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1. 直接反応理論

- 1. 核子-核子散乱:核力と位相差
- 2. ハドロン-核反応 (I): 光学模型
- 3. ハドロン-核反応 (II): インパルス近似
- 4. ハドロン-核反応 (III): グリーン関数法
- 5. (高エネルギー核反応: グラウバー模型、ハドロン共鳴)

2. (輸送模型)

- 1. 時間依存平均場理論(含:非相対論的平均場)
- 2. 半古典輸送模型とボルツマン方程式
- 3. 流体模型
- 3. 状態方程式を記述する理論模型
 - 1. 相対論的平均場理論
 - 2. 核子相関の役割 (G-matrix)
 - 3. 場の理論からのアプローチ:強結合格子 QCD



QCD Phase diagram and Nuclear Matter EOS

Phase diagram and EOS

= Two important aspects of Nuclear Matter

- Dense nuclear matter has rich physics
 - → Many-body theory, Exotic compositions, CEP, Astrophysical applications, ...



Nuclear matter EOS

NPQS

Subjects in Nuclear, Quark-Hadron, Particle, Astro, and Condensed Matter Physics !

What is EOS?

- Equation of State (EOS) of Ideal Gas (理想気体の状態方程式) $PV = NkT \rightarrow P = \rho T \quad (\rho = N/V, k=1)$
- Self-binding system \rightarrow Null pressure density (ρ_0) exists.

$$P = P(\rho, T, ...), \quad E/A = -\epsilon_0 + \frac{K}{18\rho_0^2}(\rho - \rho_0)^2 + \cdots$$

 ϵ_0 : Saturation E. (~-16 MeV), ρ_0 : Saturation density (~0.16 fm⁻³), K: incompressibility (~ 200-300 MeV)



One of the "Ultimate" Goals in Nuclear Physics



Nuclear force on the Lattice

- BS wave function → Lattice NN Pot.
 - Starting from wall source, and measure Bethe-Salpeter ampl.
 - By using Schrodinger-type Eq., NN potential is obtained.
- Lot of achievements !
 - One pion exchange potential tail. 1000
 - Repulsive core from quark Pauli principle.
 - YN potential, MB potential, …
- **Needs further studies for EOS** $\overset{\mathfrak{H}}{\triangleright}$

S. Aoki, T. Hatsuda, N. Ishii, PTP 123('10)89 Ishii, Aoki, Hatsuda, PRL 99 ('07) 022001 Nemura et al, arXiv:1005.5352 [hep-lat] H. Nemura, Ishii, Aoki, Hatsuda, PLB673('09)136.



Ab Initio Calculations

- Chiral EFT + RG evolution to low momenta
 - N3LO NN + NNLO 3N force E. Epelbaum, H.-W. Hammer, U.-G. Meißner, RMP81('09)1773.
 - 3N force → ρ dep. NN force S.K.Bogner, T.T.S.Kuo, A.Schwenk, PRep386('03)1.
- Neutron matter results
 - Consistent with other "rigorous" results such as APR

A.Akmal, V.R.Pandharipande, D.G.Ravenhall, PRC58('98)1804.

- → Understanding of the origin of phen. 3-body repl. in APR.
- Related work:
 - QMC on the lattice
 T. Abe, R. Seki, PRC79('09)054002.





K. Hebeler, A. Schwenk, arXiv:0911.0483

Symmetry Energy (1)

Recent data suggest that EOS becomes softer in asymmetric nuclear matter.

 $K = K_{sym} + K_{asy} \delta^2$, $K_{asy} \sim -550 \text{ MeV}$ $E_{sym} \simeq 31.6 (\rho / \rho_0)^{1.05} \text{ MeV}$

(c.f. E_{sym} ~ 23 MeV from Mass formula)

- Isoscalar Giant Monopole Resonance (ISGMR) of Sn isotopes
 - ISGMR in Isotope chain (¹¹²Sn ~ ¹²⁴Sn) is systematically studied.



ØQ:



T. Li, U. Garg, et al., PRC81('10), 034309.

核多体系物理学, 2010/11/11

Symmetry Energy (2)





X-ray measurements of Neutron Stars

- Neutron star mass (M)-radius (R) curve uniquely(*) determines NS matter EOS.
 - Radius measurement: flux + temperature → apparent radius



Eddington flux would give another info.

Ozel, Baym & Guver, arXiv: 1002.3153 [astro-ph.HE]

• Bayesian TOV inversion \rightarrow EOS



A. W. Steiner, J. M. Lattimer, Ed. Brown, arXiv:1005.0811





Tolman-Oppenheimer-Volkoff (TOV) equation

TOV Eq. = General Relativistic Balance of pressure and gravity



Neutron Star Mass = M(R) where P(R)=0

When you make a new EOS, please check the NS mass !



Black Hole Formation (Failed Supernova)



Sumiyoshi, Ishizuka, AO, Yamada, Suzuki, 2009

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Black Hole Formation

■ Black Hole Formation: (ρ_B , T, Y_e) ~ (4 ρ_0 , 70 MeV, 0.2) → Hyperon fraction ~ 10 %

(K. Sumiyoshi, C. Ishizuka, AO, S. Yamada, H. Suzuki, ApJ690(09)L43)







Relativistic Mean Field



Theories/Models for Nuclear Matter EOS

- Ab initio Approach
 - LQCD, GFMC, Variational, DBHF, G-matrix
 → Not easy to handle, Not satisfactory for phen. purposes
- Mean Field from Effective Interactions ~ Nuclear Density Fuctionals
 - Skyrme Hartree-Fock(-Bogoliubov)
 - Non.-Rel.,Zero Range, Two-body + Three-body (or ρ-dep. two-body)
 - In HFB, Nuclear Mass is very well explained (Total B.E. ΔE ~ 0.6 MeV)
 - Causality is violated at very high densities.
 - Relativistic Mean Field
 - Relativistic, Meson-Baryon coupling, Meson self-energies
 - Successful in describing pA scattering (Dirac Phenomenology)



Relativistic Mean Field (1)

- Relativistic Mean Field
 - = Nuclear scalar and vector mean field generated by mesons
 - \rightarrow Why do we use relativistic framework ?
 - Nuclear Force is mediated by mesons

 → Let's consider meson-baryon system !
 (Entrance of Hadron Physics)

---Ο σ, ω, ρ,...

- We are also interested in Dense Matter EOS

 → Sound velocity exceeds the Speed of Light (=c) with Non.-Rel.
 MF
- Success of "Dirac Phenomenology" (Dirac Eq. for pA scattering → Spin Observables)
 - → Strong Scalar and Vector Mean Fields are preferable to explain Spin Observables
- DBHF (Dirac-Brueckner-Hatree-Fock)

RMF is a good starting point as a framework of hadronic system including Nuclei and Nuclear Matter

Dirac Phenomenology

E.D. Cooper, S. Hama, B.C. Clark, R.L. Mercer, PRC47('93),297



EOS in Dirac-Brueckner-Hartree-Fock

R. Brockmann, R. Machleidt, PRC42('90),1965

- Non Relativistic Brueckner Calculation → Nuclear Saturation Point cannot be reproduced (Coester Line)
- Relativistic Approach (DBHF)
 - → Relativity gives additional repulsion, leading to successful description of the saturation point.





Relativistic Mean Field (2)

- Mean Field treatment of meson field operator
 - = Meson ield operator is replaced with its expectation value $\varphi(r) \rightarrow \langle \varphi(r) \rangle$

Ignoring fluctuations compared with the expectation value may be a good approximation at strong condensate.

- Which Hadrons should be included in RMF ?
 - Baryons (1/2+) p, n, $\Lambda, \Sigma, \Xi, \Delta, ...$
 - Scalar Mesons (0+) $\sigma(600)$, $f_0(980)$, $a_0(980)$, ...
 - Vector Mesons (1-) ω(783), ρ(770), φ(1020),
 - Pseuso Scalar (0-) π, K, η, η',
 - Axial Vector (1+) a_1, \dots

We require that the meson field can have uniform expectation values in nuclear matter.

 \rightarrow Scalar and Time-Component of Vector Mesons (σ , ω , ρ ,)

$\sigma \omega$ Model (1)

Serot, Walecka, Adv.Nucl.Phys.16 (1986),1

Consider only σ and ω mesons

Lagrangian

00

$$\begin{split} L = \bar{\psi} \left(i \, \gamma^{\mu} \partial_{\mu} - M + g_s \sigma - g_v \omega \right) \psi \\ + \frac{1}{2} \partial_{\mu} \sigma \, \partial^{\mu} \sigma - \frac{1}{2} m_s^2 \sigma^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_v^2 \omega_{\mu} \omega^{\mu} \\ \left(F_{\mu\nu} = \partial_{\mu} \omega_{\nu} - \partial_{\nu} \omega_{\mu} \right) \end{split}$$

σ, ω

Equation of Motion

$$\frac{\partial}{\partial x^{\mu}} \left[\frac{\partial L}{\partial (\partial_{\mu} \phi_{i})} \right] - \frac{\partial L}{\partial \phi_{i}} = 0$$

$$\sigma : \left[\partial_{\mu} \partial^{\mu} + m_{s}^{2} \right] \sigma = g_{s} \overline{\psi} \psi$$

Euler-Lagrange Equation

EOM of ω (for beginners)

Euler-Lagrange Eq.

$$\partial_{\mu}F^{\mu\nu} + m_{\nu}^{2}\omega^{\nu} = g_{\nu}\bar{\psi}\gamma^{\nu}\psi$$

Divergence of LHS and RHS

$$\partial_{\nu}\partial_{\mu}F^{\mu\nu} + m_{\nu}^{2}(\partial_{\nu}\omega^{\nu}) = m_{\nu}^{2}(\partial_{\nu}\omega^{\nu}) = g_{\nu}(\partial_{\nu}\bar{\psi}\gamma^{\nu}\psi) = 0$$

LHS: derivatives are sym. and $F_{\mu\nu}$ is anti-sym. RHS: Baryon Current = Conserved Current

Put it in the Euler-Lagrange Eq.

$$\partial_{\mu}F^{\mu\nu} = \partial_{\mu}(\partial^{\mu}\omega^{\nu} - \partial^{\nu}\omega^{\mu}) = \partial_{\mu}\partial^{\mu}\omega^{\nu} - \partial^{\nu}(\partial_{\mu}\omega^{\mu}) = \partial_{\mu}\partial^{\mu}\omega^{\nu}$$



Schroedinger Eq. for Upper Component

Dirac Equation for Nucleons

$$i\gamma\partial -\gamma^0 U_v - M - U_s \psi = 0$$
, $U_v = g_\omega \omega$, $U_s = -g_\sigma \sigma$

Decompose 4 spinor into Upper and Lower Components

$$\begin{array}{ccc} E - U_v - M - U_s & i\sigma \cdot \nabla \\ -i\sigma \cdot \nabla & -E + U_v - M - U_s \end{array} \left(\begin{array}{c} f \\ g \end{array} \right) = 0 & \begin{array}{c} g = \frac{-i}{E + M + U_s - U_v} (\sigma \cdot \nabla) f \\ (E - M - U_v - U_s) f = -i(\sigma \cdot \nabla) g \end{array} \right)$$

Erase Lower Component (assuming spherical sym.)

$$-i(\sigma \cdot \nabla)g = -(\sigma \cdot \nabla)\frac{1}{X}(\sigma \cdot \nabla)f = -\frac{1}{X}\nabla^{2}f - \frac{1}{r}\left[\frac{d}{dr}\frac{1}{X}\right](\sigma \cdot r)(\sigma \cdot \nabla)f = -\nabla\frac{1}{X}\nabla f + \frac{1}{r}\left[\frac{d}{dr}\frac{1}{X}\right](\sigma \cdot l)f$$
$$(\sigma \cdot r)(\sigma \cdot \nabla) = (r \cdot \nabla) + i\sigma \cdot (r \times \nabla) = r \cdot \nabla - \sigma \cdot l$$

Schroedinger-like" Eq. for Upper Component

$$-\nabla \frac{1}{E+M+U_s-U_v} \nabla f + \left(U_s+U_v+U_{LS}(\sigma \cdot l)\right) f = (E-M)f$$

 $U_{LS} = \frac{1}{r} \left[\frac{d}{dr} \frac{1}{E + M + U_s - U_v} \right] < 0$ on surface

(Us,Uv)~(-350 MeV,280 MeV)→Small Central(Us+Uv), Large LS

Various Ways to Evaluate Non.-Rel. Potential

From Single Particle Energy

$$\begin{split} \left(\begin{split} & \chi^0(E - U_v) + i \, \gamma \cdot \nabla - (M + U_s) \right) \psi = 0 \quad \rightarrow \quad (E - U_v)^2 = p^2 + (M + U_s)^2 \\ & \rightarrow E = \sqrt{p^2 + (M + U_s)^2} + U_v \approx E_p + \frac{M}{E_p} U_s + U_v + \frac{p^2}{2 \, E_p^3} U_s^2 \\ & (E_p = \sqrt{p^2 + M^2}) \end{split}$$

Schroedinger Equivalent Potential (Uniform matter)

$$-\frac{\nabla^2}{2M}f + \left[U_s + \frac{E}{M}U_v + \frac{U_s^2 - U_v^2}{2M}\right]f = \frac{E + M}{2M}(E - M)f$$
$$U_{\text{SEP}} \approx U_s + \frac{E}{M}U_v$$

Anyway, slow baryons feel Non.-Rel. Potential,

$$U \approx U_s + U_v = -g_s \sigma + g_v \omega$$



Nuclear Matter in σω Model

Serot, Walecka, Adv.Nucl.Phys.16 (1986),1

Uniform Nuclear Matter



核多体系物理学, 2010/11/11

$\sigma \omega$ model --- pros and cons

- Pros (merit)
 - Foundation is clear: based on the success of Dirac phen. and DBHF.
 - Simple description of scalar and vector potential in σ and ω mesons.
 - Saturation is well described in two parameters.
 - Natural explanation of large LS potential in nuclei.
- Cons (shortcomings)
 - Relation with the bare NN interaction is not clear.
 - Especially, pion effects are not included.
 - Symmetry energy is too small.
 - Incompressibility is too large (K ~ 600-700 MeV) (c.f. Empirical value K ~ (200-300) MeV)
 - Chiral symmetry is not respected.



High Quality RMF models

Variety of the RMF models

→ MB couplings, meson masses, meson self-energies

- σN , ωN , ρN couplings are well determined \rightarrow almost no model deps. in Sym. N.M. at low ρ
- ω⁴ term is introduced to simulate DBHF results of vector pot. *TM*: Y. Sugahara, H. Toki, NPA579('94)557; R. Brockmann, H. Toki, PRL68('92)3408.
- σ^3 and σ^4 terms are introduced to soften EOS at ρ_0 .

J. Boguta, A.R.Bodmer NPA292('77)413, NL1:P.-G.Reinhardt, M.Rufa, J.Maruhn, W.Greiner, J.Friedrich, ZPA323('86)13. NL3: G.A.Lalazissis, J.Konig, P.Ring, PRC55('97)540.

 \rightarrow Large differences are found at high ρ





TIP QS

K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.

How to determine Non-Linear terms ? (1)

Method 1: Fit as many as known observables

EOS, Nuclear B.E., High density EOS from HIC, Vector potential in DBHF, Neutron Star, ...



How to determine Non-Linear terms ? (2)

- Method (2): Fix parameters by using symmetry, such as the *Chiral Symmetry*
- Chiral Symmetry
 - Fundamental symmetry of massless QCD, and its spontaneous breaking generates hadron masses. Nambu, Jona-Lasinio ('61)
 - Many of the linear σ models are unstable against finite density (chiral collapse).
 → Log type chiral potential Sahu, Tsubakihara, AO('10), Tsubakihara, AO('07), Tsubakihara et al.('10)
 - Non-linear representation (chiral pert.) leads to density dependent coupling from one- and two-pion exchanges. *Kaiser, Fritsch, Weise ('02), Finelli, Kaiser, Vretener, Weise ('04)*





RMF with Hyperons --- A hypernuclei

Why Λ ?

- Λ is expected to appear in NS.
- Coupling with π, σ, ... are different

 → detailed study of Λ hypernnuclei
 will tell us what makes MF
 (OBEP or π)
- Coupling with mesons : $x_M = g_{M\Lambda}/g_{MN}$ quark counting: $x_{\sigma} \sim 2/3$ π exchanges: $x_{\sigma} \sim 1/3$ \rightarrow Which is true ?
- Single Λ hypernuclei
 - Λ Sep. E. \rightarrow U_{Λ} ~ -30 MeV ~ 2/3 U_N
 - $\rightarrow \ \mbox{We can fit them by changing} \\ g_{\sigma\Lambda}, g_{\omega\Lambda}, g_{\zeta\Lambda}, ...$





 Tsubakihara,
 H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.

 Y
 TP

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RMF with Hyperons --- Double A hypernuclei

Nagara event $\Delta B_{\Lambda\Lambda} \sim 1.0$ MeV (weakly attractive)

• TM & NL-SH based RMF *H. Shen, F. Yang, H. Toki, PTP115('06)325.* Model 1: $x_{\sigma} = 0.621, x_{\omega} = 2/3 \text{ (no } \zeta, \varphi)$ Model 2: $R_{\zeta} = g_{\zeta\Lambda} / g_{\sigma N} = 0.56 - 0.57,$ $R_{\phi} = g_{\phi\Lambda} / g_{\omega N} = -\sqrt{2/3}$

• Chiral SU(3) RMF K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206. SU(3)_f for vector coupling $x_{\omega} = 0.64, R_{\phi} = 0.504$ Det. (KMT) int. mixes σ and ς M. Kobayashi, T. Maskawa, PTP44('70)1422 G. 't Hooft, PRD14('76)3432. $\rightarrow x_{\sigma} = 0.335, R_{\varsigma} = 0.509$





Hyperon Composition in Dense Matter

- **Hyperon start to emerge at (2-3)** ρ_0 in Neutron Star Matter !
- Hyperon composition in NS is sensitive to Hyperon potential.
 - $U_{\Lambda} \sim -30$ MeV: Well-known
 - U_E ~ -(12-15) MeV (K⁻,K⁺) reaction, twin hypernuclei
 P. Khaustov et al. (E885),PRC61('00)054603; S. Aoki et al., PLB355('95)45.
 - $U_{\Sigma} \sim -30$ MeV (Old conjecture) $\rightarrow \Sigma^{-}$ appears prior to Λ
 - U_Σ > 0 (repulsive) → No Σ in NS
 Σ atom (phen. fit), QF prod.
 H. Noumi et al., PRL89('02)072301;
 T. Harada, Y. Hirabayashi, NPA759('05)143;
 M. Kohno et al. PRC74('06)064613.



J. Schaffner-Bielich, NPA804('08)309.



Hyperon Composition in Dense Matter

- Comparison of Hyperon Composition
 - U_{Σ} =-30 MeV, U_{Ξ} = -28 MeV \rightarrow SU(3) sym. matter at $\rho_{B} \sim 10 \rho_{0}$ Schaffner, Mishustin ('94)
 - U_Σ=+30 MeV, U_Ξ = -15 MeV→ Σ baryons are strongly suppressed.
 C.Ishizuka, AO, K.Tsubakihara, K.Sumiyoshi, S.Yamada, JPG35('08)085201.

Neutron Star Matter



Σ atom data



C.J.Batty, E.Friedman, A.Gal, PLB335('94)273 **Batty's DD potential is very** repulsive inside nuclei. \rightarrow No Σ baryon in dense matter.







1PQS

THEORETICAL PHYSIC

K.Tsubakihara, H.Maekawa, AO, EPJA33('07)295.

Σ atom in RMF



Σ atom and Neutron Star

- Σ may not feel *very* repulsive potential in neutron star....
 - ρ^{γ} -type fit \rightarrow very repulsive
 - RMF fit → small isovector potential
 - \rightarrow QF prod. may support the latter. Σ^{-} would appear in NS.



T. Harada, Y. Hirabayashi, NPA767('06)206



Neutron Star Mass

- Large fraction of hyperons softenes EOS at ρ_B > (0.3-0.4) fm⁻³
 - NS star max. mass red. $\sim 1 M_{sun}$.
 - RMF generally predicts stiff EOS at high density. (Scalar attraction saturation, or Z-graph in NR view.)
 - Some of RMF with Y do not support 1.44 M_{sun}.
- Additional Repulsion at high ρ ?
 - Vector mass mod.
 → stronger repulsion at high ρ.
 M. Naruki et al., PRL96('06)092301.

Another term such as NNωσ.

C. Ishizuka, AO, K. Tsubakihara, K. Sumiyoshi, S. Yamada, JPG35('08)085201. Katsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.



Bruckner-Hartree-Fock theory with Hyperons

- Microscopic G-matrix calculation with realistic NN, YN potential and microscopic (or phen.) 3N force (or 3B force).
 - Interaction dep. (V18, N93, ...) is large → Need finite nuclear info. E.Hiyama, T.Motoba, Y.Yamamoto, M.Kamimura / M.Tamura et al.
 - NS collapses with hyperons w/o 3BF.



H.J.Schulze, A.Polls, A.Ramos, I.Vidana, PRC73('06),058801.



S. Nishizaki, T. Takatsuka, Y. Yamamoto, PTP108('02)703.

Summary

- Nuclear Matter EOS is important in various aspects of Nuclear Physics
- Relativistic Mean Field may be a good starting point to describe hadronic (baryon and meson) systems.
 - Relativistic → Saturation, Causality
 - Based on successes of Dirac Phenomenology and DBHF
 - Covariant Density Functional

 \rightarrow It is desirable to obtain E/V (energy density) in fundamental theories.

(Renormalizability is not required.)

- We can re-write RMF equations in Schroedinger-like eqs. We may consider it as a method to parameterize DF in a transparent manner.
- Higher order terms / Density dependence of the coupling constants (not mentioned) → Necessary for precise description of nuclei, but need foundations of extension.

