}] + [\frac{d}{dl}\boldsymbol{M}_2^l, \boldsymbol{\lambda}_2^{l(1)}] + [\boldsymbol{M}_2^l, \frac{d}{dl}\boldsymbol{\lambda}_2^{l(1)}]) \\
&= \frac{1}{2}([\boldsymbol{Q_B}, \xi\circ\frac{d}{dl}\boldsymbol{m}_3^{l(1)}] + [[\boldsymbol{Q_B}, \boldsymbol{\nu}_2^l], \boldsymbol{\lambda}_2^{l(1)}] + [\boldsymbol{M}_2^l, \boldsymbol{\nu}_2^l]) \\
&= \frac{1}{2}([\boldsymbol{Q_B}, \xi\circ([\boldsymbol{Q_B}, \frac{d}{dl}\boldsymbol{\lambda}_3^{l(1)}] + [\frac{d}{dl}\boldsymbol{m}_2^l, \boldsymbol{\lambda}_2^{l(1)}] + [\boldsymbol{m}_2^l, \frac{d}{dl}\boldsymbol{\lambda}_2^{l(1)}])] + [\boldsymbol{Q_B}, [\boldsymbol{\nu}_2^l, \boldsymbol{\lambda}_2^{l(1)}]]) + [\boldsymbol{M}_2^l, \boldsymbol{\nu}_2^l] \\
&= \frac{1}{2}[\boldsymbol{Q_B}, (2\xi\circ[\boldsymbol{Q_B}, \xi\circ\frac{d}{dl}\boldsymbol{m}_3^l] + \xi\circ[[\boldsymbol{Q_B}, \boldsymbol{\mu}_2^l], \boldsymbol{\lambda}_2^{l(1)}] + \xi\circ[\boldsymbol{m}_2^l, \boldsymbol{\nu}_2^l] + [\boldsymbol{\nu}_2^l, \boldsymbol{\lambda}_2^{l(1)}])] + [\boldsymbol{M}_2^l, \boldsymbol{\nu}_2^l] \\
&= [\boldsymbol{Q_B}, (\xi\circ[\boldsymbol{Q_B}, \xi\circ[\boldsymbol{Q_B}, \boldsymbol{\mu}_3^l]] + \xi\circ[\boldsymbol{Q_B}, \xi\circ[\boldsymbol{m}_2^l, \boldsymbol{\mu}_2^l]] + \frac{1}{2}\xi\circ[[\boldsymbol{Q_B}, \boldsymbol{\mu}_2^l], \boldsymbol{\lambda}_2^{l(1)}] \\
&\quad + \frac{1}{2}\xi\circ[\boldsymbol{m}_2^l, \boldsymbol{\nu}_2^l] + \frac{1}{2}[\boldsymbol{\nu}_2^l, \boldsymbol{\lambda}_2^{l(1)}])] + [\boldsymbol{M}_2^l, \boldsymbol{\nu}_2^l]
\end{aligned}$$

$$\Rightarrow \quad \begin{aligned}
\boldsymbol{\nu}_3^l =& \xi\circ[\boldsymbol{Q_B}, \xi\circ[\boldsymbol{Q_B}, \boldsymbol{\mu}_3^l]] + \xi\circ[\boldsymbol{Q_B}, \xi\circ[\boldsymbol{m}_2^l, \boldsymbol{\mu}_2^l]] \\
&+ \frac{1}{2}\xi\circ[[\boldsymbol{Q_B}, \boldsymbol{\mu}_2^l], \boldsymbol{\lambda}_2^{l(1)}] + \frac{1}{2}\xi\circ[\boldsymbol{m}_2^l, \boldsymbol{\nu}_2^l] + \frac{1}{2}[\boldsymbol{\nu}_2^l, \boldsymbol{\lambda}_2^{l(1)}]
\end{aligned}$$

まとめ

- 超弦場の再定義

  $$\Psi_{l+\epsilon} = \Psi_l - \epsilon\big[\nu_2^l\big(\Psi_l, \Psi_l\big) + \nu_3^l\big(\Psi_l, \Psi_l, \Psi_l\big) + \dots\big]$$

  を構成できた。

- これにより Witten 型超SFTの弦場 Ψ_∞ と Kugo-Zwiebach 型超SFTの弦場 Ψ_0 を繋ぐことができる。

$$\Psi_\infty = \Psi_0 - \int_0^\infty dl\, \nu_2^l(\Psi_0, \Psi_0) - \int_0^\infty dl\, \nu_3^l(\Psi_0, \Psi_0, \Psi_0)$$

$$+ \int_0^\infty dl' \int_0^{l'} dl\, \nu_2^l(\nu_2^l(\Psi_0, \Psi_0), \Psi_0) + \dots$$

$$+ \text{higher orders}$$