On a Possible Importance of Nuclear Liquid-Gas Phase Transition in Supernova Nucleosynthesis

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1. Heavy Element Synthesis — When, Where and How ?

- ⋆ s- and r-processes
- ⋆ A-distribution in the Universe
- ⋆ Phase Diagram of Nuclear Matter
- ⋆ Possible Importance of LG-process

2. Simple Model Calculation

- ⋆ Model (I): Relativistic Mean Field (RMF)
- ⋆ Model (II): Statistical Model of Fragments
- ⋆ Does the Supernova Evolution Path hit LG Coexistence Region ?
- ⋆ Isotope Distribution in the Universe

3. Summary and Discussion

⋆ Synthesis of Heavy Elements

– When, Where and How ?

- Slow Neutron Capture process (s-process)
  - ⋆ Stable Nuclei upto ^{209}Bi
  - ⋆ Neutron Flux in Stars: Understanding is not complete
- Rapid Neutron Capture process (r-process)
  - ⋆ Heavy Neutron Rich Nuclei
  - ⋆ Most Probable Site
    = Hot bubble region of Massive Supernovae
  - ⋆ Requires Very High Entropy/Baryon, $S/B \simeq (110 - 400)$
    (Woosley et al. 1994, Meyer and Brown 1997, Terasawa and Kajino 1999)
- Problems
  - ⋆ Why is the Elements Dist. Universal ?
  - ⋆ How are the Heavy Proton Rich Nuclei formed ?

☞ SOMETHING ELSE ?

- Hints ?
  - ⋆ Background A-distribution in the Universe:
    ⋯ Power Law Behavior in addition to Exponential
  - ⋆ Phase Diagram of Nuclear Matter
    ⋯ Unstable (L-G coexistence) Region

☞ Fragm. through LG Phase Tr.
may be important

⋆ Simple Model (I): RMF + Adiabatic Path

Assumption:

- ⋆ Infinitely Large Liquid and Gas phase coexist.
- ⋆ Lepton to Baryon ratio is conserved.
  (νs are still trapped.)
- ⋆ Entropy per Baryon is conserved.

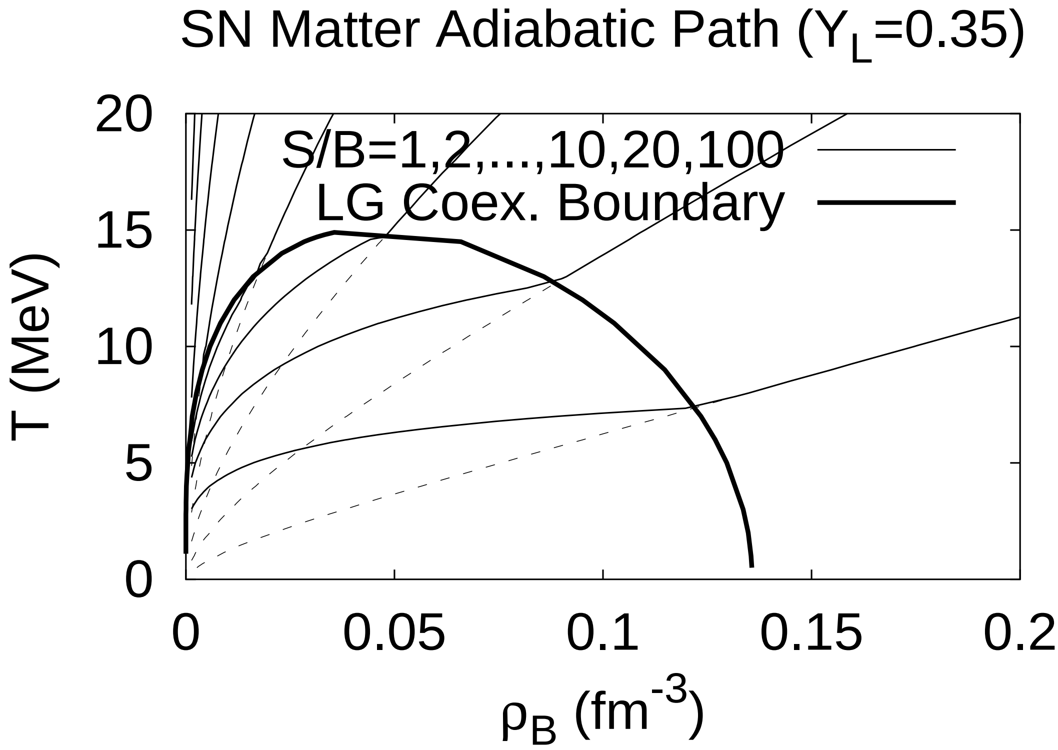
• Relativistic Mean Field (RMF)

- ⋆ Tokyo Metro. Univ. Parameter (TM1)
  - ∘ Fit B.E. of stable and unstable n-rich nuclei
  - ∘ Applied to SN Explosion
- ⋆ Lagrangian (σ, ω, ρ with $\sigma^3, \sigma^4, \omega^4$ terms)

$$\begin{aligned}
\mathcal{L}^{RMF} &= \mathcal{L}_B + \mathcal{L}_M + \mathcal{L}_{BM} \\
\mathcal{L}_B &= \sum_i \bar{\psi}_i (i\gamma^\mu \partial_\mu - M_i)\psi_i \ , \\
\mathcal{L}_M &= -\frac{1}{2}\sum_s m_s^2 \phi_s^2 + \frac{1}{2}\sum_v m_v^2 V_s^2 - U(\phi, V) \ , \\
\mathcal{L}_{BM} &= \sum_{s,i} g_{si} \bar{\psi}_i \phi_s \psi_i - \sum_{v,i} g_{vi} \psi_i^\dagger V_v \psi_i \ .
\end{aligned}$$

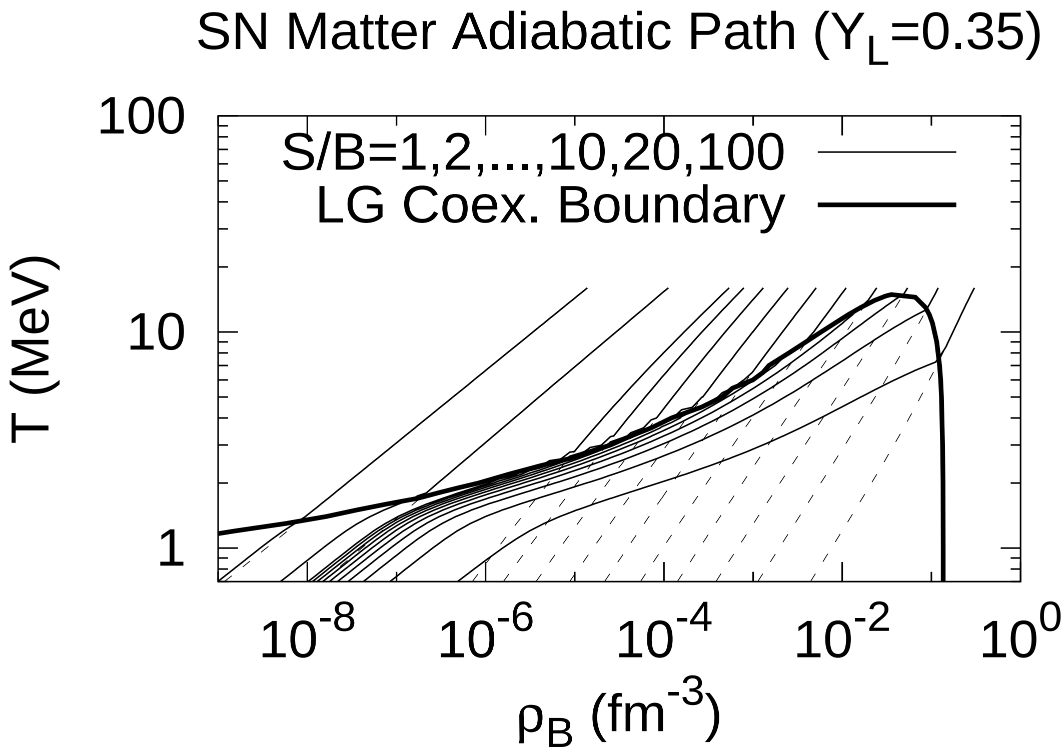
- ⋆ Phase Coexistence
  - ∘ Many Chemical Potentials $(B, Q, L, ...)$
    Maxwell Construction → Gibbs Condition

- Adiabatic Path in SN Matter Evolution
  - ⋆ Important Parameters:
    - ∘ Lepton to Baryon Ratio: $Y_l \equiv N_l/B = (0.3 - 0.4)$ (Takatsuka)
    - ∘ Entropy per Baryon: $S/B \geq 10$ for Ejection
  - ⋆ Constituents
    - ∘ $n, p, e, \nu_e(, \mu, \nu_\mu, ...)$



☞ Does it hit LG Coex. Region ?

Adiabatic Path at Low Densities



Yes, it hits LG Coex. Region even at $S/B > 10$!

- Why, Problems and To Do

⋆ At low T, Entropy is mainly carried by Leptons $(e^{\pm}, \nu, \bar{\nu})$.
↔ $S/B \sim (3-6)$ at 1 A GeV HIC

⋆ With lepton chemical pot., larger proton ratio can be supported than in neutron star matter.
→ Gains Sym. Energy in Liquid phase

⋆ p/n ratios in L and G are different, and phases are assumed to be of infinite size.
→ Coulomb Energy !

☞ Estimate based on Finite Nuclei is necessary.

⋆ Simple Model (II): Statistical Model

Stat. Model in HIC
$\simeq$ Nuclear Stat. Equil. (NSE) in Astrophys.

- Statistical Model of Fragments

Stat. Equil. between Fragments
$\cdots$ Fragment-based Grandcanonical Model

$$\begin{aligned}
\rho_f(A,Z) &= g(T) \int \frac{d^3p}{(2\pi\hbar)^3} \exp(-(E_f-\mu_f)/T) \\
E_f &= \frac{p^2}{2M_f} - B_f(A,Z) + V_c(A,Z) \\
\mu_f &= Z\,\mu_p + N\,\mu_n
\end{aligned}$$

$g(T)$: g.s (+ Disc. Levels) + Bethe formula
V_c : Average Interfrag. Coulomb Pot.

(μ_p,μ_n): Fixed from (ρ_p,ρ_n)

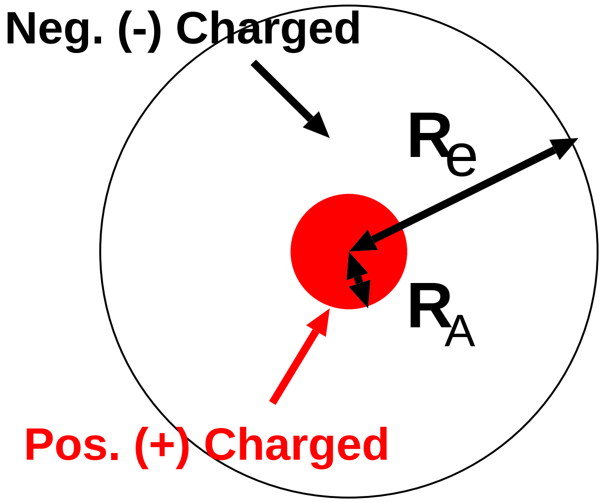
In SN Matter, Extension of Mass Table is needed.

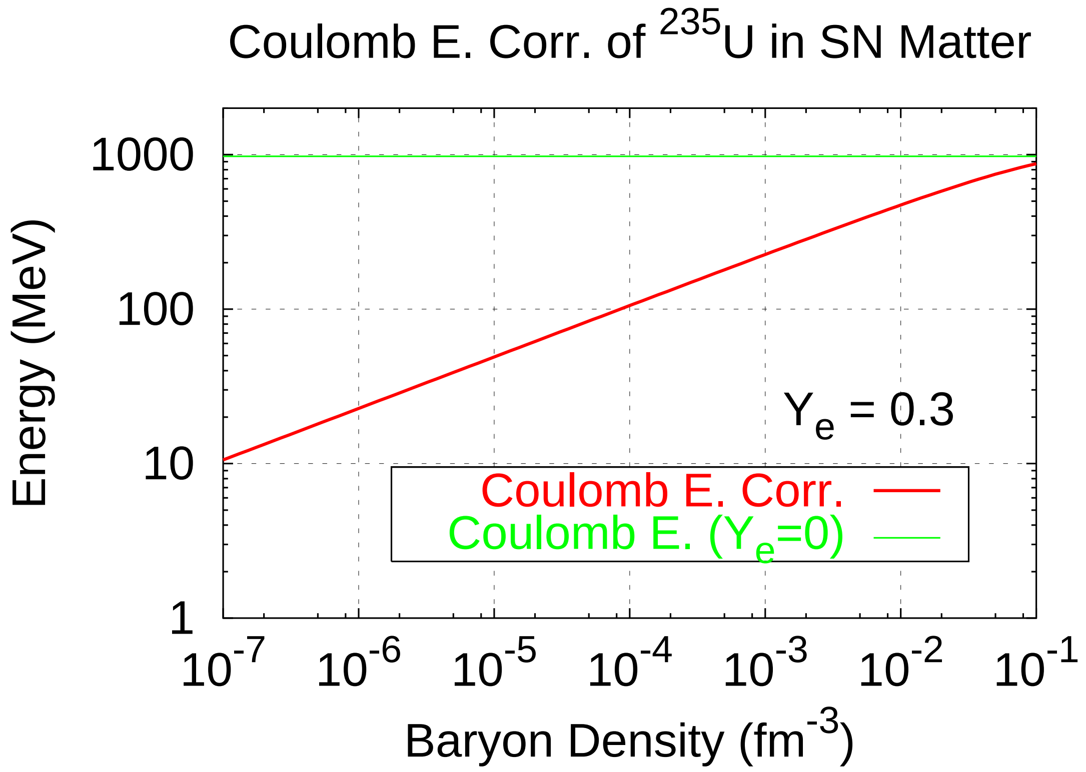
- ⋆ Electron Screening
  → Stabilize Proton Rich nuclei
- ⋆ Large Neutron Chem. Pot.
  → Stabilize Neutron Rich nuclei

- Binding Energy Correction in SN Matter

Coulomb E. Correction from Electron Screening

$$V_c(A,Z) = a_c \frac{Z^2}{A}\left(1 - \frac{3}{2}\eta + \frac{1}{2}\eta^3\right) \quad , \qquad \eta = R_A/R_e$$

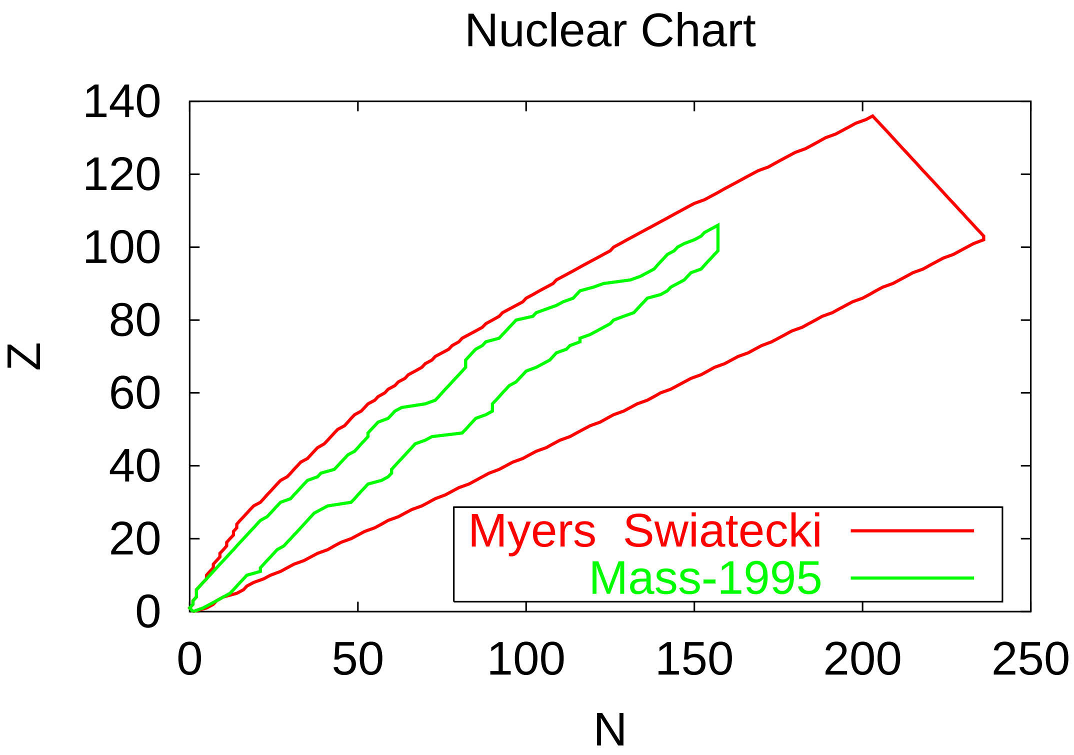




☞ Large Binding Energy Corr. even at low ρ_B

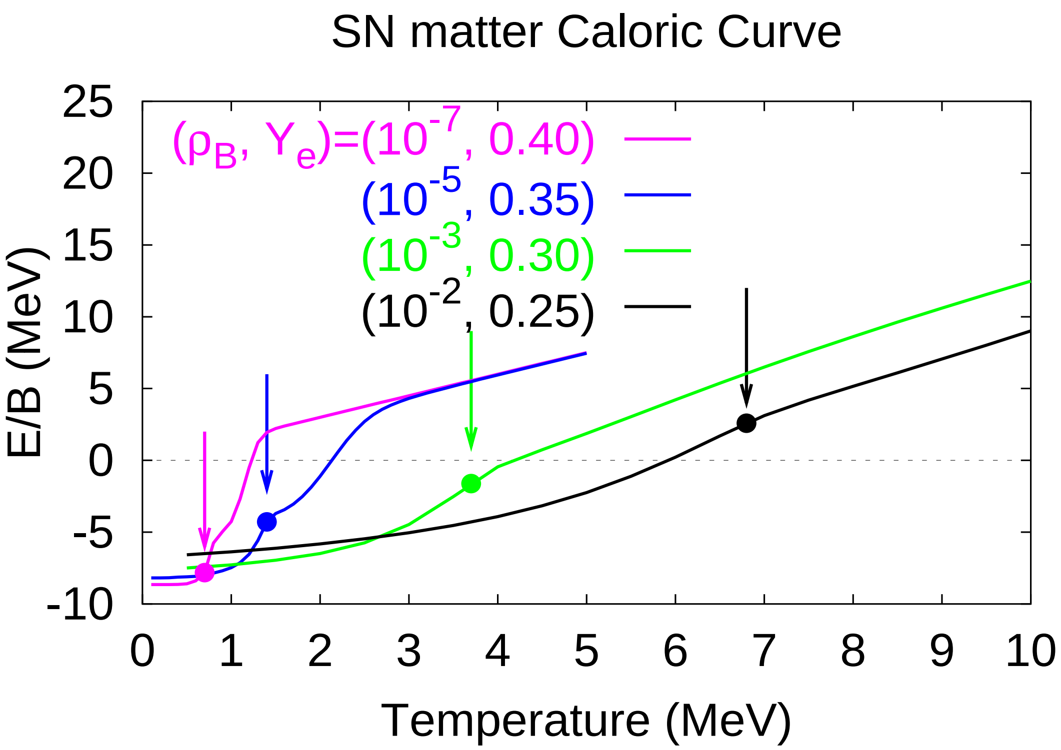
- Mass Table Extension

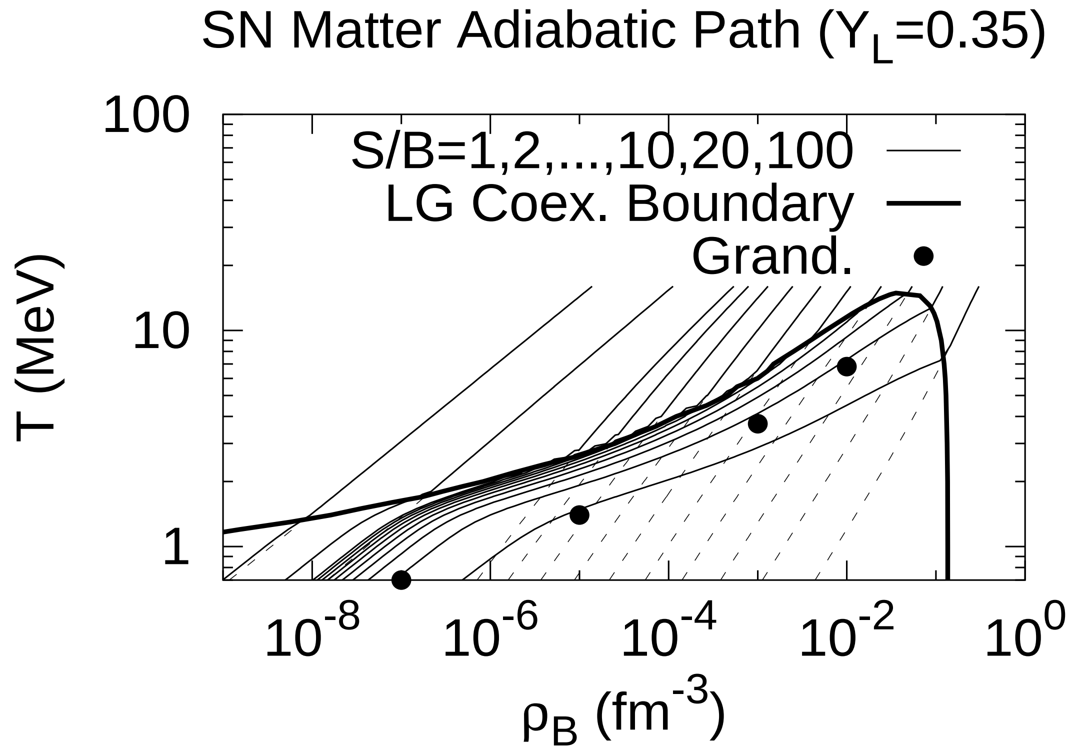
Mass Formula: Myers & Swiatecki 1995
Extended Thomas-Fermi + Shell corr.
→ 9000 nuclei



Let's Go

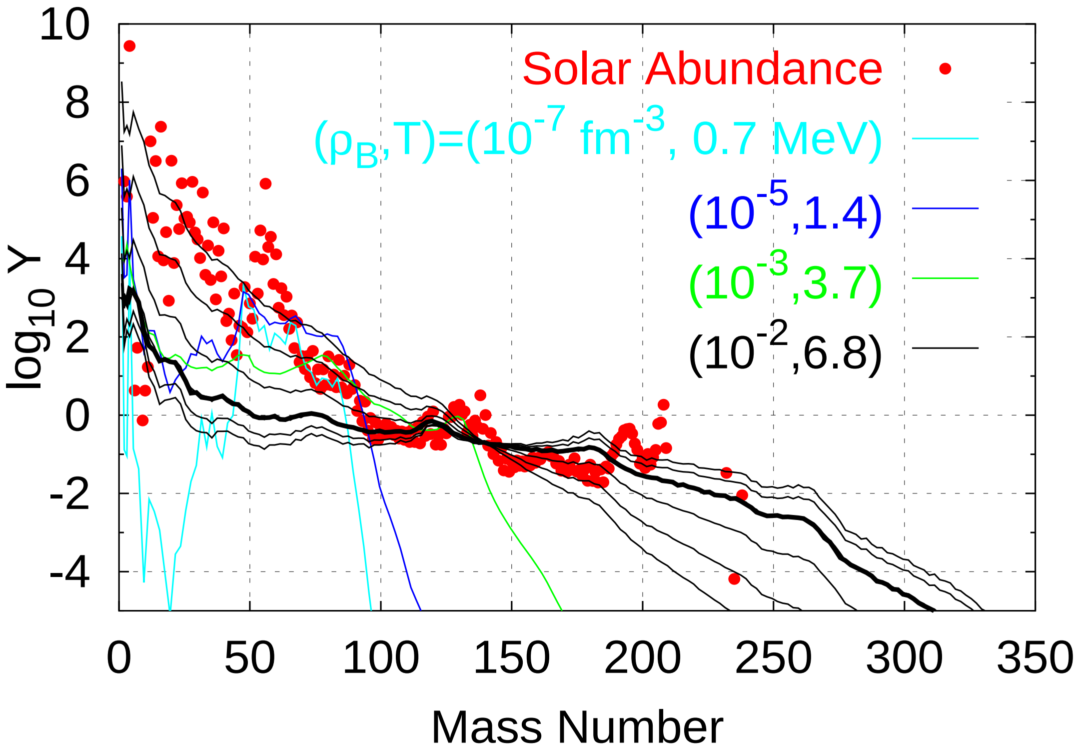
- Caloric Curve in Stat. Model

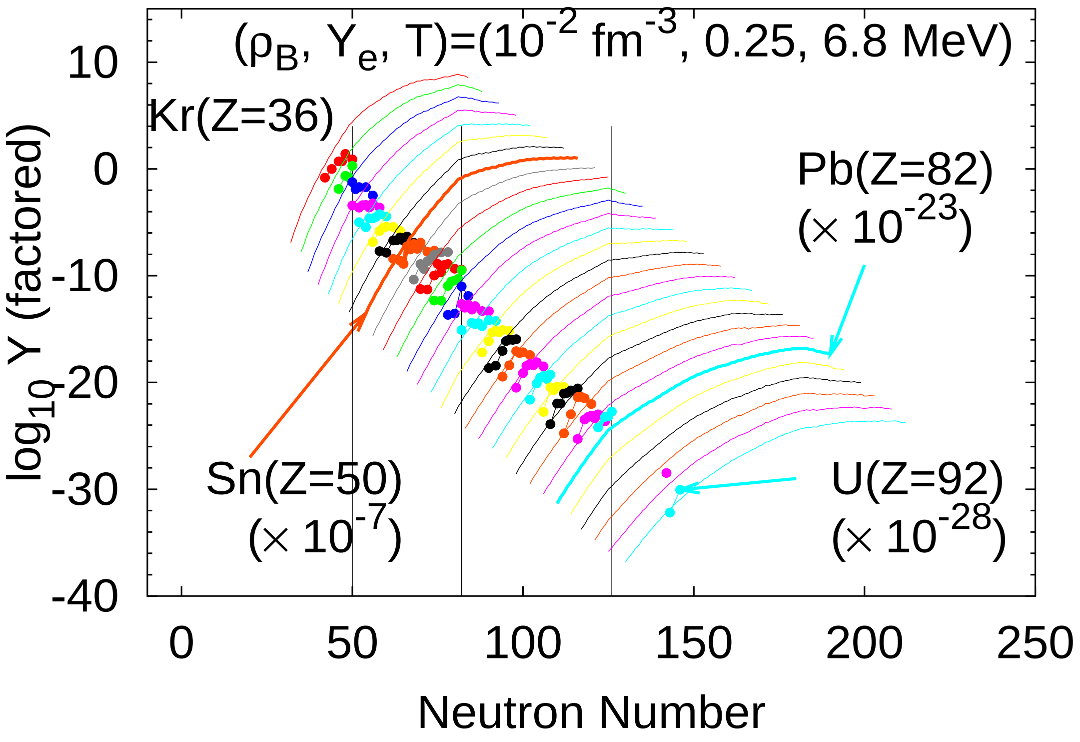




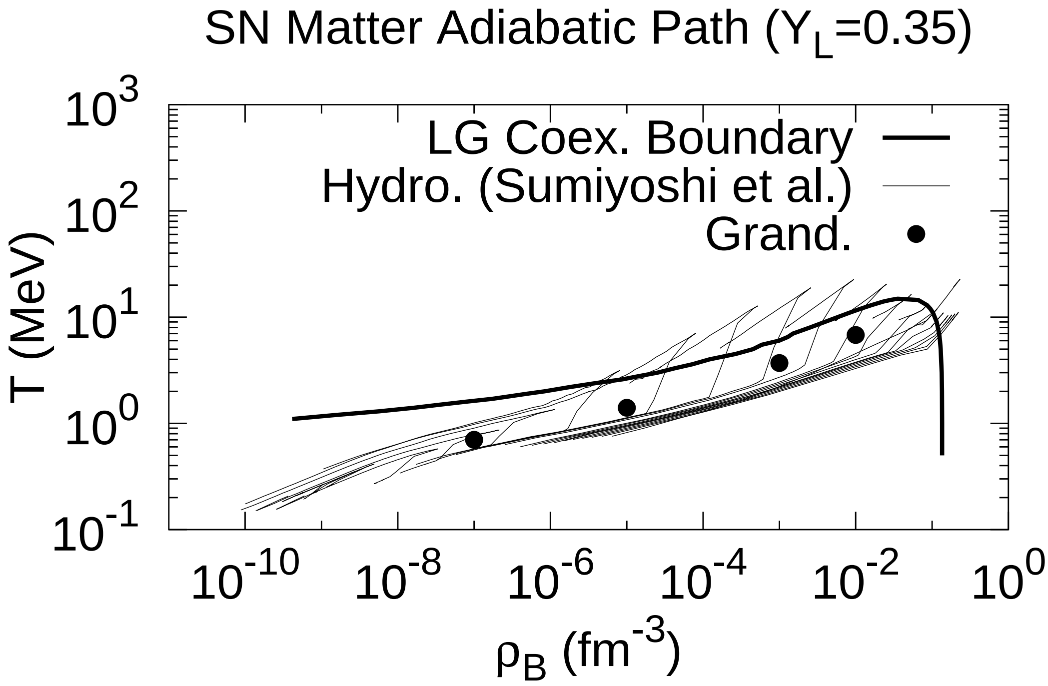
Finite Size: Reduce T_c,
but still in the Ejection Range

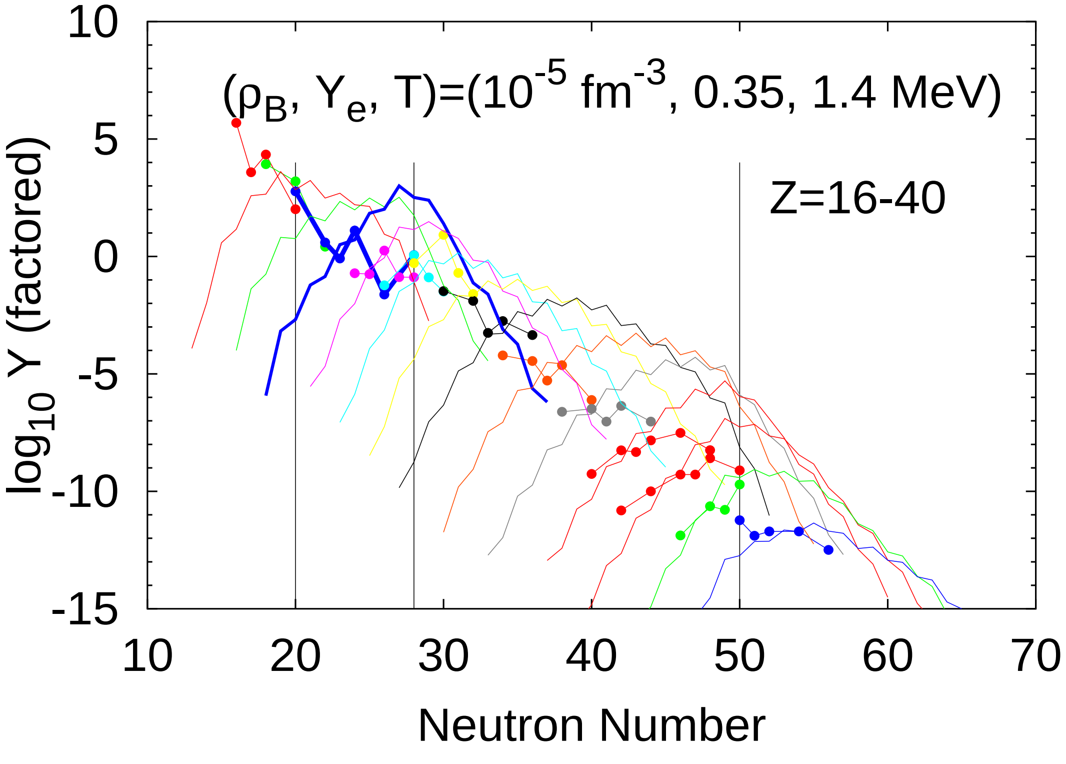
- Mass and Isotope Distribution





☞ Finite Size: Reduce T_c,
but still in the Ejection Range


SN Matter Adiabatic Path (Y_L=0.35)
T (MeV)
ρ_B (fm^{-3})
LG Coex. Boundary
Hydro. (Sumiyoshi et al.)
Grand.
10^3
10^2
10^1
10^0
10^{-1}
10^{-10}
10^{-8}
10^{-6}
10^{-4}
10^{-2}
10^0


(ρ_B, Y_e, T)=(10^{-5} fm^{-3}, 0.35, 1.4 MeV)
Z=16-40
$\log_{10}$ Y (factored)
Neutron Number
10
5
0
-5
-10
-15
10
20
30
40
50
60
70

⋆ Summary and Discussion

• Summary

1. Heavy Elements production
   through Nuclear Liquid-Gas phase transition
   at around the surface of Supernova Core (LG process)
   may be important.
   - ⋆ A Distribution in the Universe
   - ⋆ Phase Diagram of Nuclear Matter
   - ⋆ Heavy Proton Rich Nuclei
2. Simple Model Calculation (I):
   — RMF + Adiabatic path —
   - ⋆ $Y_l, S/B$ are constant along the path
   - ⋆ Critical Temperature of Nuclear LGpt
     $T_c \simeq 16$ MeV
   - ⋆ Ejection Path would experience LG coex. region.
3. Simple Model Calculation (II):
   — Statistical Model —
   - ⋆ Extension of Nuclear Mass Table
   - ⋆ Coulomb Energy Corr. in SN Matter
   - ⋆ Finite Size effects reduces T_c,
     but still in the Ejection Range
   - ⋆ Solar Isotope Dist.: seems "Thermal"
4. Unified treatment of mean field and statistical features
   would be necessary.