

# 核多体系物理学

- 授業の概要・目的：核子・ハドロン・クォークからなる多体系の性質を状態方程式、および核反応論の観点から議論する。核物質の状態方程式を記述するために必要となる核多体理論(平均場理論、G-matrix、熱場の理論、強結合格子 QCD)、ハイパー核生成反応や重イオン反応を理解する上で必要とされる原子核核反応理論(直接反応、輸送模型等)、等の理論の枠組について解説すると共に、これらについての最近の研究成果についても紹介する。
- 授業計画と内容  
核子・ハドロン・クォーク物質の相互作用と状態方程式について以下の内容で講義する。
  1. 状態方程式を記述する理論模型
    - (a) 核物質の状態方程式と QCD 相図研究の概観
    - (b) 場の理論からのアプローチ (南部 - ヨナラシニヨ模型、強結合格子 QCD)
    - (c) 相対論的平均場理論
  2. 輸送理論
    - (a) 時間依存平均場理論、半古典輸送模型とボルツマン方程式、流体模型、
    - (b) 古典ヤンミルズ場のダイナミクス
    - (c) 高エネルギー重イオン衝突の概観と輸送理論の適用例。
  3. 直接反応理論
    - (a) 核子 - 核子散乱、核力と位相差、有効相互作用 ( G-matrix)
    - (b) ハドロン - 核反応 ( 光学模型、インパルス近似、グリーン関数法 )、  
ハイパー核・中間子核生成反応の概観と直接反応理論の適用例
    - (c) 高エネルギー核反応 ( グラウバー模型、ハドロン共鳴 )
- 成績評価の方法・基準：履修状況及びレポートにより総合評価する。

参考書 *Theoretical Nuclear and Subnuclear Physics, John Dirk Walecka (World Scientific)*

講義のスライド <http://www2.yukawa.kyoto-u.ac.jp/~ohnishi/Lec2012/>

# *Phenomenological approach to dense hyperon mixed matter EOS*

**Akira Ohnishi (YITP, Kyoto Univ.)**

*YIPQS Long-term workshop*

*Dynamics and Correlations in Exotic Nuclei (DCEN2011)*

*20th September - 28th October, 2011*

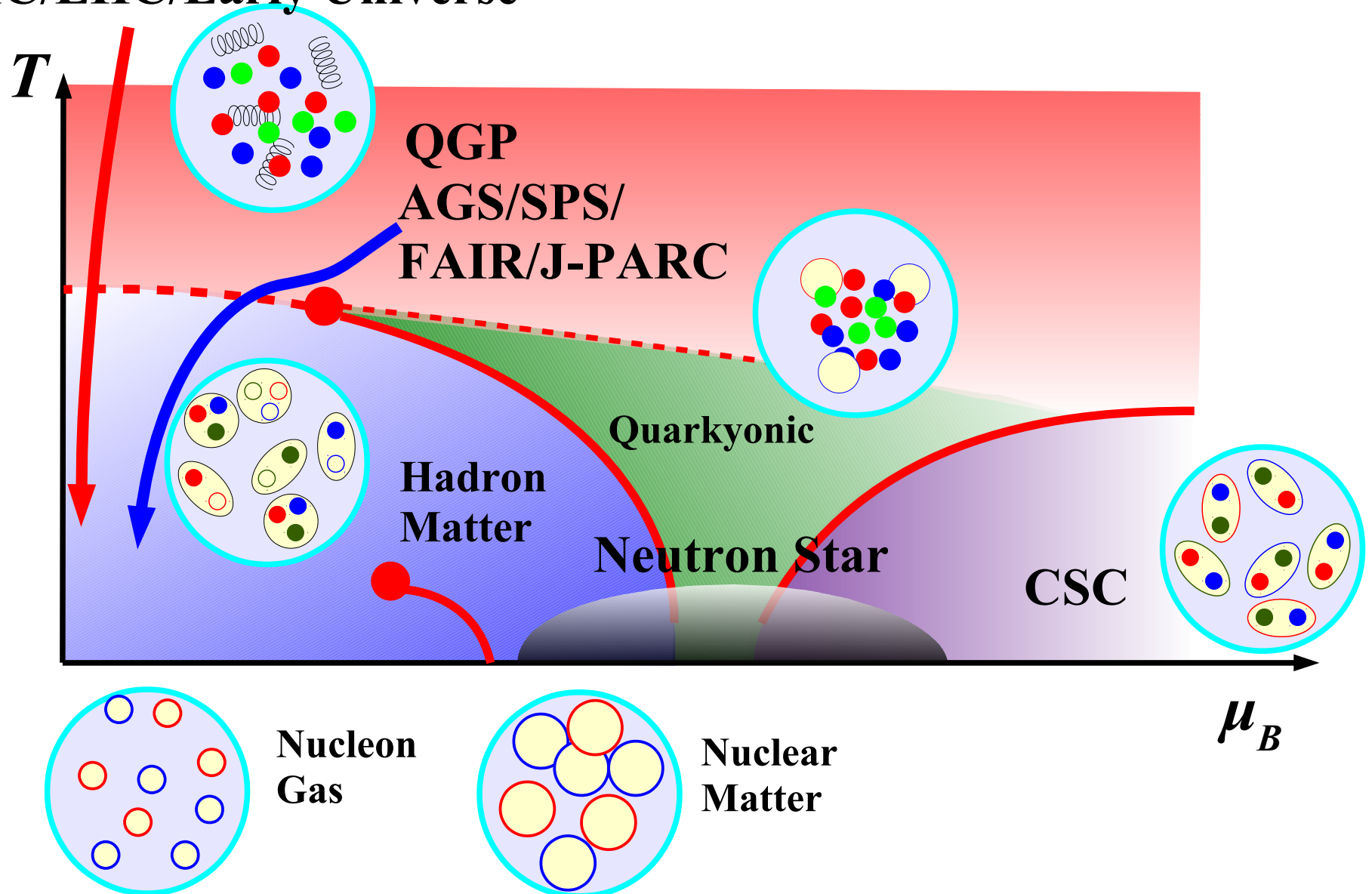
*Yukawa Institute for Theoretical Physics, Kyoto, Japan*

- Introduction
- Relativistic Mean Field for Hypernuclei and Hyperonic Matter
- Do hyperons survive in  $1.97 M_{\odot}$  neutron star ?
- Summary

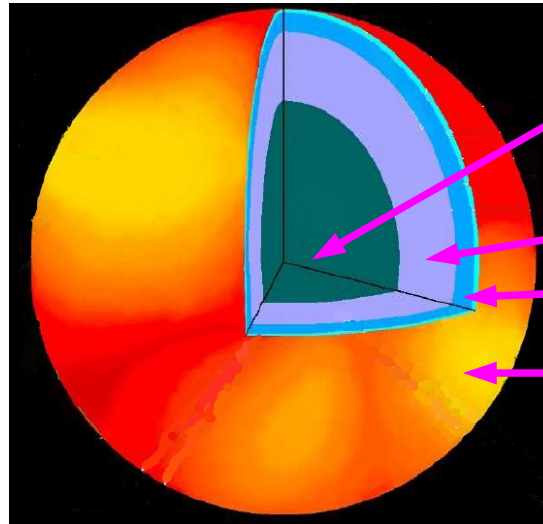


# QCD Phase Diagram

RHIC/LHC/Early Universe



# Neutron Star Composition

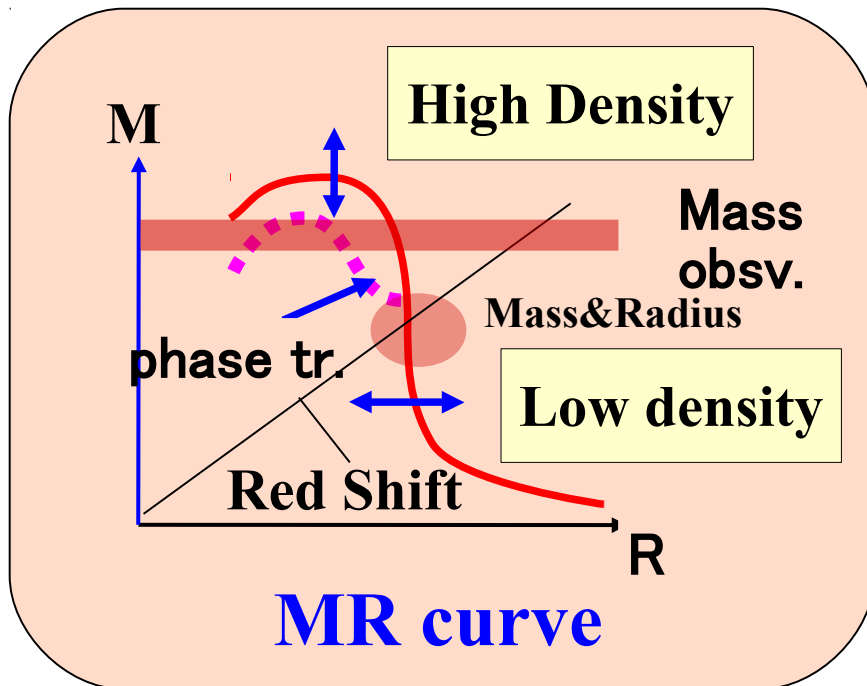


Hyperons, mesons, quarks

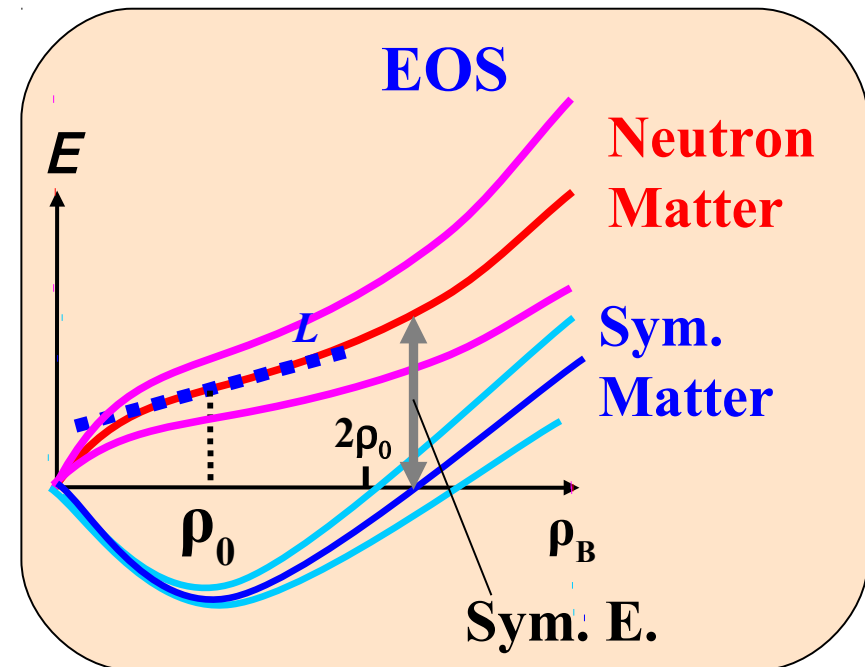
Asym. nuclear matter+elec.+ $\mu$

Nuclei+neutron gas+elec.

Nuclei + elec.

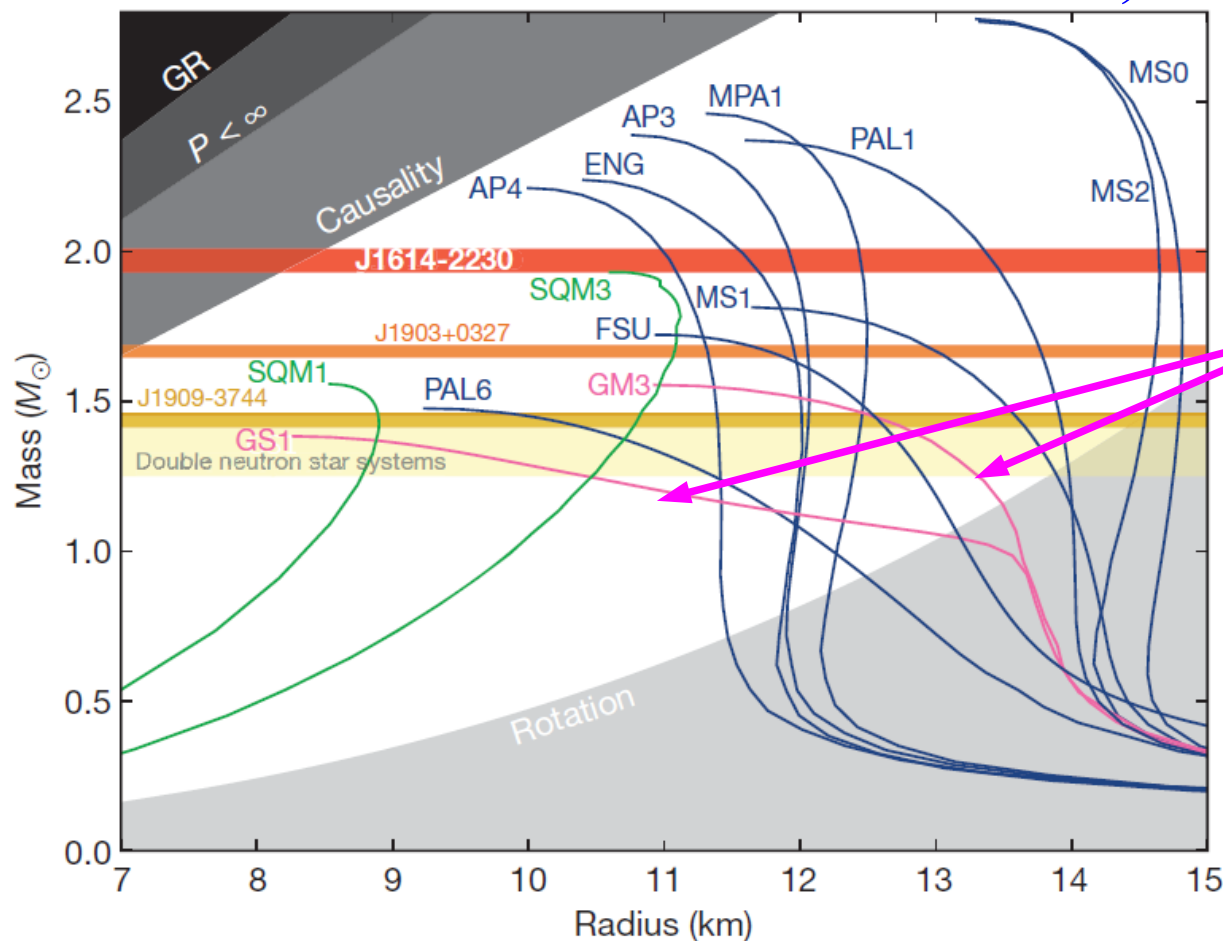


TOV eq.



# $1.97 \pm 0.04 M_{\odot}$ Neutron Star

Demorest et al., *Nature* 467 (2010) 1081 (Oct.28, 2010).



**EOS with  
Strange Hadrons**

signature. We calculate the pulsar mass to be  $(1.97 \pm 0.04)M_{\odot}$ , which rules out almost all currently proposed<sup>2-5</sup> hyperon or boson condensate equations of state ( $M_{\odot}$ , solar mass). Quark matter can support a star this massive only if the quarks are strongly interacting and are therefore not 'free' quarks<sup>12</sup>.

# *Hyperons in Dense Matter*

- **Hyperons are HOT now !**
  - **What makes NS matter core ? Nucleons ? Quarks ? Hyperons ?**
  - **How can we suppress hyperon appearance in NS ?  
or How can hyperonic matter be so stiff ?  
or Which inter-quark interaction supports  $1.97 M_{\odot}$  NS ?**

*We stick to hyperonic matter (rather than quark matter),  
and discuss possible mechanism  
to stiffen the EOS at high density.*

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## *Relativistic Mean Field*

# Theories/Models for Nuclear Matter EOS

## ■ Ab initio Approaches to Nuclear Matter

- LQCD-MC: Not (yet) applicable to cold dense matter,  $A \leq 4$   
SC-LQCD: Nuclear matter does not bound
- Variational, BHF: Need phen. 3-body repulsion to reproduce saturation point.
- GFMC: Limited to be  $A \leq 12$ .
- DBHF: Good, but  $E/A$  is not enough. Not yet extensively investigated.

→ Not easy to handle, Not yet satisfactory for phen. purposes

## ■ Mean Field Models (~ Nuclear Density Functional approach)

- Skyrme Hartree-Fock(-Bogoliubov)
  - ◆ Non.-Rel., Zero Range, Two-body + Three-body (or  $\rho$ -dep. two-body)
  - ◆ Nuclear Mass is very well explained (HFB, Total B.E.  $\Delta E \sim 0.6$  MeV)
  - ◆ Causality is violated at very high densities.

### ● Relativistic Mean Field

- ◆ Relativistic, Meson-Baryon coupling, Meson self-energies
- ◆ Successful in describing pA scattering (Dirac Phenomenology)



# Relativistic Mean Field (1)

## ■ Relativistic Mean Field

= Nuclear scalar and vector mean field generated by mesons

→ Why do we use relativistic framework ?

- Nuclear Force is mediated by mesons

→ Let's consider meson-baryon system !  
(Entrance of Hadron Physics)

- We are also interested in Dense Matter EOS

→ Sound velocity exceeds the Speed of Light (=c) with Non.-Rel. MF

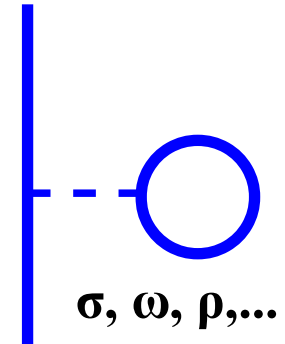
- Success of “Dirac Phenomenology”

(Dirac Eq. for pA scattering → Spin Observables)

→ Strong Scalar and Vector Mean Fields are preferable to explain Spin Observables

- DBHF (Dirac-Brueckner-Hatree-Fock)

→ Successful description of nuclear matter saturation point based on bare NN interactions

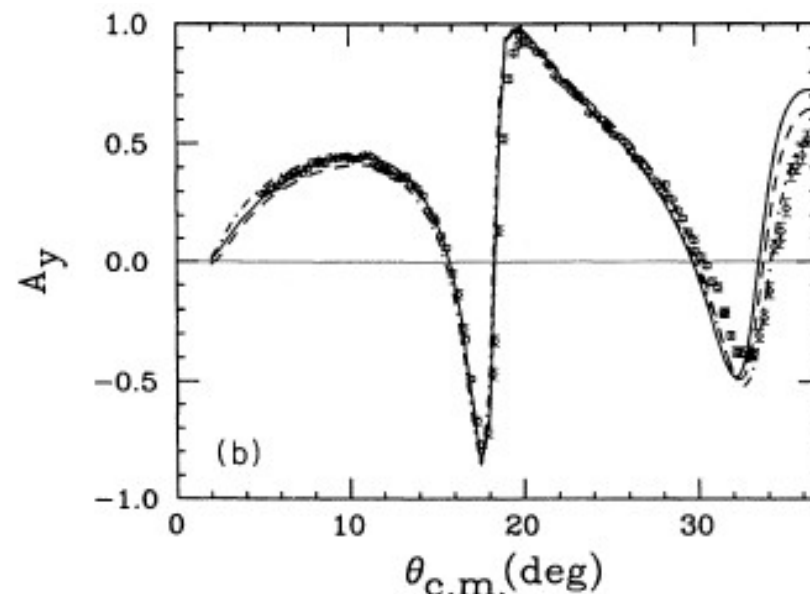
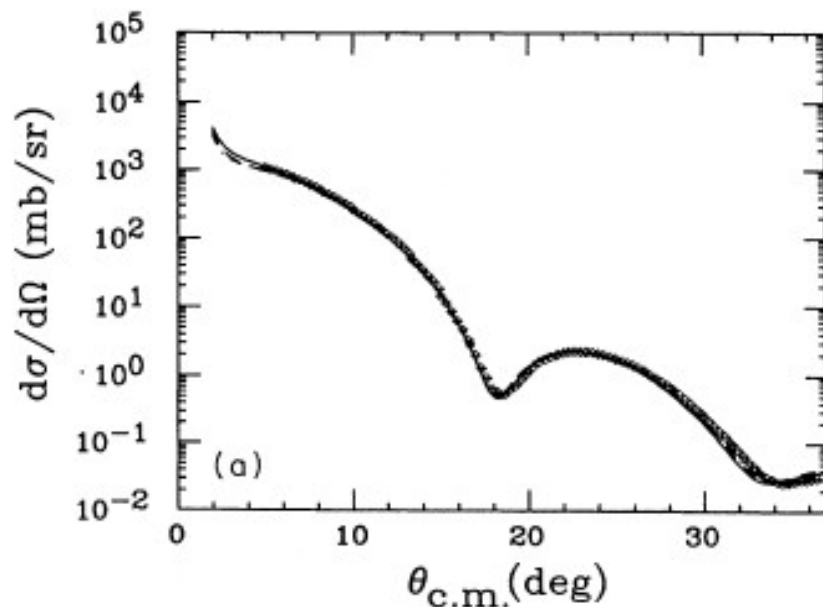
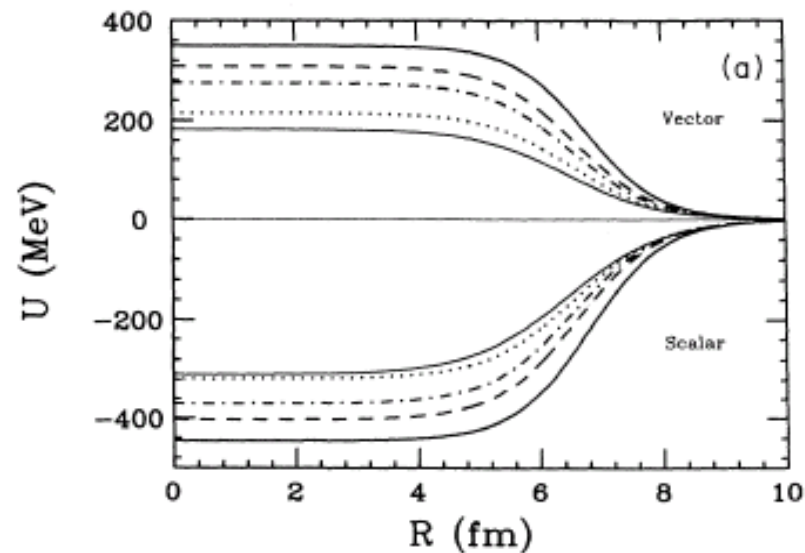


*RMF is a good starting point as a framework of hadronic system including Nuclei and Nuclear Matter*

# Dirac Phenomenology

*E.D. Cooper, S. Hama, B.C. Clark, R.L. Mercer, PRC47('93),297*

- Dirac Eq. with  
Scalar + Vector pA potential  
(-400 MeV + 350 MeV)  
→ Cross Section, Spin Observables



# EOS in Dirac-Brueckner-Hartree-Fock

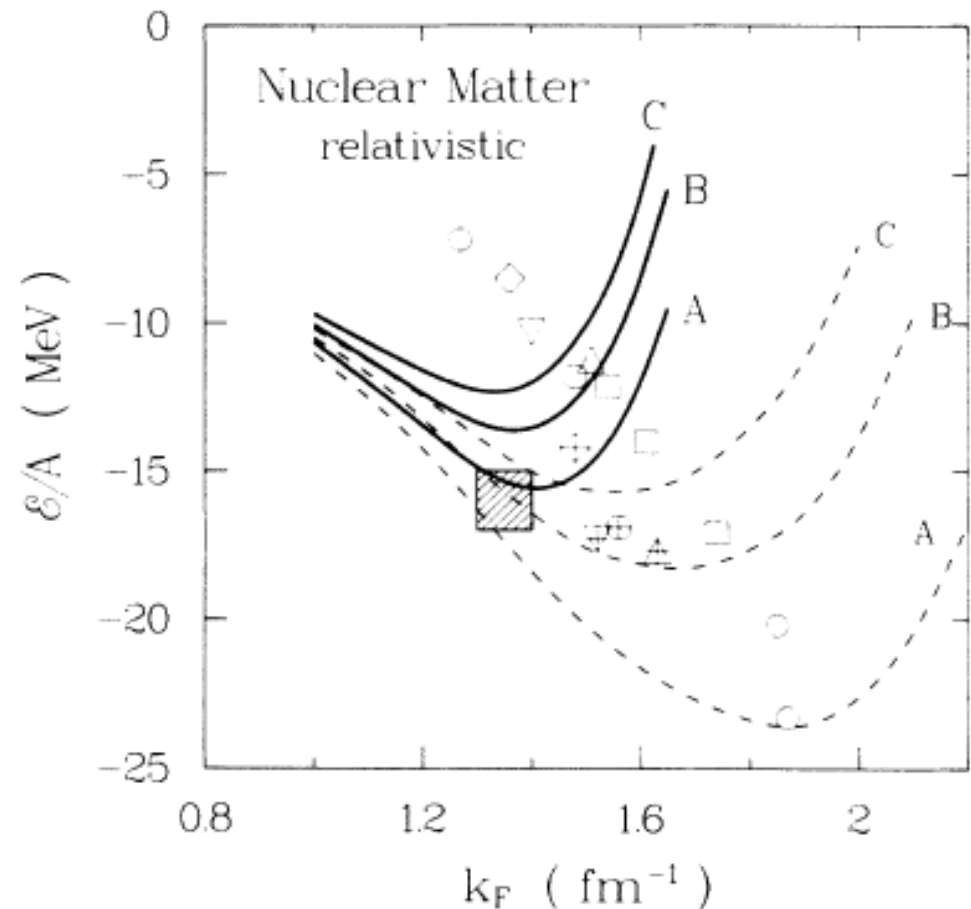
*R. Brockmann, R. Machleidt, PRC42('90),1965*

- **Non Relativistic Brueckner Calculation**

→ **Nuclear Saturation Point cannot be reproduced (Coester Line)**

- **Relativistic Approach (DBHF)**

→ **Relativity gives additional repulsion, leading to successful description of the saturation point.**



## *Relativistic Mean Field (2)*

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- **Mean Field treatment of meson field operator**  
= Meson field operator is replaced with its expectation value

$$\varphi(\mathbf{r}) \rightarrow \langle \varphi(\mathbf{r}) \rangle$$

Ignoring fluctuations compared with the expectation value may be a good approximation at strong condensate.

- **Which Hadrons should be included in RMF ?**

- **Baryons (1/2+)**       $p, n, \Lambda, \Sigma, \Xi, \Delta, \dots$
- **Scalar Mesons (0+)**       $\sigma(600), f_0(980), a_0(980), \dots$
- **Vector Mesons (1-)**       $\omega(783), \rho(770), \phi(1020), \dots$
- **Pseudo Scalar (0-)**       $\pi, K, \eta, \eta', \dots$
- **Axial Vector (1+)**       $a_1, \dots$

We require that the meson field can have uniform expectation values in nuclear matter.

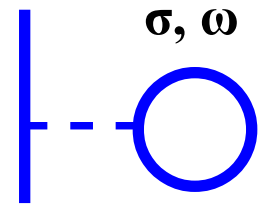
→ Scalar and Time-Component of Vector Mesons ( $\sigma, \omega, \rho, \dots$ )

# $\sigma\omega$ Model (1)

Serot, Walecka, *Adv.Nucl.Phys.16 (1986),1*

- Consider only  $\sigma$  and  $\omega$  mesons
- Lagrangian

$$L = \bar{\Psi} (i \gamma^\mu \partial_\mu - M + g_s \sigma - g_v \gamma^\mu \omega_\mu) \Psi$$
$$+ \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_s^2 \sigma^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_v^2 \omega_\mu \omega^\mu$$
$$(F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu)$$



- Equation of Motion

- Euler-Lagrange Equation

$$\frac{\partial}{\partial x^\mu} \left[ \frac{\partial L}{\partial (\partial_\mu \Phi_i)} \right] - \frac{\partial L}{\partial \Phi_i} = 0$$

$$\sigma : \left[ \partial_\mu \partial^\mu + m_s^2 \right] \sigma = g_s \bar{\Psi} \Psi$$

$$\omega : \partial_\mu F^{\mu\nu} + m_v^2 \omega^\nu = g_v \bar{\Psi} \gamma^\nu \Psi \quad \rightarrow \quad \left[ \partial_\mu \partial^\mu + m_v^2 \right] \omega^\nu = g_v \bar{\Psi} \gamma^\nu \Psi$$

$$\Psi : \left[ \gamma^\mu (i \partial_\mu - g_v V_\mu) - (M - g_s \sigma) \right] \Psi = 0$$

# *EOM of $\omega$ (for beginners)*

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## ■ Euler-Lagrange Eq.

$$\partial_{\mu} F^{\mu\nu} + m_{\nu}^2 \omega^{\nu} = g_{\nu} \bar{\psi} \gamma^{\nu} \psi$$

## ■ Divergence of LHS and RHS

$$\partial_{\nu} \partial_{\mu} F^{\mu\nu} + m_{\nu}^2 (\partial_{\nu} \omega^{\nu}) = m_{\nu}^2 (\partial_{\nu} \omega^{\nu}) = g_{\nu} (\partial_{\nu} \bar{\psi} \gamma^{\nu} \psi) = 0$$

**LHS: derivatives are sym. and  $F_{\mu\nu}$  is anti-sym.**

**RHS: Baryon Current = Conserved Current**

## ■ Put it in the Euler-Lagrange Eq.

$$\partial_{\mu} F^{\mu\nu} = \partial_{\mu} (\partial^{\mu} \omega^{\nu} - \partial^{\nu} \omega^{\mu}) = \partial_{\mu} \partial^{\mu} \omega^{\nu} - \partial^{\nu} (\partial_{\mu} \omega^{\mu}) = \partial_{\mu} \partial^{\mu} \omega^{\nu}$$

# Schroedinger Eq. for Upper Component

## Dirac Equation for Nucleons

$$(i \gamma \partial - \gamma^0 U_v - M - U_s) \psi = 0 \quad , \quad U_v = g_\omega \omega \quad , \quad U_s = -g_\sigma \sigma$$

## Decompose 4 spinor into Upper and Lower Components

$$\begin{pmatrix} E - U_v - M - U_s & i \sigma \cdot \nabla \\ -i \sigma \cdot \nabla & -E + U_v - M - U_s \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix} = 0 \quad \begin{aligned} g &= \frac{-i}{E + M + U_s - U_v} (\sigma \cdot \nabla) f \\ (E - M - U_v - U_s) f &= -i (\sigma \cdot \nabla) g \end{aligned}$$

## Erase Lower Component (assuming spherical sym.)

$$-i (\sigma \cdot \nabla) g = -(\sigma \cdot \nabla) \frac{1}{X} (\sigma \cdot \nabla) f = -\frac{1}{X} \nabla^2 f - \frac{1}{r} \left[ \frac{d}{dr} \frac{1}{X} \right] (\sigma \cdot r) (\sigma \cdot \nabla) f = -\nabla \frac{1}{X} \nabla f + \frac{1}{r} \left[ \frac{d}{dr} \frac{1}{X} \right] (\sigma \cdot l) f$$

$$(\sigma \cdot r) (\sigma \cdot \nabla) = (r \cdot \nabla) + i \sigma \cdot (r \times \nabla) = r \cdot \nabla - \sigma \cdot l$$

## “Schroedinger-like” Eq. for Upper Component

$$-\nabla \frac{1}{E + M + U_s - U_v} \nabla f + (U_s + U_v + U_{LS} (\sigma \cdot l)) f = (E - M) f$$

$$U_{LS} = \frac{1}{r} \left[ \frac{d}{dr} \frac{1}{E + M + U_s - U_v} \right] < 0 \quad \text{on surface}$$

(Us, Uv) ~ (-350 MeV, 280 MeV)

→ Small Central (Us + Uv), Large LS (Us - Uv)

# Various Ways to Evaluate Non.-Rel. Potential

## ■ From Single Particle Energy

$$\left( \gamma^0 (E - U_v) + i \boldsymbol{\gamma} \cdot \nabla - (M + U_s) \right) \psi = 0 \rightarrow (E - U_v)^2 = p^2 + (M + U_s)^2$$
$$\rightarrow E = \sqrt{p^2 + (M + U_s)^2} + U_v \approx E_p + \frac{M}{E_p} U_s + U_v + \frac{p^2}{2 E_p^3} U_s^2$$
$$(E_p = \sqrt{p^2 + M^2})$$

## ■ Schroedinger Equivalent Potential (Uniform matter)

$$-\frac{\nabla^2}{2M} f + \left[ U_s + \frac{E}{M} U_v + \frac{U_s^2 - U_v^2}{2M} \right] f = \frac{E + M}{2M} (E - M) f$$

$$U_{\text{SEP}} \approx U_s + \frac{E}{M} U_v$$

Anyway, slow baryons feel Non.-Rel. Potential,

$$U \approx U_s + U_v = -g_s \sigma + g_v \omega$$



# Nuclear Matter in $\sigma\omega$ Model

Serot, Walecka, *Adv.Nucl.Phys.*16 (1986),1

## Uniform Nuclear Matter

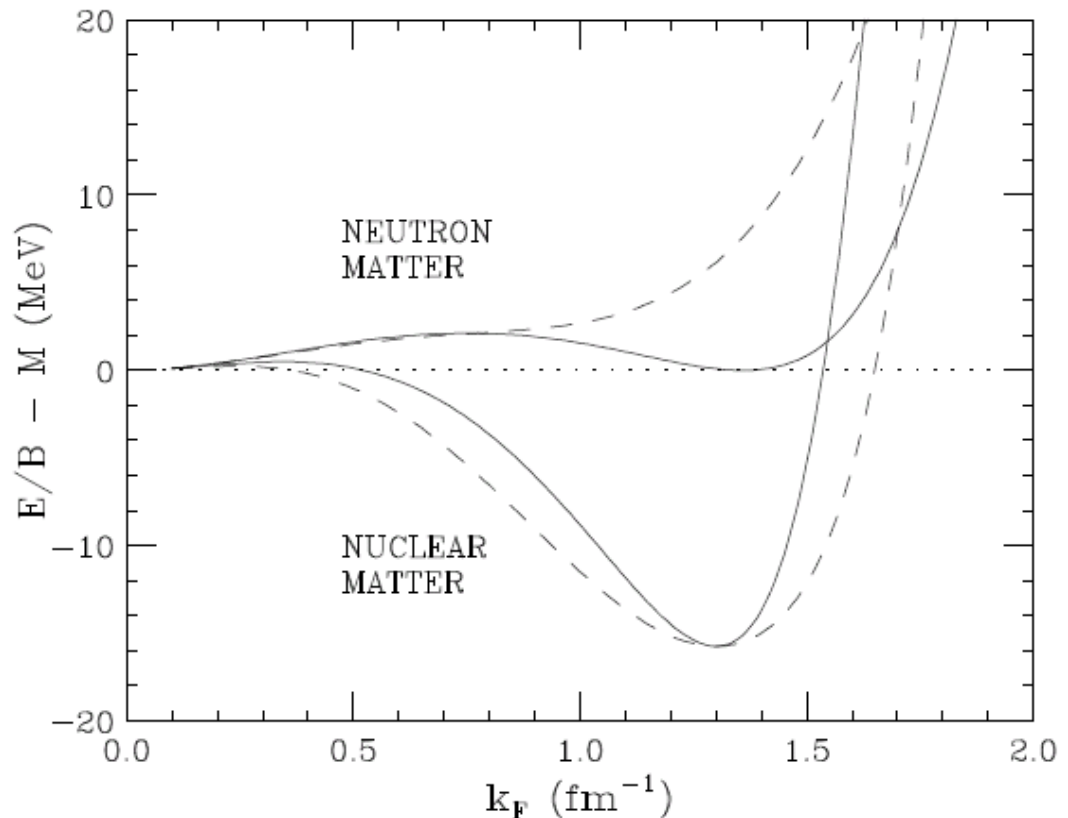
$$E/V = \gamma_N \int^{P_F} \frac{d^3 p}{(2\pi)^2} E^* + \frac{1}{2} m_s^2 \sigma^2 - \frac{1}{2} m_v^2 \omega^2 + g_v \rho_B \omega$$

$$\sigma = \frac{g_s}{m_s^2} \rho_s = \gamma_N \frac{g_s}{m_s^2} \int^{P_F} \frac{d^3 p}{(2\pi)^2} \frac{M^*}{E^*} \quad (M^* = M + U_s = M - g_s \sigma, \quad E^* = \sqrt{p^2 + M^{*2}})$$

$$\omega = \frac{g_v}{m_v^2} \rho_B = \gamma_N \frac{g_v}{m_v^2} \int^{P_F} \frac{d^3 p}{(2\pi)^3}$$

$\gamma_N =$  Nucleon degeneracy  
(=4 in sym. nuclear matter)

**Problem: EOS is too stiff**  
 **$K \sim (500-600) \text{ MeV}!$**   
 **$\rightarrow$  How can we solve ?**



# *$\sigma$ $\omega$ model --- pros and cons*

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## ■ Pros (merit)

- Foundation is clear: based on the success of Dirac phen. and DBHF.
- Simple description of scalar and vector potential in  $\sigma$  and  $\omega$  mesons.
- Saturation is well described in two parameters.
- Natural explanation of large LS potential in nuclei.

## ■ Cons (shortcomings)

- Relation with the bare NN interaction is not clear.
- Especially, pion effects are not included.
- Symmetry energy is too small.
- Incompressibility is too large ( $K \sim 600\text{-}700$  MeV)  
(c.f. Empirical value  $K \sim (200\text{-}300)$  MeV)
- Chiral symmetry is not respected.

# High Quality RMF models

## ■ Variety of the RMF models

→ MB couplings, meson masses, meson self-energies

- $\sigma N$ ,  $\omega N$ ,  $\rho N$  couplings are well determined

→ almost no model deps. in Sym. N.M. at low  $\rho$

- $\omega^4$  term is introduced to simulate DBHF results of vector pot.

*TM: Y. Sugahara, H. Toki, NPA579('94)557;*

*R. Brockmann, H. Toki, PRL68('92)3408.*

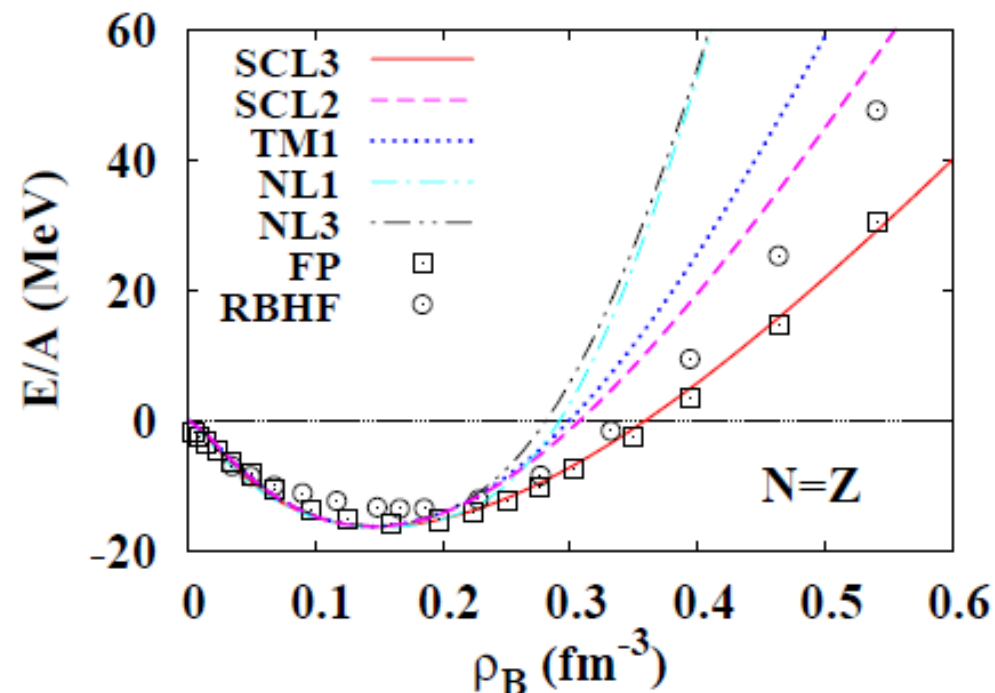
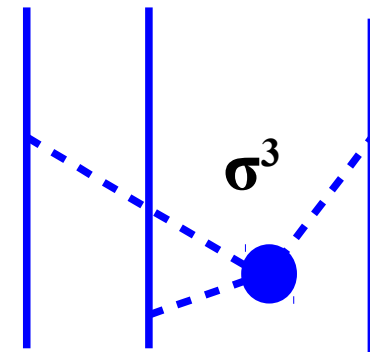
- $\sigma^3$  and  $\sigma^4$  terms are introduced to soften EOS at  $\rho_0$ .

*J. Boguta, A.R. Bodmer NPA292('77)413,*

*NL1: P.-G. Reinhardt, M. Rufa, J. Maruhn, W. Greiner, J. Friedrich, ZPA323('86)13.*

*NL3: G.A. Lalazissis, J. König, P. Ring, PRC55('97)540.*

→ Large differences are found at high  $\rho$

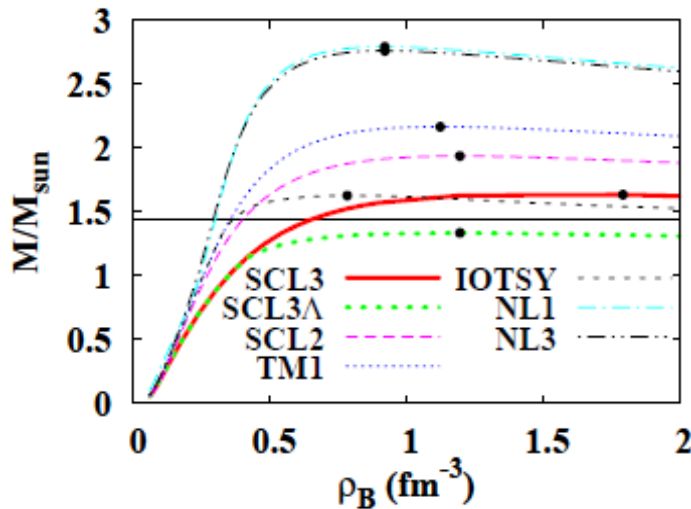
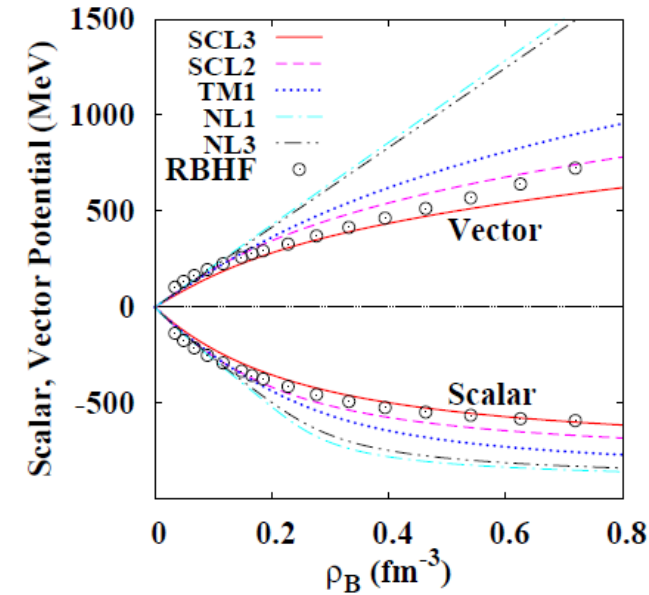
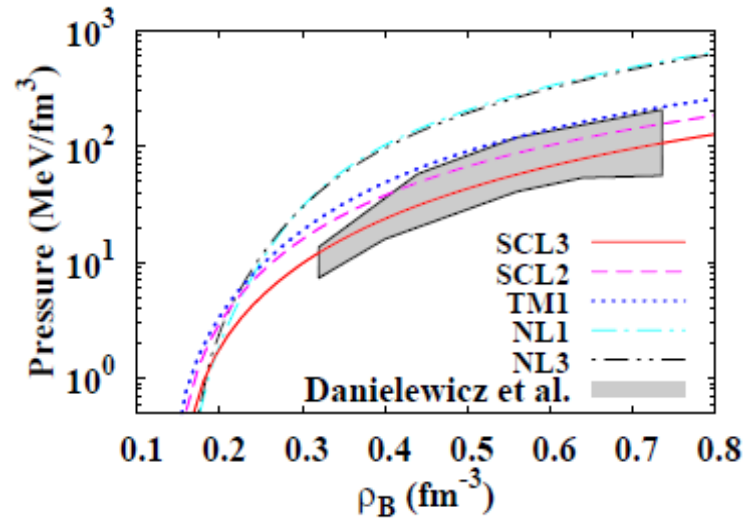
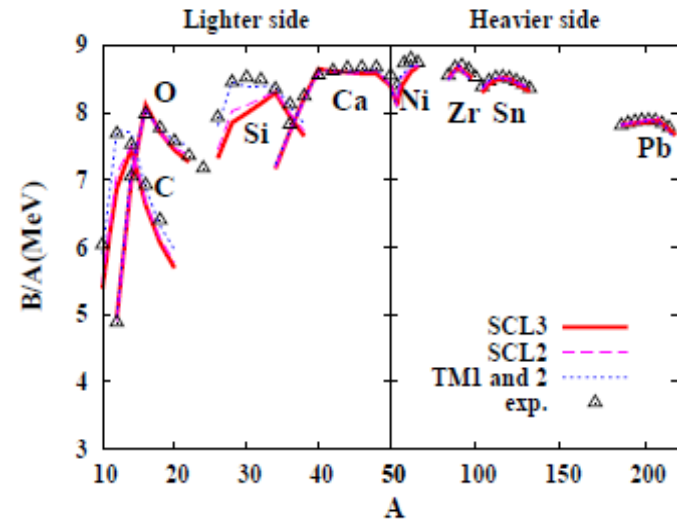


*K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.*

# How to determine Non-Linear terms ? (1)

## Method 1: Fit as many as known observables

- EOS, Nuclear B.E., High density EOS from HIC, Vector potential in DBHF, Neutron Star, ...



*P. Danielewicz, R. Lacey, W. G. Lynch, Science 298('02)1592.*

*R. Brockmann, R. Machleidt, PRC 42('90)1965.*

*K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC 81('10)065206.*

# How to determine Non-Linear terms ? (2)

- **Method (2):** Fix parameters by using symmetry, such as the *Chiral Symmetry*

- **Chiral Symmetry**

- **Fundamental symmetry of massless QCD, and its spontaneous breaking generates hadron masses.**

*Nambu, Jona-Lasinio ('61)*

- **Many of the linear  $\sigma$  models are unstable against finite density (chiral collapse).**

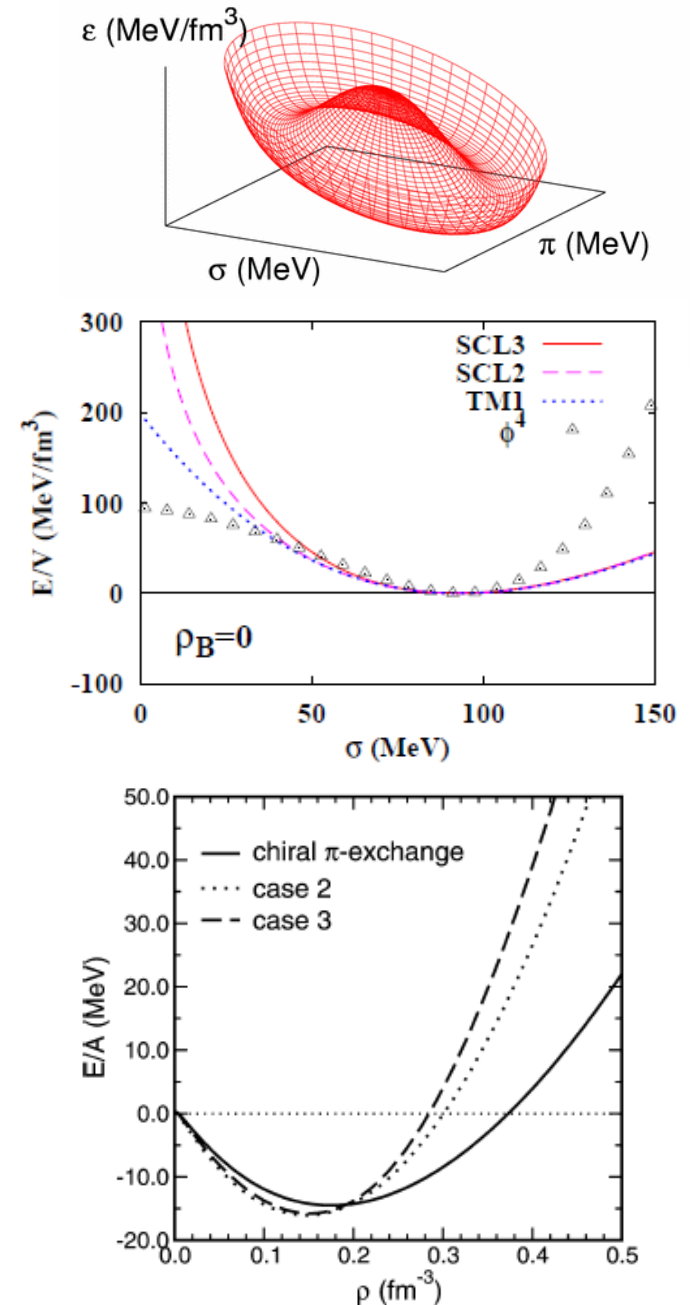
→ **Log type chiral potential**

*Sahu, Tsubakihara, AO('10), Tsubakihara, AO('07), Tsubakihara et al.('10)*

- **Non-linear representation (chiral pert.) leads to density dependent coupling from one- and two-pion exchanges.**

*Kaiser, Fritsch, Weise ('02),*

*Finelli, Kaiser, Vretener, Weise ('04)*



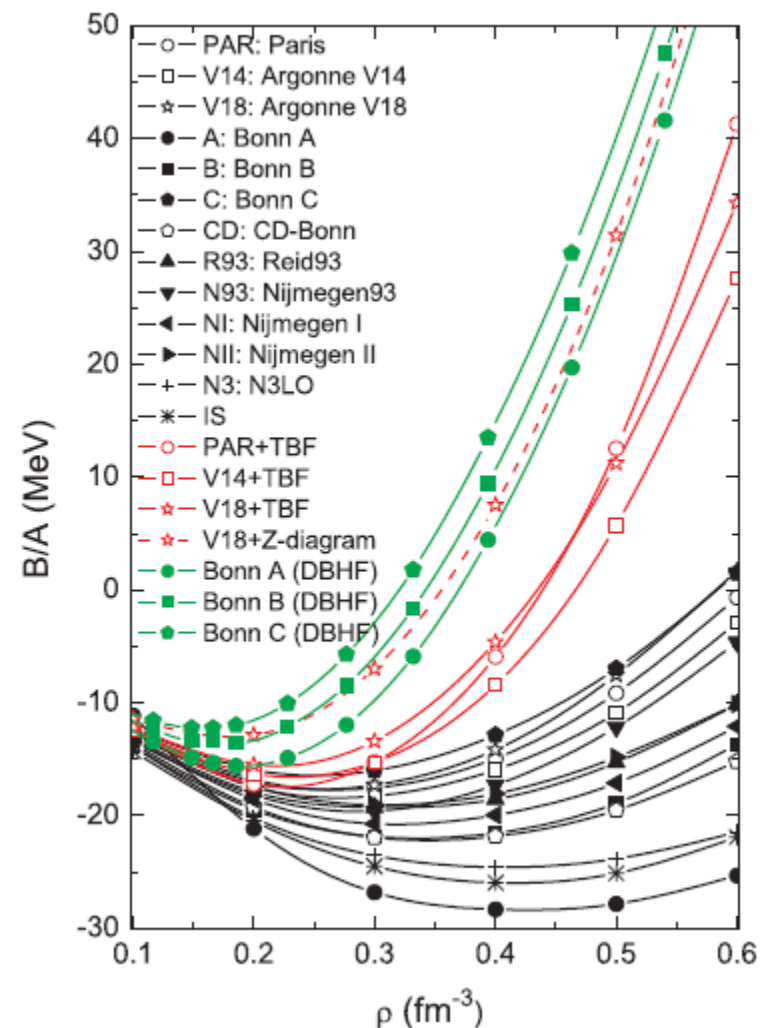
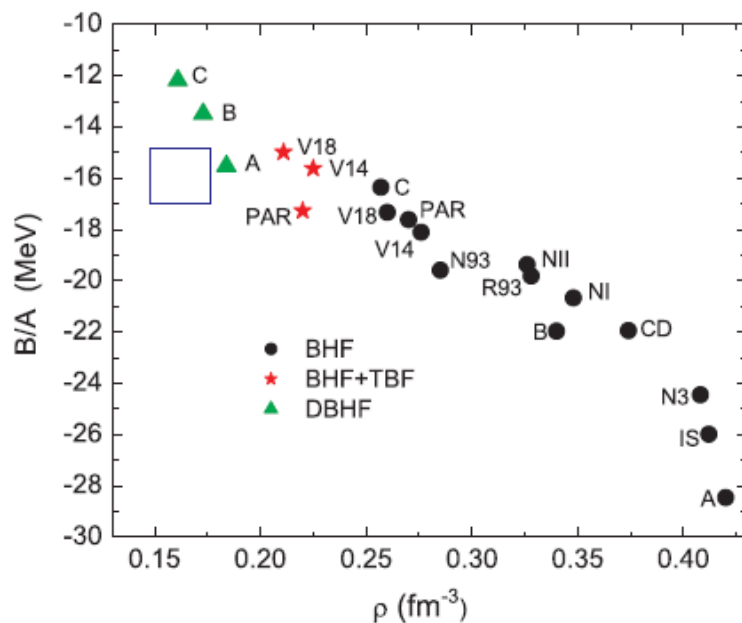
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*Dense Nuclear Matter EOS  
with Hyperon Admixture*

# Bruckner-Hartree-Fock

- Self-consistent treatment of Effective interaction (G-matrix) in the Bruckner Theory and Single particle energy from G-matrix

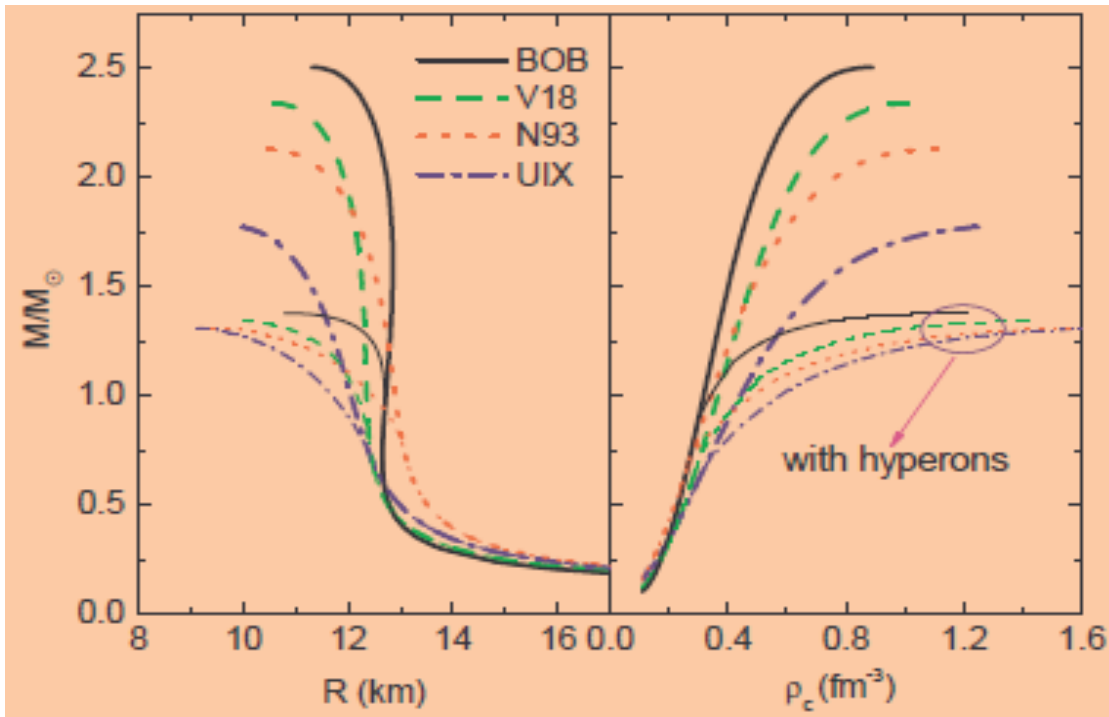
- Need 3-body force to reproduce saturation point.
  - FY type 2  $\pi$  exchange + phen. or Z-diagram



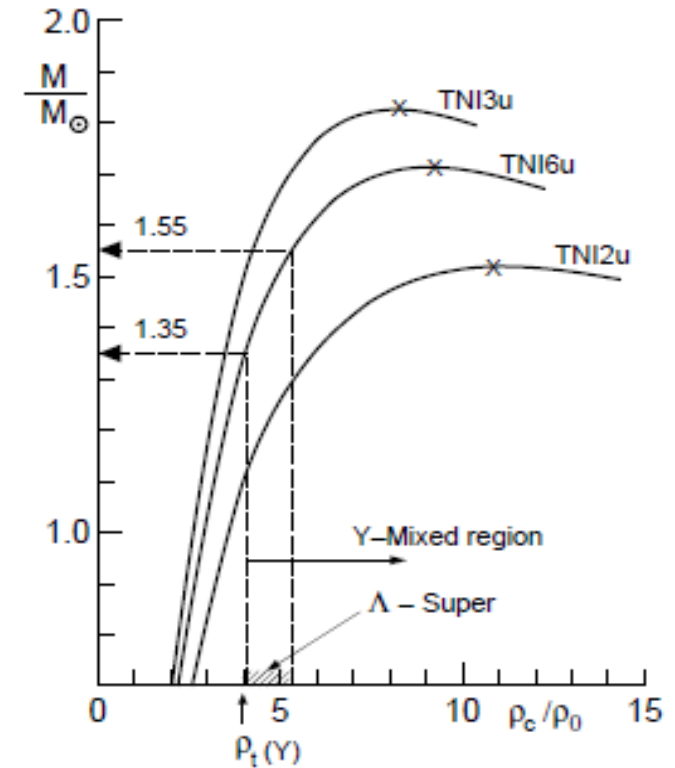
Z.H.Li, U. Lombardo, H.-J. Schulze, W. Zuo, L. W. Chen, H. R. Ma, PRC74('06)047304.

# Bruckner-Hartree-Fock theory with Hyperons

- Microscopic G-matrix calculation with realistic NN, YN potential and microscopic (or phen.) 3N force (or 3B force).
  - Interaction dep. (V18, N93, ...) is large  $\rightarrow$  Need finite nuclear info.  
*E.Hiyama, T.Motoba, Y.Yamamoto, M.Kamimura / M.Tamura et al.*
  - NS collapses with hyperons w/o 3BF.



*H.J.Schulze, A.Polls, A.Ramos, I.Vidana,  
PRC73('06),058801.*



*S. Nishizaki, T. Takatsuka,  
Y. Yamamoto, PTP108('02)703.*



# Relativistic Mean Field

## Effective Lagrangian of Baryons and Mesons + Mean Field App.

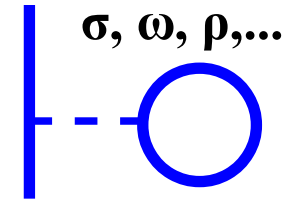
*B.D.Serot, J.D.Walecka, Adv.Nucl.Phys.16 ('86), 1*

$$L = L_B^{\text{free}} + L_M^{\text{free}} + L_{BM} + L_M^{\text{Int}}$$

$$L_M^{\text{Int}} = -U_\sigma(\sigma) + \frac{1}{4}c_\omega(\omega_\mu\omega^\mu)^2 + \dots$$

$$L_{BM} = -\sum_{B,S} g_{BS} \bar{\Psi}_B \varphi_S \Psi_B - \sum_{B,V} g_{BV} \bar{\Psi}_B \gamma^\mu V_\mu \Psi_B$$

$$L_B^{\text{free}} = \bar{\Psi}_B (i \gamma^\mu \partial_\mu - M_B) \Psi_B, \quad L_M^{\text{free}} = \sum_S \left[ \frac{1}{2} \partial^\mu \varphi_S \partial_\mu \varphi_S - \frac{1}{2} m_S^2 \varphi_S^2 \right] + \sum_V \left[ -\frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \frac{1}{2} V_\mu V^\mu \right]$$



• **Baryons and Mesons: B=N, Λ, Σ, Ξ, ..., S=σ, ζ, ..., V=ω, ρ, φ, ...**

• **Based on Dirac phenomenology & Dirac Bruckner-Hatree-Fock**

*E.D. Cooper, S. Hama, B.C. Clark, R.L. Mercer, PRC47('93),297*

*R. Brockmann, R. Machleidt, PRC42('90),1965*

• **Large scalar (att.) and vector (repl.) → Large spin-orbit pot.  
Relativistic Kinematics → Effective 3-body repulsion**

• **Non-linear terms of mesons → Bare 3-body and 4-body force**

*Boguta, Bodmer ('77), NL1:Reinhardt, Rufa, Maruhn, Greiner, Friedrich ('86), NL3:*

*Lalazissis, Konig, Ring ('97), TM1 and TM2: Sugahara, Toki ('94), Brockmann, Toki ('92)*

# Choice of $U_\sigma(\sigma)$

## ■ Logarithmic $\sigma$ potential

*K.Tsubakihara, AO, PTP 117 (2007) 903.*

*K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10) 065206.*

$$U_\sigma = -\frac{a_\sigma}{2} \log \det(M M^+) + \frac{b_\sigma}{2} \text{tr}(M M^+) \\ - d_\sigma (\det M + \det M^+) - \frac{c_\sigma}{4} \text{tr}(M + M^+)$$

$$M = \text{Meson matrix} = [\lambda^a (\sigma^a + i \pi^a)] / \sqrt{2}$$

- No chiral collapse, No instability at large  $\sigma$
- Log  $\sigma$  term appears from coupling to dilaton (scale anomaly)  
*E. K. Heide, S. Rudaz, and P. J. Ellis, NPA571('94)713*  
or from strong coupling limit of lattice QCD  
*N.Kawamoto, J.Smit, NPB 190 ('81)100.*
- det  $\sigma$  term (KMT interaction) represents  $U(1)_A$  anomaly  
*M.Kobayashi, T.Maskawa, PTP44('70)1422; M.Kobayashi, H.Kondo, T. Maskawa, PTP 45('71)1955; G. 't Hooft, PRD 14 ('76)3432.*

# *RMF is a phenomenological MODEL !*

---

- **Baryon one-loop approximation (Hartree approximation) makes RMF a phenomenological model.**
  - **We need DATA and AB INITIO results.**
    - **Saturation point ( $\rho_0$  and  $E/A(\rho_0)$ ) from mass formula**
    - **Nuclear binding energies**
    - **$U_v$  and  $U_s$  from DBHF results**
    - **$P(\rho_B)$  from heavy-ion data**
    - **$\Lambda$  separation energy from single  $\Lambda$  hypernuclear data**
    - **$\Lambda\Lambda$  bond energy from double  $\Lambda$  hypernuclear data**
    - **$\Sigma$  atomic shift**
    - **$\Sigma$  and  $\Xi$  potential depth from quasi-free production data**
    - ***Pure neutron matter EOS from ab initio calculations (not used here)***

# RMF models

## ■ Variety of the RMF models

→ MB couplings, meson masses, meson self-energies

- $\sigma N$ ,  $\omega N$ ,  $\rho N$  couplings are well determined

→ almost no model deps. in Sym. N.M. at low  $\rho$

- $\omega^4$  term is introduced to simulate DBHF results of vector pot.

*TM1&2: Y. Sugahara, H. Toki, NPA579('94)557;*

*R. Brockmann, H. Toki, PRL68('92)3408.*

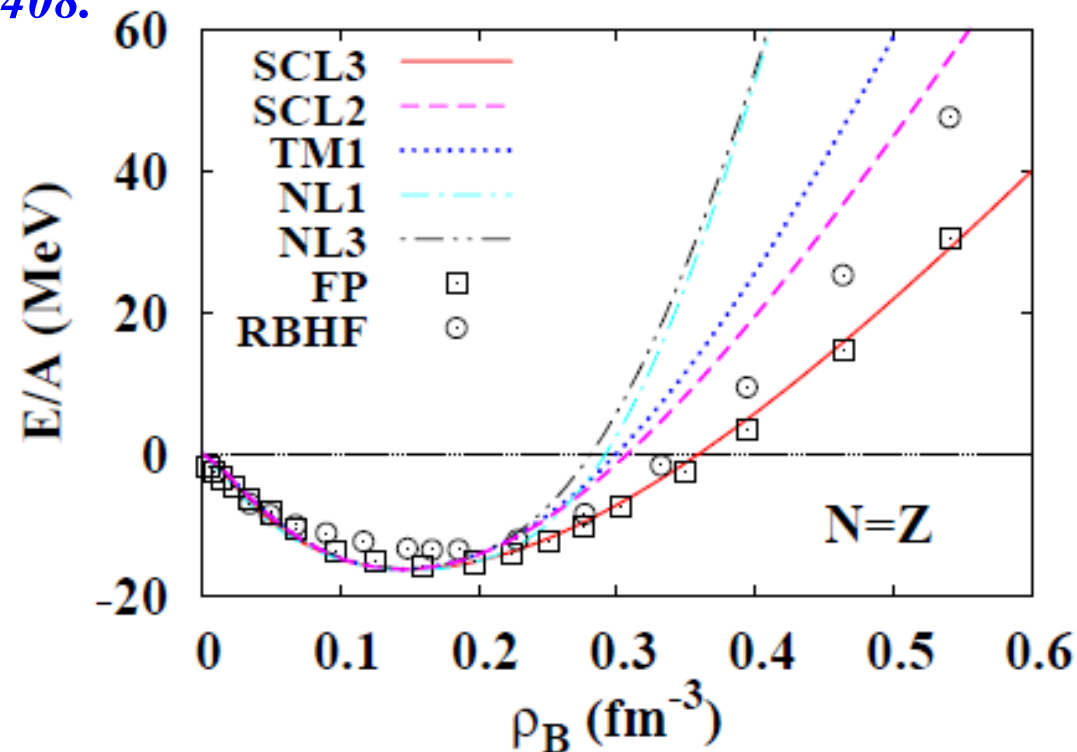
- $\sigma^3$  and  $\sigma^4$  terms are introduced to soften EOS at  $\rho_0$ .

*J. Boguta, A.R. Bodmer NPA292('77)413,*

*NL1: P.-G. Reinhardt, M. Rufa, J. Maruhn, W. Greiner, J. Friedrich, ZPA323('86)13.*

*NL3: G.A. Lalazissis, J. Konig, P. Ring, PRC55('97)540.*

→ Large differences are found at high  $\rho$



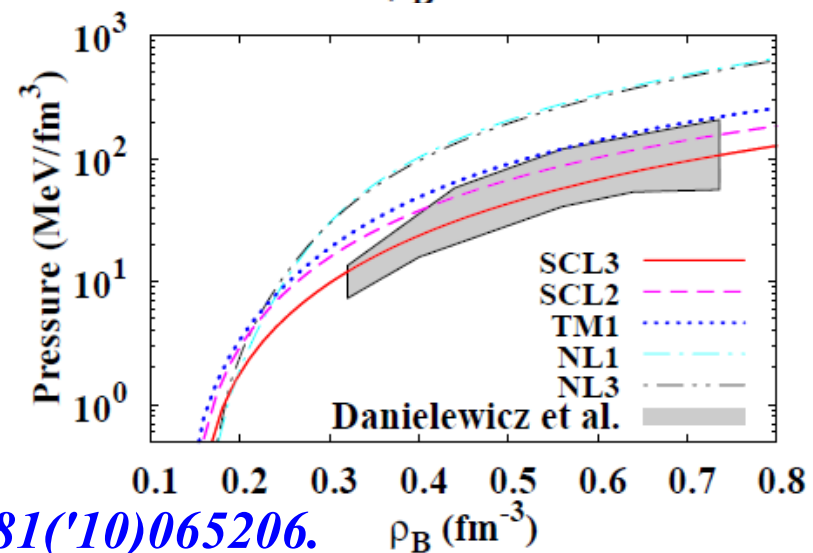
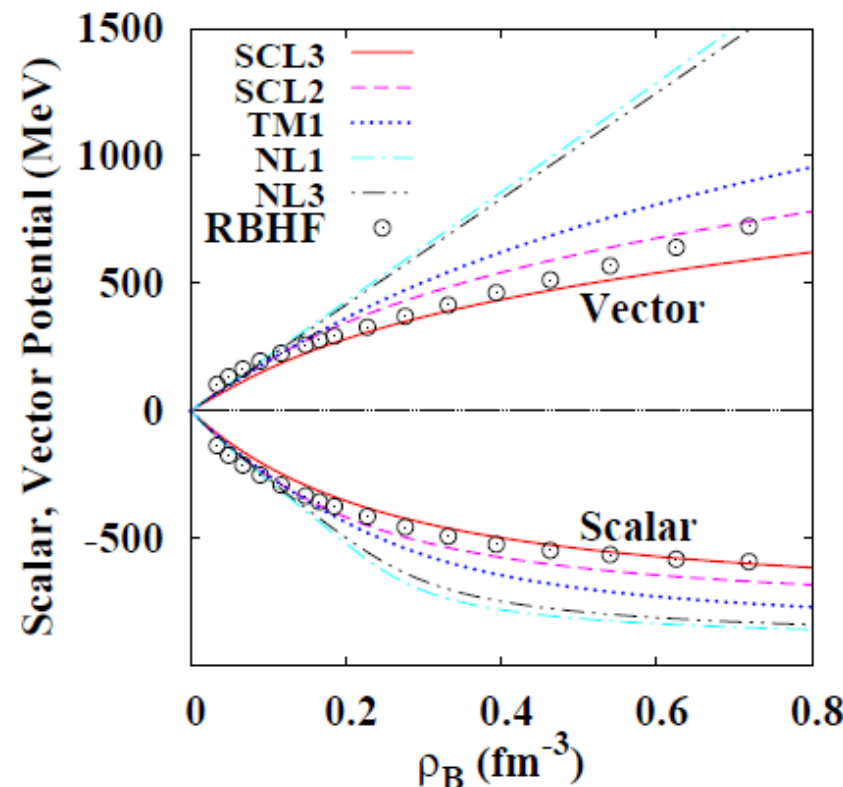
*K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.*

# Vector potential in RMF

- Vector potential from  $\omega$  dominates at high density !

$$U_v(\rho_B) = g_\omega \omega \sim \frac{g_\omega^2}{m_\omega^2} \rho_B$$

- Dirac-Bruckner-Hartree-Fock shows suppressed vector potential at high  $\rho_B$ .  
*R. Brockmann, R. Machleidt, PRC42('90)1965.*
- Collective flow in heavy-ion collisions suggests pressure at high  $\rho_B$ .  
*P. Danielewicz, R. Lacey, W. G. Lynch, Science298('02)1592.*
- Self-interaction of  $\omega \sim c_\omega (\omega_\mu \omega^\mu)^2$   
→ DBHF results & Heavy-ion data



*K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.*

# RMF with Hyperons (Single $\Lambda$ hypernuclei)

## ■ RMF for $\Lambda$ hypernuclei

$x \sim 1/3$ : R. Brockmann, W. Weise, PLB69('77)167; J. Boguta and S. Bohrman, PLB102('81)93.

$x \sim 2/3$ : N. K. Glendenning, PRC23('81)2757, PLB114('82)392;

Tensor: Y. Sugahara, H. Toki, PTP92('94)803; H. Shen, F. Yang, H. Toki, PTP115('06)325;

J. Mares, B. K. Jennings, PRC49('94)2472.

$\rho$ -dep. coupling: H. Lenske, Lect. Notes Phys. 641('04)147; C. M. Keil, F. Hofmann, H. Lenske, PRC 61('00)064309.

$SU(3)$  or  $SU(6)$  ( $\zeta, \varphi$ ): J. Schaffner, C. B. Dover, A. Gal, C. Greiner, H. Stoecker, PRL71('93)1328;

Schaffner et al., Ann.Phys.235('94)35; J. Schaffner, I. N. Mishustin, PRC 53('96)1416.

Chiral  $SU(3)$  RMF: K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.

- Sep. E. of  $\Lambda$  is well fitted by  $U_{\Lambda} \sim -30 \text{ MeV} \sim 2/3 U_N$

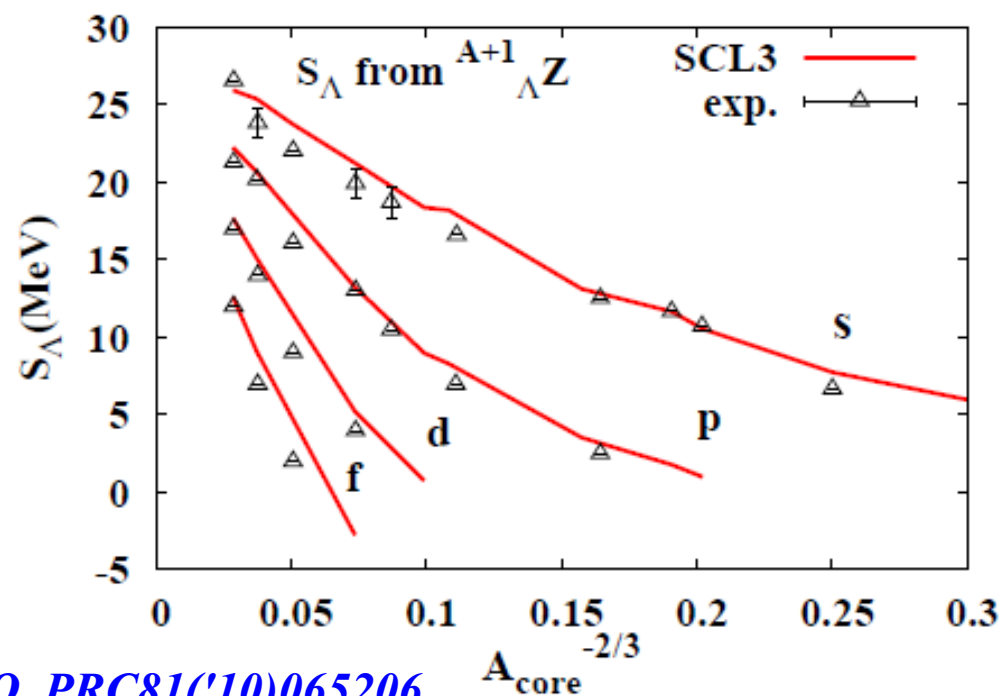
- Coupling with mesons

$$x_M = g_{M\Lambda} / g_{MN}$$

quark counting:  $x_{\sigma} \sim 2/3$

$\pi$  exchanges:  $x_{\sigma} \sim 1/3$

→ Which is true ?



K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.

# RMF with Hyperons (Double $\Lambda$ hypernuclei)

## ■ Nagara event $\Delta B_{\Lambda\Lambda} \sim 1.0$ MeV (weakly attractive)

### ● TM & NL-SH based RMF

*H. Shen, F. Yang, H. Toki, PTP115('06)325.*

Model 1:  $x_\sigma = 0.621$ ,  $x_\omega = 2/3$  (no  $\zeta$ ,  $\varphi$ )

Model 2:  $R_\zeta = g_{\zeta\Lambda} / g_{\zeta N} = 0.56-0.57$ ,  $R_\varphi = g_{\varphi\Lambda} / g_{\varphi N} = -\sqrt{2}/3$

### ● Chiral SU(3) RMF

*K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.*

SU(3)f for vector coupling

$x_\omega = 0.64$ ,  $R_\varphi = 0.504$

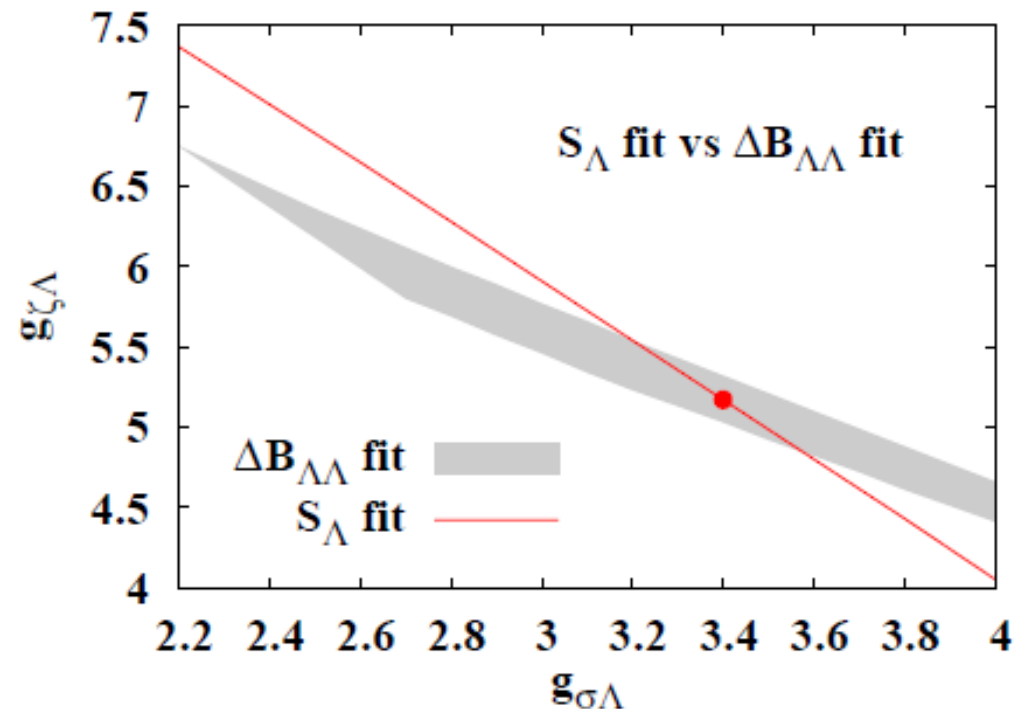
Det. (KMT) int. mixes  $\sigma$  and  $\zeta$

*M. Kobayashi, T. Maskawa,*

*PTP44('70)1422;*

*G. 't Hooft, PRD14('76)3432.*

→  $x_\sigma = 0.335$ ,  $R_\zeta = 0.509$



# Hyperon Composition in Dense Matter

■ Hyperon start to emerge at  $(2-3)\rho_0$  in Neutron Star Matter !

■ Hyperon composition in NS is sensitive to Hyperon potential.

●  $U_\Lambda \sim -30$  MeV: Well-known

●  $U_\Sigma \sim -(12-15)$  MeV

( $K^-, K^+$ ) reaction, twin hypernuclei

*P. Khaustov et al. (E885), PRC61('00)054603;*

*S. Aoki et al., PLB355('95)45.*

●  $U_\Sigma \sim -30$  MeV (Old conjecture)

→  $\Sigma^-$  appears prior to  $\Lambda$

●  $U_\Sigma > 0$  (repulsive) → No  $\Sigma$  in NS

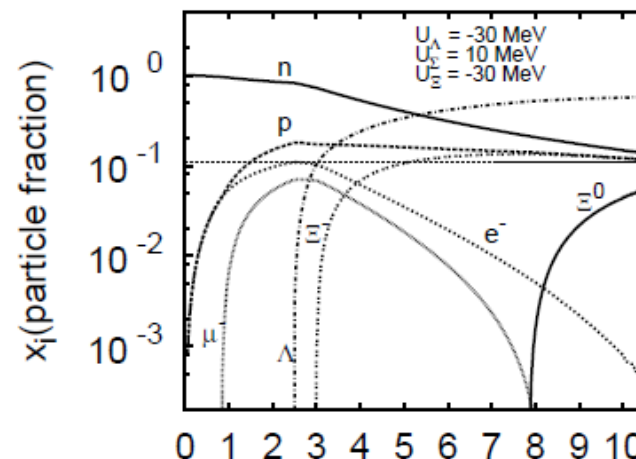
$\Sigma$  atom (phen. fit), QF prod.

*S. Balberg, A. Gal, NPA625('97)435;*

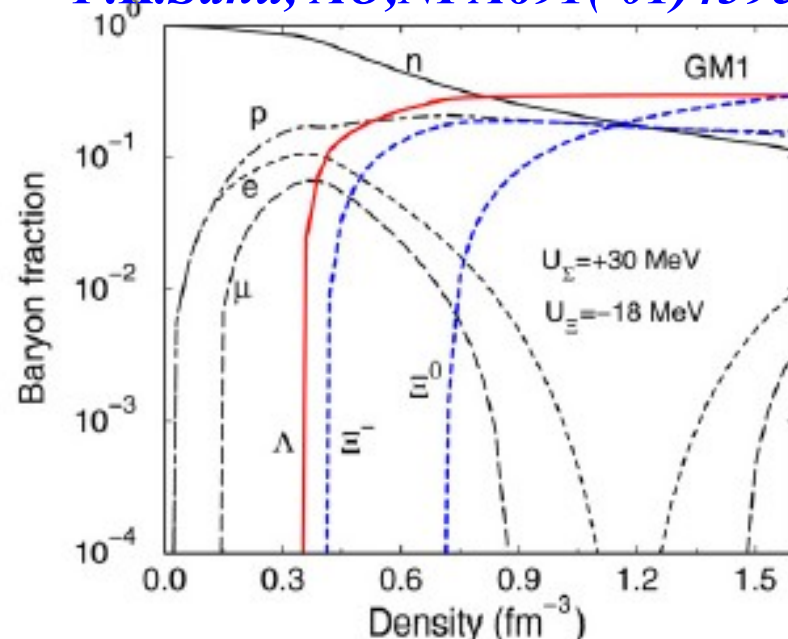
*H. Noumi et al., PRL89('02)072301;*

*T. Harada, Y. Hirabayashi, NPA759('05)143;*

*M. Kohno et al. PRC74('06)064613.*



*P.K.Sahu, AO, NPA691('01)439c*



*J. Schaffner-Bielich, NPA804('08)309.*

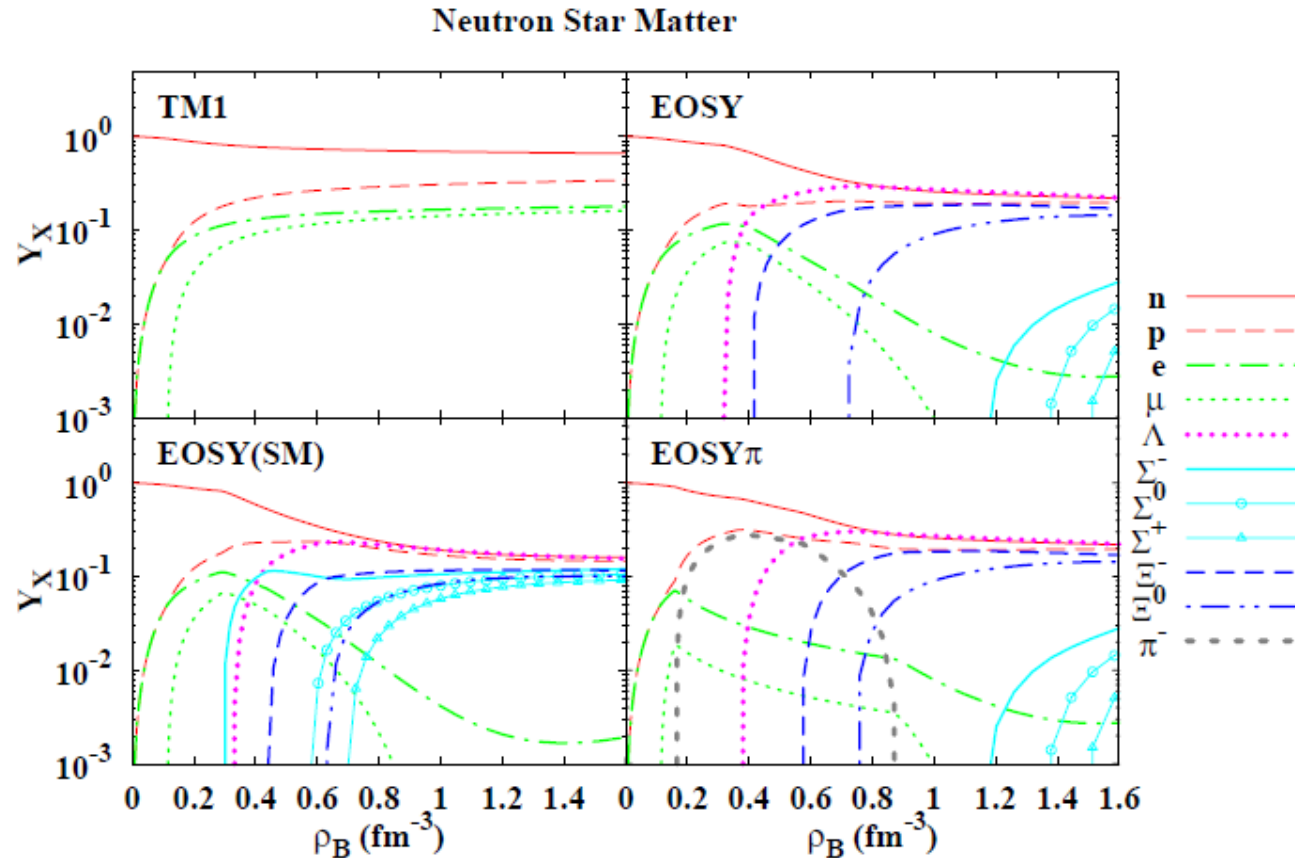


# Hyperon Composition in Dense Matter

## ■ Comparison of Hyperon Composition

- $U_{\Sigma} = -30$  MeV,  $U_{\Xi} = -28$  MeV  $\rightarrow$  SU(3) sym. matter at  $\rho_B \sim 10 \rho_0$   
Schaffner, Mishustin ('94)
- $U_{\Sigma} = +30$  MeV,  $U_{\Xi} = -15$  MeV  $\rightarrow$   $\Sigma$  baryons are strongly suppressed.  
*C.Ishizuka, AO, K.Tsubakihara, K.Sumiyoshi, S.Yamada, JPG35('08)085201.*

$\rightarrow$  Does  $\Sigma$  play no role in NS ?



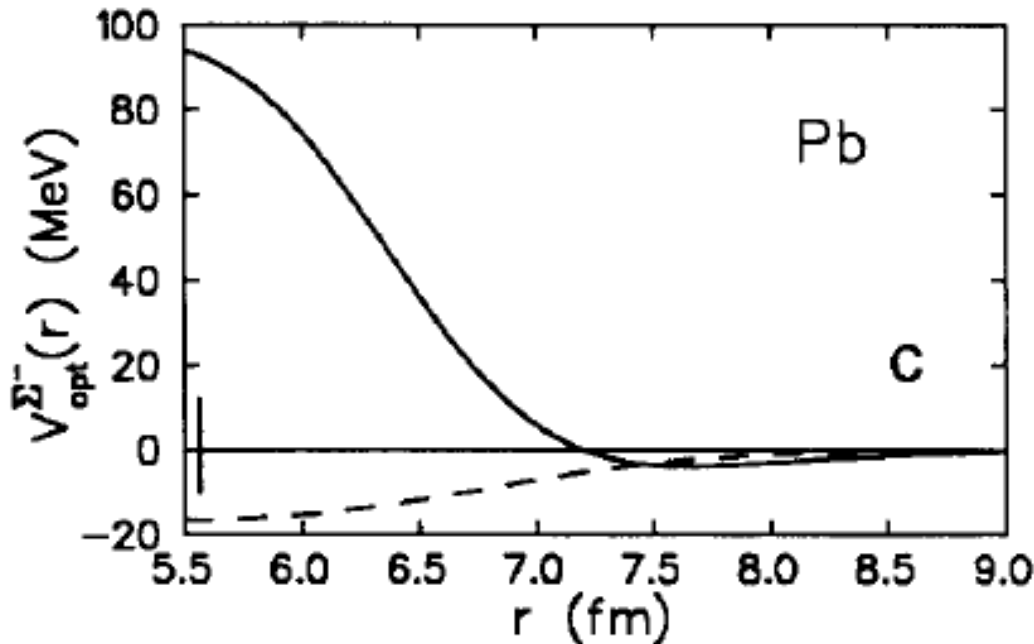
# $\Sigma^-$ atom data

- $\Sigma^-$  atom data suggested repulsion in the interior of nuclei !

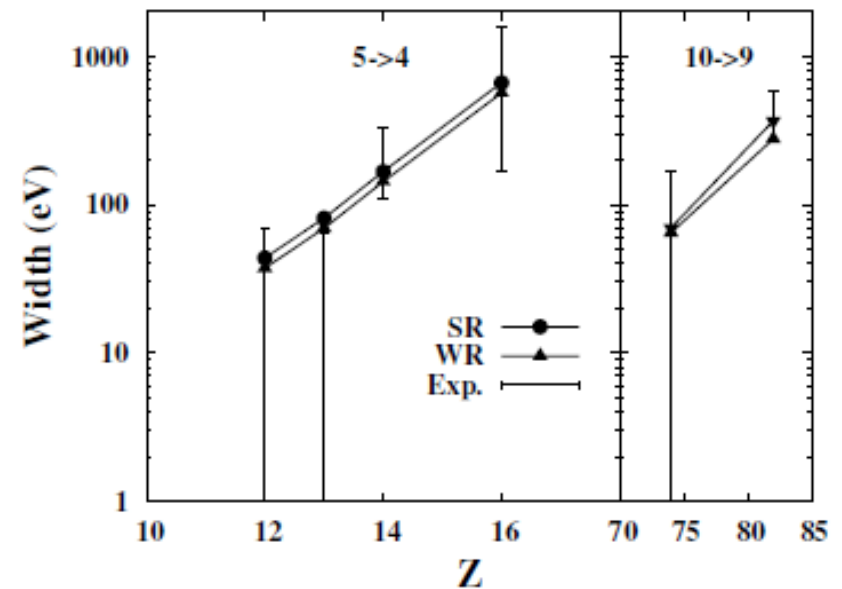
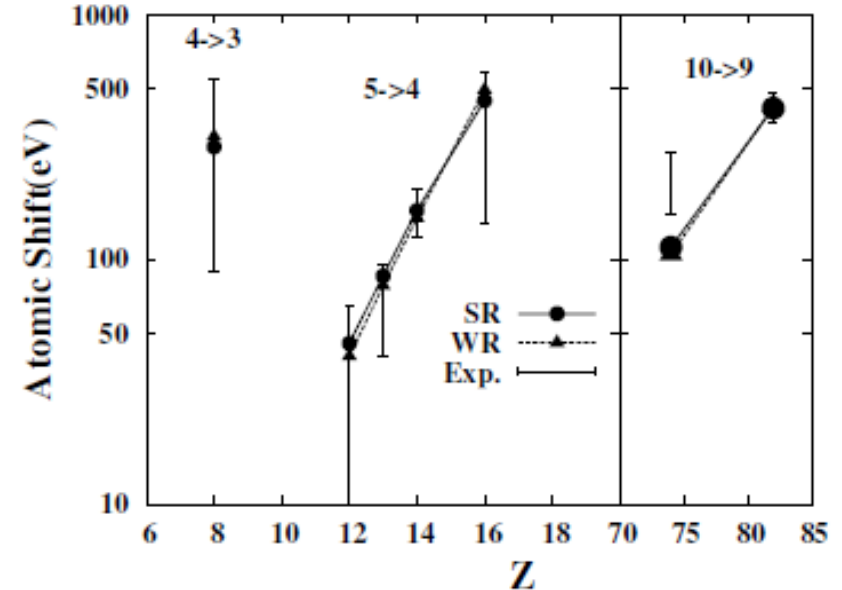
*C.J.Batty, E.Friedman, A.Gal, PLB335('94)273*

Batty's DD potential is very repulsive inside nuclei.

→ No  $\Sigma$  baryon in dense matter.



*J.Mares, E.Friedman, A.Gal, B.K.Jennings, NPA594('95)311.*



*K.Tsubakihara, H.Maekawa, AO, EPJA33('07)295.*

# $\Sigma^-$ atom in RMF

## ■ RMF fit of Si and Pb $\Sigma^-$ atom

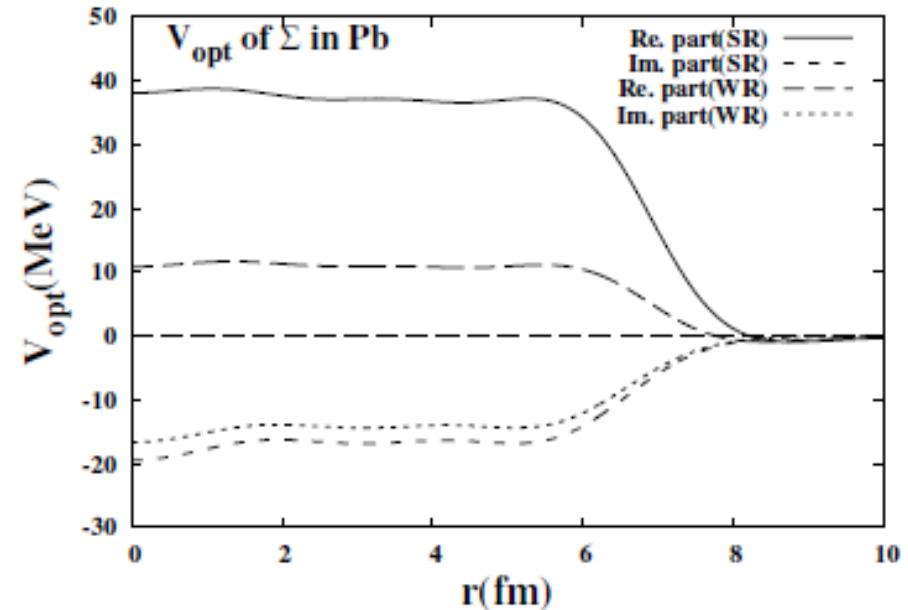
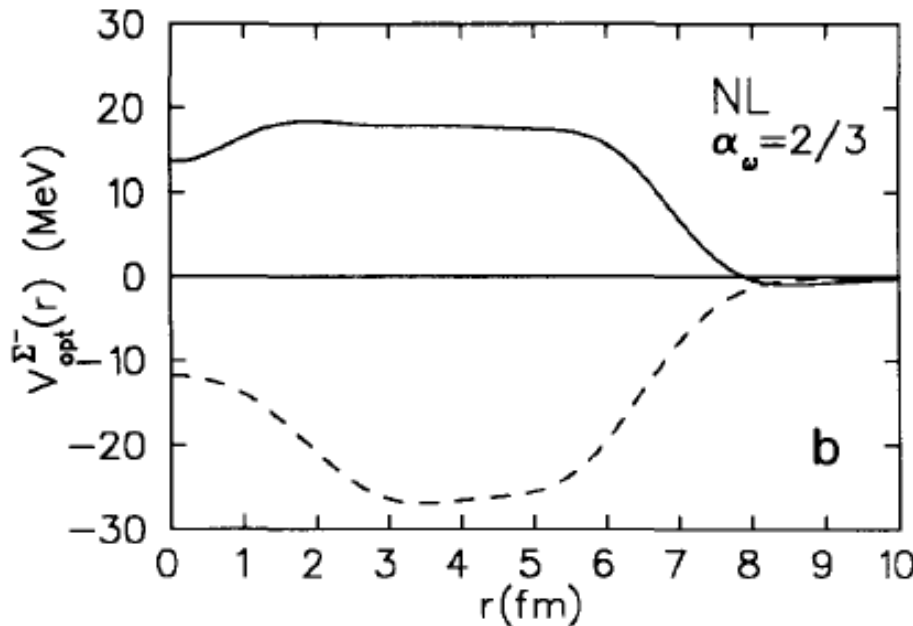
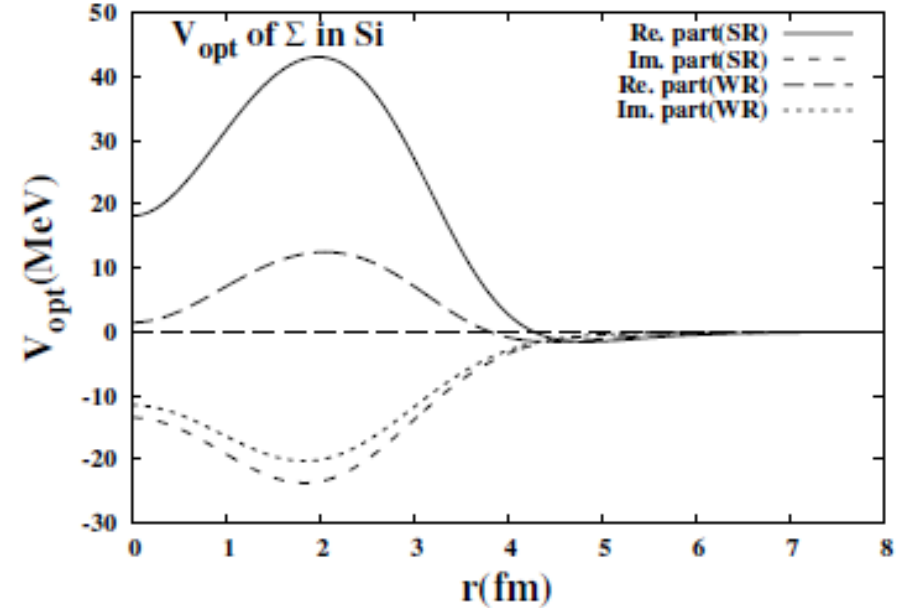
$$\alpha_\omega = g_{\omega\Sigma} / g_{\omega N} \sim 2/3(\text{M}), 0.69(\text{T})$$

$$\alpha_\rho = g_{\rho\Sigma} / g_{\rho N} \sim 2/3(\text{M}), 0.434(\text{T})$$

*J.Mares, E.Friedman, A.Gal, B.K.Jennings,*

*NPA594('95)311; Tsubakihara et al.('10)*

- Much smaller  $g_{\rho\Sigma}$  than naïve SU(3) ( $g_{\rho\Sigma} / g_{\rho N} = 2$ ), which has been applied in some of previous works.

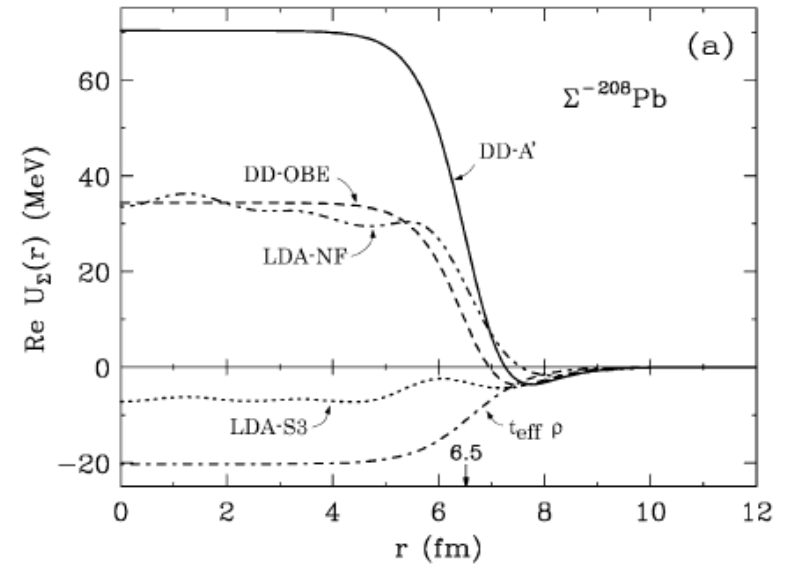


# $\Sigma$ atom and Neutron Star

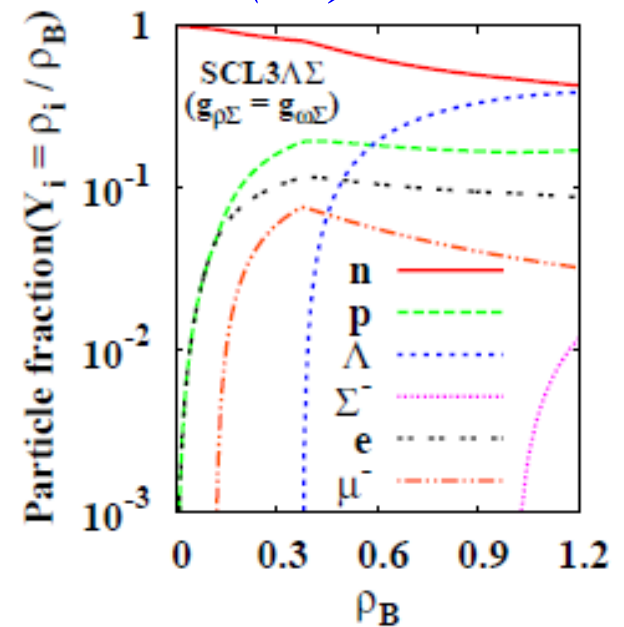
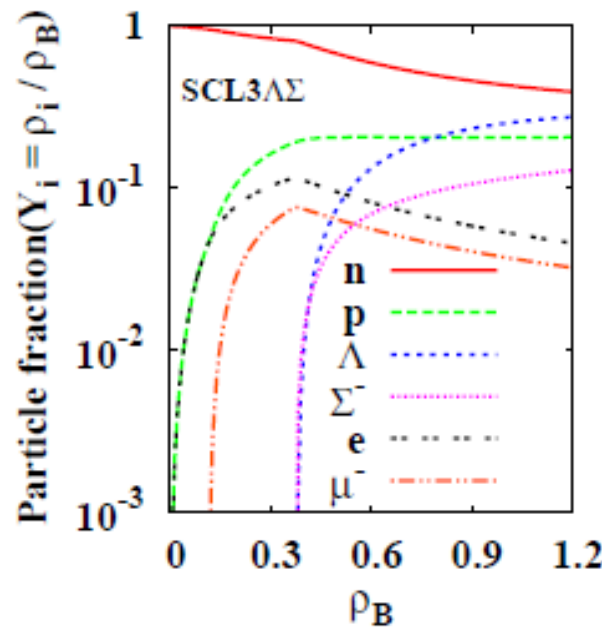
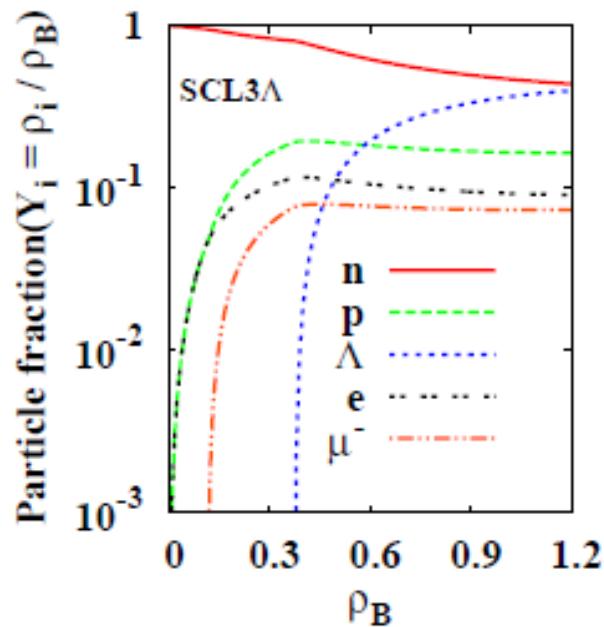
■  $\Sigma$  may not feel *very* repulsive potential in neutron star....

- $\rho^\gamma$ -type fit  $\rightarrow$  very repulsive
- RMF fit  $\rightarrow$  small isovector potential

$\rightarrow$  QF prod. may support the latter.  
 $\Sigma^-$  would appear in NS.



*T. Harada, Y. Hirabayashi, NPA767('06)206*

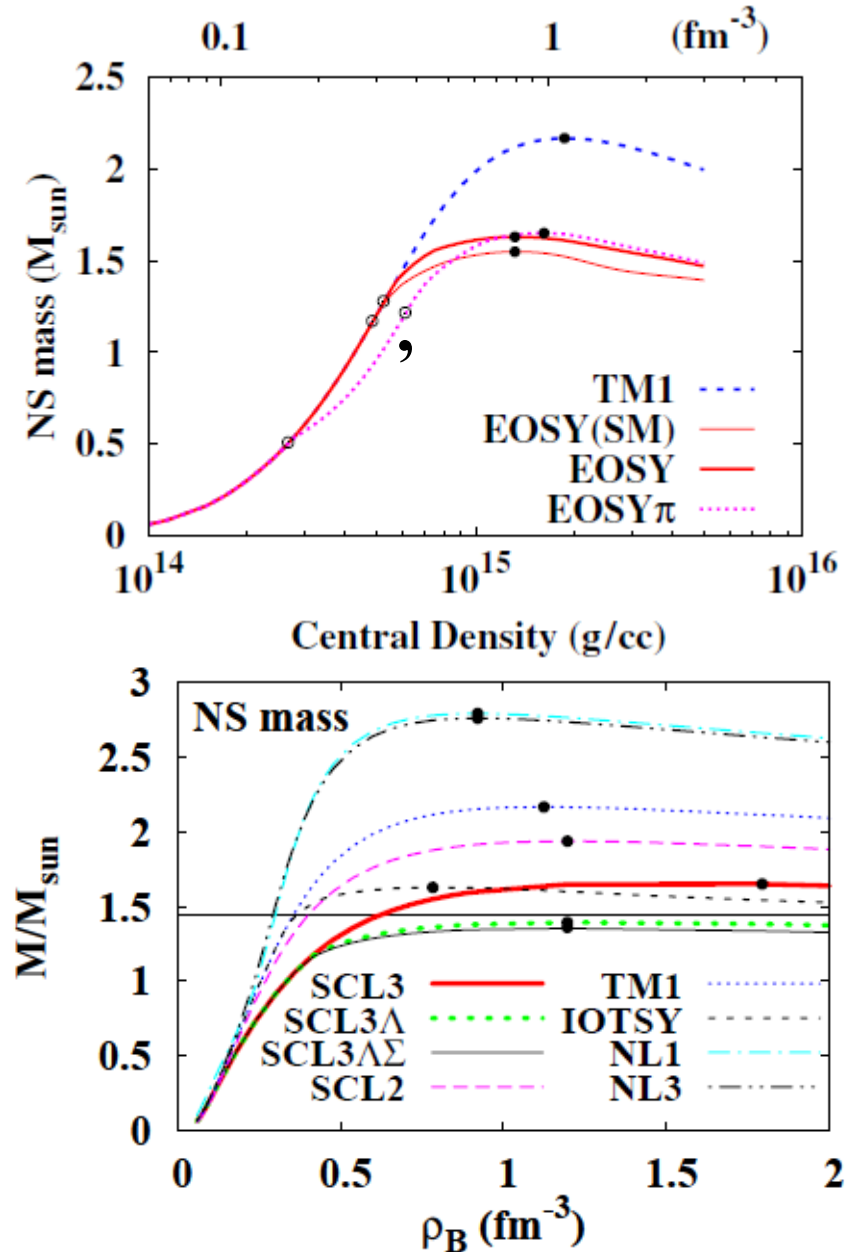


*K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.*

*Ohnishi @ DCEN2011, Sep.20-Oct.28, 2011, YITP, Kyoto, Japan*

# Neutron Star Mass

- Large fraction of hyperons softens EOS at  $\rho_B > (0.3-0.4) \text{ fm}^{-3}$ 
  - NS star max. mass red.  $\sim 1 M_{\text{sun}}$ .
  - RMF generally predicts stiff EOS at high density.  
(Scalar attraction saturation, or Z-graph in NR view.)
  - Some of RMF with Y do not support  $1.44 M_{\text{sun}}$ .
- Additional Repulsion at high  $\rho$  ?
  - Vector mass mod.  
→ stronger repulsion at high  $\rho$ .  
*M. Naruki et al., PRL96('06)092301.*
  - Another term such as  $NN\omega\sigma$ .



*C. Ishizuka, AO, K. Tsubakihara, K. Sumiyoshi, S. Yamada, JPG35('08)085201.*

*K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.*

# *RMF is a phenomenological MODEL !*

- **Baryon one-loop approximation (Hartree approximation) makes RMF a phenomenological model.**
  - We need **DATA** and **AB INITIO** results.
- ● **Saturation point (  $\rho_0$  and  $E/A(\rho_0)$ ) from mass formula**
- ● **Nuclear binding energies**
- ●  **$U_v$  and  $U_s$  from DBHF results**
- ●  **$P(\rho_B)$  from heavy-ion data**
- ●  **$\Lambda$  separation energy from single  $\Lambda$  hypernuclear data**
- ●  **$\Lambda\Lambda$  bond energy from double  $\Lambda$  hypernuclear data**
- ●  **$\Sigma$  atomic shift**
- ●  **$\Sigma$  and  $\Xi$  potential depth from quasi-free production data**
  - *Pure neutron matter EOS from ab initio calculations (not used here)*
- △ ● **Neutron Star Max. Mass  $\sim 1.40 M_{\odot}$ , a little smaller  $1.44 M_{\odot}$ .**

*The Judgement Day, Oct. 28, 2010.*

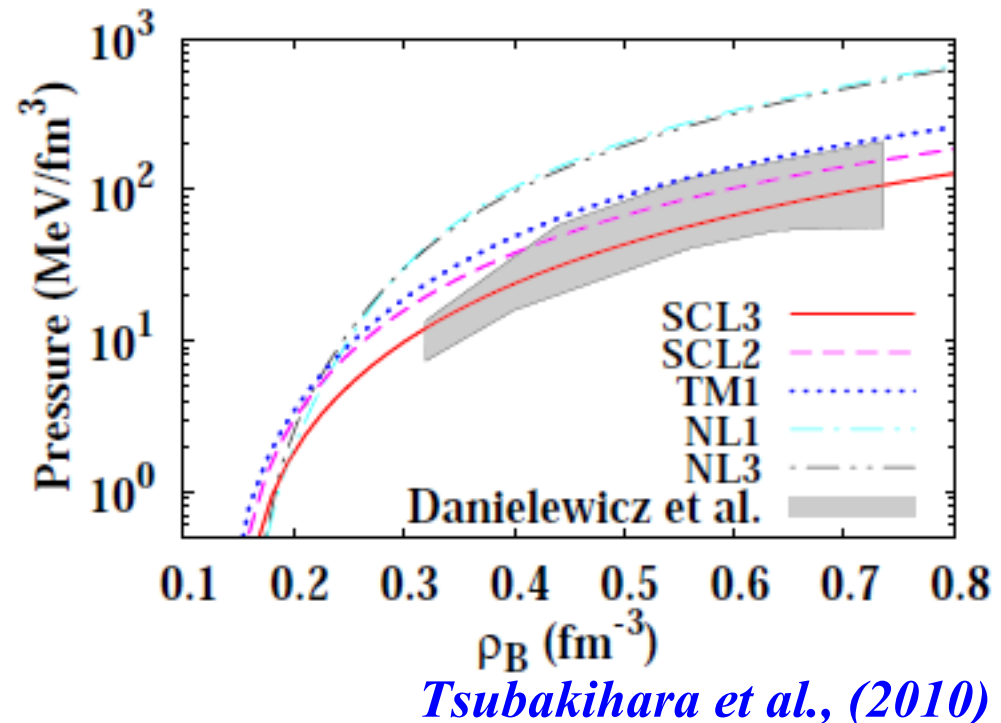
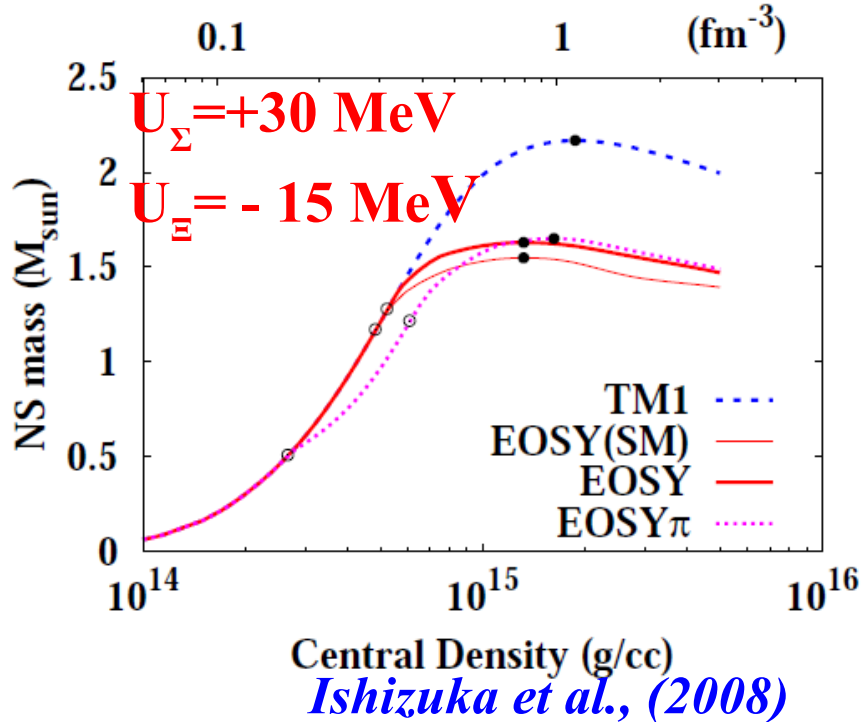
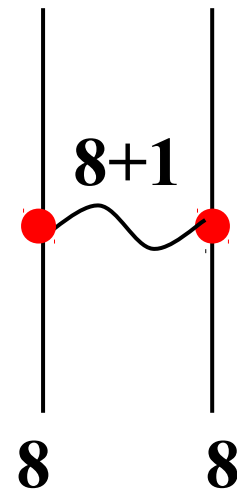
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*Can hyperon survive in  $1.97 M_{\odot}$   
neutron star ?*

# Which type of EOSs are rejected ?

## Rejected Hyperonic Matter EOS

- Relativistic Mean Field model
  - GM3: Glendenning & Moszkowski (1991)(npY)
  - GS1: Glendenning & Schaffner-Bielich (1999)(npK)
- Coupling  $\sim$  Quark Counting ( $g_{\omega Y}/g_{\omega N} \sim 2/3$ )
- Even with rel. effects, we cannot support  $1.97M_{\odot}$  as long as we respect hypernuclear & HIC data.



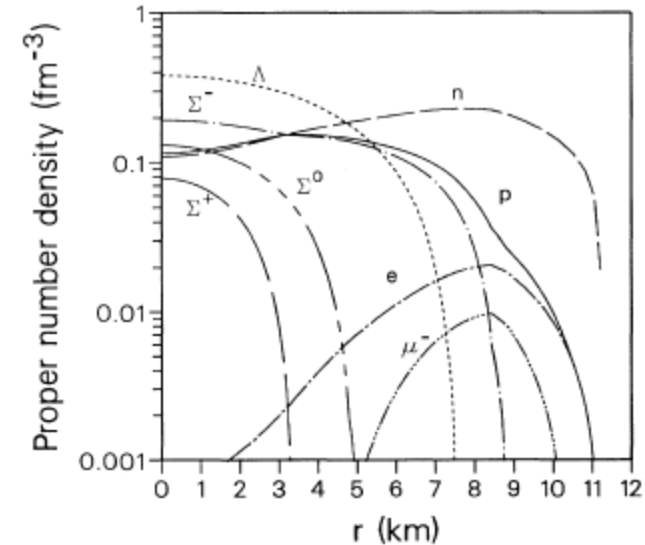
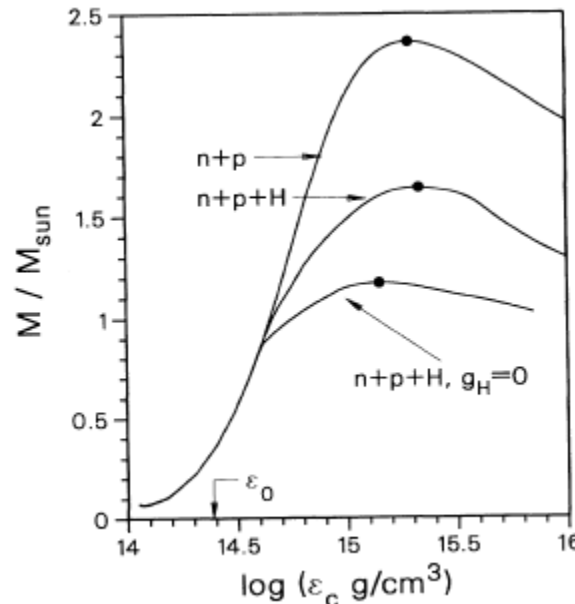
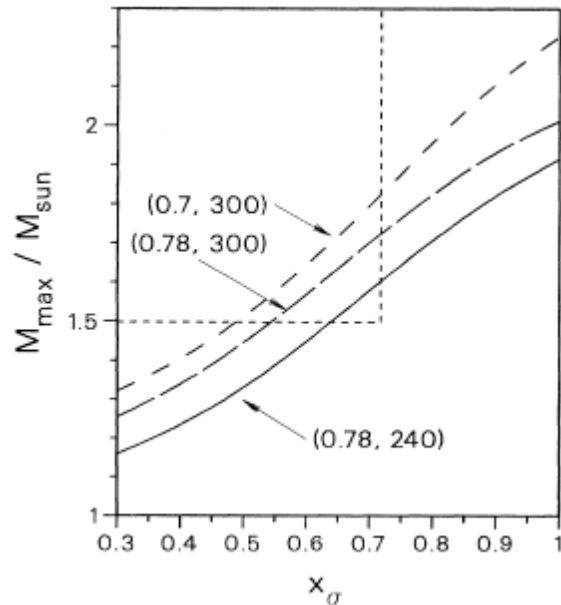


## ■ RMF with hyperons

- $n, p, Y, \sigma, \omega, \rho / \sigma^3, \sigma^4$
- Give  $x_\sigma = g_{\sigma Y} / g_{\sigma N}$  and fix  $x_\omega = g_{\omega Y} / g_{\omega N}$  to fit  $\Lambda$  separation energy.
- $x_\sigma = 0.6 \rightarrow m^*/m=0.7, x_\omega=0.653$   
(similar to quark number counting result,  $x=2/3$ )  
 $\rightarrow M_{\max} \sim 1.6 M_\odot$

TABLE I. Values of the hyperon-to-nucleon scalar and vector coupling that are compatible with the binding of  $-28$  MeV for  $\Lambda$  hyperons in nuclear matter for two values of the nucleon (Dirac) effective mass at saturation density.

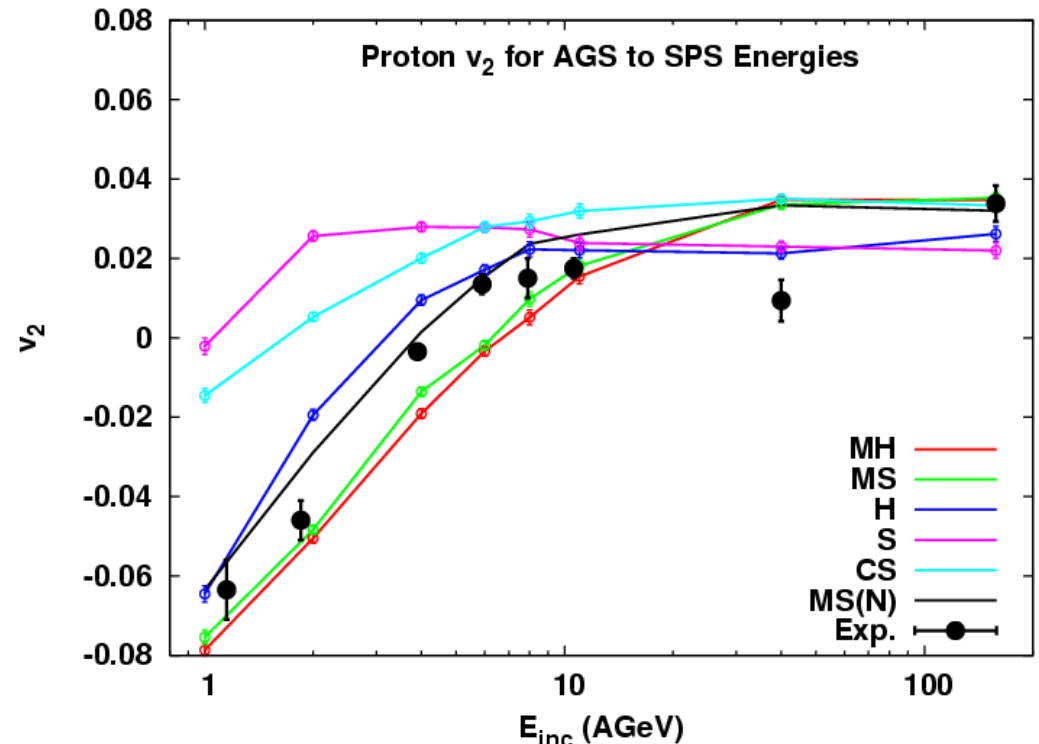
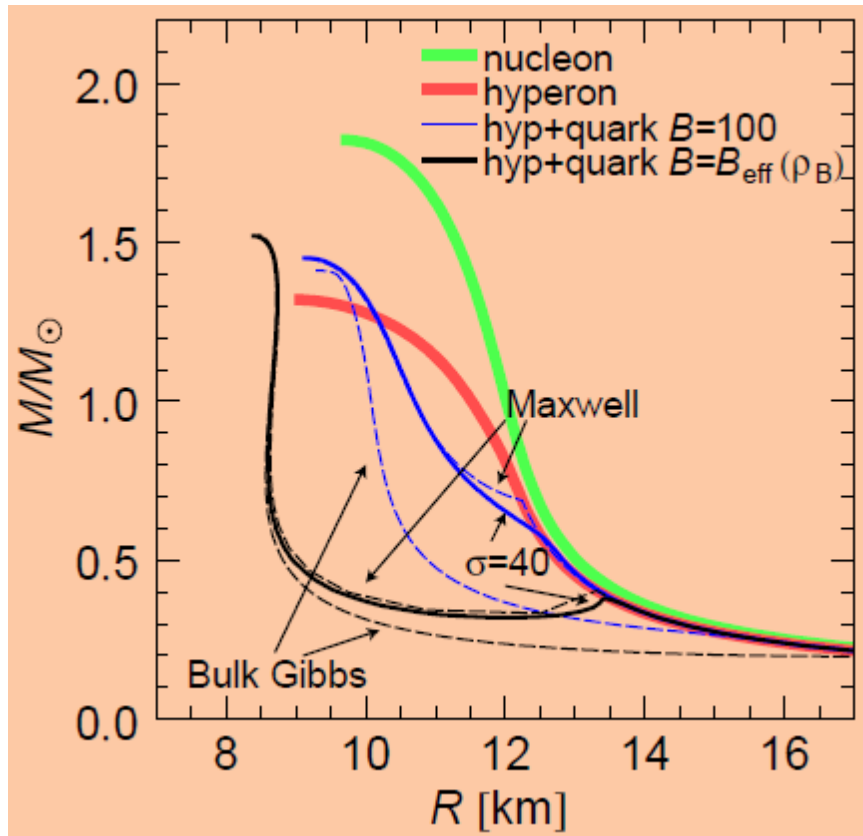
$x_\sigma$	$x_\omega$	$m^*/m=0.7$	$m^*/m=0.78$
0.2		0.131	0.091
0.3		0.261	0.233
0.4		0.392	0.375
0.5		0.522	0.517
0.6		0.653	0.568
0.7		0.783	0.800
0.8		0.913	0.942
0.9		1.04	1.08
1		1.17	1.23



*N.K.Glendenning, S.A.Moszkowski, PRL67('91)2414*

# How can we solve it ?

- No Hyperons, No Kaons  
→ How can it be consistent with YN interaction ?
- Stiff nuclear matter EOS + transition to quark matter at small  $\rho_B$   
→ How can it be consistent with HIC data at AGS-SPS energies ?
- Three-body force for baryons, quarks, ...



*M. Isse, A.O., N. Otuka, P. K. Sahu,  
Y. Nara, PRC72 ('05)064908*

*H.-J. Schulze, NFQCD10*

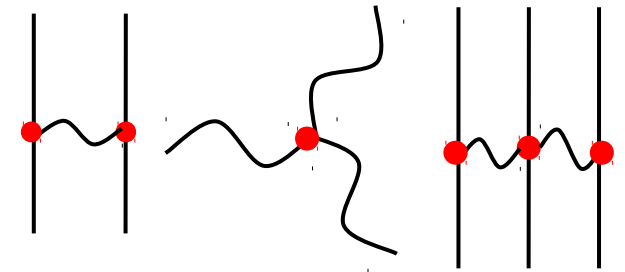
## ■ Three-baryon coupling term

$$L = L_B^{\text{free}} + L_M^{\text{free}} + L_{BM} + L_M^{\text{Int}} + \delta L$$

$$\delta L = -U_\sigma(\sigma) - \frac{1}{2} c_{\sigma\omega} \sigma \omega_\mu \omega^\mu - \frac{1}{4} c_{\omega\omega} (\omega_\mu \omega^\mu)^2$$

$$- \sum_B \bar{\Psi}_B \left[ g_{\sigma\sigma B} \sigma^2 + g_{\sigma\omega B} \sigma \omega_\mu \gamma^\mu + g_{\omega\omega B} \omega_\mu \omega^\mu \right] \Psi_B$$

$\mathbf{v} = 3$  terms



- BBMM terms are ignored in standard RMF.

(They can be absorbed in other terms by field re-definitions.)

*R.D.Furnstahl, B.D.Serot, H.-B. Tang, NPA615 ('97)441*

- But field re-definition modifies the order of NDA.

Naïve dimensional analysis (NDA)

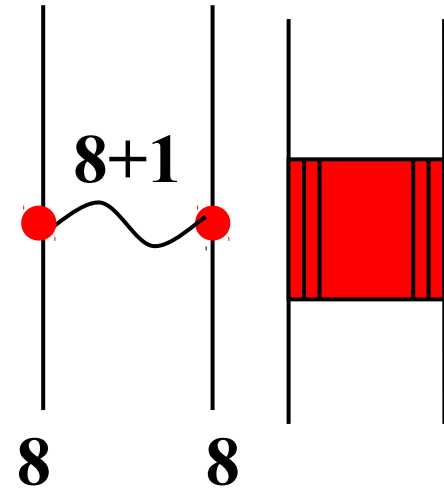
$$\mathbf{v} = \mathbf{B}/2 + \mathbf{M} + \mathbf{d}$$

(B, M, d=# of baryon and non-NG boson field, derivatives to NG fields)

- Higher  $\mathbf{v}$  terms are found to be suppressed at  $\rho \sim \rho_0$ , but they will contribute more at high densities.

# $SU(3)_f$ “violating” coupling

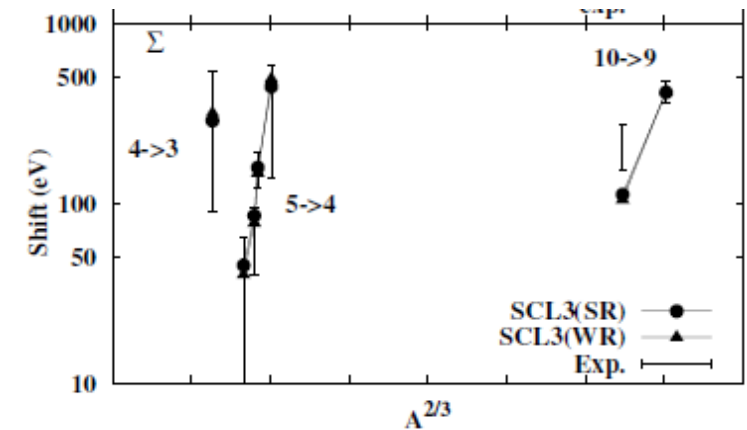
- Naïve RMF assumption = BM coupling follows  $SU(3)_f$ .
- Short range BB interaction comes from quark Pauli blocking + one-gluon exch.  
*Oka, Yazaki; Faessler et al.; Fujiwara et al.; HAL QCD collab.*
- Short-range BB repulsion is sensitive to (S,T) in the s-channel. When we include those interactions in the bosonized form, BM coupling violates  $SU(3)_f$ .



$$V = \sum_{\alpha, \beta} (\bar{\Psi} \bar{\Psi})_{\alpha} \Gamma_{\alpha\beta} (\Psi \Psi)_{\beta} \rightarrow -\frac{1}{2} \sum_{\alpha} m_{\alpha}^2 \omega_{\alpha}^2 + \sum_{\alpha} g_{\alpha} \omega_{\alpha} (\Psi \Gamma \Psi)_{\alpha}$$

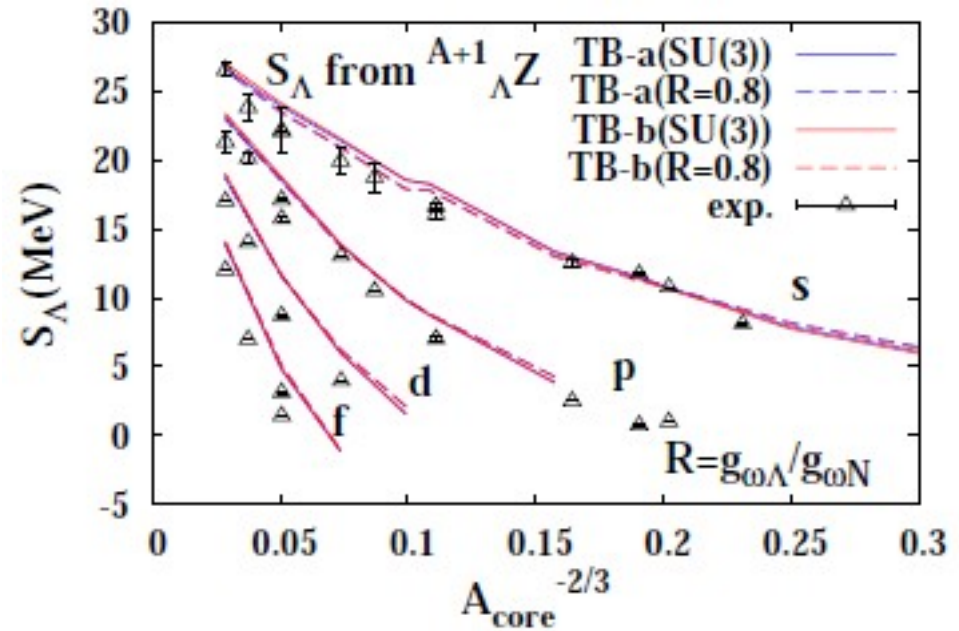
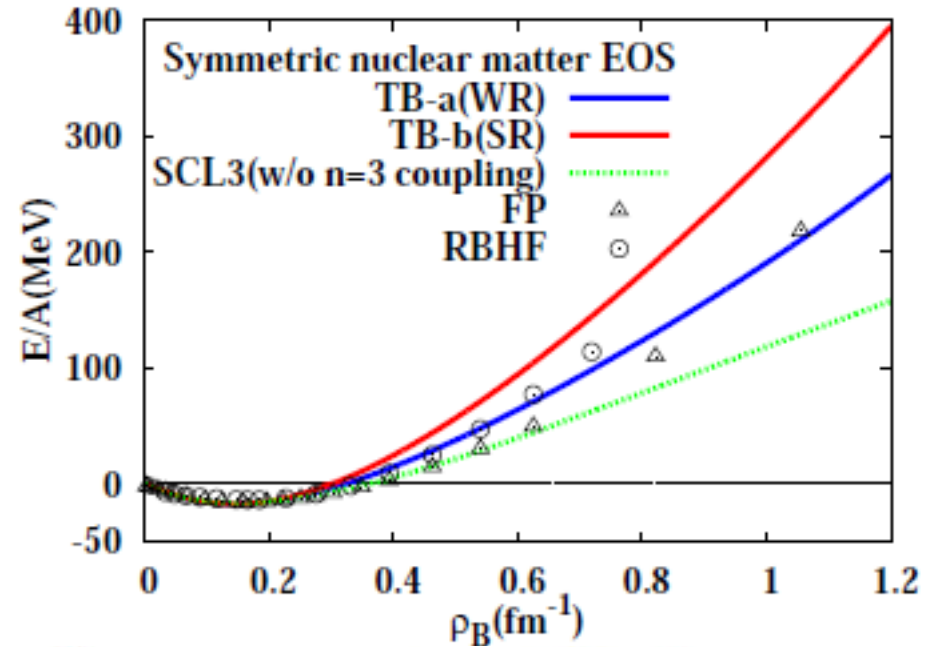
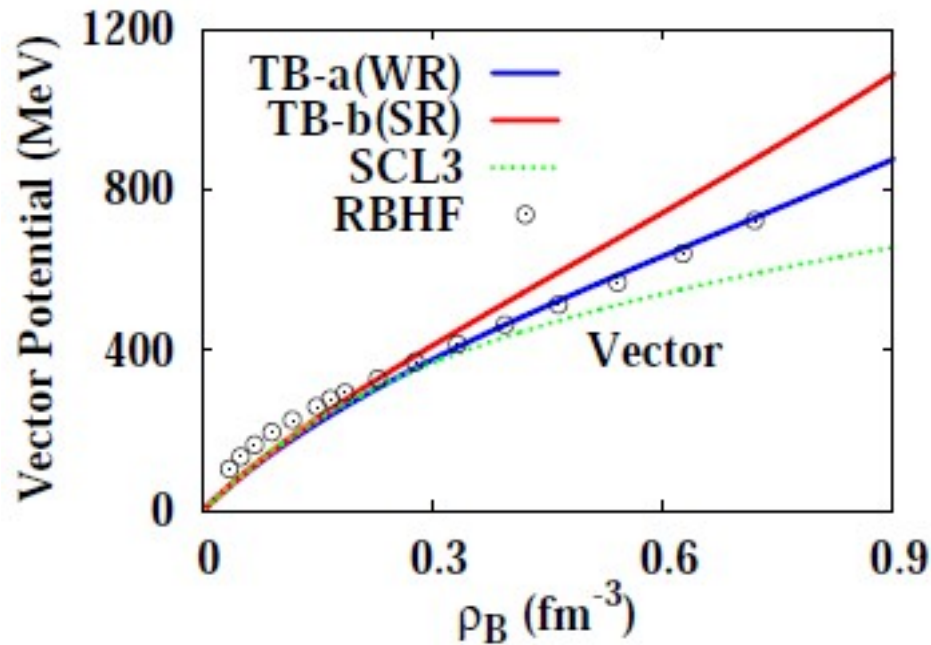
E.g.,  $\Sigma$  atomic shift

$\rightarrow g_{\sigma\Sigma} \sim g_{\sigma\Sigma} (SU(3)) \times (0.2-0.3)$



*Tsubakihara et al., (2010)*

- Nucleon vector potential  $U_V(\rho)$  in DBHF: Non-linear behavior in  $\rho_B$ .  
 → EOS becomes gradually stiffer at high density !



# RMF with 3BF + $SU(6)_{sf}$ “violation”

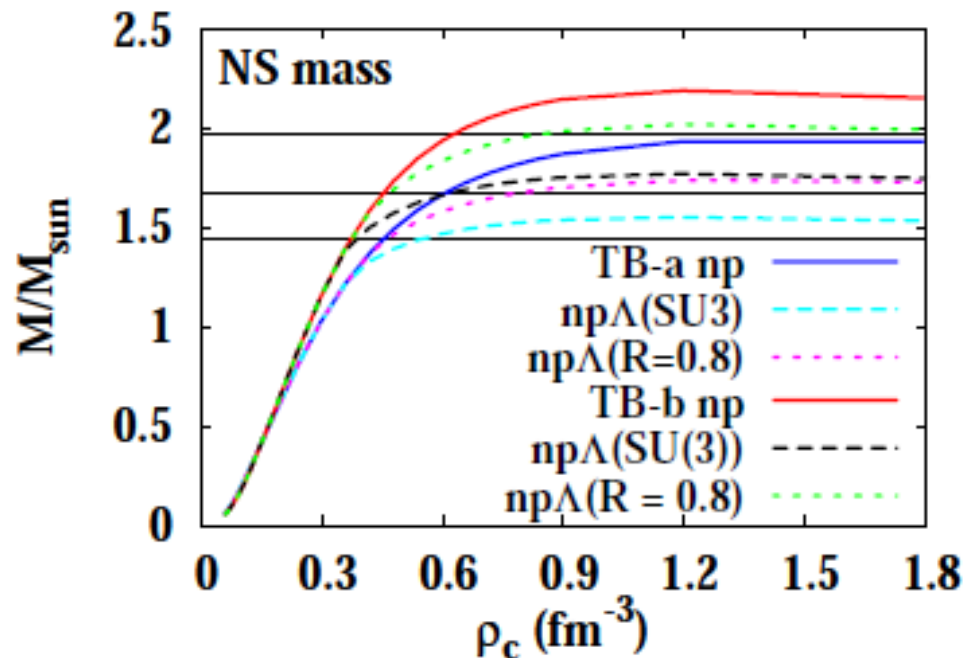
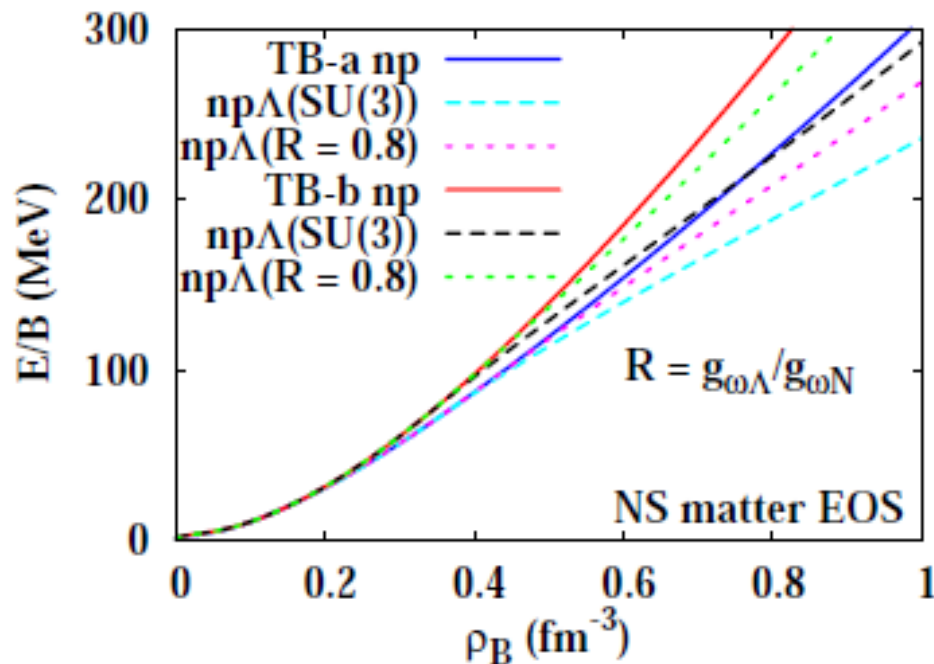
Tsubakihara, AO, Hyp2012 proc.

## Two types of modification

- 3-baryon repulsion  $\rightarrow$  EOS becomes stiff gradually at high density.  
(Fitting meson mass (E325) and  $U_v$  in RBHF)

- $R = g_{\omega\Lambda} / g_{\omega N} \sim 0.8$  ( $\sim 2/3$  (SU(3)))

$\rightarrow M_{\max} \sim 2.02 M_{\odot}$  with hyperons ( $\sim 1.4 M_{\odot}$  w/o 3BF, violation)



# Summary

- Hyperons in dense matter is still an important problem.
- Standard RMF with hyperons cannot support  $1.97 M_{\odot}$  neutron star.
  - Various data / DBHF results can be fitted in RMF.
  - Vector Coupling  $\sim SU(3)_f$ , linear BM coupling ( $\bar{B}MB$ )
- RMF with 3BF +  $SU(3)_f$  “violation” may help to support the heavy NS.
  - Atomic shift data of  $\Sigma$  atom suggests the “violation” of  $SU(3)_f$ .  
Similar trend is seen in previous RMF with hyperons.  
*R.Brockmann, W.Weise, PLB69('77)167; J.Boguta, S.Bohrmann, PLB102('81)93.*
  - Ab initio calculations with induced/bare 3B force are helpful for phen. approaches to fix parameters.
  - Discussions during DCEN 2011 were encouraging.  
(Importance of 3BF, large effects of 3BF at  $\rho_0$ )
- Can we support  $1.97 M_{\odot}$  neutron star with hyperons ?  
→ Open problem.

# レポート問題

## ■ Sec. 1(b) [ 〆切 12/28、大西部屋まで。 ]

- ボソン化した NJL 模型の作用から出発して、ゼロ温度 ( $T=0$ ) での有効ポテンシャルを求めよ。
  - ◆ 発展：有限温度・有限密度 (有限化学ポテンシャル) での有効ポテンシャルを構成子クォーク質量で 2 次まで展開し、2 次相転移線が  $T^2 + \mu_B^2 / 3\pi^2 = T_c^2$  で与えられることを示せ。
- リンク積分を利用して、Wilson ループの期待値を強結合領域で求めよ。強結合極限での結果に加えて、 $1/g^2$  補正がどのように与えられるか考察せよ。
- bag 模型状態方程式において相転移が起こる温度を bag constant  $B$  を用いて表せ。ハドロン相は 3 つのパイオンを考え、クォーク・グルーオン相ではクォークのフレーバー数を 3 とする。

## ■ Sec. 1(c)

- 相対論的平均場理論 ( $\sigma\omega$  模型等) において、核子の four spinor の上 2 成分が満たす方程式を導き、スピン軌道力の表式を与えよ。
- 相対論的平均場理論 ( $\sigma\omega$  模型等) において、エネルギー密度の表式を求めよ。また余裕があれば、核物質の飽和点を満たすように  $\sigma_N, \omega_N$  の結合定数を与えよ。飽和点は  $\rho_0 = 0.15 \text{ fm}^{-3}$ ,  $E/A = -16 \text{ MeV}$  とする。  
(後半は多少の数値計算が必要である。)



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*Thank you for your attention !*

# Chiral Symmetry

- Fundamental symmetry of massless QCD, and its spontaneous breaking generates hadron masses.

*Nambu, Jona-Lasinio ('61)*

- Many of the linear  $\sigma$  models are unstable against finite density (chiral collapse).

→ Log type chiral potential

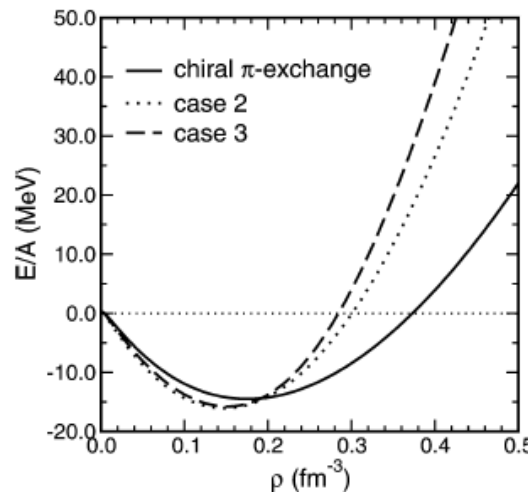
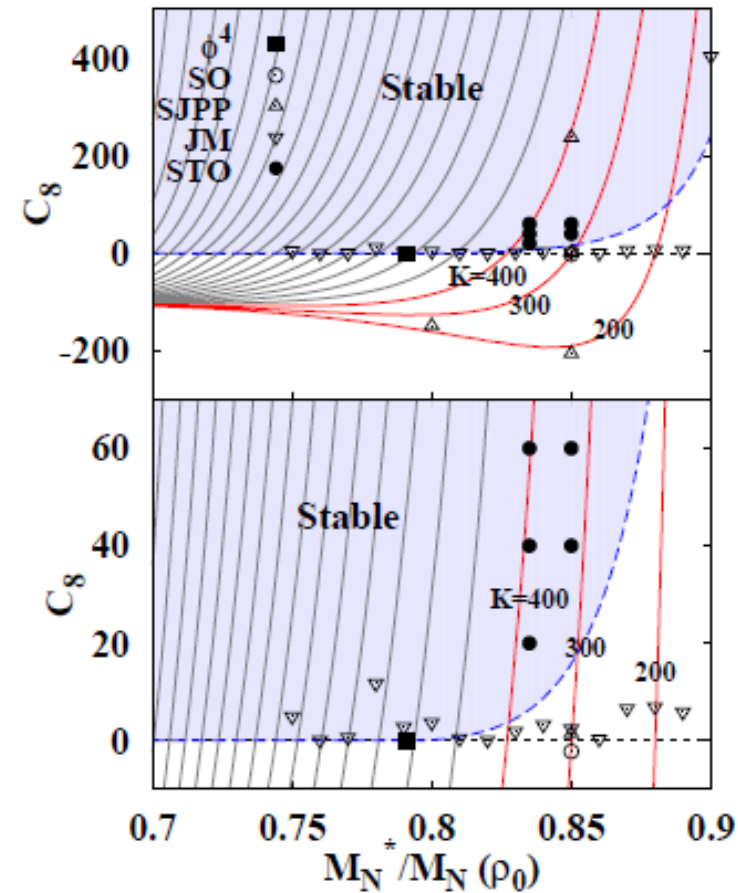
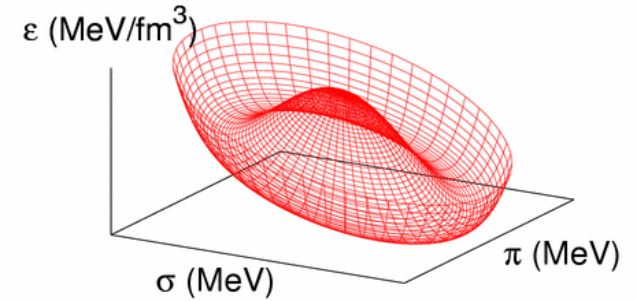
*Sahu, Tsubakihara, AO('10), Tsubakihara, AO('07)*

- Non-linear representation (chiral pert.) leads to density dependent coupling from pion loops.

*Kaiser, Fritsch, Weise ('02),*

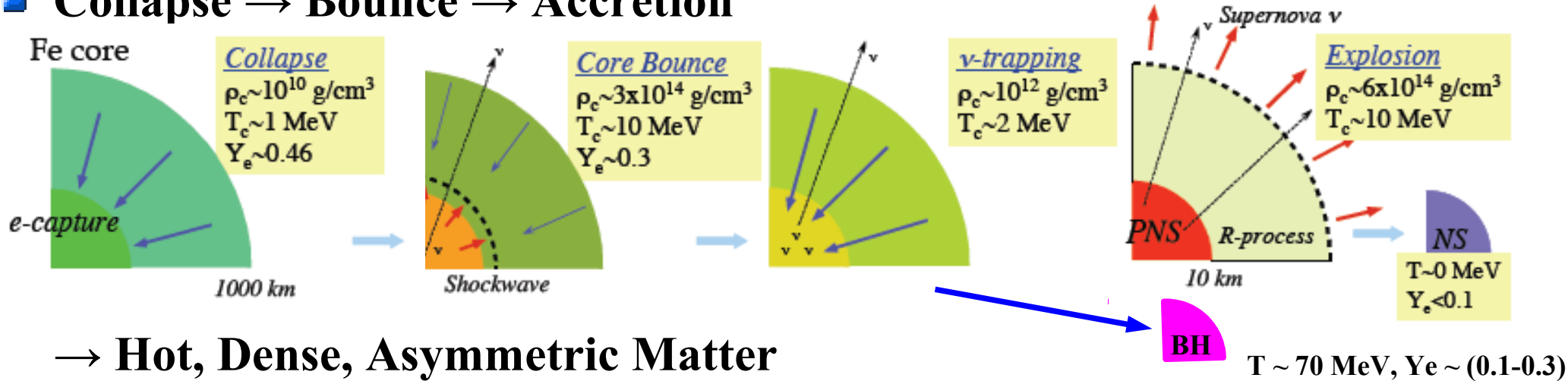
*Finelli, Kaiser, Vretener*

*Weise ('04)*



# Dynamical Black Hole Formation

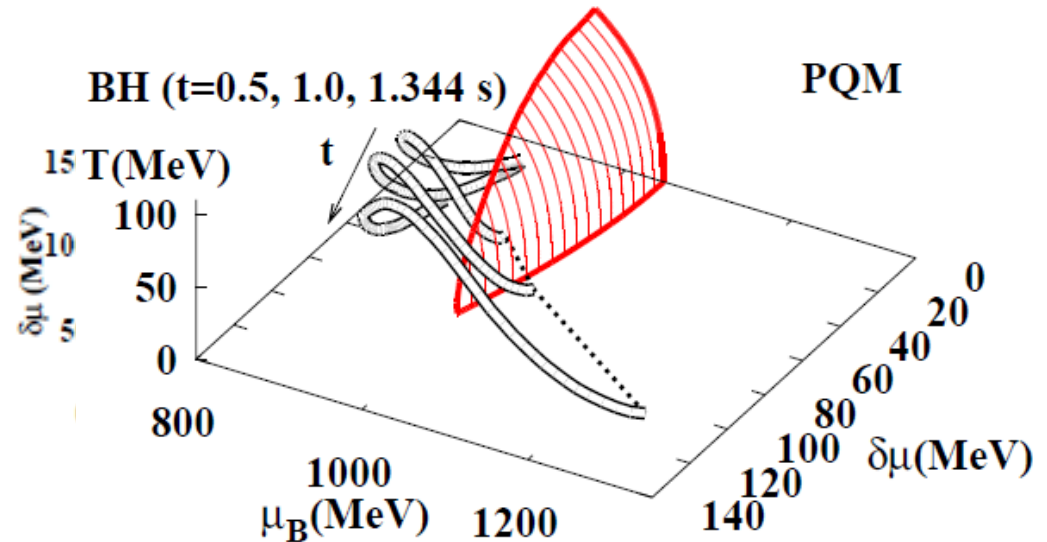
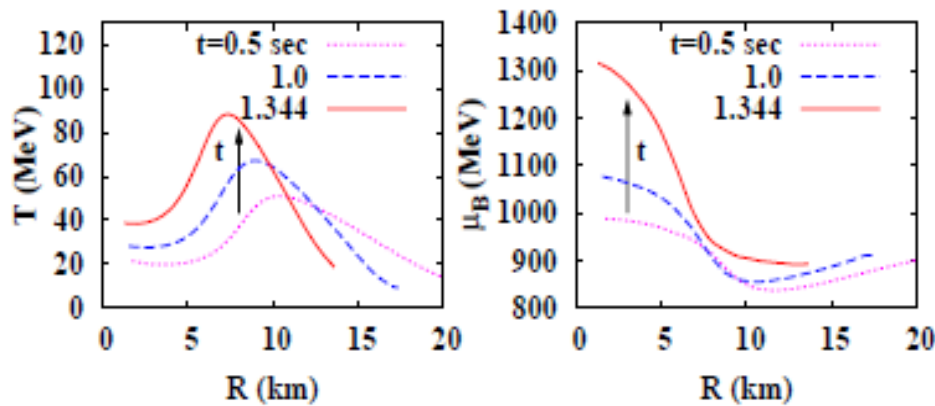
## ■ Collapse → Bounce → Accretion



→ Hot, Dense, Asymmetric Matter

$$T \sim 70 \text{ MeV}, \mu_B \sim 1300 \text{ MeV}, \delta\mu = \mu_e/2 \sim 130 \text{ MeV}$$

→ CP may be reachable



*K. Sumiyoshi, et al., ('06); K. Sumiyoshi, C. Ishizuka, AO, S. Yamada, H. Suzuki ('09)  
AO, H. Ueda, T. Z. Nakano, M. Ruggieri, K. Sumiyoshi, PLB in press.*