

高密度物質と中性子星の物理 *Physics of Neutron Star Matter*

京大基研 大西 明

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1. 中性子星の基本的性質
2. 状態方程式を記述する理論模型
3. 対称エネルギーと非対称核物質の状態方程式
4. ハイパー核物理と高密度核物質の状態方程式
5. 中性子星におけるエキゾチック自由度

九州大学集中講義 7/8-10



M-R Relation and EOS

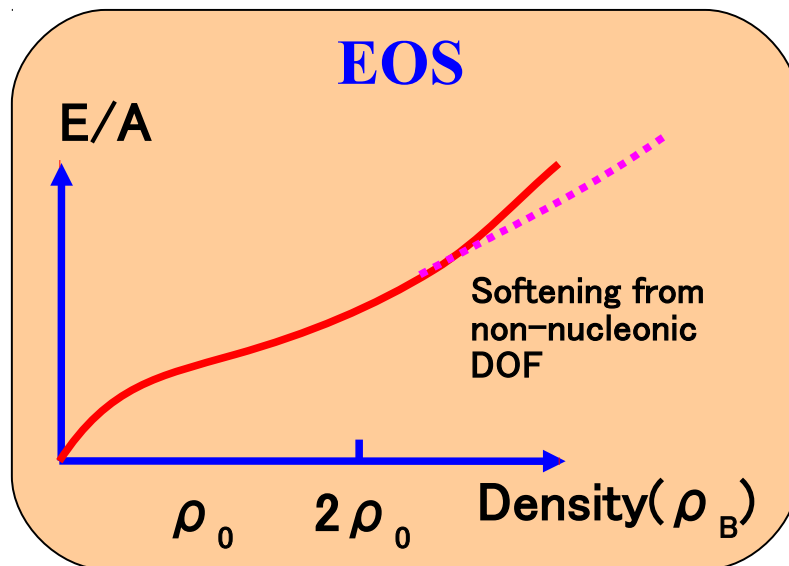
■ Solving TOV eq.

starting from the “initial” condition, $\varepsilon(r=0) = \varepsilon_c = \text{given}$
until the “boundary” condition $P(r)=0$ is satisfied.

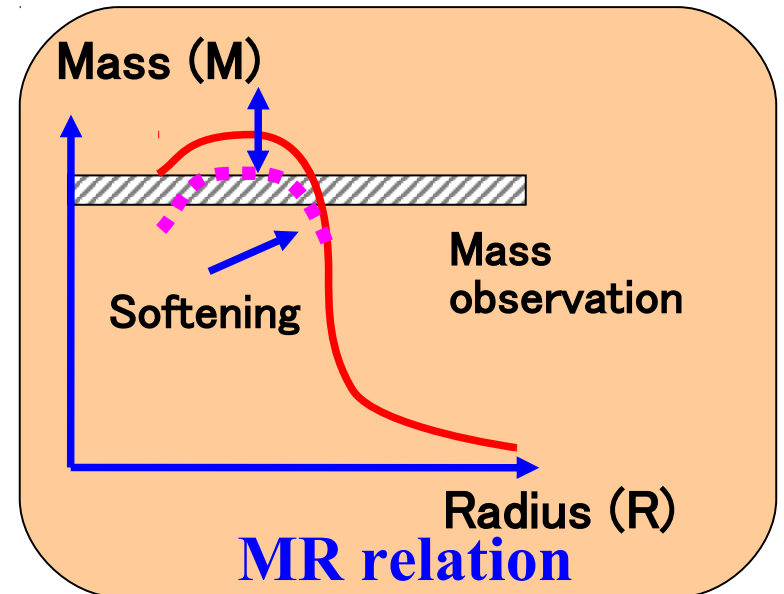
→ M and R are the functions of $\varepsilon(r=0)$
and functionals of EOS, $P=P(\varepsilon)$.

$$M = M(\varepsilon_c)[P(\varepsilon)] \quad , \quad R = R(\varepsilon_c)[P(\varepsilon)]$$

→ **M-R curve and NS matter EOS : 1 to 1 correspondence**



TOV Eq.



Nuclear Mass

■ Bethe-Weizsacker mass formula

Nuclear binding energy is roughly given by Liquid drop.

Nuclear size measurement $\rightarrow R = r_0 A^{1/3}$

$$B(A, Z) = \underbrace{a_v A}_{\text{Volume}} - \underbrace{a_s A^{2/3}}_{\text{Surface}} - \underbrace{a_c \frac{Z^2}{A^{1/3}}}_{\text{Coulomb}} - \underbrace{a_a \frac{(N - Z)^2}{A}}_{\text{Symmetry}} + \underbrace{a_p \frac{\delta_p}{A^{1/2}}}_{\text{Paring}}$$

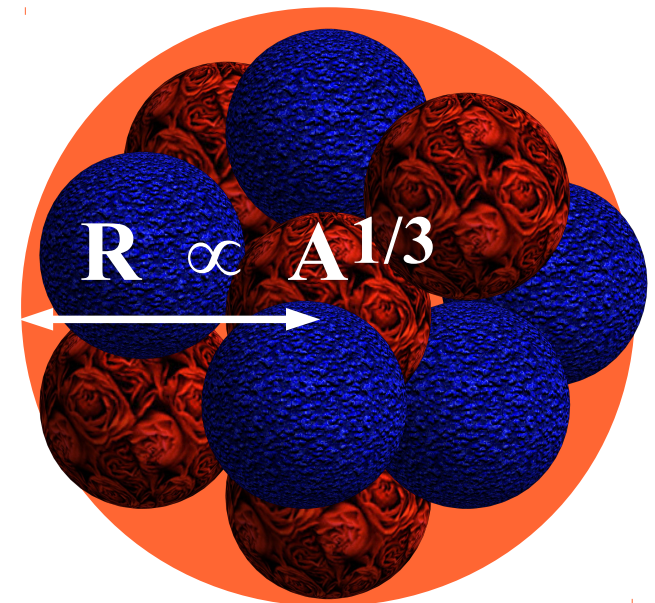
Volume	Surface	Coulomb	Symmetry	Paring
$A \propto \frac{4\pi}{3} R^3$	$A^{2/3} \propto 4\pi R^2$	$\propto \frac{Q^2}{R}$		

■ Ignore Coulomb, consider $A \rightarrow \infty$,

$$B/A = a_v(\rho) - a_a(\rho) \delta^2, \quad \delta = (N - Z)/A$$

$$a_v \simeq 16 \text{ MeV}, \quad a_a \simeq 30 \text{ MeV}$$

Coef. may depend on the number density ρ
 $\rightarrow$ Nuclear Matter EOS



Nuclear Matter EOS

■ Energy per nucleon in nuclear matter

$$E(\rho, \delta) = E_{\text{SNM}}(\rho) + E_{\text{Sym}}(\rho) \delta^2, \quad \delta = (N - Z) / A$$

■ Saturation point (ρ_0, E_0)

$$\rho_0 \sim 0.15 \text{ fm}^{-3}$$

$$E_0 = -a_v \sim -16 \text{ MeV}$$

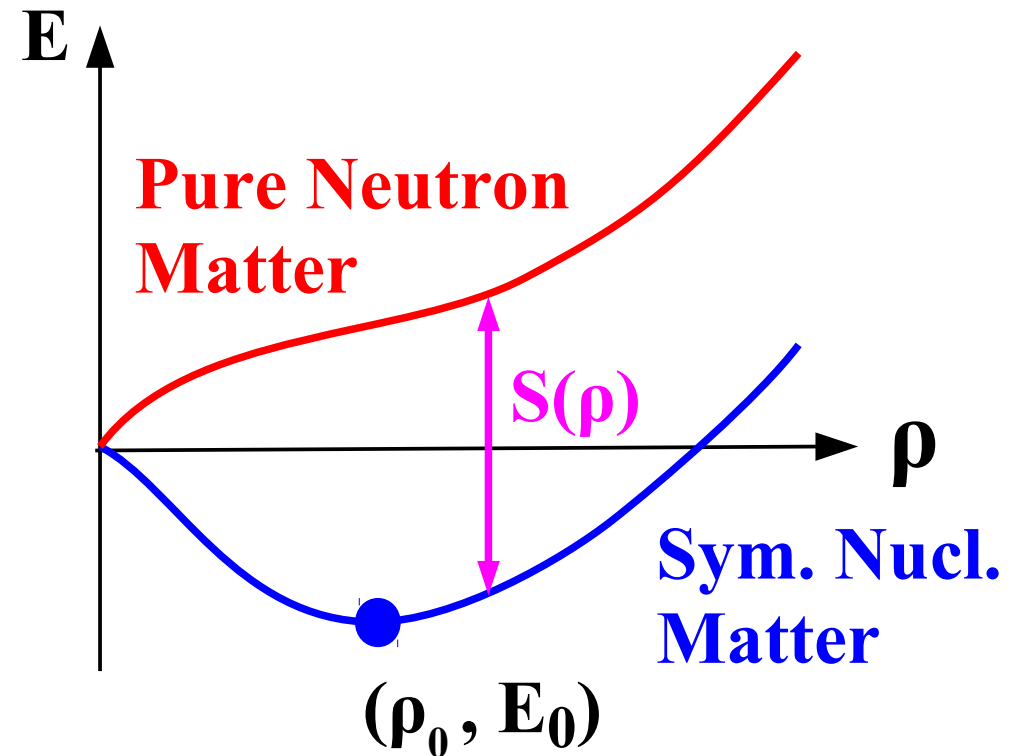
(nuclear radius and mass)

■ Symmetry energy

$$\begin{aligned} S(\rho) &= E_{\text{PNM}}(\rho) - E_{\text{SNM}}(\rho) \\ &= E(\rho, \delta=1) - E(\rho, \delta=0) \end{aligned}$$

$$S_0 = S(\rho_0) \sim 30 \text{ MeV}$$

(mass formula)



Nuclear Matter EOS can be, in principle, determined by terrestrial (laboratory) nuclear physics experiments !

Nuclear Matter EOS

- Additional two important parameters: **K** and **L**
- Pressure is given by the derivative of **E** via ρ

$$P = \rho^2 (\partial E / \partial \rho)$$

At ρ_0 , **L** determines **P**

$$P = \rho_0 L / 3 \quad (\text{at } \rho = \rho_0)$$

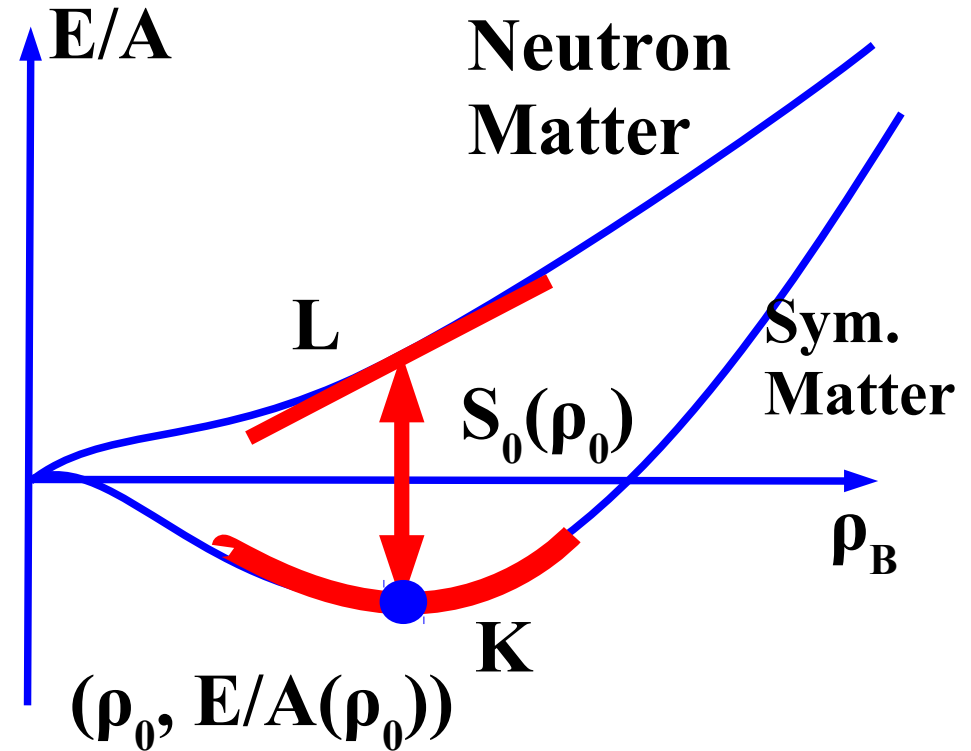
$$E/A(\rho, \delta) = \varepsilon(\rho) + E_{\text{sym}}(\rho) \delta^2 + O(\delta^4)$$

Symmetric Nuclear Matter

$$\varepsilon(\rho) = \varepsilon(\rho_0) + \frac{K(\rho - \rho_0)^2}{18\rho_0^2} + O((\rho - \rho_0)^3)$$

Symmetry Energy ($\delta = (N - Z)/A = 1 - 2Y_p$)

$$E_{\text{sym}}(\rho) = S_0 + \frac{L(\rho - \rho_0)}{3\rho_0} + \frac{K_{\text{sym}}(\rho - \rho_0)^2}{18\rho_0^2} + O((\rho - \rho_0)^3)$$



Neutron Star Matter EOS

- What happens in low-density uniform neutron star matter ?
 - Constituents = proton, neutron and electron
 - Charge neutrality $\rightarrow$ # of electrons = # of protons ($\rho_e = \rho_p = \rho(1 - \delta)/2$)

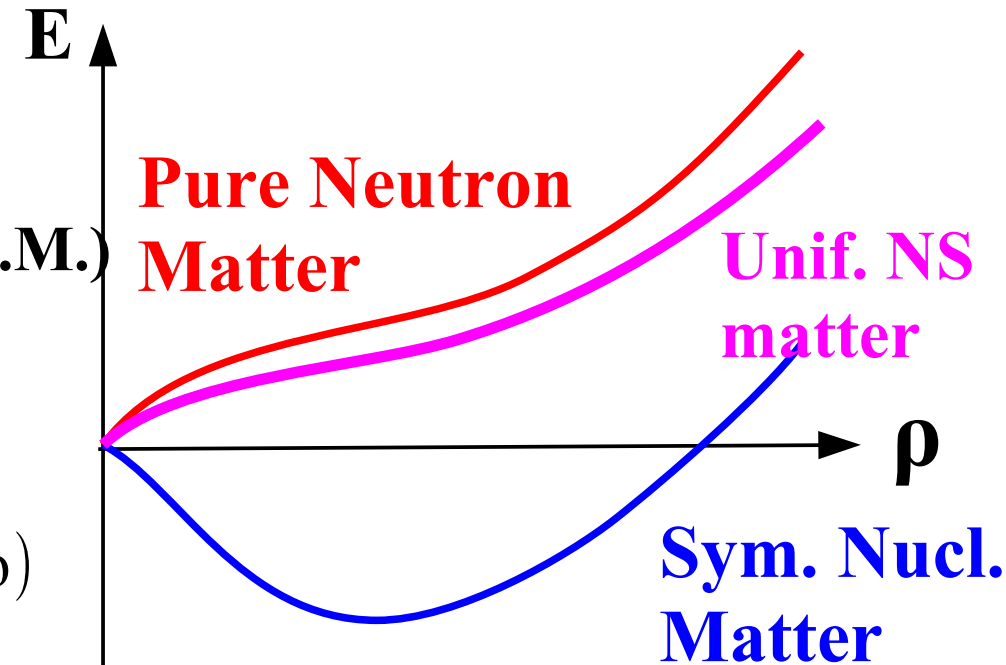
$$E_{\text{NSM}}(\rho) = E_{\text{NM}}(\rho, \delta) + E_e(\rho_e = \rho_p)$$

$$= E_{\text{SNM}}(\rho) + S(\rho)\delta^2 + \frac{\Delta M}{2}\delta + \frac{3}{8}\hbar k_F(1 - \delta)^{4/3}$$

(electron mass neglected,
neutron-proton mass diff. incl.
 k_F = Fermi wave num. in Sym. N.M.)

- δ is optimized to minimize energy per nucleon

$$E_{\text{NSM}}(\rho) \leq E_{\text{NM}}(\rho, \delta = 1) = E_{\text{PNM}}(\rho)$$



対称エネルギーの起源

■ Fermi Gas model での核子あたりの運動エネルギー

$$E_{\text{sym}, K} = \frac{Z}{A} \frac{3}{5} \frac{\hbar^2 k_{\text{F}p}^2}{2m} + \frac{N}{A} \frac{3}{5} \frac{\hbar^2 k_{\text{F}n}^2}{2m} = \frac{3}{5} E_F \frac{1}{2} \left[(1 - \delta)^{5/3} + (1 + \delta)^{5/3} \right]$$

$$\simeq \frac{3}{5} E_F + \frac{1}{3} E_F \delta^2 + \mathcal{O}(\delta^4)$$

$a_{\text{sym}}(\text{FG}) = E_F/3 \sim 11 \text{ MeV}$ となり、質量公式の $a_{\text{sym}} \sim 23 \text{ MeV}$ と比べて半分程度。残りは相互作用。

■ 残りの半分の対称エネルギーを RMF で評価してみましょう。

$$\Delta E_{\text{sym}, \rho} = \frac{1}{2} \frac{m_\rho^2 R^2}{\rho_B} = \frac{1}{2} \frac{g_\rho^2}{m_\rho^2} \rho_B \delta^2 = \Delta a_{\text{sym}} \delta^2 \quad \left(R = \frac{g_\rho (\rho_n - \rho_p)}{m_\rho^2} = \frac{g_\rho \rho_B \delta}{m_\rho^2} \right)$$

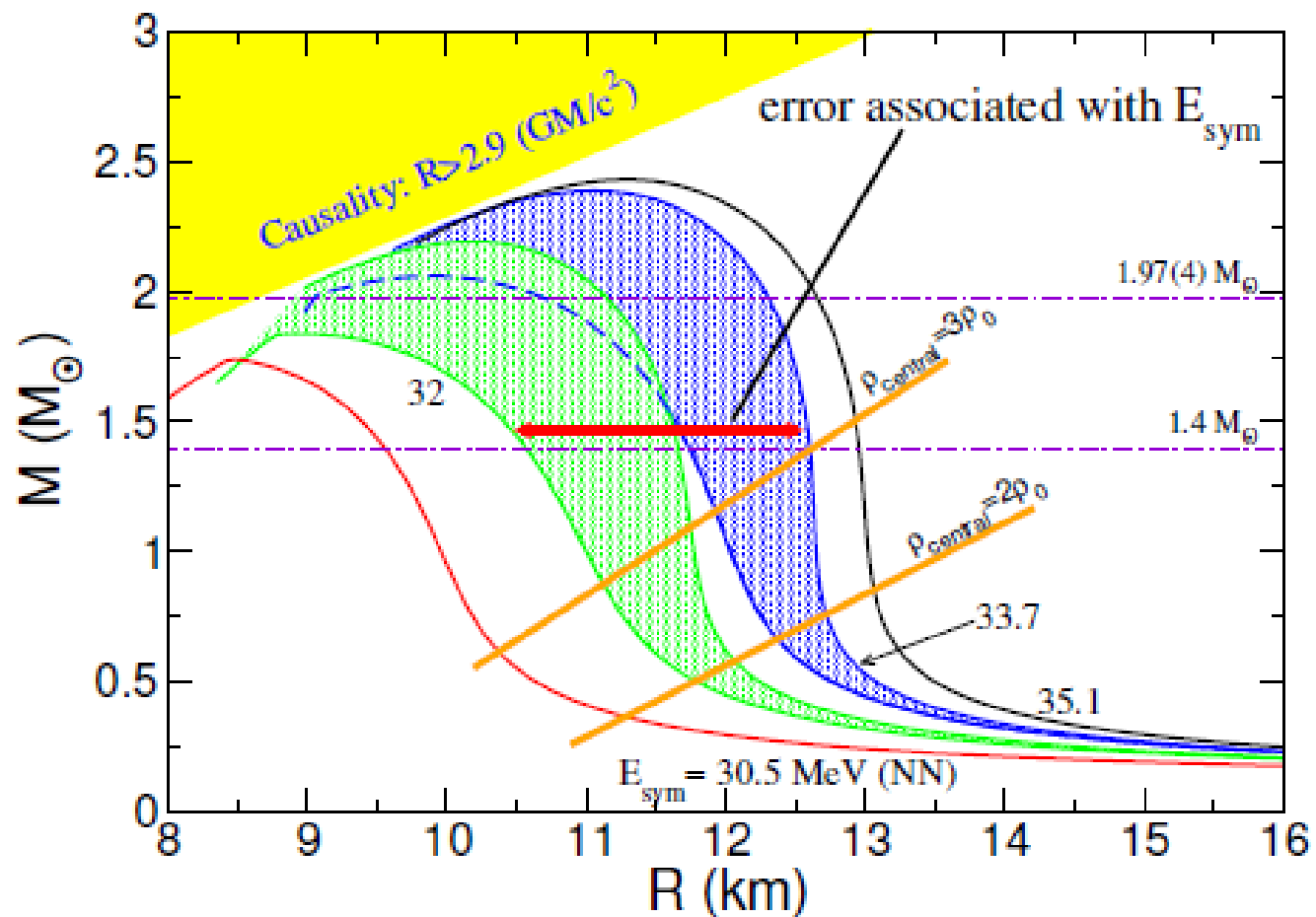
$$g_\rho^2 = \frac{2 m_\rho^2 \Delta a_{\text{sym}}}{\rho_B} \simeq (4.3)^2 \quad (a_{\text{sym}} = 30 \text{ MeV}) \quad \leftarrow \text{RMF par. より少し小さめ}$$

$$L \simeq E_F + 3 \Delta a_{\text{sym}} \simeq 90 \text{ MeV}$$

$\leftarrow$ Optimal value より少し大きい

Symmetry Energy affects MR Relation of NS

- Nuclear pressure at ρ_0 comes **ONLY** from E_{sym} , then E_{sym} dominates pressure around ρ_0 !
- 5 MeV Difference in E_{sym} results in (3-4) km difference in R_{NS} prediction.



Gandol, Carlson, Reddy, PRC 032801, 85 (2012).

Symmetry Energy

■ Summary of Nuclear Symmetry Energy workshop

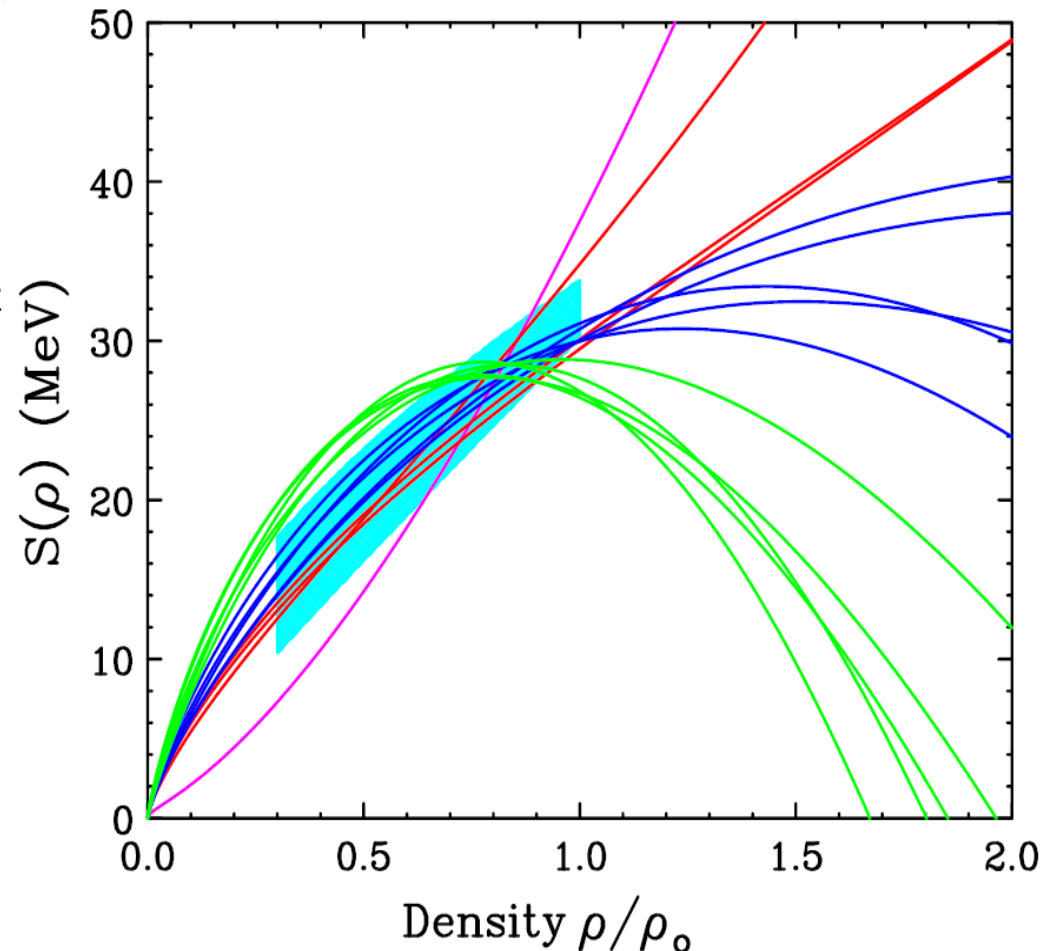
NuSym11 <http://www.smith.edu/nusym11>

$$E_{\text{sym}}(\rho_0) = 31\text{-}34 \text{ MeV}, L = 50\text{-}110 \text{ MeV}$$

extracted from various observations.

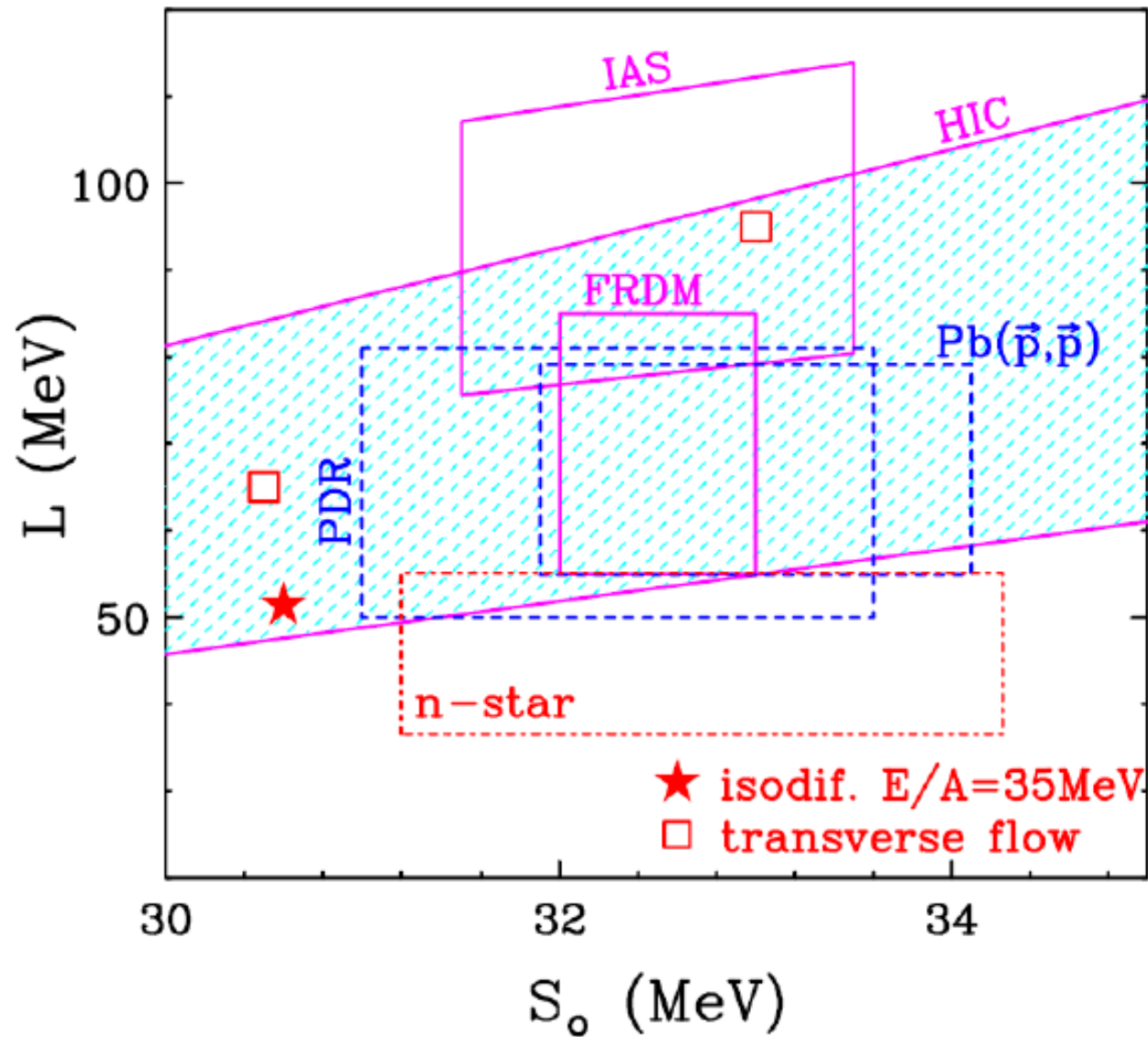
- Mass formula *Moller ('10)*
- Isobaric Analog State
Danielewicz, Lee ('11)
- Pygmy Dipole Resonance
Carbone+ ('10)
- Isospin Diffusion
Tsang et al. ('04)
- Neutron Skin thickness
J.Zenihiro+ ('10)

- これらの多くは ρ_0 以下の密度での E_{sym} に敏感。



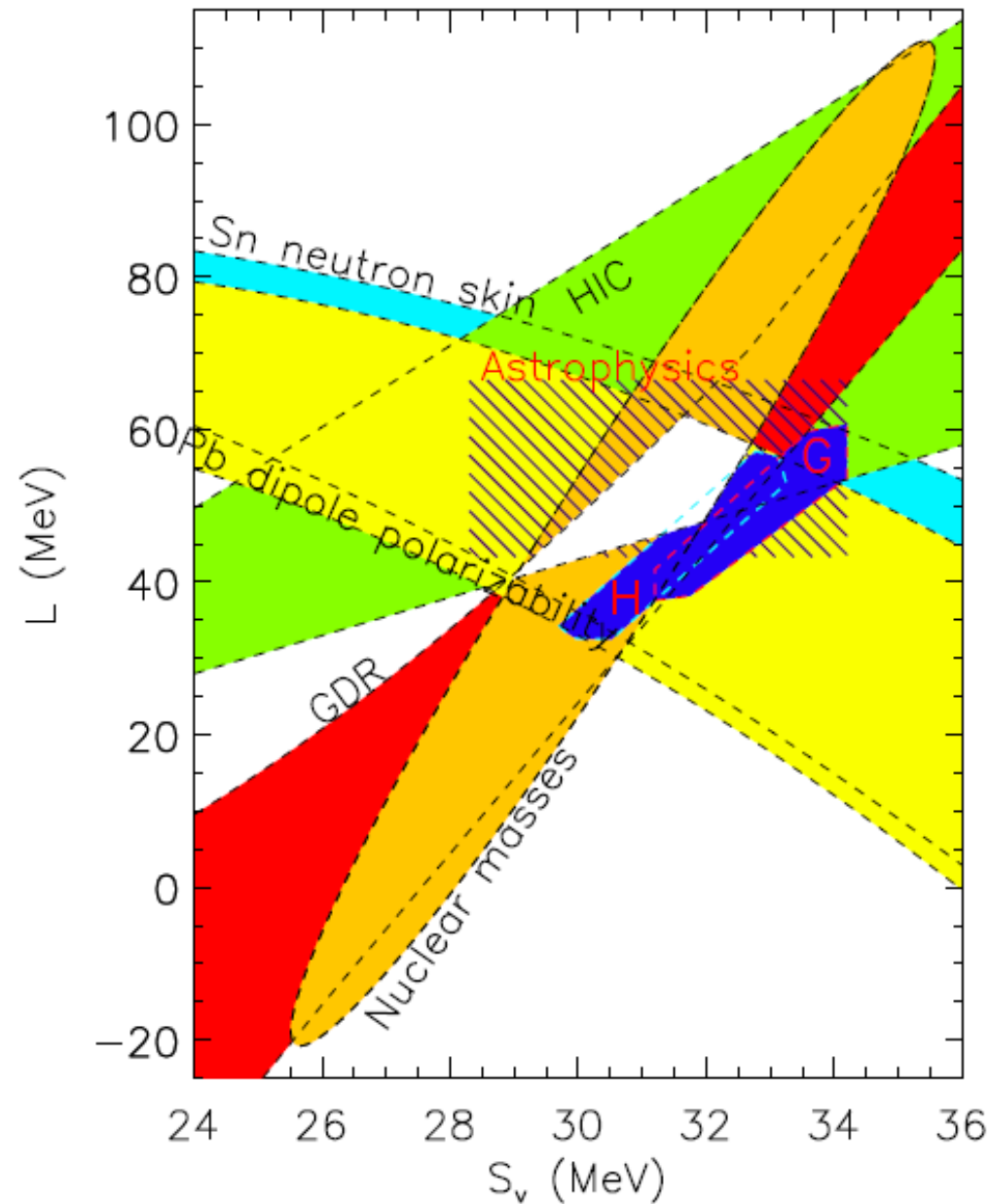
M. B. Tsang et al., Phys. Rev. C 86 (2012) 015803.

Nuclear Symmetry Energy (NuSYM 2011)



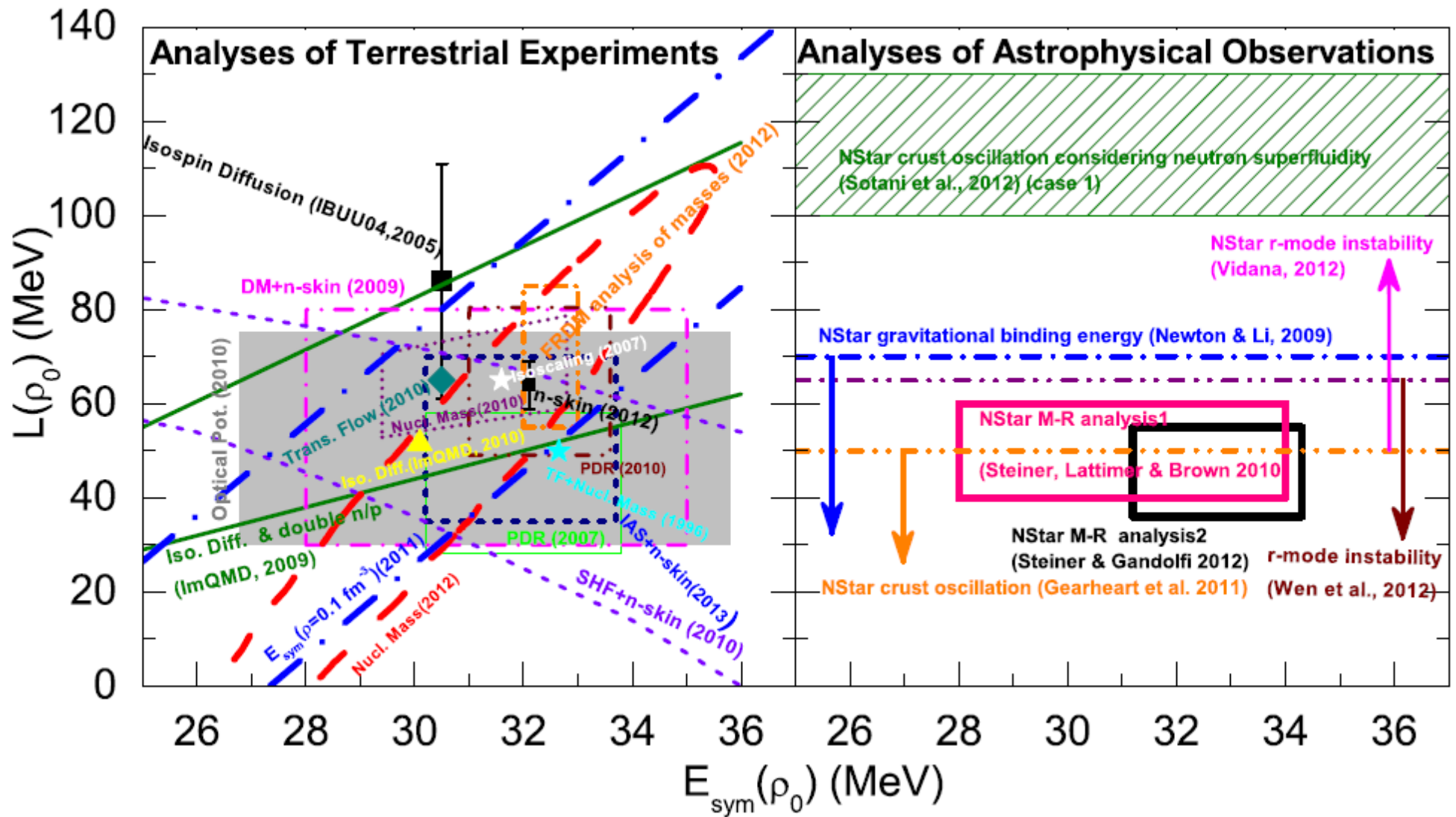
Tsang et al. ('12): NuSYM 2011

Nuclear Symmetry Energy (Lattimer-Lim, 2013)



Lattimer, Lim ('13)

Nuclear Symmetry Energy (NuSYM 2013)



B. A. Li et al. ('13)

Nuclear Mass

- Larger symmetry energy $\rightarrow$ B.E. of n-rich nuclei become smaller.

- Volume and surface symmetry energy

$$E_{\text{sym}}(A) = a_a(A) = S_v - S_s A^{-1/3}$$
$$(\delta = (N - Z)/A = 1 - 2Y_p)$$

$$B(A, Z) = a_v A - a_s A^{2/3} - a_c \frac{Z^2}{A^{1/3}} - a_a \frac{(N - Z)^2}{A} + a_p \frac{\delta_p}{A^\gamma}$$

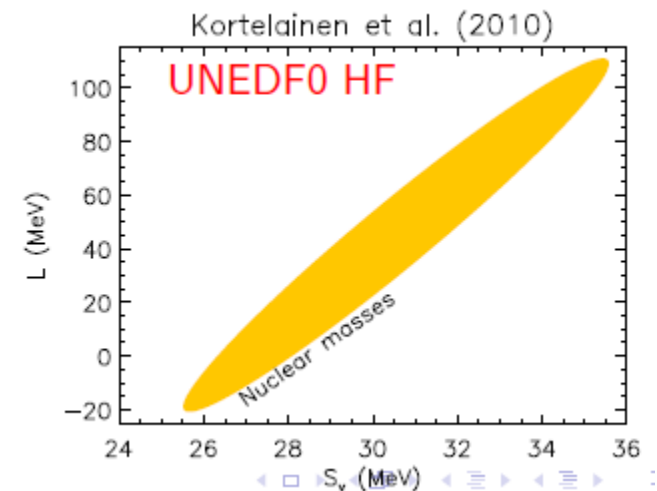
- Finite Range Droplet Model

*P. Moller, W.D. Myers, H. Sagawa, S. Yoshida,
Phys. Rev. Lett. 108, 052501 (2012)*

$$S_v = 32.5 \pm 0.5 \text{ MeV}, L = 70 \pm 15 \text{ MeV}$$

- Density Functional (UNEDF)

Kortelainen et al. (2010)



Lattimer, Lim ('13)

- When L and S_v are linearly correlated as

$$L = a S_v + b,$$

Symmetry energy is given as

$$E_{\text{sym}}(x) = S_v + L(x-1)/3 = S_v (1 + a(x-1)/3) + \text{const. (indep. of } S_v)$$
$$(x = \rho / \rho_0)$$

→ That observable determines symmetry energy most effectively at $x = 1 - 3/a$.

- Nuclear mass: $a \sim 14$

原子核質量は $x \sim 1 - 3/14 = 0.78$ 近辺の対称エネルギーをよく決める

Pigmy Dipole Resonance

■ E1 response of nuclei

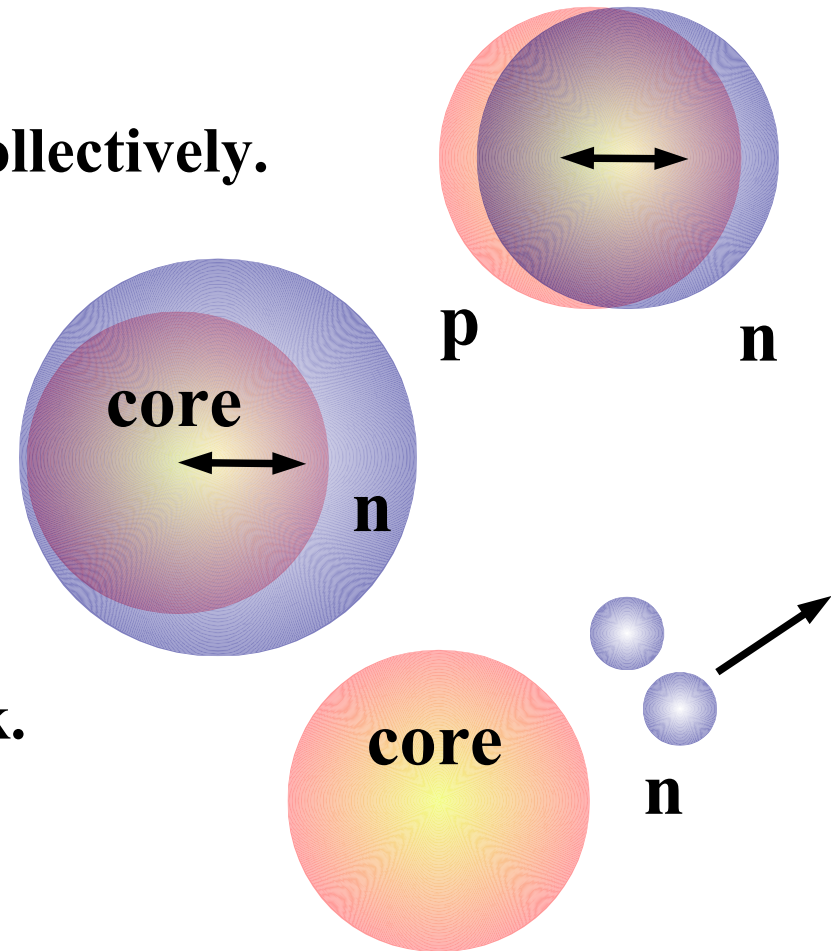
- Giant Resonance: p and n oscillates collectively.

$$E^* \sim 80 A^{-1/3} \text{ MeV}$$

- Pigmy Dipole Resonance:
Core oscillates in neutron skin/halo

$$E^* \sim (5-10) \text{ MeV}$$

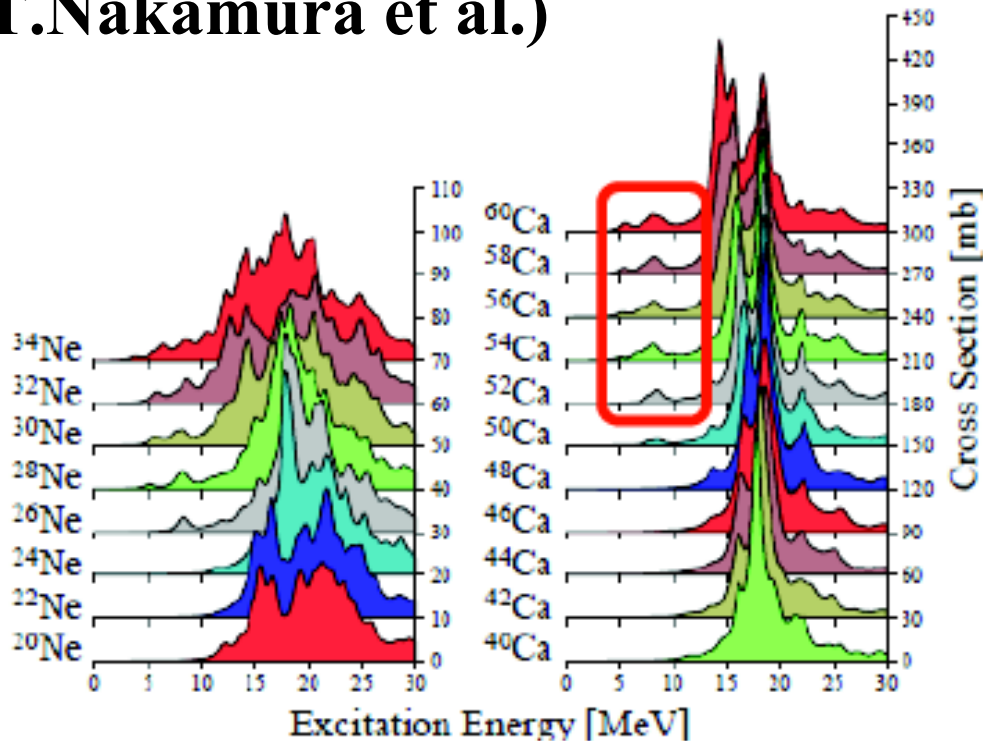
- Soft E1 excitation
When n wf is extended,
direct dissociation σ also shows a peak.



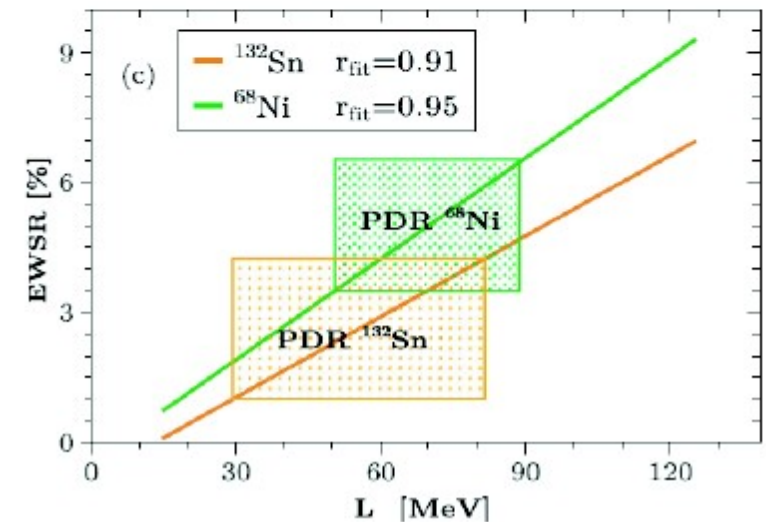
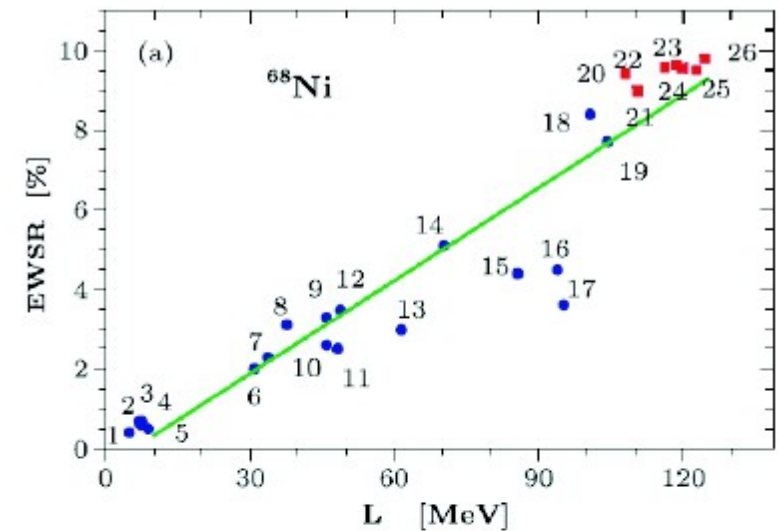
PDR would be sensitive to E_{sym}

PDR of neutron skin nuclei

- Energy Weighted Sum Rule value of PDR would have linear dep. on L
- PDR of very neutron rich nuclei will be measured at RIBF-SAMURAI (T.Nakamura et al.)



T.Inakura, T.Nakatsukasa, K.Yabana
Phys.Rev. C 84, 021302(R) (2011)

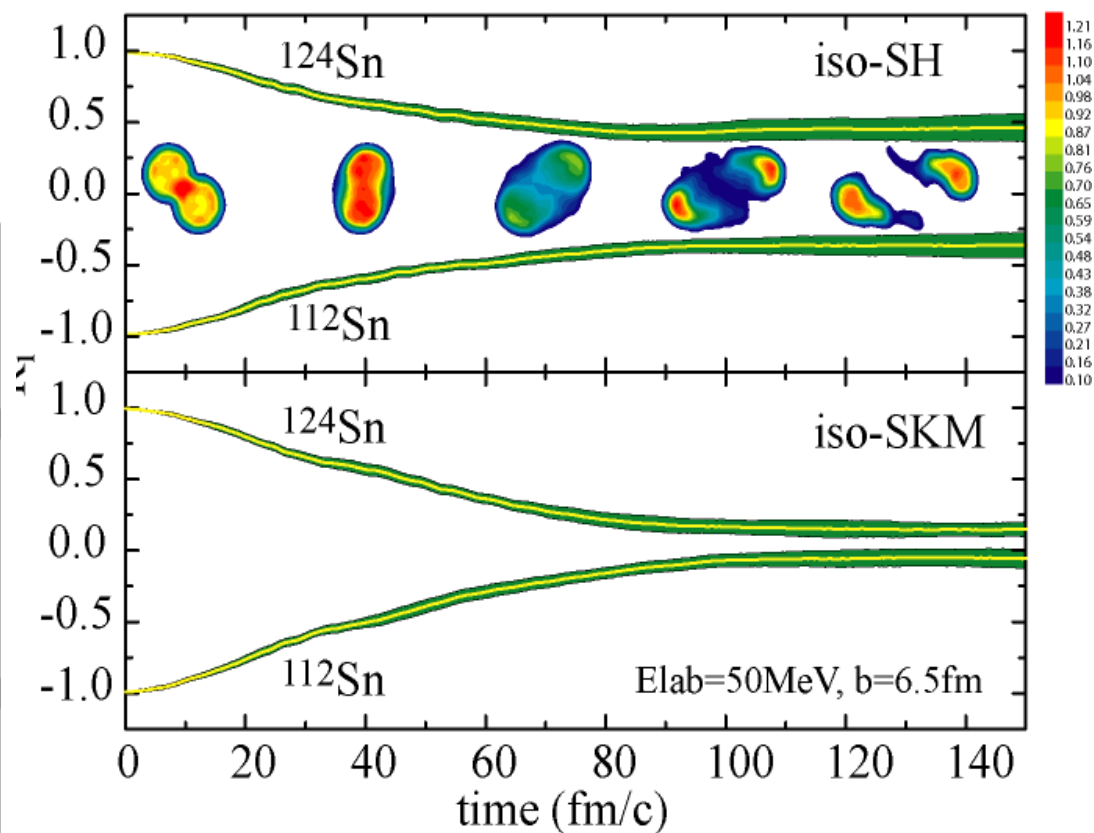
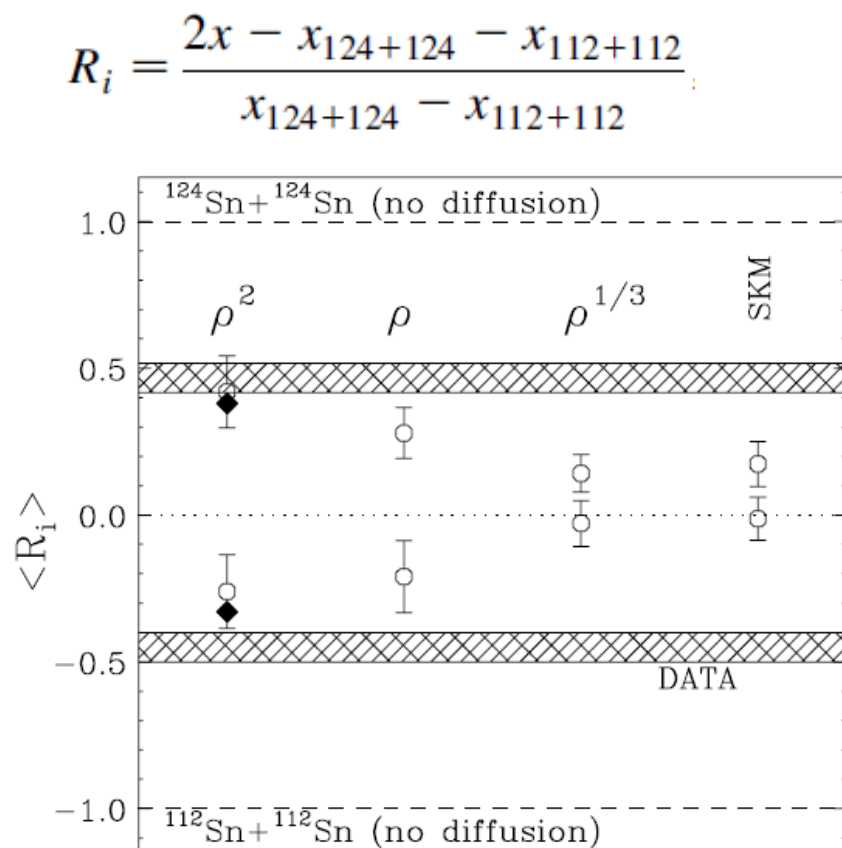


P. Adrich et al., *PRL* 95, 132501
 (2005). (GSI) $^{130,132}\text{Sn}$

O.Wieland et al., *PRL* 102, 092502
 (2009). (GSI) ^{68}Ni

Isospin Diffusion

- Collision of nuclei having different n/p ratio
→ fragments with medium n/p ratio will be formed
(Isospin diffusion)
- Driving force of isospin diffusion
= Symmetry energy



Tsang et al. ('04)

Skin Thickness & Dipole Polarizability

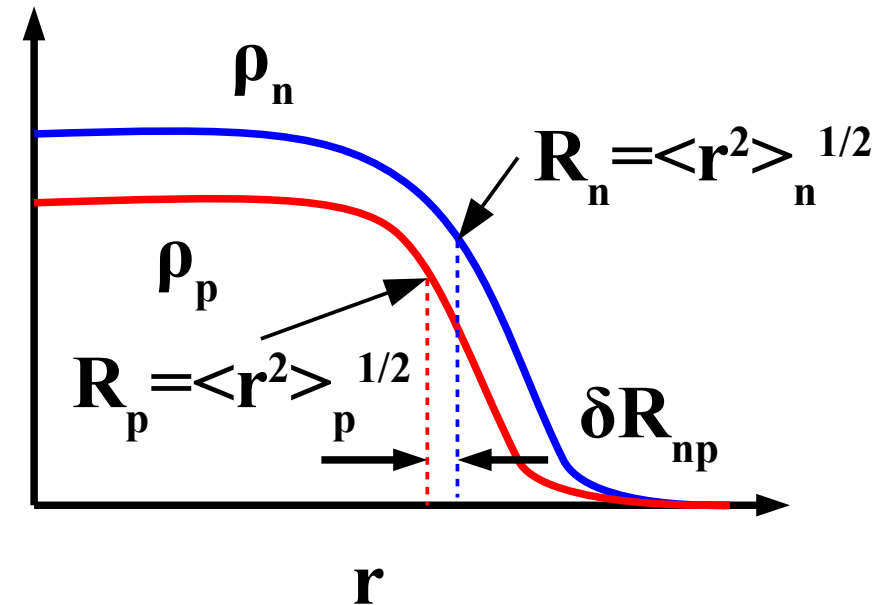
■ Skin Thickness δR_{np}

● Larger L

→ Small E_{sym} at low ρ

Large E_{sym} at high ρ

→ Larger δR_{np}



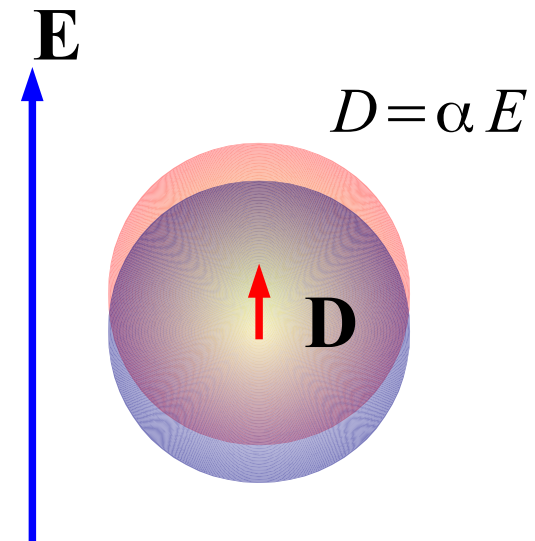
■ Electric Dipole Polarizability α

$$H = H_0 - e E \sum_{i \in p} x_i = H_0 - E \hat{D}$$

$$|\psi\rangle = |0\rangle - \sum_{n>0} \frac{|n\rangle \langle n|V|0\rangle}{E_n - E_0} + O(E^2)$$

$$D = \langle \psi | \hat{D} | \psi \rangle = 2E \sum_{n>0} \frac{\langle 0 | \hat{D} | n \rangle \langle n | \hat{D} | 0 \rangle}{E_n - E_0}$$

$$\alpha = \frac{8\pi}{9} \int \frac{dB(E1)}{\omega}$$

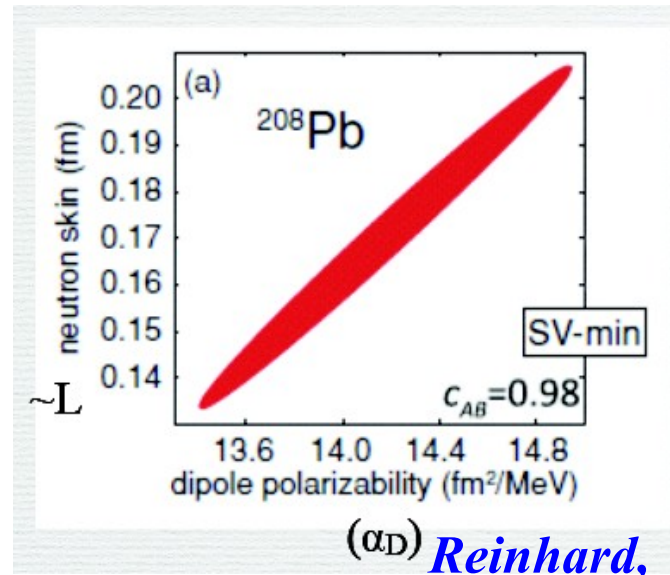


Skin Thickness & Dipole Polarizability

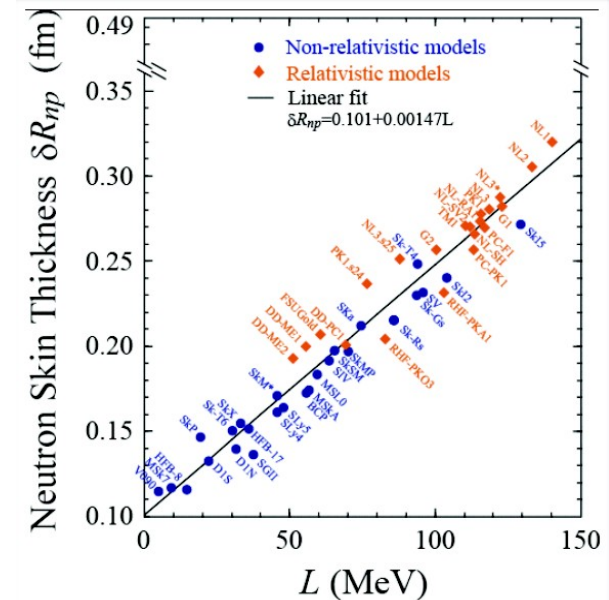
- Strong corr. btw α and skin thickness (smaller restoring force $\rightarrow$ soft)

- Skin thickness is also correlated with L .

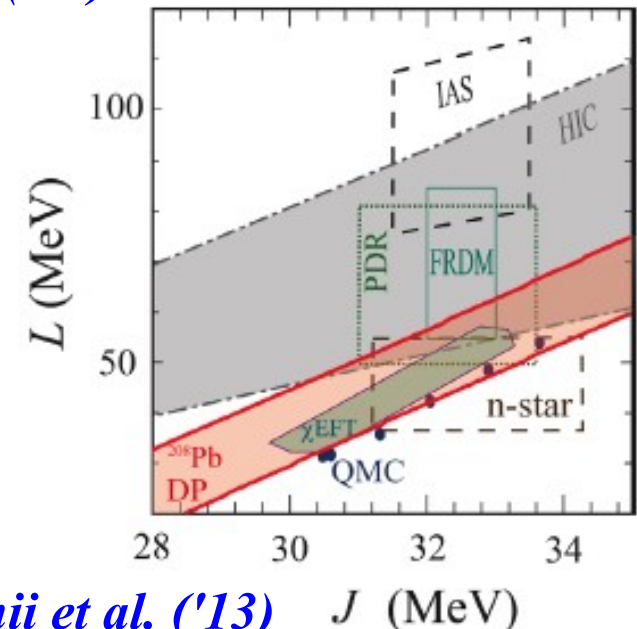
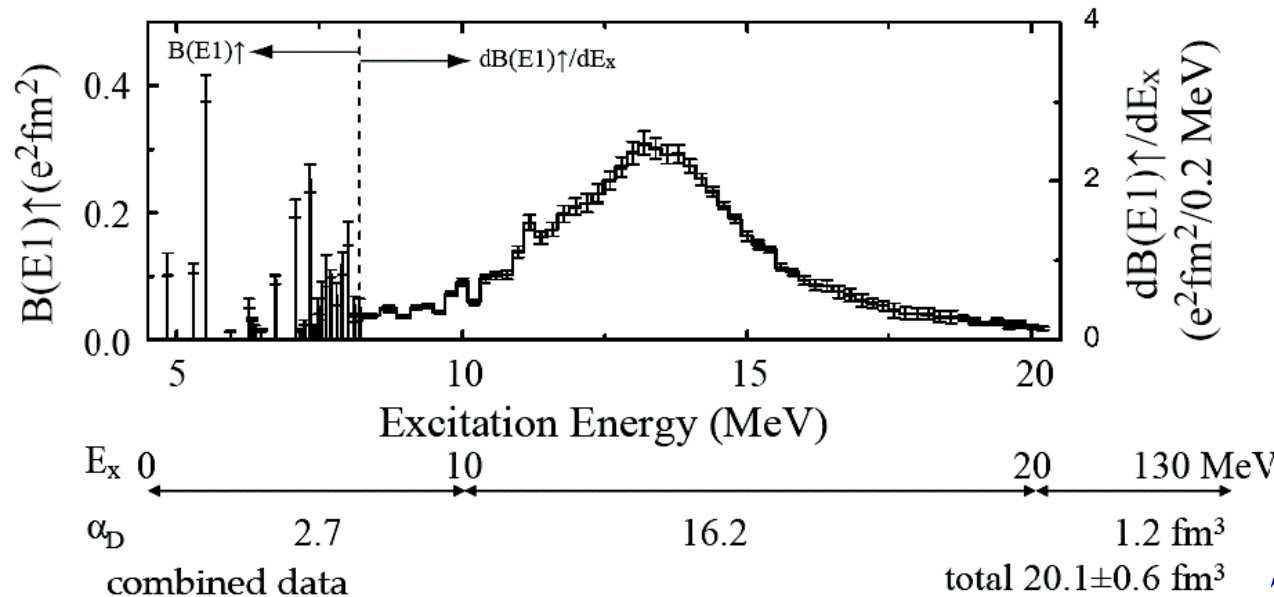
- Precise data from RCNP



Reinhard,
Nazarewicz ('10)

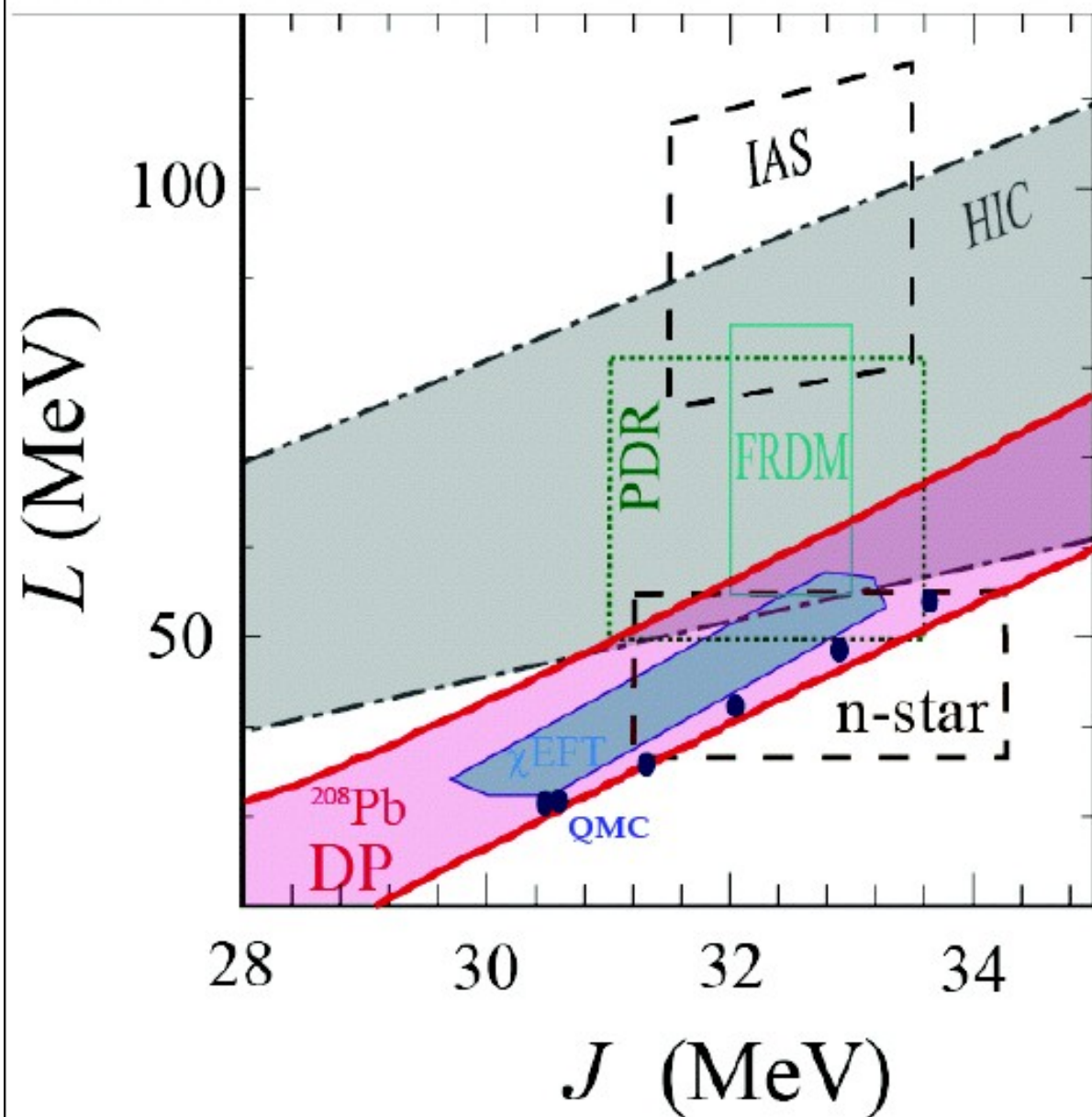


Roca-Maza et al. ('11)



Tamii et al. ('13)

Constraints on J and L



AT *et al.*, to be published in EPJA.

M.B. Tsang *et al.*, PRC86, 015803 (2012).

I. Tews *et al.*, PRL110, 032504 (2013)

DP: Dipole Polarizability (this work)

HIC: Heavy Ion Collision

PDR: Pygmy Dipole Resonance

IAS: Isobaric Analogue State

FRDM: Finite Range Droplet

Model (nuclear mass analysis)

n-star: Neutron Star Observation

(A.W. Steiner *et al.*,)

χEFT : Chiral Effective Field Theory

QMC: Quantum Monte-Carlo Calc.

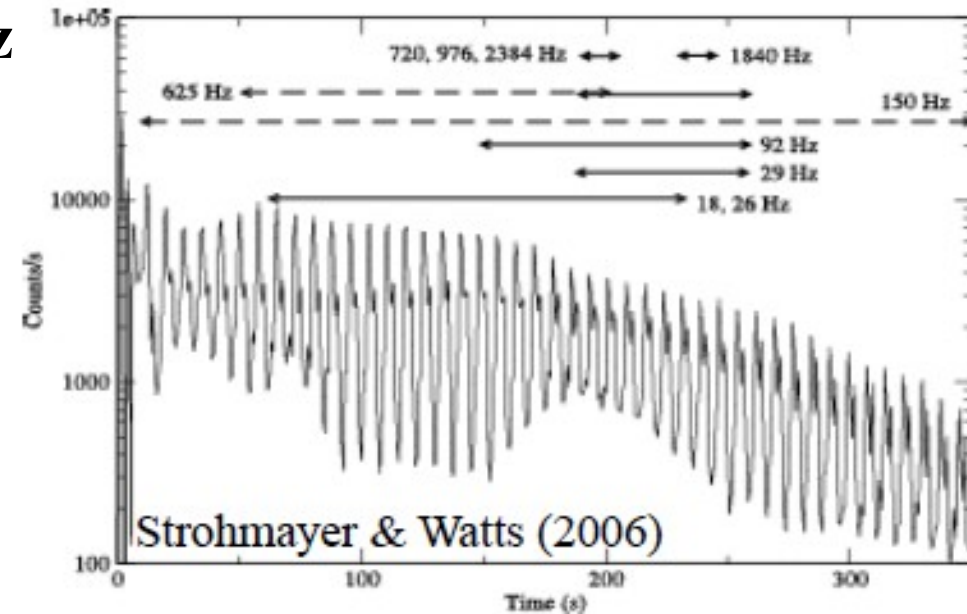
Quasi Periodic Oscillation of Neutron Stars

■ QPOs in afterglow of giant flares from soft-gamma repeaters (SGRs) (*Barat+ 83, Israel+ 05, Strohmayer & Watts 05, Watts & Strohmayer 06*)

- SGR 0526-66 (5th/3/1979) : 43 Hz

- SGR 1900+14 (27th/8/1998) :
28, 54, 84, 155 Hz

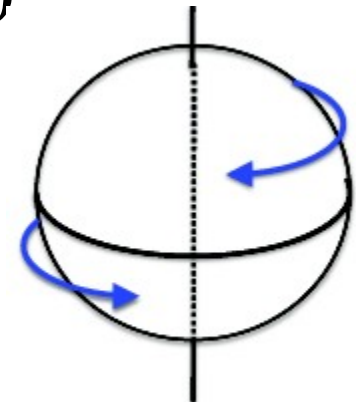
- SGR 1806-20 (27th/12/2004) :
18, 26, 30, 92.5, 150,
626.5, 1837 Hz



■ Asteroseismology

- From star quake to stellar properties (M, R, B, EOS ...)

- Low frequency (e.g. 28 Hz) requires long wave mode.
→ Torsional oscillations of the crust



QPO and Symmetry Energy

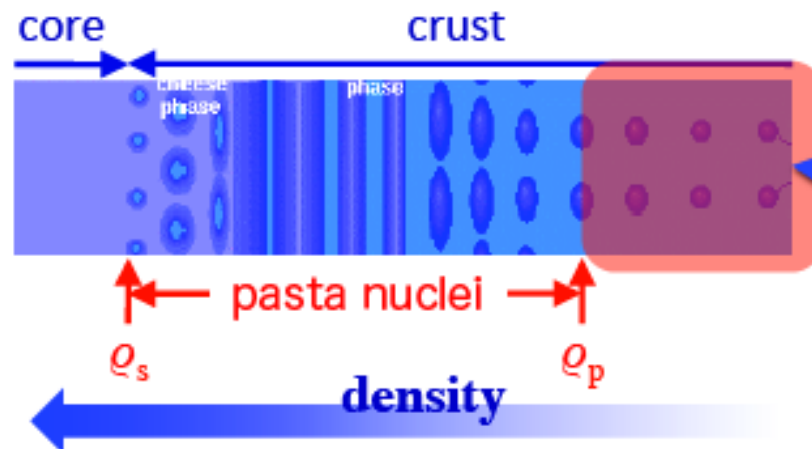
■ Torsional oscillations (ねじれ振動)

- incompressible (no density perturbations)

- Frequency ${}_l t_0 = \sqrt{l(l+1)} \frac{v_s}{2\pi R}$, $v_s = \sqrt{\mu/\rho}$

μ : shear modulus, v_s : shear velocity
(Hansen & Cioff 1980)

- Shear modulus of bcc lattice depends on nuclear charge
→ Dependence on the symmetry energy



for bcc lattice (Strohmayer+ 1991)

$$\mu = 0.1194 \frac{n_i (Ze)^2}{a}$$

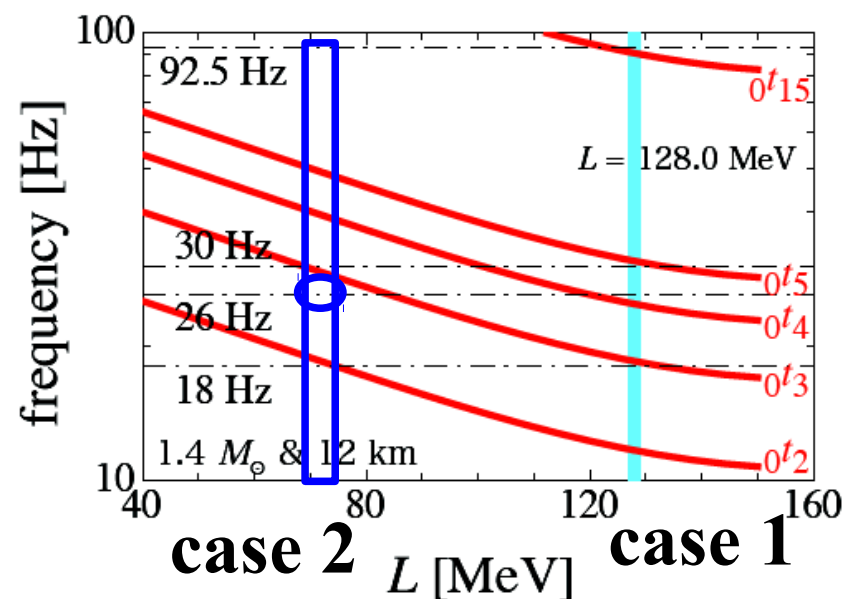
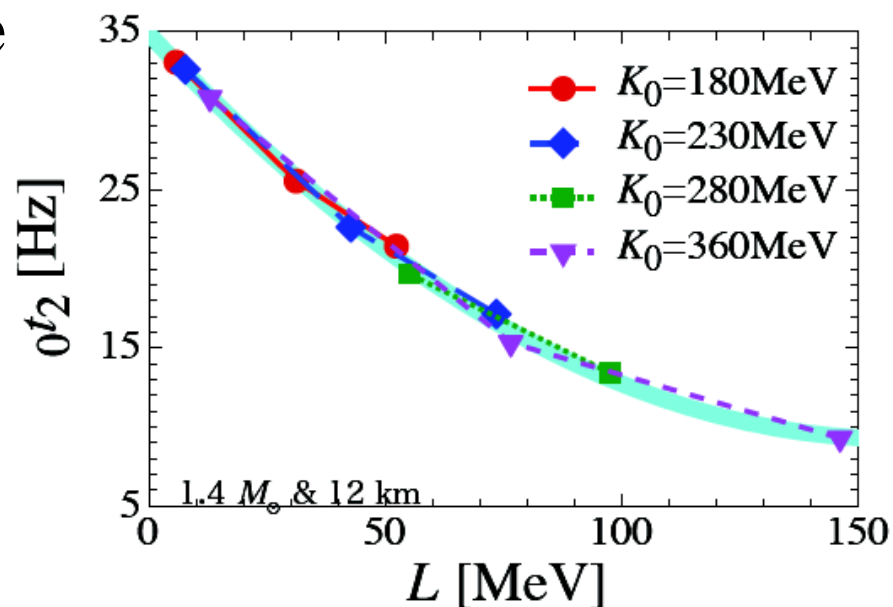
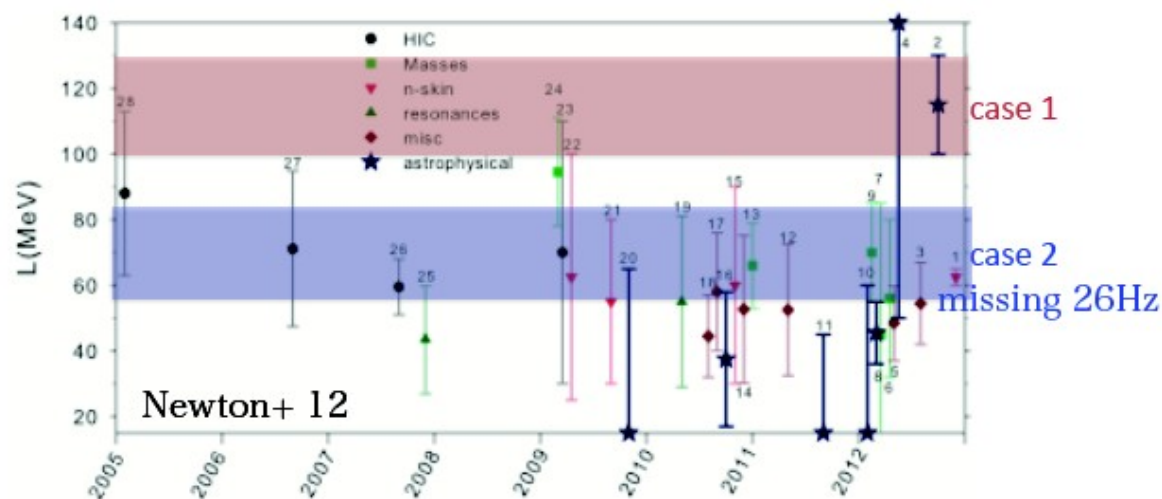
n_i : number density of quark droplet

Z : charge of quark droplet

a : Wigner-Seitz radius

QPO and Symmetry Energy

- For a given set of (M,R), we can solve TOV equation from the surface.
- Torsional oscillation frequencies are calculated using EOSs with various (L,K).
Oyamatsu, Iida ('03,'07)
- Compared with observed freq.
→ constrains on L (small dep. on K)



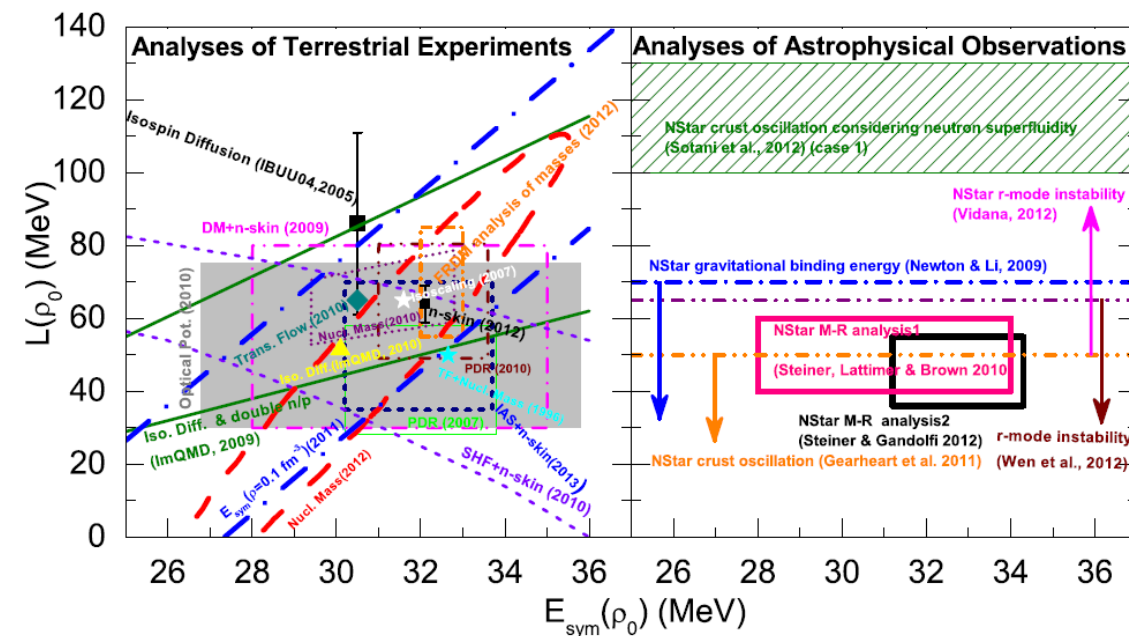
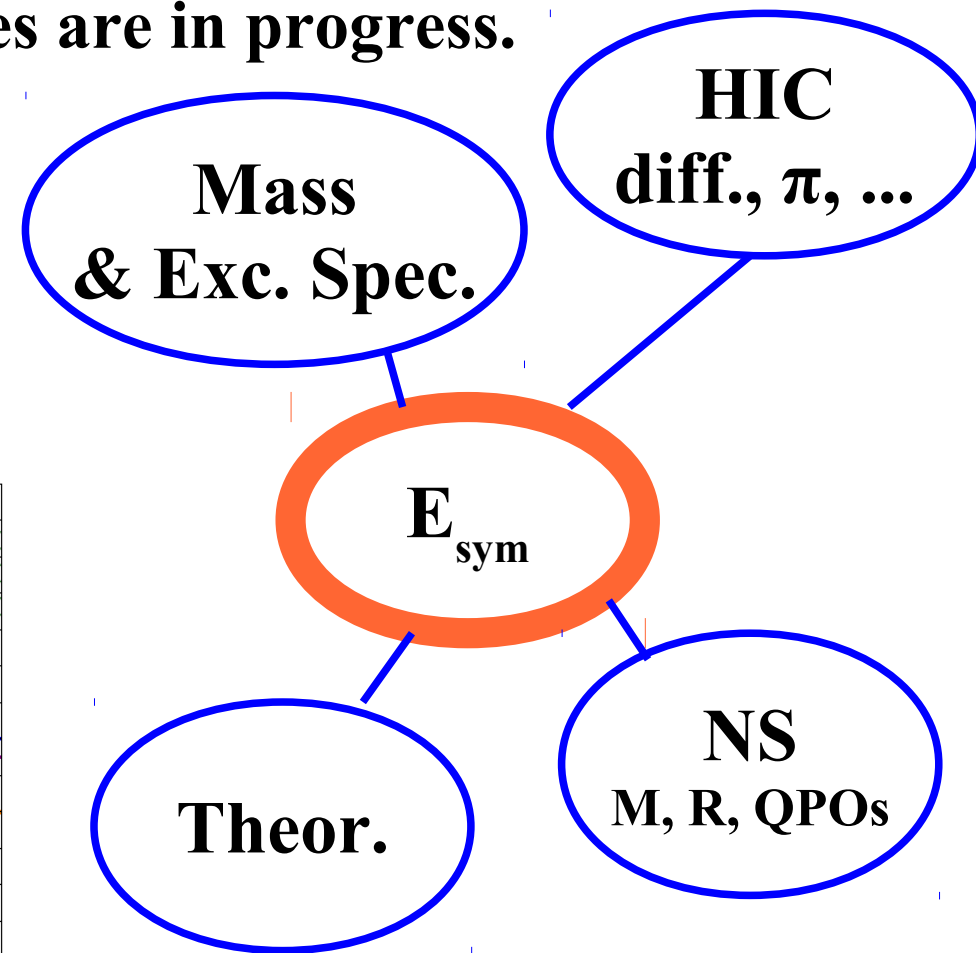
Large L or missing 26 Hz

Sotani+ '13

Yinishi @ Kyushu U., 2014 23

Summary

- Symmetry energy is decisive in neutron star matter EOS, and is related to various properties of nuclei.
- Experimental & Theoretical studies are in progress.
- If you have a new idea to determine E_{sym} , please propose as soon as possible.



- 中性子、陽子、電子のみからなる中性子星物質を考える。電子の質量を無視すると、核子あたりのエネルギーは、Lecture 1 で示したように

$$E_{\text{NSM}}(\rho) = E_{\text{SNM}}(\rho) + S(\rho)\delta^2 + \frac{\Delta M}{2}\delta + \frac{3}{8}\hbar k_F(1-\delta)^{4/3}$$

と与えられる。ここで $\Delta M = M_n - M_p$ 、 k_F は同じ密度での対象物質のフェルミ波数である。非対称度 δ は、核子あたりのエネルギーが最小になるように選ばれる。

- 上の表式を導け。
- 核子あたりのエネルギーが最小となる非対称度 δ を求めよ。
(3次方程式を解くこととなる。 $S(\rho)$, k_F , ΔM は与えられているとしてよい。)
- 今回の講義において、中性子物質の物理の課題の中で各自が興味を持った項目をあげ、その理由を述べよ。

Thank you !