核多体系物理学

担当：大西　明、八田佳孝（基礎物理学研究所）

- **授業の概要・目的：**核子・ハドロン・クォークからなる多体系の性質を量子色力学（QCD）、状態方程式、および核反応論の観点から議論する。強い相互作用の基本理論であるQCDの基本的性質、核物質の状態方程式を記述するために必要となる核多体理論（平均場理論、G-matrix、熱場の理論、強結合格子QCD）、ハイパー核生成反応や重イオン反応を理解する上で必要とされる原子核核反応理論（直接反応、輸送模型等）、等の理論の枠組について解説すると共に、これらについての最近の研究成果についても紹介する。
- **授業計画と内容：**量子色力学、および核子・ハドロン・クォーク物質の相互作用と状態方程式について以下の内容で講義する。

1. 量子色力学(QCD)の基本的性質
   QCD作用と対称性、摂動論的QCD、発展方程式、カラーグラス描像

八田

2. 状態方程式とQCD相図を記述する理論模型
   - 核力と位相差、有効相互作用、核物質の状態方程式、平均場理論、
   - 有限温度での場の理論、南部-ヨナラシニョ模型、強結合格子QCD
3. 原子核反応理論
   - 核子-核子散乱、ハドロン-原子核反応、
   - 流体力学、輸送理論、
   - ハイパー核・中間子核生成反応の概観と直接反応

大西

- **成績評価の方法・基準：** 履修状況及びレポートにより総合評価する。

参考書 Quark Gluon Plasma, K.Yagi, T.Hatsuda, Y.Miake (CAMBRIDGE).
格子上の場の理論、青木慎也（シュプリンガー・ジャパン）
クォーク・ハドロン物理学入門，国広悌二（サイエンス社）

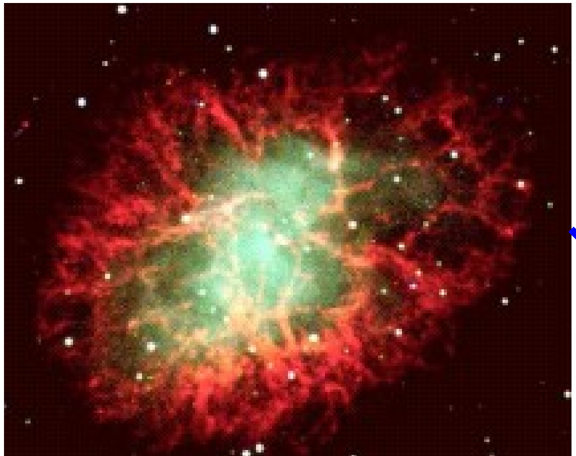
状態方程式やQCD 相図を記述する理論模型

- 2.1 核物質の状態方程式概観 (10/20)
  - なぜ状態方程式か？中性子星パズル、対称エネルギー
  - 状態方程式を記述する理論模型
- 2.2 核力と位相差、有効相互作用 (10/27)
- 2.3 平均場理論：相対論的平均場 (RMF) 模型を中心に。(11/10)
- 2.4 カイラル相転移と南部 - ヨナラシニョ (NJL) 模型 (11/17)
  - 有限温度・密度の場の理論入門
  - 経路積分表示、ユークリッド時空、松原和、自由場の分配関数
  - カイラル対称性、Nambu-Jona-Lasinio (NJL) 模型、カイラル相転移
- 2.5 格子上の場の理論入門 (→ 青木さんの講義)
  - 格子 QCD 、Plaquette 作用、格子 Fermion 、リンク積分、
  - 強結合格子 QCD 、Area Law 、強結合展開、ポリアコフ・ループ
- 2.6 高密度物質の QCD 有効模型 (12/1)
  - Bag 模型、Quark-Meson 模型、Polyakov loop extended Quark-Meson (PQM) 模型、Polyakov loop extended NJL (PNJL) 模型

Why do we study Nuclear Matter EOS ?

Why do we study Nuclear Matter EOS ?

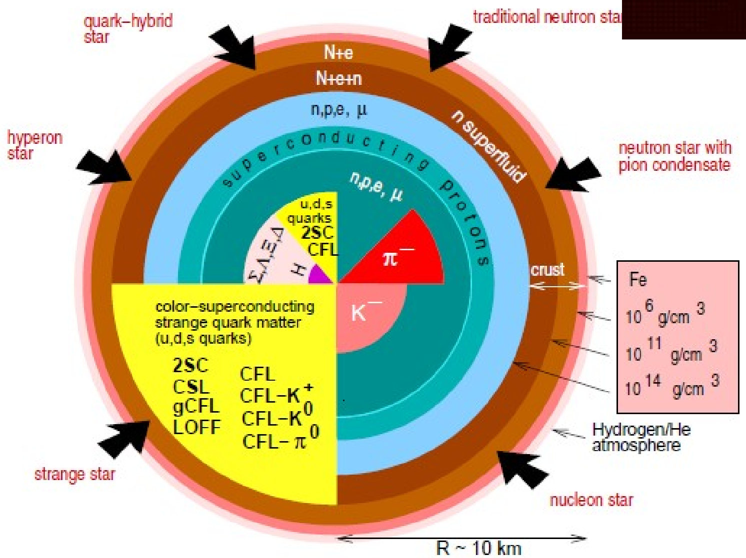
- **Answer 2: Since nuclear matter EOS is decisive in compact astrophysical objects such as neutron stars, supernovae, and black hole formation, EOS is important to understand where atomic elements are made.**

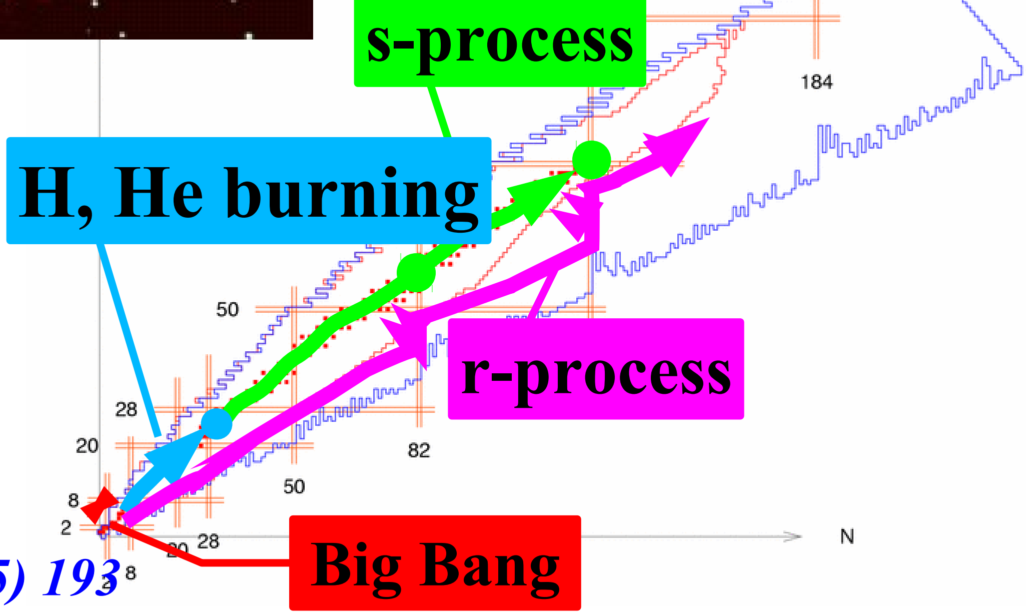


Supernova

Nucleosynthesis

Neutron Star





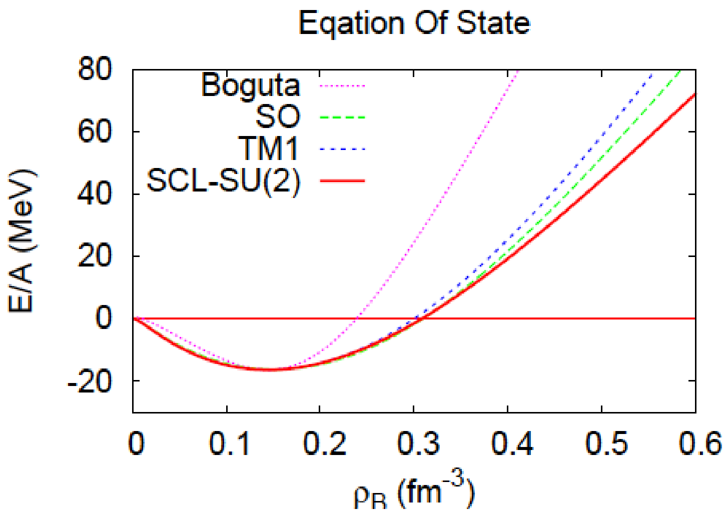
F. Weber, Prog. Part. Nucl. Phys. 54 (2005) 193

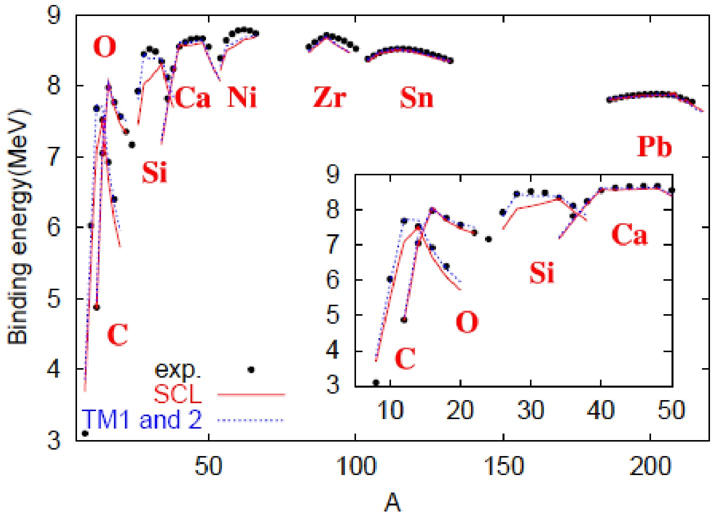
Why do we study Nuclear Matter EOS ?

- **Answer 1: Since bulk nuclear properties are mainly determined by nuclear matter EOS, it is important for nuclear physics.**
  - **Nuclear Radius → Saturation of Density**
    $R_A = r_0 A^{1/3}$ (r_0=1.2 fm)
  - **Nuclear Binding Energy (Bethe-Weizsacker Formula)**

$$B(A,Z) = a_{vol} A - a_{surf} A^{2/3} - a_{Coulomb} \frac{Z^2}{A^{1/3}} - a_{sym} \frac{(N-Z)^2}{A} + a_{pair} \delta(A,Z) A^{-3/4}$$

Nuclear Matter





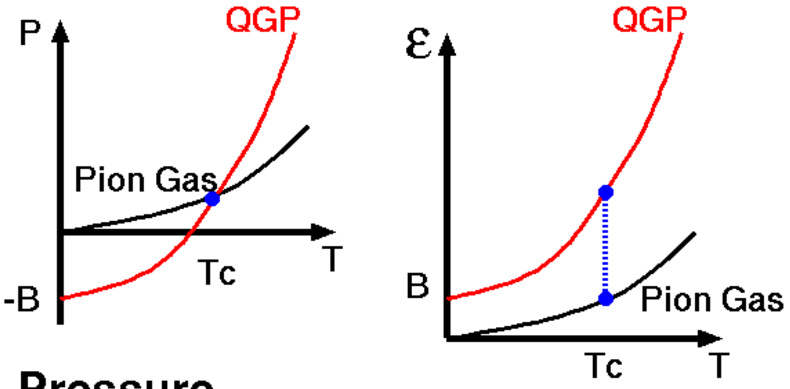
Why do we study Nuclear Matter EOS ?

- **Answer 3: Since the EOS should have singularity (or at least sudden change) at phase boundary, it would be possible to catch the signal of phase transition in nuclear collisions.**

- **Pressure and Energy Density of Free Massless Gas**

$$P=\frac{\pi^2}{90}N_B T^4 \quad , \quad \epsilon=\frac{\pi^2}{30}N_B T^4$$

N_B = Bosonic DOF (7/8 for Fermions)

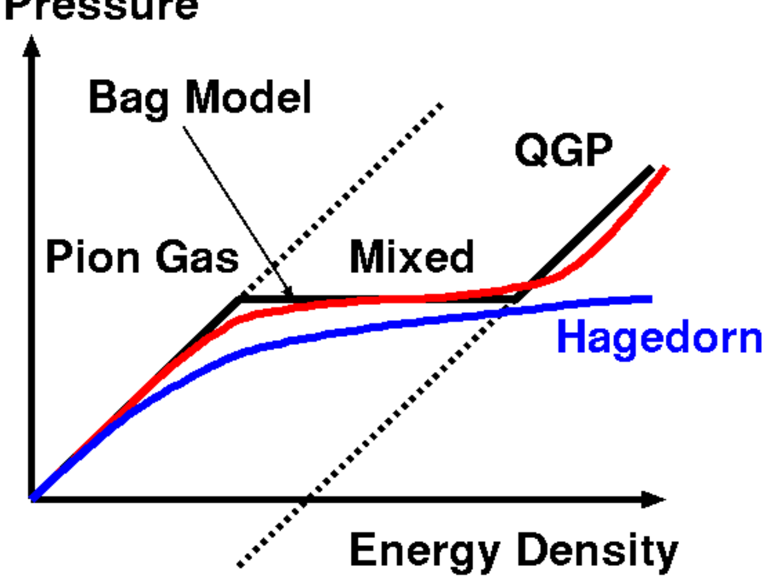


- **Hadron Gas ~ 3 pions (N_B=3)**

$$P_\pi=\frac{\pi^2}{30}T^4 \quad , \quad \epsilon_\pi=\frac{\pi^2}{10}T^4$$

- **QGP N_B=16(gluon)+24 x 7/8 (quarks) and Bag Pressure**

$$P_{QGP}=\frac{37\pi^2}{90}T^4-B \quad \epsilon_{QGP}=\frac{37\pi^2}{30}T^4+B$$



Nuclear Matter EOS

- **Energy per nucleon in nuclear matter**

$$E(\rho,\delta)=E_{\mathrm{SNM}}(\rho)+E_{\mathrm{Sym}}(\rho)\delta^2 \quad , \quad \delta=(N-Z)/A$$

- **Saturation point (ρ_0, E_0)**

  $\rho_0 \sim 0.15\ \mathrm{fm}^{-3}$

  $E_0 = -a_v \sim -16$ MeV

  (nuclear radius and mass)

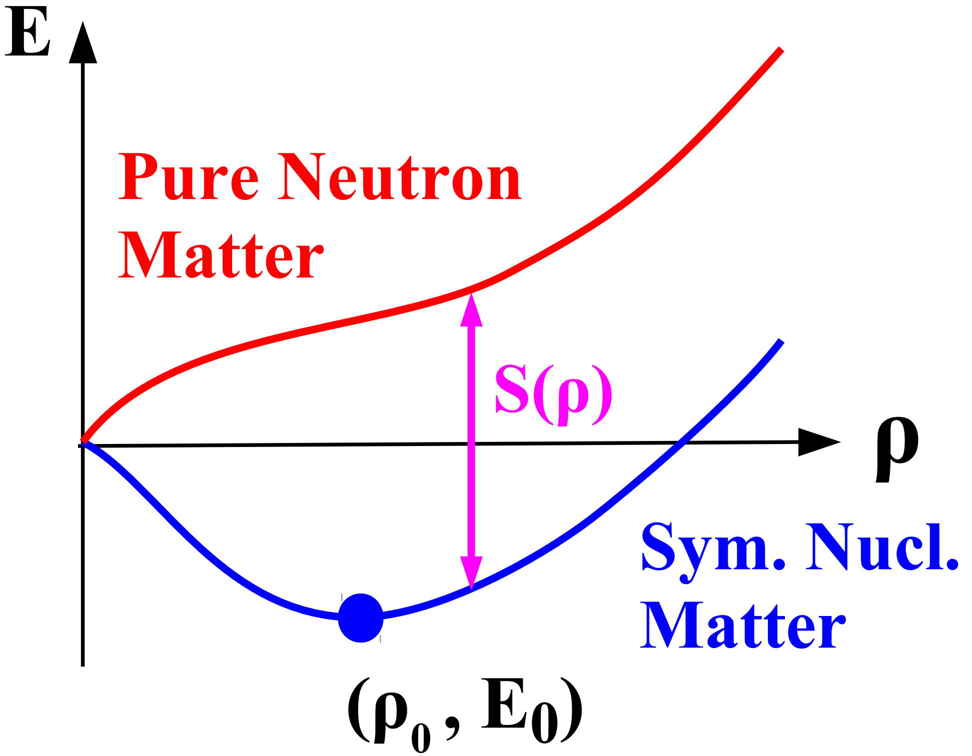
- **Symmetry energy**

  $S(\rho) = E_{\mathrm{PNM}}(\rho) - E_{\mathrm{SNM}}(\rho)$

  $= E(\rho, \delta=1) - E(\rho, \delta=0)$

  $S_0 = S(\rho_0) \sim 30$ MeV

  (mass formula)



Nuclear Matter EOS can be, in principle, determined by terrestrial (laboratory) nuclear physics experiments !

Nuclear Matter EOS

- **Additional two important parameters: K and L**
- **Pressure is given by the derivative of E via ρ**

$$P=\rho^2(\partial E/\partial\rho)$$

At ρ_0, L determines P

$$P=\rho_0 L/3\,(\mathrm{at}\quad \rho=\rho_0)$$

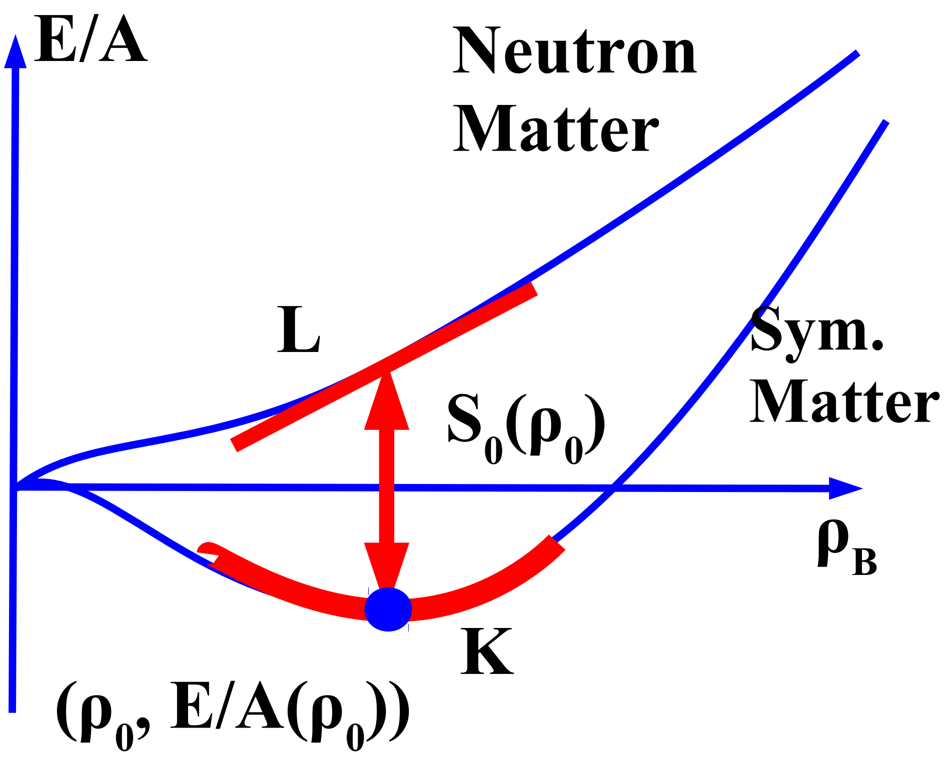
$$E/A(\rho,\delta)=\varepsilon(\rho)+E_{\mathrm{sym}}(\rho)\delta^2+O(\delta^4)$$

Symmetric Nuclear Matter

$$\varepsilon(\rho)=\varepsilon(\rho_0)+\frac{K(\rho-\rho_0)^2}{18\rho_0^2}+O((\rho-\rho_0)^3)$$

Symmetry Energy $(\delta=(N-Z)/A=1-2Y_p)$

$$E_{\mathrm{sym}}(\rho)=S_0+\frac{L(\rho-\rho_0)}{3\rho_0}+\frac{K_{\mathrm{sym}}(\rho-\rho_0)^2}{18\rho_0^2}$$
$$+O((\rho-\rho_0)^3)$$



Neutron Star Matter EOS

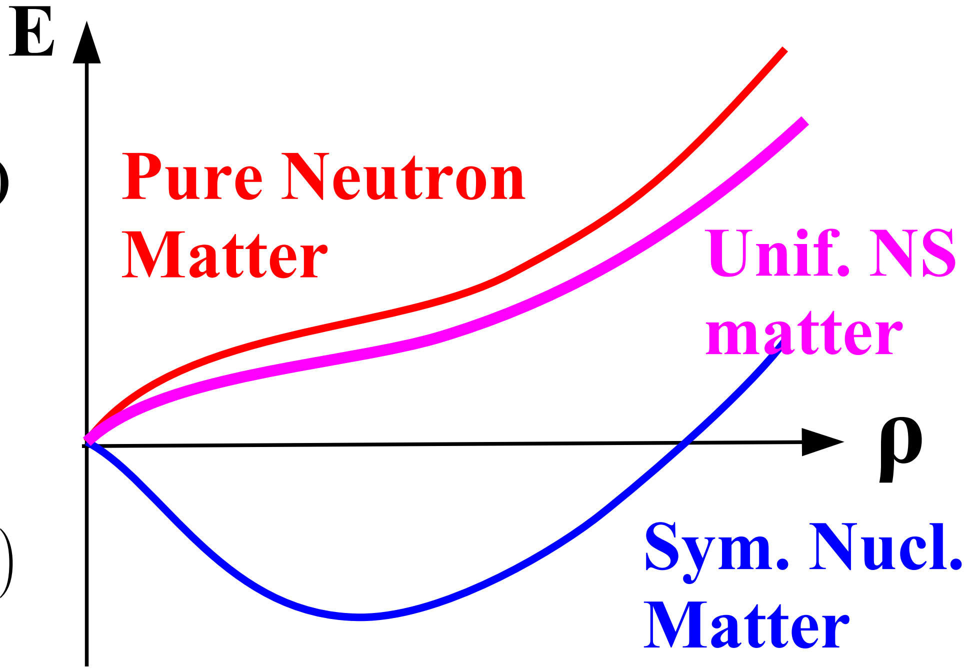
- **What happens in low-density uniform neutron star matter ?**
  - **Constituents = proton, neutron and electron**
  - **Charge neutrality → # of electons= # of protons ($\rho_e=\rho_p=\rho(1-\delta)/2$)**

$$E_{\mathrm{NSM}}(\rho)=E_{\mathrm{NM}}(\rho,\delta)+E_e(\rho_e=\rho_p)$$
$$=E_{\mathrm{SNM}}(\rho)+S(\rho)\delta^2+\frac{\Delta M}{2}\delta+\frac{3}{8}\hbar k_F(1-\delta)^{4/3}$$

**(electron mass neglected,
neutron-proton mass diff. incl.
k_F= Fermi wave num. in Sym. N.M.)**

  - **δ is optimized to minimize energy per nucleon**

$$E_{\mathrm{NSM}}(\rho)\leq E_{\mathrm{NM}}(\rho,\delta=1)=E_{\mathrm{PNM}}(\rho)$$



対称エネルギーの起源

- **Fermi Gas model での核子あたりの運動エネルギー**

$$E_{\mathrm{sym},K} = \frac{Z}{A}\frac{3}{5}\frac{\hbar^2 k_{\mathrm{F}p}^2}{2m} + \frac{N}{A}\frac{3}{5}\frac{\hbar^2 k_{\mathrm{F}n}^2}{2m} = \frac{3}{5}E_{\mathrm{F}}\frac{1}{2}\left[(1-\delta)^{5/3} + (1+\delta)^{5/3}\right]$$

$$\simeq \frac{3}{5}E_{\mathrm{F}} + \frac{1}{3}E_{\mathrm{F}}\delta^2 + \mathcal{O}(\delta^4)$$

a_{sym} (FG)=E_F/3 ~ 11 MeV となり、質量公式の a_{sym} ~ 23 MeV (surface を考えると a_{sym} (vol) ~ 30 MeV) と比べて半分程度。残りは相互作用。

- **残りの半分の対称エネルギーを RMF で評価してみましょう。**

$$\Delta E_{\mathrm{sym},\rho} = \frac{1}{2}\frac{m_\rho^2 R^2}{\rho_B} = \frac{1}{2}\frac{g_\rho^2}{m_\rho^2}\rho_B\delta^2 = \Delta a_{\mathrm{sym}}\delta^2 \quad \left(R = \frac{g_\rho(\rho_n - \rho_p)}{m_\rho^2} = \frac{g_\rho \rho_B \delta}{m_\rho^2} \right)$$

$$g_\rho^2 = \frac{2 m_\rho^2 \Delta a_{\mathrm{sym}}}{\rho_B} \simeq (4.3)^2 \quad (a_{\mathrm{sym}} = 30\,\mathrm{MeV})$$

← RMF par. より少し小さめ

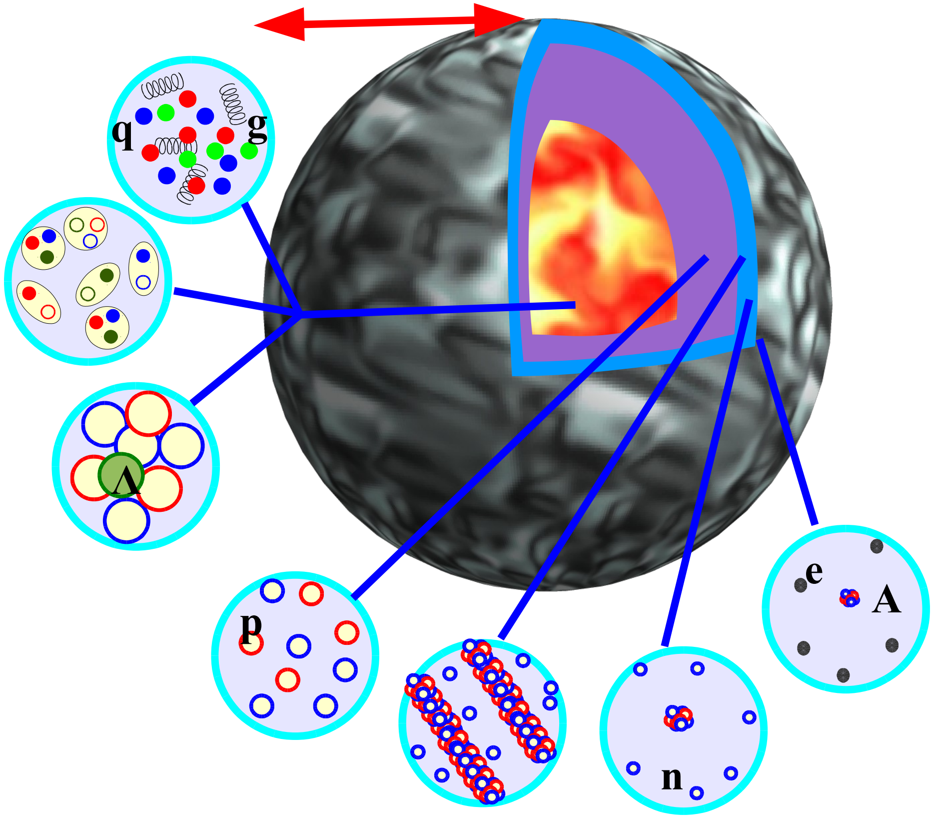
$$L \simeq E_F + 3\Delta a_{\mathrm{sym}} \simeq 90\,\mathrm{MeV}$$

← Optimal value より少し大きい

Neutron Star

Star supported by nuclear force

R ~ 10 km



$M \sim 1.4\, M_\odot$, $\rho_c \sim (3\text{-}10)\, \rho_0$

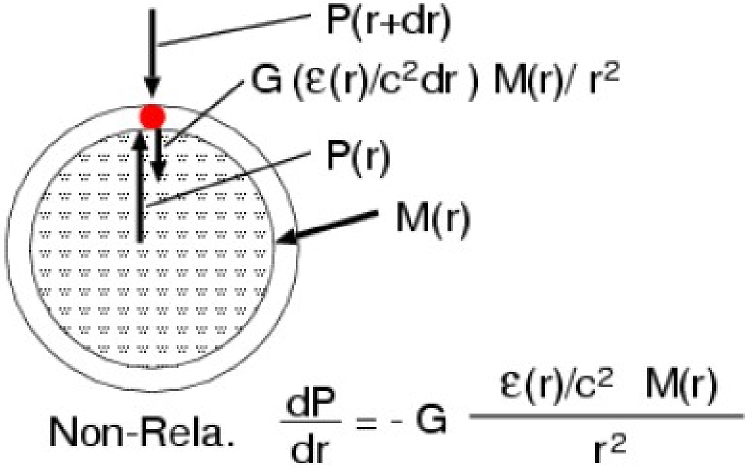
Wide density range → various constituents
NS = high-energy astrophysical objects
and laboratories of dense matter.

M-R curve and EOS

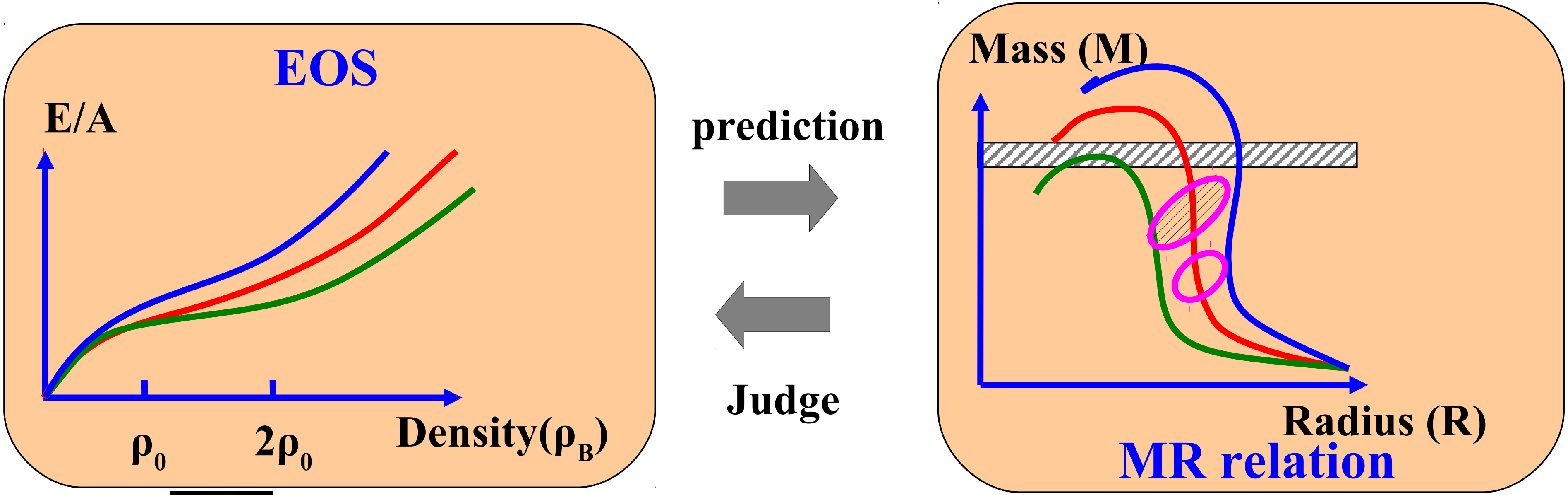
- **M-R curve and NS matter EOS has 1 to 1 correspondence**
  - **TOV(Tolman-Oppenheimer-Volkoff) equation =GR Hydrostatic Eq.**

$$\frac{dP}{dr}=-G\frac{(\varepsilon/c^2+P/c^2)(M+4\pi r^3 P/c^2)}{r^2(1-2GM/rc^2)}$$

$$\frac{dM}{dr}=4\pi r^2\varepsilon/c^2,\quad P=P(\varepsilon)\quad(\mathrm{EOS})$$

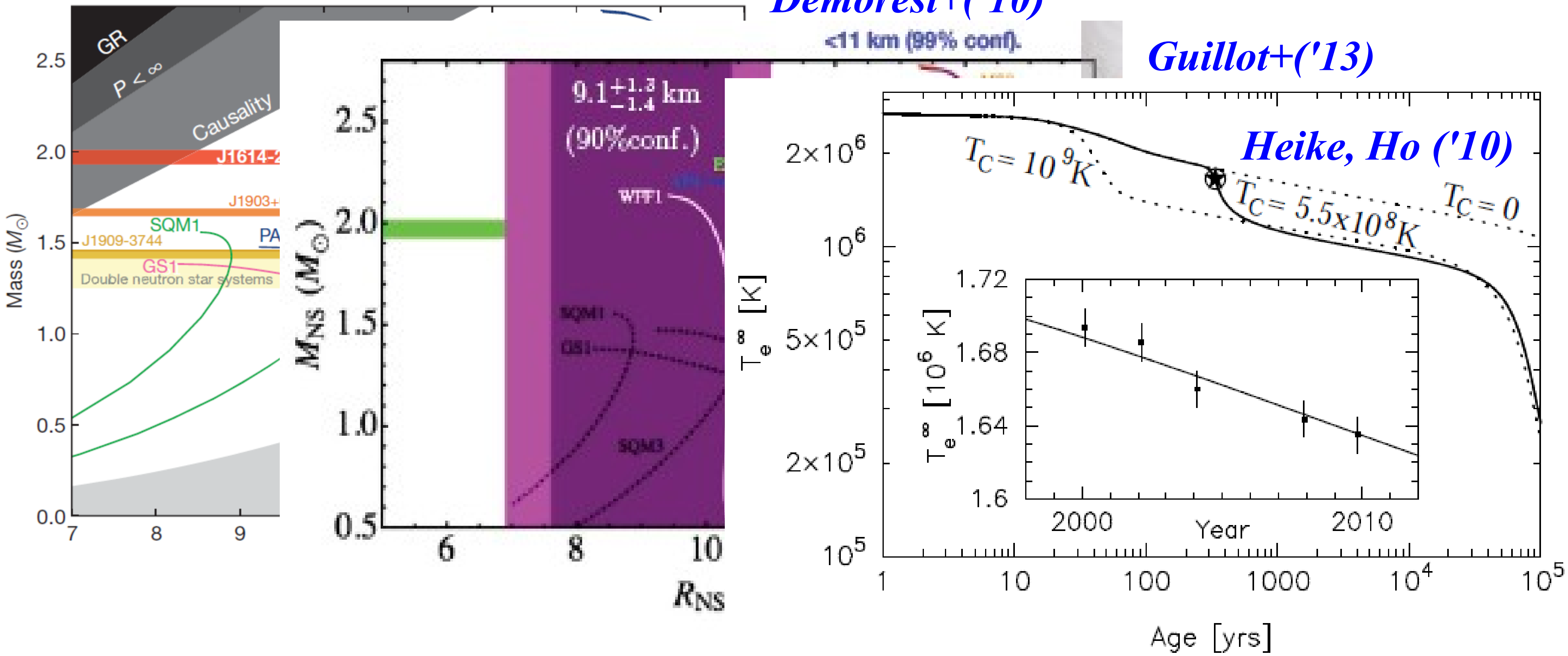


Non-Rela. $\frac{dP}{dr}=-G\frac{\varepsilon(r)/c^2\ \ M(r)}{r^2}$



Current Big Puzzles in NS Physics

- **Massive NS puzzle (2 $M_\odot$ NS ?)**
- **Compact NS puzzle (9-10 km NS ?)**
- **Rapid NS cooling mystery (CasA cools too fast ?), Origin of Strong Mag. Field, …..**



Demorest+('10)

Guillot+('13)

Heike, Ho ('10)

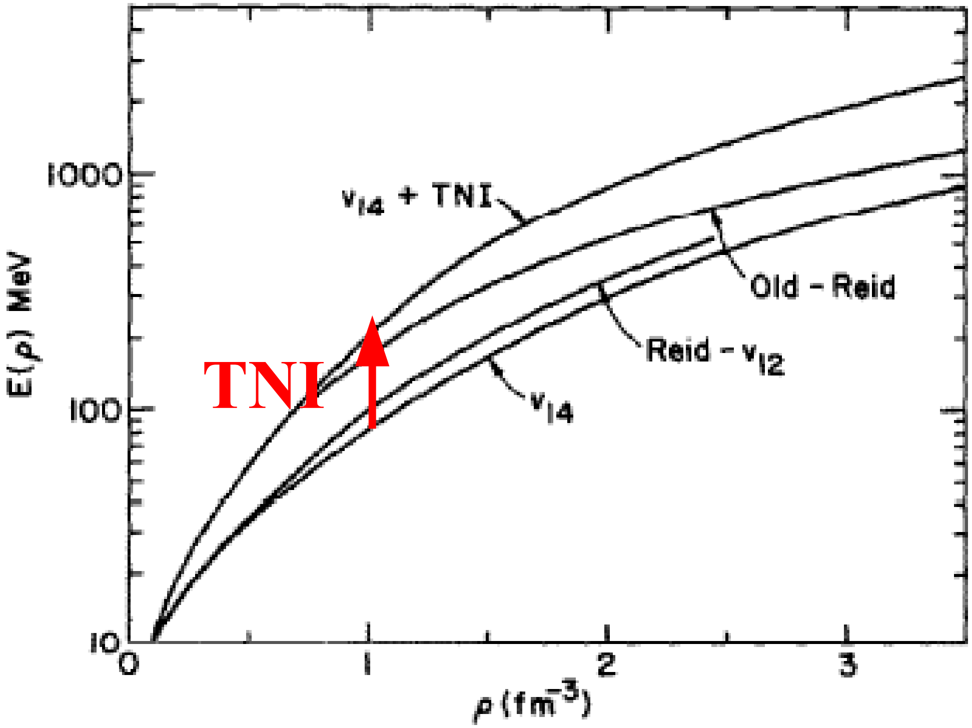
Nuclear Matter EOS Theories

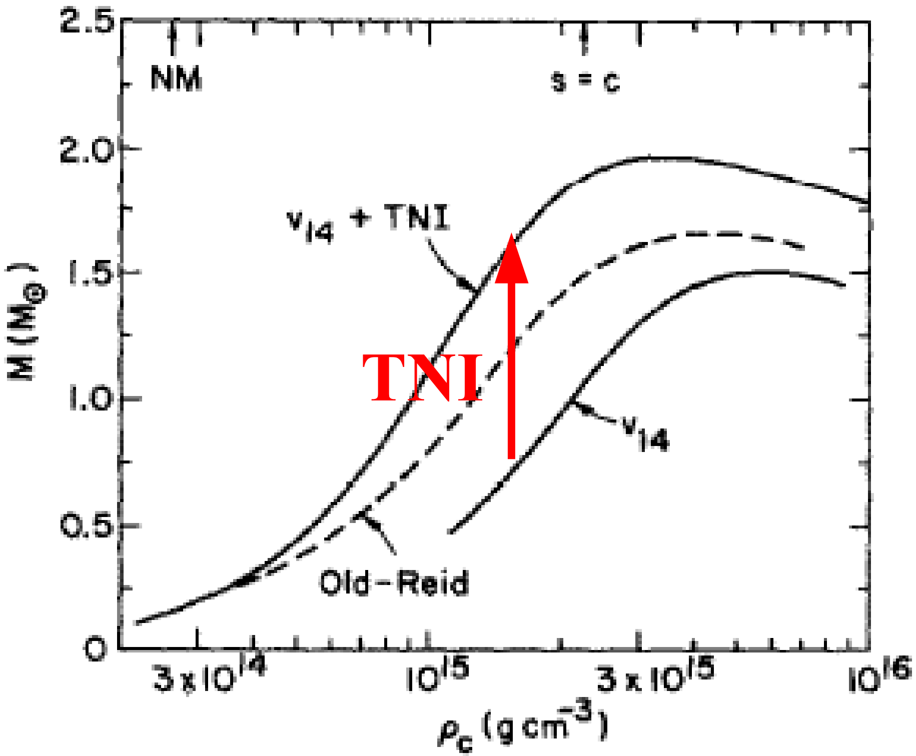
Theories/Models for Nuclear Matter EOS

- **Ab initio Approaches**
  - **LQCD, GFMC, Variational, BHF, DBHF, G-matrix, ...**
- **Mean Field from Effective Interactions ~ Nuclear Density Fuctionals**
  - **Skyrme Hartree-Fock(-Bogoliubov)**
    - **Non.-Rel.,Zero Range, Two-body + Three-body (or ρ-dep. two-body)**
    - **In HFB, Nuclear Mass is very well explained (Total B.E. $\Delta E \sim 0.6$ MeV)**
    - **Causality is violated at very high densities.**
  - **Relativistic Mean Field**
    - **Relativistic, Meson-Baryon coupling, Meson self-energies**
    - **Successful in describing pA scatering (Dirac Phenomenology)**

Variational Calculations (1)

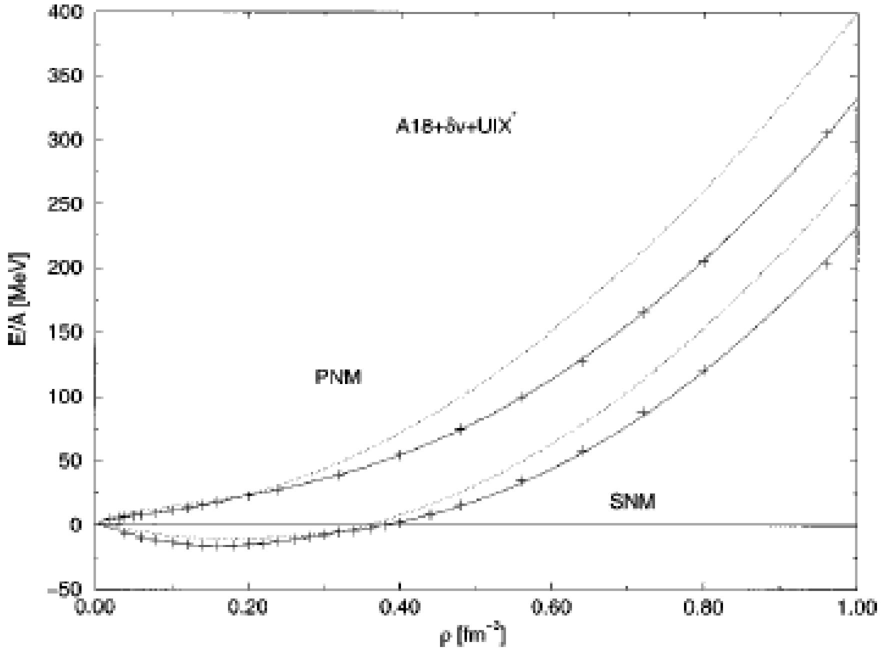
- **Variational Calculation starting from bare nuclear force**
  B. Friedman, V.R. Pandharipande, NPA361('81)502
  - **Argonne v14 + TNI (TNR+TNA)**
    (TNI/TNR/TNA: three-nucleon int./repulsion/attraction)

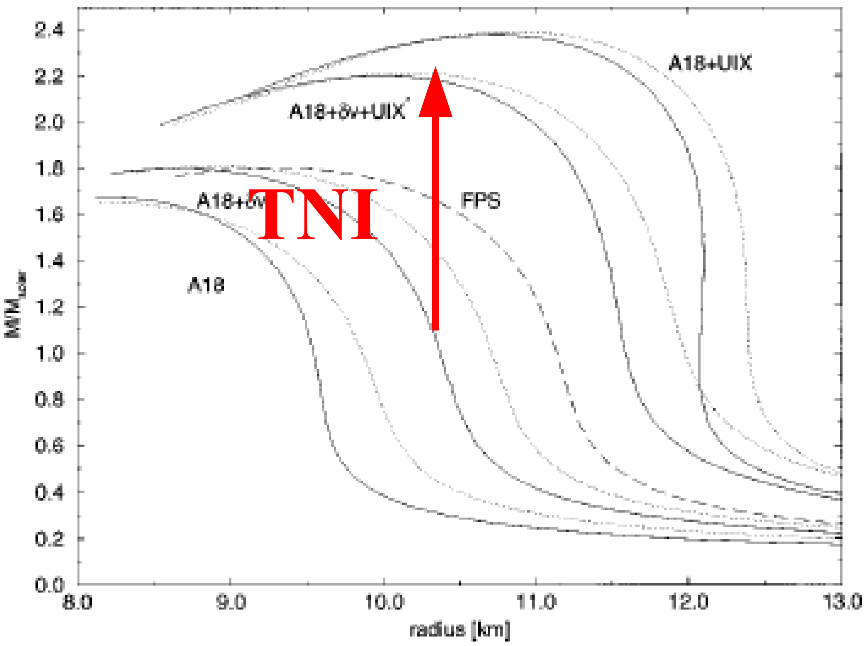




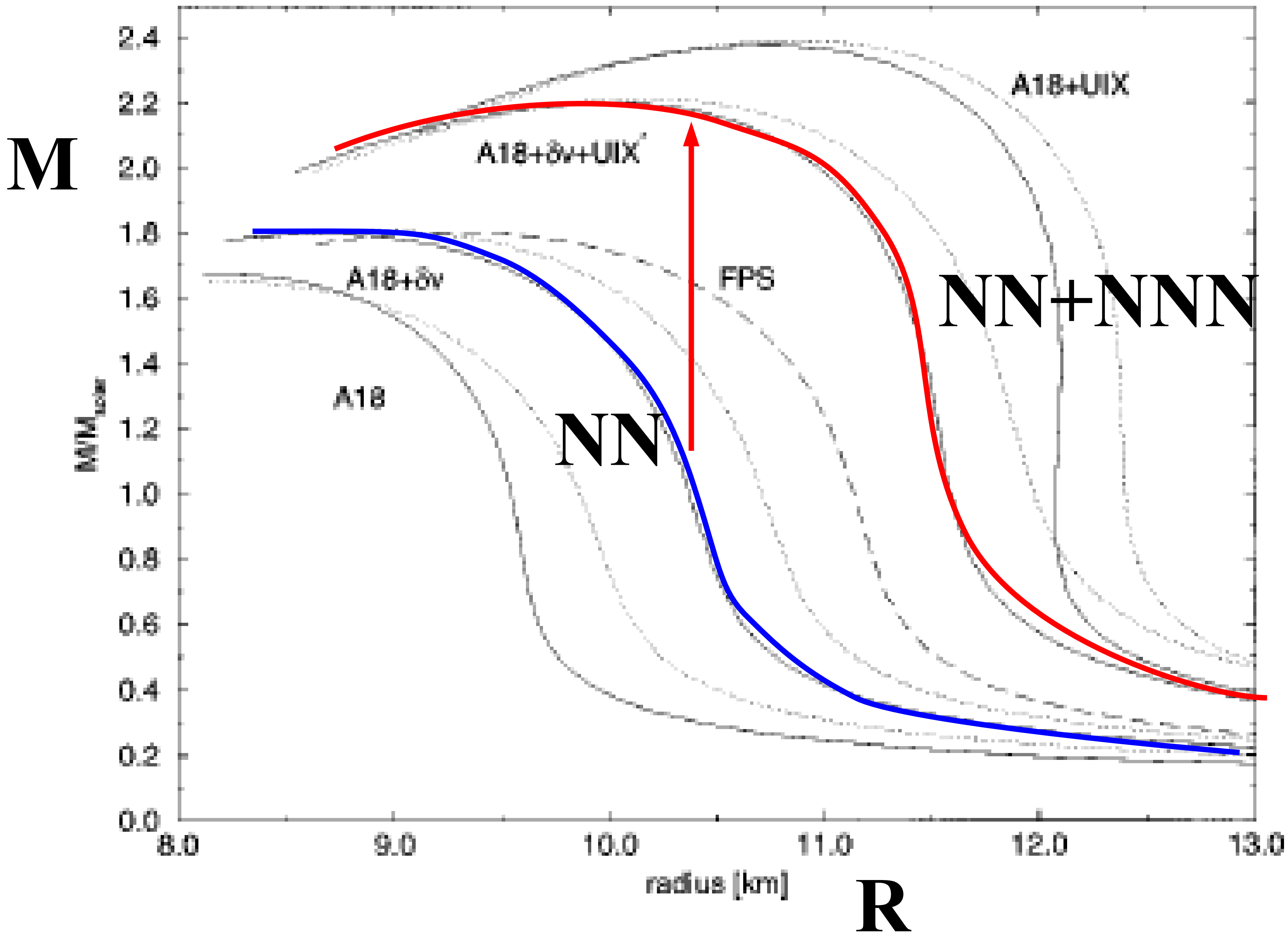
Variational Calculation (2)

- **Variational chain summation method**
  A. Akmal, V.R.Pandharipande, D.G. Ravenhall, PRC58('98)1804
  - **v18, relativistic correction, TNI**
  - **Existence of neutral pion condensation at $\rho_B > 0.2\ \mathrm{fm}^{-3}$**



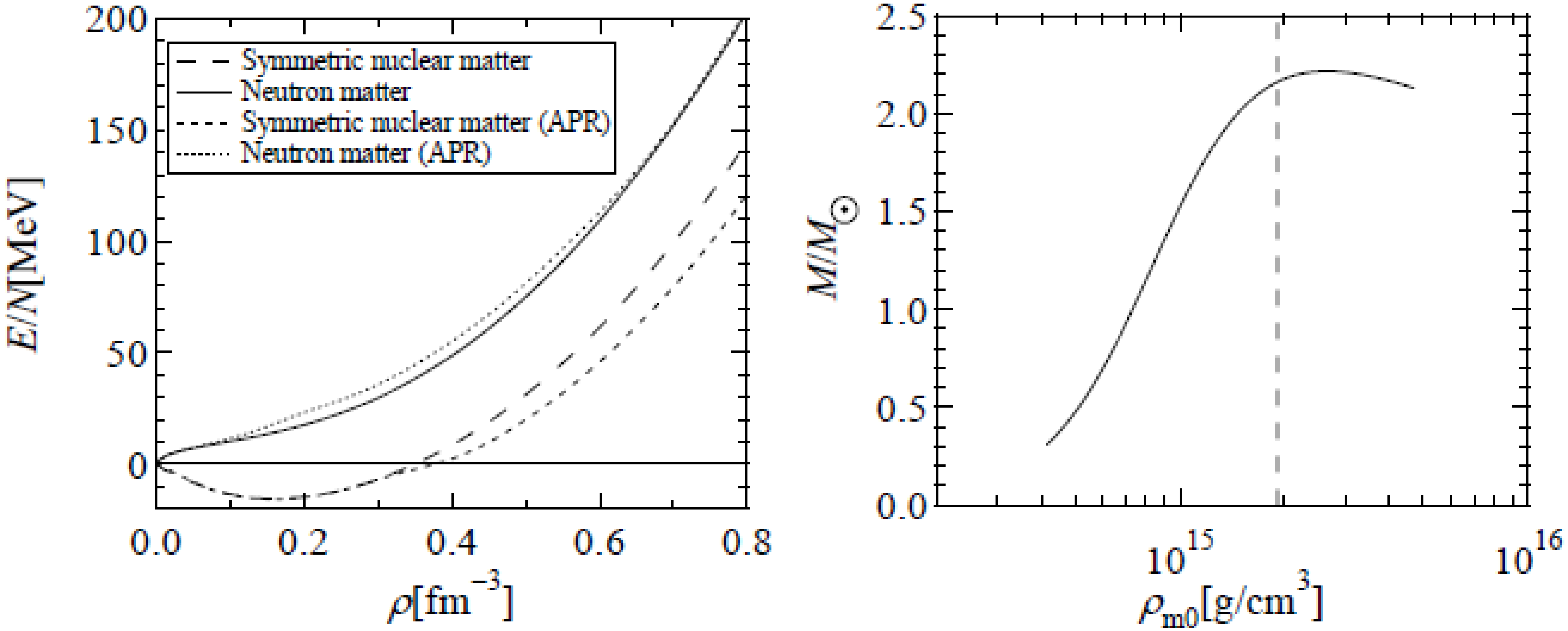


APR


M
R
NN
NN+NNN
A18+UIX
A18+δv+UIX*
A18+δv
FPS
A18
M/M_{solar}
radius [km]
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
8.0
9.0
10.0
11.0
12.0
13.0

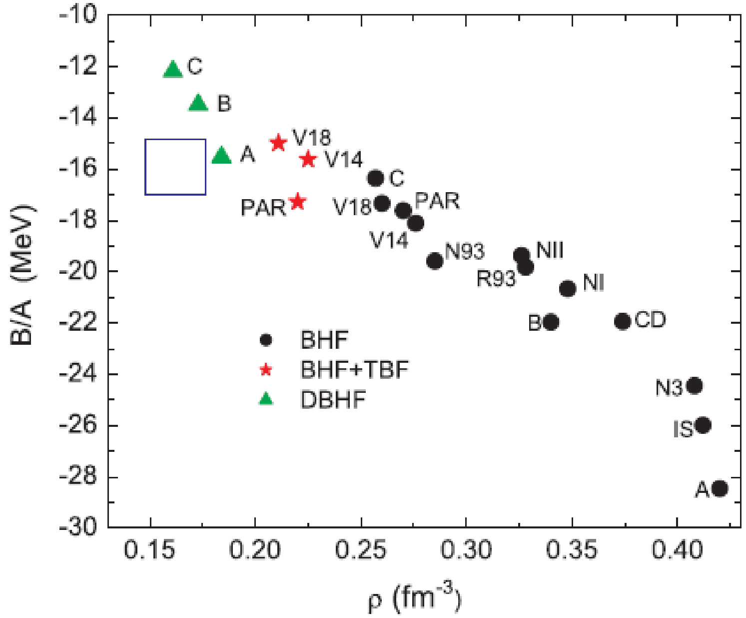
Variational Calculation (3)

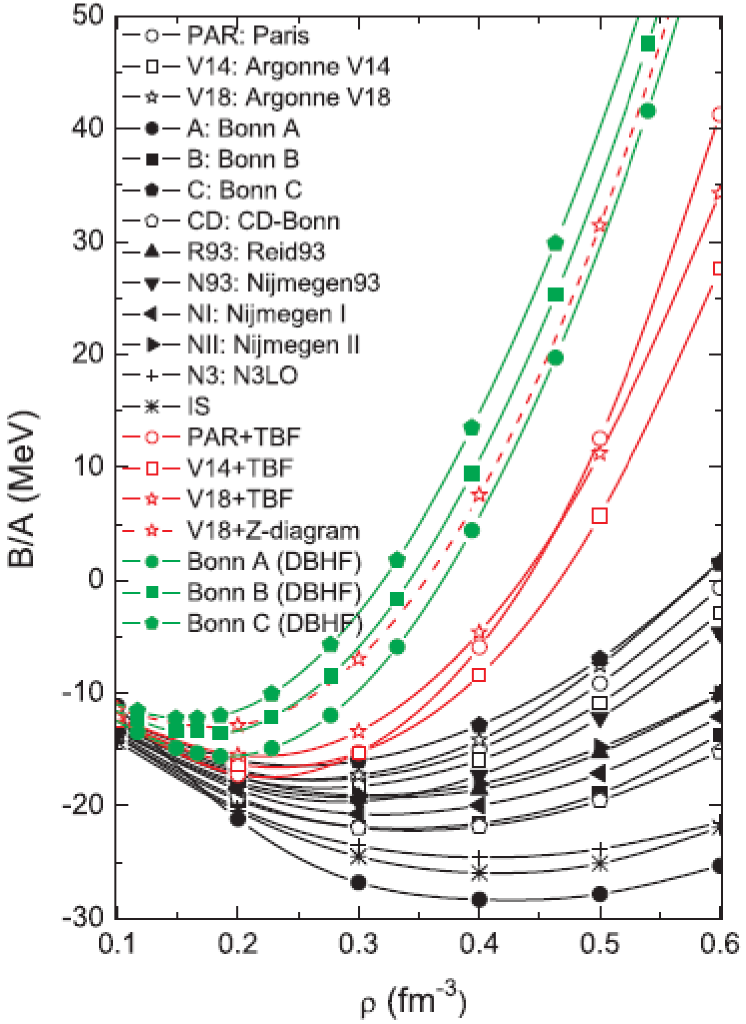
- **Variational Calculation using v18+UIX**
  H. Kanzawa, K. Oyamatsu, K. Sumiyoshi, M. Takano, NPA791 ('07) 232
  - **Similar to APR, but healing-distance condition is required.**
    → no π^0 condensation



Bruckner-Hartree-Fock

- **Self-consistent treatment of**
  Effective interaction (G-matrix) in the Bruckner Theory
  and Single particle energy from G-matrix
- **Need 3-body force to reproduce saturation point.**
  → FY type 2 π exchange
  + phen. or Z-diagram

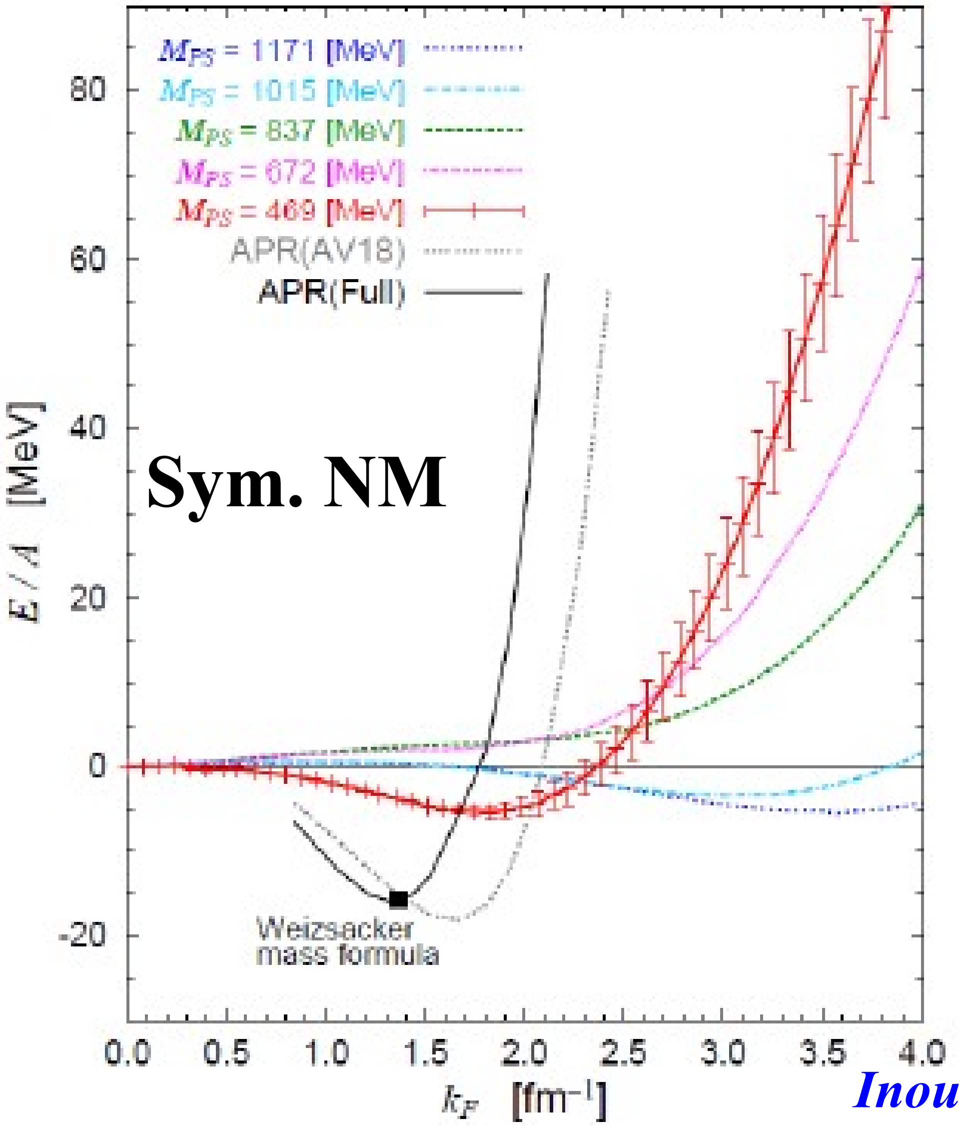


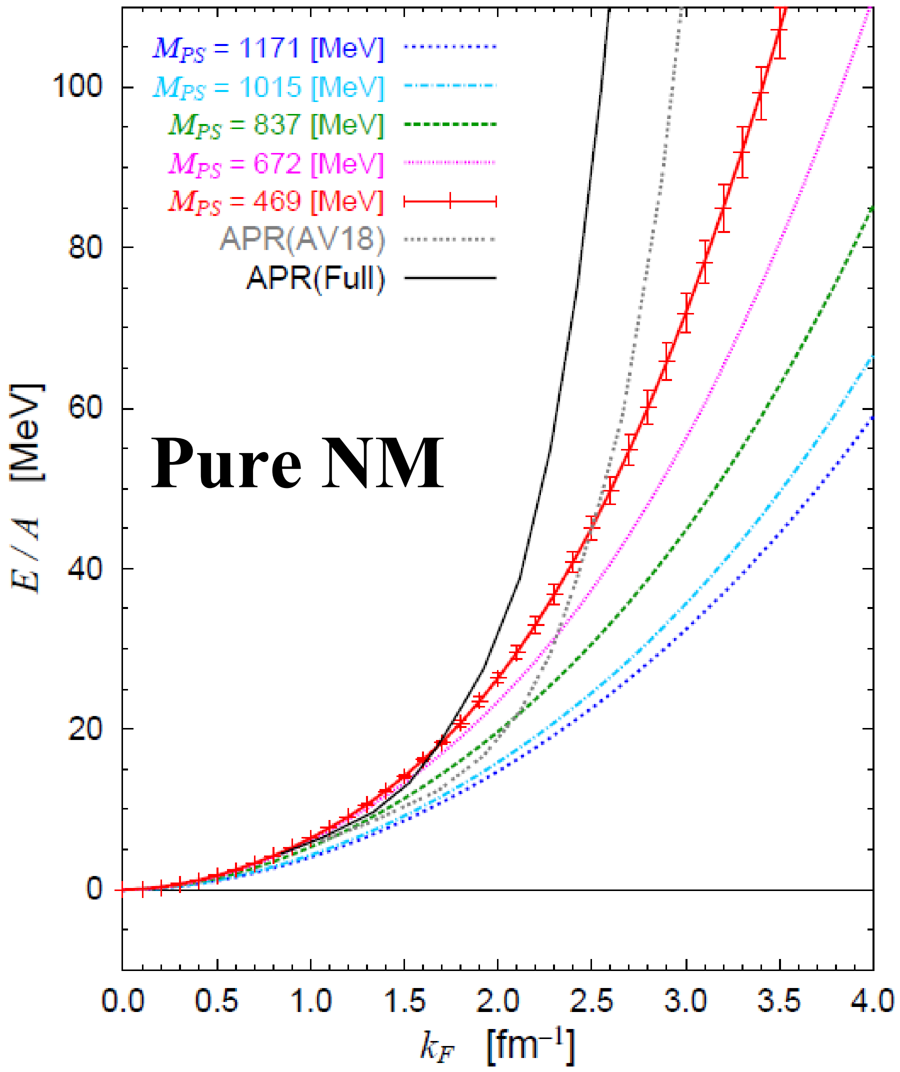


Z.H.Li, U. Lombardo, H.-J. Schulze, W. Zuo, L. W. Chen, H. R. Ma, PRC74('06)047304.

EOS from lattice NN force

- **格子 QCD 核力を用いた高密度状態方程式 (LQCD+BHF)**
  NN force: $^1S_0$, $^3S_1$, $^3D_1$ only





Inoue et al. (HAL QCD Coll.), PRL111 ('13)112503

A simple model

Simple parametrized EOS

- **Skyrme int. motivated parameterization**

$$E_{\mathrm{SNM}} = \frac{3}{5}E_F(\rho) + \frac{\alpha}{2}\left(\frac{\rho}{\rho_0}\right) + \frac{\beta}{2+\gamma}\left(\frac{\rho}{\rho_0}\right)^{1+\gamma}$$

$$\alpha = \frac{2}{\gamma}\left(E_0(1+\gamma) - \frac{E_F(\rho_0)(1+3\gamma)}{5}\right) , \quad \beta = \frac{2+\gamma}{\gamma}\left[-E_0 + \frac{1}{5}E_F(\rho_0)\right] .$$

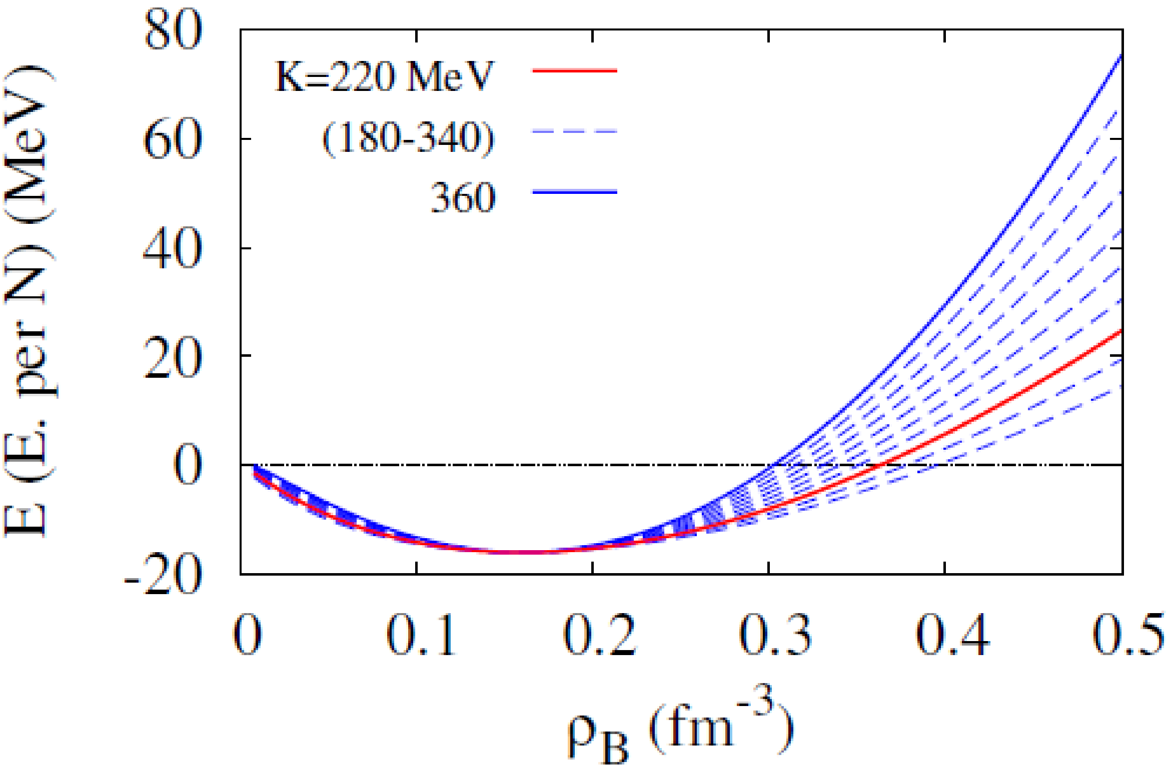
$$K = \frac{1+3\gamma}{5}E_F(\rho_0) - 3E_0(1+\gamma) .$$

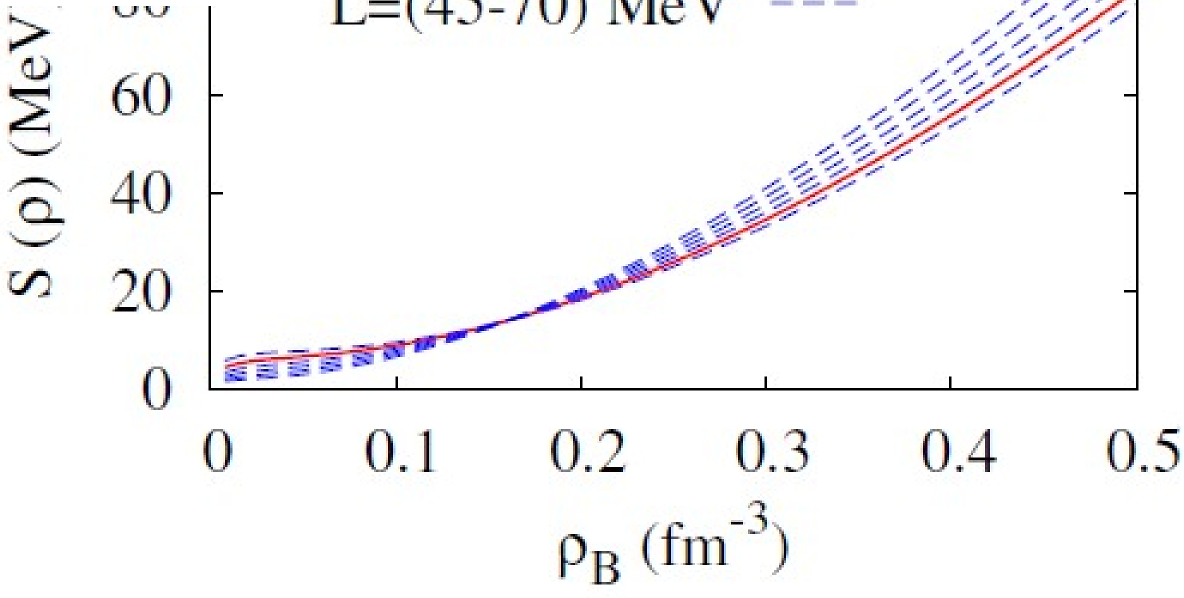
- **Symmetry energy parameterization**

$$S(\rho) = \frac{1}{3}E_F(\rho) + \left[S_0 - \frac{1}{3}E_F(\rho_0)\right]\left(\frac{\rho}{\rho_0}\right)^{\gamma_{\mathrm{sym}}}$$

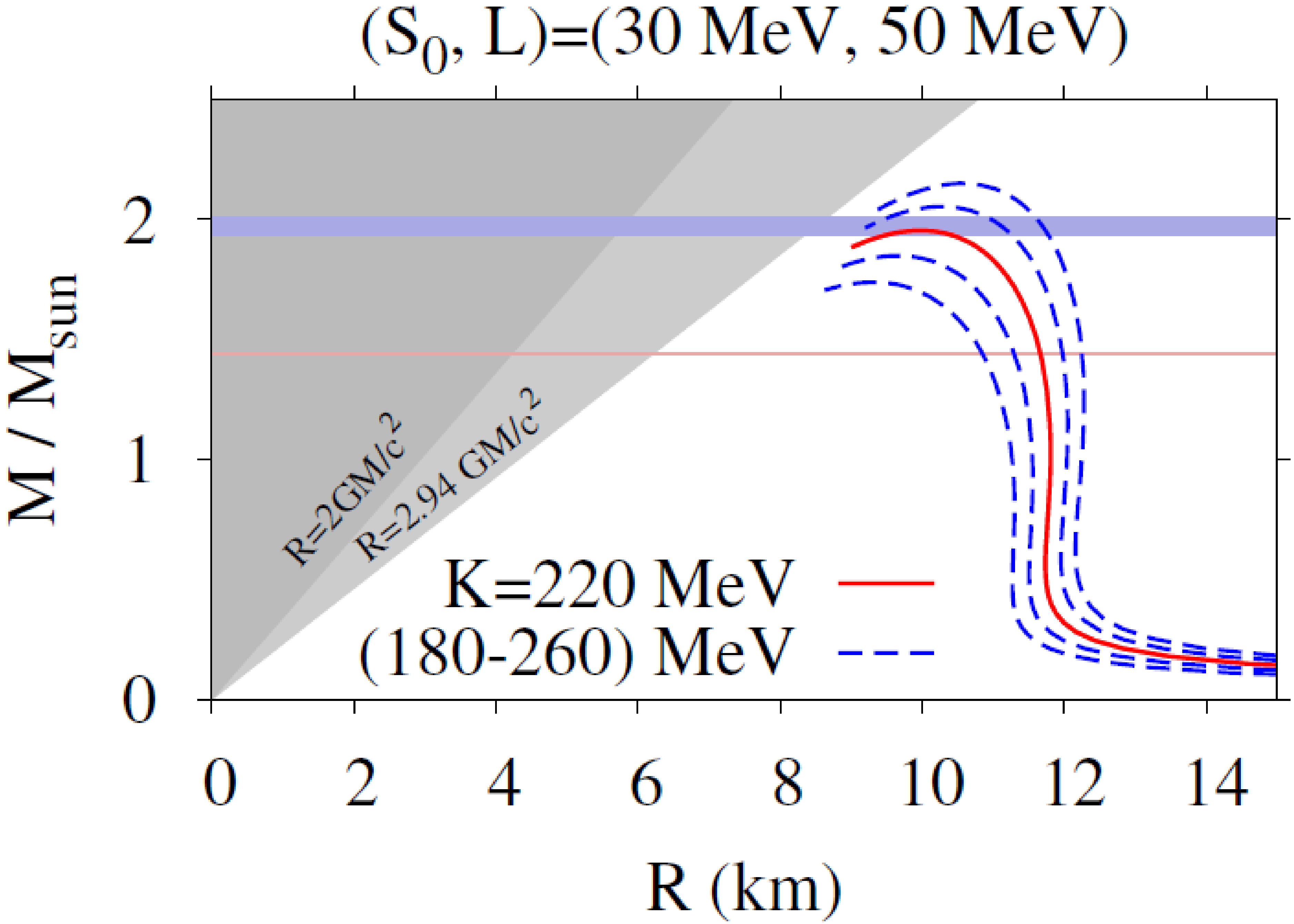
$$\gamma_{\mathrm{sym}} = \frac{L - \frac{2}{3}E_F(\rho_0)}{3S_0 - E_F(\rho_0)}$$

Simple parametrized EOS

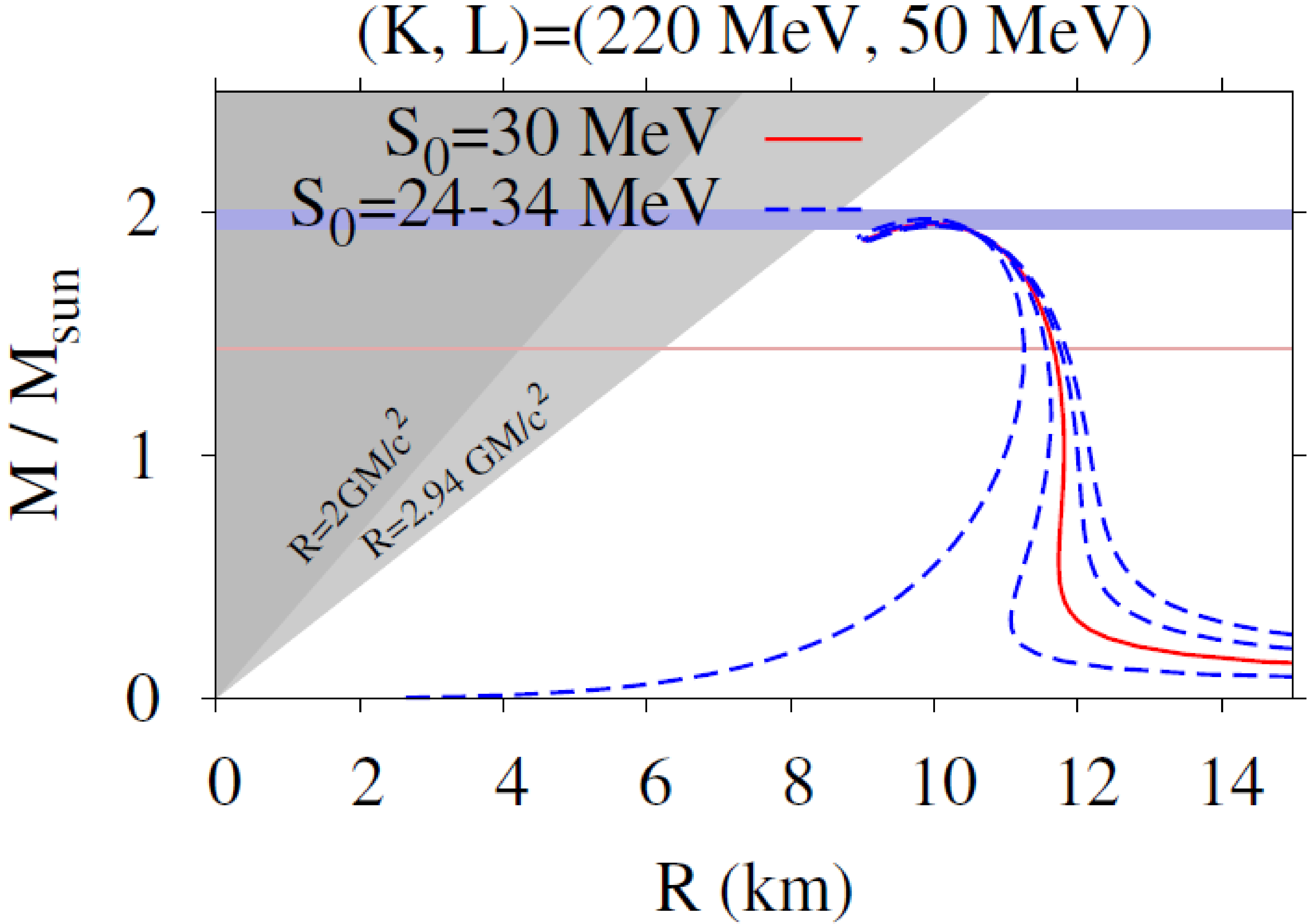




Simple parametrized EOS



Simple parametrized EOS



Simple parametrized EOS