高密度物質と中性子星の物理
Physics of Neutron Star Matter

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1. 中性子星の基本的性質
2. 状態方程式を記述する理論模型
3. 対称エネルギーと非対称核物質の状態方程式
4. ハイパー核物理と高密度核物質の状態方程式
5. 中性子星におけるエキゾチック自由度

九州大学集中講義 7/8-10

Why do we study Nuclear Matter EOS ?

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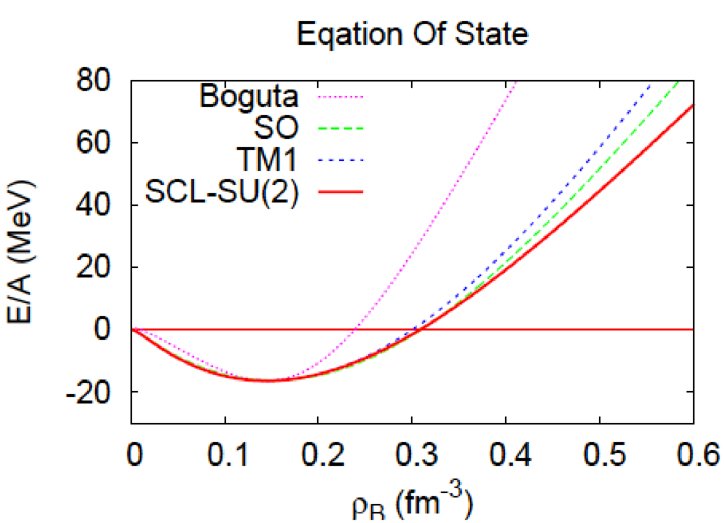
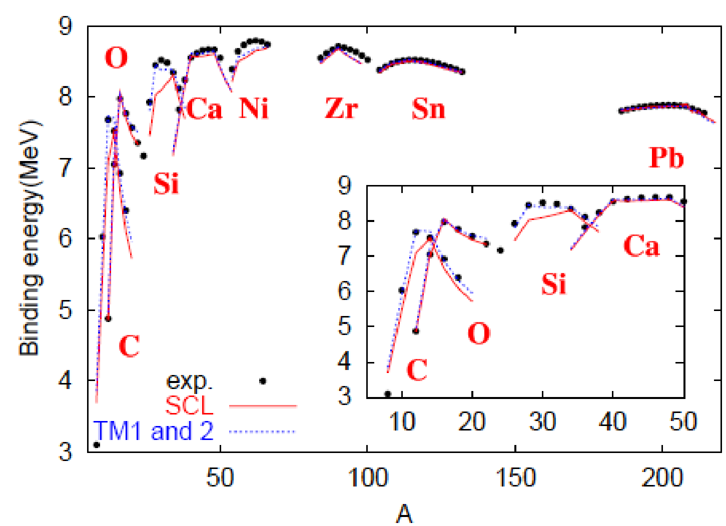
- **Answer 1: Since bulk nuclear properties are mainly determined by nuclear matter EOS, it is important for nuclear physics.**
  - **Nuclear Radius → Saturation of Density**

    $R_A = r_0 A^{1/3}$ (r_0=1.2 fm)
  - **Nuclear Binding Energy (Bethe-Weizsacker Formula)**

$$B(A,Z) = a_{vol} A - a_{surf} A^{2/3} - a_{Coulomb} \frac{Z^2}{A^{1/3}} - a_{sym} \frac{(N-Z)^2}{A} + a_{pair} \delta(A,Z) A^{-3/4}$$

Nuclear Matter



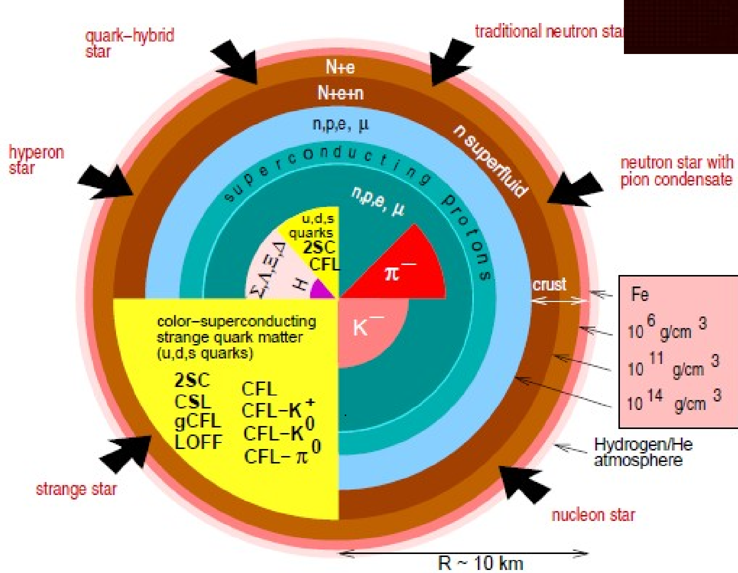


Why do we study Nuclear Matter EOS ?

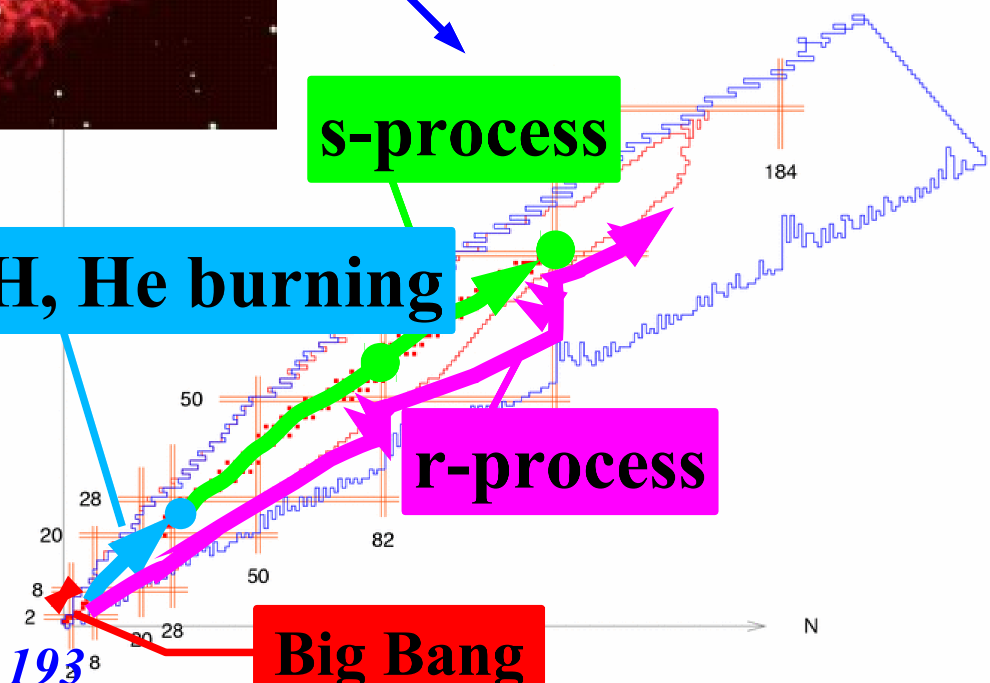
- **Answer 2: Since nuclear matter EOS is decisive in compact astrophysical objects such as neutron stars, supernovae, and black hole formation, EOS is important to understand where atomic elements are m**

Supernova

Neutron Star



Nucleosynthesis



F. Weber, Prog. Part. Nucl. Phys. 54 (2005) 193

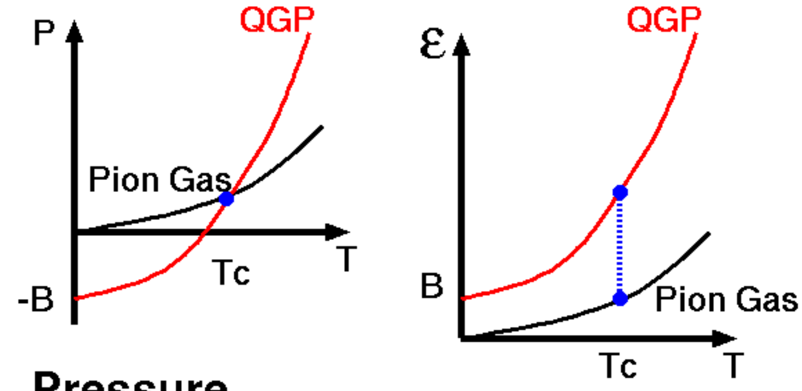
Why do we study Nuclear Matter EOS ?

- **Answer 3: Since the EOS should have singularity (or at least sudden change) at phase boundary, it would be possible to catch the signal of phase transition in nuclear collisions.**

- **Pressure and Energy Density of Free Massless Gas**

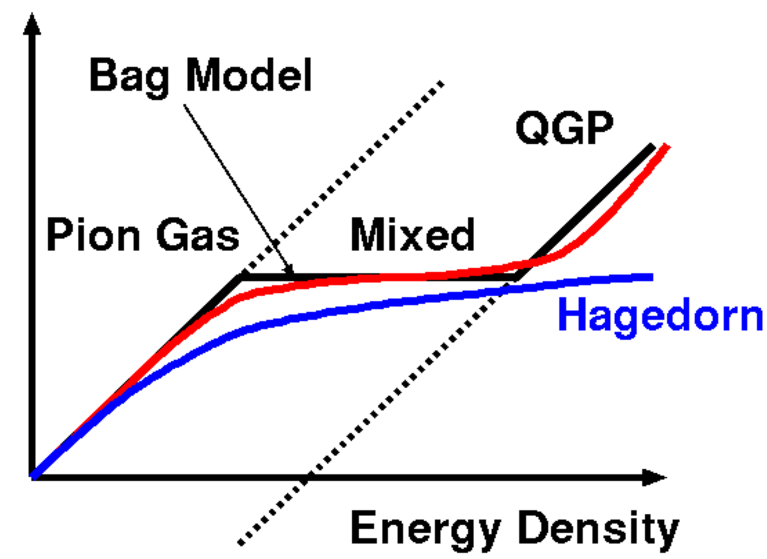
$$P=\frac{\pi^2}{90}N_B T^4 \quad , \quad \epsilon=\frac{\pi^2}{30}N_B T^4$$

N_B = Bosonic DOF (7/8 for Fermions)



- **Hadron Gas ~ 3 pions (N_B=3)**

$$P_\pi=\frac{\pi^2}{30}T^4 \quad , \quad \epsilon_\pi=\frac{\pi^2}{10}T^4$$



- **QGP N_B=16(gluon)+24 x 7/8 (quarks) and Bag Pressure**

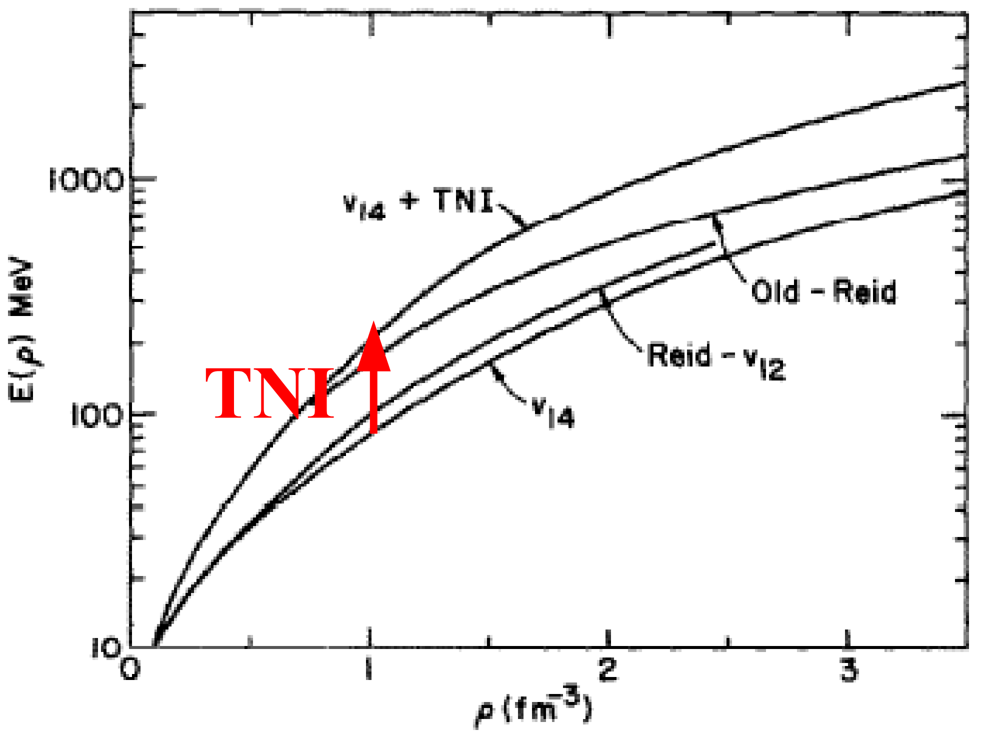
$$P_{QGP}=\frac{37\pi^2}{90}T^4-B \quad \epsilon_{QGP}=\frac{37\pi^2}{30}T^4+B$$

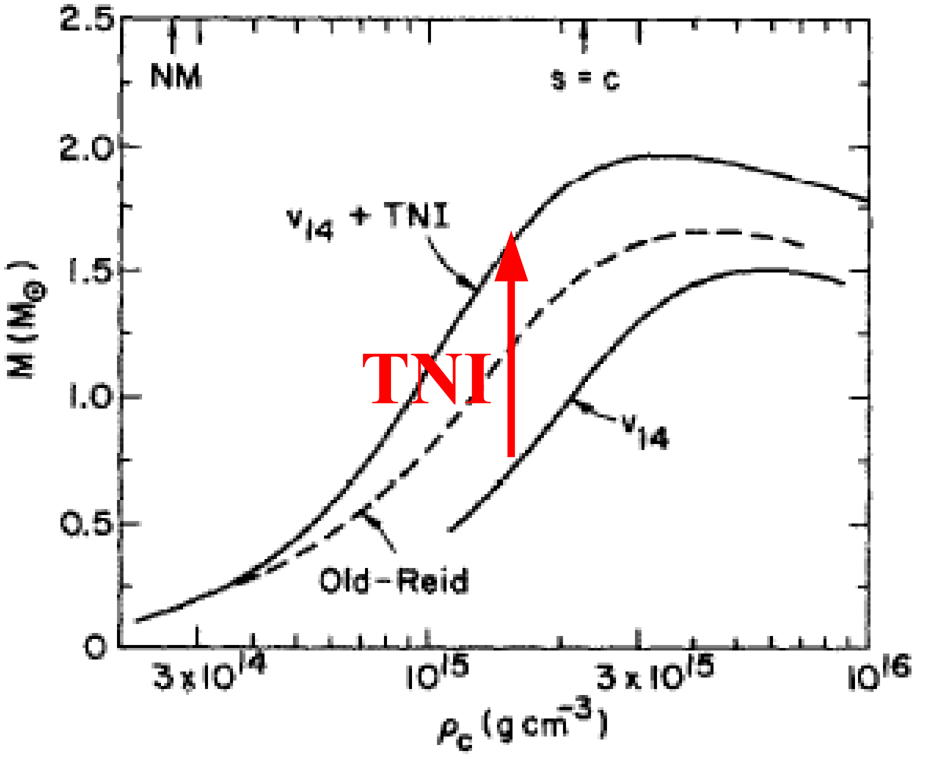
Theories/Models for Nuclear Matter EOS

- **Ab initio Approaches**
  - **LQCD, GFMC, Variational, BHF, DBHF, G-matrix**
    → Not easy to handle, Not satisfactory for phen. purposes
- **Mean Field from Effective Interactions ~ Nuclear Density Fuctionals**
  - **Skyrme Hartree-Fock(-Bogoliubov)**
    - **Non.-Rel.,Zero Range, Two-body + Three-body (or ρ-dep. two-body)**
    - **In HFB, Nuclear Mass is very well explained (Total B.E. $\Delta E \sim 0.6$ MeV)**
    - **Causality is violated at very high densities.**
  - **Relativistic Mean Field**
    - **Relativistic, Meson-Baryon coupling, Meson self-energies**
    - **Successful in describing pA scatering (Dirac Phenomenology)**

Variational Calculations (1)

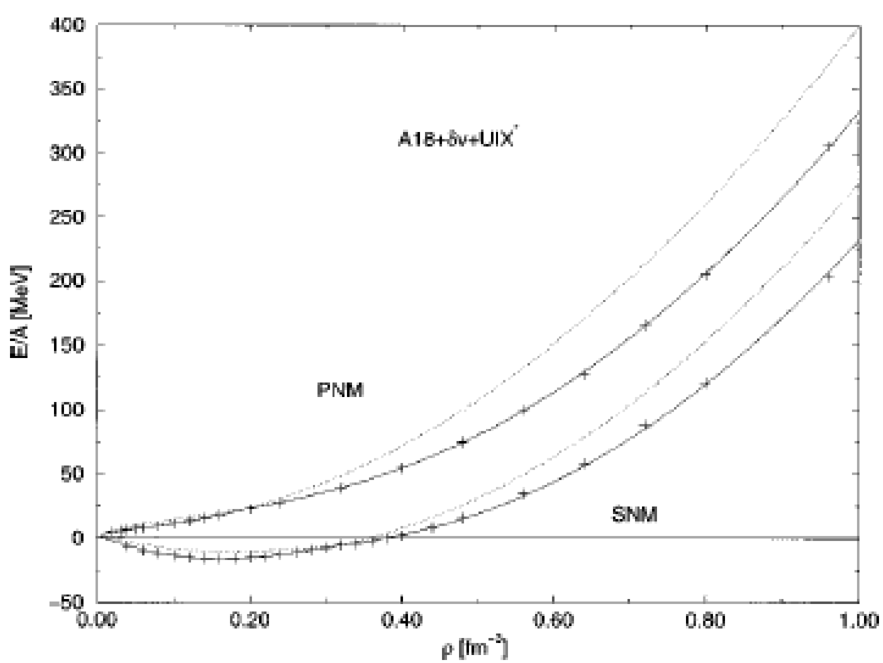
- **Variational Calculation starting from bare nuclear force**
  B. Friedman, V.R. Pandharipande, NPA361('81)502
  - **Argonne v14 + TNI (TNR+TNA)**
    (TNI/TNR/TNA: three-nucleon int./repulsion/attraction)

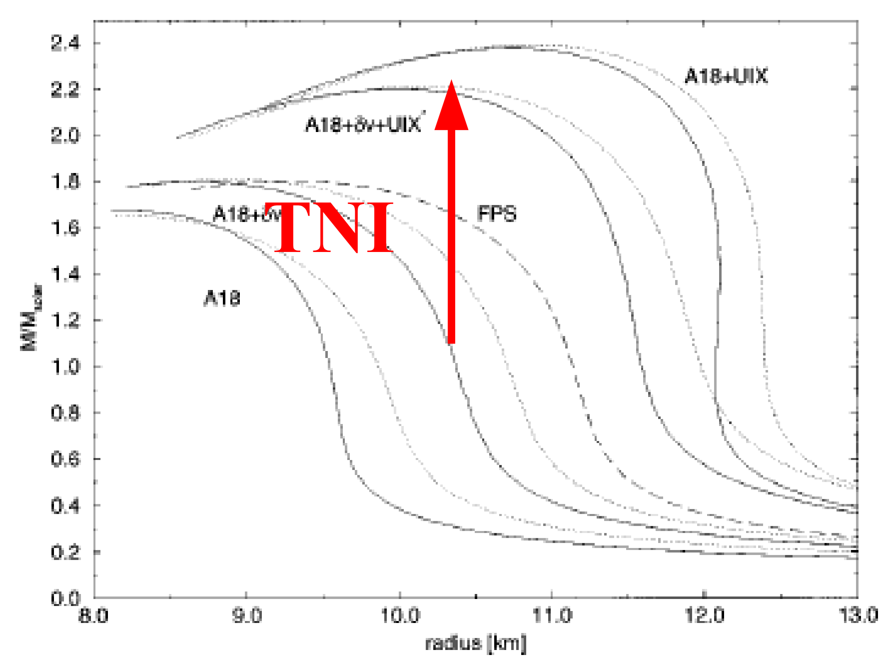




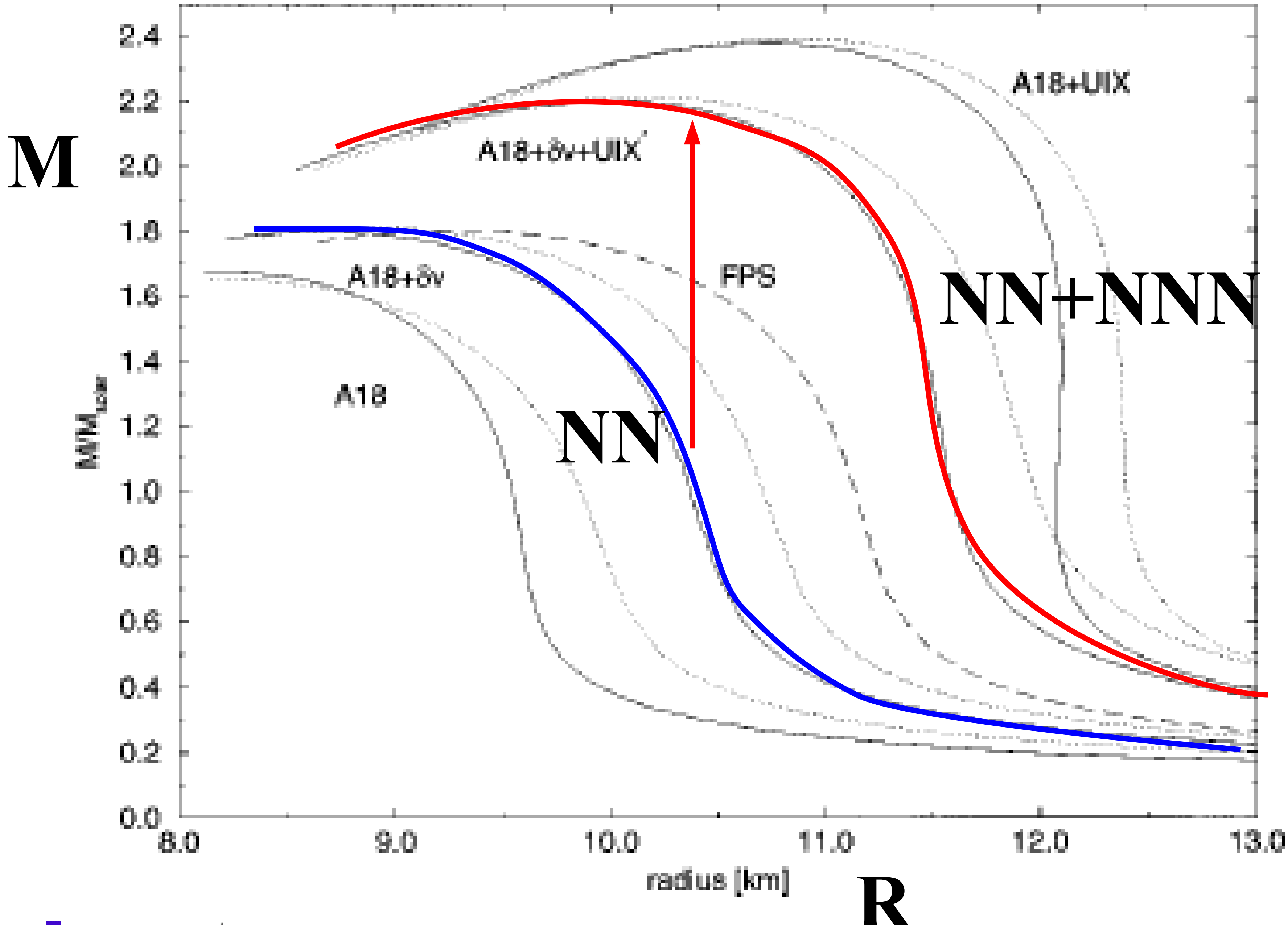
Variational Calculation (2)

- **Variational chain summation method**
  A. Akmal, V.R.Pandharipande, D.G. Ravenhall, PRC58('98)1804
  - **v18, relativistic correction, TNI**
  - **Existence of neutral pion condensation at $\rho_B > 0.2\ \mathrm{fm}^{-3}$**



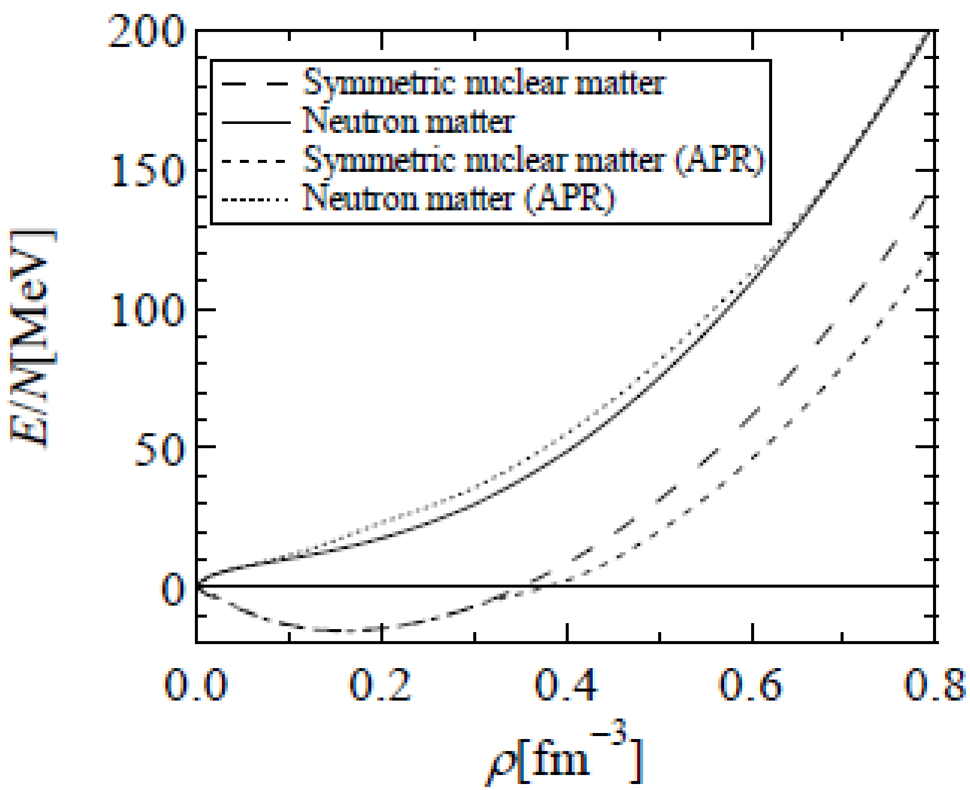


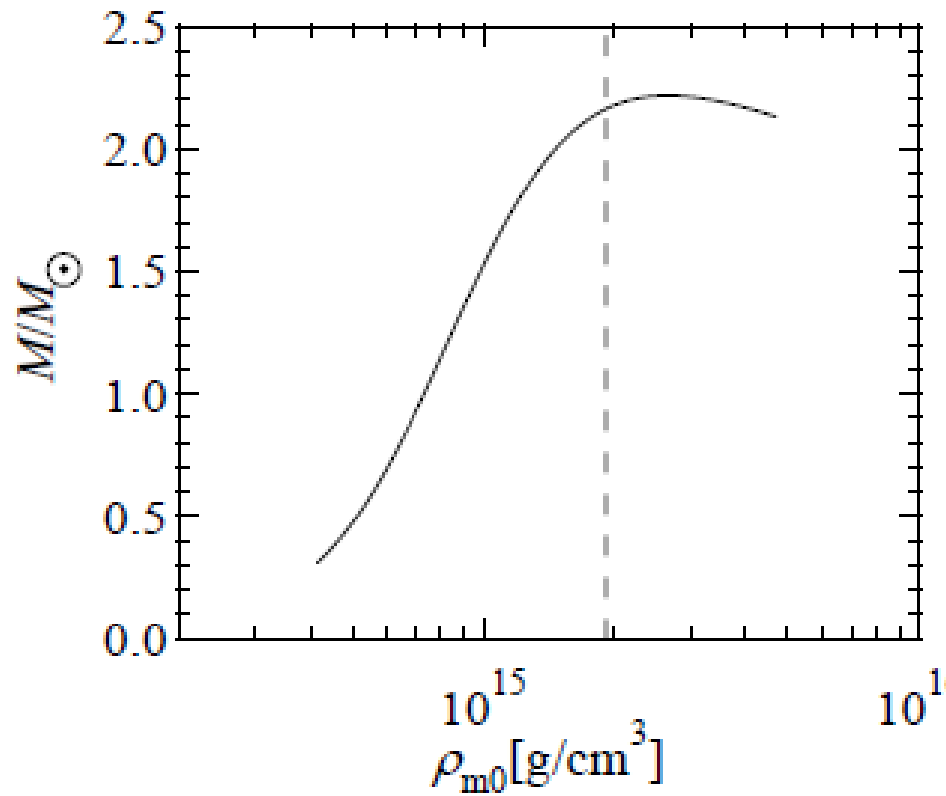
APR


M
R
M/M_solar
radius [km]
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
8.0
9.0
10.0
11.0
12.0
13.0
A18+UIX
A18+δv+UIX*
A18+δv
FPS
A18
NN
NN+NNN

Variational Calculation (3)

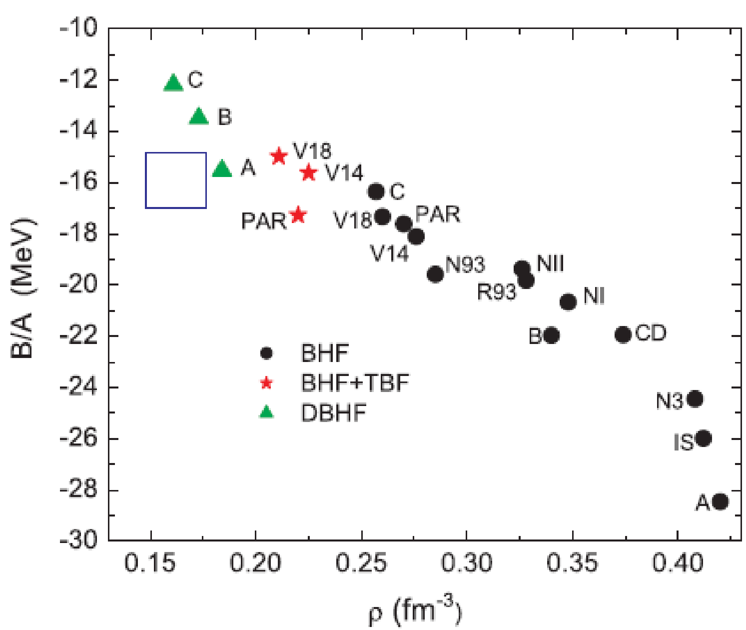
- **Variational Calculation using v18+UIX**
  H. Kanzawa, K. Oyamatsu, K. Sumiyoshi, M. Takano, NPA791 ('07) 232
  - **Similar to APR, but healing-distance condition is required.**
    → no π^0 condensation

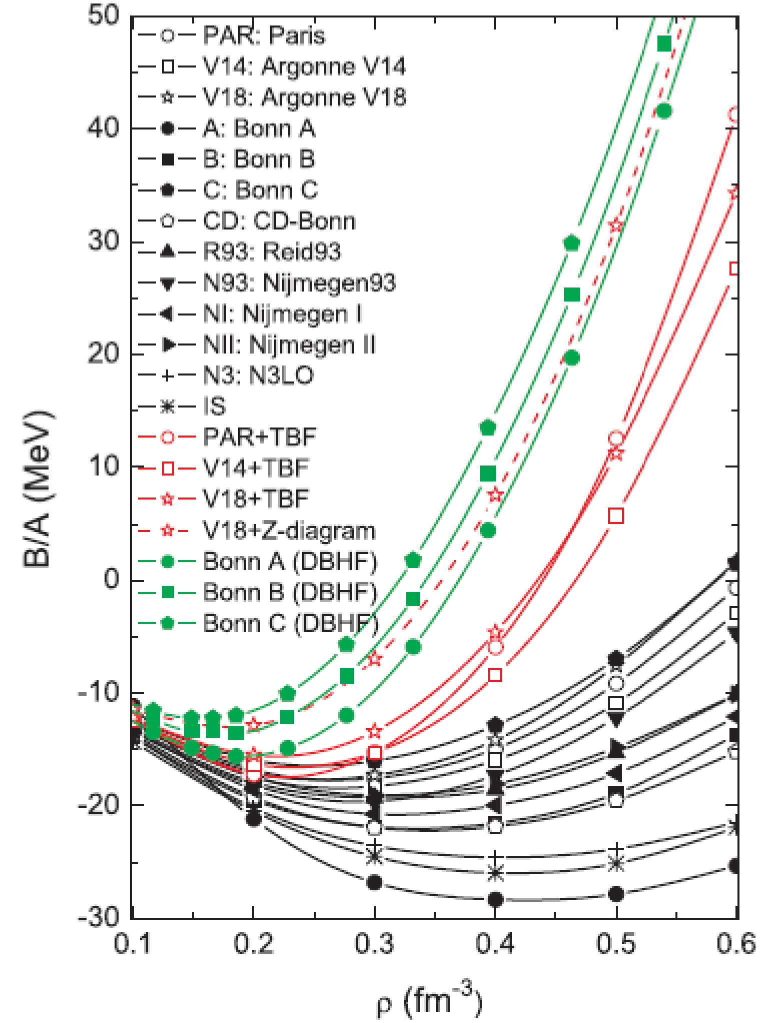




Bruckner-Hartree-Fock

- **Self-consistent treatment of**
  Effective interaction (G-matrix) in the Bruckner Theory
  and Single particle energy from G-matrix
- **Need 3-body force to reproduce saturation point.**
  → FY type 2 π exchange
  + phen. or Z-diagram

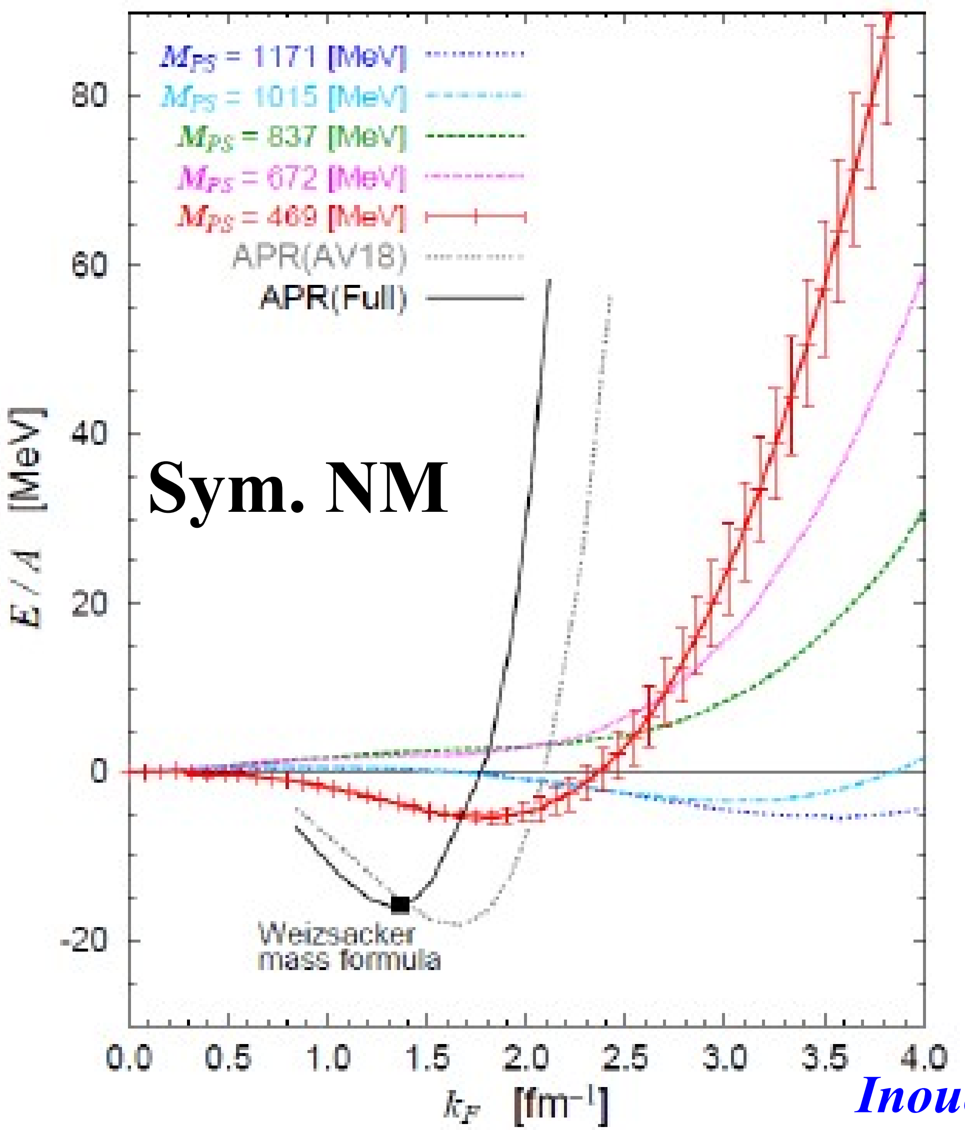


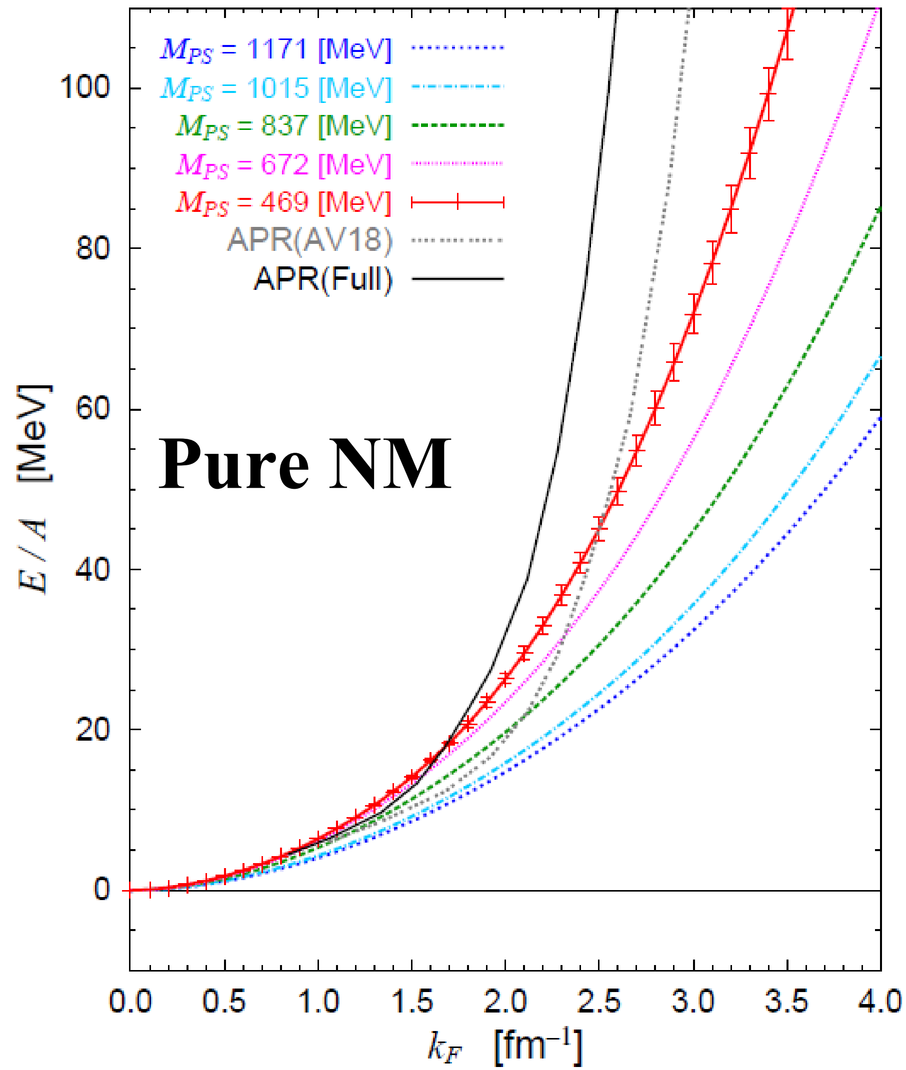


Z.H.Li, U. Lombardo, H.-J. Schulze, W. Zuo, L. W. Chen, H. R. Ma, PRC74('06)047304.

EOS from lattice NN force

- **格子 QCD 核力を用いた高密度状態方程式 (LQCD+BHF)**
  NN force: $^1S_0$, $^3S_1$, $^3D_1$ only





Inoue et al. (HAL QCD Coll.), PRL111 ('13)112503

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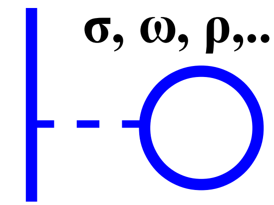
Relativistic Mean Field

Relativistic Mean Field (1)

- **Effective Lagrangian of Baryons and Mesons + Mean Field App.**

  B.D.Serot, J.D.Walecka, Adv.Nucl.Phys.16 ('86), 1



$$L = L_B^{\rm free} + L_M^{\rm free} + L_{BM} + L_M^{\rm Int}$$

$$L_M^{\rm Int} = -U_\sigma(\sigma) + \frac{1}{4} c_\omega (\omega_\mu \omega^\mu)^2 + \cdots$$

$$L_{BM} = -\sum_{B,S} g_{BS} \bar{\Psi}_B \varphi_S \Psi_B - \sum_{B,V} g_{BV} \bar{\Psi}_B \gamma^\mu V_\mu \Psi_B$$

$$L_B^{\rm free} = \bar{\Psi}_B (i\gamma^\mu \partial_\mu - M_B) \Psi_B \ , \quad L_M^{\rm free} = \sum_S \left[\frac{1}{2} \partial^\mu \varphi_S \partial_\mu \varphi_S - \frac{1}{2} m_S^2 \varphi_S^2\right] + \sum_V \left[-\frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \frac{1}{2} V_\mu V^\mu\right]$$

- **Baryons and Mesons: B=N, Λ, Σ, Ξ, ..., S= σ, ς, ..., V= ω, ρ, φ, …**
- **Based on Dirac phenomenology & Dirac Bruckner-Hatree-Fock**

  E.D. Cooper, S. Hama, B.C. Clark, R.L. Mercer, PRC47('93),297
  R. Brockmann, R. Machleidt, PRC42('90),1965

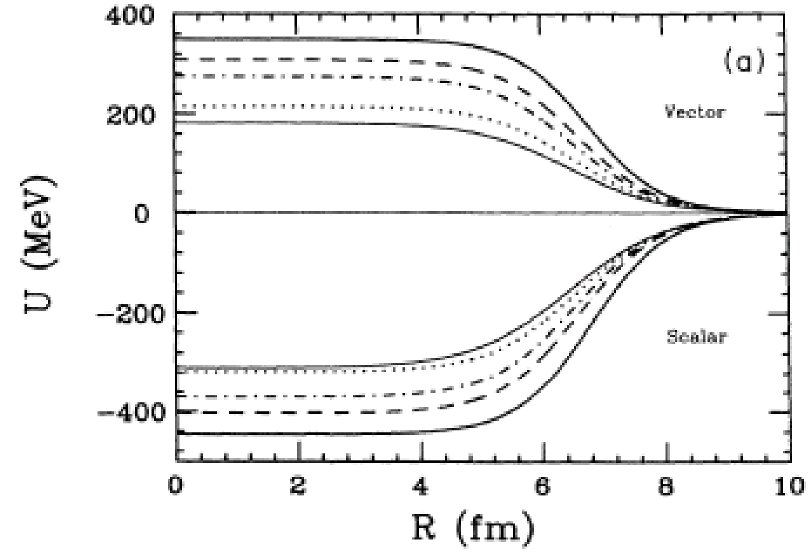
- **Large scalar (att.) and vector (repl.) → Large spin-orbit pot.**
  Relativistic Kinematics → Effective 3-body repulsion
- **Non-linear terms of mesons → Bare 3-body and 4-body force**

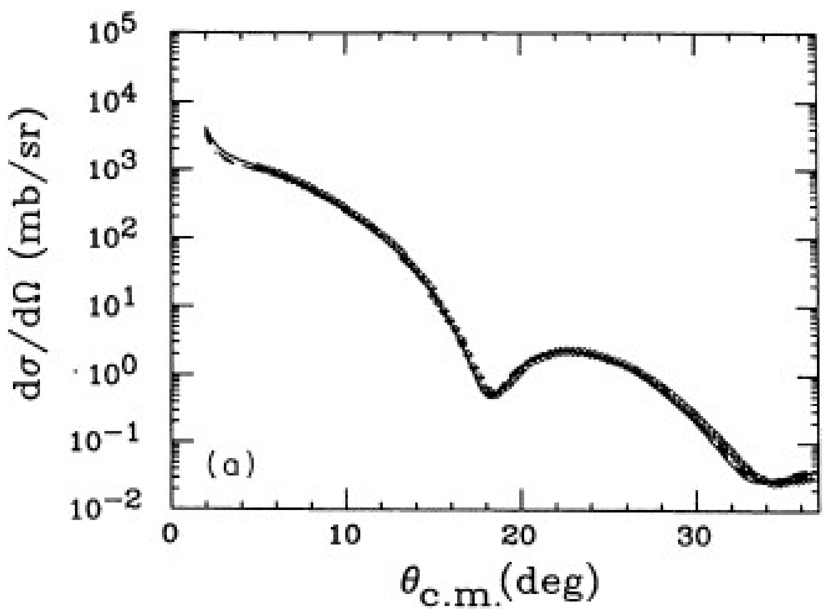
  Boguta, Bodmer ('77), NL1:Reinhardt, Rufa, Maruhn, Greiner, Friedrich ('86), NL3: Lalazissis, Konig, Ring ('97),TM1 and TM2: Sugahara, Toki ('94), Brockmann, Toki ('92)

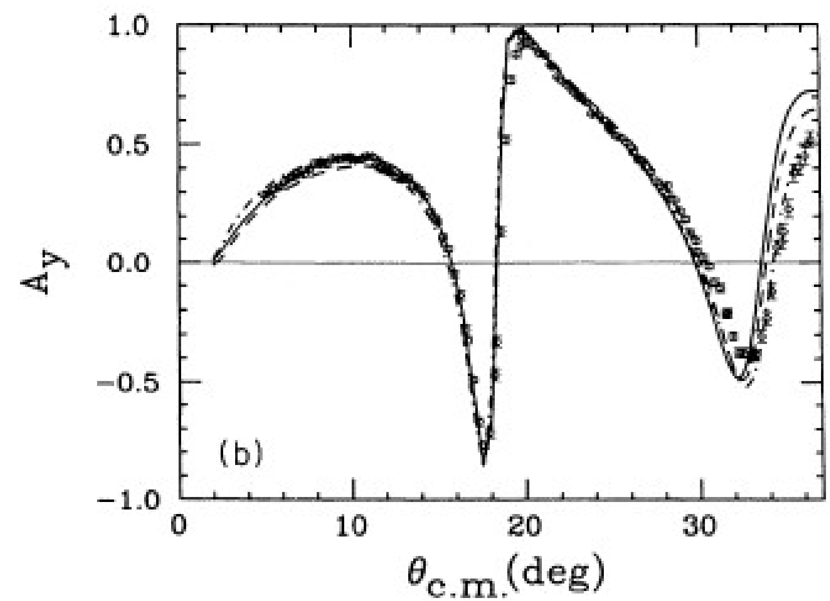
Dirac Phenomenology

E.D. Cooper, S. Hama, B.C. Clark, R.L. Mercer, PRC47('93),297

- **Dirac Eq. with Scalar + Vector pA potential (-400 MeV + 350 MeV) → Cross Section, Spin Observables**



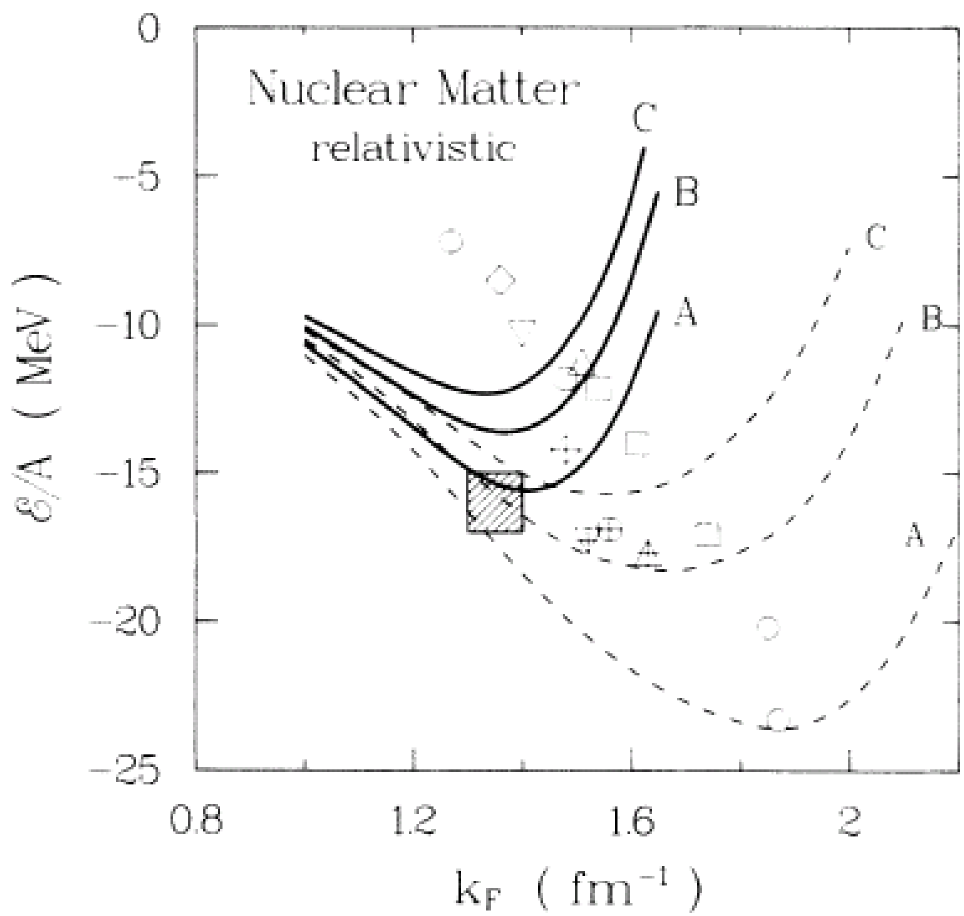




EOS in Dirac-Brueckner-Hartree-Fock

R. Brockmann, R. Machleidt, PRC42('90),1965

- **Non Relativistic Brueckner Calculation**
  → Nuclear Saturation Point cannot be reproduced (Coester Line)
- **Relativistic Approach (DBHF)**
  → Relativity gives additional repulsion, leading to successful description of the saturation point.



Relativistic Mean Field (2)

- **Mean Field treatment of meson field operator**
  = Meson ield operator is replaced with its expectation value
  $$\varphi(r) \rightarrow \langle\varphi(r)\rangle$$
  Ignoring fluctuations compared with the expectation value may be a good approximation at strong condensate.

- **Which Hadrons should be included in RMF ?**
  - **Baryons (1/2+)** p, n, Λ, Σ, Ξ, Δ, ...
  - **Scalar Mesons (0+)** σ(600), f_0(980), a_0(980), ...
  - **Vector Mesons (1-)** ω(783), ρ(770), ϕ(1020), ...
  - **Pseuso Scalar (0-)** π , K, η, η', ...
  - **Axial Vector (1+)** a_1, ...

  We require that the meson field can have uniform expectation values in nuclear matter.
  → Scalar and Time-Component of Vector Mesons (σ , ω, ρ, ...)

σω Model (1)

Serot, Walecka, Adv.Nucl.Phys.16 (1986),1

- **Consider only σ and ω mesons**
- **Lagrangian**

$$L=\bar{\psi}\left(i\gamma^{\mu}\partial_{\mu}-M+g_s\sigma-g_v\omega\right)\psi$$
$$+\frac{1}{2}\partial_{\mu}\sigma\,\partial^{\mu}\sigma-\frac{1}{2}m_s^2\sigma^2-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}m_v^2\omega_{\mu}\omega^{\mu}$$
$$\left(F_{\mu\nu}=\partial_{\mu}\omega_{\nu}-\partial_{\nu}\omega_{\mu}\right)$$

- **Equation of Motion**
  - **Euler-Lagrange Equation**

$$\frac{\partial}{\partial x^{\mu}}\left[\frac{\partial L}{\partial(\partial_{\mu}\phi_i)}\right]-\frac{\partial L}{\partial\phi_i}=0$$

$$\sigma:\left[\partial_{\mu}\partial^{\mu}+m_s^2\right]\sigma=g_s\bar{\psi}\psi$$

$$\omega:\partial_{\mu}F^{\mu\nu}+m_v^2\omega^{\nu}=g_v\bar{\psi}\gamma^{\nu}\psi\quad\rightarrow\quad\left[\partial_{\mu}\partial^{\mu}+m_v^2\right]\omega^{\nu}=g_v\bar{\psi}\gamma^{\nu}\psi$$

$$\psi:\left[\gamma^{\mu}\left(i\partial_{\mu}-g_vV_{\mu}\right)-\left(M-g_s\sigma\right)\right]\psi=0$$

EOM of ω (for beginners)

- **Euler-Lagrange Eq.**

$$\partial_\mu F^{\mu\nu} + m_v^2 \omega^\nu = g_v \bar{\psi} \gamma^\nu \psi$$

- **Divergence of LHS and RHS**

$$\partial_\nu \partial_\mu F^{\mu\nu} + m_v^2 (\partial_\nu \omega^\nu) = m_v^2 (\partial_\nu \omega^\nu) = g_v (\partial_\nu \bar{\psi} \gamma^\nu \psi) = 0$$

LHS: derivatives are sym. and $F_{\mu\nu}$ is anti-sym.

RHS: Baryon Current = Conserved Current

- **Put it in the Euler-Lagrange Eq.**

$$\partial_\mu F^{\mu\nu} = \partial_\mu (\partial^\mu \omega^\nu - \partial^\nu \omega^\mu) = \partial_\mu \partial^\mu \omega^\nu - \partial^\nu (\partial_\mu \omega^\mu) = \partial_\mu \partial^\mu \omega^\nu$$

Schroedinger Eq. for Upper Component (1)

- **Dirac Equation for Nucleons**

$$\left(i\gamma\partial-\gamma^0 U_v - M - U_s\right)\psi=0 \quad ,$$
$$U_v = g_\omega \omega \quad , \quad U_s = -g_\sigma \sigma$$

- **Decompose 4 spinor into Upper and Lower Components**

$$\begin{pmatrix} E-U_v-M-U_s & i\sigma\cdot\nabla \\ -i\sigma\cdot\nabla & -E+U_v-M-U_s \end{pmatrix}\begin{pmatrix} f \\ g \end{pmatrix}=0$$

$$g=\frac{-i}{E+M+U_s-U_V}(\sigma\cdot\nabla)f$$
$$\left(E-M-U_v-U_s\right)f=-i(\sigma\cdot\nabla)g$$

Schroedinger Eq. for Upper Component (2)

- **Erase Lower Component (assuming spherical sym.)**

$$-i(\sigma\cdot\nabla)g = -(\sigma\cdot\nabla)\frac{1}{X}(\sigma\cdot\nabla)f = -\frac{1}{X}\nabla^2 f - \frac{1}{r}\left[\frac{d}{dr}\frac{1}{X}\right](\sigma\cdot r)(\sigma\cdot\nabla)f$$

$$= -\nabla\frac{1}{X}\nabla f + \frac{1}{r}\left[\frac{d}{dr}\frac{1}{X}\right](\sigma\cdot l)f$$

$$(\sigma\cdot r)(\sigma\cdot\nabla) = (r\cdot\nabla) + i\sigma\cdot(r\times\nabla) = r\cdot\nabla - \sigma\cdot l$$

- **"Schroedinger-like" Eq. for Upper Component**

$$-\nabla\frac{1}{E+M+U_s-U_v}\nabla f + \left(U_s + U_v + U_{LS}(\sigma\cdot l)\right)f = (E-M)f$$

$$U_{LS} = \frac{1}{r}\left[\frac{d}{dr}\frac{1}{E+M+U_s-U_v}\right] < 0 \quad \text{on surface}$$

$(U_s, U_v) \sim$ (-350 MeV, 280 MeV)

→Small Central(U_s+U_v), Large LS (U_s-U_v)

Various Ways to Evaluate Non.-Rel. Potential

- **From Single Particle Energy**

$$\left(\gamma^0(E-U_v)+i\gamma\cdot\nabla-(M+U_s)\right)\psi=0 \ \rightarrow \ (E-U_v)^2=p^2+(M+U_s)^2$$

$$\rightarrow E=\sqrt{p^2+(M+U_s)^2}+U_v \approx E_p+\frac{M}{E_p}U_s+U_v+\frac{p^2}{2E_p^3}U_s^2$$

$$(E_p=\sqrt{p^2+M^2})$$

- **Schroedinger Equivalent Potential (Uniform matter)**

$$-\frac{\nabla^2}{2\mathrm{M}}f+\left[U_s+\frac{E}{M}U_v+\frac{U_s^2-U_v^2}{2\mathrm{M}}\right]f=\frac{E+M}{2\mathrm{M}}(E-M)f$$

$$U_{\mathrm{SEP}}\approx U_s+\frac{E}{M}U_v$$

Anyway, slow baryons feel Non.-Rel. Potential,

$$U\approx U_s+U_v=-g_s\sigma+g_v\omega$$

Nuclear Matter in σω Model

Serot, Walecka, Adv.Nucl.Phys.16 (1986),1

- **Uniform Nuclear Matter**

$$E/V = \gamma_N \int^{P_F} \frac{d^3p}{(2\pi)^2} E^* + \frac{1}{2} m_s^2 \sigma^2 - \frac{1}{2} m_v^2 \omega^2 + g_v \rho_B \omega$$

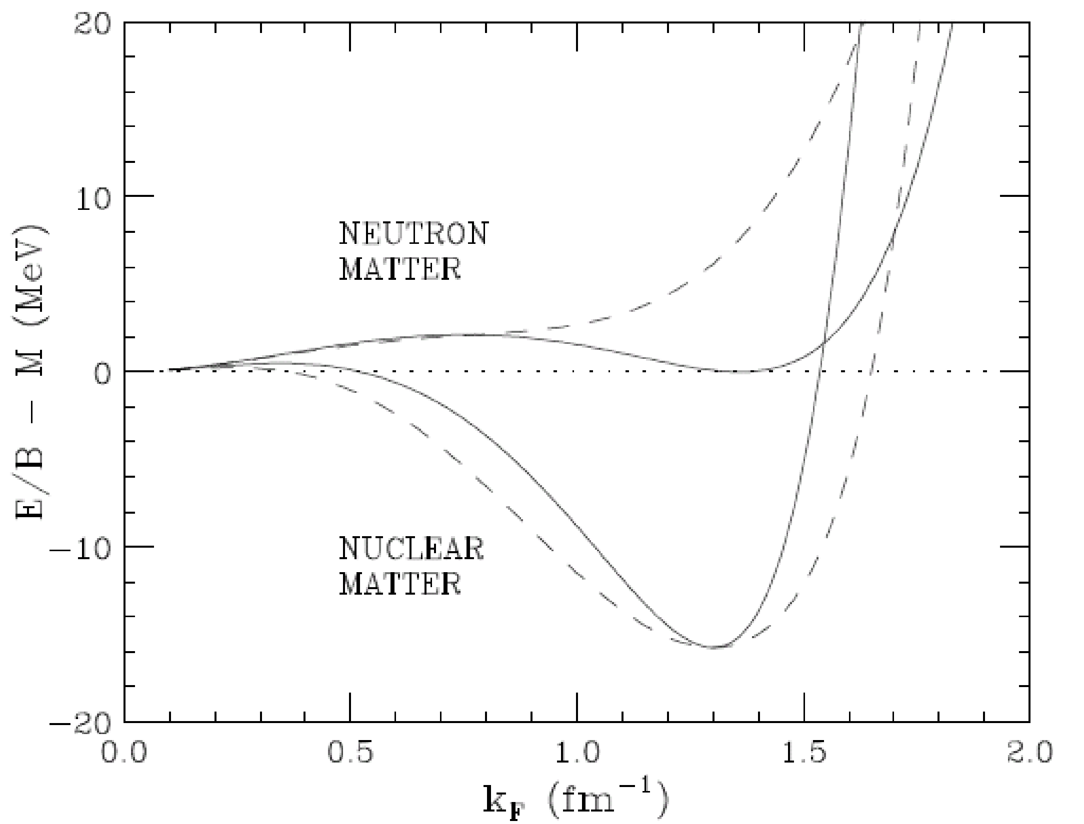
$$\left(M^* = M + U_s = M - g_s \sigma\,,\quad E^* = \sqrt{p^2 + M^{*2}}\right)$$

$$\sigma = \frac{g_s}{m_s^2} \rho_s = \frac{g_s}{m_s^2} \int^{P_F} \frac{\gamma_N\, d^3p}{(2\pi)^2} \frac{M^*}{E^*}$$

$$\omega = \frac{g_v}{m_v^2} \rho_B = \gamma_N \frac{g_v}{m_v^2} \int^{P_F} \frac{d^3p}{(2\pi)^3}$$

γ_N = Nucleon degeneracy
(=4 in sym. nuclear matte

Problem: EOS is too stiff
K ~ (500-600) MeV !
→ How can we avoid it ?



RMF with Non-Linear Meson Int. Terms

Boguta, Bodmer ('77), NL1:Reinhardt, Rufa, Maruhn, Greiner, Friedrich ('86), NL3: Lalazissis, Konig, Ring ('97),TM1 and TM2: Sugahara, Toki ('94), Brockmann, Toki ('92)

- **Too stiff EOS in the simplest RMF (σω model) is improved by introducing non-linear terms (σ^4, ω^4)**
  - **Fit B.E. of Stable as well as Unstable (n-rich) Nuclei**
  - **Three Mesons (σ,ω,ρ) are included**
  - **Meson Self-Energy Term (σ,ω)**

$$
\begin{aligned}
\mathcal{L} =& \bar{\psi}_N \left(i\partial\!\!\!/ - M - g_\sigma \sigma - g_\omega \omega\!\!\!/ - g_\rho \tau^a \rho\!\!\!/^a \right) \psi_N \\
&+ \frac{1}{2}\partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 \boxed{- \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4} \\
&- \frac{1}{4} W^{\mu\nu} W_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4} R^{a\mu\nu} R^a_{\mu\nu} + \frac{1}{2} m_\rho^2 \rho^{a\mu} \rho^a_\mu + \boxed{\frac{1}{4} c_3 \left(\omega_\mu \omega^\mu \right)^2} \\
&+ \bar{\psi}_e \left(i\partial\!\!\!/ - m_e \right) \psi_e + \bar{\psi}_\nu i\partial\!\!\!/ \psi_\nu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} ,
\end{aligned}
$$

$$
W_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu ,
$$

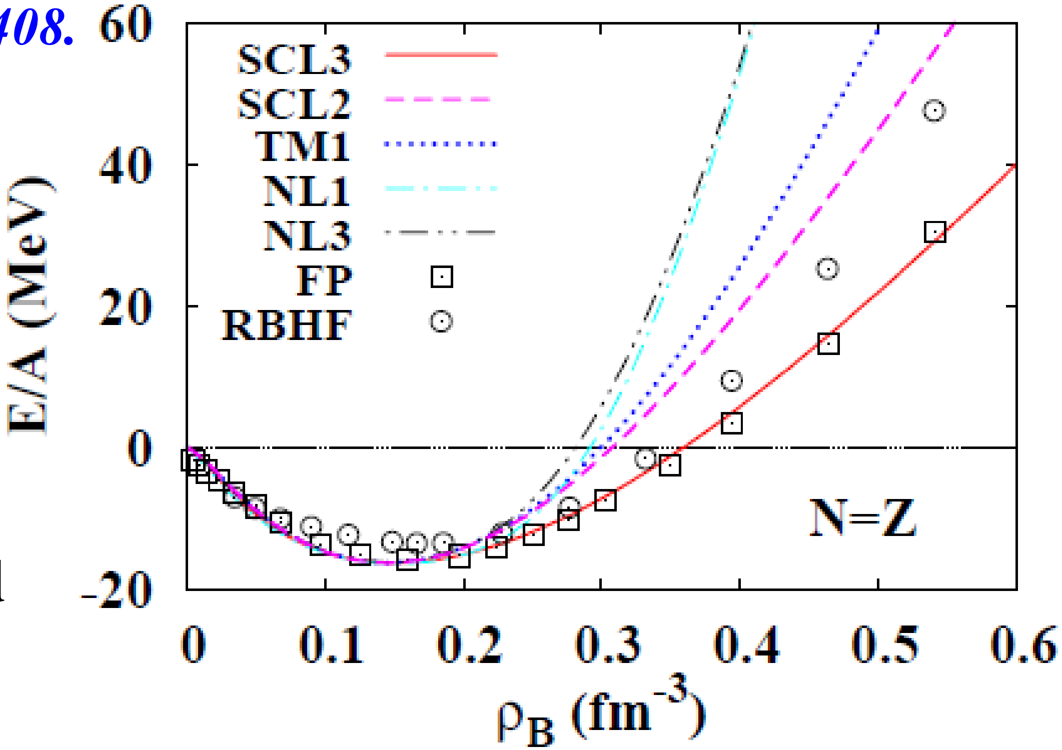
$$
R^a_{\mu\nu} = \partial_\mu \rho^a_\nu - \partial_\nu \rho^a_\mu + g_\rho \epsilon^{abc} \rho^{b\mu} \rho^{c\nu} ,
$$

$$
F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu .
$$

RMF models with Non-Linear Meson Int. Terms

- **Variety of the RMF models**
  → MB couplings, meson masses, meson self-energies
  - **σN, ωN, ρN couplings are well determined**
    → almost no model deps. in Sym. N.M. at low ρ
  - **ω^4 term is introduced to simulate DBHF results of vector pot.**
    TM1&2: Y. Sugahara, H. Toki, NPA579('94)557;
    R. Brockmann, H. Toki, PRL68('92)3408.
  - **σ^3 and σ^4 terms are introduced to soften EOS at ρ_0.**
    J. Boguta, A.R.Bodmer NPA292('77)413,
    NL1:P.-G.Reinhardt, M.Rufa, J.Maruhn, W.Greiner, J.Friedrich, ZPA323('86)13.
    NL3: G.A.Lalazissis, J.Konig, P.Ring, PRC55('97)540.

    → Large differences are found at high ρ



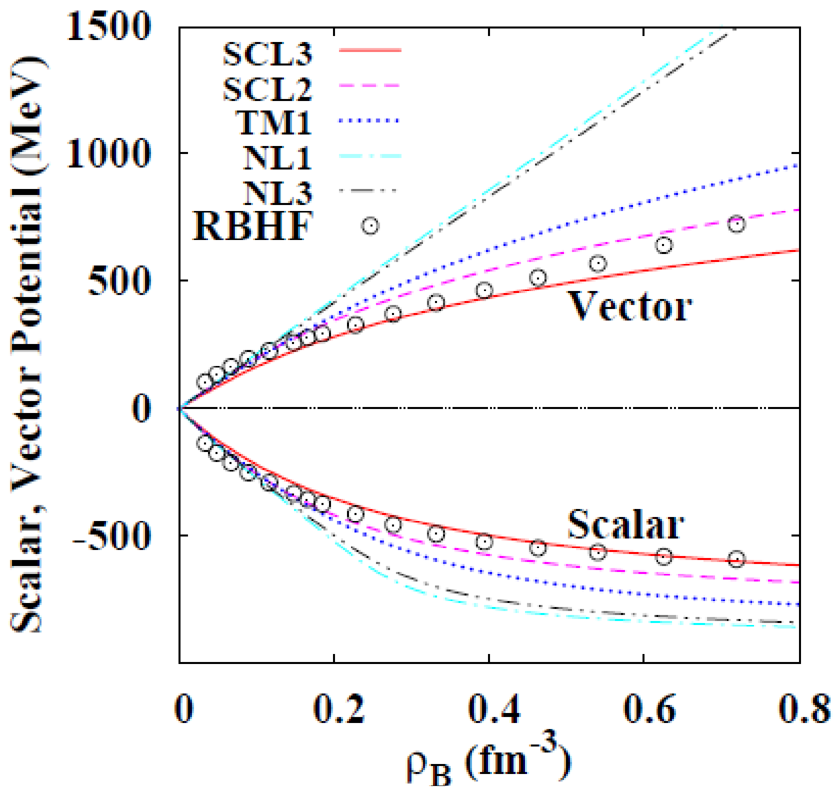
K. Tsubakihara, H. Maekawa, H. Matsumiya, AO,PRC81('10)065206.

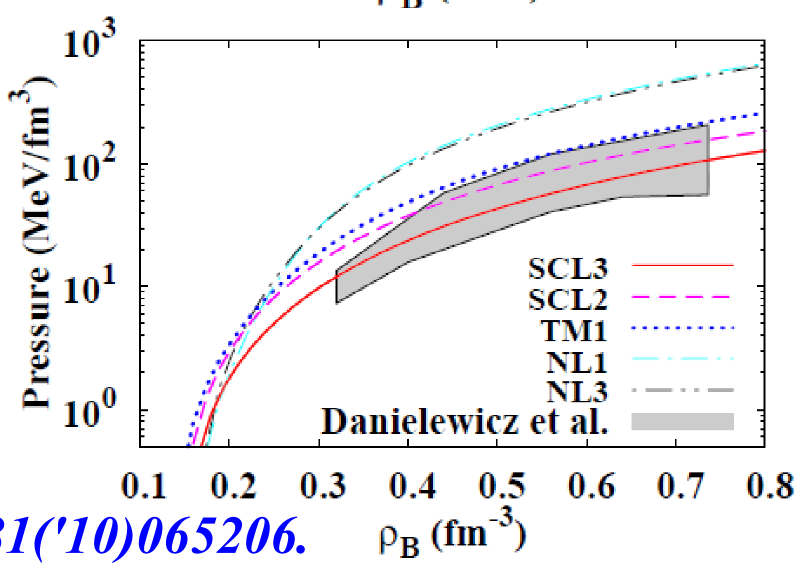
Vector potential in RMF

- **Vector potential from ω dominates at high density !**

$$U_v(\rho_B) = g_\omega \omega \sim \frac{g_\omega^2}{m_\omega^2}\rho_B$$

  - **Dirac-Bruckner-Hartree-Fock shows suppessed vector potential at high ρ_B.**
    R. Brockmann, R. Machleidt, PRC42('90)1965.
  - **Collective flow in heavy-ion collisions suggests pressure at high ρ_B.**
    P. Danielewicz, R. Lacey, W. G. Lynch, Science298('02)1592.
  - **Self-interaction of ω ~ $c_\omega(\omega_\mu\omega^\mu)^2$**
    → DBHF results & Heavy-ion data

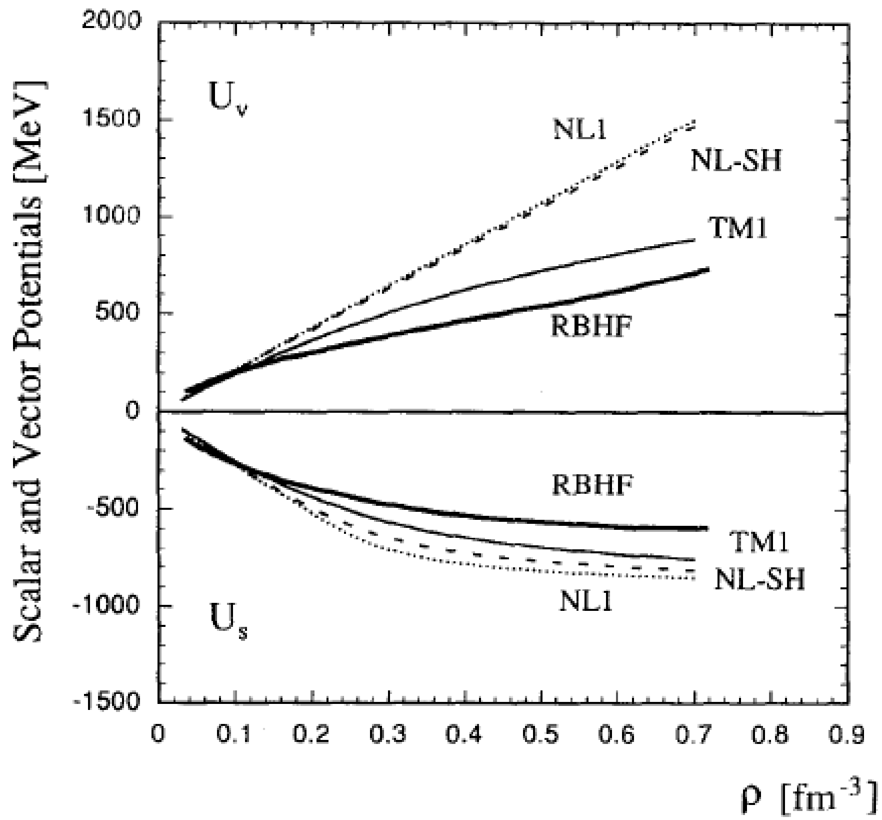


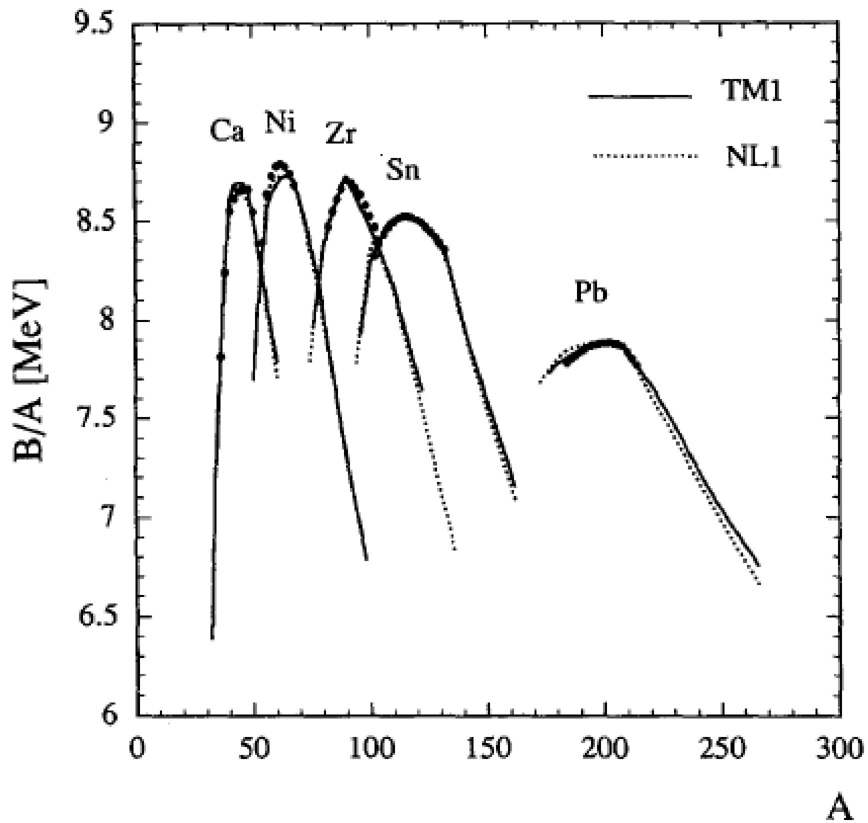


K. Tsubakihara, H. Maekawa, H. Matsumiya, AO,PRC81('10)065206.

TM1

- **TM1 Sugahara, Toki ('94)**
  - **Fit vector potential in RBHF by introducing ω^4 term.**
  - **Fit binding energies of neutron-rich nuclei**

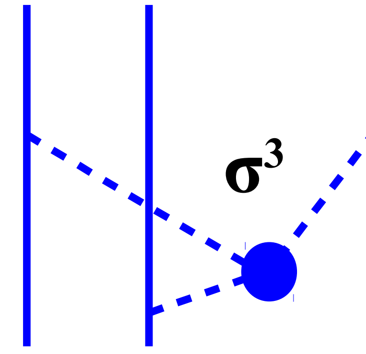


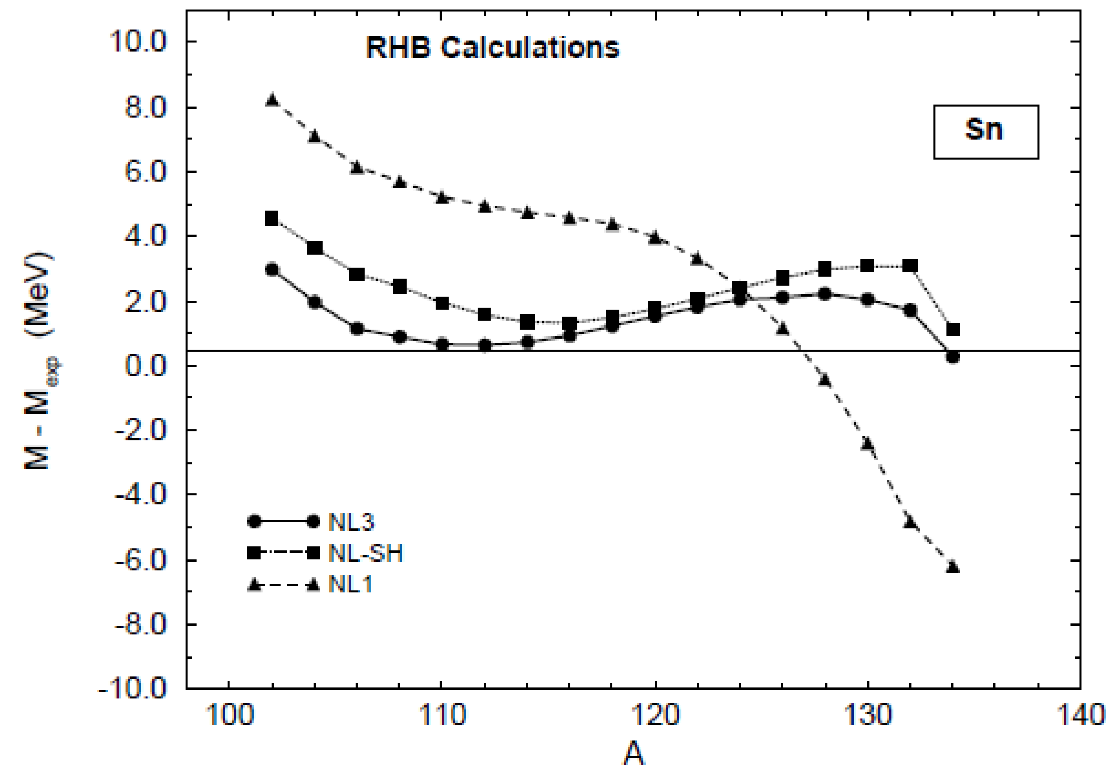


TM1: Sugahara, Toki ('94)

High Quality RMF models

- いくつかの RMF パラメータによる計算は、
  「質量公式」に迫る精度で原子核質量を記述！
  → High Quality RMF models.
  TM, NL1, NL3, ….
  - 全質量で 1-2 MeV の誤差 (NL3)
  - Linear coupling (σN, ωN, ρN), self-energy in σ, ω
  - 場合によっては結合定数の密度依存性を導入。





NL3: Lalazissis, Konig, Ring, PRC55 ('97)540

RMF with Non-Linear Meson Int. Terms

- **Are the Lagrangian parameters are well determined ?**

$$\mathcal{L} = \mathcal{L}_{\text{free}}(\psi, \sigma, \omega, \rho, ...) + \bar{\psi}\left[g_\sigma \sigma - g_\omega \gamma^0 \omega - g_\rho \tau_z \gamma^0 \rho\right]\psi + c_\omega \omega^4/4 - V_\sigma(\sigma) \ , \quad (3)$$

$$V_\sigma = \begin{cases} \frac{1}{3} g_3 \sigma^3 + \frac{1}{4} g_4 \sigma^4 & \text{(NL1, NL3, TM1)} \\ -a_\sigma f_{\text{SCL}}(\sigma/f_\pi) & \text{(SCL)} \end{cases} , \quad (4)$$

  - **Linear terms, Meson-Nucleon Coupling → Well determined**
  - **Negative Coef. of $\sigma^4 < 0$ in some of RMF models→ Vacuum is unstable**
  - **Meson interaction terms → Different in RMF parameterization**

TABLE II: RMF parameters

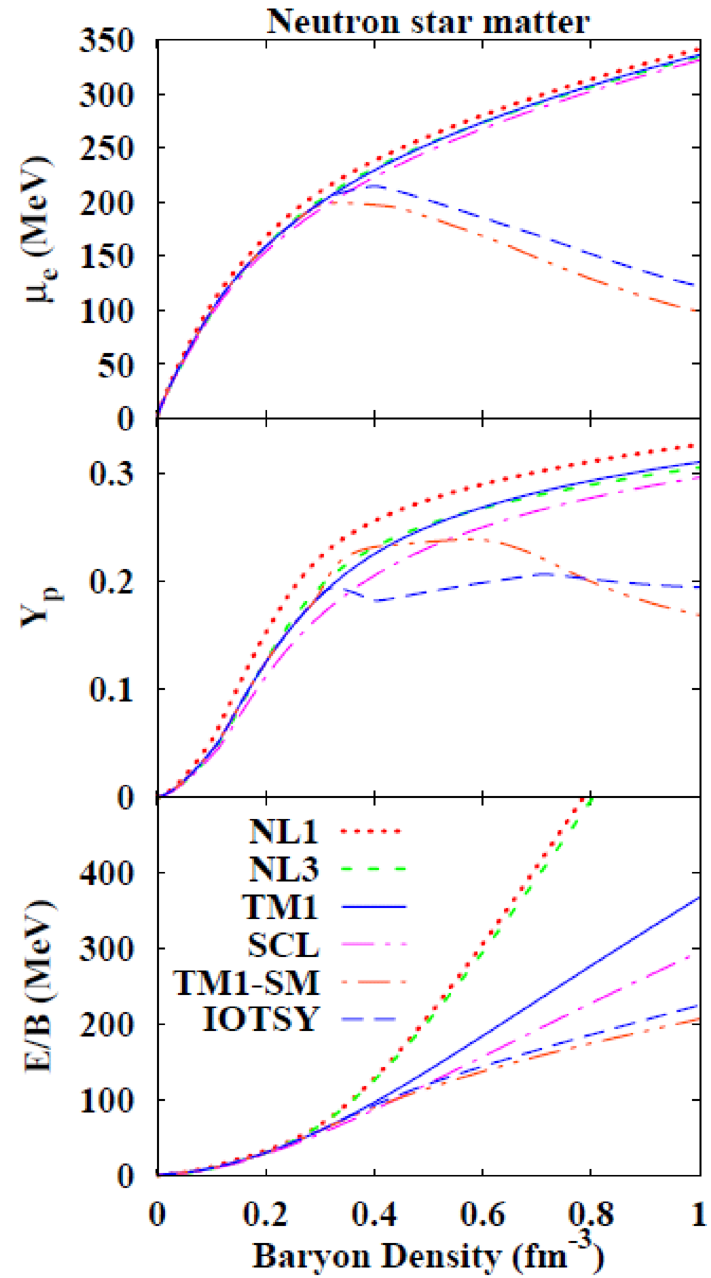
	$g_{\sigma N}$	$g_{\omega N}$	$g_{\rho N}$	g_3(MeV)	g_4	c_ω	m_σ(MeV)	m_ω(MeV)	m_ρ(MeV)
NL1[18]	10.138	13.285	4.976	2401.9	-36.265	0	492.25	795.359	763
NL3[19]	10.217	12.868	4.474	2058.35	-28.885	0	508.194	782.501	763
TM1[6]	10.0289	12.6139	4.6322	1426.466	0.6183	71.3075	511.198	783	770
SCL[20](*1)	10.08	13.02	4.40	1255.88	13.504	200	502.63	783	770

(*1): g_3 and g_4 are from the expansion of f_{SCL}.

AO, Jido, Sekihara, Tsubakihara, in prep.

Neutron Star Matter EOS

- **Difference in non-linear meson terms generate different predictions of EOS at high densities**



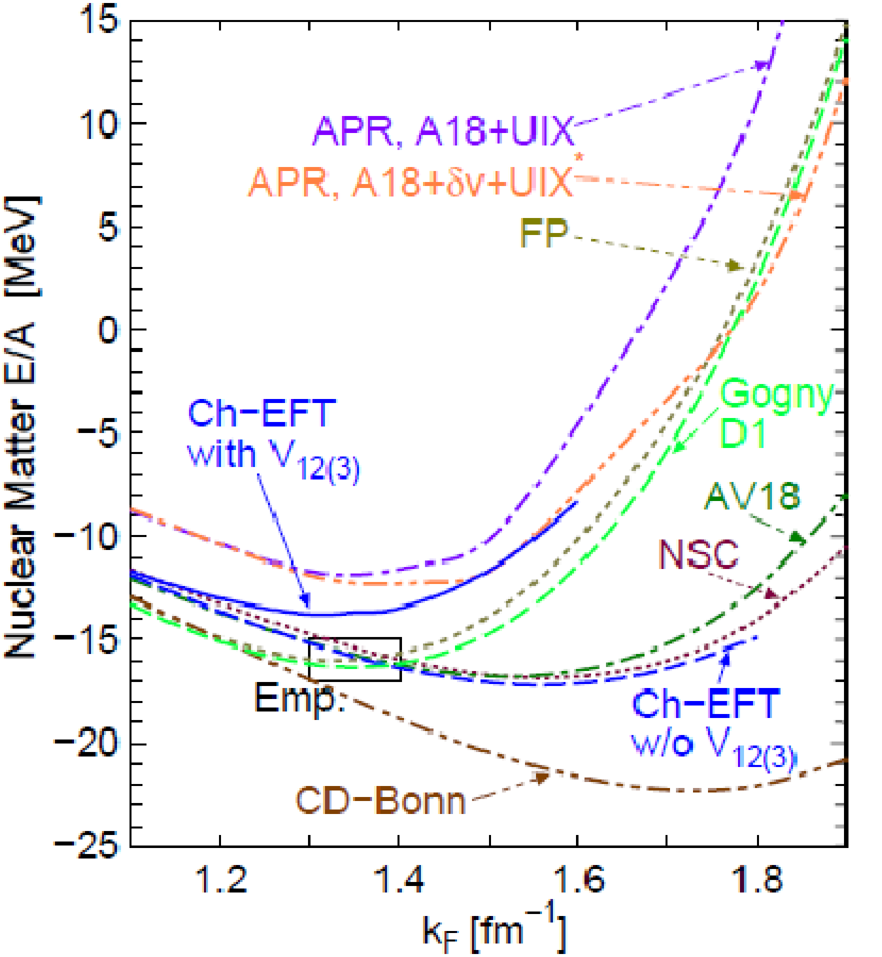
How can we fix non-linear terms ?

AO, Jido, Sekihara, Tsubakihara, Phys. Rev. C 80 (2009), 038202.

Ch-EFT EOS

- **Phen. models need inputs from**
  Experimental Data and/or Microscopic (Ab initio) Calc.
- **Recent Ch-EFT EOS is promising !**
  NN (N3LO)+3NF(N2LO)

M.Kohno ('13)

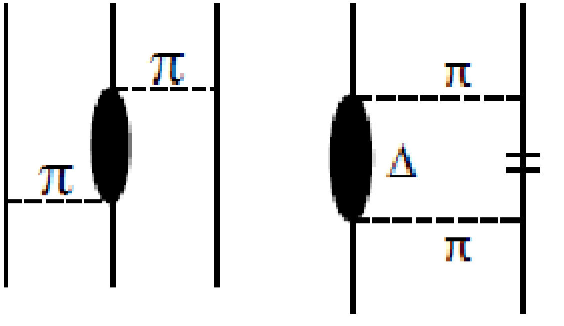


M. Kohno, PRC 88 ('13) 064005

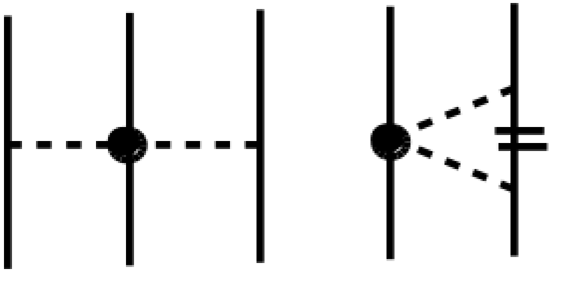
"Universal" mechanism of "Three-body" repulsion

- "Universal" 3-body repulsion is necessary to support NS.
  Nishizaki, Takatsuka, Yamamoto ('02)
- Mechanism of "Universal" Three-Baryon Repulsion.
  - "σ"-exchange ~ two pion exch. w/ res.
  - Large attraction from
    two pion exchange is suppressed
    by the Pauli blocking
    in the intermediate stage.
    Kohno ('13)

Physical Picture



χ EFT



"Universal" TBR
- *Coupling to Res. (hidden DOF)*
- *Reduced "σ" exch. pot. ?*

Summary of Lecture 2

- **Nuclear Matter EOS is important in many subjects of physics.**
  - **Bulk nuclear properties (B.E., radius)**
  - **Dense Matter in Compact Astrophysical Objects**
  - **High-Energy Heavy-Ion Collisions**
- **Relativistic Mean Field models**
  - **Simple description of nucleon scalar and vector potentials in terms of meson fields.**
  - **With non-linear meson interaction terms, nuclear binding energies (and radii) are well explained.**
  - **Ambiguities of non-linear couplings bring large differences of EOS at high densities, especially in asymmetric nuclear matter.**
- **It is promising to utilize the results of G-matrix based on Chiral EFT (2 and 3 nucleon force), which reproduces the saturation density in an “ab initio” manner.**

Report 問題

- 中性子、陽子、電子のみからなる中性子星物質を考える。電子の質量を無視すると、核子あたりのエネルギーは、Lecture 1 で示したように

$$E_{\rm NSM}(\rho)=E_{\rm SNM}(\rho)+S(\rho)\delta^2+\frac{\Delta M}{2}\delta+\frac{3}{8}\hbar k_F(1-\delta)^{4/3}$$

  と与えられる。ここで $\Delta M=M_n-M_p$、k_F は同じ密度での対称核物質のフェルミ波数である。非対称度 δ は、核子あたりのエネルギーが最小になるように選ばれる。
  - 上の表式を導け。
  - 核子あたりのエネルギーが最小となる非対称度 δ を求めよ。(３次方程式を解くこととなる。$S(\rho)$, k_F, ΔM は与えられているとしてよい。)
- 今回の講義において、中性子物質の物理の課題の中で各自が興味を持った項目をあげ、その理由を述べよ。

Thank you !

Field Theory at Finite T & ρ
– Short Course –

経路積分

- **量子力学での経路積分 (Path integral)**
  - **時刻 t_i で位置 q_i にいた粒子が時刻 t_f で位置 q_f に到着する振幅**

$$S_{fi}=\langle q_f, t_f|\exp[-i\hat{H}(t_f-t_i)]|q_i, t_i\rangle=\int Dq\exp(iS[q])$$
$$S[q]=\int_{q(t_i)=q_i, q(t_f)=q_f} dt\, L(q,\dot{q})$$

経路 $q(t)$ についての和 → 経路積分

  - **特徴**
    - **演算子の代わりに通常の数 (c- 数) で表せる。**
    - **作用 S の構成時に正準交換関係を用いることにより「量子論」の性質を取り込む。**
- **場の理論＝各点での場の振幅 φ(x,t) を座標とする量子力学**

$$S_{fi}=\langle \Psi_f|\exp[-i\hat{H}(t_f-t_i)]|\Psi_i\rangle=\int D\phi\exp(iS[\phi])$$
$$S[\phi]=\int_{\Psi(t_i)=\Psi_i, \Psi(t_f)=\Psi_f} d^4x\, L(\phi,\partial_\mu\phi)$$

分配関数とユークリッド化

分配関数

$$Z=\sum_n \exp(-E_n/T)=\sum_n \langle n|\exp[-\hat{H}/T]|n\rangle$$

$$=\sum_n \langle n|\exp[-i\hat{H}(t_f-t_i)]|n\rangle_{t_f-t_i=-i/T}=\int D\phi \exp(-S_E[\phi])$$

$$S_E[\phi]=\int_0^\beta d\tau\, d^3x\, L_E(\phi,\partial_i\phi,\partial_\tau\phi)\ |_{\phi(x,\beta)=\phi(x,0)}$$

$$L_E(\phi,\partial_i\phi,\partial_\tau\phi)=-L(\phi,\partial_i\phi,i\partial_t\phi)$$

$$t=-i\tau,\quad \partial_\tau=-i\partial_t,\ \beta=1/T$$

$$iS=i\int_0^{-i\beta}dt\int d^3x\, L=\int_0^\beta d\tau\, d^3x\, L=-\int_0^\beta d\tau\, d^3x\, L_E$$

- **統計力学の分配関数は虚時間発展の振幅の和である。**
- **全ての状態について和 → τ=0, β で周期境界条件をつけて任意の φ(x,t) について足し合わせる。**

Example: Scalar Field

- **Lagrangian density**

$$L=\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi-\frac{1}{2}m^2\phi^2-U(\phi)$$

Euler-Lagrange equation (principle of least action)

$$\partial_\mu\left[\frac{\partial L}{\partial(\partial_\mu\phi)}\right]-\frac{\partial L}{\partial\phi}=0\ \rightarrow\ \partial_\mu\partial^\mu\phi+m^2\phi+\frac{\partial U}{\partial\phi}=0\,(\text{Klein-Gordon eq.})$$

- **Euclidean Lagrangian**
  - **Euclid 化のルール** $t = -i\tau$, $x_4=\tau$, $g_{\mu\nu}=(1,1,1,1)$, $L_E=-L$

$$L_E=\frac{1}{2}\partial_\mu\phi\,\partial_\mu\phi+\frac{1}{2}m^2\phi^2+U(\phi)$$

→ 相互作用がない場合に実際に経路積分してみましょう。

Partition Func. of Free Scalar Field

- **自由スカラー場の分配関数**
  - **有限のサイズの箱 (体積 *V*) の中で自由スカラー場 (*U*=0)**
  - **フーリエ変換**

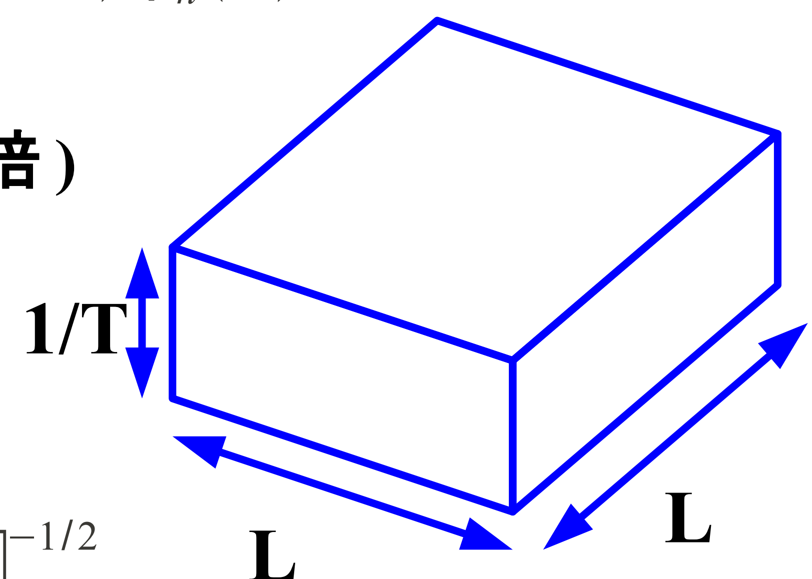
$$\phi(\tau, \boldsymbol{x}) = \frac{1}{\sqrt{V/T}} \sum_{n,\boldsymbol{k}} \exp(-i\omega_n\tau + i\boldsymbol{k}\cdot\boldsymbol{x})\phi_n(\boldsymbol{k})$$

Periodic boudnary condition $\omega_n = 2\pi n T,\ \ k_i = 2\pi n_i / L$

Euclidean action $S_E = \frac{1}{2}\sum_{n,\boldsymbol{k}} (\omega_n^2 + \boldsymbol{k}^2 + m^2)\phi_n^2(\boldsymbol{k})$

  - **フーリエ変換はユニタリー変換だから、**
    積分の測度は変わらない。(高々定数倍)

$$D\phi = N \prod_{n,\boldsymbol{k}} d\phi_n(\boldsymbol{k})$$



  - **ガウス積分 → 分配関数**

$$Z = \int D\phi\, e^{-S_E} = N \prod_{n,\boldsymbol{k}} \sqrt{2\pi}\left[\omega_n^2 + \boldsymbol{k}^2 + m^2\right]^{-1/2}$$

Partition Func. of Free Scalar Field (cont.)

- 自由エネルギー

$$\Omega = -T\log Z = \frac{1}{2}\sum_{\boldsymbol{k}}\left[T\sum_n \log(\omega_n^2 + \underbrace{\boldsymbol{k}^2 + m^2}_{E_k^2})\right] + \text{const.}$$

$$= \frac{1}{2}\sum_{\boldsymbol{k}} I(E_{\boldsymbol{k}}, T) + \text{const.}$$

- 松原和 (Matsubara Frequency summation)

$$\sum_n \frac{1}{a^2 + \bar{n}^2} = \frac{\pi}{2a} \times \begin{cases} \coth(\pi a/2) & (\bar{n} = 2\text{n}) \\ \tanh(\pi a/2) & (\bar{n} = 2\text{n}+1) \end{cases}$$

$$\frac{\partial I(E_k, T)}{\partial E_k} = \sum_n \frac{2TE_k}{\omega_n^2 + E_k^2} = \cdots = \frac{1+\exp(-E_k/T)}{1-\exp(-E_k/T)}$$

$$I(E_k, T) = E_{\boldsymbol{k}} + 2T\log\left(1-\exp(-E_k/T)\right) + \text{const.}$$

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Partition Func. of Free Scalar Field (cont.)

- 自由エネルギー (グランド･ポテンシャル)

$$\Omega = \sum_{\boldsymbol{k}} \left\{ \frac{E_{\boldsymbol{k}}}{2} + T \log\left(1 - e^{-E_{\boldsymbol{k}}/T}\right) \right\} + \text{const.}$$

$$= V \int \frac{d^3 k}{(2\pi)^3} \left[\frac{E_{\boldsymbol{k}}}{2} + T \log\left(1 - e^{-E_{\boldsymbol{k}}/T}\right) \right]$$

ゼロ点エネルギー ($\hbar\omega/2$)　　熱的励起

ゼロ点エネルギー部分を無視して部分積分すると、
通常の圧力を得る。

$$P = -\Omega / V = \int \frac{d^3 k}{(2\pi)^3} \frac{\boldsymbol{k}\cdot\boldsymbol{v}}{3} \frac{e^{-E_{\boldsymbol{k}}/T}}{1 - e^{-E_{\boldsymbol{k}}/T}} \qquad \left(\boldsymbol{v} = \frac{\partial E_{\boldsymbol{k}}}{\partial \boldsymbol{k}} \right)$$

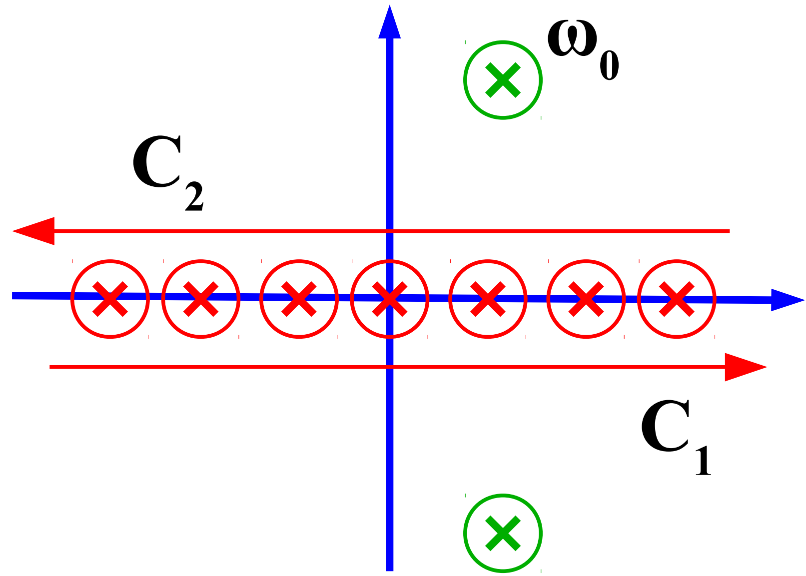
場の理論 → Euclid 化 + Imag. Time → 統計力学

Matsubara Frequency Summation

- **Contour integral technique**

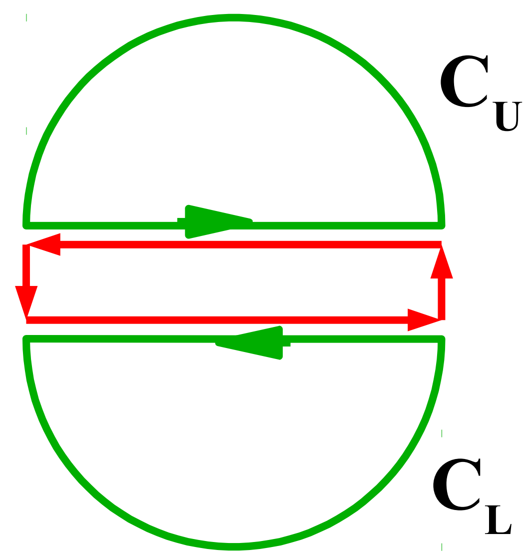
$$S = T\sum_n g(\omega_n = 2\pi n T, \pi(2n+1)T)$$

$$= \pm\int_{C_1+C_2} \frac{dz}{2\pi}\frac{g(z)}{e^{i\beta z}\mp 1} = \mp i\sum_{\omega_0}\frac{\mathrm{Res}\, g(\omega_0)}{e^{i\beta\omega_0}\mp 1}$$



(g: meromorphic (有理型),
no pole on real axis,
decreases faster than 1/ω at ω → ∞)

- **Applicable to more general cases !**
- **Anti-periodic condition → Fermi-Dirac dist.**



- **Example:** $g(\omega)=1/(\omega^2+E^2)$
→ $\omega_0 = \pm iE$**, Res** $g = \pm 1/2iE$

$$S = \frac{1}{2E}\frac{e^{\beta E}\pm 1}{e^{\beta E}\mp 1}$$

$$C_1+C_2+C_U+C_L=0$$

Fermion

- **Lagrangian**

$$L=\bar{N}\left(i\gamma^{\mu}\partial_{\mu}-m\right)N$$

- **Euclidean**

$$(x_{\mu})_E=(\tau=it\,,\boldsymbol{x}),\quad (\gamma_{\mu})_E=(\gamma_4=i\gamma^0,\boldsymbol{\gamma})$$

$$L_E=\bar{N}\left(-i\gamma_{\mu}\partial_{\mu}+m\right)N$$

- **Grassman number**
経路積分において、フェルミオンは反可換な Grassmann 数

$$\int d\chi\cdot 1=\text{anti-comm. constant}=0\quad,\quad\int d\chi\cdot\chi=\text{comm. constant}\equiv 1$$

$$\int d\chi\, d\bar{\chi}\exp\left[\bar{\chi}A\chi\right]=\int d\chi\, d\bar{\chi}\frac{1}{N!}(\bar{\chi}A\chi)^N=\cdots=\det A$$

$$=\exp\left[-(-\log\det A)\right]$$

Bi-linear Fermion action leads to -log(det A) effective action

- **Example: Relativistic Mean Field (RMF)**

$$L=\bar{\psi}\left(i\gamma^{\mu}\partial_{\mu}-m-\Sigma\right)\psi+L_{\mathrm{meson}}(\Phi) \quad (\Phi=\sigma,\omega,\rho)$$
$$\Sigma=g_{\sigma}\sigma+\gamma^{0}\left(g_{\omega}\omega^{0}+g_{\rho}\rho^{0}\boldsymbol{\tau}\right)$$

- **Euclid 化 + 化学ポテンシャルの導入**

$$Z=\int D\psi D\bar{\psi} D\Phi \ \exp\left[-\int d^4x\left(L-\mu\psi^{+}\psi\right)\right]$$
$$=\int D\psi D\bar{\psi} D\Phi \ \exp\left[-\int d^4x\left\{\bar{\psi}D\psi+L_{\mathrm{meson}}(\Phi)\right\}\right]$$
$$=\int D\Phi \exp\left[-S_{\mathrm{eff}}(\Phi;T,\mu)\right]$$
$$D=-i\gamma\partial-\mu\gamma^{0}+m+\Sigma$$

- **有効作用**

$$S_{\mathrm{eff}}=S_{\mathrm{eff}}^{(F)}+S_{\mathrm{meson}}=-\sum_{n,\boldsymbol{k}}\log\det D_{n,\boldsymbol{k}}+\int d^4x\, L_{\mathrm{meson}}$$

RMF (cont.)

- 一様な場を仮定 → Fourier 変換により D をブロック対角化

$$D_{n,\boldsymbol{k}} = \gamma^0\left(-i\omega_n - \left(\mu - V^0\right)\right) + \boldsymbol{\gamma}\cdot\boldsymbol{k} + M + g_\sigma\sigma$$

$$\to \det D = \left[\left(\omega_n + i\mu^*\right)^2 + E^{*2}\right]^2$$

$$\mu^* = \mu - g_\omega\omega^0 - g_\rho\rho^0\boldsymbol{\tau},\quad E^* = \sqrt{\boldsymbol{k}^2 + M^{*2}},\quad M^* = m + g_\sigma\sigma$$

- 松原振動数和を実行

$$F_{\mathrm{eff}}^{(F)} = -\frac{d_f}{2}\int\frac{d^3k}{(2\pi)^3}\left[E^* + T\log\left(1 + e^{-(E^*-\mu^*)/T}\right) + T\log\left(1 + e^{-(E^*+\mu^*)/T}\right)\right]$$

ゼロ点　　粒子 (核子)　　反粒子 (反核子)

- 温度 0 の場合

$$F_{\mathrm{eff}}^{(F)} = -\frac{d_f}{2}\int^{\Lambda}\frac{d^3k}{(2\pi)^3}E^* + d_N\int^{k_F}\frac{d^3k}{(2\pi)^3}E^* - \mu^*\rho_B \quad \left(d_N = d_f/2\right)$$

ゼロ点エネルギーは核子のループから現れる
(RMF では通常無視)