核多体系物理学

- **授業の概要・目的：** 核子・ハドロン・クォークからなる多体系の性質を状態方程式、および核反応論の観点から議論する。核物質の状態方程式を記述するために必要となる核多体理論（平均場理論、G-matrix 、熱場の理論、強結合格子 QCD ）、ハイパー核生成反応や重イオン反応を理解する上で必要とされる原子核核反応理論（直接反応、輸送模型等）、等の理論の枠組について解説すると共に、これらについての最近の研究成果についても紹介する。

- **授業計画と内容**
  核子・ハドロン・クォーク物質の相互作用と状態方程式について以下の内容で講義する。

1. 状態方程式を記述する理論模型
   (a) 核物質の状態方程式と QCD 相図研究の概観
   (b) 場の理論からのアプローチ (南部 - ヨナラシニョ模型、強結合格子 QCD)
   (c) 相対論的平均場理論

2. 輸送理論
   (a) 時間依存平均場理論、半古典輸送模型とボルツマン方程式、流体模型、
   (b) 古典ヤンミルズ場のダイナミクス
   (c) 高エネルギー重イオン衝突の概観と輸送理論の適用例。

3. 直接反応理論
   (a) 核子 - 核子散乱、核力と位相差、有効相互作用 (G-matrix)
   (b) ハドロン - 核反応 (光学模型、インパルス近似、グリーン関数法) 、
   ハイパー核・中間子核生成反応の概観と直接反応理論の適用例
   (c) 高エネルギー核反応 (グラウバー模型、ハドロン共鳴)

- **成績評価の方法・基準：** 履修状況及びレポートにより総合評価する。

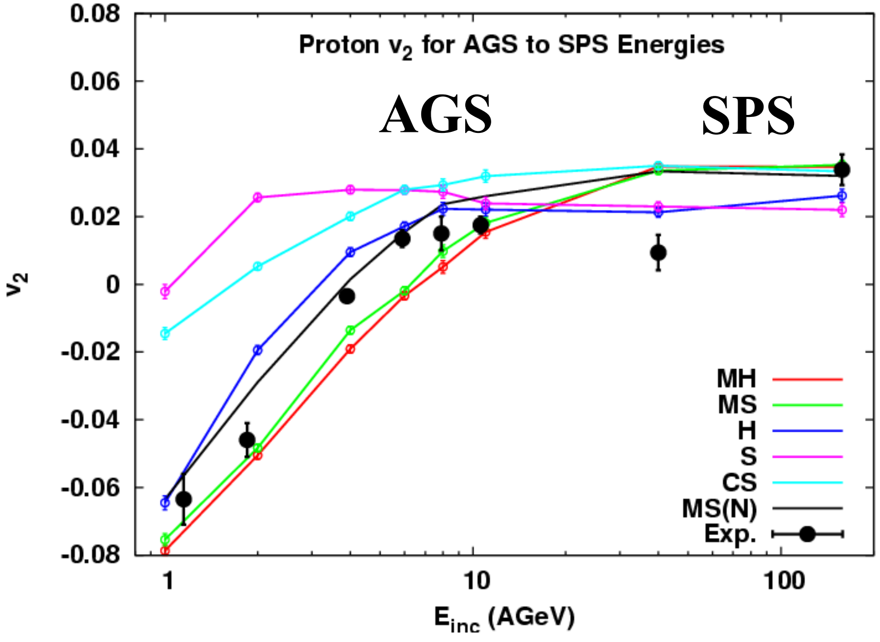
参考書 *Theoretical Nuclear and Subnuclear Physics, John Dirk Walecka (World Scientific)*

講義のスライド **http://www2.yukawa.kyoto-u.ac.jp/~ohnishi/Lec2012/**

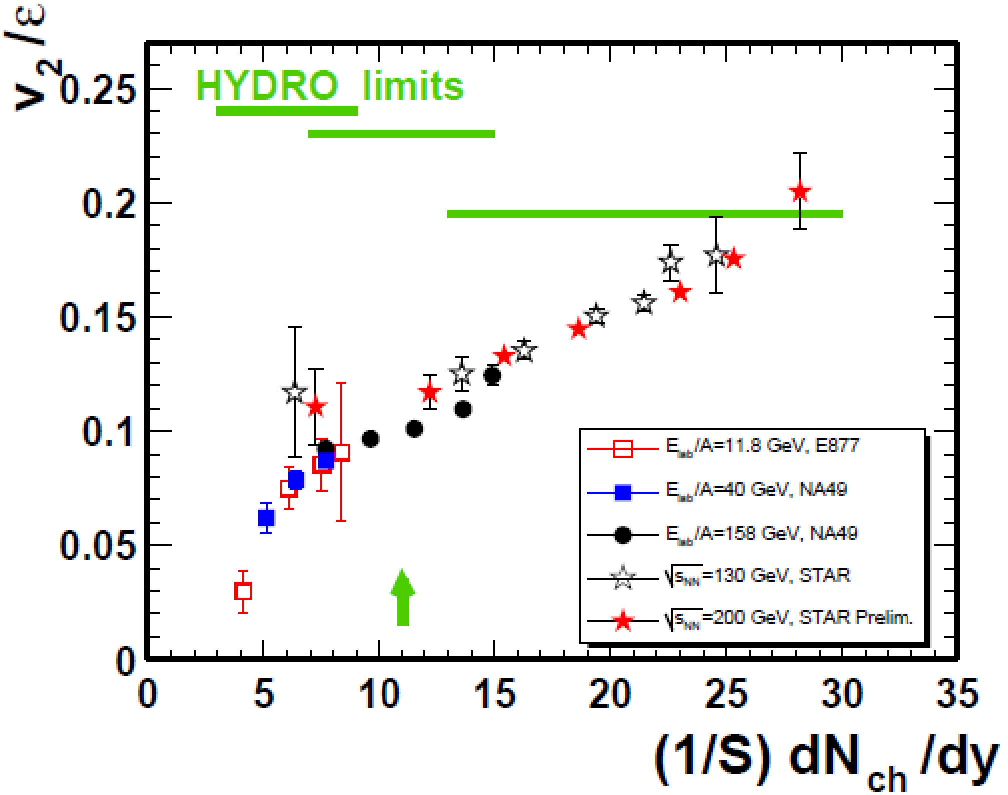
Nuclear Transport Models and Heavy-Ion Collision

Hydrodynamics vs Transport

- **$\sqrt{s_{NN}} < 20$ GeV → Transport model calculation explains v2 data.**
- **RHIC (& LHC) → Hydrodynamics is successful.**



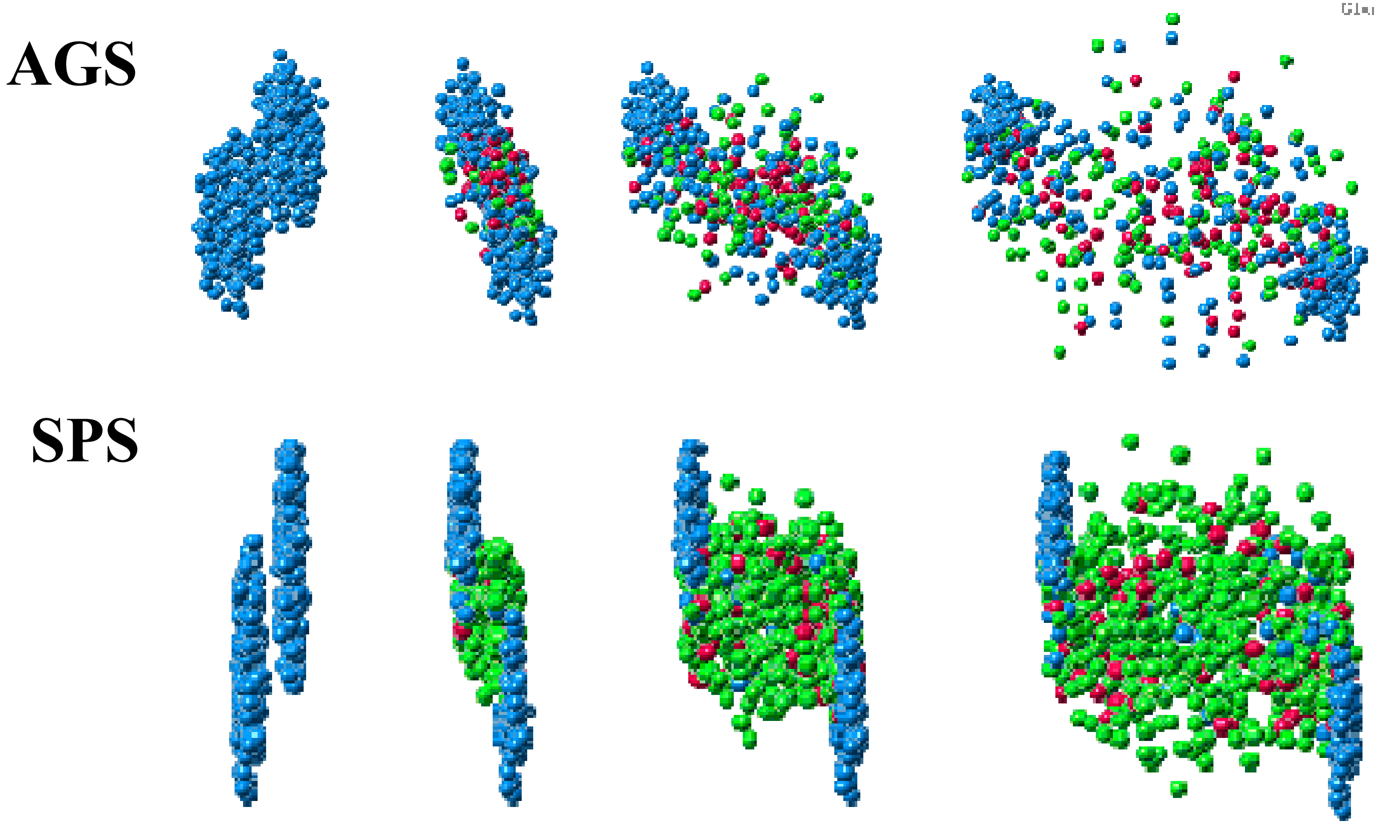
M. Isse, AO, N. Otuka, P.K. Sahu, Y. Nara, PRC72 ('05) 064908 [nucl-th/0502058]



U. W. Heinz, AIP Conf.Proc. 739 ('05) 163 [nucl-th/0407067]

Heavy-Ion Collisions at Einc ~ (1-100) A GeV

- **Study of Hot and Dense Hadronic Matter**
  → Particle Yield, Collective Dynamics (Flow), EOS, ...



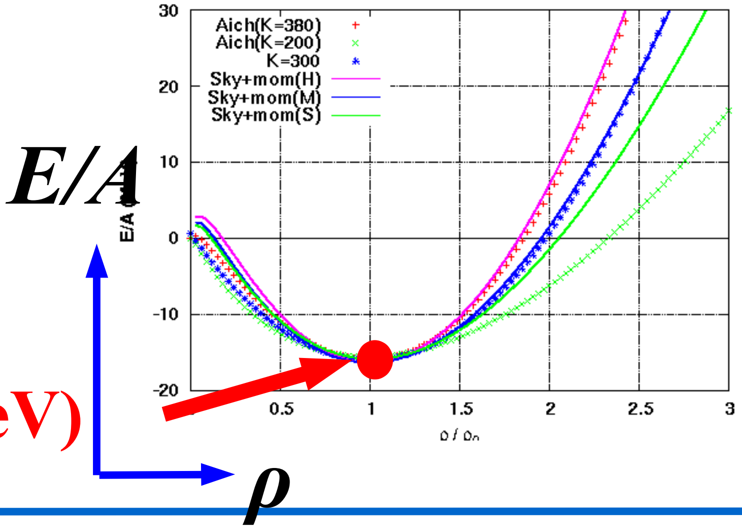
JAMming on the Web, linked from http://www.jcprg.org/

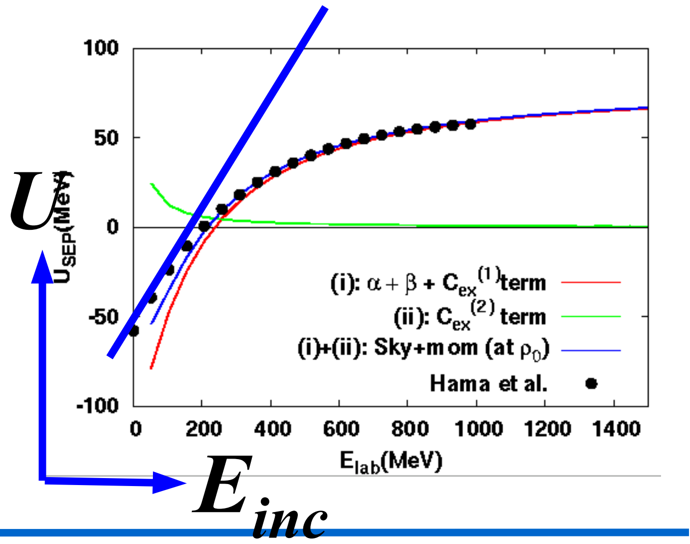
Nuclear Mean Field

- **MF has on both of ρ and p-deps.**
  - **ρ dep.: (ρ_0, E/A) = (0.15 fm^{-3}, -16.3 MeV) is known**
    Stiffness is not known well
  - **p dep.: Global potential up to E=1 GeV is known from pA scattering**
    $U(\rho_0, E) = U(\rho_0, E=0)+0.3\,E$
- **Ab initio Approach; LQCD, GFMC, DBHF, G-matrix, ...**
  → Not easy to handle, Not satisfactory for phen. purposes
- **Effective Interactions (or Energy Functionals):**
  Skyrme HF, RMF, ...

$U(E)=U(0)+0.3E$

$(\rho_0\,, E/A)$
$=(0.15\ \mathrm{fm}^{-3}\,, -16.3\ \mathrm{MeV})$





HIC Transport Models: Major Four Origins

- *Nuclear Mean Field Dynamics*
  - **Basic Element of Low Energy Nuclear Physics, and Critically Determines High Density EOS / Collective Flows**
  - **TDHF → Vlasov → BUU**
- *NN two-body (residual) interaction*
  - **Main Source of Particle Production**
  - **Intranuclear Cascade Models**
- *Partonic Interaction and String Decay*
  - **Main Source of high pT Particles at Collider Energies**
  - **JETSET + (previous) PYTHIA (Lund model) → (new) PYTHIA**
- *Relativistic Hydrodynamics*
  - **Most Successful Picture at RHIC**

Nuclear Mean Field Dynamics

TDHF and Vlasov Equation

- **Time-Dependent Mean Field Theory (e.g., TDHF)**

$$i\hbar \frac{\partial \phi_i}{\partial t} = h\phi_i$$

- **Density Matrix**

$$\rho(r, r') = \sum_i^{Occ} \phi_i(r)\phi_i^*(r') \quad \rightarrow \quad \boldsymbol{\rho_W = f} \text{ \textbf{\textit{(phase space density)}}}$$

- **TDHF for Density Matrix**

$$i\hbar \frac{\partial \rho}{\partial t} = [h, \rho] \quad \rightarrow \quad \boldsymbol{\frac{\partial f}{\partial t} = \{h_W, f\}_{P.B.} + O(\hbar^2)}$$

- **Wigner Transformation and Wigner-Kirkwood Expansion**

(Ref.: Ring-Schuck)

$$O_W(r, p) \equiv \int d^3s \exp(-ip\cdot s/\hbar) < r + s/2|O|r - s/2 >$$

$$(AB)_W = A_W \exp(i\hbar\Lambda) B_W \quad \Lambda \equiv \nabla'_r \cdot \nabla_p - \nabla'_p \cdot \nabla_r \quad (\nabla' \text{acts on the left})$$

$$[A, B]_W = 2\mathrm{i} A_W \sin(\hbar\Lambda/2) B_W = i\hbar\{A_W, B_W\}_{P.B.} + O(\hbar^3)$$

Test Particle Method

- **Vlasov Equation**

$$\frac{\partial f}{\partial t} - \{h_W, f\}_{P.B.} = \frac{\partial f}{\partial t} + v \cdot \nabla_r f - \nabla U \cdot \nabla_p f = 0$$

- **Classical Hamiltonian**

$$h_W(r, p) = \frac{p^2}{2\mathrm{m}} + U(r, p)$$

- **Test Particle Method (C. Y. Wong, 1982)**

$$f(r, p) = \frac{1}{N_0} \sum_i^{A N_0} \delta(r - r_i)\, \delta(p - p_i) \quad \rightarrow \quad \frac{dr_i}{dt} = \nabla_p h_w, \quad \frac{dp_i}{dt} = -\nabla_r h_w,$$

Mean Field Evolution can be simulated
by Classical Test Particles
→ Opened a possibility to Simulate High Energy HIC
including Two-Body Collisions in Cascade

BUU (Boltzmann-Uehling-Uhlenbeck) Equation

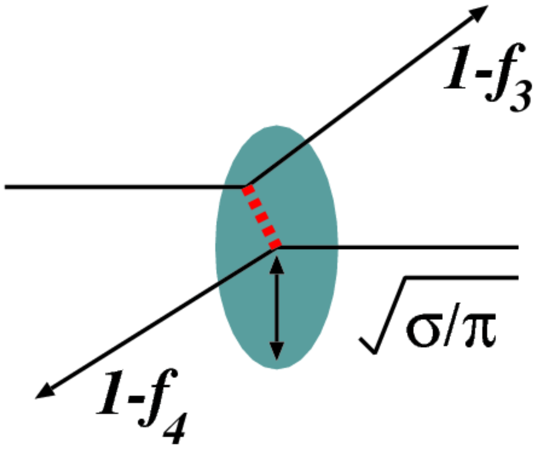
- **BUU Equation** **(Bertsch and Das Gupta, Phys. Rept. 160(88), 190)**

$$\frac{\partial f}{\partial t} + v \cdot \nabla_r f - \nabla U \cdot \nabla_p f = I_{coll}[f]$$

$$I_{coll}[f] = -\frac{1}{2} \int \frac{d^3 p_2 \, d\Omega}{(2\pi\hbar)^3} v_{12} \frac{d\sigma}{d\Omega}$$

$$\times \quad [f f_2 (1-f_3)(1-f_4) - f_3 f_4 (1-f)(1-f_2)]$$

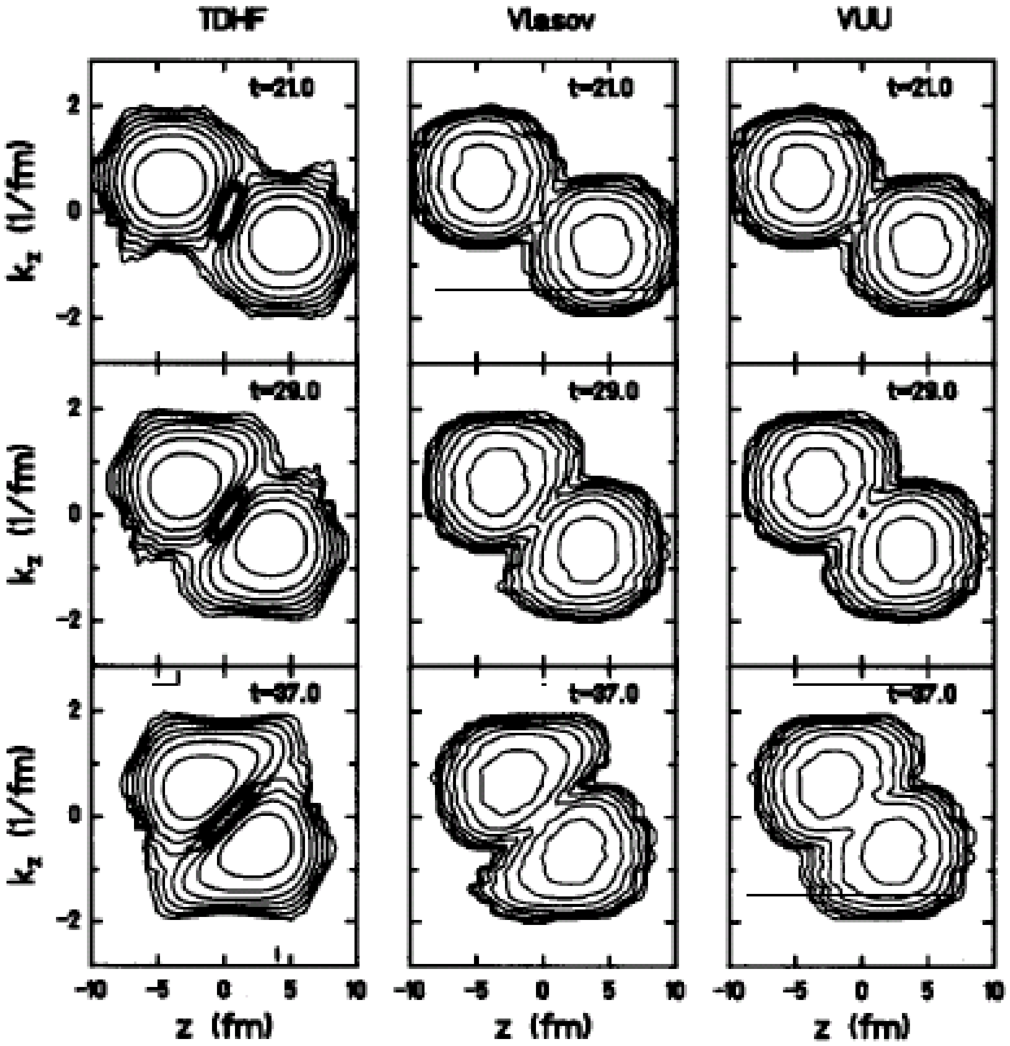
- **Incorporated Physics in BUU**
  - **Mean Field Evolution**
  - **(Incoherent) Two-Body Collisions**
  - **Pauli Blocking in Two-Body Collisions**



O *One-Body Observables (Particle Spectra, Collective Flow, ..)*
X *Event-by-Event Fluctuation (Fragment, Intermittency, ...)*

Comarison of TDHF, Vlasov and BUU(VUU)

- **Ca+Ca, 40 A MeV**
  (Cassing-Metag-Mosel-Niita, Phys. Rep. 188 (1990) 363).



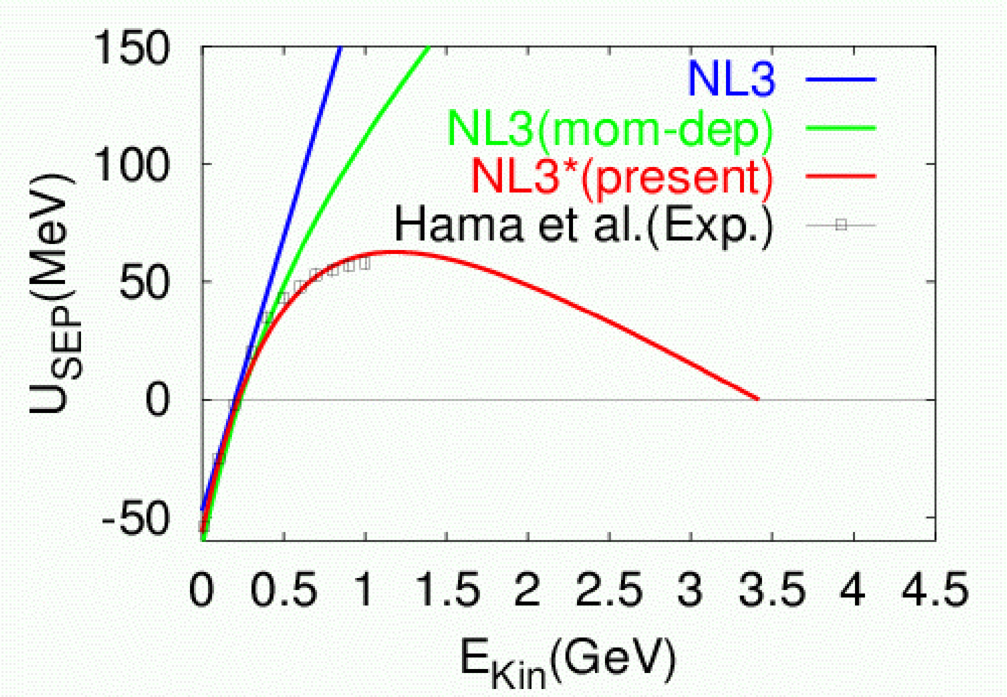
Relativistic Mean Field (II)

- **Dirac Equation** $\left(i\gamma\partial-\gamma^0 U_v - M - U_s\right)\psi=0\ ,\quad U_v=g_\omega\omega\ ,\quad U_s=-g_\sigma\sigma$
- **Schroedinger Equivalent Potential**

$$\begin{pmatrix} E-U_v-M-U_s & i\sigma\cdot\nabla \\ -i\sigma\cdot\nabla & -E+U_v-M-U_s \end{pmatrix}\begin{pmatrix} f \\ g \end{pmatrix}=0$$

$$U_{sep}\sim U_S+\frac{E}{m}U_V=-g_\sigma\sigma+\frac{E}{m}g_\omega\omega$$

$$=-\frac{g_\sigma^2}{m_\sigma^2}\rho_s+\frac{E}{m}\frac{g_\omega^2}{m_\omega^2}\rho_B$$



Saturation: -Scalar+Baryon Density
Linear Energy Dependence: Good at Low Energies,
Bad at High Energies (We need cut off !)

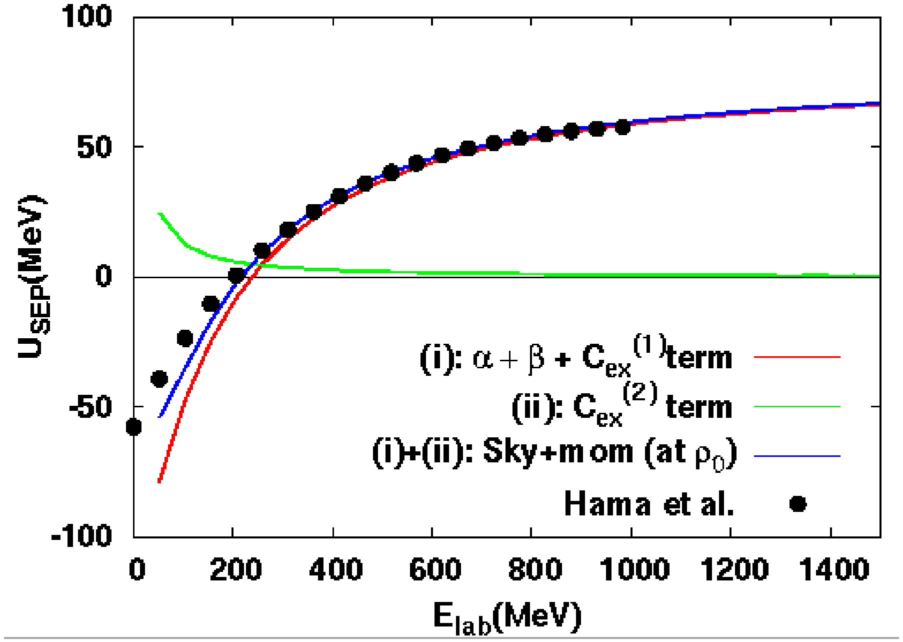
(Sahu, Cassing, Mosel, AO, Nucl. Phys. A672 (2000), 376.)

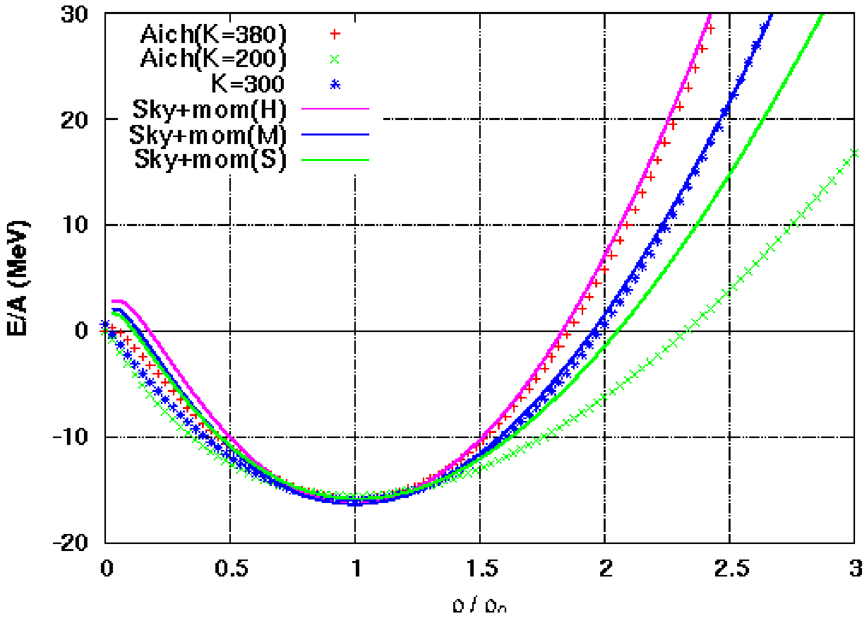
Phenomenological Mean Field

- **Skyrme type ρ-Dep. + Lorentzian p-Dep. Potential**

$$V=\sum_i V_i=\int d^3r\left[\frac{\alpha}{2}\left(\frac{\rho}{\rho_0}\right)^2+\frac{\beta}{\gamma+1}\left(\frac{\rho}{\rho_0}\right)^{\gamma+1}\right]$$

$$+\sum_k\int d^3r\,d^3p\,d^3p'\,\frac{C_{ex}^{(k)}}{2\rho_0}\,\frac{f(r,p)\,f(r,p')}{1+(p-p')^2/\mu_k^2}$$



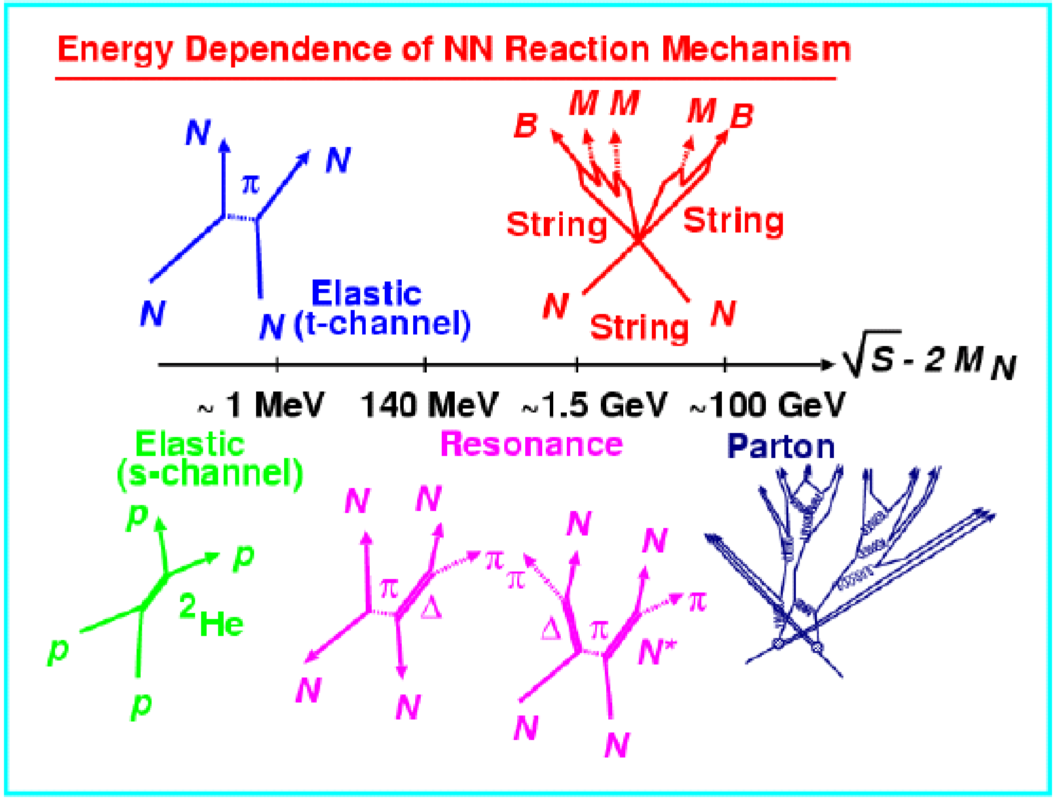


Isse, AO, Otuka, Sahu, Nara, Phys.Rev. C 72 (2005), 064908

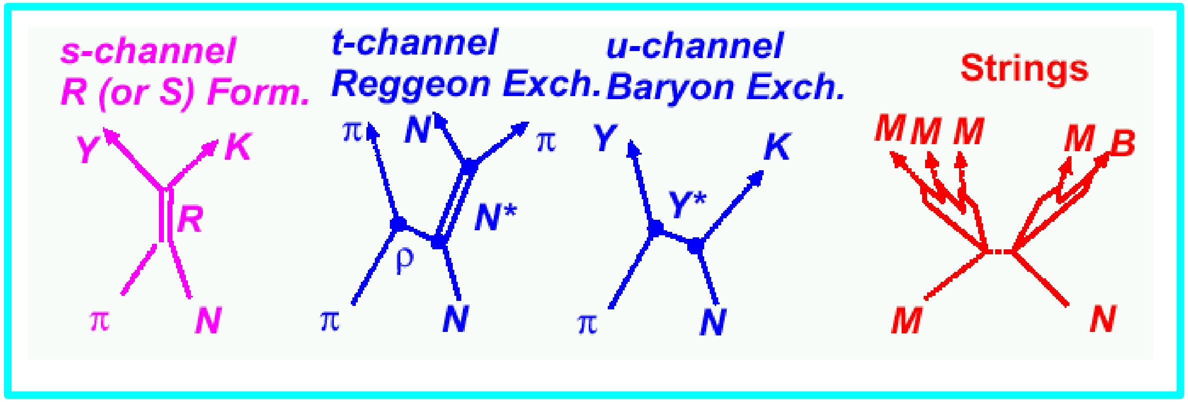
Collision Term and Particle Production

Baryon-Baryon and Meson-Baryon Collisions

- **NN collision mechanism**
  Elastic
  → Resonance
  → String
  → Jet

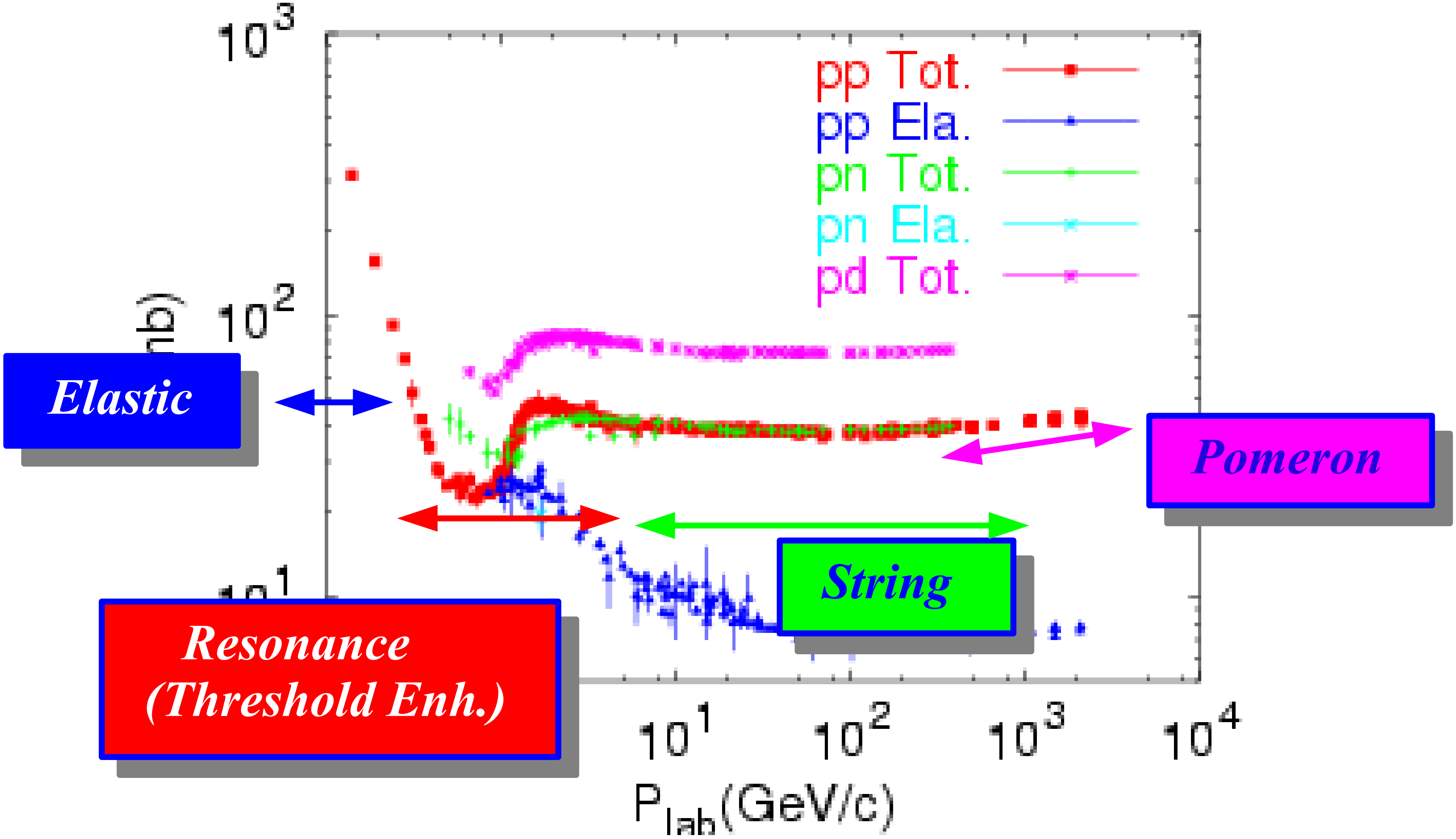


- **Meson-Nucleon Collision**
  → s-channel Resonance
  → t-(u-) channel Res.
  → String formation



NN Cross Sections

From Particle Data Group



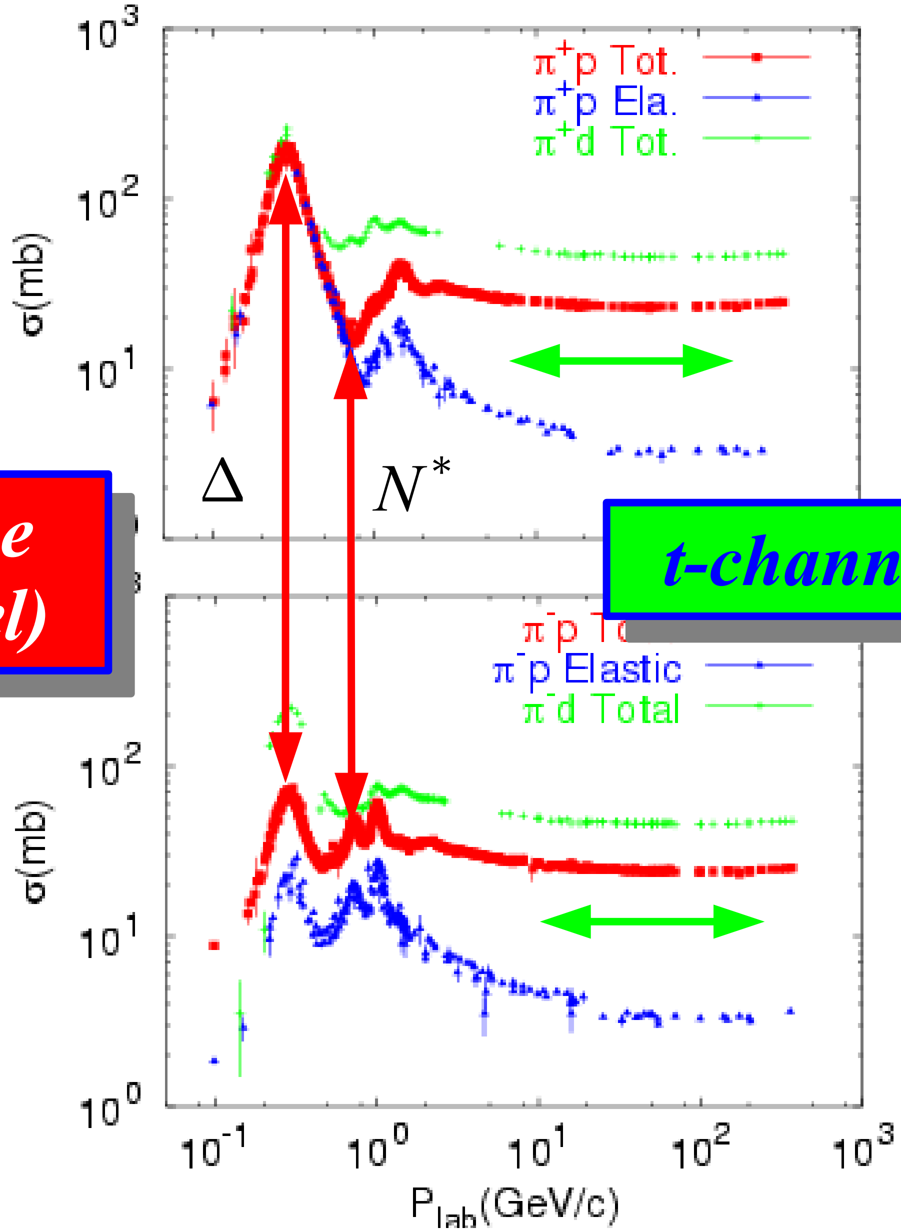
Elastic

Resonance (Threshold Enh.)

String

Pomeron

Meson-Baryon Cross Section



Resonance (s-channel)

t-channel and String

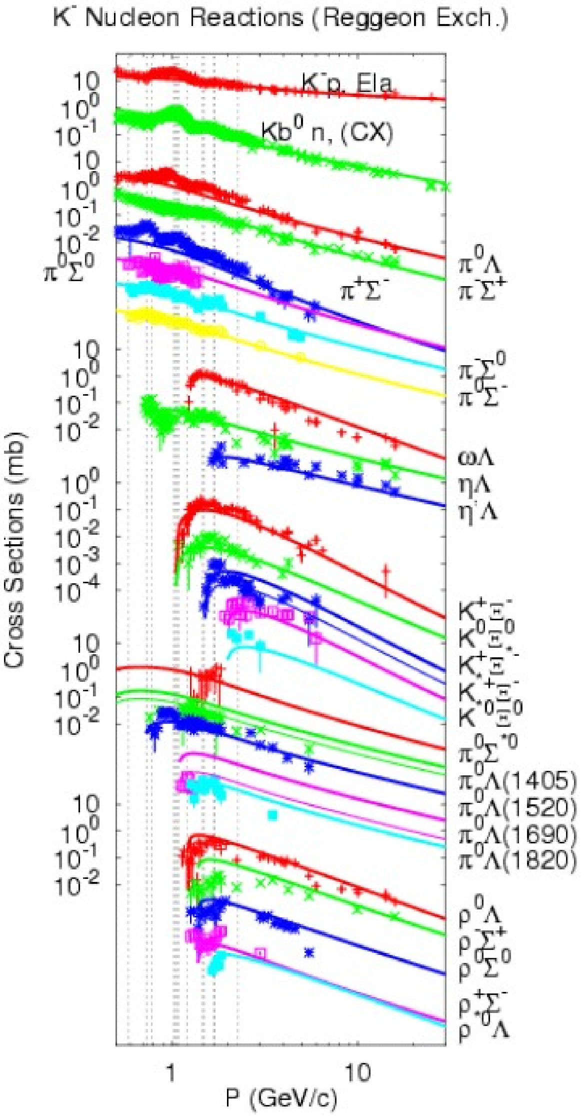
Reggeon Exchange

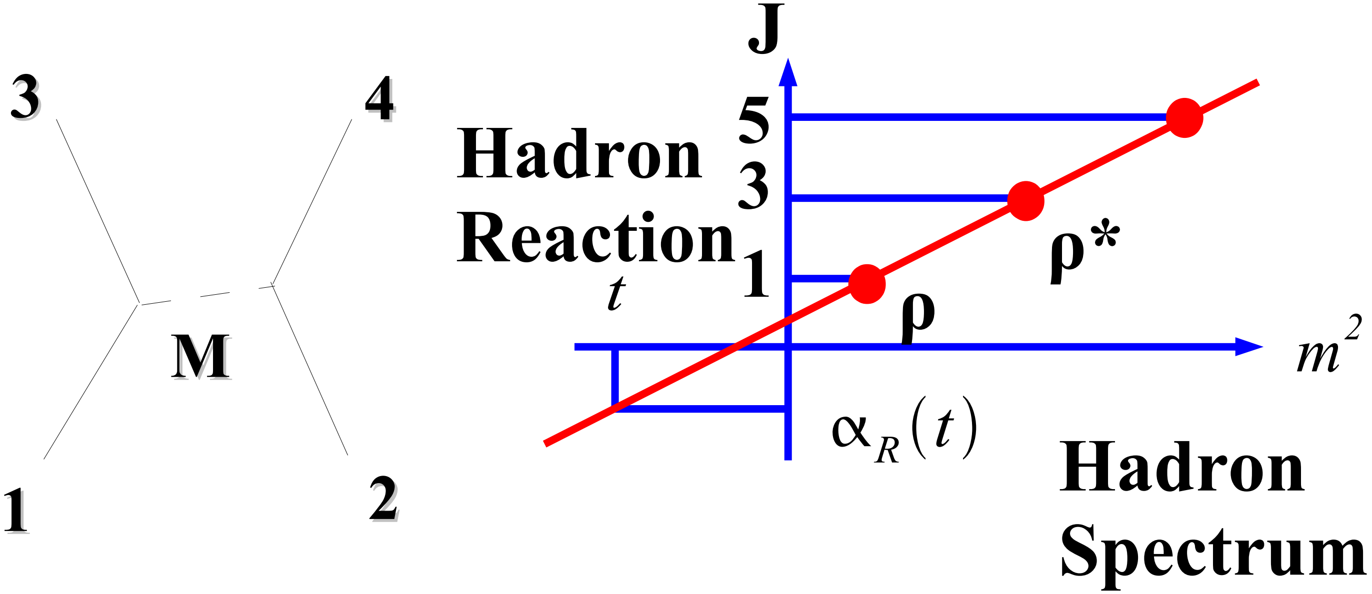
(Barger and Cline (Benjamin, 1969), H. Sorge, PRC (1995), RQMD2.1)

- **Regge Trajectory** $J=\alpha_R(t) \sim \alpha_R(0)+\alpha'_R(0)\,t$
- **2 to 2 Cross Section**

$$\frac{d\sigma}{d\Omega}=\frac{p_f}{64\pi s\, p_i}|M(s,t)|^2$$

$$M(s,t) \sim \sum_R \frac{(p_i p_f)^J}{t-M_R} \sim F(t)\exp\left[\alpha_R(t)\log(s/s_0)\right]$$



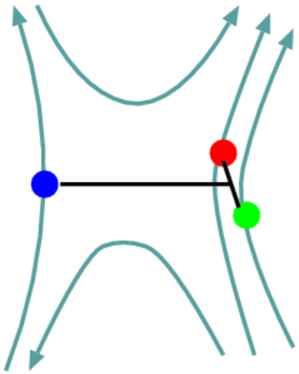


String formation and decay

- **What does the regge trajectory suggest ?**
  → Existence of (color- or hadron-)String !

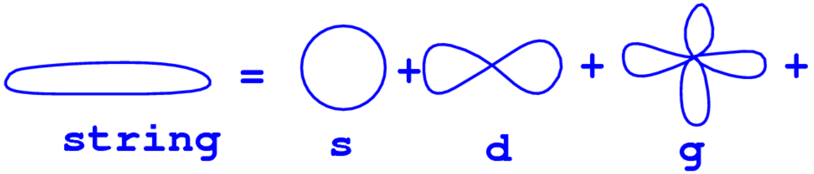
$$M = 2\int_0^R \frac{\kappa\, dr}{\sqrt{1-(r/R)^2}} = \pi\kappa R\ ,\quad J = 2\int_0^R r\times\frac{\kappa\, dr}{\sqrt{1-(r/R)^2}}\,\frac{r}{R} = \frac{\pi\kappa R^2}{2}\pi$$

$$\rightarrow\ \boldsymbol{J = \frac{M^2}{2\pi\kappa}}$$

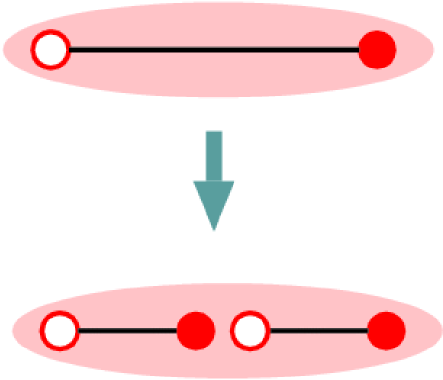


- **String Tension**

$$\frac{1}{2\pi\kappa} = \alpha'_R(0) \approx 0.9\,\mathrm{GeV}^{-2}\ \rightarrow\ \kappa \approx 1\,\mathrm{GeV/fm}$$



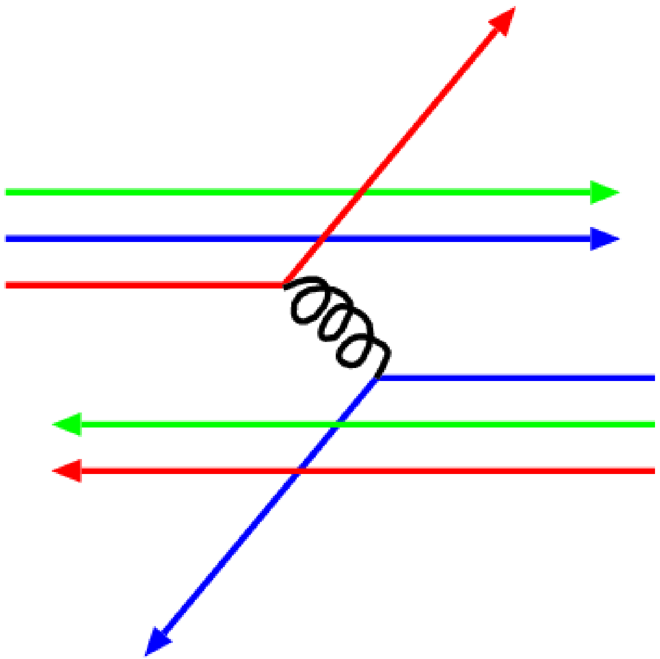
- **String decay**
  Extended String
  → Large E stored
  → q qbar pair creation (Schwinger mech.)

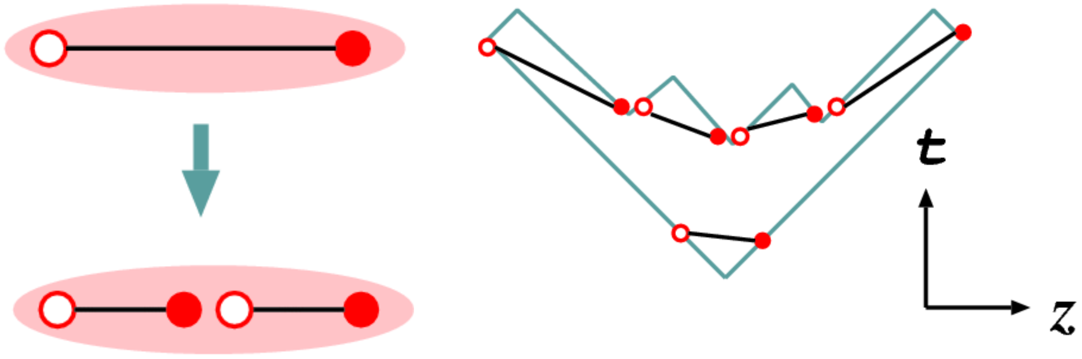


String = Coherent superposition of hadron resonances with various J

Jet Production

- **Elastic Scattering of Partons (mainly) with One Gluon Exch.**
- **Color Exch. between Hadrons**
  → Complex color flux starting from leading partons
  → many hadron production
  → Jet production
- ***PYTHIA***
  - **Event Generator of High Energy Reactions**
    → Jet production +String decay
    for QCD processes





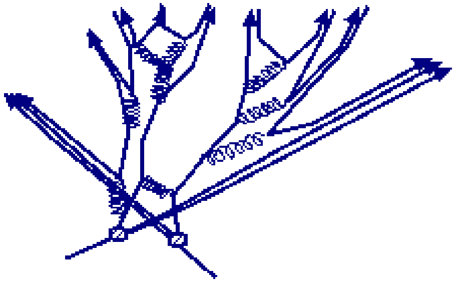
(T. Sjostrand et al., Comput. Phys. Commun. 135 (2001), 238.)

JAM (Jet AA Microscopic transport model)

Nara, Otuka, AO, Niita, Chiba, Phys. Rev. C61 (2000), 024901.

- **Hadron-String Cascade with Jet production**
  - **hh collision with Res. up to m < 2 GeV (3.5 GeV) for M (B)**
  - **String excitation and decay**
  - **String-Hadron collisions are simulated by hh collisions in the formation time.**
  - **jet production is incl. using PYTHIA**
  - **Secondary partonic int.:**
    NOT incl.
  - **Color transparency:**
    NOT taken care of

Resonance
+ String
+ Jet



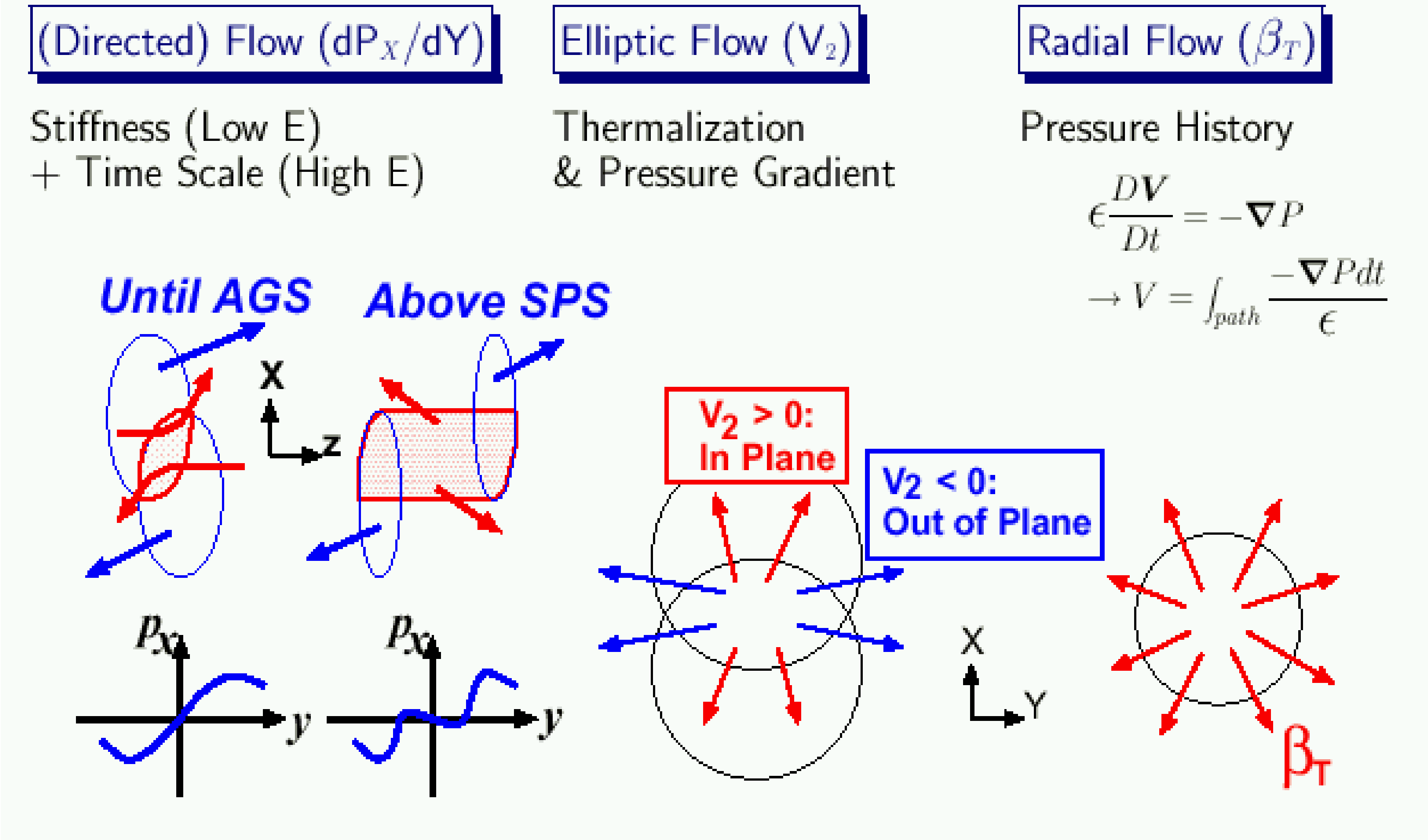
Collective Flows at AGS and SPS Energies

Collective Flow and EOS: Old Problem ?

- **1970's-1980's: First Suggestions and Measurement**
  - **Hydrodynamics suggested the Exsitence of Flow.**
  - **Strong Collective Flow suggests Hard EOS**
- **1980's-1990's: Deeper Discussions in Wider E_{inc} Range**
  - **Momentum Dep. Pot. can generate Strong Flows.**
  - **Einc deps. implies the importance of Momentum Deps.**
  - **Flow Measurement up to AGS Energies.**
- **2000's: Extention to SPS and RHIC Energies**
  - **EOS is determined with Mom. AND Density Dep. Pot. ?**

Old but New (Continuing) Problem !

What is Collective Flow ?



(Directed) Flow (dP_X/dY)

Stiffness (Low E)
+ Time Scale (High E)

Elliptic Flow (V_2)

Thermalization
& Pressure Gradient

Radial Flow (β_T)

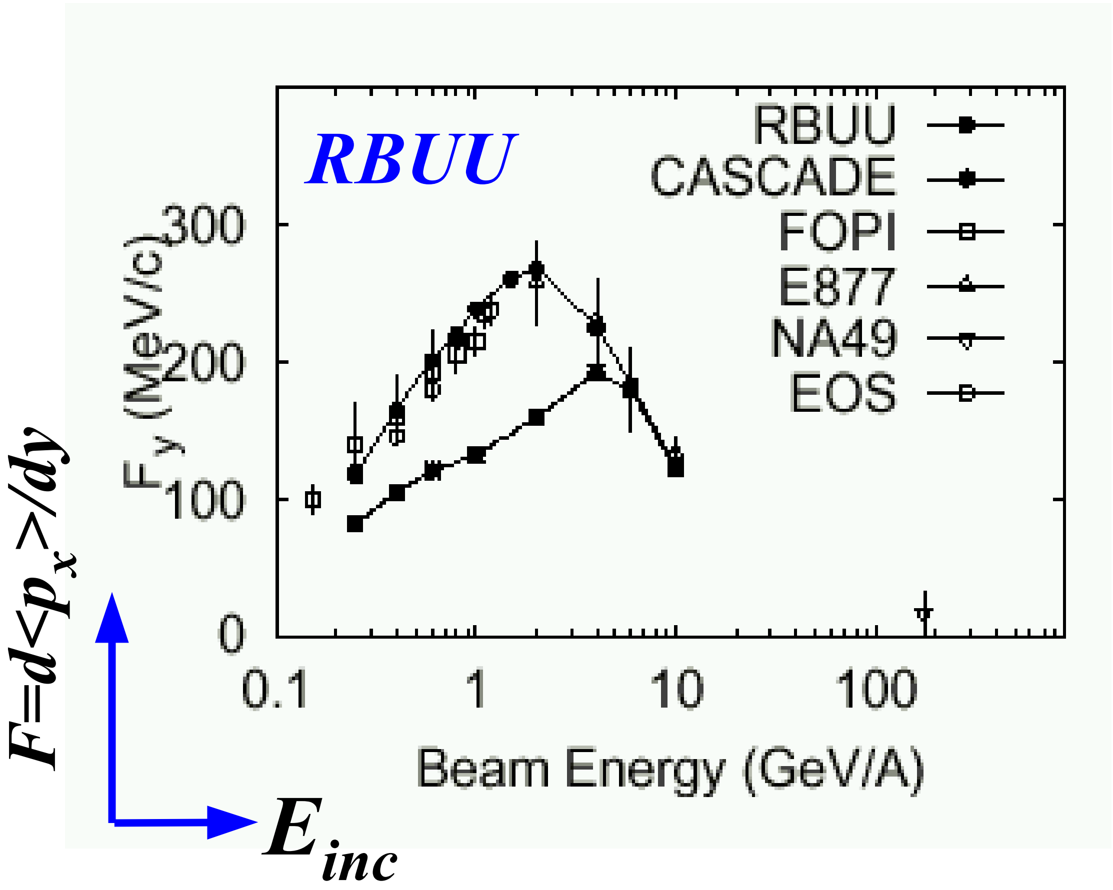
Pressure History

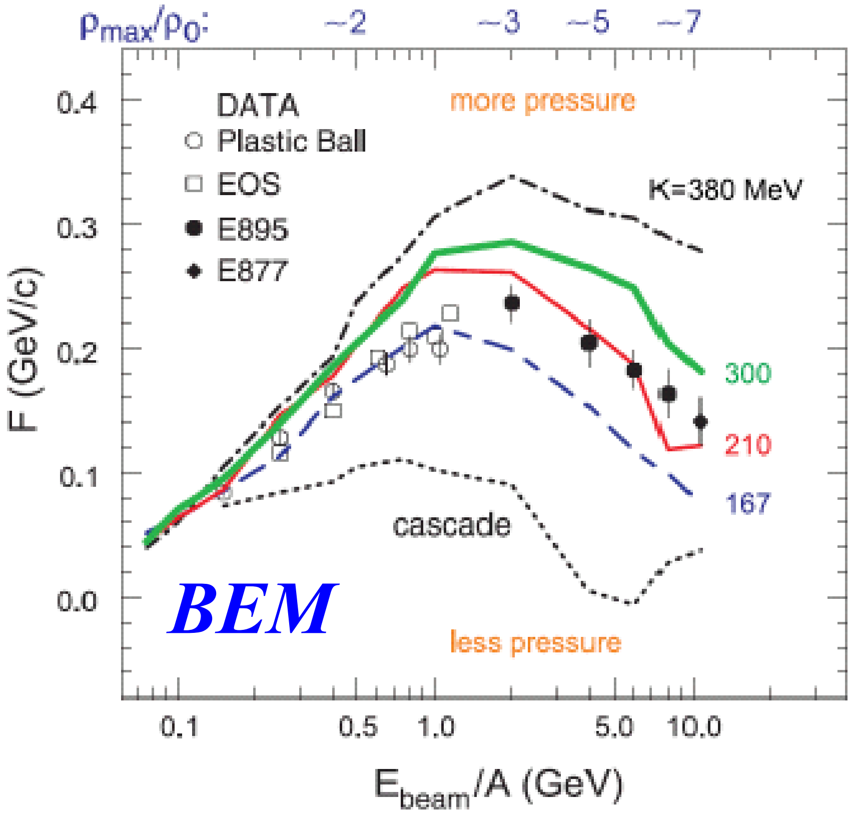
$$\epsilon \frac{D\mathbf{V}}{Dt} = -\nabla P$$

$$\rightarrow V = \int_{path} \frac{-\nabla P dt}{\epsilon}$$

Side Flow at AGS Energies

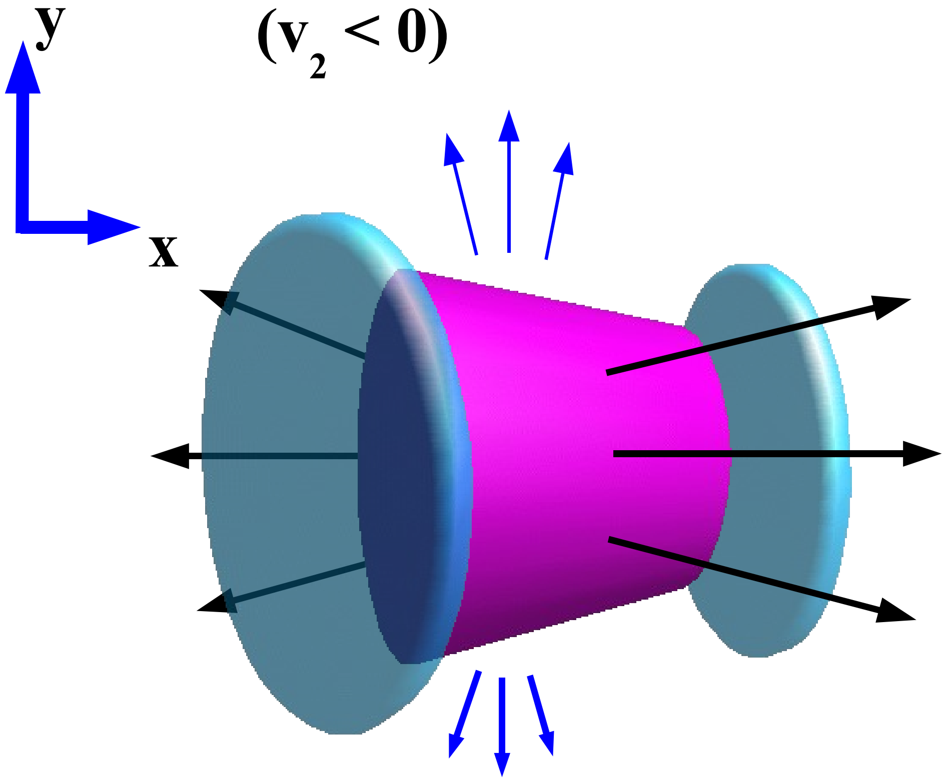
- **Relativistic BUU (RBUU) model:** ***K ~ 300 MeV***
  (Sahu, Cassing, Mosel, AO, Nucl. Phys. A672 (2000), 376.)
- **Boltzmann Equation Model (BEM):** ***K=167~210 MeV***
  (P. Danielewicz, R. Lacey, W.G. Lynch, Science 298(2002), 1592.)





Elliptic Flow

- **What is Elliptic Flow ? → Anisotropy in P space**
- **Hydrodynamical Picture**
  - **Sensitive to the Pressure Anisotropy in the Early Stage**
  - **Early Thermalization is Required for Large V2**



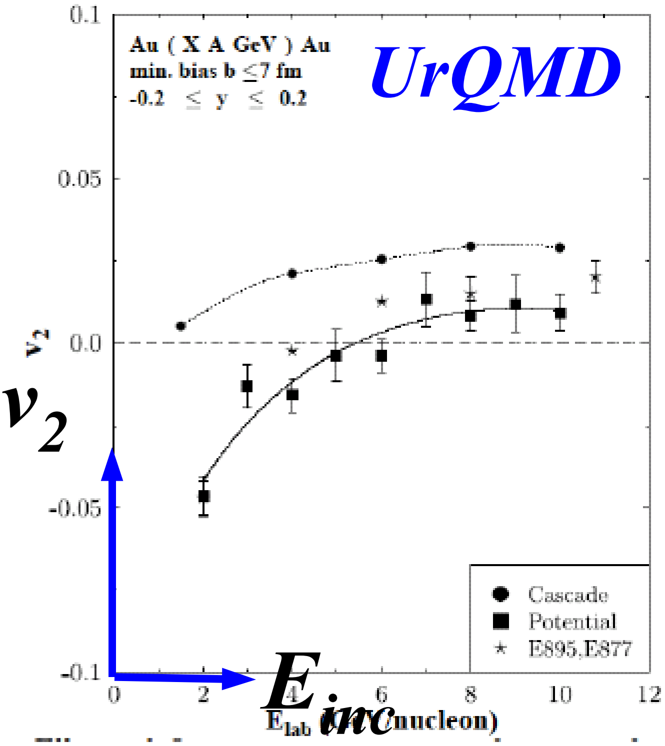
Out-of-Plane Flow
($v_2 < 0$)

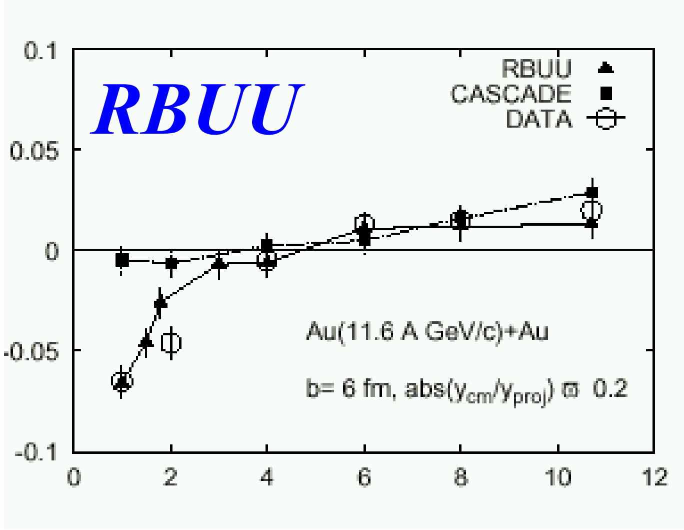
$$v_2 \equiv \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle = \langle \cos 2\phi \rangle$$

In-Plane Flow
($v_2 > 0$)

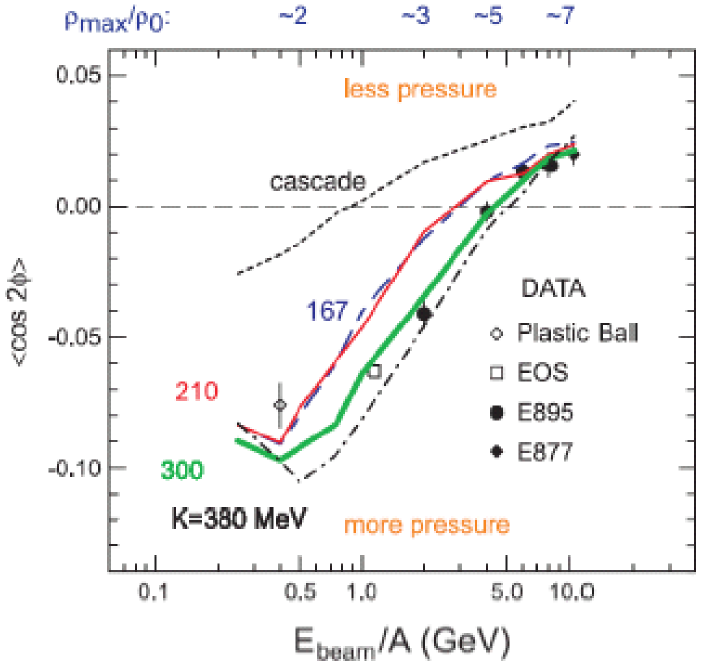
Elliptic Flow at AGS

- **Strong Squeezing Effects at low E (2-4 A GeV)**
  - **UrQMD: Hard EOS (S.Soff et al., nucl-th/9903061)**
  - **RBUU (Sahu-Cassing-Mosel-AO, 2000): K ~ 300 MeV**
  - **BEM(Danielewicz2002): K = 167 → 300 MeV**



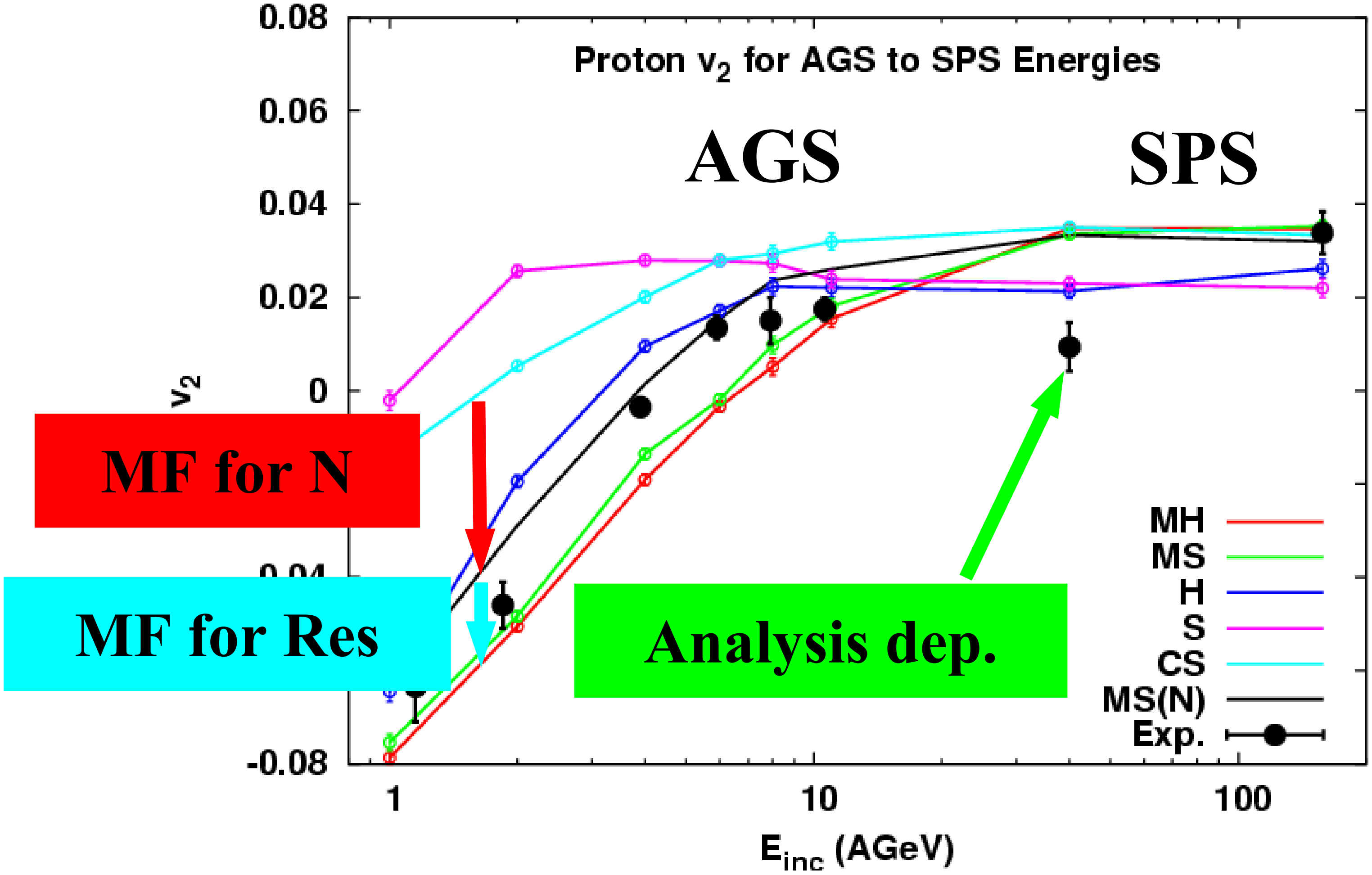


BEM



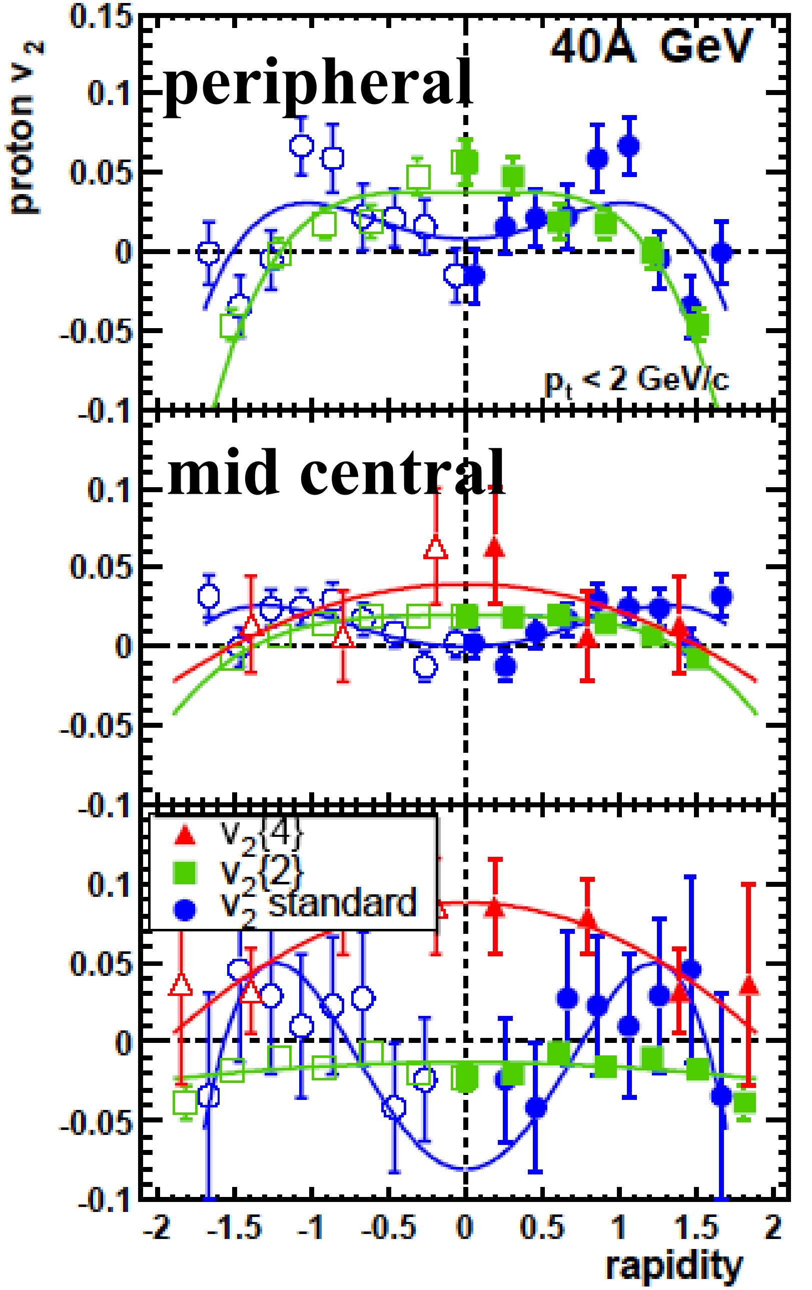
Elliptic Flow from AGS to SPS

- **JAM-MF with p dep. MF explains proton v2 at 1-158 A GeV**
  - **v2 is not very sensitive to K (incompressibility)**
  - **Data lies between MS(B) and MS(N)**



Dip of V_2 at 40 A GeV: Phase Transition ?

- Dip of V_2 at 40 A GeV may be a signal of QCD phase transition at high baryon density.
  (Cassing et al.)
- However, the data is too sensitive to the way of the analysis (reaction plane/two particle correlation).
  - We have to wait for better data.



Flow and EOS; to be continued

- **In addition to the ambiguities in in-medium cross sections, Res.-Res. cross sections, we have model dependence.**
  - **RBUU** *(e.g. Sahu, Cassing, Mosel, AO, Nucl. Phys. A672 (2000), 376.)*
    - **In RMF, Strong cut-off for meson-N coupling in RMF → Smaller EOS dep.**
  - **Scalar potential interpretation in BUU**
    Larionov,Cassing,Greiner,Mosel, PRC62,064611('00), Danielewicz, NPA673,375('00)

$$\varepsilon(\boldsymbol{p},\rho)=\sqrt{[m+U_s(\boldsymbol{p},\rho)]^2+\boldsymbol{p}^2}\quad=\sqrt{m^2+\boldsymbol{p}^2}+U(\boldsymbol{p},\rho)$$

    - **Due to the Scalar potential nature, EOS dependence is smaller.**
  - **Scalar/Vector Combination** *Danielewicz, Lacey, Lynch, Science 298('02), 1592*

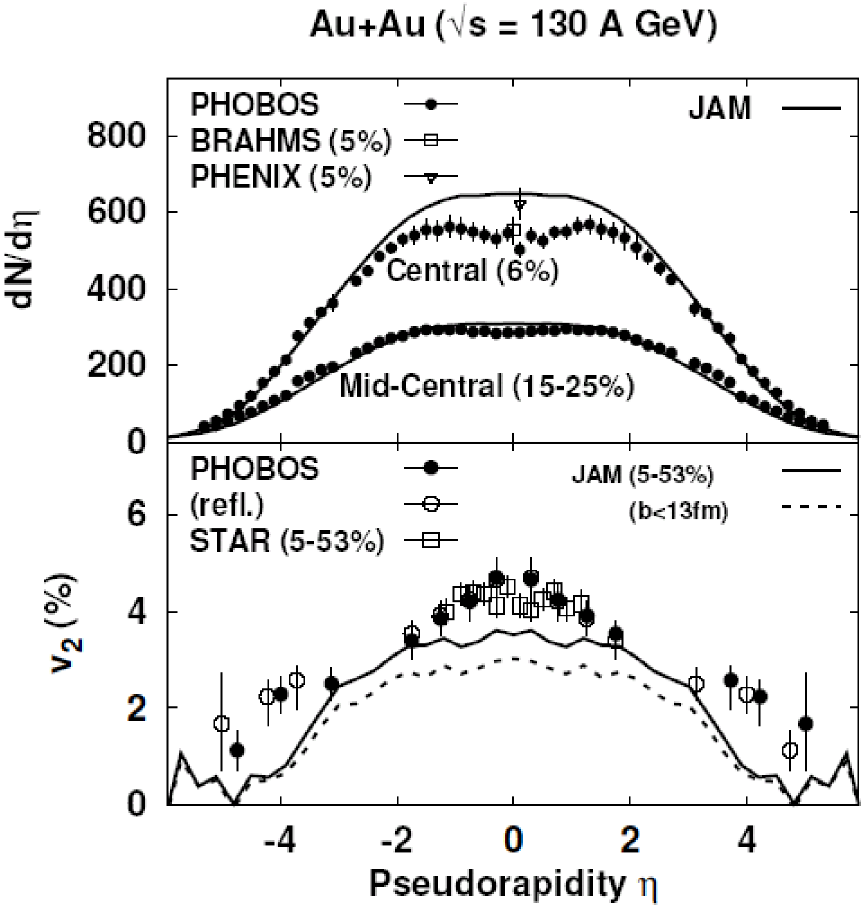
$$\varepsilon(p,\rho)=m+\int_0^p dp'v^*(p',\rho)+\widetilde{U}(\rho),\quad v^*(p,\rho)=\frac{p}{\sqrt{p^2+[m^*(p,\rho)]^2}}.$$

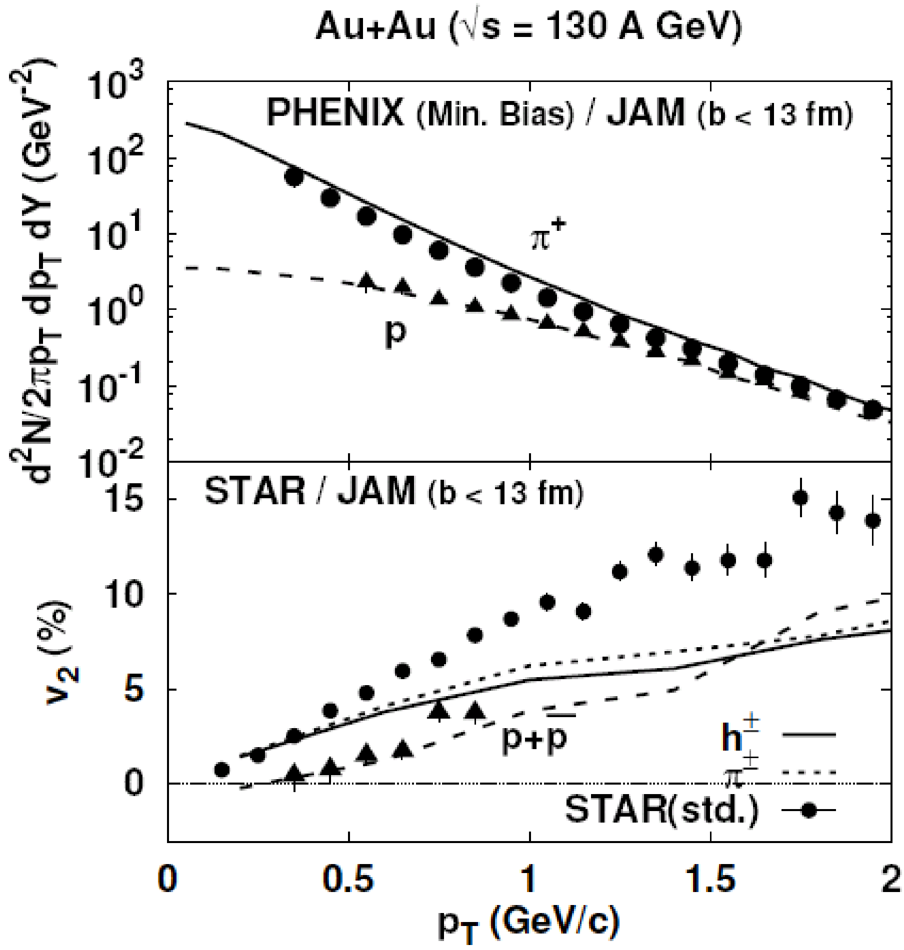
    - **Relatively Strong EOS dependence even at high energy**
  - **JAM-RQMD/S** *Isse, AO, Otuka, Sahu, Nara, PRC 72 (2005), 064908*
    - **Similar to the Scalar model BUU**

Elliptic Flow @ RHIC

Elliptic Flow in Hadron-String Cascade (I)

- **Hadron-String Cascade (JAM) @ RHIC**
  - **Hadron Yield is reasonably explained up to 2 GeV/c (10-20 % error)**
  - **v2 is underestimated (20-30 % (integrated), 50 % (p_T > 1 GeV)**

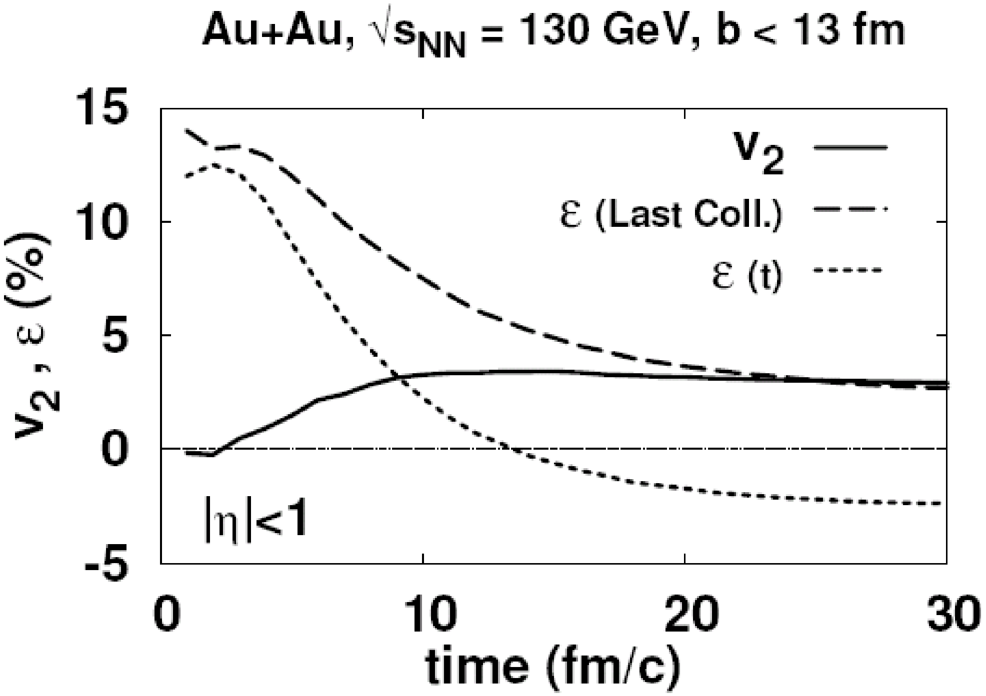


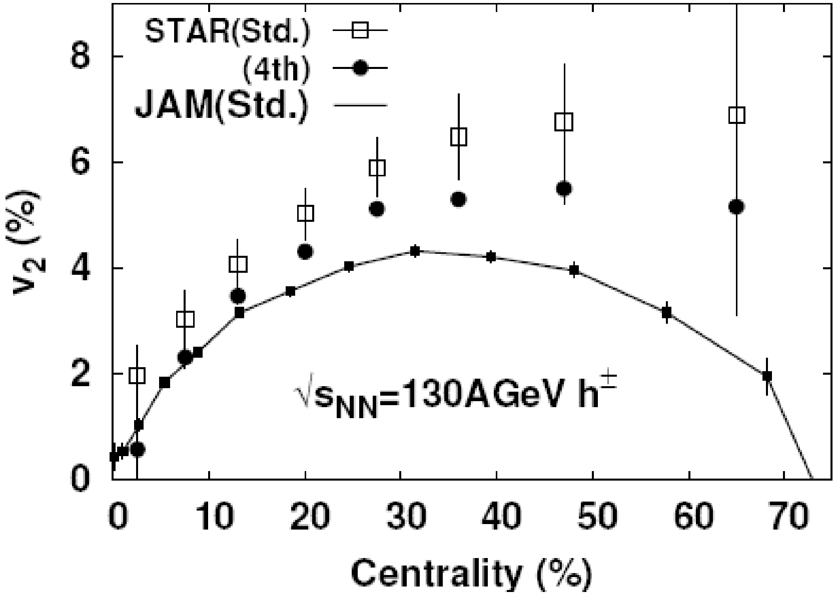


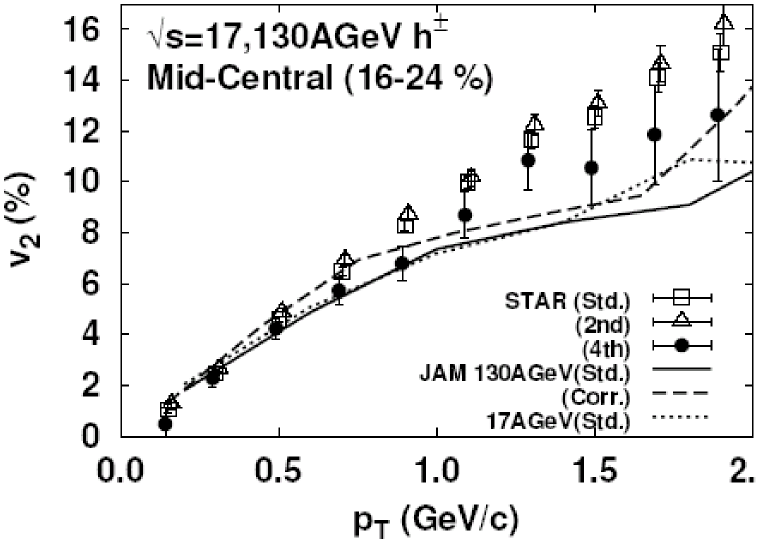
Elliptic Flow in Hadron-String Cascade (II)

- **Why do we underestimate v2 in Hadron-String Cascade ?**
  - v2 growth time is long (~ 10 fm/c), due to hadron formation time (τ~ 1 fm/c). → much longer than hydro

Sahu-Isse-AO-Otuka-Phatak 2006







Results of Parton Cascade

- **Unexpectedly high parton cross sections of σ =5-6 mb have to be assumed in parton cascades in order to reproduce the elliptic flow.**

ZI-WEI LIN AND C. M. KO PHYSICAL REVIEW C **65** 034904

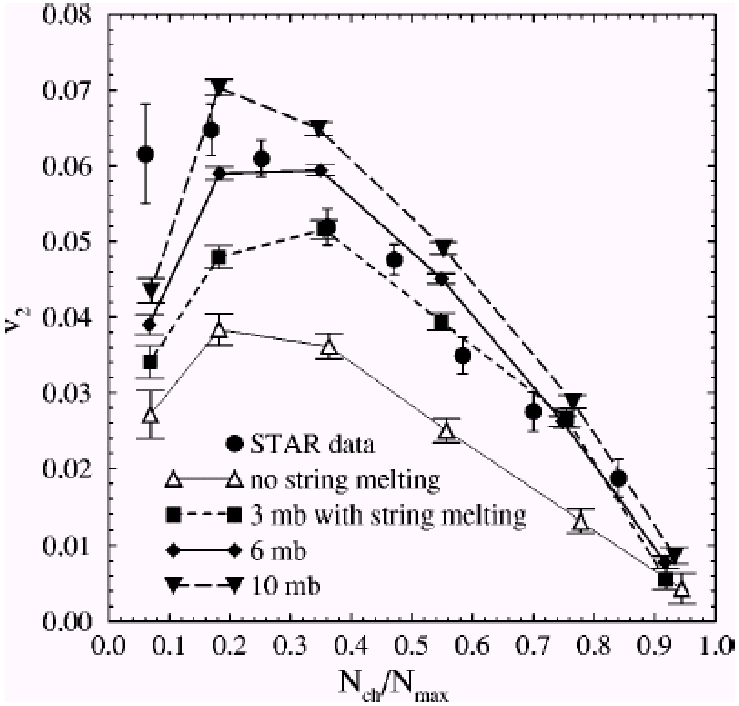


FIG. 3. Impact parameter dependence of elliptic flow at 130 A GeV. The data from the STAR collaboration [7] are shown by filled circles, while the theoretical results for different partonic dynamics are given by curves.

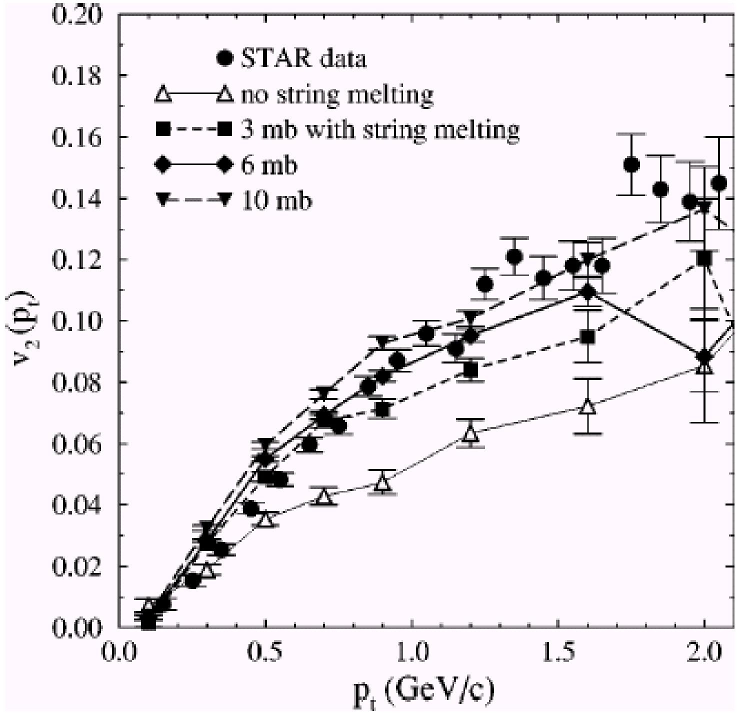
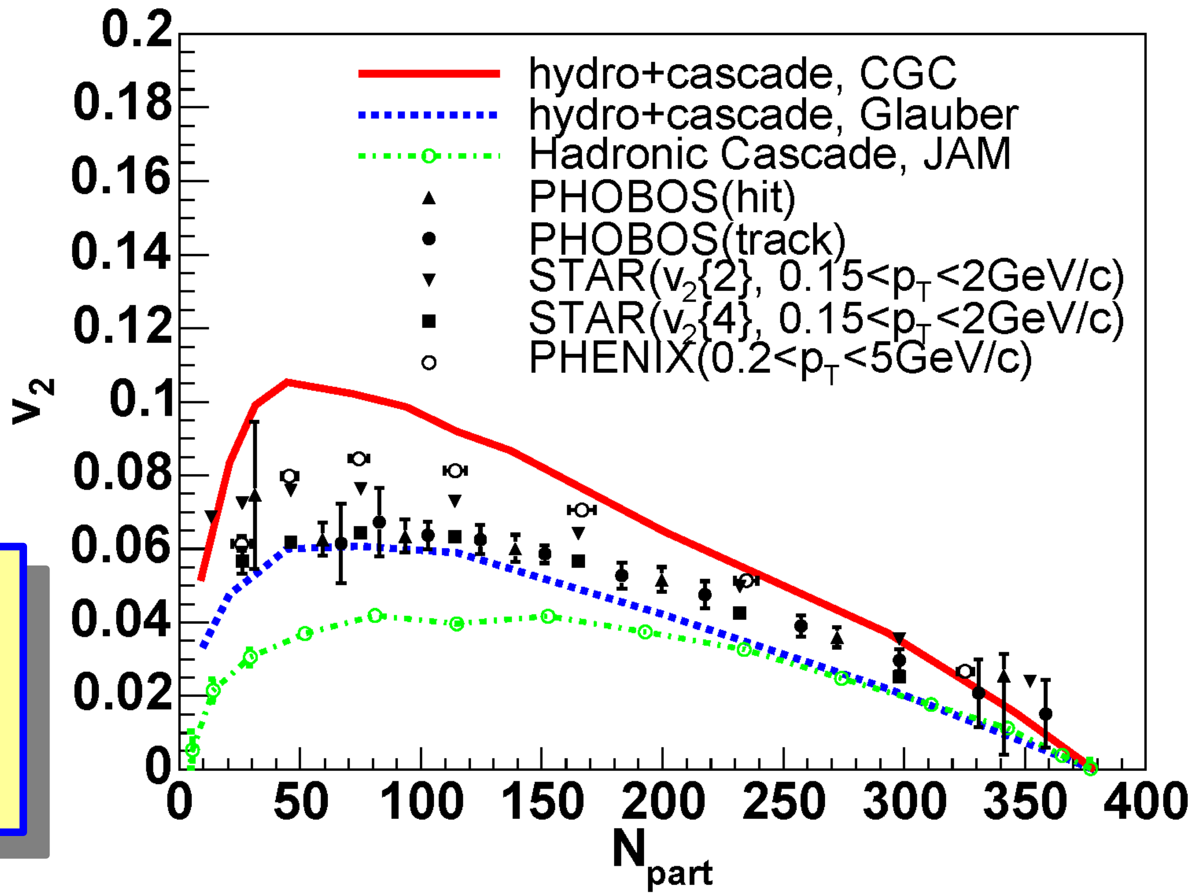


FIG. 4. Transverse momentum dependence of elliptic flow at 130 A GeV. Circles are the STAR data for minimum-bias Au+Au collisions [7], and curves represent the minimum-bias results for charged particles within $\eta \in (-1.3, 1.3)$ from the AMPT model.

Cascade vs Hydro @ RHIC: Au+Au

- **Comparison of v2 as a function of N_{part}**
  - **Cascade predict smaller v2 in peripheral collisions**
  - **Data lies between hydro results with two different initial condition CGC (Color Glass Condensate) and Glauber type initial condition.**

Hydro is better,
CGC may be realized
in central collisions.



Relativistic Hydrodynamics

Relativistic Hydrodynamics

- EOM: Conservation Laws

$\partial_\mu T^{\mu\nu} = 0$ **Energy Momentum Conservation**

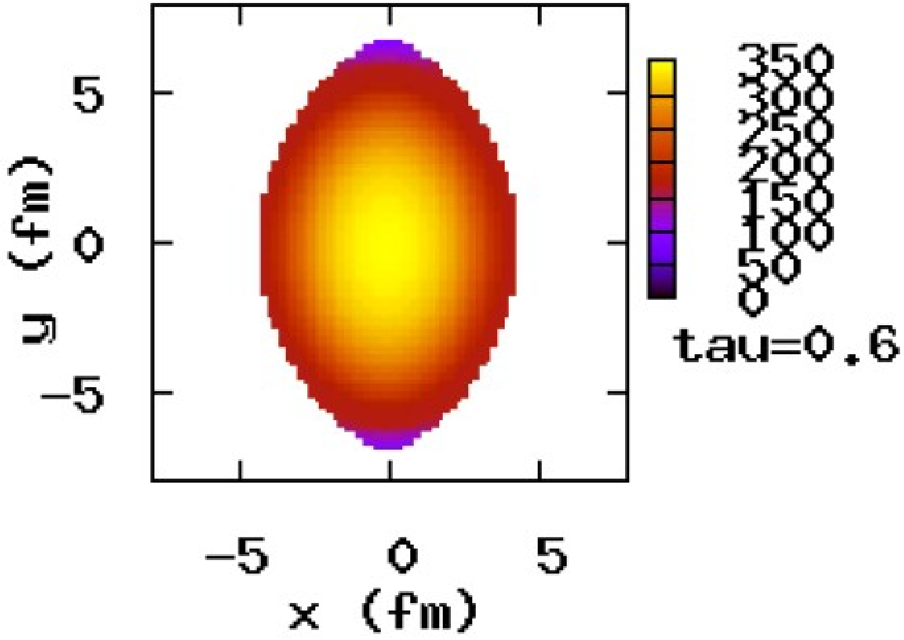
$\partial_\mu (n_i u^\mu) = 0$ **Conservation of Charge (Baryon, Strangeness, ...)**

$$T^{\mu\nu} = (e + P) u^\mu u^\nu - P g^{\mu\nu}$$

e : energy density, P: *pressure,*

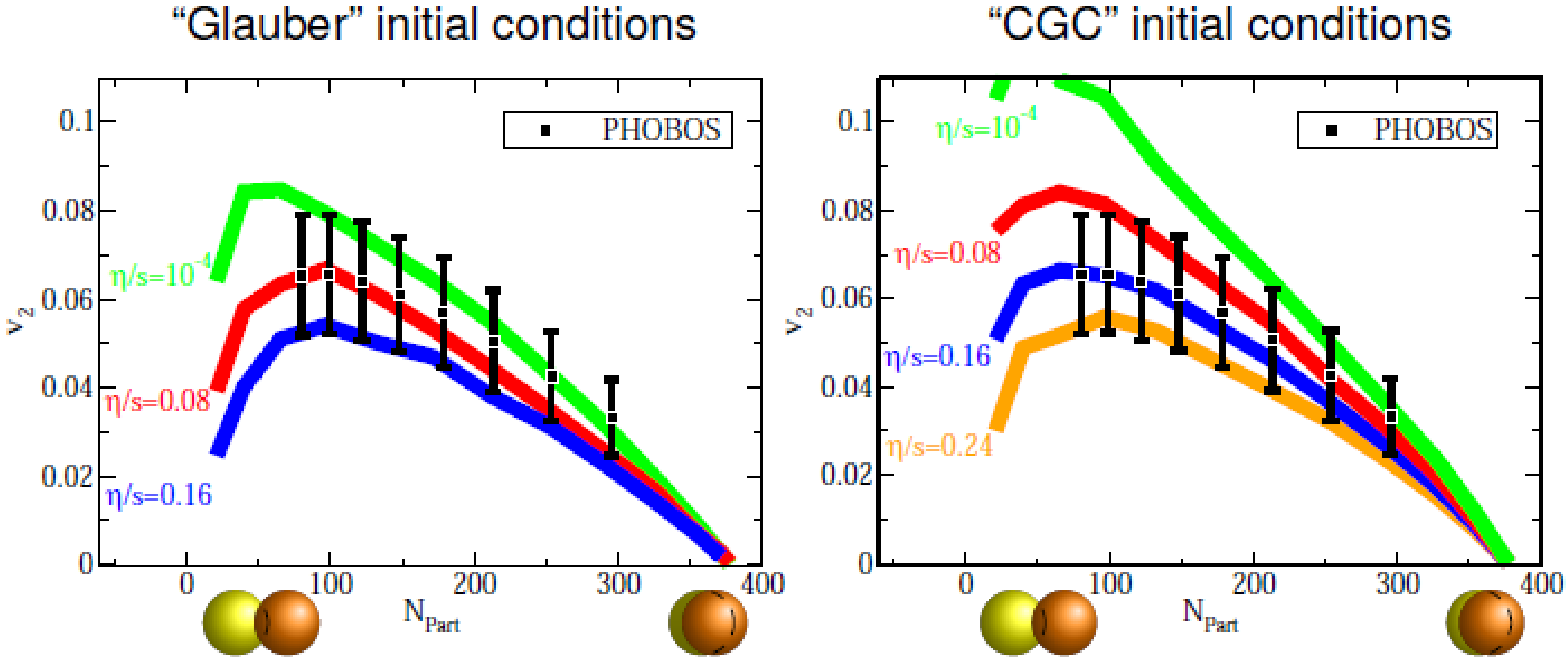
u^μ: :four velocity $\gamma(1, v)$,

n_i: :number density



T. Hirano, Y. Nara, NPA743, 305 (2004)
T. Hirano, K. Tsuda, PRC 66, 054905(2002)

η/s FROM FLOW (HISTORICAL)



“Glauber” initial conditions

“CGC” initial conditions

(ML & Romatschke, Phys.Rev. C78 (2008) 034915)

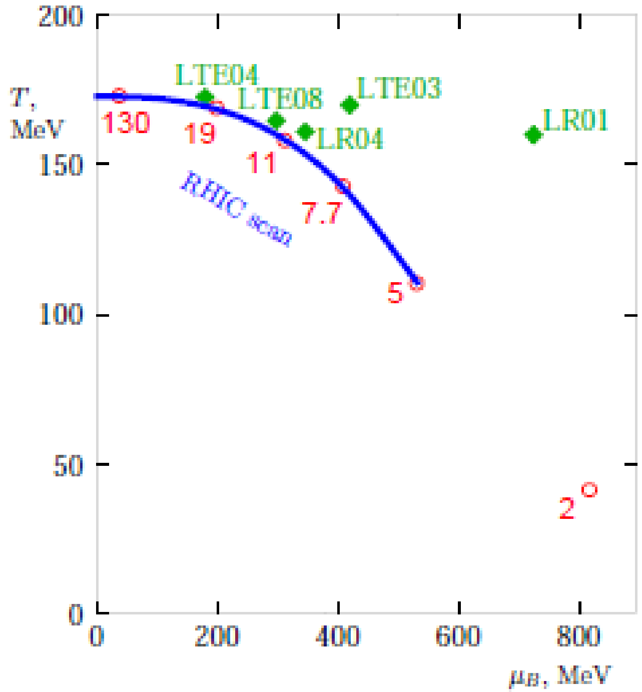
- Best extraction of η/s by comparing viscous hydro to flow data
- Largest uncertainty from unknown initial condition

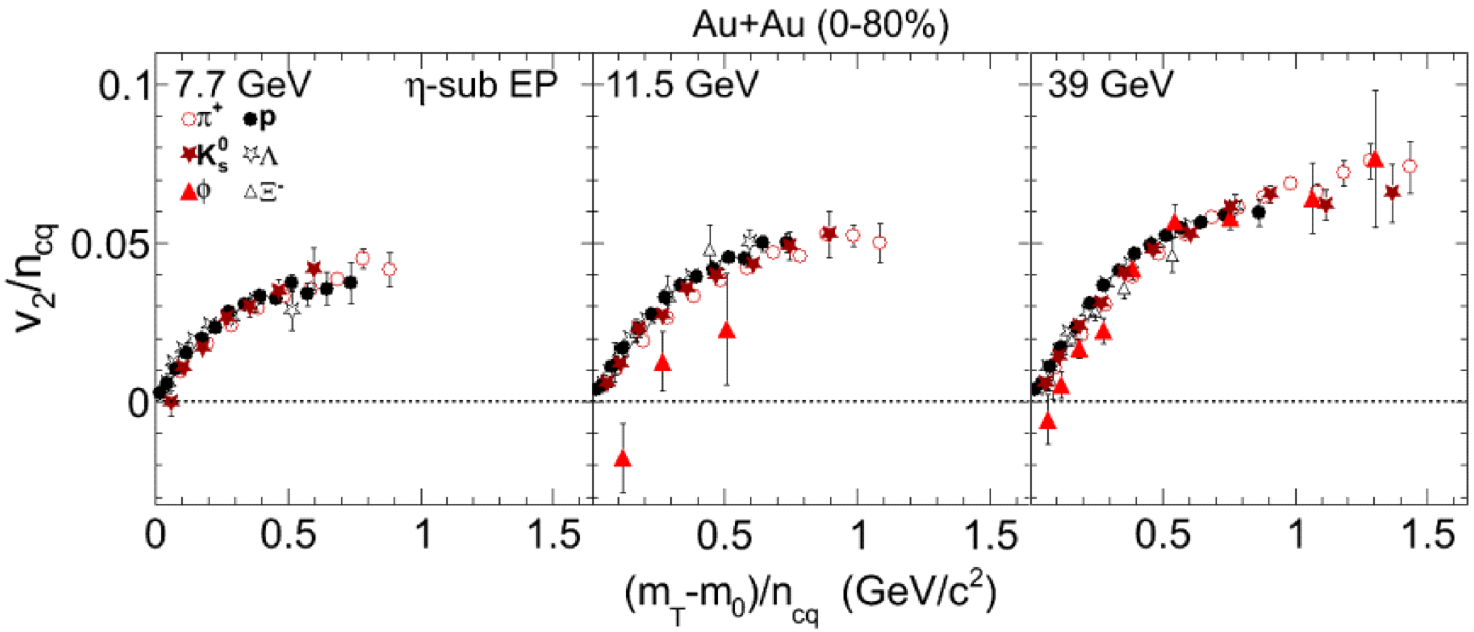
Summary

- **Heavy-ion collisions up to SPS energies seems to be reasonably described by using hadron-string cascade such as JAM model, while HIC at RHIC requires earlier thermalization (larger anisotropic pressure).**
- **There are many things to do in high-energy heavy-ion collision physics.**
  - **AGS-FAIR-SPS energies**
    Nuclear matter EOS, Baryon rich QGP, Strangeness enh., …
    Puzzle: Is QGP formed below top SPS energy ($\sqrt{s}$ = 17 GeV) ?
  - **RHIC-LHC energies**
    → Consistent understandings are not yet achieved,
    and we still have many puzzles
    - **Pre-equilibrium dynamics**
    - **QGP properties ($\eta/s = (1\text{-}2)/4\pi$, Parton energy loss, ….)**

Critical Point Search at RHIC

- **Beam Energy Scan (BES) program at RHIC**
  - **$\sqrt{s_{NN}}$ = 5-200 GeV**
    → μ_B < 500 MeV at Chemical Freeze-out
  - **First stage results of BES @ RHIC**
    → Quark number scaling of v_2 works
    and $v_2(p_T)$ saturates
    for $\sqrt{s_{NN}}$ > 39 GeV





Backups

When and where is QGP formed ?

- **Incident Energy**
  - **AGS: Strangeness Enh. (High baryon ρ effect ?)**
  - **SPS:**
    J/ψ suppression (QGP?), Low mass dilepton enh. (chiral sym.)
    Hydro overestimate v_2 data
  - **RHIC:**
    Jet quenching, Strong v_2, Quark number scaling of v_2, ...
    Hadronic Cascade underestimate v_2 data

  → Bulk QGP formation seems to start between SPS and RHIC

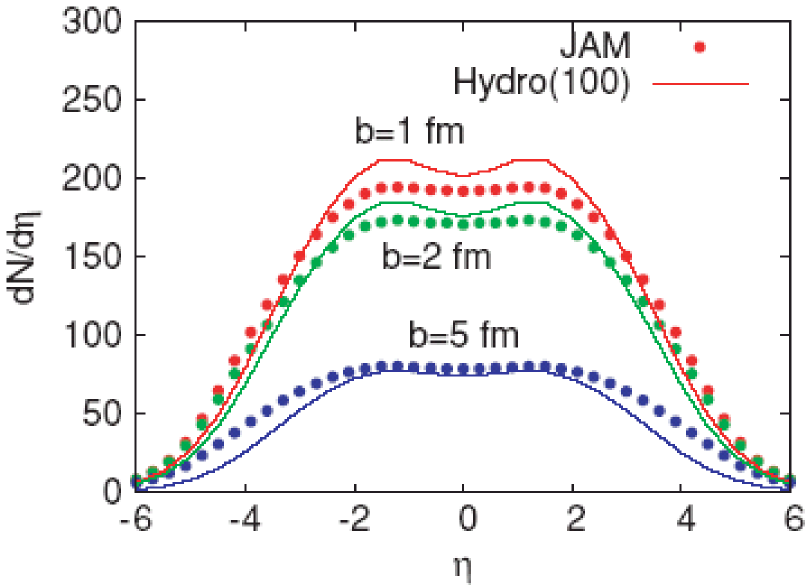
- **Proj./Targ. Mass dependence**
  - **Au+Au: v_2(Casc.) $<$ v_2(hydro) $\sim$ v_2(data)**
  - **Cu+Cu: Recently Measured**

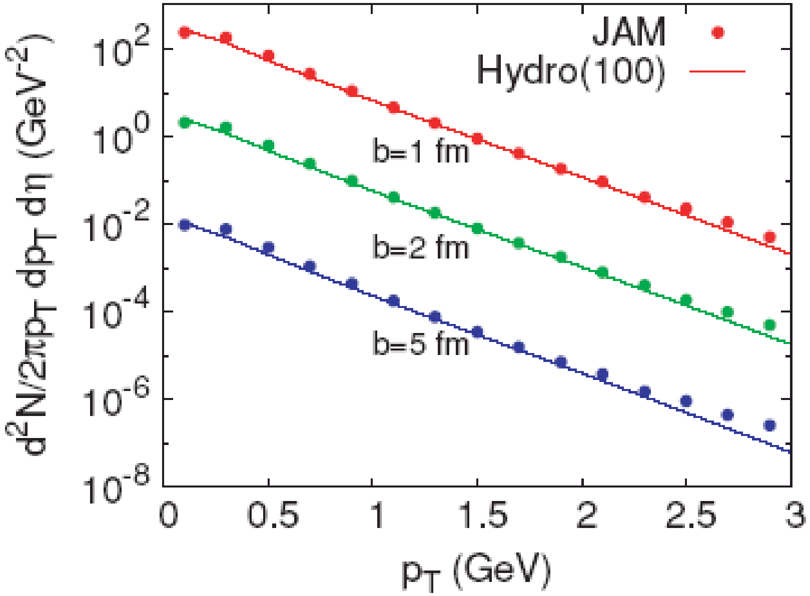
Predictions of Cu+Cu Collisions @ RHIC (I)

- **Single particle spectra**
  - **Cascade (JAM) and Hydro predict almost the same single particle spectra**

    $dN/d\eta$, $d^2N/p_T dp_T d\eta$
- **Surprising ?**
  - **Initial Cond. of Hydro is tuned to fit *$dN/d\eta$* (~ Energy per rapidity)**
  - **Cascade use fitted σ_{NN}**
  - **Themailzation is expected at Low p_T (long time before particle production)**

    → Coincidence may not be surprising

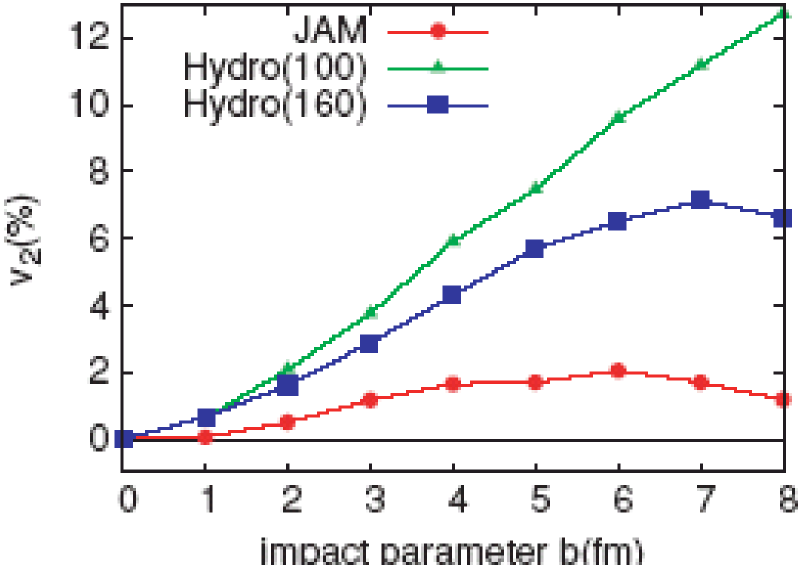


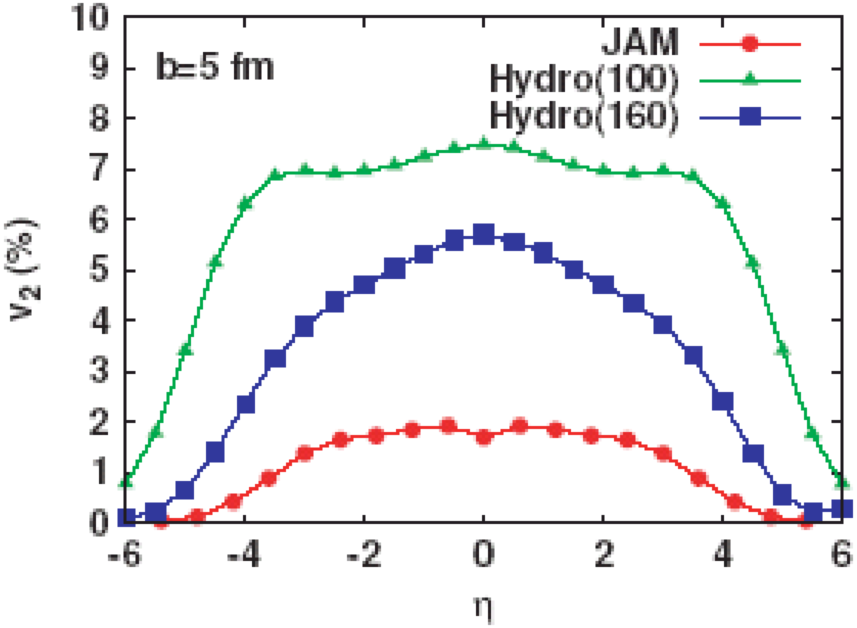


Hirano, Isse, Nara, AO, Yoshino, Phys. Rev. C 72(2005), 041901

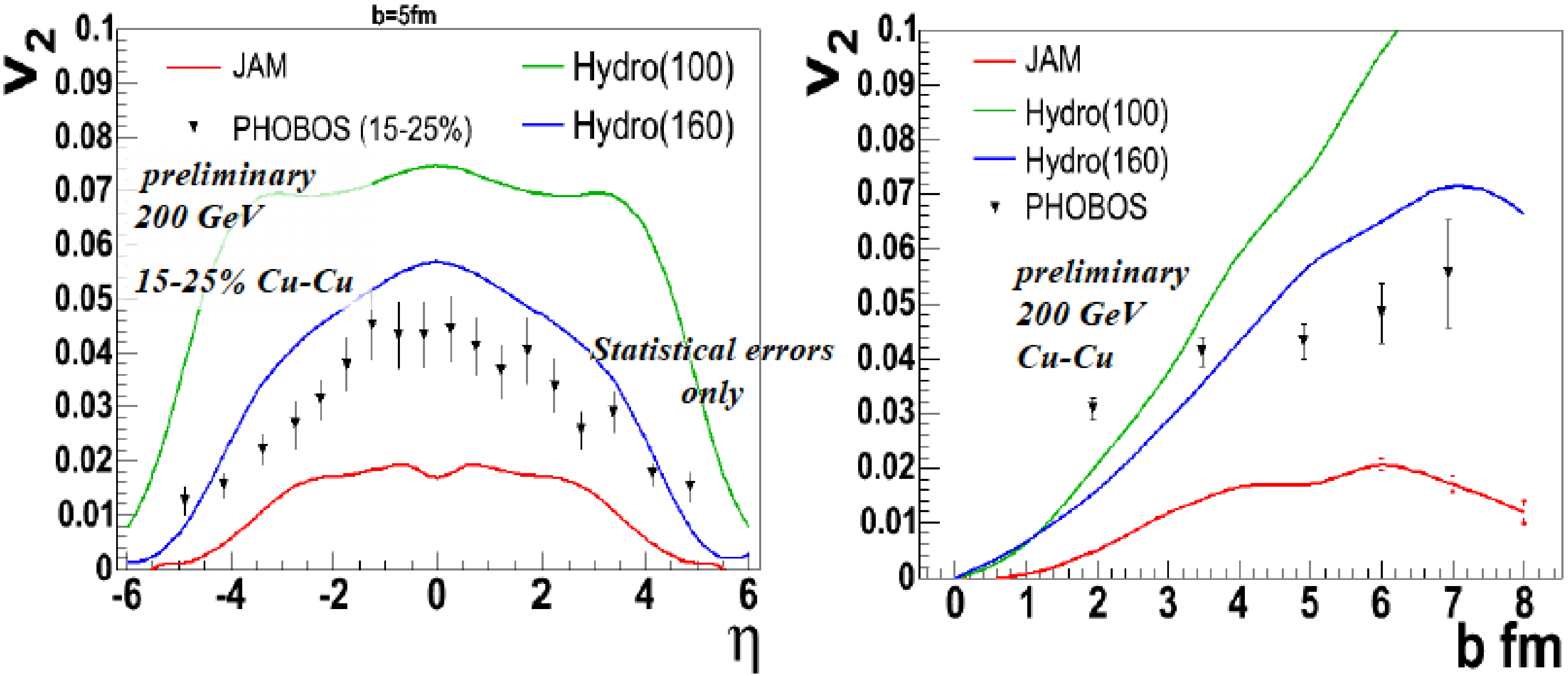
Predictions of Cu+Cu Collisions @ RHIC (II)

- **Calculations were done BEFORE the data are opened to public.**
- **Cascade and Hydro predict very different Elliptic Flow !**
  - **Cascade: small v2**
    → Small int. in the early stage
  - **Hydro: large v2**
    → Strong int. after $\tau=\tau_0 \sim 0.6$ fm/c
- **T^{th} dependence**
  - **T^{th} = 160 MeV ~ Tc = 170 MeV**
    → short time of expansion
    in the hadron phase
  - **T^{th} = 100 MeV < Tc = 170 MeV**
    → long time of expansion





Compared to JAM Model

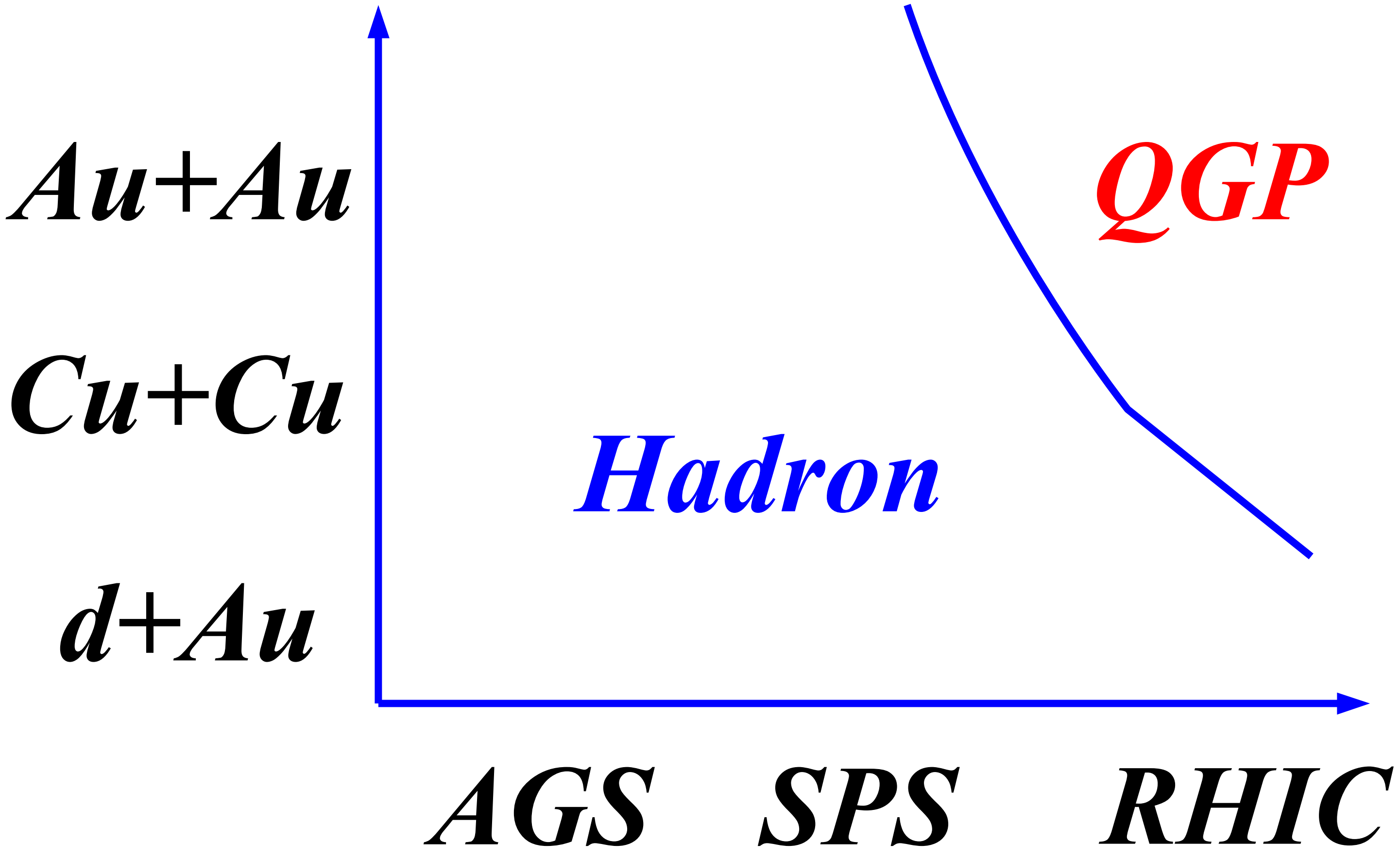


Cu-Cu more like Hydro than JAM hadron string cascade model

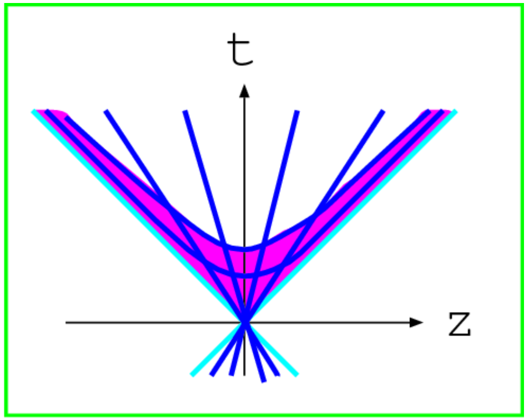
Here JAM uses a 1 fm/c formation time. Hydro (160) has kinetic freezeout temperature at 160 MeV

After Data are opened, ...

- **Hydro wins Cascade at RHIC even for Cu+Cu collisions in the initial stage evolution.....**
- **"Reaction Phase Diagram" seems to be .....**



Relativistic Hydrodynamics (II)



- **One more condition is necessary**
  → *Equation of State $P = P(e, n_i)$ is needed*
  - **Independent Variables: e, P, v, n_i → 6**
  - **Independent Equations: 4+1 =5**
- **Solve Hydro. in Bjorken Variables (τ, η_s, x, y) → Save CPU a lot !**
  - **Most of the Dynamics is govered by τ during $\tau < 10$ fm/c**
  - **η_s approximately corresponds to η, and fixed by inc. E.**
- **Parameters**
  - **τ_0 (thermalization time), T^{ch} (chem. F.O.) → Au+Au $dN/d\eta$ fit**
  - **T^{th}: Free Parameter**
- **Initial Condition: Glauber type / Color Glass Condensate**