

高密度物質と中性子星の物理

Physics of Neutron Star Matter

京大基研 大西 明

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- 中性子星の基本的性質
- 状態方程式を記述する理論模型
- 対称エネルギーと非対称核物質の状態方程式
- ハイパー核物理と高密度核物質の状態方程式
- 中性子星におけるエキゾチック自由度
- Supplementary Contents
 - 実験・観測・理論で解き明かす中性子星物質状態方程式
 - 重イオン衝突とハイパー核から中性子星へ

大阪大学集中講義 7/30-8/1



Why do we study Nuclear Matter EOS ?

Why do we study Nuclear Matter EOS ?

- Answer 1: Since bulk nuclear properties are mainly determined by nuclear matter EOS, it is important for nuclear physics.

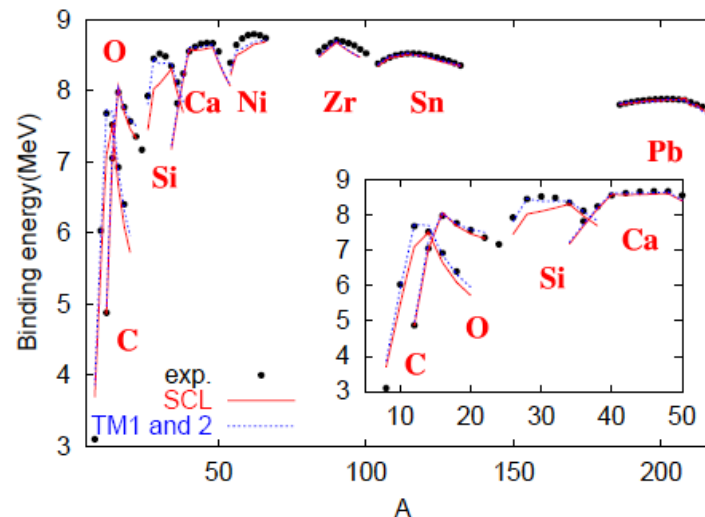
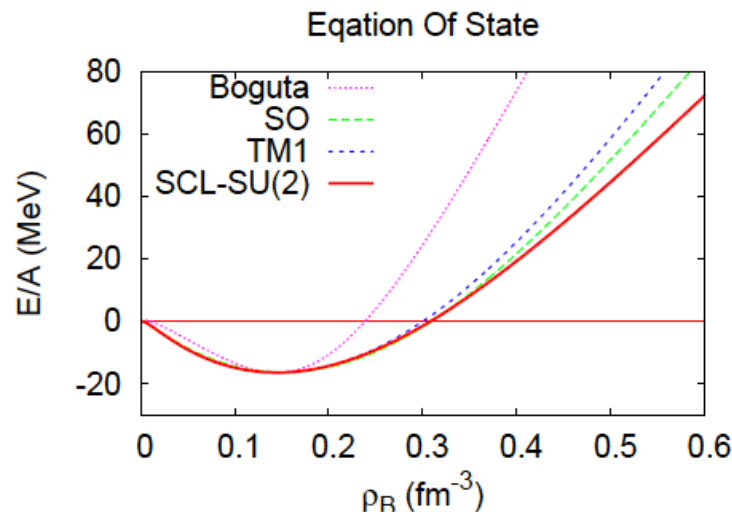
- Nuclear Radius → Saturation of Density

$$R_A = r_0 A^{1/3} \quad (r_0 = 1.2 \text{ fm})$$

- Nuclear Binding Energy (Bethe-Weizsacker Formula)

$$B(A, Z) = \underbrace{a_{\text{vol}} A}_{\text{Nuclear Matter}} - a_{\text{surf}} A^{2/3} - a_{\text{Coulomb}} \frac{Z^2}{A^{1/3}} - a_{\text{sym}} \frac{(N-Z)^2}{A} + a_{\text{pair}} \delta(A, Z) A^{-3/4}$$

Nuclear Matter



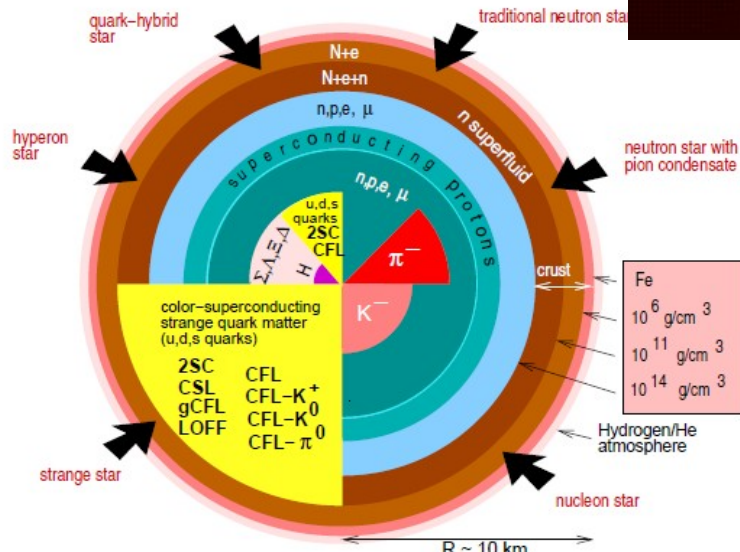
Why do we study Nuclear Matter EOS ?

- Answer 2: Since nuclear matter EOS is decisive in compact astrophysical objects such as neutron stars, supernovae, and black hole formation, EOS is important to understand where atomic elements are made

Neutron Star

Supernova

Nucleosynthesis

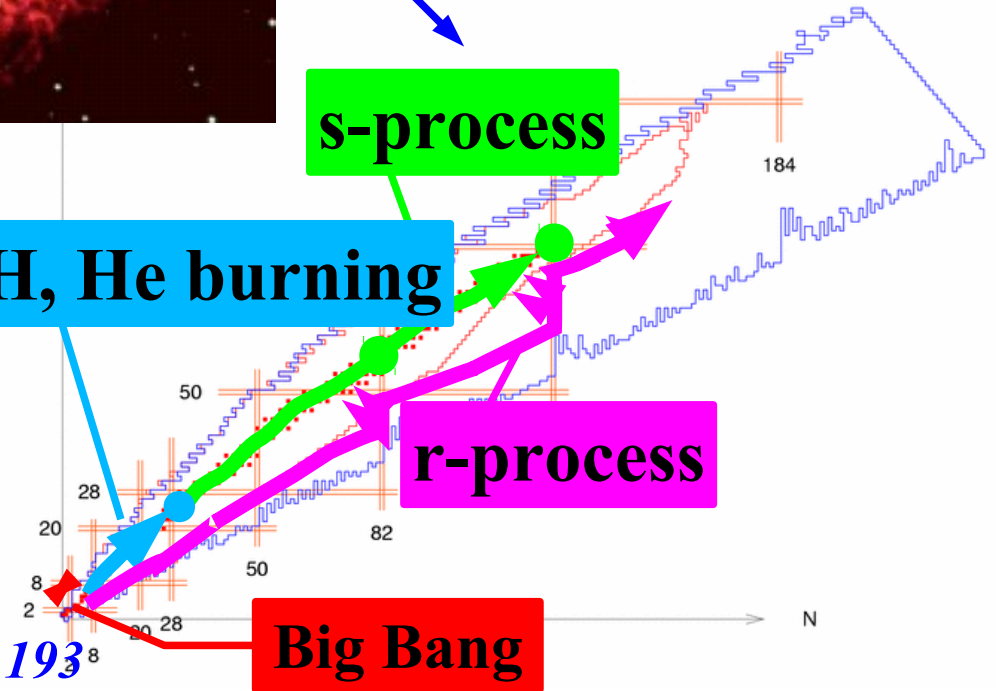


H, He burning

s-process

r-process

Big Bang



F. Weber, Prog. Part. Nucl. Phys. 54 (2005) 193

Why do we study Nuclear Matter EOS ?

- **Answer 3:** Since the EOS should have singularity (or at least sudden change) at phase boundary, it would be possible to catch the signal of phase transition in nuclear collisions.

- **Pressure and Energy Density of Free Massless Gas**

$$P = \frac{\pi^2}{90} N_B T^4, \quad \epsilon = \frac{\pi^2}{30} N_B T^4$$

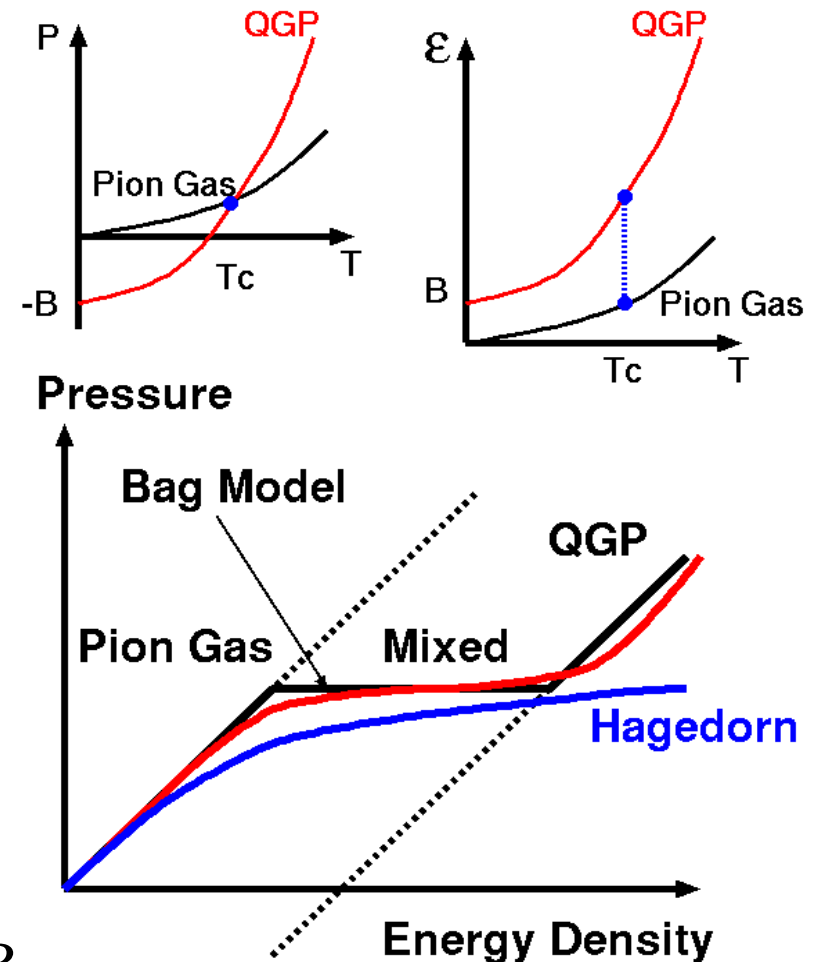
N_B = Bosonic DOF (7/8 for Fermions)

- **Hadron Gas ~ 3 pions ($N_B=3$)**

$$P_\pi = \frac{\pi^2}{30} T^4, \quad \epsilon_\pi = \frac{\pi^2}{10} T^4$$

- **QGP $N_B=16(\text{gluon})+24 \times 7/8$ (quarks) and Bag Pressure**

$$P_{QGP} = \frac{37\pi^2}{90} T^4 - B, \quad \epsilon_{QGP} = \frac{37\pi^2}{30} T^4 + B$$



Theories/Models for Nuclear Matter EOS

■ Ab initio Approaches

- LQCD, GFMC, Variational, BHF, DBHF, G-matrix
→ Not easy to handle, Not satisfactory for phen. purposes

■ Mean Field from Effective Interactions ~ Nuclear Density Functionals

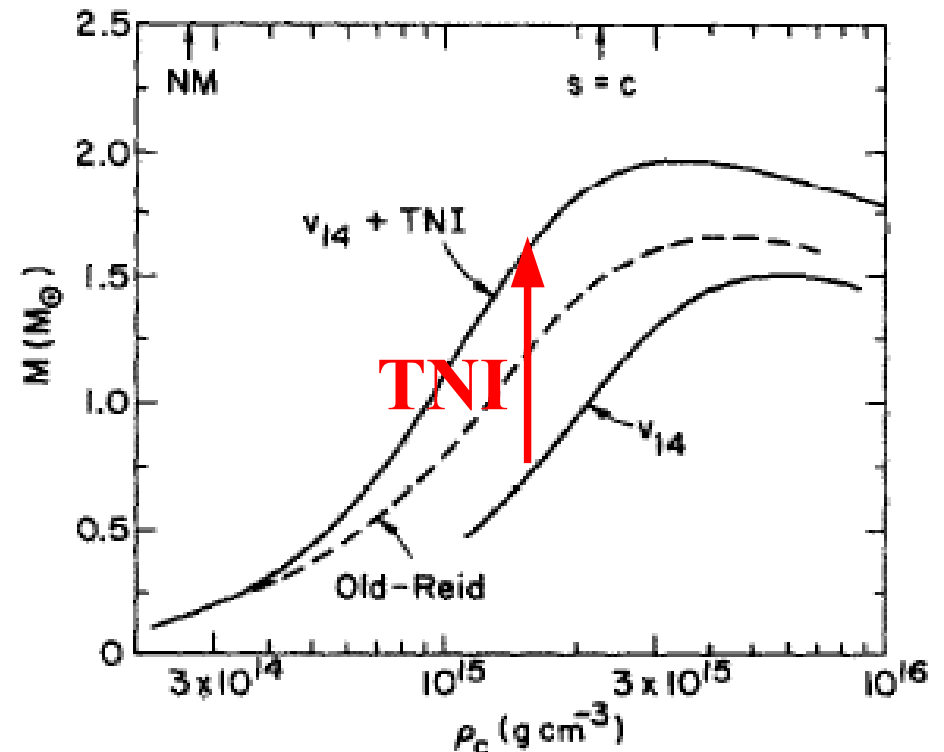
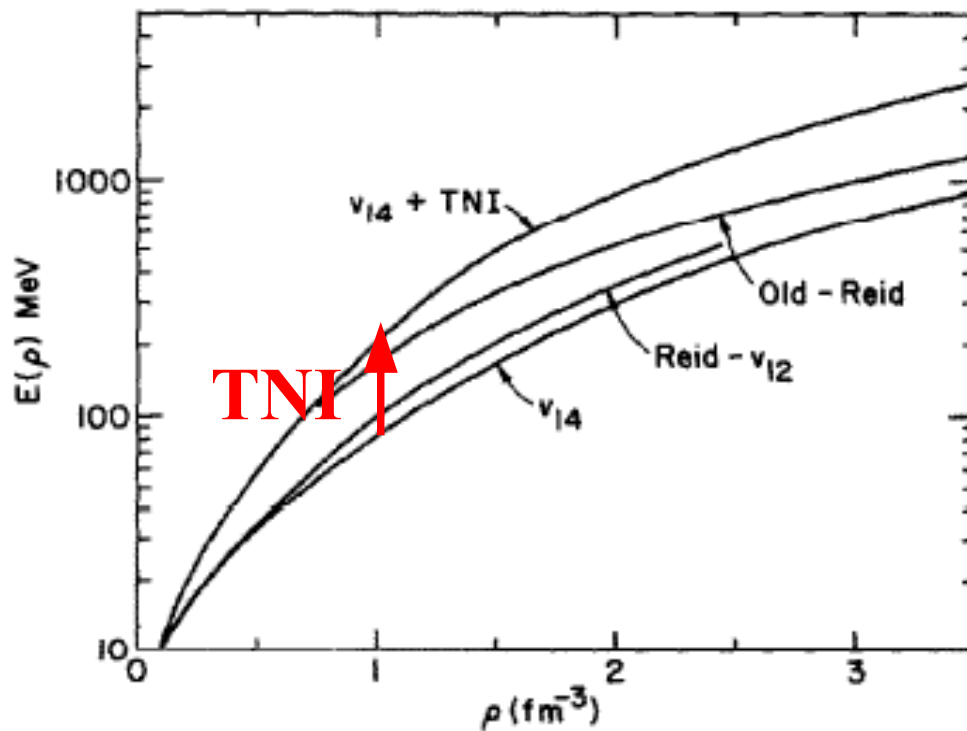
- Skyrme Hartree-Fock(-Bogoliubov)
 - ◆ Non.-Rel., Zero Range, Two-body + Three-body (or ρ -dep. two-body)
 - ◆ In HFB, Nuclear Mass is very well explained (Total B.E. $\Delta E \sim 0.6$ MeV)
 - ◆ Causality is violated at very high densities.
- Relativistic Mean Field
 - ◆ Relativistic, Meson-Baryon coupling, Meson self-energies
 - ◆ Successful in describing pA scattering (Dirac Phenomenology)

Variational Calculations (1)

■ Variational Calculation starting from bare nuclear force

B. Friedman, V.R. Pandharipande, NPA361('81)502

- Argonne v14 + TNI (TNR+TNA)
(TNI/TNR/TNA: three-nucleon int./repulsion/attraction)

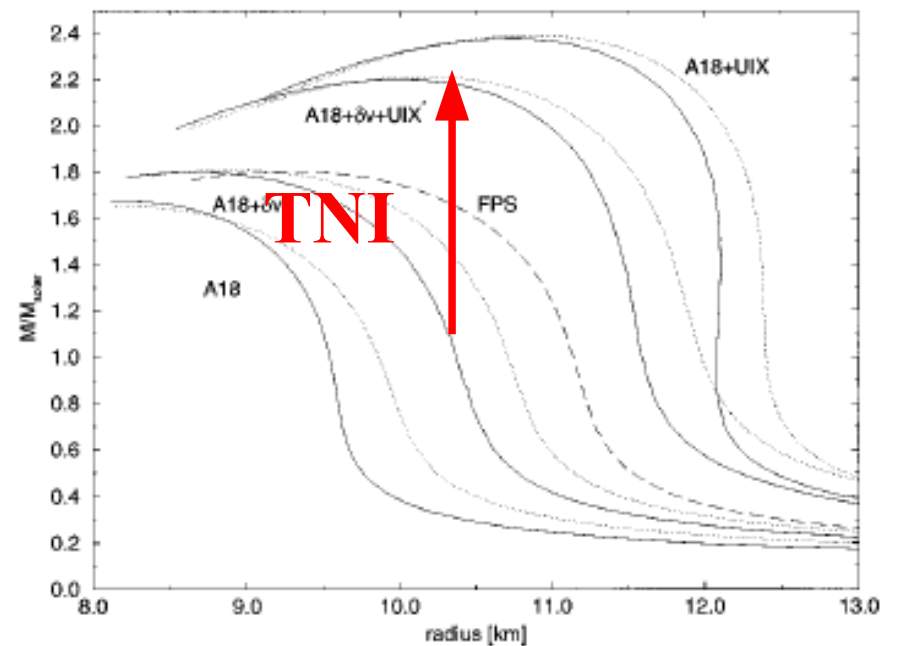
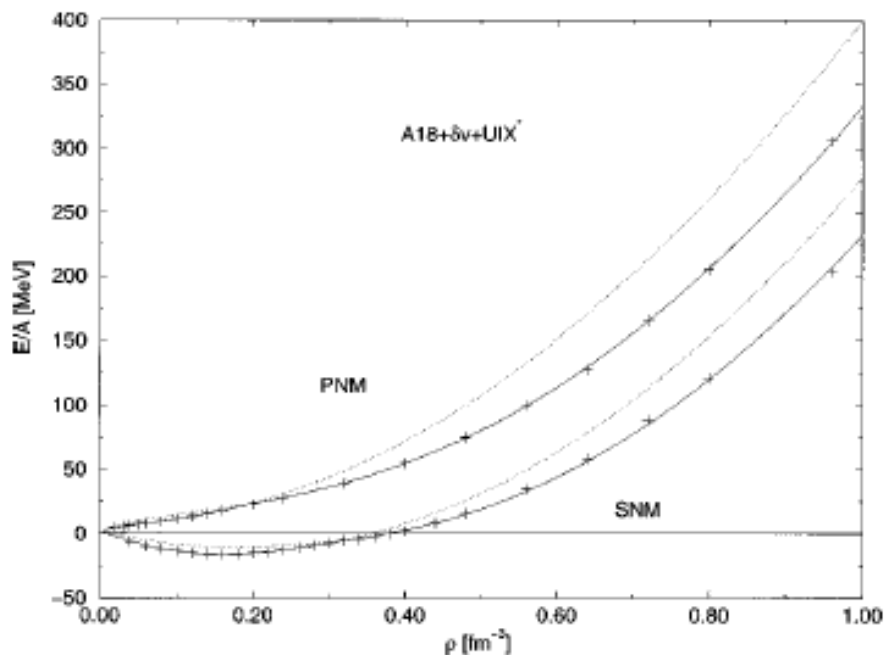


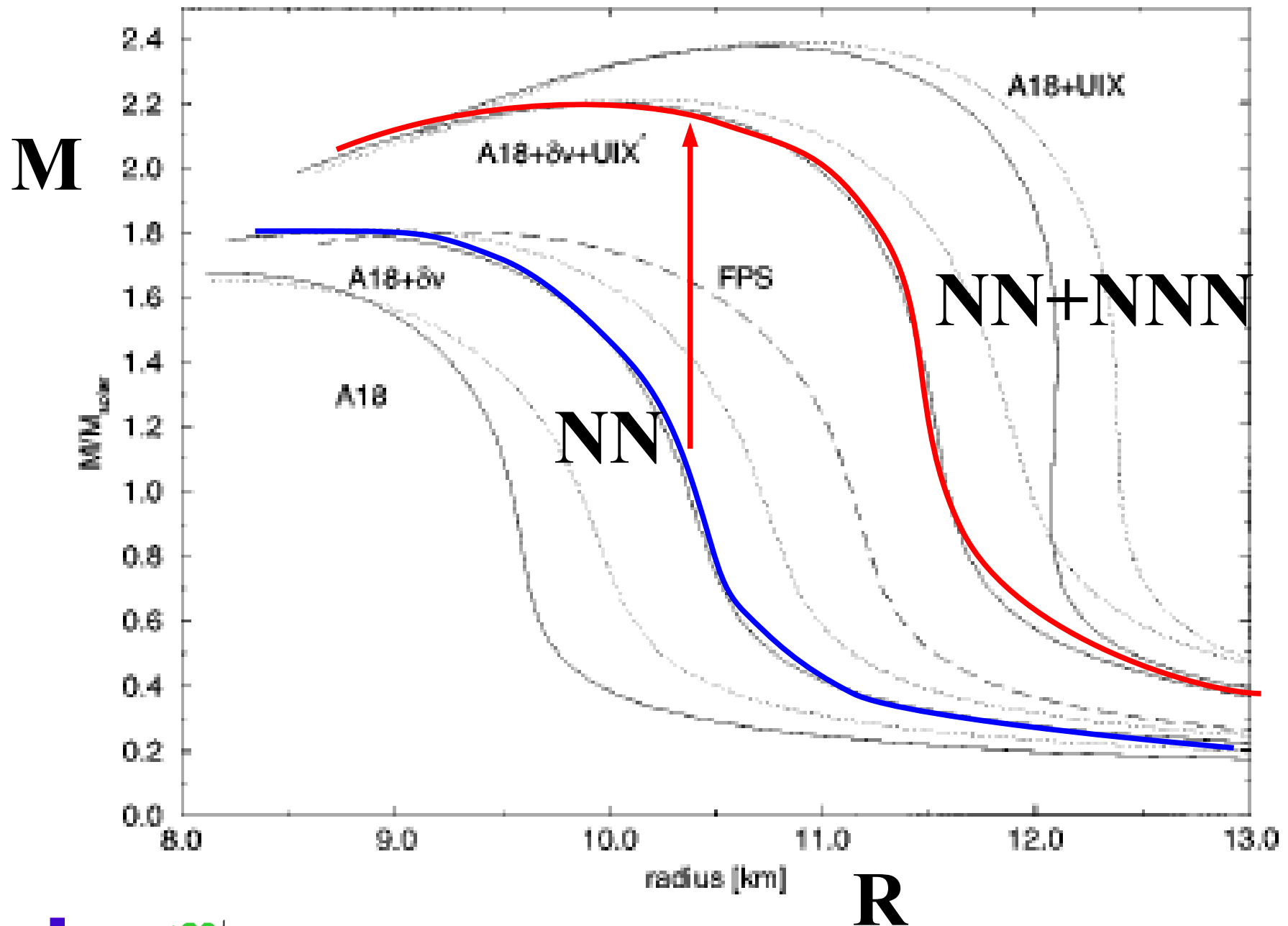
Variational Calculation (2)

■ Variational chain summation method

A. Akmal, V.R.Pandharipande, D.G. Ravenhall, PRC58('98)1804

- v18, relativistic correction, TNI
- Existence of neutral pion condensation at $\rho_B > 0.2 \text{ fm}^{-3}$



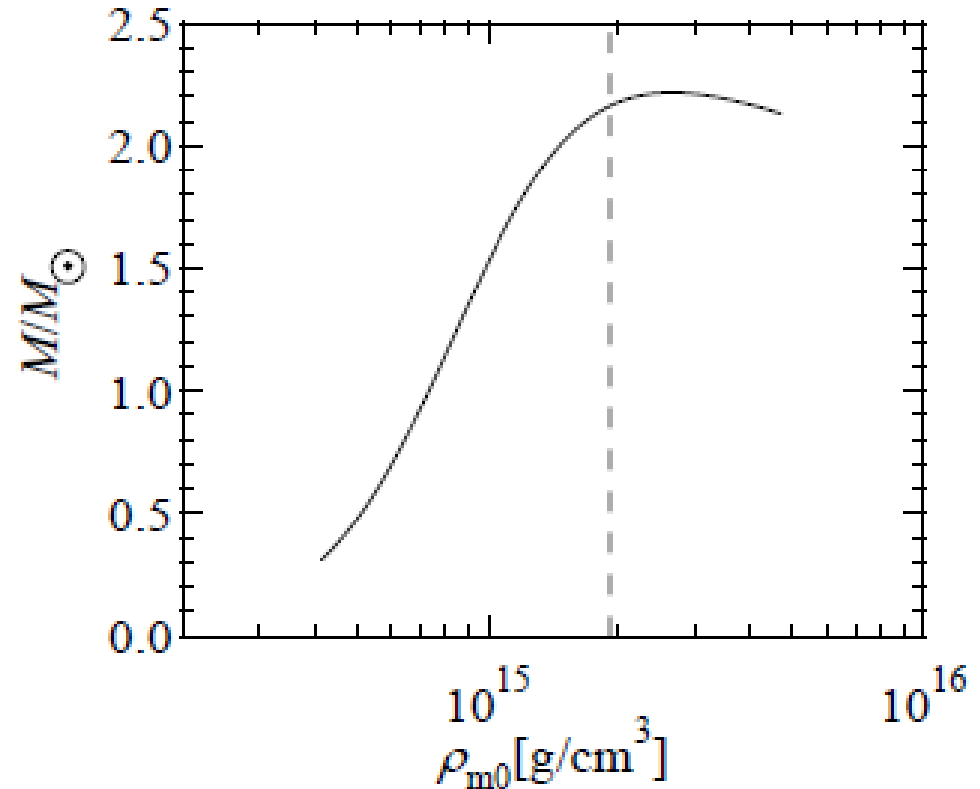
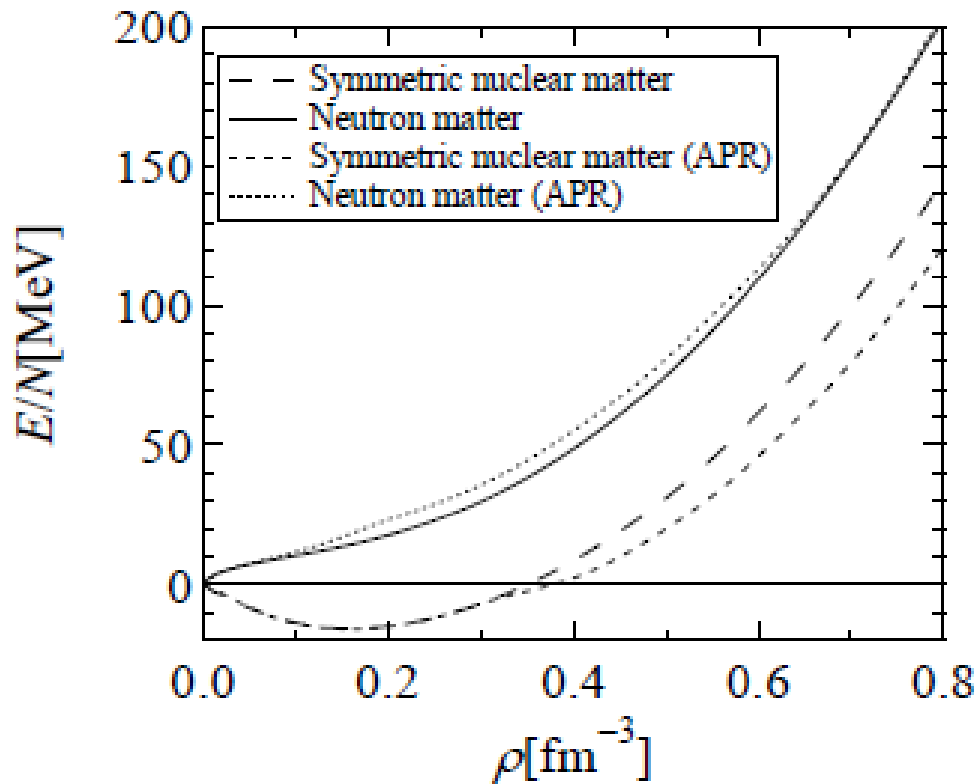


Variational Calculation (3)

■ Variational Calculation using v18+UIX

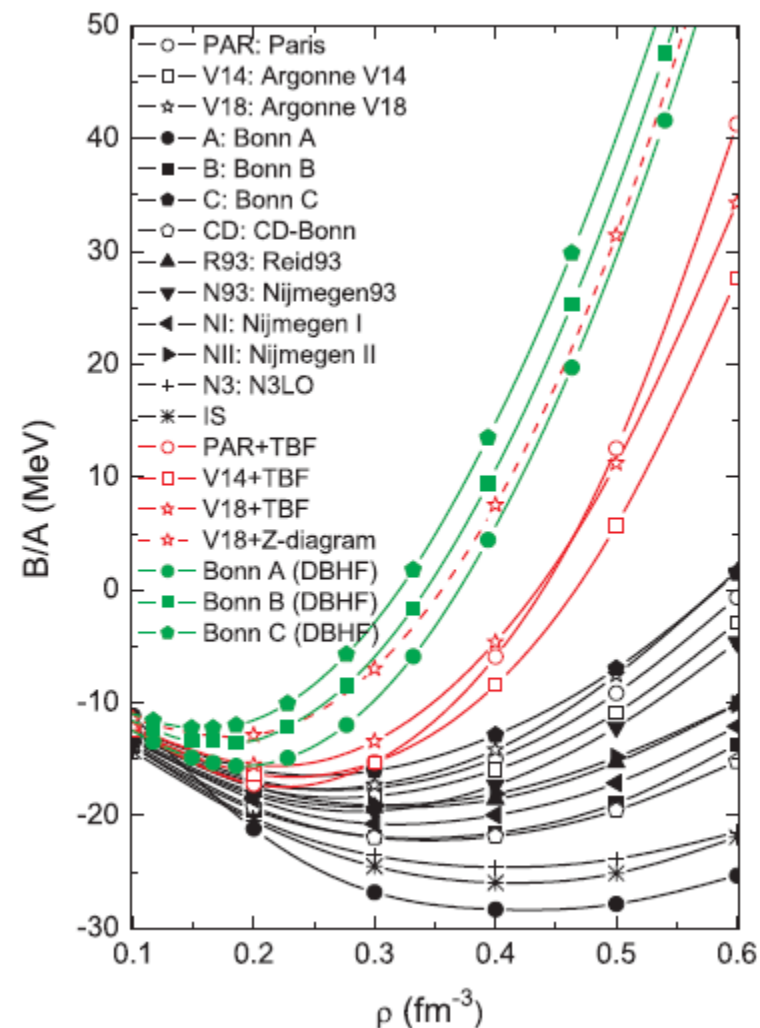
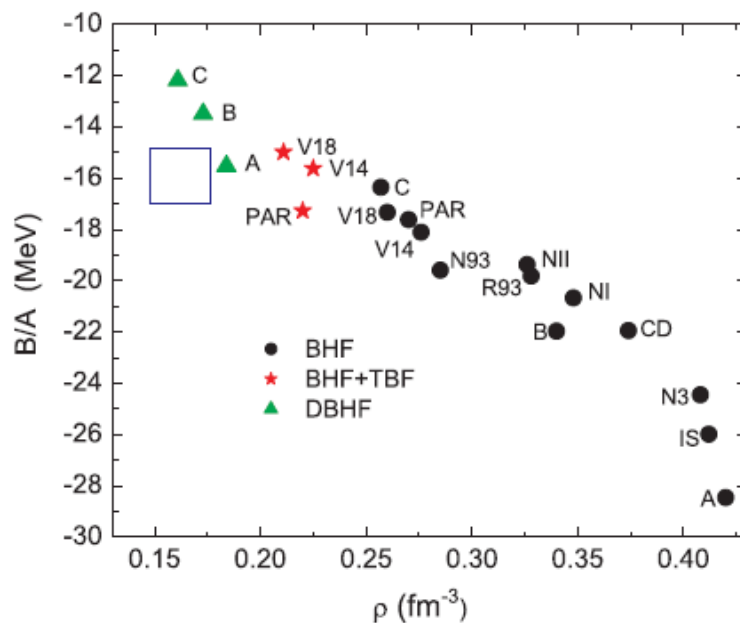
H. Kanzawa, K. Oyamatsu, K. Sumiyoshi, M. Takano, NPA791 ('07) 232

- Similar to APR, but healing-distance condition is required.
→ no π^0 condensation



Bruckner-Hartree-Fock

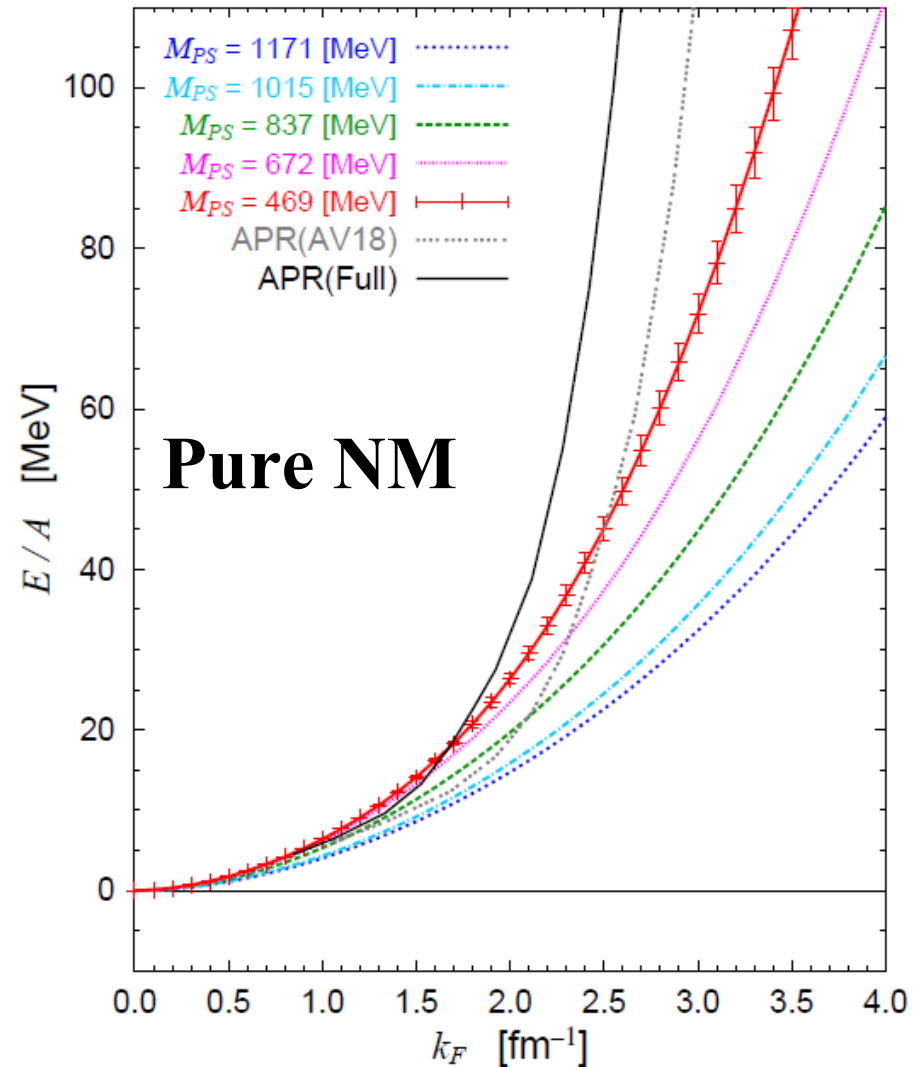
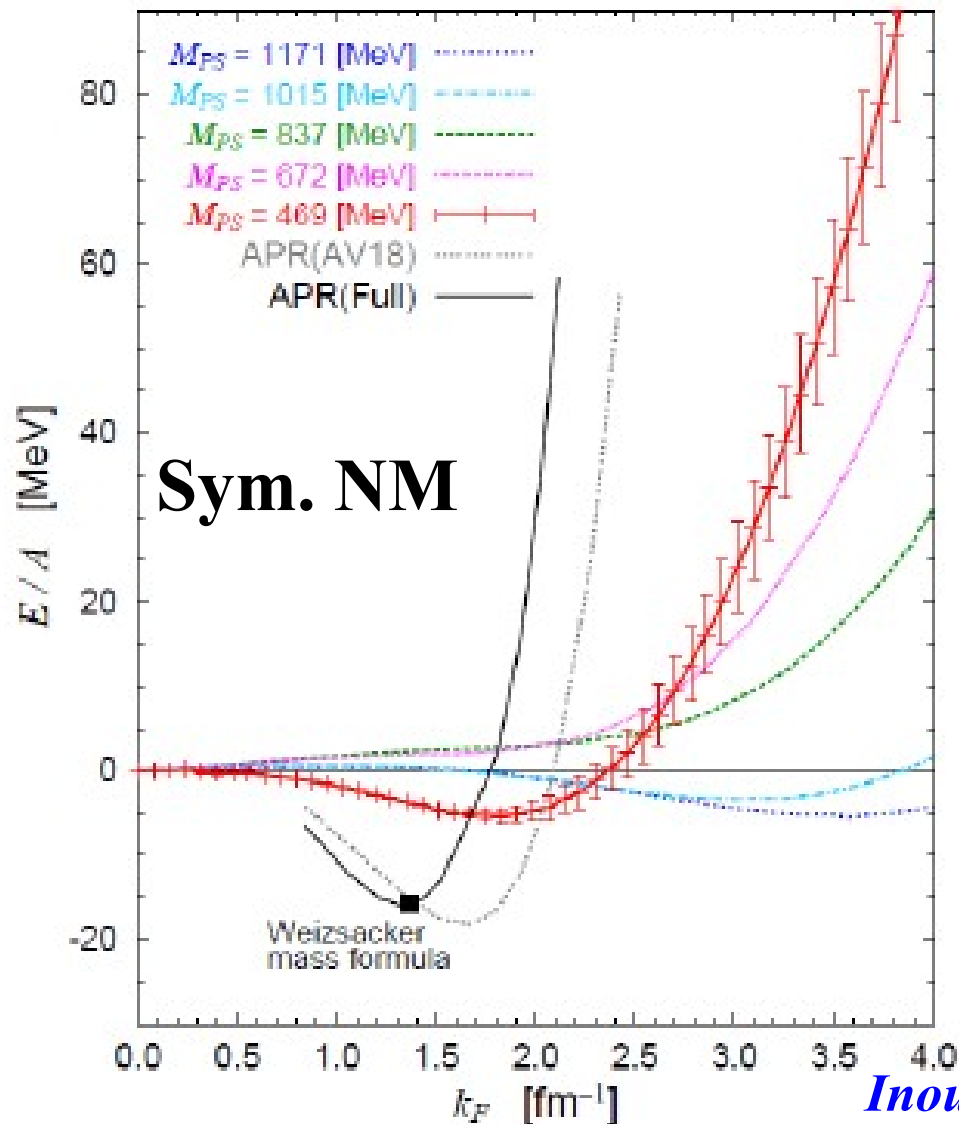
- Self-consistent treatment of
Effective interaction (G-matrix) in the Bruckner Theory
and Single particle energy from G-matrix
- Need 3-body force to reproduce
saturation point.
→ FY type 2 π exchange
+ phen. or Z-diagram



Z.H.Li, U. Lombardo, H.-J. Schulze, W. Zuo, L. W. Chen, H. R. Ma, PRC74('06)047304.

EOS from lattice NN force

- 格子 QCD 核力を用いた高密度状態方程式 (LQCD+BHF)
 NN force: 1S_0 , 3S_1 , 3D_1 only



Inoue et al. (HAL QCD Coll.), PRL111 ('13)112503

A simple model

Simple parametrized EOS

■ Skyrme int. motivated parameterization

$$E_{\text{SNM}} = \frac{3}{5} E_F(\rho) + \frac{\alpha}{2} \left(\frac{\rho}{\rho_0} \right) + \frac{\beta}{2 + \gamma} \left(\frac{\rho}{\rho_0} \right)^{1+\gamma}$$

$$\alpha = \frac{2}{\gamma} \left(E_0(1 + \gamma) - \frac{E_F(\rho_0)(1 + 3\gamma)}{5} \right), \quad \beta = \frac{2 + \gamma}{\gamma} \left[-E_0 + \frac{1}{5} E_F(\rho_0) \right].$$

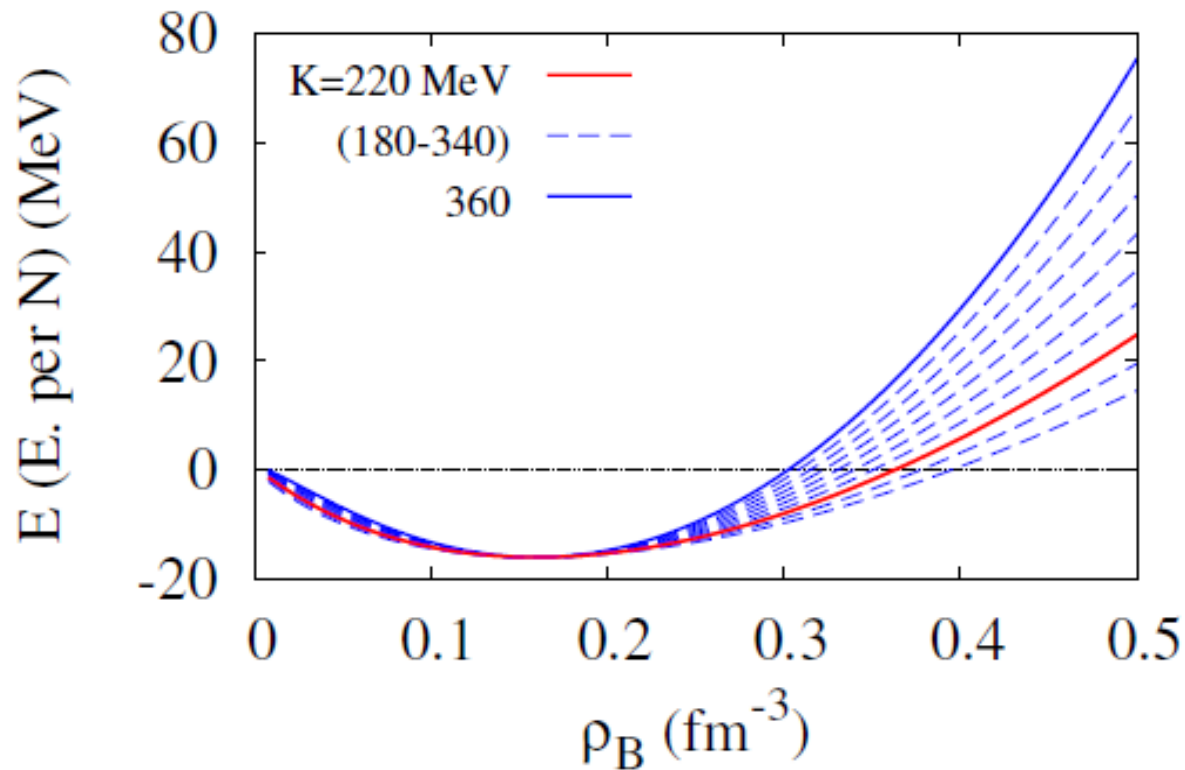
$$K = \frac{1 + 3\gamma}{5} E_F(\rho_0) - 3E_0(1 + \gamma).$$

■ Symmetry energy parameterization

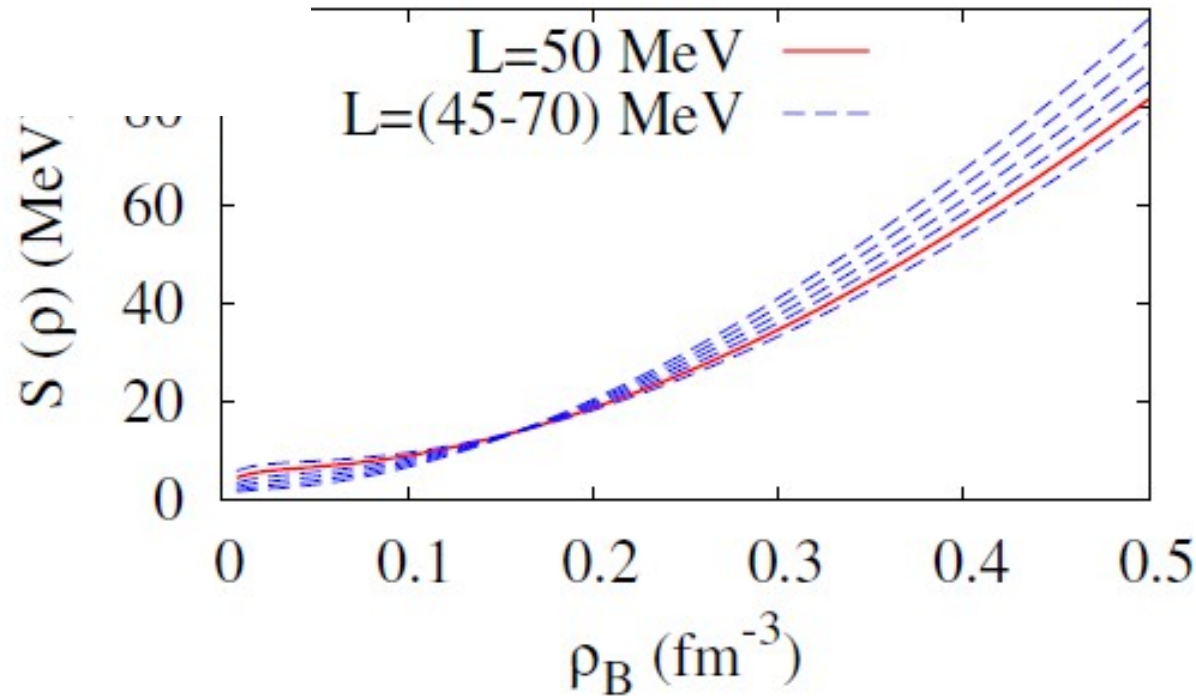
$$S(\rho) = \frac{1}{3} E_F(\rho) + \left[S_0 - \frac{1}{3} E_F(\rho_0) \right] \left(\frac{\rho}{\rho_0} \right)^{\gamma_{\text{sym}}}$$

$$\gamma_{\text{sym}} = \frac{L - \frac{2}{3} E_F(\rho_0)}{3S_0 - E_F(\rho_0)}$$

Simple parametrized EOS

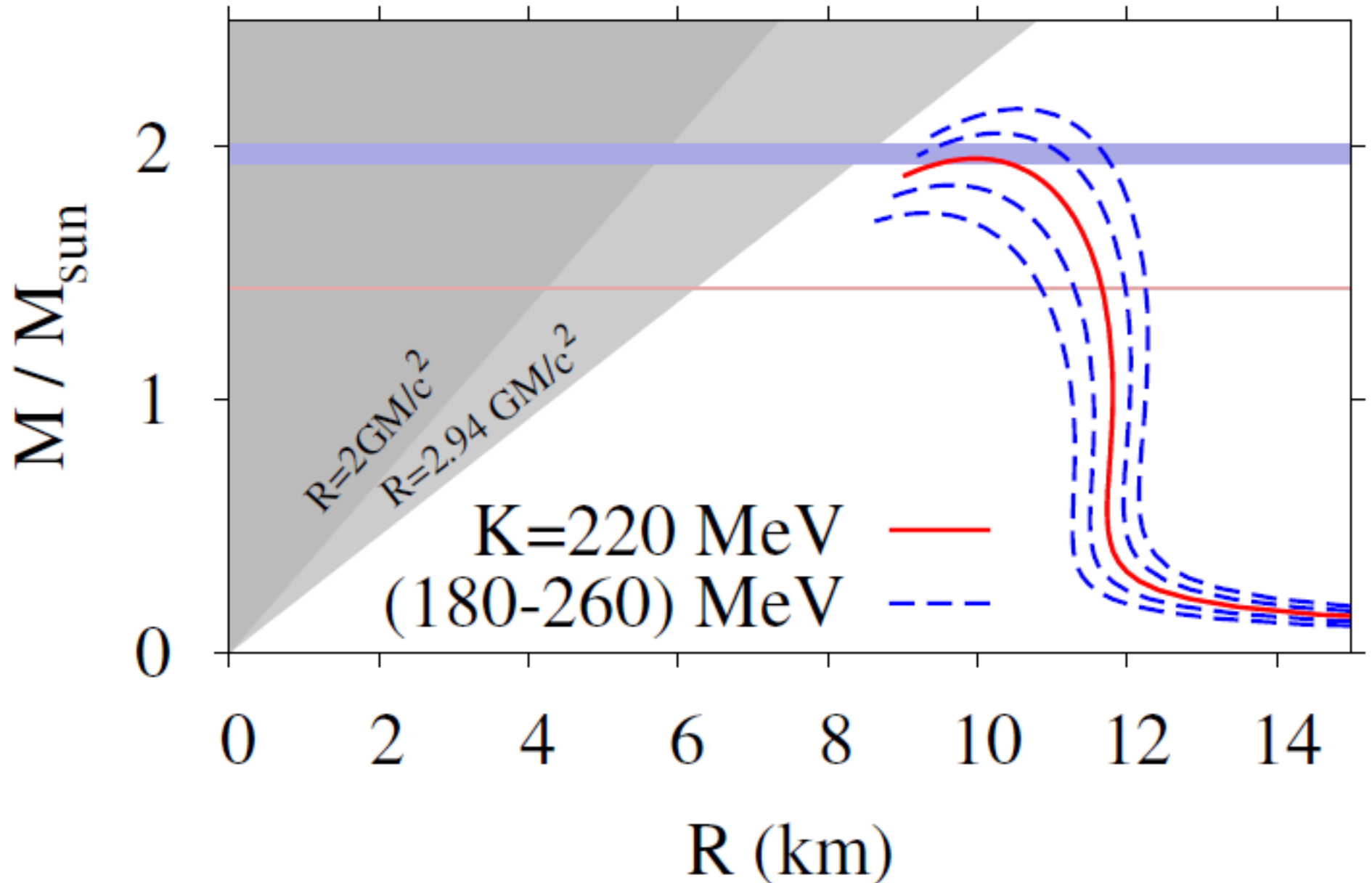


$K=220$ MeV, $S_0=30$ MeV



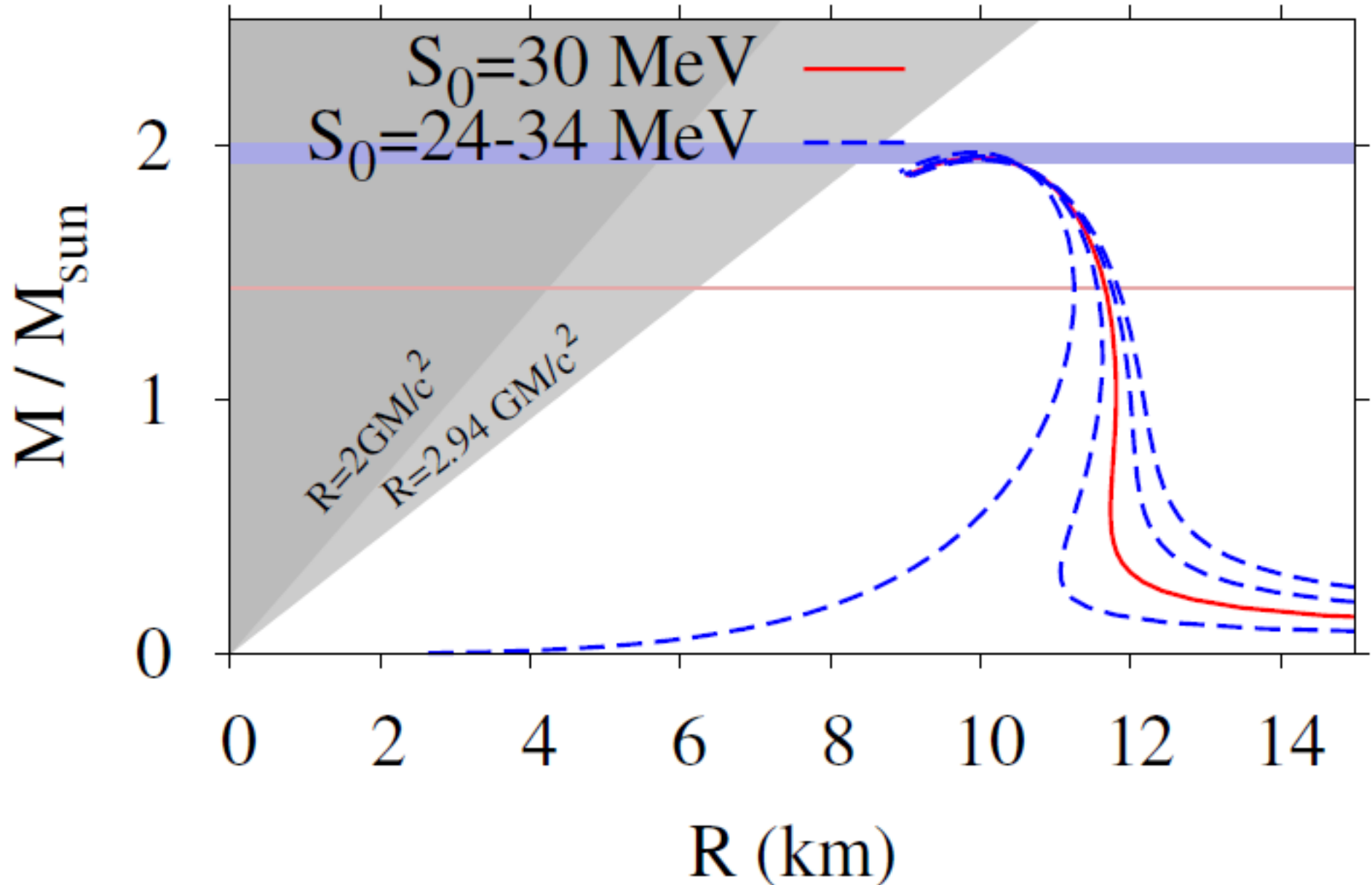
Simple parametrized EOS

$$(S_0, L) = (30 \text{ MeV}, 50 \text{ MeV})$$



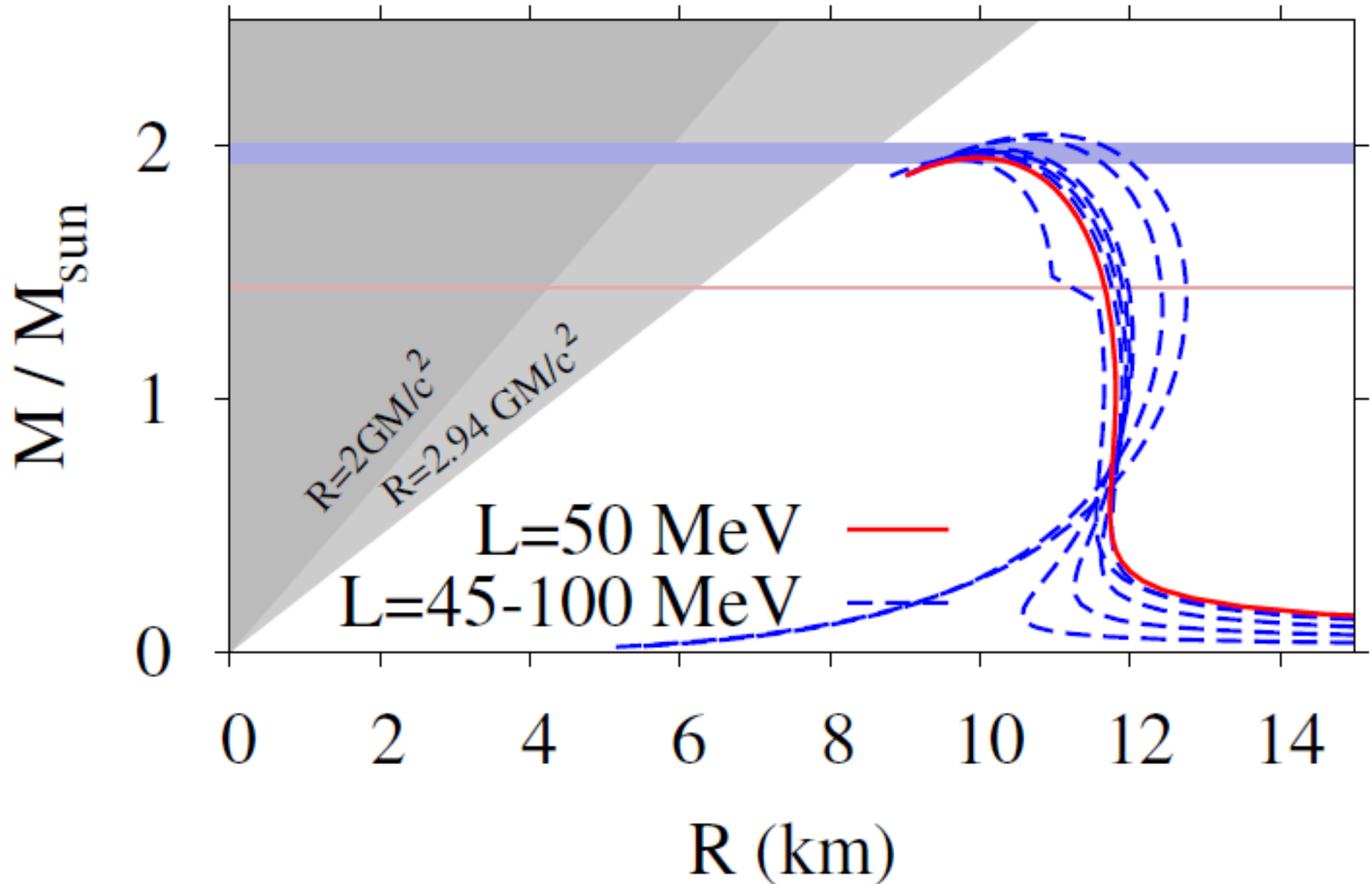
Simple parametrized EOS

$(K, L)=(220 \text{ MeV}, 50 \text{ MeV})$



Simple parametrized EOS

$$(S_0, K)=(30 \text{ MeV}, 220 \text{ MeV})$$



Theories/Models for Nuclear Matter EOS

■ Ab initio Approach

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■ Mean Field from Effective Interactions ~ Nuclear Density Functionals

- Skyrme Hartree-Fock(-Bogoliubov)
 - ◆ Non.-Rel., Zero Range, Two-body + Three-body (or ρ -dep. two-body)
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● Relativistic Mean Field

- ◆ Relativistic, Meson-Baryon coupling, Meson self-energies
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Relativistic Mean Field

Relativistic Mean Field (1)

■ Effective Lagrangian of Baryons and Mesons + Mean Field App.

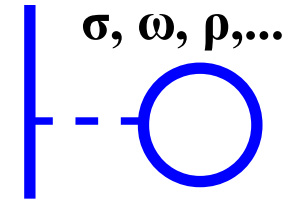
B.D.Serot, J.D.Walecka, Adv.Nucl.Phys.16 ('86), 1

$$L = L_B^{\text{free}} + L_M^{\text{free}} + L_{BM} + L_M^{\text{Int}}$$

$$L_M^{\text{Int}} = -U_\sigma(\sigma) + \frac{1}{4}c_\omega(\omega_\mu\omega^\mu)^2 + \dots$$

$$L_{BM} = -\sum_{B,S} g_{BS} \bar{\Psi}_B \varphi_S \Psi_B - \sum_{B,V} g_{BV} \bar{\Psi}_B \gamma^\mu V_\mu \Psi_B$$

$$L_B^{\text{free}} = \bar{\Psi}_B (i \gamma^\mu \partial_\mu - M_B) \Psi_B, \quad L_M^{\text{free}} = \sum_S \left[\frac{1}{2} \partial^\mu \varphi_S \partial_\mu \varphi_S - \frac{1}{2} m_S^2 \varphi_S^2 \right] + \sum_V \left[-\frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \frac{1}{2} m_V^2 V_\mu V^\mu \right]$$



● **Baryons and Mesons:** B=N, Λ , Σ , Ξ , ..., S= σ , ς , ..., V= ω , ρ , φ , ...

● **Based on Dirac phenomenology & Dirac Bruckner-Hatree-Fock**

E.D. Cooper, S. Hama, B.C. Clark, R.L. Mercer, PRC47('93),297

R. Brockmann, R. Machleidt, PRC42('90),1965

● **Large scalar (att.) and vector (repl.) → Large spin-orbit pot.**

Relativistic Kinematics → Effective 3-body repulsion

● **Non-linear terms of mesons → Bare 3-body and 4-body force**

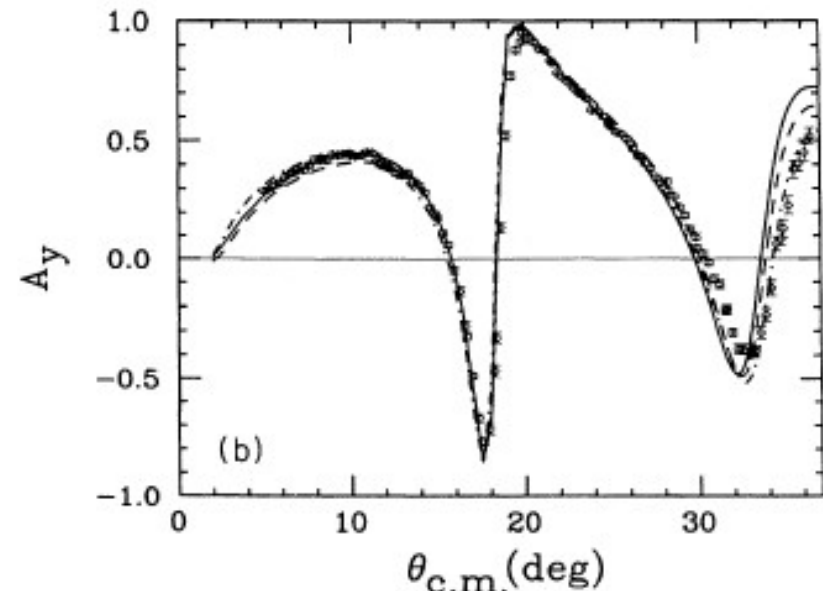
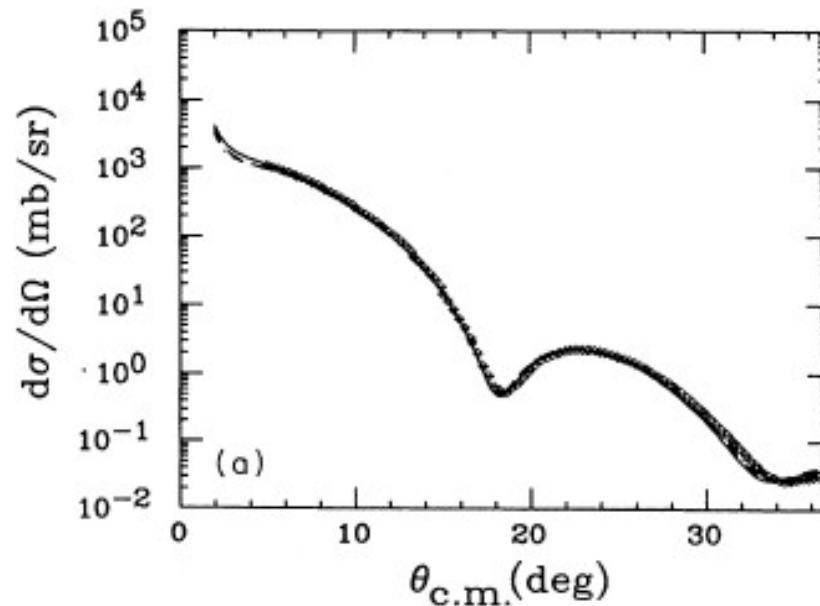
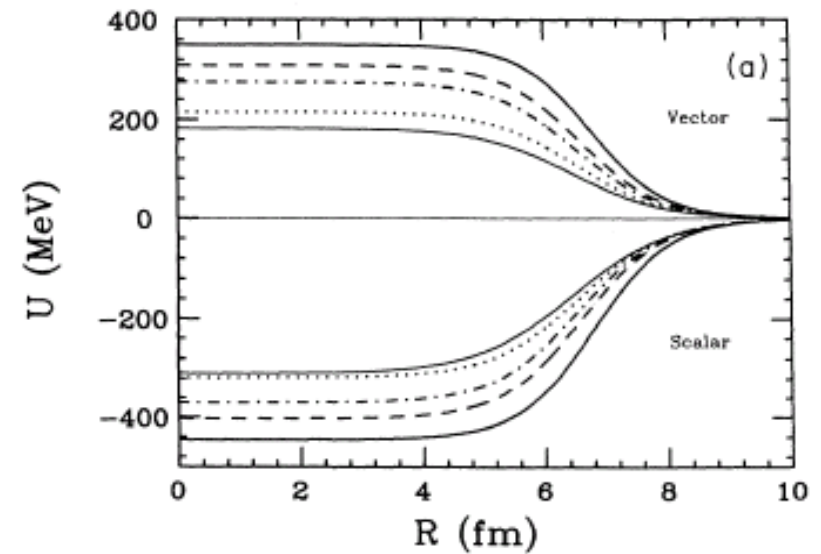
Boguta, Bodmer ('77), NL1:Reinhardt, Rufa, Maruhn, Greiner, Friedrich ('86), NL3:

Lalazissis, Konig, Ring ('97), TM1 and TM2: Sugahara, Toki ('94), Brockmann, Toki ('92)

Dirac Phenomenology

E.D. Cooper, S. Hama, B.C. Clark, R.L. Mercer, PRC47('93),297

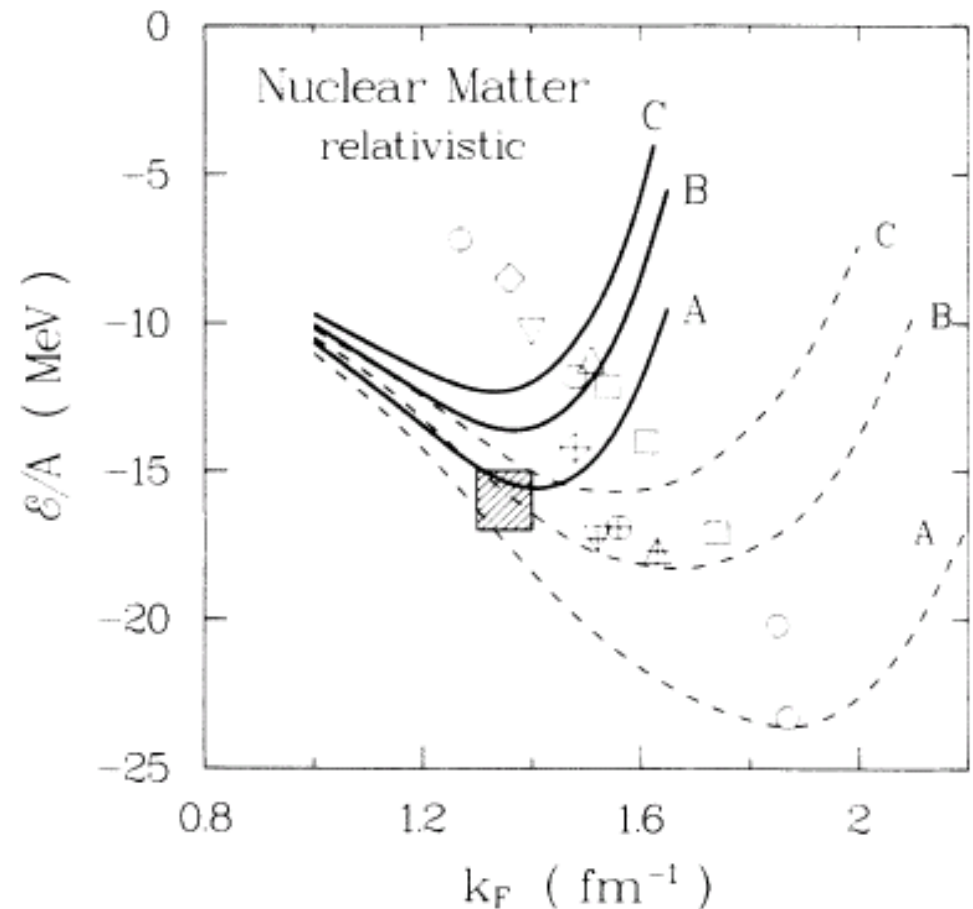
- Dirac Eq. with
Scalar + Vector pA potential
(-400 MeV + 350 MeV)
→ Cross Section, Spin Observable



EOS in Dirac-Brueckner-Hartree-Fock

R. Brockmann, R. Machleidt, PRC42('90),1965

- **Non Relativistic Brueckner Calculation**
→ **Nuclear Saturation Point cannot be reproduced (Coester Line)**
- **Relativistic Approach (DBHF)**
→ **Relativity gives additional repulsion, leading to successful description of the saturation point.**



Relativistic Mean Field (2)

- **Mean Field treatment of meson field operator**
= Meson field operator is replaced with its expectation value
 $\phi(r) \rightarrow \langle \phi(r) \rangle$

Ignoring fluctuations compared with the expectation value may be a good approximation at strong condensate.

- **Which Hadrons should be included in RMF ?**
 - **Baryons (1/2+)** $p, n, \Lambda, \Sigma, \Xi, \Delta, \dots$
 - **Scalar Mesons (0+)** $\sigma(600), f_0(980), a_0(980), \dots$
 - **Vector Mesons (1-)** $\omega(783), \rho(770), \phi(1020), \dots$
 - **Pseudo Scalar (0-)** $\pi, K, \eta, \eta', \dots$
 - **Axial Vector (1+)** $a_1, \dots$

We require that the meson field can have uniform expectation values in nuclear matter.

→ Scalar and Time-Component of Vector Mesons ($\sigma, \omega, \rho, \dots$)

$\sigma\omega$ Model (1)

Serot, Walecka, Adv.Nucl.Phys.16 (1986),1

- Consider only σ and ω mesons

- Lagrangian

$$L = \bar{\Psi} (i \gamma^\mu \partial_\mu - M + g_s \sigma - g_v \omega) \Psi \\ + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_s^2 \sigma^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_v^2 \omega_\mu \omega^\mu \\ (F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu)$$

- Equation of Motion

- Euler-Lagrange Equation

$$\frac{\partial}{\partial x^\mu} \left[\frac{\partial L}{\partial (\partial_\mu \Phi_i)} \right] - \frac{\partial L}{\partial \Phi_i} = 0$$

$$\sigma : \left[\partial_\mu \partial^\mu + m_s^2 \right] \sigma = g_s \bar{\Psi} \Psi$$

$$\omega : \partial_\mu F^{\mu\nu} + m_v^2 \omega^\nu = g_v \bar{\Psi} \gamma^\nu \Psi \quad \rightarrow \quad \left[\partial_\mu \partial^\mu + m_v^2 \right] \omega^\nu = g_v \bar{\Psi} \gamma^\nu \Psi$$

$$\Psi : \left[\gamma^\mu (i \partial_\mu - g_v V_\mu) - (M - g_s \sigma) \right] \Psi = 0$$

EOM of ω (for beginners)

■ Euler-Lagrange Eq.

$$\partial_\mu F^{\mu\nu} + m_\nu^2 \omega^\nu = g_\nu \bar{\psi} \gamma^\nu \psi$$

■ Divergence of LHS and RHS

$$\partial_\nu \partial_\mu F^{\mu\nu} + m_\nu^2 (\partial_\nu \omega^\nu) = m_\nu^2 (\partial_\nu \omega^\nu) = g_\nu (\partial_\nu \bar{\psi} \gamma^\nu \psi) = 0$$

LHS: derivatives are sym. and $F_{\mu\nu}$ is anti-sym.

RHS: Baryon Current = Conserved Current

■ Put it in the Euler-Lagrange Eq.

$$\partial_\mu F^{\mu\nu} = \partial_\mu (\partial^\mu \omega^\nu - \partial^\nu \omega^\mu) = \partial_\mu \partial^\mu \omega^\nu - \partial^\nu (\partial_\mu \omega^\mu) = \partial_\mu \partial^\mu \omega^\nu$$

Schroedinger Eq. for Upper Component (1)

■ Dirac Equation for Nucleons

$$\left(i \gamma \partial - \gamma^0 U_v - M - U_s \right) \psi = 0 \quad ,$$
$$U_v = g_\omega \omega \quad , \quad U_s = -g_\sigma \sigma$$

■ Decompose 4 spinor into Upper and Lower Components

$$\begin{pmatrix} E - U_v - M - U_s & i \sigma \cdot \nabla \\ -i \sigma \cdot \nabla & -E + U_v - M - U_s \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix} = 0$$

$$g = \frac{-i}{E + M + U_s - U_v} (\sigma \cdot \nabla) f$$

$$(E - M - U_v - U_s) f = -i (\sigma \cdot \nabla) g$$

Schroedinger Eq. for Upper Component (2)

■ Erase Lower Component (assuming spherical sym.)

$$\begin{aligned}
 -i(\sigma \cdot \nabla) g &= -(\sigma \cdot \nabla) \frac{1}{X} (\sigma \cdot \nabla) f = -\frac{1}{X} \nabla^2 f - \frac{1}{r} \left[\frac{d}{dr} \frac{1}{X} \right] (\sigma \cdot r) (\sigma \cdot \nabla) f \\
 &= -\nabla \frac{1}{X} \nabla f + \frac{1}{r} \left[\frac{d}{dr} \frac{1}{X} \right] (\sigma \cdot l) f
 \end{aligned}$$

$$(\sigma \cdot r)(\sigma \cdot \nabla) = (r \cdot \nabla) + i \sigma \cdot (r \times \nabla) = r \cdot \nabla - \sigma \cdot l$$

■ “Schroedinger-like” Eq. for Upper Component

$$-\nabla \frac{1}{E + M + U_s - U_v} \nabla f + \left(U_s + U_v + U_{LS} (\sigma \cdot l) \right) f = (E - M) f$$

$$U_{LS} = \frac{1}{r} \left[\frac{d}{dr} \frac{1}{E + M + U_s - U_v} \right] < 0 \quad \text{on surface}$$

$$(U_s, U_v) \sim (-350 \text{ MeV}, 280 \text{ MeV})$$

→ Small Central ($U_s + U_v$), Large LS ($U_s - U_v$)

Various Ways to Evaluate Non.-Rel. Potential

■ From Single Particle Energy

$$\begin{aligned} \left(\gamma^0 (E - U_v) + i \gamma \cdot \nabla - (M + U_s) \right) \psi &= 0 \rightarrow (E - U_v)^2 = p^2 + (M + U_s)^2 \\ \rightarrow E &= \sqrt{p^2 + (M + U_s)^2} + U_v \approx E_p + \frac{M}{E_p} U_s + U_v + \frac{p^2}{2 E_p^3} U_s^2 \\ (E_p &= \sqrt{p^2 + M^2}) \end{aligned}$$

■ Schroedinger Equivalent Potential (Uniform matter)

$$\begin{aligned} -\frac{\nabla^2}{2M} f + \left[U_s + \frac{E}{M} U_v + \frac{U_s^2 - U_v^2}{2M} \right] f &= \frac{E + M}{2M} (E - M) f \\ U_{\text{SEP}} &\approx U_s + \frac{E}{M} U_v \end{aligned}$$

Anyway, slow baryons feel Non.-Rel. Potential,

$$U \approx U_s + U_v = -g_s \sigma + g_v \omega$$

Nuclear Matter in $\sigma\omega$ Model

Serot, Walecka, *Adv.Nucl.Phys.*16 (1986),1

■ Uniform Nuclear Matter

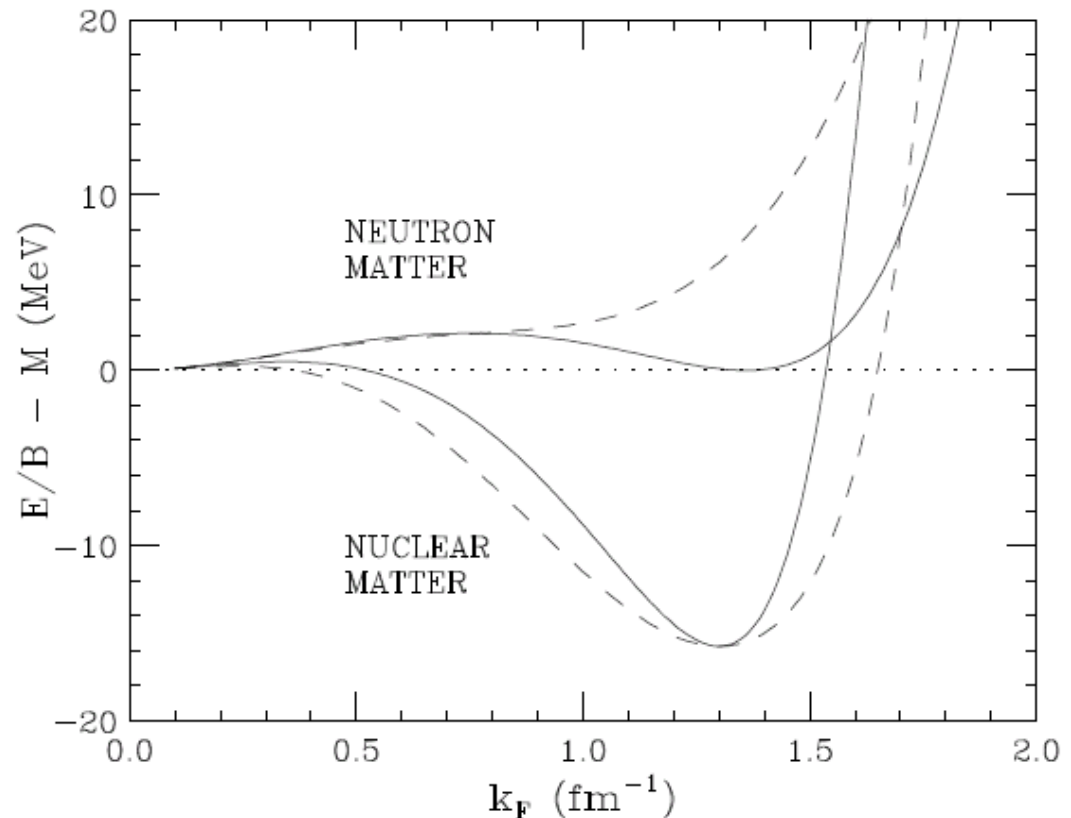
$$E/V = \gamma_N \int^{P_F} \frac{d^3 p}{(2\pi)^2} E^* + \frac{1}{2} m_s^2 \sigma^2 - \frac{1}{2} m_v^2 \omega^2 + g_v \rho_B \omega$$

$$\sigma = \frac{g_s}{m_s^2} \rho_s = \frac{g_s}{m_s^2} \int^{P_F} \frac{\gamma_N d^3 p}{(2\pi)^2} \frac{M^*}{E^*} \quad (M^* = M + U_s = M - g_s \sigma, \quad E^* = \sqrt{p^2 + M^{*2}})$$

$$\omega = \frac{g_v}{m_v^2} \rho_B = \gamma_N \frac{g_v}{m_v^2} \int^{P_F} \frac{d^3 p}{(2\pi)^3}$$

γ_N = Nucleon degeneracy
(=4 in sym. nuclear matter)

Problem: EOS is too stiff
 $K \sim (500-600) \text{ MeV}!$
 $\rightarrow$ How can we avoid it ?



RMF with Non-Linear Meson Int. Terms

*Boguta, Bodmer ('77), NL1:Reinhardt, Rufa, Maruhn, Greiner, Friedrich ('86),
NL3: Lalazissis, Konig, Ring ('97), TM1 and TM2: Sugahara, Toki ('94), Brockmann, Toki ('92)*

- Too stiff EOS in the simplest RMF ($\sigma\omega$ model) is improved by introducing non-linear terms (σ^4, ω^4)
- Fit B.E. of Stable as well as Unstable (n-rich) Nuclei
- Three Mesons (σ, ω, ρ) are included
- Meson Self-Energy Term (σ, ω)

$$\begin{aligned}\mathcal{L} = & \bar{\psi}_N (i\partial - M - g_\sigma \sigma - g_\omega \not{\omega} - g_\rho \tau^a \not{\rho}^a) \psi_N \\ & + \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 \\ & - \frac{1}{4} W^{\mu\nu} W_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4} R^{a\mu\nu} R_{\mu\nu}^a + \frac{1}{2} m_\rho^2 \rho^{a\mu} \rho_\mu^a + \frac{1}{4} c_3 (\omega_\mu \omega^\mu)^2 \\ & + \bar{\psi}_e (i\partial - m_e) \psi_e + \bar{\psi}_\nu i\partial \psi_\nu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} , \\ W_{\mu\nu} = & \partial_\mu \omega_\nu - \partial_\nu \omega_\mu , \\ R_{\mu\nu}^a = & \partial_\mu \rho_\nu^a - \partial_\nu \rho_\mu^a + g_\rho \epsilon^{abc} \rho^{b\mu} \rho^{c\nu} , \\ F_{\mu\nu} = & \partial_\mu A_\nu - \partial_\nu A_\mu .\end{aligned}$$

RMF models with Non-Linear Meson Int. Terms

■ Variety of the RMF models

→ MB couplings, meson masses, meson self-energies

- σN , ωN , ρN couplings are well determined

→ almost no model deps. in Sym. N.M. at low ρ

- ω^4 term is introduced to simulate DBHF results of vector pot.

TM1&2: Y. Sugahara, H. Toki, NPA579('94)557;

R. Brockmann, H. Toki, PRL68('92)3408.

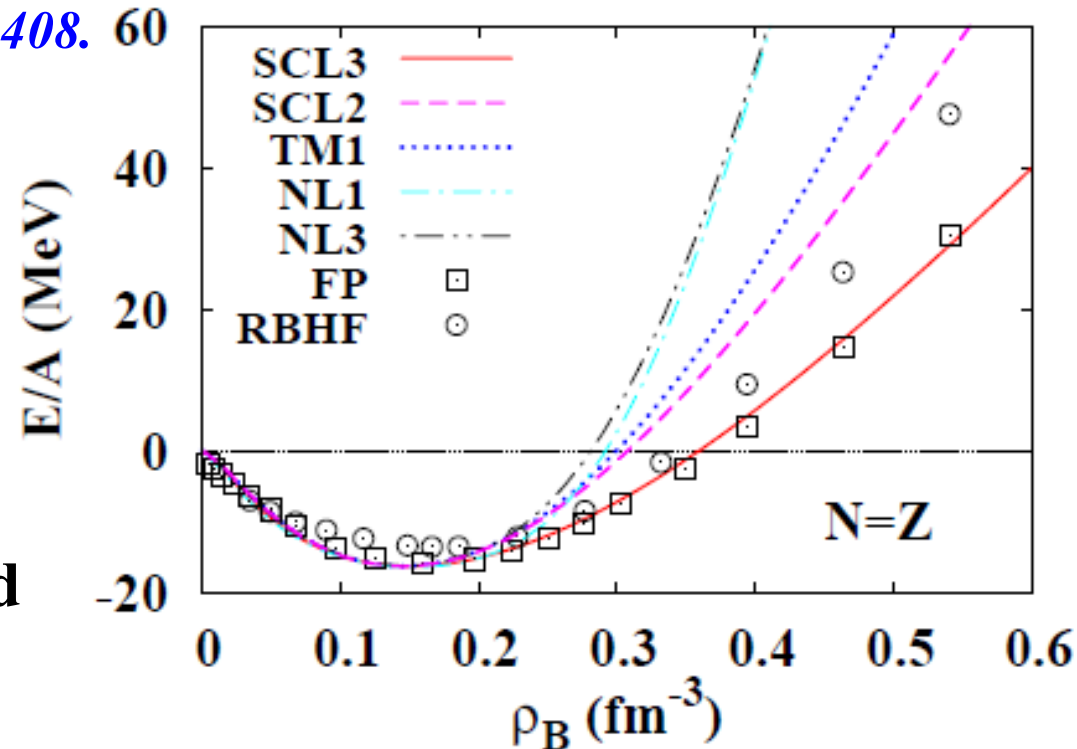
- σ^3 and σ^4 terms are introduced to soften EOS at ρ_0 .

J. Boguta, A.R. Bodmer NPA292('77)413,

NL1: P.-G. Reinhardt, M. Rufa, J. Maruhn, W. Greiner, J. Friedrich, ZPA323('86)13.

NL3: G.A. Lalazissis, J. König, P. Ring, PRC55('97)540.

→ Large differences are found at high ρ



K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.

Vector potential in RMF

- Vector potential from ω dominates at high density !

$$U_v(\rho_B) = g_\omega \omega \sim \frac{g_\omega^2}{m_\omega^2} \rho_B$$

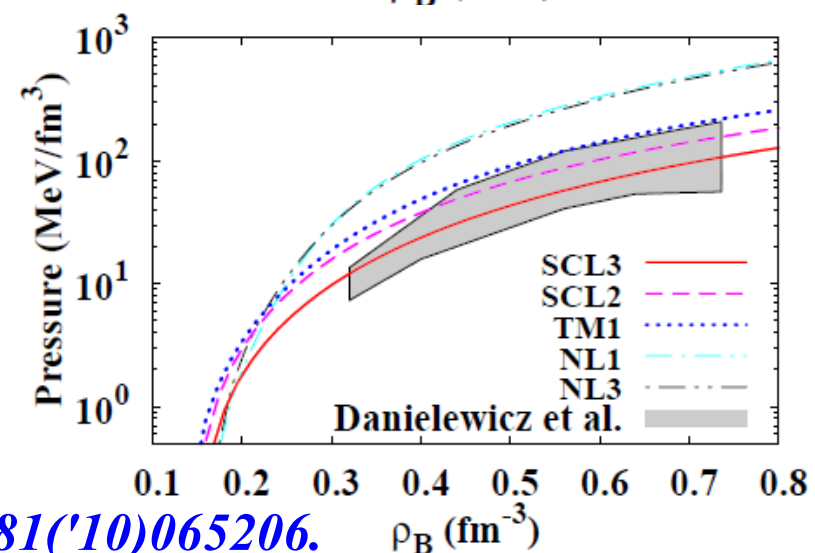
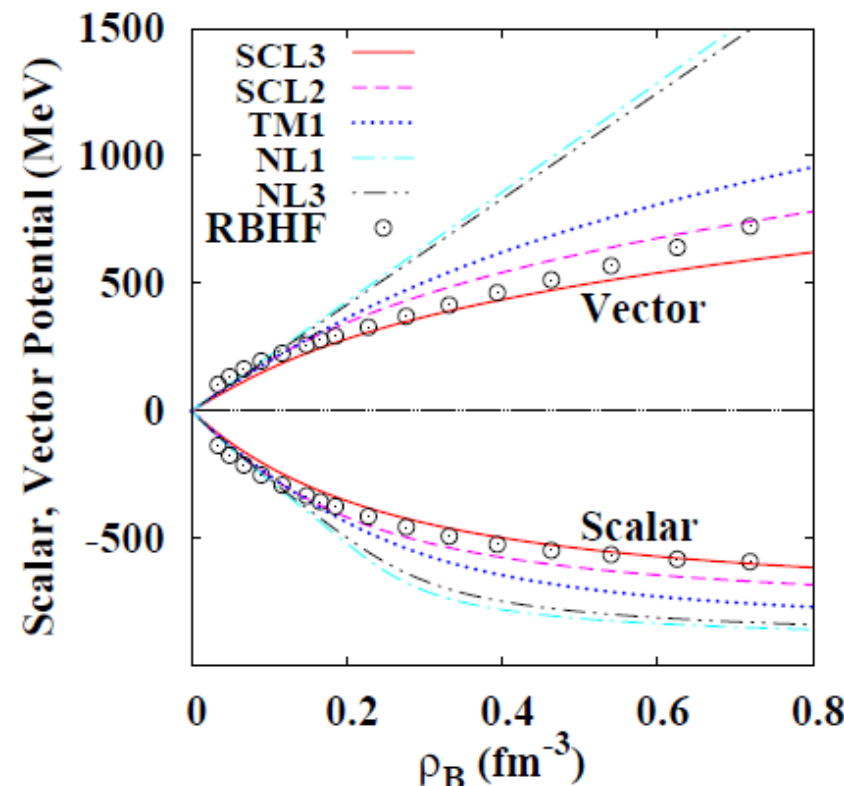
- Dirac-Bruckner-Hartree-Fock shows suppressed vector potential at high ρ_B .

R. Brockmann, R. Machleidt, PRC42('90)1965.

- Collective flow in heavy-ion collisions suggests pressure at high ρ_B .

P. Danielewicz, R. Lacey, W. G. Lynch, Science298('02)1592.

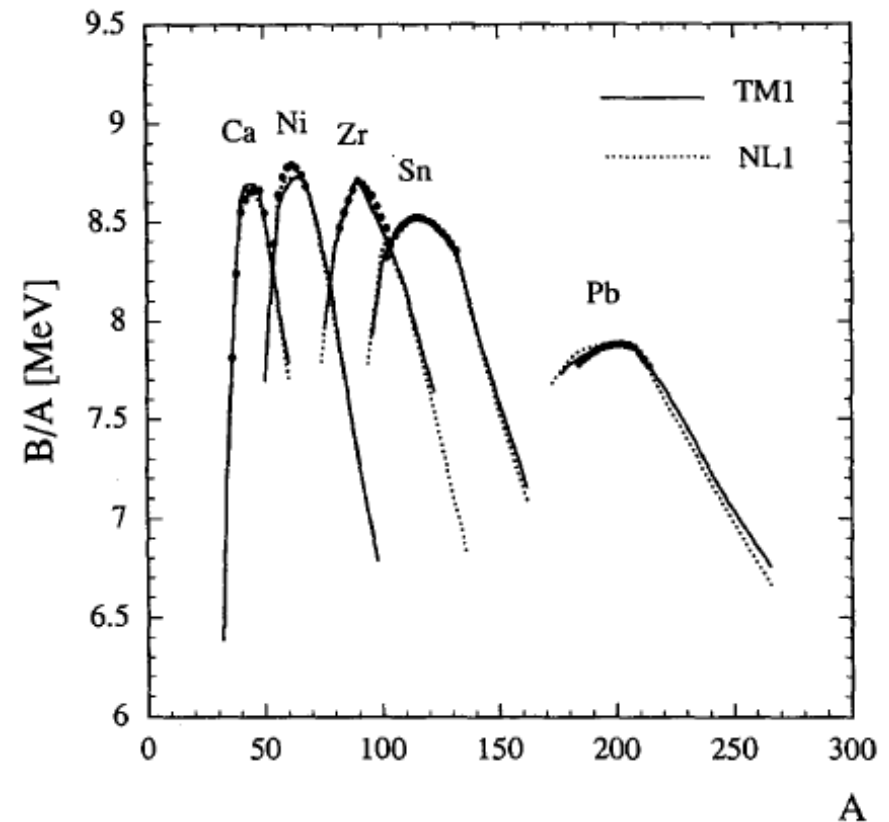
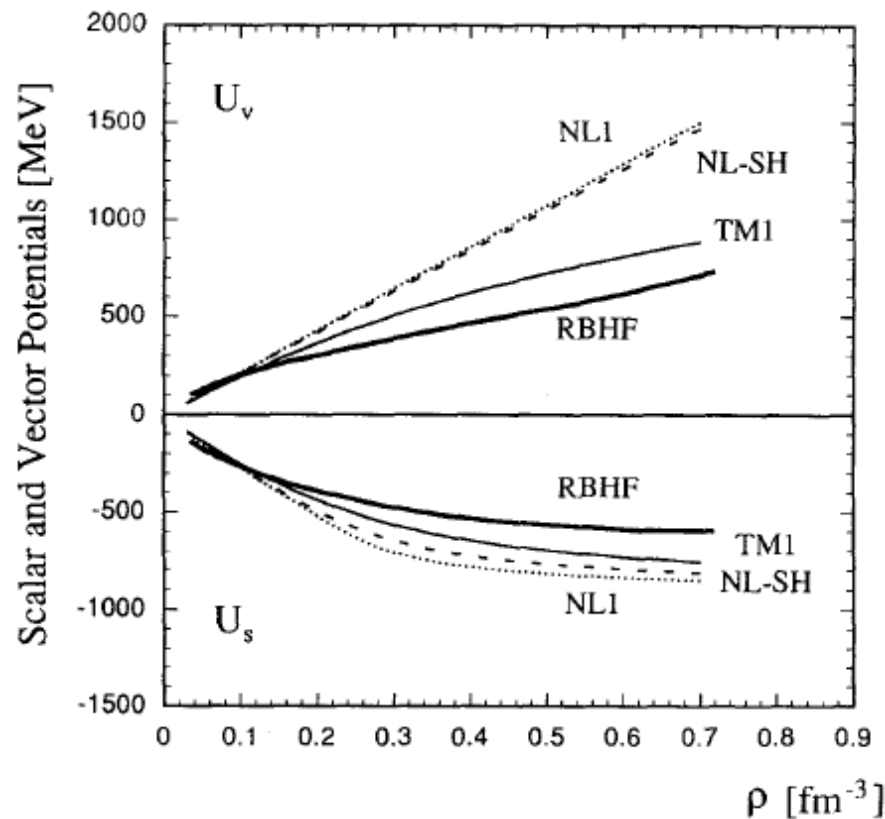
- Self-interaction of $\omega \sim c_\omega (\omega_\mu \omega^\mu)^2$
→ DBHF results & Heavy-ion data



K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.

■ TM1 Sugahara, Toki ('94)

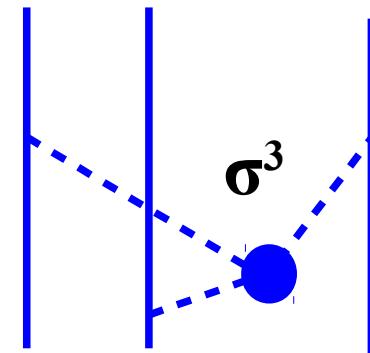
- Fit vector potential in RBHF by introducing ω^4 term.
- Fit binding energies of neutron-rich nuclei



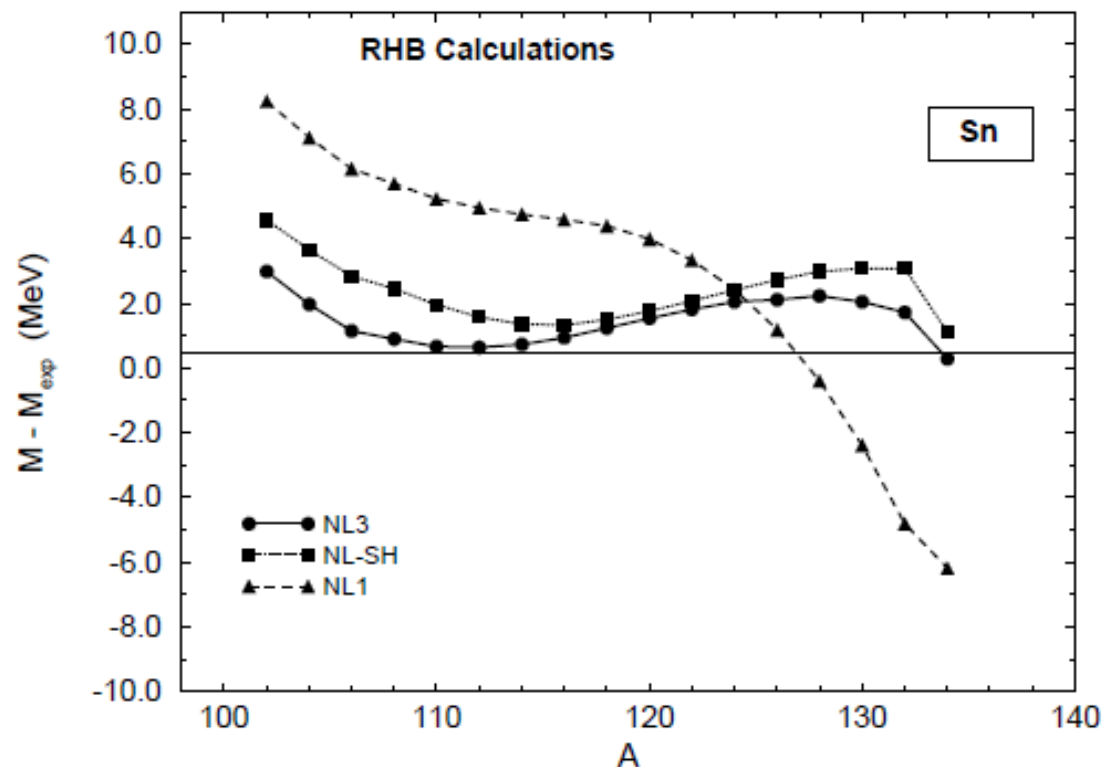
TM1: Sugahara, Toki ('94)

High Quality RMF models

- いくつかの RMF パラメータによる計算は、「質量公式」に迫る精度で原子核質量を記述！
→ High Quality RMF models.
TM, NL1, NL3,



- 全質量で 1-2 MeV の誤差 (NL3)
- Linear coupling (σN , ωN , ρN), self-energy in σ , ω
- 場合によっては結合定数の密度依存性を導入。



NL3: Lalazissis, Konig, Ring, PRC55 ('97)540

RMF with Non-Linear Meson Int. Terms

- Are the Lagrangian parameters are well determined ?

$$\mathcal{L} = \mathcal{L}_{\text{free}}(\psi, \sigma, \omega, \rho, \dots) + \bar{\psi} [g_{\sigma}\sigma - g_{\omega}\gamma^0\omega - g_{\rho}\tau_z\gamma^0\rho] \psi + c_{\omega}\omega^4/4 - V_{\sigma}(\sigma), \quad (3)$$

$$V_{\sigma} = \begin{cases} \frac{1}{3}g_3\sigma^3 + \frac{1}{4}g_4\sigma^4 & (\text{NL1, NL3, TM1}) \\ -a_{\sigma}f_{\text{SCL}}(\sigma/f_{\pi}) & (\text{SCL}) \end{cases}, \quad (4)$$

- Linear terms, Meson-Nucleon Coupling → Well determined
- Negative Coef. of $\sigma^4 < 0$ in some of RMF models → Vacuum is unstable
- Meson interaction terms → Different in RMF parameterization

TABLE II: RMF parameters

	$g_{\sigma N}$	$g_{\omega N}$	$g_{\rho N}$	$g_3(\text{MeV})$	g_4	c_{ω}	$m_{\sigma}(\text{MeV})$	$m_{\omega}(\text{MeV})$	$m_{\rho}(\text{MeV})$
NL1[18]	10.138	13.285	4.976	2401.9	-36.265	0	492.25	795.359	763
NL3[19]	10.217	12.868	4.474	2058.35	-28.885	0	508.194	782.501	763
TM1[6]	10.0289	12.6139	4.6322	1426.466	0.6183	71.3075	511.198	783	770
SCL[20](*)	10.08	13.02	4.40	1255.88	13.504	200	502.63	783	770

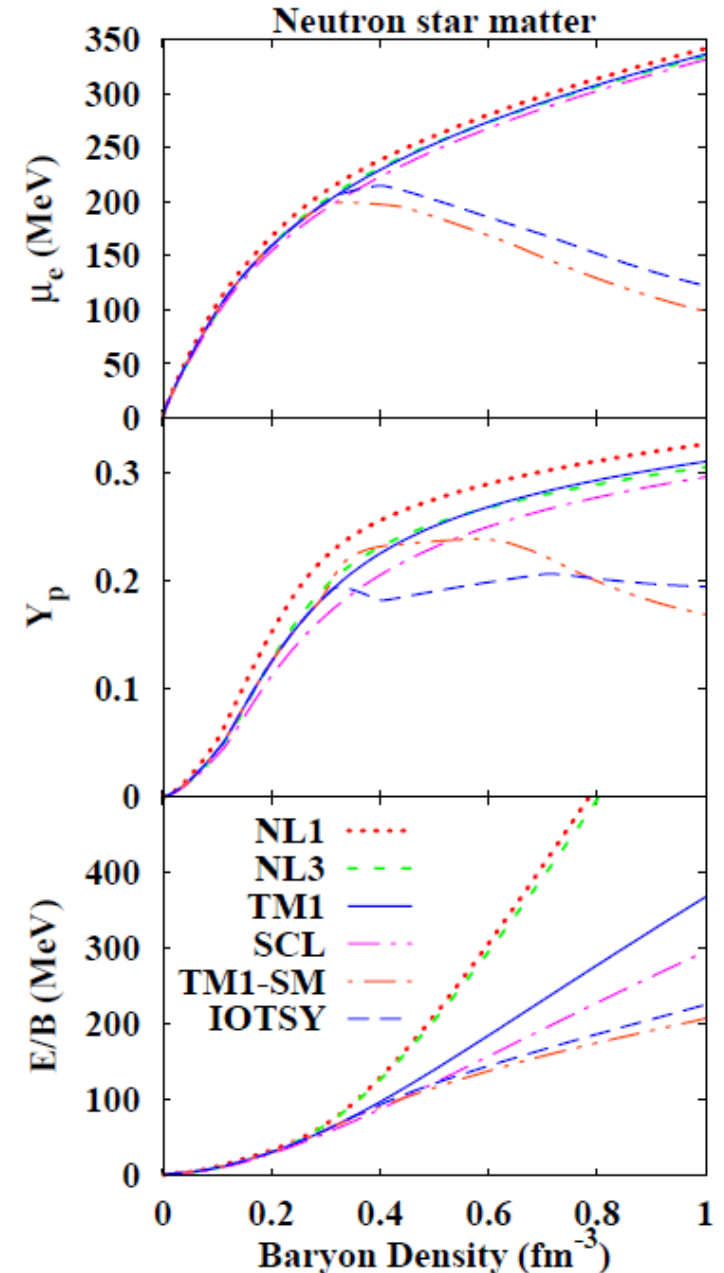
(*)1): g_3 and g_4 are from the expansion of f_{π} .

AO, Jido, Sekihara, Tsubakihara, in prep.

Neutron Star Matter EOS

- Difference in non-linear meson terms generate different predictions of EOS at high densities

How can we fix non-linear terms ?

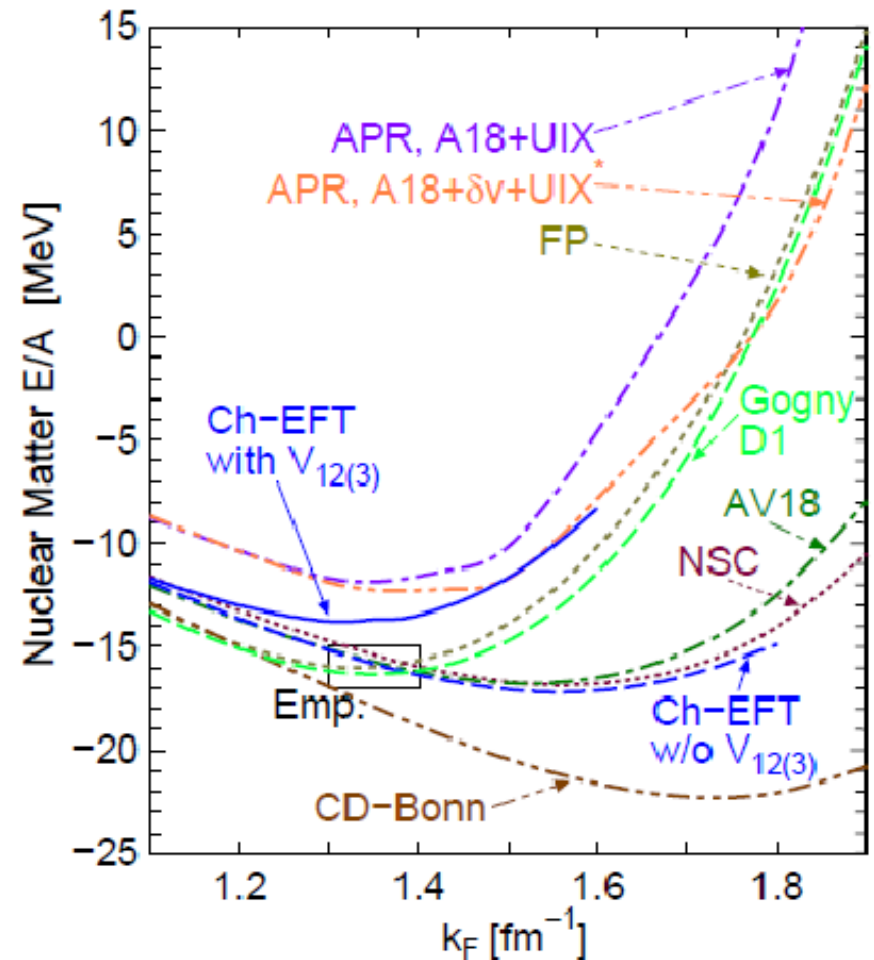


AO, Jido, Sekihara, Tsubakihara, *Phys. Rev. C* 80 (2009), 038202.

Ch-EFT EOS

- Phen. models need inputs from
Experimental Data and/or Microscopic (Ab initio) Calc.
- Recent Ch-EFT EOS is promising !
NN (N3LO)+3NF(N2LO)

M.Kohno ('13)

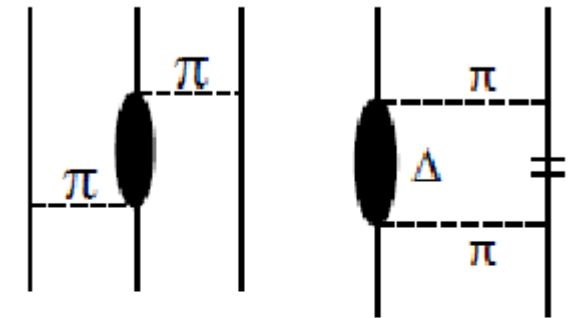


M. Kohno, PRC 88 ('13) 064005

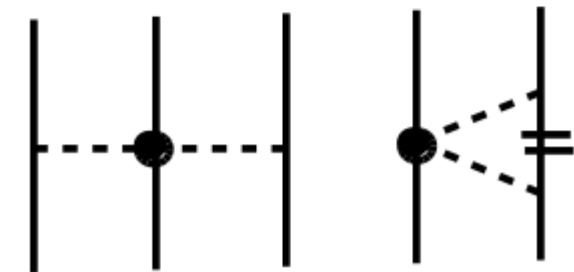
“Universal” mechanism of “Three-body” repulsion

- “Universal” 3-body repulsion is necessary to support NS.
Nishizaki, Takatsuka, Yamamoto (‘02)
- Mechanism of “Universal” Three-Baryon Repulsion.
 - “ σ ”-exchange $\sim$ two pion exch. w/ res.
 - Large attraction from two pion exchange is suppressed by the Pauli blocking in the intermediate stage.
Kohn ('13)

Physical Picture



χ EFT



“Universal” TBR

- *Coupling to Res. (hidden DOF)*
- *Reduced “ σ ” exch. pot. ?*

Summary of Lecture 2

- **Nuclear Matter EOS is important in many subjects of physics.**
 - Bulk nuclear properties (B.E., radius)
 - Dense Matter in Compact Astrophysical Objects
 - High-Energy Heavy-Ion Collisions
- **Relativistic Mean Field models**
 - Simple description of nucleon scalar and vector potentials in terms of meson fields.
 - With non-linear meson interaction terms, nuclear binding energies (and radii) are well explained.
 - Ambiguities of non-linear couplings bring large differences of EOS at high densities, especially in asymmetric nuclear matter.
- **It is promising to utilize the results of G-matrix based on Chiral EFT (2 and 3 nucleon force), which reproduces the saturation density in an “ab initio” manner.**

Thank you !

Field Theory at Finite T & ρ
– Short Course –

■ 量子力学での経路積分 (Path integral)

- 時刻 t_i で位置 q_i にいた粒子が時刻 t_f で位置 q_f に到着する振幅

$$S_{fi} = \langle q_f, t_f | \exp[-i \hat{H}(t_f - t_i)] | q_i, t_i \rangle = \int Dq \exp(iS[q])$$
$$S[q] = \int_{q(t_i)=q_i, q(t_f)=q_f} dt L(q, \dot{q})$$

経路 $q(t)$ についての和 $\rightarrow$ 経路積分

- 特徴

- ◆ 演算子の代わりに通常の数 (c-数) で表せる。
- ◆ 作用 S の構成時に正準交換関係を用いることにより「量子論」の性質を取り込む。

■ 場の理論＝各点での場の振幅 $\varphi(x,t)$ を座標とする量子力学

$$S_{fi} = \langle \Psi_f | \exp[-i \hat{H}(t_f - t_i)] | \Psi_i \rangle = \int D\phi \exp(iS[\phi])$$
$$S[\phi] = \int_{\Psi(t_i)=\Psi_i, \Psi(t_f)=\Psi_f} d^4x L(\phi, \partial_\mu \phi)$$

■ 分配関数

$$\begin{aligned} Z &= \sum_n \exp(-E_n/T) = \sum_n \langle n | \exp[-\hat{H}/T] | n \rangle \\ &= \sum_n \langle n | \exp[-i \hat{H}(t_f - t_i)] | n \rangle_{t_f - t_i = -i/T} = \int D\phi \exp(-S_E[\phi]) \end{aligned}$$

$$S_E[\phi] = \int_0^\beta d\tau d^3x L_E(\phi, \partial_i \phi, \partial_\tau \phi) \Big|_{\phi(x, \beta) = \phi(x, 0)}$$

$$L_E(\phi, \partial_i \phi, \partial_\tau \phi) = -L(\phi, \partial_i \phi, i \partial_t \phi)$$

$$t = -i\tau, \quad \partial_\tau = -i \partial_t, \quad \beta = 1/T$$

$$iS = i \int_0^{-i\beta} dt \int d^3x L = \int_0^\beta d\tau d^3x L = - \int_0^\beta d\tau d^3x L_E$$

- 統計力学の分配関数は虚時間発展の振幅の和である。
- 全ての状態について和 $\rightarrow \tau=0, \beta$ で周期境界条件をつけて任意の $\varphi(x, t)$ について足し合わせる。

Example: Scalar Field

■ Lagrangian density

$$L = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - U(\phi)$$

Euler-Lagrange equation (principle of least action)

$$\partial_\mu \left[\frac{\partial L}{\partial (\partial_\mu \phi)} \right] - \frac{\partial L}{\partial \phi} = 0 \rightarrow \partial_\mu \partial^\mu \phi + m^2 \phi + \frac{\partial U}{\partial \phi} = 0 \text{ (Klein-Gordon eq.)}$$

■ Euclidean Lagrangian

- Euclid 化のルール $t = -i\tau, x_4 = \tau, g_{\mu\nu} = (1, 1, 1, 1), L_E = -L$

$$L_E = \frac{1}{2} \partial_\mu \phi \partial_\mu \phi + \frac{1}{2} m^2 \phi^2 + U(\phi)$$

→ 相互作用がない場合に実際に経路積分してみましょう。

■ 自由スカラー場の分配関数

- 有限のサイズの箱 (体積 V) の中で自由スカラー場 ($U=0$)
- フーリエ変換

$$\phi(\tau, \mathbf{x}) = \frac{1}{\sqrt{V/T}} \sum_{n, \mathbf{k}} \exp(-i \omega_n \tau + i \mathbf{k} \cdot \mathbf{x}) \phi_n(\mathbf{k})$$

Periodic boundary condition $\omega_n = 2\pi n T, \quad k_i = 2\pi n_i / L$

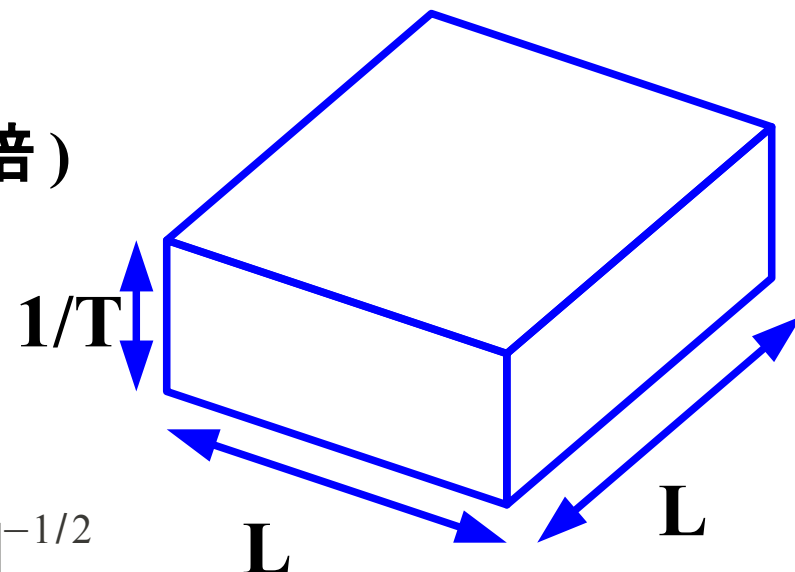
Euclidean action
$$S_E = \frac{1}{2} \sum_{n, \mathbf{k}} (\omega_n^2 + \mathbf{k}^2 + m^2) \phi_n^2(\mathbf{k})$$

- フーリエ変換はユニタリー変換だから、積分の測度が変わらない。(高々定数倍)

$$D\phi = N \prod_{n, \mathbf{k}} d\phi_n(\mathbf{k})$$

- ガウス積分 → 分配関数

$$Z = \int D\phi e^{-S_E} = N \prod_{n, \mathbf{k}} \sqrt{2\pi} [\omega_n^2 + \mathbf{k}^2 + m^2]^{-1/2}$$



■ 自由エネルギー

$$\begin{aligned}\Omega &= -T \log Z = \frac{1}{2} \sum_{\mathbf{k}} \left[T \sum_n \log(\omega_n^2 + \underbrace{\mathbf{k}^2 + m^2}_{E_k^2}) \right] + \text{const.} \\ &= \frac{1}{2} \sum_{\mathbf{k}} I(E_k, T) + \text{const.}\end{aligned}$$

■ 松原和 (Matsubara Frequency summation)

$$\begin{aligned}\sum_n \frac{1}{a^2 + \bar{n}^2} &= \frac{\pi}{2a} \times \begin{cases} \coth(\pi a/2) & (\bar{n} = 2n) \\ \tanh(\pi a/2) & (\bar{n} = 2n+1) \end{cases} \\ \frac{\partial I(E_k, T)}{\partial E_k} &= \sum_n \frac{2T E_k}{\omega_n^2 + E_k^2} = \dots = \frac{1 + \exp(-E_k/T)}{1 - \exp(-E_k/T)} \\ I(E_k, T) &= E_k + 2T \log(1 - \exp(-E_k/T)) + \text{const.}\end{aligned}$$

■ 自由エネルギー (グランド・ポテンシャル)

$$\Omega = \sum_k \left\{ \frac{E_k}{2} + T \log(1 - e^{-E_k/T}) \right\} + \text{const.}$$

$$= V \int \frac{d^3 k}{(2\pi)^3} \left[\frac{E_k}{2} + T \log(1 - e^{-E_k/T}) \right]$$

ゼロ点エネルギー ($\hbar\omega/2$) 熱的励起

ゼロ点エネルギー部分を無視して部分積分すると、
通常の圧力を得る。

$$P = -\Omega/V = \int \frac{d^3 k}{(2\pi)^3} \frac{\mathbf{k} \cdot \mathbf{v}}{3} \frac{e^{-E_k/T}}{1 - e^{-E_k/T}} \quad \left(\mathbf{v} = \frac{\partial E_k}{\partial \mathbf{k}} \right)$$

場の理論 $\rightarrow$ Euclid 化 + Imag. Time $\rightarrow$ 統計力学

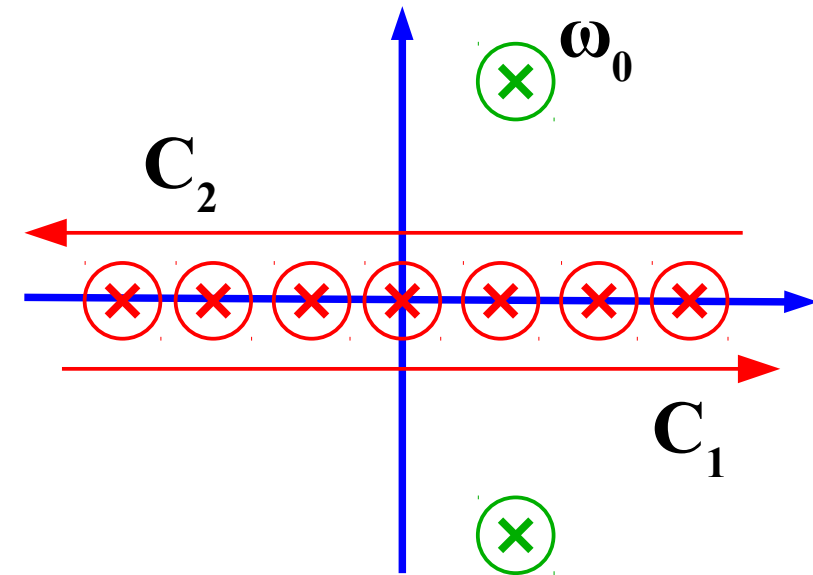
Matsubara Frequency Summation

Contour integral technique

$$S = T \sum_n g(\omega_n = 2\pi n T, \pi(2n+1)T)$$

$$= \pm \int_{C_1+C_2} \frac{dz}{2\pi} \frac{g(z)}{e^{i\beta z} \mp 1} = \mp i \sum_{\omega_0} \frac{\text{Res } g(\omega_0)}{e^{i\beta \omega_0} \mp 1}$$

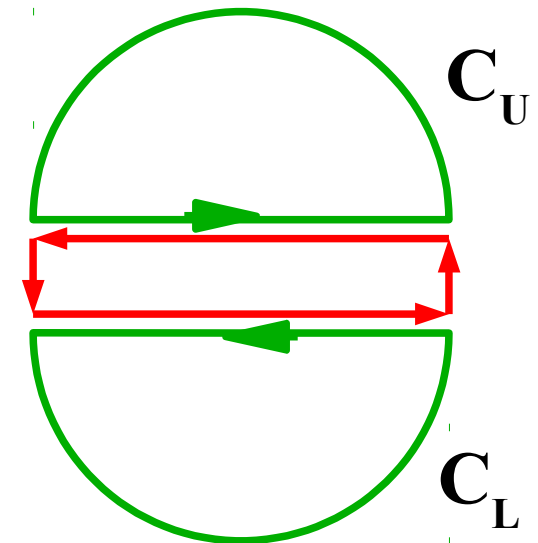
(g : meromorphic (有理型),
no pole on real axis,
decreases faster than $1/\omega$ at $\omega \rightarrow \infty$)



- Applicable to more general cases !
- Anti-periodic condition $\rightarrow$ Fermi-Dirac dist.

■ Example: $g(\omega) = 1/(\omega^2 + E^2)$
 $\rightarrow \omega_0 = \pm iE, \text{Res } g = \pm 1/2iE$

$$S = \frac{1}{2E} \frac{e^{\beta E} \pm 1}{e^{\beta E} \mp 1}$$



$$C_1 + C_2 + C_U + C_L = 0$$

■ Lagrangian

$$L = \bar{N} (i \gamma^\mu \partial_\mu - m) N$$

■ Euclidean

$$(x_\mu)_E = (\tau = it, \mathbf{x}), \quad (\gamma_\mu)_E = (\gamma_4 = i \gamma^0, \boldsymbol{\gamma})$$

$$L_E = \bar{N} (-i \gamma_\mu \partial_\mu + m) N$$

■ Grassman number

経路積分において、フェルミオンは反可換な Grassmann 数

$$\int d\chi \cdot 1 = \text{anti-comm. constant} = 0, \quad \int d\chi \cdot \chi = \text{comm. constant} \equiv 1$$

$$\begin{aligned} \int d\chi d\bar{\chi} \exp[\bar{\chi} A \chi] &= \int d\chi d\bar{\chi} \frac{1}{N!} (\bar{\chi} A \chi)^N = \cdots = \det A \\ &= \exp[-(-\log \det A)] \end{aligned}$$

Bi-linear Fermion action leads to $-\log(\det A)$ effective action

■ Example: Relativistic Mean Field (RMF)

$$L = \bar{\psi} (i \gamma^\mu \partial_\mu - m - \Sigma) \psi + L_{\text{meson}}(\Phi) \quad (\Phi = \sigma, \omega, \rho)$$

$$\Sigma = g_\sigma \sigma + \gamma^0 (g_\omega \omega^0 + g_\rho \rho^0 \tau)$$

■ Euclid 化 + 化学ポテンシャルの導入

$$\begin{aligned} Z &= \int D\psi D\bar{\psi} D\Phi \exp \left[- \int d^4x (L - \mu \psi^\dagger \psi) \right] \\ &= \int D\psi D\bar{\psi} D\Phi \exp \left[- \int d^4x \{ \bar{\psi} D\psi + L_{\text{meson}}(\Phi) \} \right] \\ &= \int D\Phi \exp \left[- S_{\text{eff}}(\Phi; T, \mu) \right] \\ D &= -i \gamma \partial - \mu \gamma^0 + m + \Sigma \end{aligned}$$

■ 有効作用

$$S_{\text{eff}} = S_{\text{eff}}^{(F)} + S_{\text{meson}} = - \sum_{n, k} \log \det D_{n, k} + \int d^4x L_{\text{meson}}$$

- 一様な場を仮定 → Fourier 変換により D をブロック対角化

$$D_{n,k} = \gamma^0 (-i\omega_n - (\mu - V^0)) + \boldsymbol{\gamma} \cdot \mathbf{k} + M + g_\sigma \sigma$$

$$\rightarrow \det D = \left[(\omega_n + i\mu^*)^2 + E^{*2} \right]^2$$

$$\mu^* = \mu - g_\omega \omega^0 - g_\rho \rho^0 \boldsymbol{\tau}, \quad E^* = \sqrt{\mathbf{k}^2 + M^{*2}}, \quad M^* = m + g_\sigma \sigma$$

- 松原振動数和を実行

$$F_{\text{eff}}^{(F)} = -\frac{d_f}{2} \int \frac{d^3 k}{(2\pi)^3} \left[E^* + T \log \left(1 + e^{-(E^* - \mu^*)/T} \right) + T \log \left(1 + e^{-(E^* + \mu^*)/T} \right) \right]$$

- 温度 0 の場合 ゼロ点 粒子 (核子) 反粒子 (反核子)

$$F_{\text{eff}}^{(F)} = -\frac{d_f}{2} \int^\Lambda \frac{d^3 k}{(2\pi)^3} E^* + d_N \int^{k_F} \frac{d^3 k}{(2\pi)^3} E^* - \mu^* \rho_B \quad (d_N = d_f/2)$$

ゼロ点エネルギーは核子のループから現れる
(RMF では通常無視)