Approaches to QCD phase diagram; effective models, strong coupling lattice QCD, and compact stars

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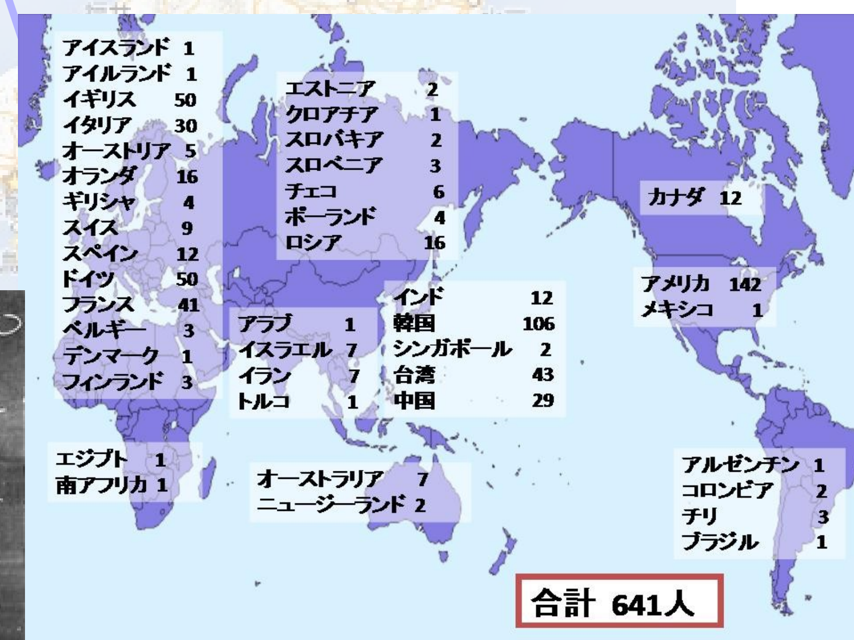
"Dense Matter 2015", JINR, Jun.29-Jul.11, 2015.
Helmholtz Int. Summer School & Dubna Int. Adv. School on Theor. Phys. / DIAS-TH, Bogoliubove Lab. of Theor. Phys,, Joint Inst. for Nucl. Research, Russia.

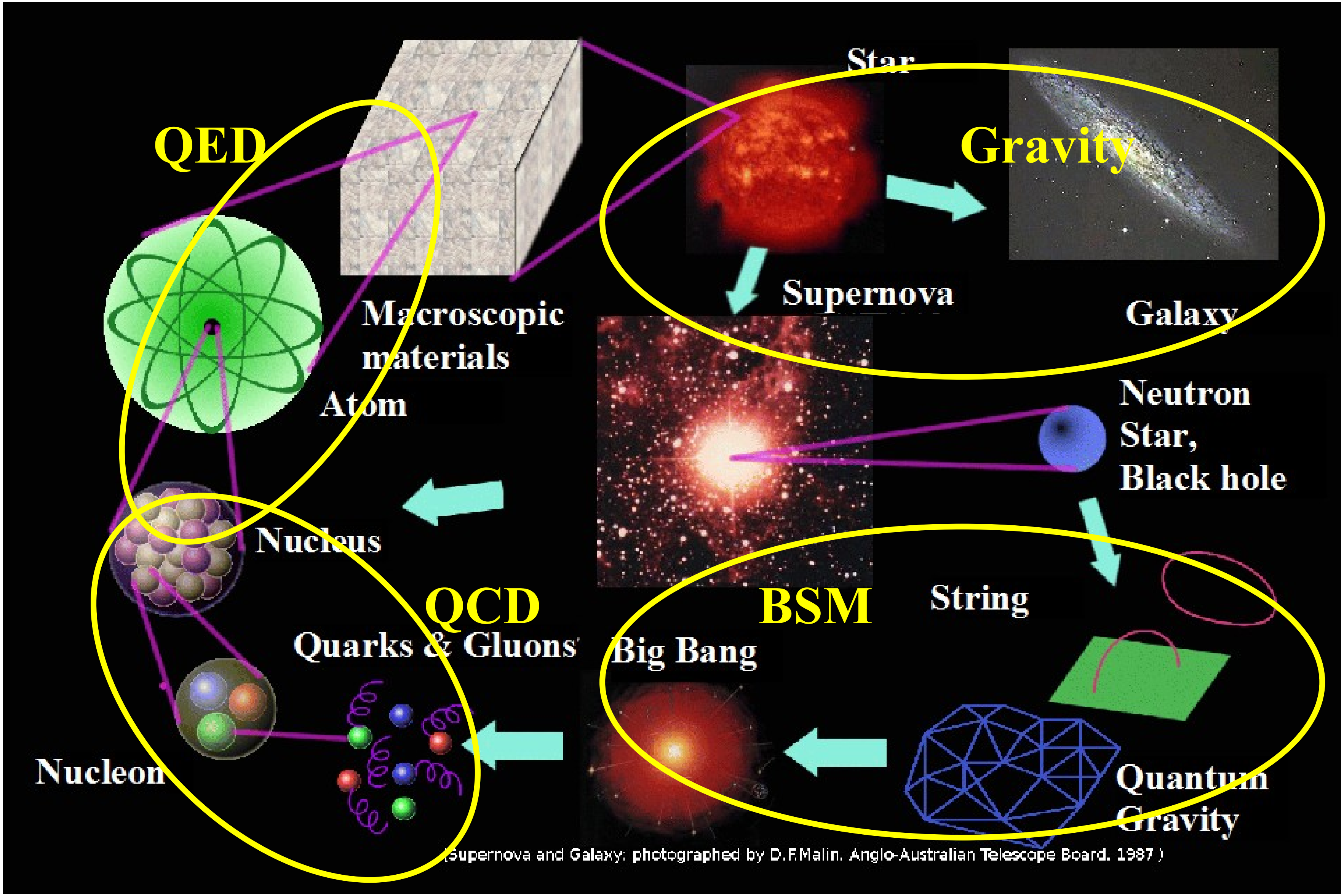
- **Lecture I : Introduction to QCD phase diagram**
  - **Spontaneous breaking and restoration of chiral symmetry.**
- **Lecture II : Approaches to QCD phase diagram**
  - **QCD phase transition and strong-coupling lattice QCD**
  - **QCD phase transition in compact star phenomena**
- **Summary**

Do you know Yukawa Institute ?

- **Yukawa Institute for Theoretical Physics, Kyoto University**
  - **Founded in 1953 to memorize Yukawa's Nobel prize (first winner in Japan).**
  - **Domestic & International Collaboration program**
    20-30 domestic workshops, ~ 10 international workshops,
    ~ 1000(?) domestic visitors, 600-700 visitors from abroad
  - **We will have a long term workshop next year,**
    "Nuclear Physics and Compact Stars 2016" (NPACS 2016).

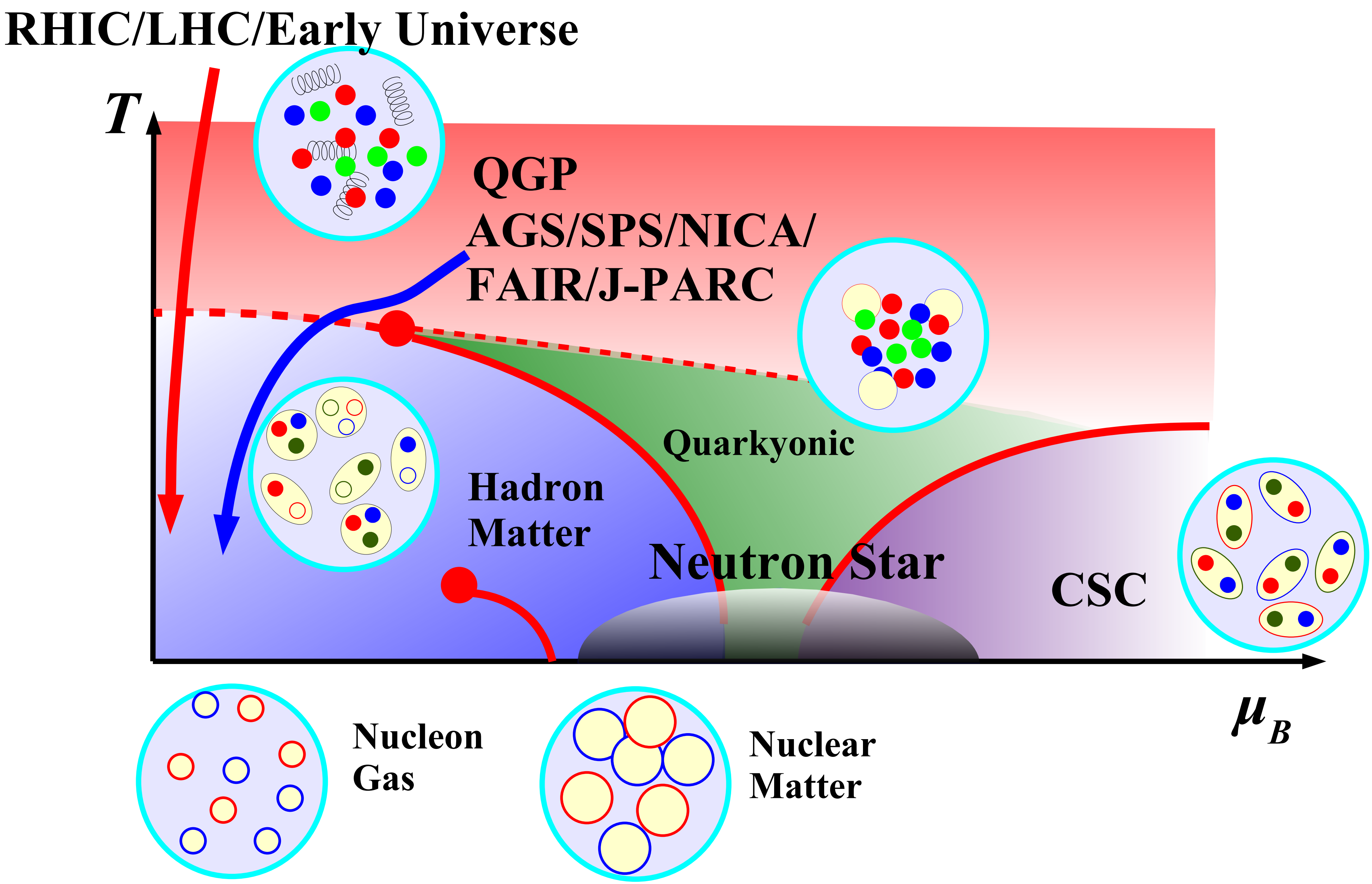


Hierarchy of Matter



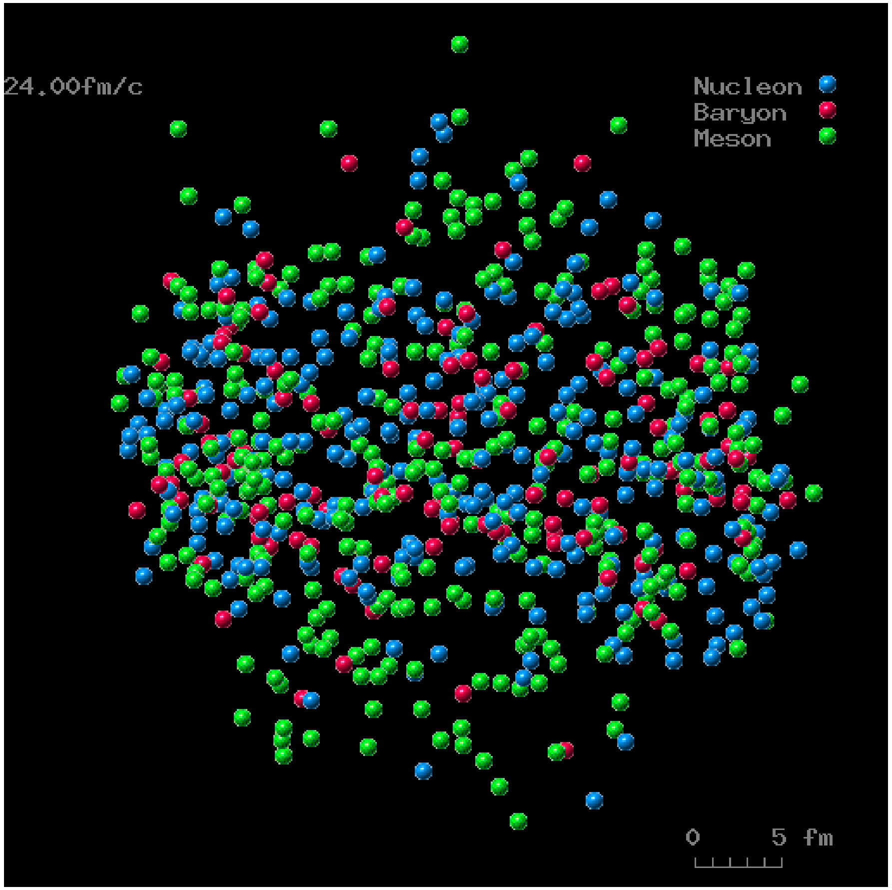
We cannot describe nuclei from quarks & gluons yet.
→ Main obstacle in describing our world from SM.

QCD Phase Diagram

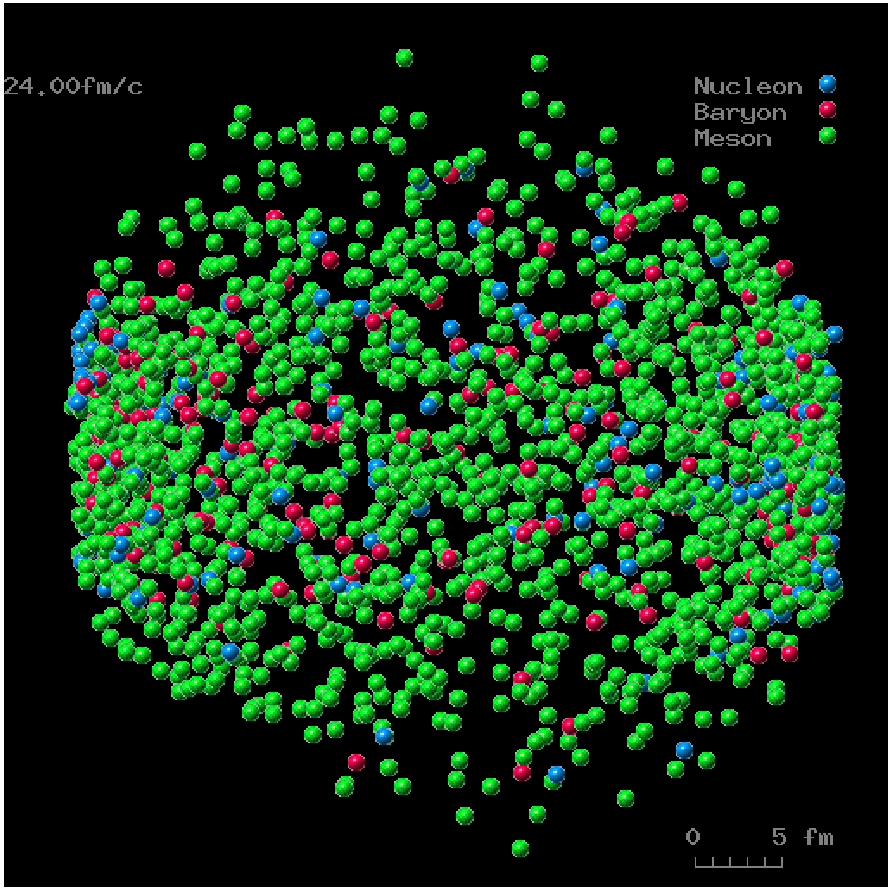


How do heavy-ion collisions look like ?

Au+Au, 10.6 A GeV

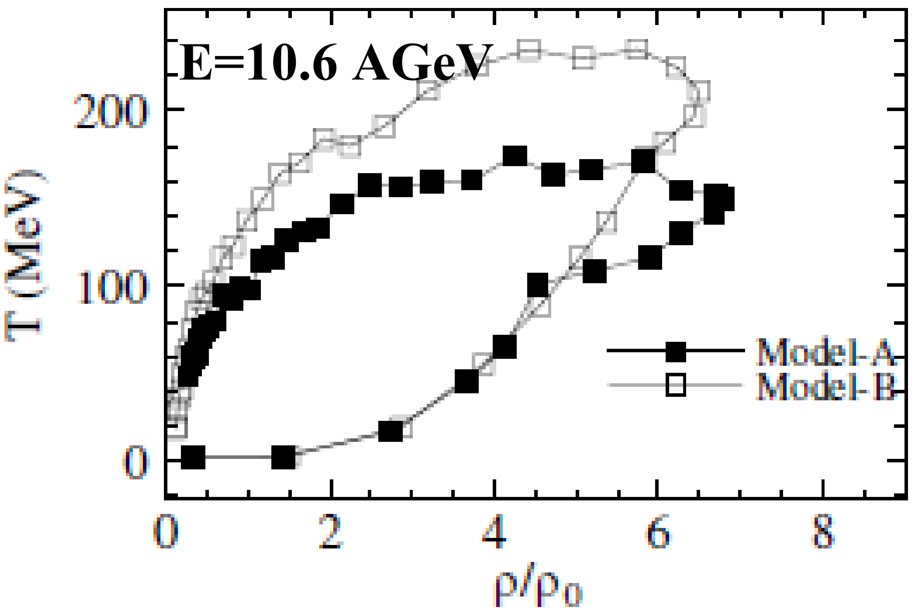


Pb+Pb, 158 A GeV

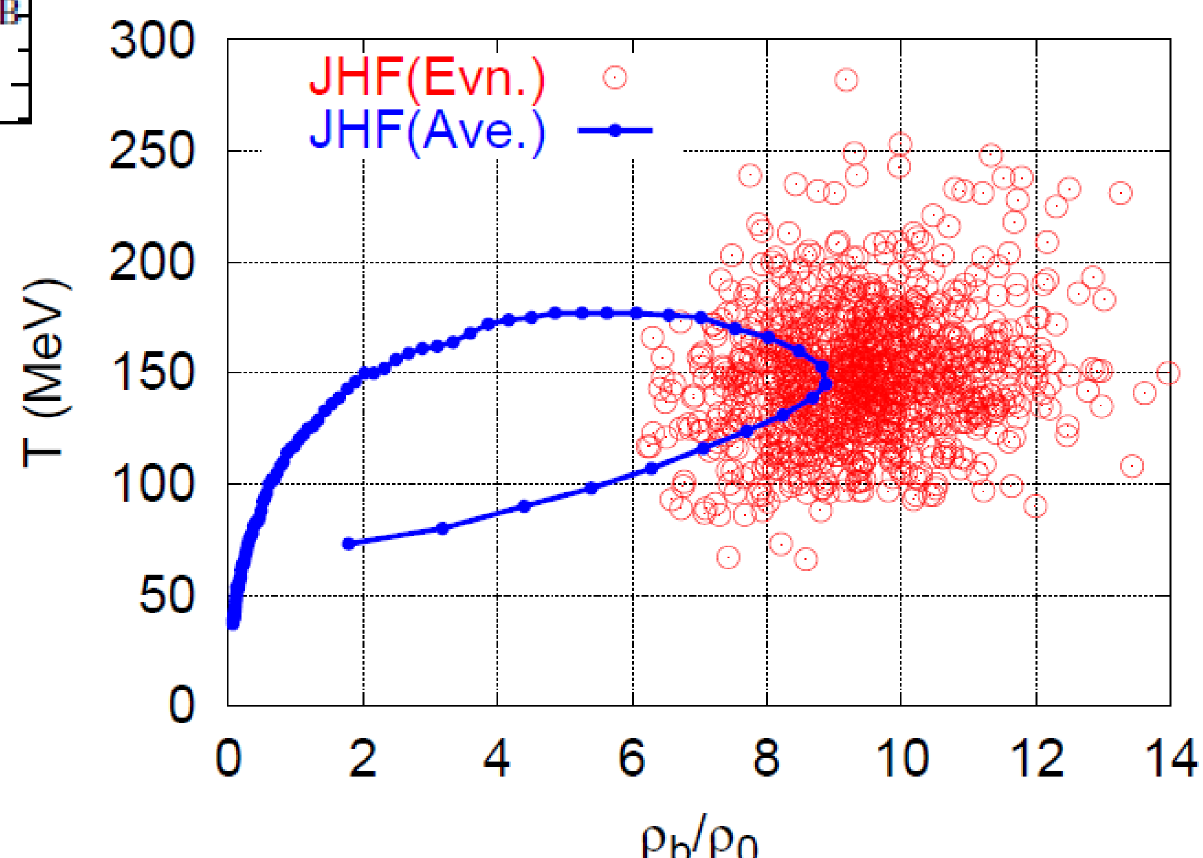


JAMming on the Web **http://www.jcprg.org/jow/**

Highest Density Matter at J-PARC ?



Nara, Otuka, AO, Maruyama ('97)

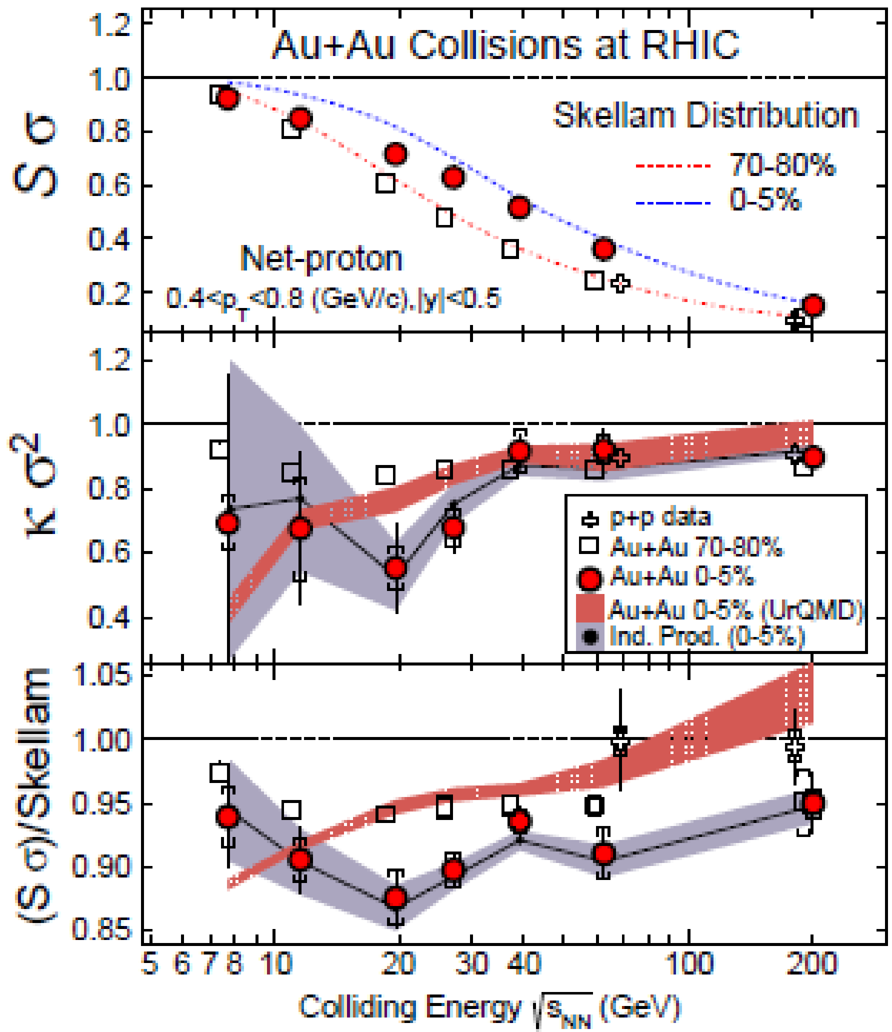


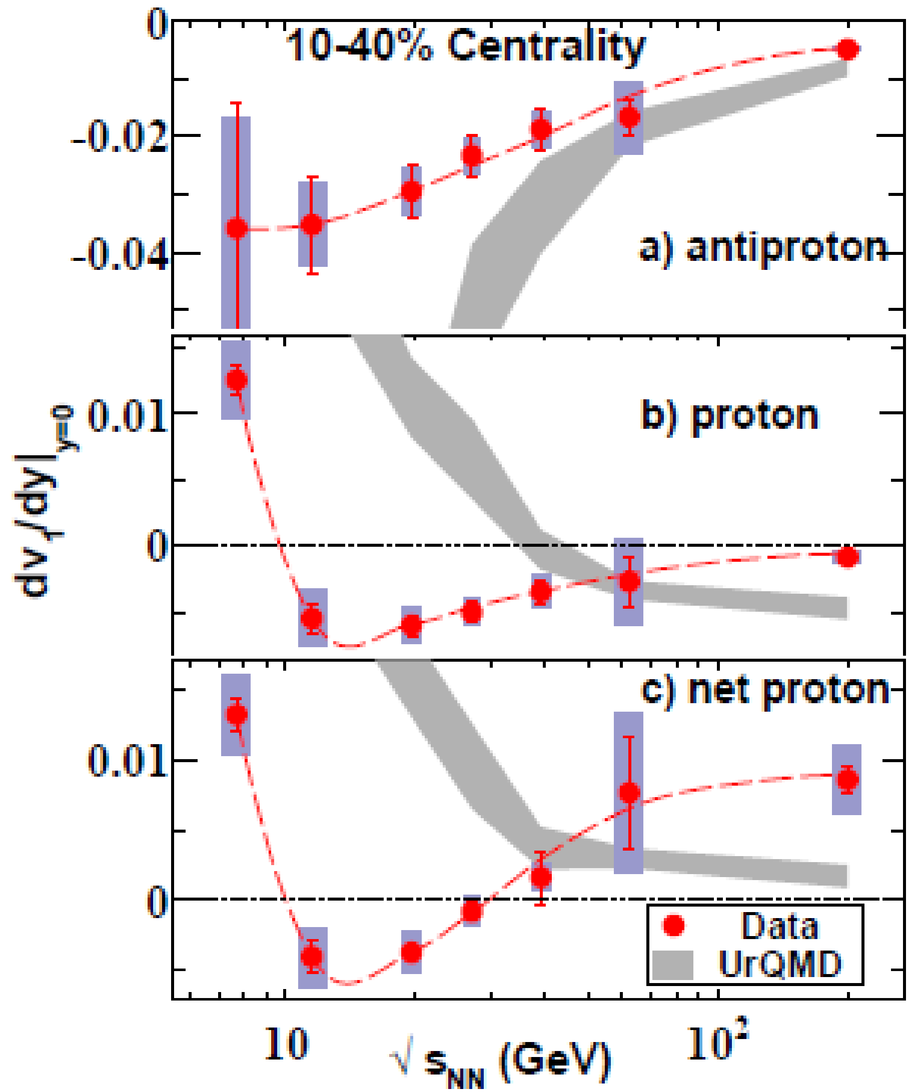
Central 1 fm^3 cube.

AO, JHF workshop (2002, unpublished)

Net-Proton Number Moments & Directed Flow

- **Non-monotonic behavior of $\kappa\sigma^2$ and dv_1/dy. CP signal ?**

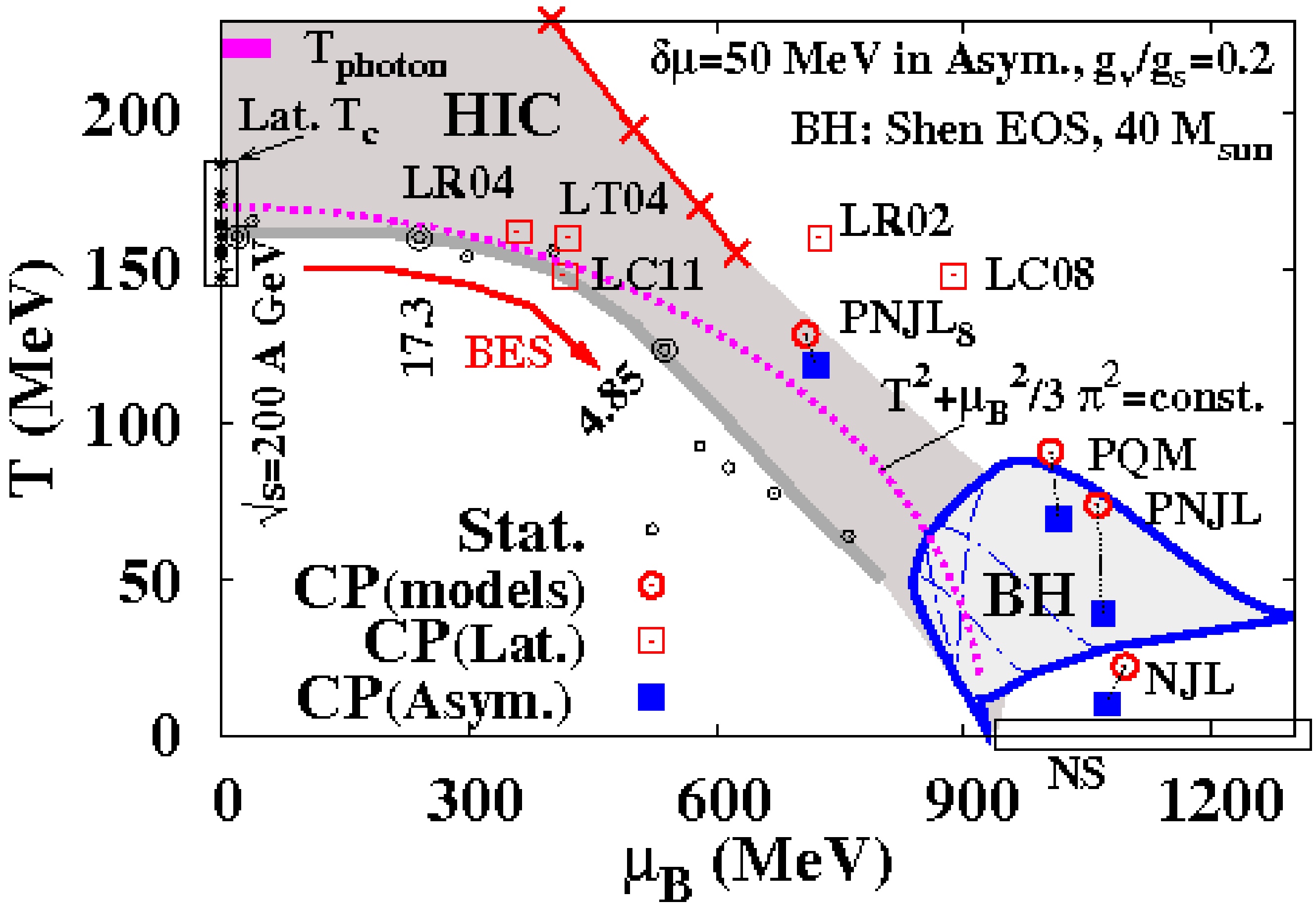




STAR Collab. (PRL 112('14)032302

STAR Collab., PRL 112('14)162301.

QCD phase diagram (Exp. & Theor. Studies)

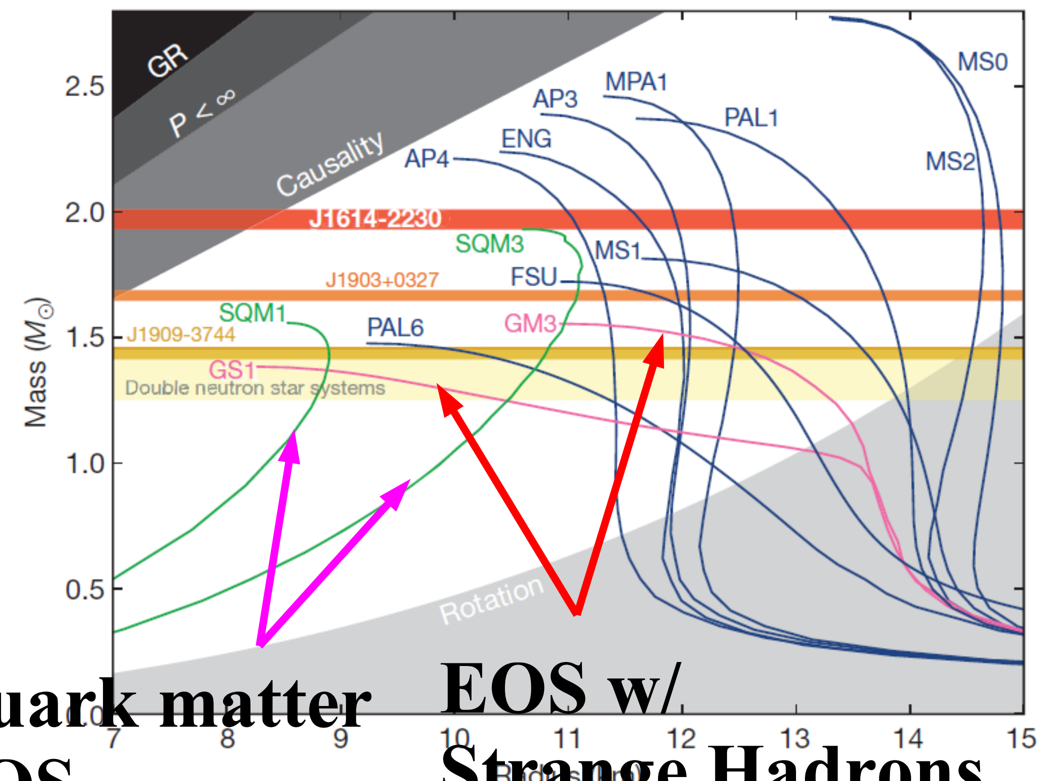


QCD phase transition is not only an academic problem, but also a subject which would be measured in HIC or Compact Stars

AO, PTPS 193('12)1

Massive Neutron Star Puzzle

- **Observation of massive neutron stars (M ~ 2 $M_\odot$)**
  - **PSR J1614-2230 (NS-WD binary), 1.97 ± 0.04 $M_\odot$**
    Demorest et al., Nature 467('10)1081 (Oct.28, 2010).
    "Kinematical" measurement (Shapiro delay, GR)
    + large inclination angle
  - **PSR J0348+0432 (NS-WS binary), 2.01 ± 0.04 $M_\odot$**
    Antoniadis et al., Science 340('13)



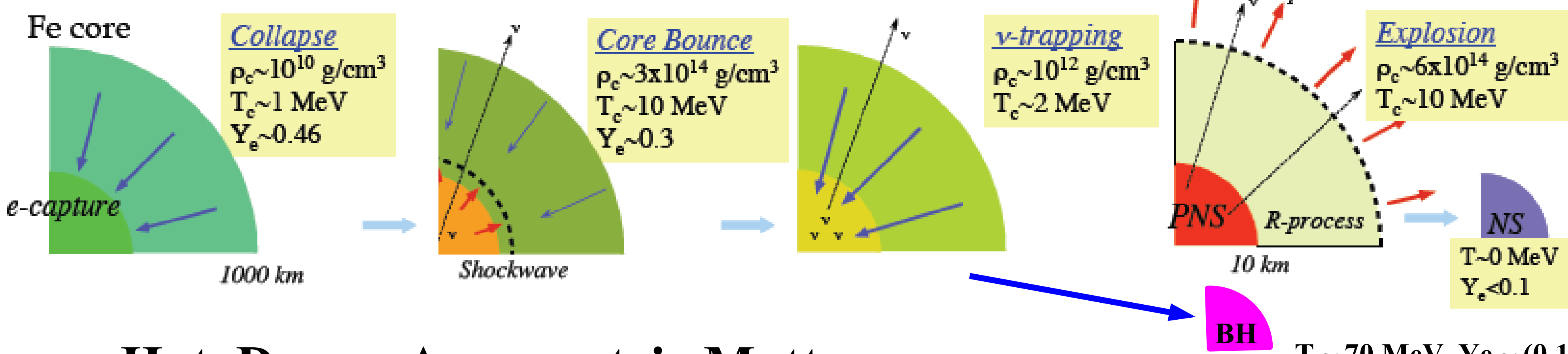
No Exotics in NS ?

Quark matter EOS

EOS w/ Strange Hadrons

Dynamical Black Hole Formation

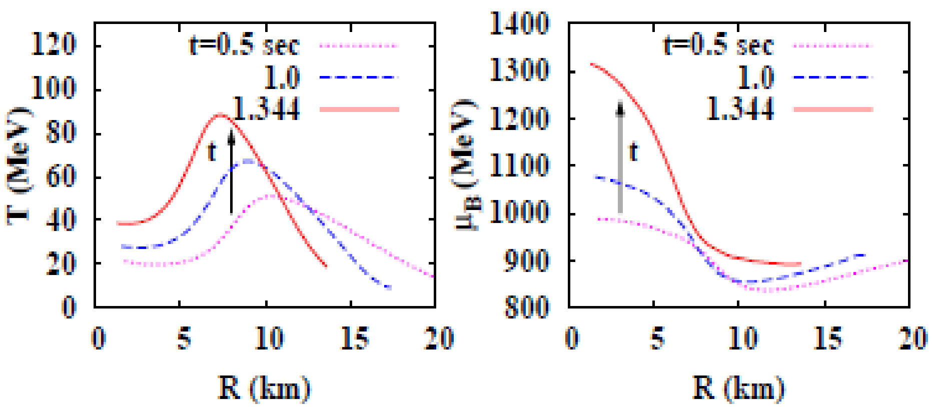
- Collapse → Bounce → Accretion

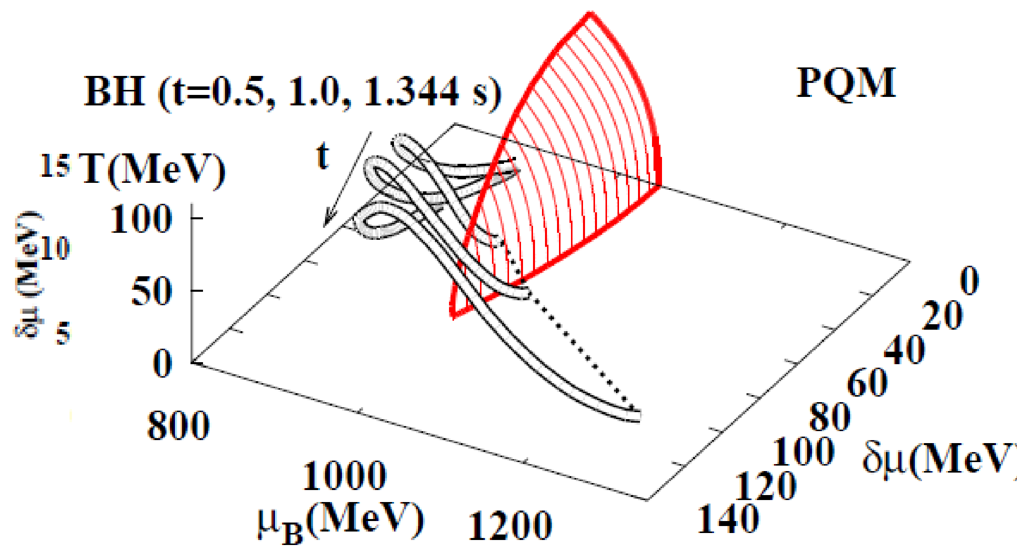


→ **Hot, Dense, Asymmetric Matter**

T ~ 70 MeV, μ_B ~ 1300 MeV, $\delta\mu=\mu_e/2$ ~ 130 MeV

→ **CP may be reachable**

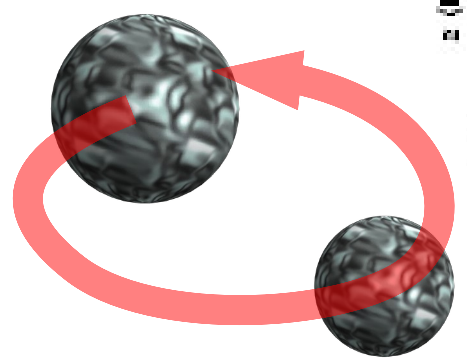


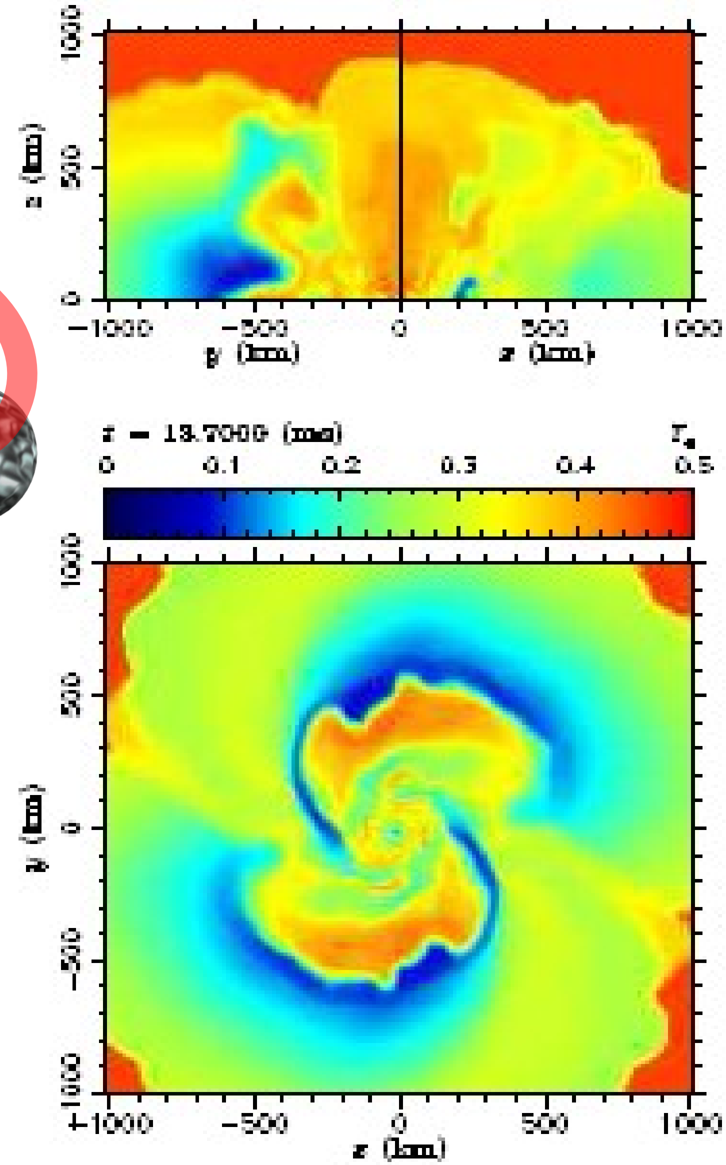


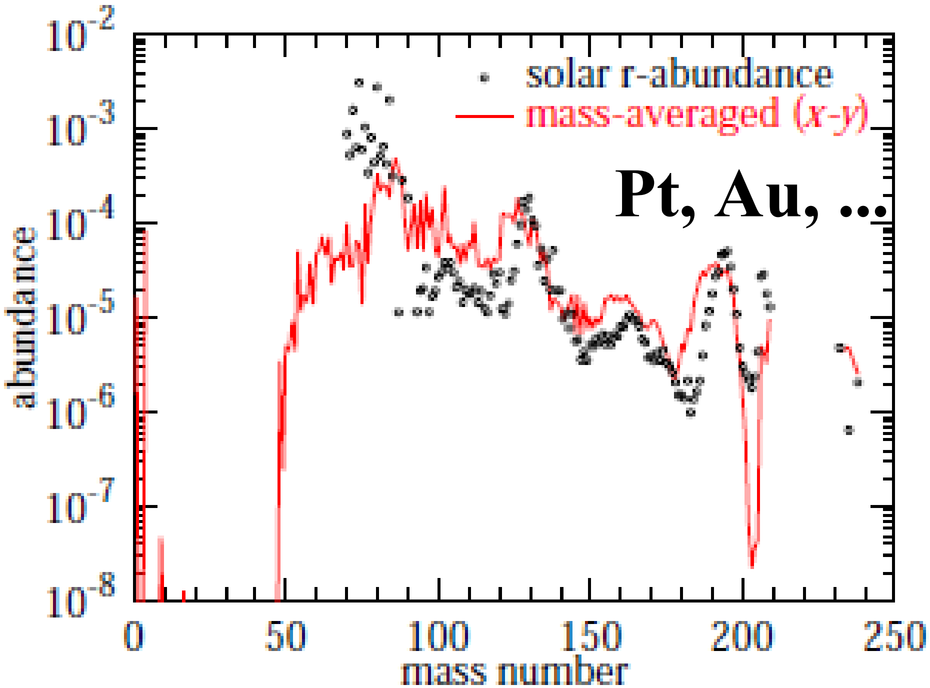
K. Sumiyoshi, et al.,('06); K.Sumiyoshi, C.Ishizuka, AO, S.Yamada, H.Suzuki ('09)
AO, H.Ueda, T.Z.Nakano, M. Ruggieri, K. Sumiyoshi ('11).

Binary Neutron Star Mergers and Nucleosynthesis

- New possibility of r-process nucleosynthesis
  - Element ratio from binary NS merger is found to reproduce Solar abundance.



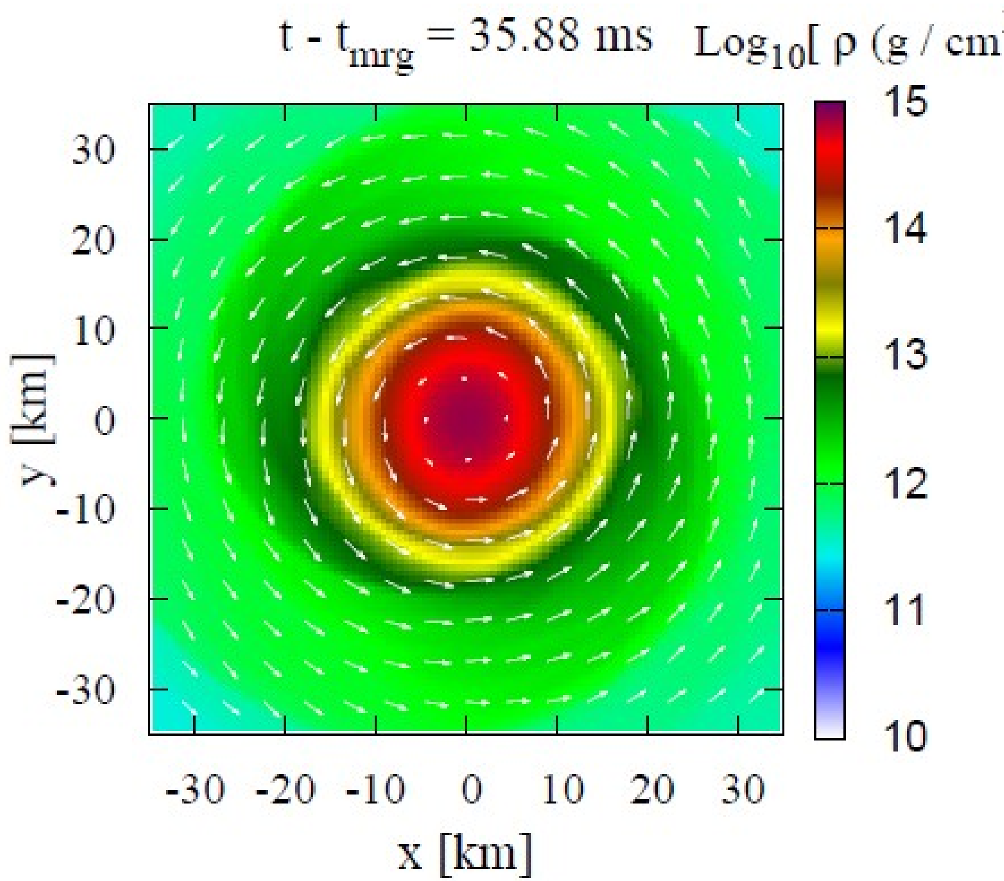


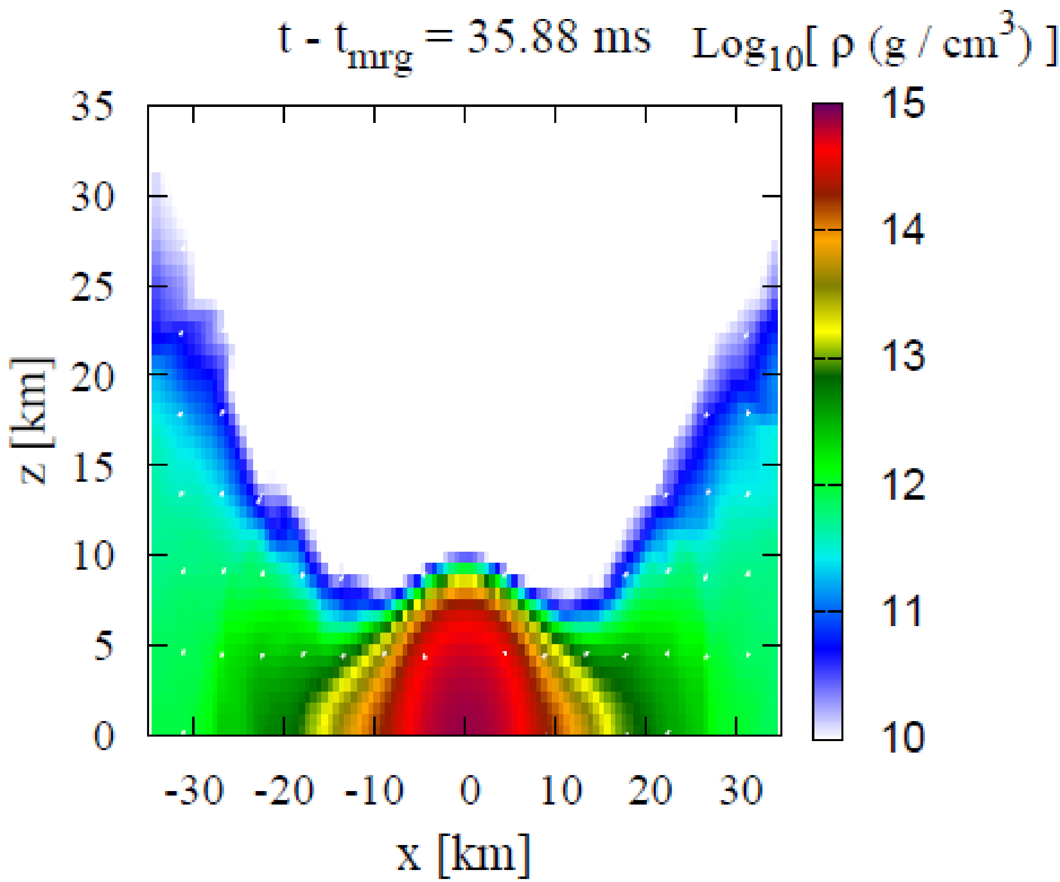


Wanajo, Sekiguchi ('14)

Binary Neutron Star Merger

- **T ~ 40 MeV, $\rho_B \sim 10^{15}$ g/cm^3 ~ 4 ρ_0 ($\rho_0 \sim 2.5 \times 10^{14}$ g/cm^3)**

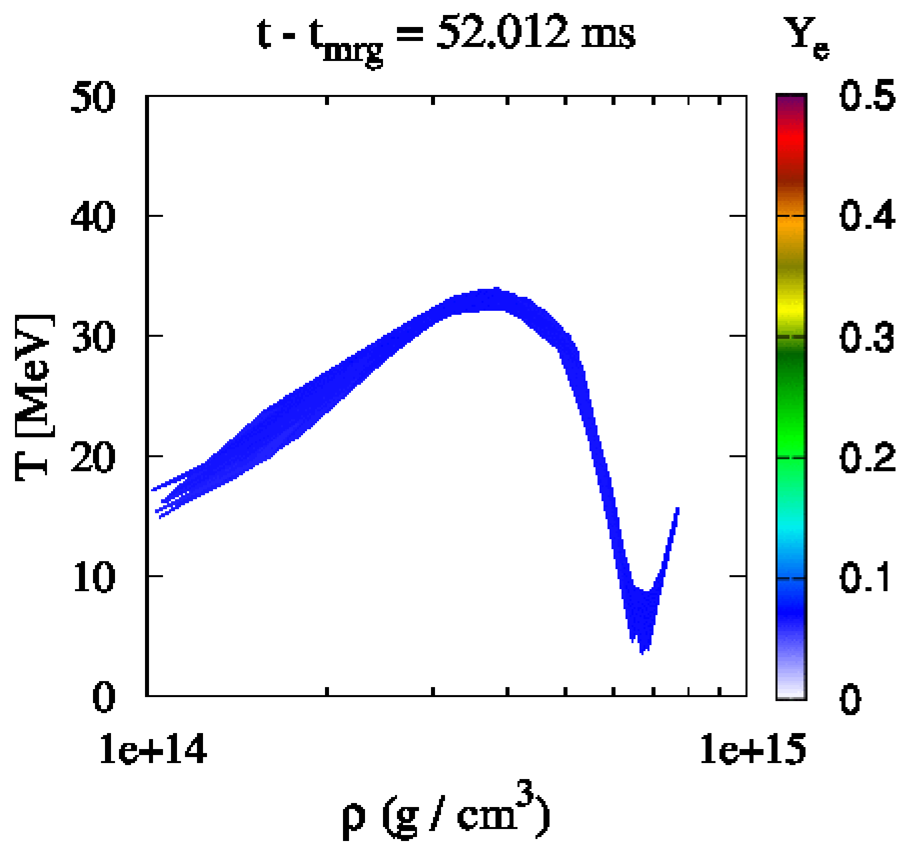




Courtesy of K. Kiuchi
Data are from Y. Sekiguchi, K. Kiuchi, K. Kyotoku, M. Shibata, PRD91('15)064059.

Binary Neutron Star Merger

- **T ~ 40 MeV, $\rho_B \sim 10^{15}$ g/cm^3 ~ 4 ρ_0 ($\rho_0 \sim 2.5 \times 10^{14}$ g/cm^3)**



Courtesy of K. Kiuchi
Data are from Y. Sekiguchi, K. Kiuchi, K. Kyotoku, M. Shibata, PRD91('15)064059.

Dense Matter

- "Dense Matter" ($\rho_B > \rho_0$) is probed in heavy-ion collisions and compact star phenomena, and we expect phase transition at some density.
- Finite density QCD is a challenge.
  - Lattice QCD has a sign problem and difficulty (at present) in predicting properties of cold dense matter.
  - Effective model approaches, Approximate approaches, and Experiments are necessary to elucidate its nature.
- Contents in Lecture I
  - Introduction
  - Spontaneous Chiral Symmetry Breaking in NJL
  - Restoration of Chiral Symmetry in NJL
  - Summary

Spontaneous Chiral Symmetry Breaking in NJL model

Chiral Symmetry in Quantum Chromodynamics

- **QCD Lagrangian**

notation: Yagi, Hatsuda, Miake

$$L=\bar{q}(i\gamma^\mu D_\mu - m)q - \frac{1}{2}\mathrm{tr}\, F_{\mu\nu}F^{\mu\nu}$$

- **Chiral symmetry: SU(N_f)$_L$ x SU(N_f)$_R$**
  - **Left- and Right-handed quarks can rotate independently**

$$q_L=(1-\gamma_5)q/2,\quad q_R=(1+\gamma_5)q/2 \to V_L q_L,\quad V_R q_R$$

$$L_q=\underbrace{\bar{q}_L(i\gamma^\mu D_\mu)q_L+\bar{q}_R(i\gamma^\mu D_\mu)q_R}_{\text{invariant}}-\underbrace{m}_{\text{small (for u, d)}}(\bar{q}_L q_R+\bar{q}_R q_L)$$

- **Chiral transf. of hadrons**

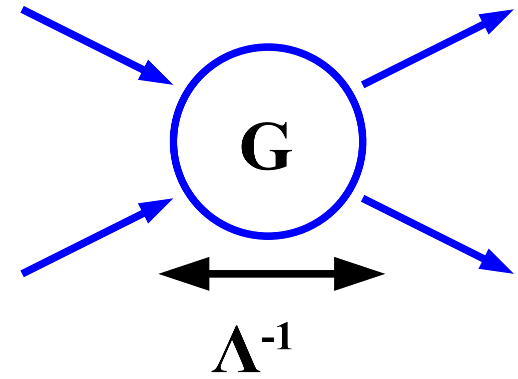
$$\sigma=\bar{q}q\ ,\quad \pi^a=\bar{q}i\gamma_5\tau^a q \ \to \begin{pmatrix}\sigma' \\ \pi'\end{pmatrix}=\begin{pmatrix}1 & -\theta \\ \theta & 1\end{pmatrix}\begin{pmatrix}\sigma \\ \pi\end{pmatrix}$$

  - **σ ($J^\pi=0^+$) and π ($J^\pi=0^-$) mix via chiral transf. but have diff. masses.**
    → Spontaneous breaking of chiral symmetry.
    (As in Bogoliubov shown in superconductor of electrons.)

Nambu-Jona-Lasinio (NJL) model

- **NJL Lagrangian**

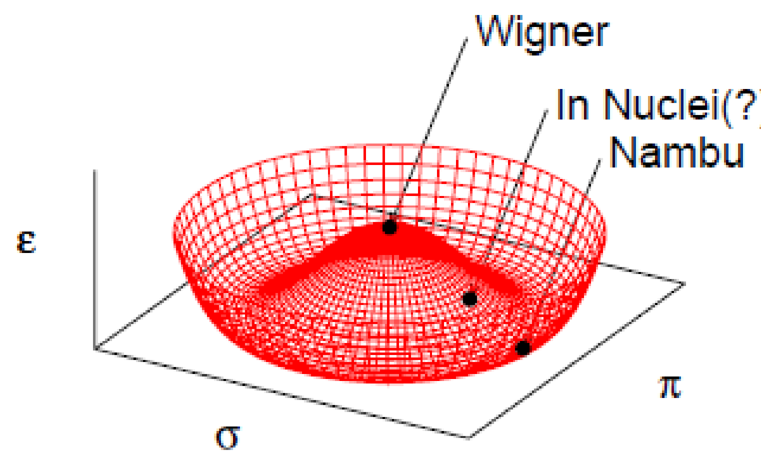
$$L=\bar{q}(i\gamma^\mu\partial_\mu-m)q+\frac{G^2}{2\Lambda^2}\left[(\bar{q}q)^2+(\bar{q}i\gamma_5\boldsymbol{\tau}q)^2\right]$$



- **Integrating out gluons and hard quarks in QCD**
  → Effective theory of quarks
  with the same symmetry as QCD

$$S=\bar{q}q\,,\ \boldsymbol{P}=\bar{q}i\gamma_5\boldsymbol{\tau}q$$

$$\rightarrow S^2+\boldsymbol{P}^2=\text{inv. under chiral transf.}$$



- **Euclidean action**

$$(x_\mu)_E=(\tau=it\,,\boldsymbol{x}),\quad (\gamma_\mu)_E=(\gamma_4=i\gamma^0,\boldsymbol{\gamma})$$

$$L_E=\bar{q}(-i\gamma_\mu\partial_\mu+m)q-\frac{G^2}{2\Lambda^2}\left[(\bar{q}q)^2+(\bar{q}i\gamma_5\boldsymbol{\tau}q)^2\right]$$

Nambu, Jona-Lasinio ('61), Hatsuda, Kunihiro ('94)

Partition Function in NJL

- **Bosonization (Hubbard-Stratonovich transf.)**

$$-\frac{G^2}{2\Lambda^2}\left[(\bar{q}q)^2+(\bar{q}i\gamma_5\boldsymbol{\tau}q)^2\right] \;\rightarrow\; \frac{\Lambda^2}{2}(\sigma^2+\boldsymbol{\pi}^2)+G\,\bar{q}(\underbrace{\sigma+i\gamma_5\boldsymbol{\tau}\cdot\boldsymbol{\pi}}_{\Sigma})q$$

- **Partition Function**

$$G \;=\; G_0 \;-\; G_0\,\Sigma\,G$$

$$Z_{\mathrm{NJL}}=\int Dq\,D\bar{q}\exp\left[-\int d^4x\,L_{\mathrm{NJL}}\right]$$

$$=\int Dq\,D\bar{q}\,D\Sigma\exp\Big[-\int d^4x\Big\{\bar{q}(\underbrace{-i\gamma\partial+\overbrace{m+G}^{M}}_{D}\Sigma)q+\frac{\Lambda^2}{2}(\sigma^2+\boldsymbol{\pi}^2)\Big\}\Big]$$

$$=\int D\Sigma\exp\left[-S_{\mathrm{eff}}(\sigma,\boldsymbol{\pi};T)\right]$$

- **Effective Action**

$$S_{\mathrm{eff}}(\Sigma;T)=-\log\det D+\int d^4x\,\frac{\Lambda^2}{2}\left[\sigma^2(x)+\boldsymbol{\pi}^2(x)\right]$$

Bosonization & Grassman Integral

- **Bosonization (Hubbard-Stratonovich transf.)**

$$\exp\left[\frac{G^2 S^2}{2\Lambda^2}\right]=\int d\sigma \exp\left[-\frac{\Lambda^2}{2}\left(\sigma-\frac{G S}{\Lambda^2}\right)^2+\frac{G^2 S^2}{2\Lambda^2}\right]$$

$$\exp\left[\frac{G^2 (P^a)^2}{2\Lambda^2}\right]=\int d\pi^a \exp\left[-\frac{\Lambda^2}{2}\left(\pi^a-\frac{G P^a}{\Lambda^2}\right)^2+\frac{G^2 (P^a)^2}{2\Lambda^2}\right]$$

- **Grassman number**

$$\int d\chi\cdot 1=\text{anti-comm. constant}=0 \quad , \quad \int d\chi\cdot\chi=\text{comm. constant}\equiv 1$$

$$\int d\chi\, d\bar\chi \exp[\bar\chi A\chi]=\int d\chi\, d\bar\chi \frac{1}{N!}(\bar\chi A\chi)^N=\cdots=\det A$$

$$=\exp[-(-\log\det A)]$$

Bi-linear Fermion action leads to -log(det A) effective action

Fermion Determinant in Mean Field Approximation

- **Mean Field approx.+Fourier transf.→ Diagonal Fermion matrix**

$$D = -i\boldsymbol{\gamma}\cdot\nabla - i\gamma_4\partial_\tau + M \to \begin{pmatrix} -i\omega + M & \boldsymbol{k}\cdot\boldsymbol{\sigma} \\ -\boldsymbol{k}\cdot\boldsymbol{\sigma} & i\omega + M \end{pmatrix} \quad (M = G\sigma = \text{const.})$$

$$\det D = \prod_{n,\boldsymbol{k}} (\omega_n^2 + \boldsymbol{k}^2 + M^2)^{d_f/2} \quad (d_f = 4N_c N_f = \text{Fermion d.o.f.})$$

- **Effective Potential**

$$F_{\text{eff}} = \Omega/V = -\frac{T}{V}\log Z = \frac{\Lambda^2}{2}\sigma^2 - \frac{T}{V}\sum_{n,\boldsymbol{k}} \log(\omega_n^2 + \boldsymbol{k}^2 + M^2)^{d_f/2}$$

$$= \frac{\Lambda^2}{2}\sigma^2 - d_f \int \frac{d^3k}{(2\pi)^3}\left[\underline{\frac{E_k}{2}} + \underline{\frac{k^2}{3E_k}\frac{1}{e^{E_k/T}+1}}\right]$$

Matsubara sum

Fermion det. → Zero point energy ($\hbar\omega/2$)+ Thermal pressure

- **Matsubara Frequency Summation**

$$I(E,T)=T\sum_n \log(\omega_n^2+E^2)$$

$$\omega_n=2n\pi T,\ \pi T(2n-1)$$

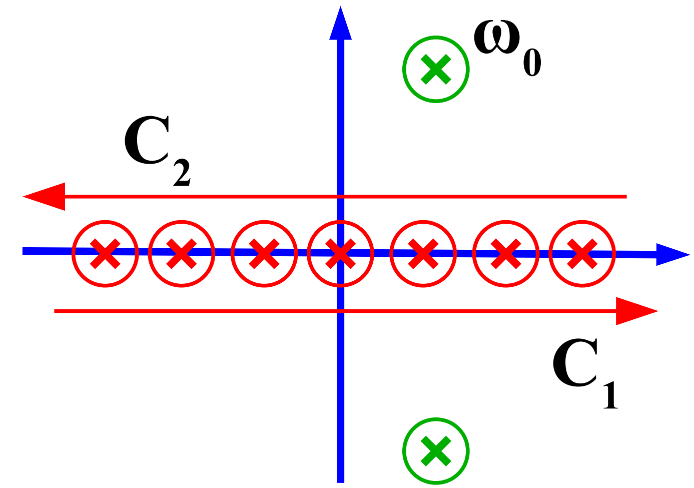
$$(\text{for bosons, fermions})$$

This is it !

$$\frac{\partial I(E,T)}{\partial E}=\sum_n \frac{2TE}{\omega_n^2+E^2}=\frac{e^{E/2T}\pm e^{-E/2T}}{e^{E/2T}\mp e^{-E/2T}}$$

$$I(E,T)=2T\log[e^{E/2T}\mp e^{-E/2T}]$$

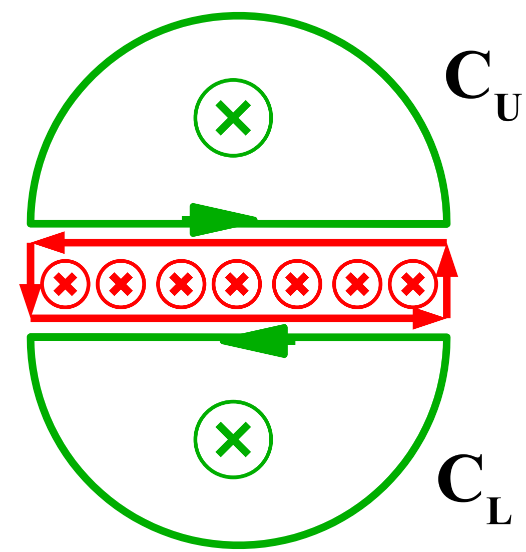
$$=E+2T\log\left(1\mp\exp(-E/T)\right)+\text{const.}$$



- **Contour integral technique**

$$S=T\sum_n g(\omega_n=2\pi nT,\ \pi(2n+1)T)$$

$$=\pm\int_{C_1+C_2}\frac{dz}{2\pi}\frac{g(z)}{e^{i\beta z}\mp 1}=\mp i\sum_{\omega_0}\frac{\operatorname{Res} g(\omega_0)}{e^{i\beta\omega_0}\mp 1}$$



$$C_1+C_2+C_U+C_L=0$$

Effective potential of NJL model

- **Effective potential (Grand pot. density)**

$$F_{\rm eff}=\Omega/V=-d_f\int\frac{d^3k}{(2\pi)^3}\left[\frac{E_k}{2}+T\log\left(1+e^{-E_k/T}\right)\right]+\frac{\Lambda^2}{2}\sigma^2$$

Zero point energy + Thermal (particle) excitation + Aux. Fields

- **Effective potential in vacuum (T=0, μ=0) in the chiral limit (m=0)**

$$F_{\rm eff}=-\frac{d_f}{2}\underbrace{\int^\Lambda\frac{d^3k}{(2\pi)^3}E_k}_{\Lambda^4 I(M/\Lambda)}+\frac{\Lambda^2}{2}\sigma^2=\Lambda^4\left[-\frac{d_f}{2}I(x)+\frac{x^2}{2G^2}\right]\quad(x=M/\Lambda)$$

$$\frac{F_{\rm eff}}{\Lambda^4}=-\frac{d_f}{16\pi^2}+\frac{x^2}{2}\boxed{\left[\frac{1}{G^2}-\frac{1}{G_c^2}\right]}+O(x^4\log x)\quad(G_c^2=8\pi^2/d_f)$$

$G>G_c$ → 2nd coef. < 0 →Spontaneous Chiral Sym. Breaking

$$I(x)=\frac{1}{16\pi^2}\left[\sqrt{1+x^2}(2+x^2)-x^4\log\frac{1+\sqrt{1+x^2}}{x}\right]\simeq\frac{1}{8\pi^2}\left[1+x^2+\frac{1}{8}x^4\left(1+4\log\frac{x}{2}\right)+O(x^6)\right]$$

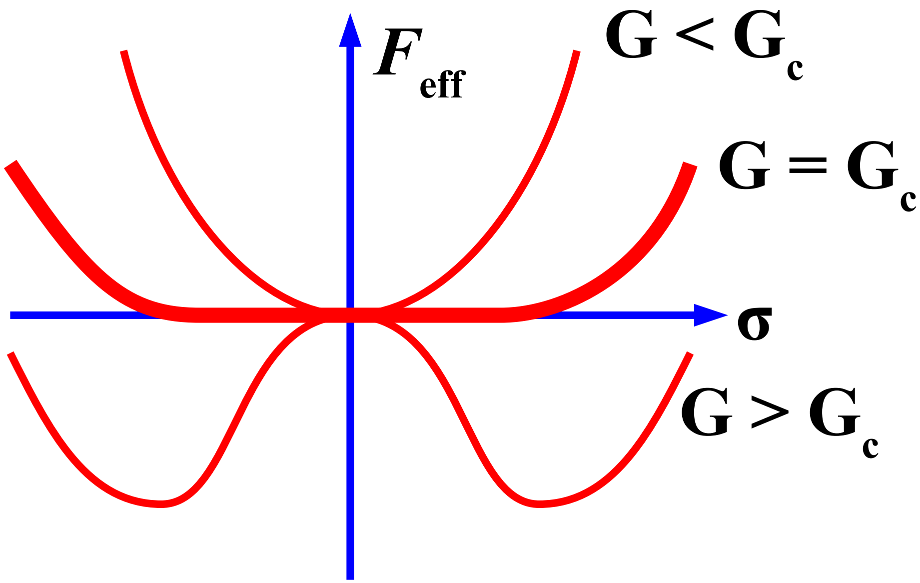
Spontaneous breaking of chiral symmetry

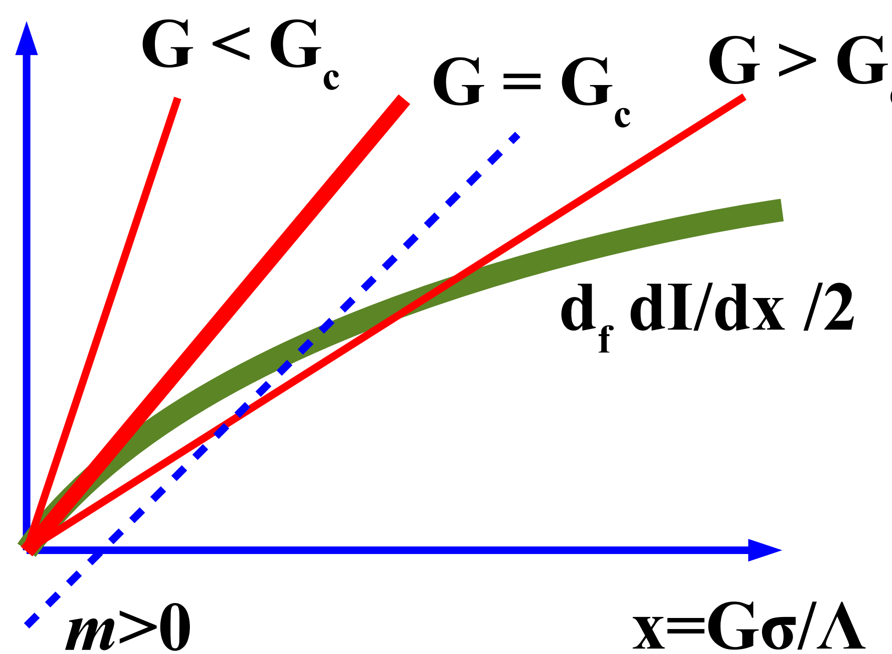
- **σ is chosen to minimize F_{eff} (Gap equation)**

$$\frac{1}{\Lambda^4}\frac{\partial F_{\mathrm{eff}}}{\partial x} = -\frac{d_f}{2}\frac{dI(x)}{dx} + \frac{x}{G^2} = 0$$

For $G>G_c$ → finite σ(~ $q^{bar}q$) solution gives min. energy state.

If the interaction is strong enough, σ(~$q^{bar}q$) condensates and quark mass is generate.(Nambu, Jona-Lasinio ('61))





Chiral phase transition at finite T and μ (Chiral Limit)

T, μ and m dependence of thermal pressure

- **Thermal pressure as a function of T, μ, and m (Fermions)**
  Kapusta ('89), Kapusta, Gale (2006)

$$P^F/d_F = \frac{7}{8}\frac{\pi^2}{90}T^4 + \frac{1}{24}\mu^2 T^2 + \frac{\mu^4}{48\pi^2}$$

Stefan-Boltzmann (m=0)

$$- \frac{m^2}{16\pi^2}\left[\frac{\pi^2}{3}T^2 + \mu^2\right]$$

m^2 term → phase transition

$$- \frac{m^4}{32\pi^2}\left[\log\left(\frac{m}{\pi T}\right) - \frac{3}{4} + \gamma_E \boxed{- H^\nu\left(\frac{\mu}{T}\right)}\right] + \mathcal{O}(m^6)$$

New

m^4 term → critical point

$$H^\nu(\nu) = \frac{7}{4}\zeta(3)\left(\frac{\nu}{\pi}\right)^2 - \frac{31}{16}\zeta(5)\left(\frac{\nu}{\pi}\right)^4 + \frac{127}{64}\zeta(7)\left(\frac{\nu}{\pi}\right)^6 + \cdots$$

Mass reduces pressure (enh. Feff) → phase transition ?

High-Temperature Expansion (1)

- **Thermal pressure (Fermions)**

$$P^F = \frac{d_F}{2}\int \frac{d^3p}{(2\pi)^3}\frac{p^2}{3\omega}\left[\frac{1}{e^{(\omega-\mu)/T}+1}+\frac{1}{e^{(\omega+\mu)/T}+1}\right]$$

$$\omega = \sqrt{p^2+m^2}$$

- **High-Temperature Expansion = Expansion in *m/T***
  - **Important to discuss chiral transition (*m* = *Gσ*)**
  - **Naive expansion does not work (non-analytic term in *m*)**
- **Kapusta method**
  - **Recursion formula: simpler integral → pressure**

$$P^F = \frac{4T^4 d_F}{\pi^2}\, h_5^F\left(y=\frac{m}{T}, \nu=\frac{\mu}{T}\right)\ , \quad \frac{dh_{n+1}}{dy} = -\frac{y}{n}h_{n-1}$$

  - **Replace integrand**

$$\frac{1}{2\omega}\left[\frac{1}{e^{\omega-\nu}+1}+\frac{1}{e^{\omega+\nu}+1}\right] = \frac{1}{2\omega} - \sum_{l=-\infty}^{\infty}\frac{1}{\omega^2+[\pi(2l-1)-i\nu]^2}$$

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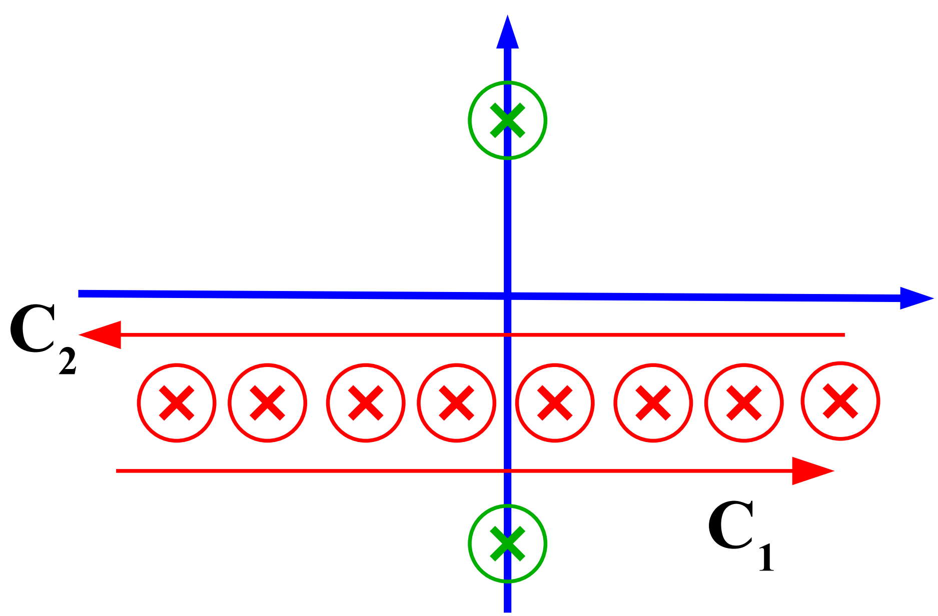
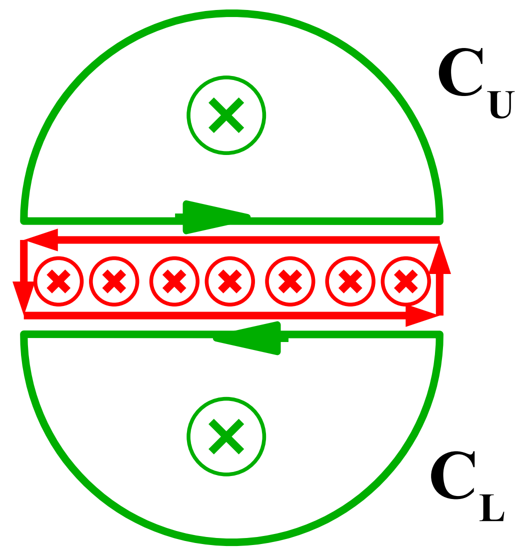
- **Following identity is obtained from contour integral.**

$$\frac{1}{2\omega}\left[\frac{1}{e^{\omega-\nu}+1}+\frac{1}{e^{\omega+\nu}+1}\right]=\frac{1}{2\omega}-\sum_{l=-\infty}^{\infty}\frac{1}{\omega^2+[\pi(2l-1)-i\nu]^2}$$

$$\oint_{\mathcal{C}_\mathrm{U}+\mathcal{C}_\mathrm{L}}\frac{dz}{2\pi}\,\frac{1}{e^{iz-\nu}+1}\,\frac{1}{z^2+\omega^2}=-\oint_{\mathcal{C}}\frac{dz}{2\pi}\,\frac{1}{e^{iz-\nu}+1}\,\frac{1}{z^2+\omega^2}$$

pole at $z = \pm i\,\omega$ **pole at $z = \pi(2\,l\,-1\,) - i\,\nu$**





$C_1+C_2+C_U+C_L=0$

High-Temperature Expansion (3)

- **Recursion relation of h-functions**

$$h_n^F(y,\nu) = \frac{1}{2(n-1)!}\int_0^\infty \frac{x^{n-1}dx}{\omega}\left\{\frac{1}{e^{\omega-\nu}+1}+\frac{1}{e^{\omega+\nu}+1}\right\}$$

$$\frac{dh_{n+1}}{dy} = -\frac{y}{n}h_{n-1}$$

  - **From h1(y, ν) , h3(0, ν), h5(0, ν), we obtain h5(y, ν) and pressure.**

- **Key function= h1(y, ν)**

$$h_1^F(y,\nu) = \lim_{L\to\infty}\int_0^{2\pi L} dx\left[\frac{1}{2\omega}-\sum_{l=-\infty}^{\infty}\frac{1}{\omega^2+[\pi(2l-1)-i\nu]^2}\right]$$

$$= -\frac{1}{2}\log\frac{y}{\pi}-\frac{1}{2}\gamma_{\mathrm{E}}-\frac{1}{2}\sum_{l=1}^{\infty}\left[\frac{\pi}{\omega_l}+\frac{\pi}{\omega_l^*}-\frac{2}{2l-1}\right]$$

$$(\omega_l = \sqrt{y^2+[\pi(2l-1)-i\nu]^2})$$

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Chiral Transition at Finite T

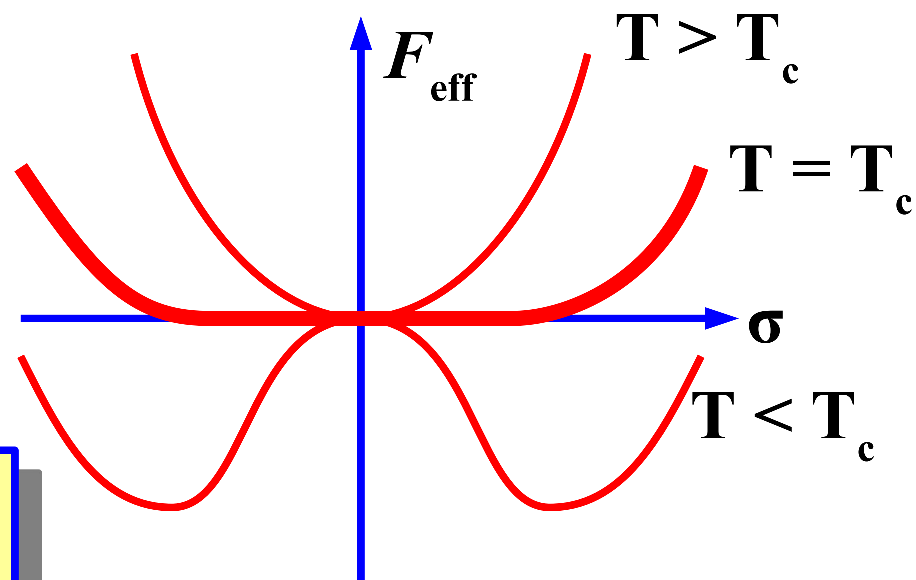
- **Effective potential at finite *T* in NJL**

$$\frac{F_{\text{eff}}}{\Lambda^4} = -\frac{d_f}{2} I(x) + \frac{x^2}{2G^2} - \frac{P^F}{\Lambda^4}$$

$$= -\frac{d_f}{16\pi^2} \underline{- \frac{d_f \pi^2}{90}\frac{7}{8}\left(\frac{T}{\Lambda}\right)^4} + \frac{x^2}{2}\left[\frac{1}{G^2} - \frac{1}{G_c^2}\left(\underline{1 - \frac{\pi^2}{3}\left(\frac{T}{\Lambda}\right)^2}\right)\right] + O(x^4)$$

Stefan-Boltzmann **Correction from T**

- **Chiral transition should occur at T < $3^{1/2}$ Λ/π.**



Chiral Transition at finite T is suggested by NJL !

Chiral Transition at Finite μ

- **Effective potential at finite μ in NJL**

$$F_{\mathrm{eff}}(m;T,\mu) = F_{\mathrm{eff}}(0;T,\mu) + \frac{c_2(T,\mu)}{2}m^2 + \frac{c_4(T,\mu)}{24}m^4 + \mathcal{O}(m^6)$$

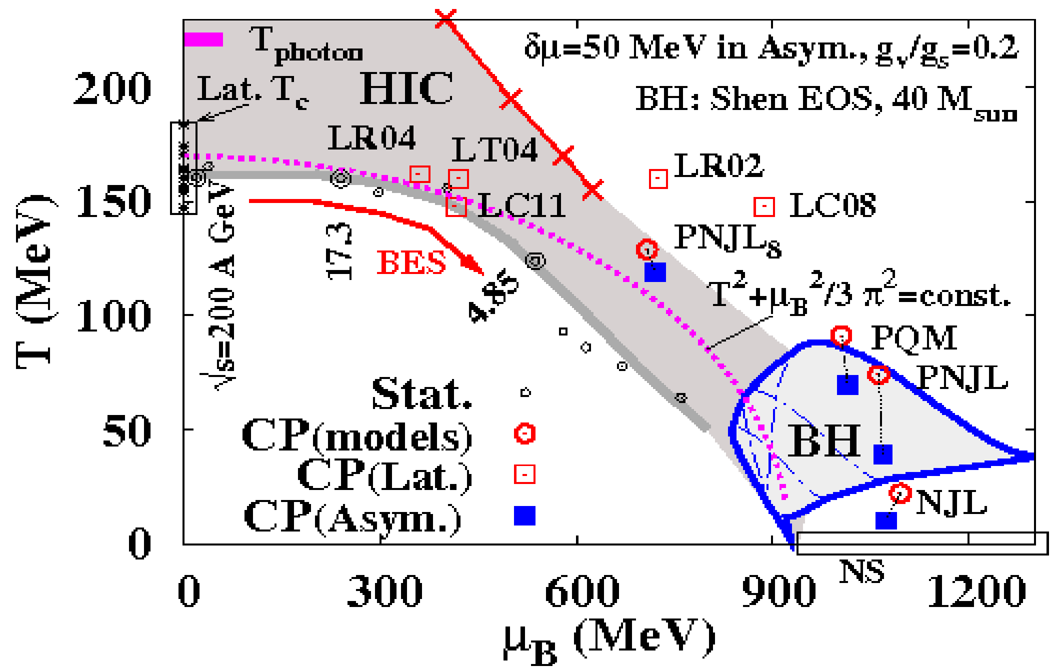
$$c_2(T,\mu) = -\frac{d_F}{24}\left[\boxed{\frac{3}{\pi^2}\Lambda^2\left(1-\frac{8\pi^2}{d_F G^2}\right)} - \left(T^2 + \frac{3}{\pi^2}\mu^2\right)\right]$$

$T_c^2(\mu=0)$

- **2nd order phase boundary**

$$T^2 + \frac{3}{\pi^2}\mu^2 = T_c^2(\mu=0)$$

Roughly matches chem. freeze-out line.



Chiral Transition at Finite μ

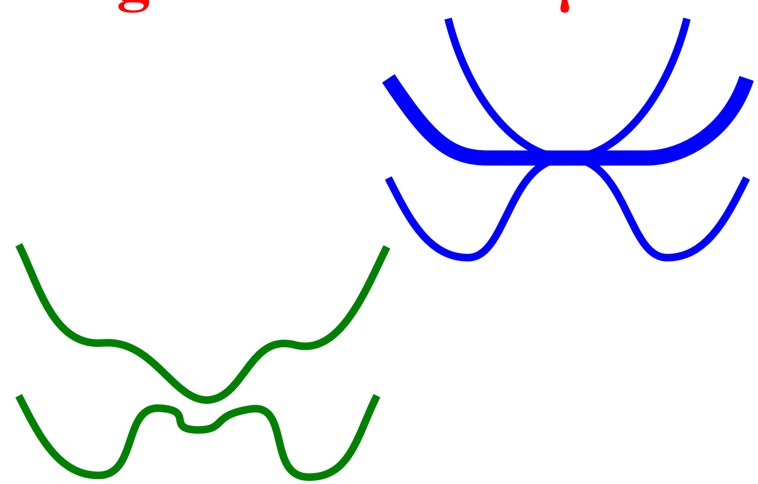
- **Effective potential at finite μ in NJL**

$$F_{\rm eff}(m;T,\mu) = F_{\rm eff}(0;T,\mu) + \frac{c_2(T,\mu)}{2} m^2 + \frac{c_4(T,\mu)}{24} m^4 + \mathcal{O}(m^6)$$

$$c_2(T,\mu) = -\frac{d_F}{24}\left[\frac{3}{\pi^2}\Lambda^2\left(1-\frac{8\pi^2}{d_F G^2}\right) - \left(T^2 + \frac{3}{\pi^2}\mu^2\right)\right]$$

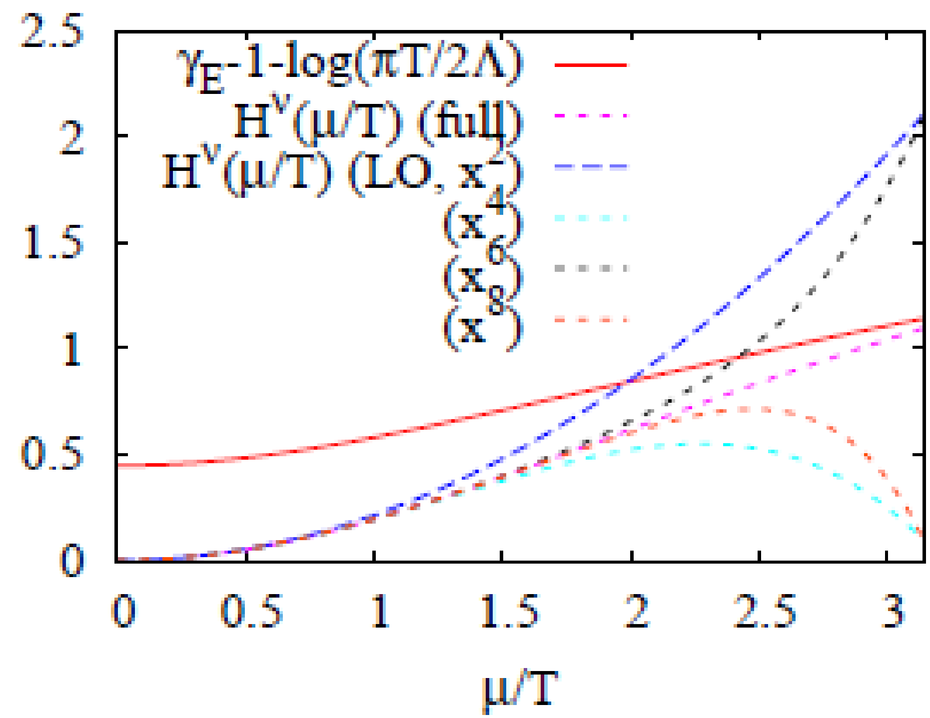
$$c_4(T,\mu) = \frac{3d_F}{4}\left[\underbrace{\gamma_E - 1 - \log\left(\frac{\pi T}{2\Lambda}\right)}_{\mu=0} \underbrace{- H^\nu(\mu/T)}_{\text{negative at finite } \mu}\right]$$

- **$c_2 = 0$ and $c_4 > 0 \rightarrow$ 2nd order**
- **$c_2 \geq 0$ and $c_4 < 0 \rightarrow$ 1st order**
- **$c_2 = 0$ and $c_4 = 0 \rightarrow$ tricritical point**



(Tri)Critical Point

- **Do we expect the existence of (Tri)Critical Point in NJL ?**
  - **Yes, as first shown by Asakawa, Yazaki ('89)**
  - **TCP in the chiral limit → CP at finite bare quark mass**
- **Estimate from high-temperature expansion**
  - **TCP: $c_2 = 0$ and $c_4 = 0$ simultaneously.**
  - **c_4 decreases as μ/T increases.**
  - **Existence is probable,
Position is sensitive
to parameters and treatment.**



Summary of Lecture I

- **We expect the existence of QCD phase transition and the critical point from chiral effective model studies. This point is discussed based on the Nambu-Jona-Lasinio model**
  - **When qq interaction is stroung enough, chiral symmetry is spontaneously broken in vacuum.**
  - **Chiral symmetry should be restored at high temperature.**
  - **Density effect reduces the 4-th coeff. in m (or σ), and we can expect the first order transition at high density.**
  - **Technical part**
    Matsubara sum, Hubbard-Stratonovich transformation, High-temperature expansion, ...
- **Since the first principle calculation of QCD has difficulties at finite densities, we need studies using effective models, approximate treatment of QCD, and of course, experiments.**

Thank you for your attention !

Abstract

QCD phase diagram is attracting much attention in these years. It is now extensively studied in the Beam Energy Scan (BES) program at RHIC, and is closely related to the beginning of our universe (big bang) and the final form of matter (neutron stars).

The robust mechanism for the QCD phase transition is the spontaneous chiral symmetry breaking and restoration. The spontaneous symmetry breaking is well understood in chiral effective models of QCD such as the Nambu-Jona-Lasinio model; zero-point energy of quarks favors finite chiral condensate. By using the high-temperature expansion, the transition temperature is found to decrease with increasing chemical potential.

In the first lecture (during the dense matter school), I explain the basic mechanism of the chiral phase transition in effective models, and the expected shape of the phase boundary using the high-temperature expansion. I also introduce the strong coupling lattice QCD, in which we can examine that the same mechanism applies to QCD at strong coupling.

Discovery of the QCD critical point and the first order phase transition at high density is one of the ultimate goals in the BES program and forthcoming FAIR and NICA fascilities, and heavy-ion programs at J-PARC. We also expect formation of dense matter in compact astrophysical phenomena, such as the neutron star core, supernova explosion, dynamical collapse to black holes, and binary neutron star mergers. From the theoretical side, it is desirable to draw the QCD phase diagram using the lattice QCD Monte-Carlo simulations. However, the sign problem in lattice QCD at finite chemical potential causes difficulty in performing precise Monte-Carlo simulations of finite density matter. There are some exceptions such as the color SU(2) QCD, finite isospin chemical potential, or imaginary chemical potential. The strong coupling lattice QCD can be one of these exceptions; while the sign problem exists, it is milder and two independent methods predict the same QCD phase boundary.

In the second lecture (during SQM), we discuss the phase diagram in effective models and the strong coupling lattice QCD, and some observables expected to appear around the critical point. We also discuss the thermodynamic conditions realized during the failed supernova, where the black hole is formed dynamically. We find that cold, dense, and isospin asymmetric matter is formed during the black hole formation, and it may be possible to sweep the QCD critical point in compact star phenomena.