核多体系物理学

担当:大西　明、八田佳孝 (基礎物理学研究所)

- 授業の概要・目的 : 核子・ハドロン・クォークからなる多体系の性質を量子色力学(QCD)、状態方程式、および核反応論の観点から議論する。強い相互作用の基本理論である QCD の基本的性質、核物質の状態方程式を記述するために必要となる核多体理論(平均場理論、G-matrix 、熱場の理論、強結合格子 QCD)、ハイパー核生成反応や重イオン反応を理解する上で必要とされる原子核核反応理論(直接反応、輸送模型等)、等の理論の枠組について解説すると共に、これらについての最近の研究成果についても紹介する。

- 授業計画と内容 : 量子色力学、および核子・ハドロン・クォーク物質の相互作用と状態方程式について以下の内容で講義する。

1. 量子色力学 (QCD) の基本的性質
   QCD 作用と対称性、摂動論的 QCD 、発展方程式、カラーグラス描像

八田

2. 状態方程式と QCD 相図を記述する理論模型
   - 核力と位相差、有効相互作用、核物質の状態方程式、平均場理論、
   - 有限温度での場の理論、南部 - ヨナラシニョ模型、強結合格子 QCD
3. 原子核反応理論
   - 核子 - 核子散乱、ハドロン - 原子核反応、
   - 流体力学、輸送理論、
   - ハイパー核・中間子核生成反応の概観と直接反応

大西

- 成績評価の方法・基準 : 履修状況及びレポートにより総合評価する。

参考書 Quark Gluon Plasma, K.Yagi, T.Hatsuda, Y.Miake (CAMBRIDGE).
格子上の場の理論、青木慎也 (シュプリンガー・ジャパン)
クォーク・ハドロン物理学入門, 国広悌二 (サイエンス社)

状態方程式やQCD 相図を記述する理論模型

- 2.1 核物質の状態方程式概観 (10/20)
  - なぜ状態方程式か？中性子星パズル、対称エネルギー
  - 状態方程式を記述する理論模型
- 2.2 核力と位相差、有効相互作用 (10/27)
- 2.3 平均場理論 : 相対論的平均場 (RMF) 模型を中心に。(11/10)
- 2.4 カイラル相転移と南部 - ヨナラシニョ (NJL) 模型 (11/17)
  - 有限温度･密度の場の理論入門
  - 経路積分表示、ユークリッド時空、松原和、自由場の分配関数
  - カイラル対称性、Nambu-Jona-Lasinio (NJL) 模型、カイラル相転移
- 2.5 格子上の場の理論入門 (→ 青木さんの講義)
  - 格子 QCD 、Plaquette 作用、格子 Fermion 、リンク積分、
  - 強結合格子 QCD 、Area Law 、強結合展開、ポリアコフ･ループ
- 2.6 高密度物質の QCD 有効模型 (12/1)
  - Bag 模型、Quark-Meson 模型、Polyakov loop extended Quark-Meson (PQM) 模型、Polyakov loop extended NJL (PNJL) 模型

核力と位相差、有効相互作用 (white board にて)

Relativistic Mean Field

Relativistic Mean Field (1)

- **Effective Lagrangian of Baryons and Mesons + Mean Field App.**

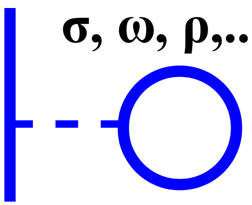
  B.D.Serot, J.D.Walecka, Adv.Nucl.Phys.16 ('86), 1

$$L = L_B^{\mathrm{free}} + L_M^{\mathrm{free}} + L_{BM} + L_M^{\mathrm{Int}}$$

$$L_M^{\mathrm{Int}} = -U_\sigma(\sigma) + \frac{1}{4} c_\omega (\omega_\mu \omega^\mu)^2 + \cdots$$

$$L_{BM} = -\sum_{B,S} g_{BS} \bar{\psi}_B \varphi_S \psi_B - \sum_{B,V} g_{BV} \bar{\psi}_B \gamma^\mu V_\mu \psi_B$$

$$L_B^{\mathrm{free}} = \bar{\psi}_B (i \gamma^\mu \partial_\mu - M_B) \psi_B \ , \quad L_M^{\mathrm{free}} = \sum_S \left[\frac{1}{2} \partial^\mu \varphi_S \partial_\mu \varphi_S - \frac{1}{2} m_S^2 \varphi_S^2\right] + \sum_V \left[-\frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \frac{1}{2} m_v^2 V_\mu V^\mu\right]$$



- **Baryons and Mesons: B=N, Λ, Σ, Ξ, ...,   S= σ, ς, ...,   V= ω, ρ, φ, …**
- **Based on Dirac phenomenology & Dirac Bruckner-Hatree-Fock**

  E.D. Cooper, S. Hama, B.C. Clark, R.L. Mercer, PRC47('93),297
  R. Brockmann, R. Machleidt, PRC42('90),1965

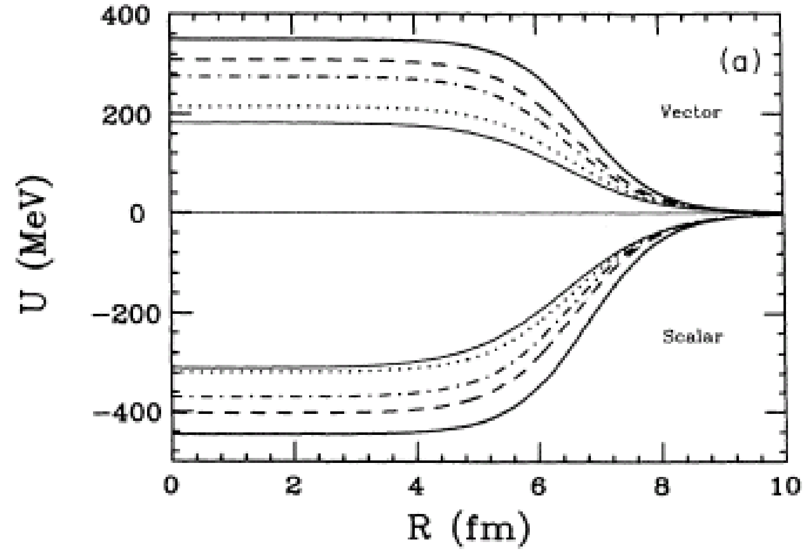
- **Large scalar (att.) and vector (repl.) → Large spin-orbit pot.**
  Relativistic Kinematics → Effective 3-body repulsion
- **Non-linear terms of mesons → Bare 3-body and 4-body force**

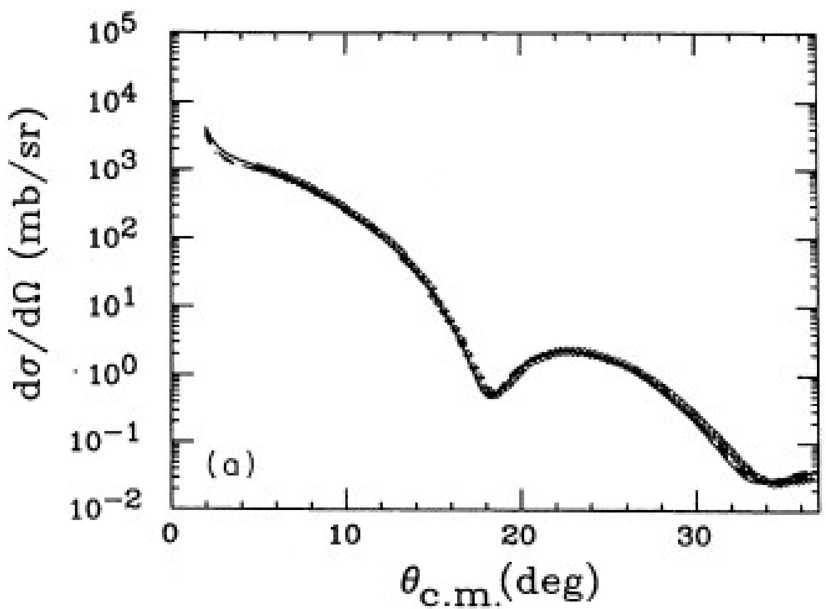
  Boguta, Bodmer ('77), NL1:Reinhardt, Rufa, Maruhn, Greiner, Friedrich ('86), NL3: Lalazissis, Konig, Ring ('97),TM1 and TM2: Sugahara, Toki ('94), Brockmann, Toki ('92)

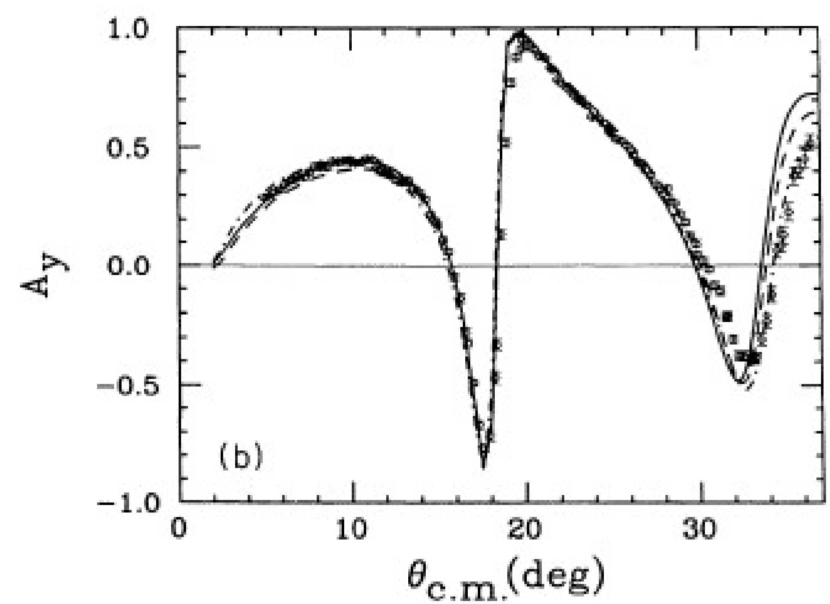
Dirac Phenomenology

E.D. Cooper, S. Hama, B.C. Clark, R.L. Mercer, PRC47('93),297

- **Dirac Eq. with**
  Scalar + Vector pA potential
  (-400 MeV + 350 MeV)
  → Cross Section, Spin Observables



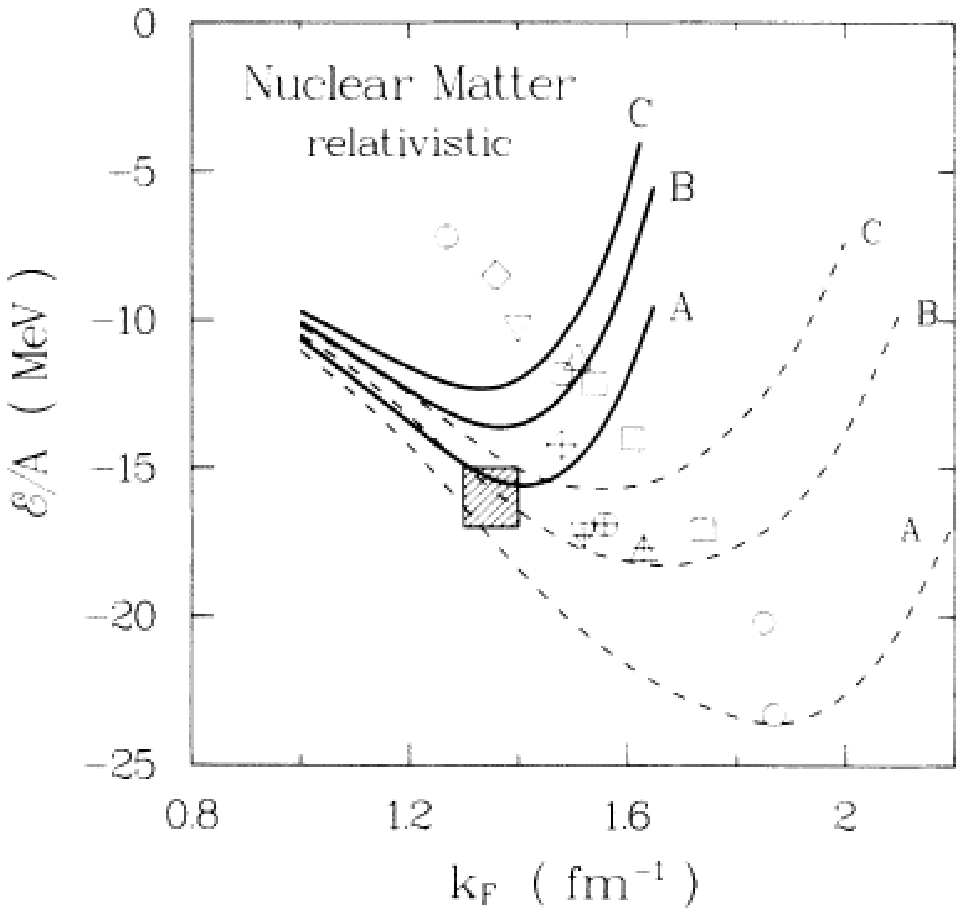




EOS in Dirac-Brueckner-Hartree-Fock

R. Brockmann, R. Machleidt, PRC42('90),1965

- **Non Relativistic Brueckner Calculation**
  → Nuclear Saturation Point cannot be reproduced (Coester Line)
- **Relativistic Approach (DBHF)**
  → Relativity gives additional repulsion, leading to successful description of the saturation point.



Relativistic Mean Field (2)

- **Mean Field treatment of meson field operator**
  = Meson ield operator is replaced with its expectation value
  $$\varphi(r) \to \langle\varphi(r)\rangle$$
  Ignoring fluctuations compared with the expectation value may be a good approximation at strong condensate.

- **Which Hadrons should be included in RMF ?**
  - **Baryons (1/2+)** p, n, Λ, Σ, Ξ, Δ, ...
  - **Scalar Mesons (0+)** σ(600), f_0(980), a_0(980), ...
  - **Vector Mesons (1-)** ω(783), ρ(770), ϕ(1020), ...
  - **Pseuso Scalar (0-)** π , K, η, η', ...
  - **Axial Vector (1+)** a_1, ...

  We require that the meson field can have uniform expectation values in nuclear matter.
  → Scalar and Time-Component of Vector Mesons (σ , ω, ρ, ...)

σω Model (1)

Serot, Walecka, Adv.Nucl.Phys.16 (1986),1

- **Consider only σ and ω mesons**
- **Lagrangian**

$$L=\bar{\psi}\left(i\gamma^\mu\partial_\mu - M + g_s\sigma - g_v\omega\right)\psi$$
$$+\frac{1}{2}\partial_\mu\sigma\,\partial^\mu\sigma - \frac{1}{2}m_s^2\sigma^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_v^2\omega_\mu\omega^\mu$$
$$\left(F_{\mu\nu}=\partial_\mu\omega_\nu - \partial_\nu\omega_\mu\right)$$

- **Equation of Motion**
  - **Euler-Lagrange Equation**

$$\frac{\partial}{\partial x^\mu}\left[\frac{\partial L}{\partial(\partial_\mu\phi_i)}\right]-\frac{\partial L}{\partial\phi_i}=0$$

$$\sigma:\left[\partial_\mu\partial^\mu + m_s^2\right]\sigma = g_s\bar{\psi}\psi$$

$$\omega:\partial_\mu F^{\mu\nu} + m_v^2\omega^\nu = g_v\bar{\psi}\gamma^\nu\psi \quad\rightarrow\quad \left[\partial_\mu\partial^\mu + m_v^2\right]\omega^\nu = g_v\bar{\psi}\gamma^\nu\psi$$

$$\psi:\left[\gamma^\mu\left(i\partial_\mu - g_v V_\mu\right) - \left(M - g_s\sigma\right)\right]\psi = 0$$

EOM of ω (for beginners)

- **Euler-Lagrange Eq.**

$$\partial_\mu F^{\mu\nu} + m_v^2 \omega^\nu = g_v \bar{\psi} \gamma^\nu \psi$$

- **Divergence of LHS and RHS**

$$\partial_\nu \partial_\mu F^{\mu\nu} + m_v^2 (\partial_\nu \omega^\nu) = m_v^2 (\partial_\nu \omega^\nu) = g_v (\partial_\nu \bar{\psi} \gamma^\nu \psi) = 0$$

LHS: derivatives are sym. and $F_{\mu\nu}$ is anti-sym.

RHS: Baryon Current = Conserved Current

- **Put it in the Euler-Lagrange Eq.**

$$\partial_\mu F^{\mu\nu} = \partial_\mu (\partial^\mu \omega^\nu - \partial^\nu \omega^\mu) = \partial_\mu \partial^\mu \omega^\nu - \partial^\nu (\partial_\mu \omega^\mu) = \partial_\mu \partial^\mu \omega^\nu$$

Schroedinger Eq. for Upper Component (1)

- **Dirac Equation for Nucleons**

$$\left(i\gamma\partial-\gamma^0 U_v - M - U_s\right)\psi=0 \quad ,$$
$$U_v = g_\omega \omega \quad , \quad U_s = -g_\sigma \sigma$$

- **Decompose 4 spinor into Upper and Lower Components**

$$\begin{pmatrix} E-U_v-M-U_s & i\sigma\cdot\nabla \\ -i\sigma\cdot\nabla & -E+U_v-M-U_s \end{pmatrix}\begin{pmatrix} f \\ g \end{pmatrix}=0$$

$$g=\frac{-i}{E+M+U_s-U_V}(\sigma\cdot\nabla)f$$
$$(E-M-U_v-U_s)f=-i(\sigma\cdot\nabla)g$$

Schroedinger Eq. for Upper Component (2)

- **Erase Lower Component (assuming spherical sym.)**

$$-i(\sigma\cdot\nabla)g=-(\sigma\cdot\nabla)\frac{1}{X}(\sigma\cdot\nabla)f=-\frac{1}{X}\nabla^2 f-\frac{1}{r}\left[\frac{d}{dr}\frac{1}{X}\right](\sigma\cdot r)(\sigma\cdot\nabla)f$$

$$=-\nabla\frac{1}{X}\nabla f+\frac{1}{r}\left[\frac{d}{dr}\frac{1}{X}\right](\sigma\cdot l)f$$

$$(\sigma\cdot r)(\sigma\cdot\nabla)=(r\cdot\nabla)+i\sigma\cdot(r\times\nabla)=r\cdot\nabla-\sigma\cdot l$$

- **"Schroedinger-like" Eq. for Upper Component**

$$-\nabla\frac{1}{E+M+U_s-U_v}\nabla f+\left(U_s+U_v+U_{LS}(\sigma\cdot l)\right)f=(E-M)f$$

$$U_{LS}=\frac{1}{r}\left[\frac{d}{dr}\frac{1}{E+M+U_s-U_v}\right]<0 \quad \text{on surface}$$

$(U_s,U_v)\sim$ (-350 MeV, 280 MeV)

$\rightarrow$Small Central(U_s+U_v), Large LS (U_s-U_v)

- **From Single Particle Energy**

$$\left(\gamma^0(E-U_v)+i\gamma\cdot\nabla-(M+U_s)\right)\psi=0 \;\rightarrow\; (E-U_v)^2=p^2+(M+U_s)^2$$

$$\rightarrow E=\sqrt{p^2+(M+U_s)^2}+U_v\approx E_p+\frac{M}{E_p}U_s+U_v+\frac{p^2}{2E_p^3}U_s^2$$

$$(E_p=\sqrt{p^2+M^2})$$

- **Schroedinger Equivalent Potential (Uniform matter)**

$$-\frac{\nabla^2}{2\mathrm{M}}f+\left[U_s+\frac{E}{M}U_v+\frac{U_s^2-U_v^2}{2\mathrm{M}}\right]f=\frac{E+M}{2\mathrm{M}}(E-M)f$$

$$U_{\mathrm{SEP}}\approx U_s+\frac{E}{M}U_v$$

Anyway, slow baryons feel Non.-Rel. Potential,

$$U\approx U_s+U_v=-g_s\sigma+g_v\omega$$

Nuclear Matter in σω Model

Serot, Walecka, Adv.Nucl.Phys.16 (1986),1

- **Uniform Nuclear Matter**

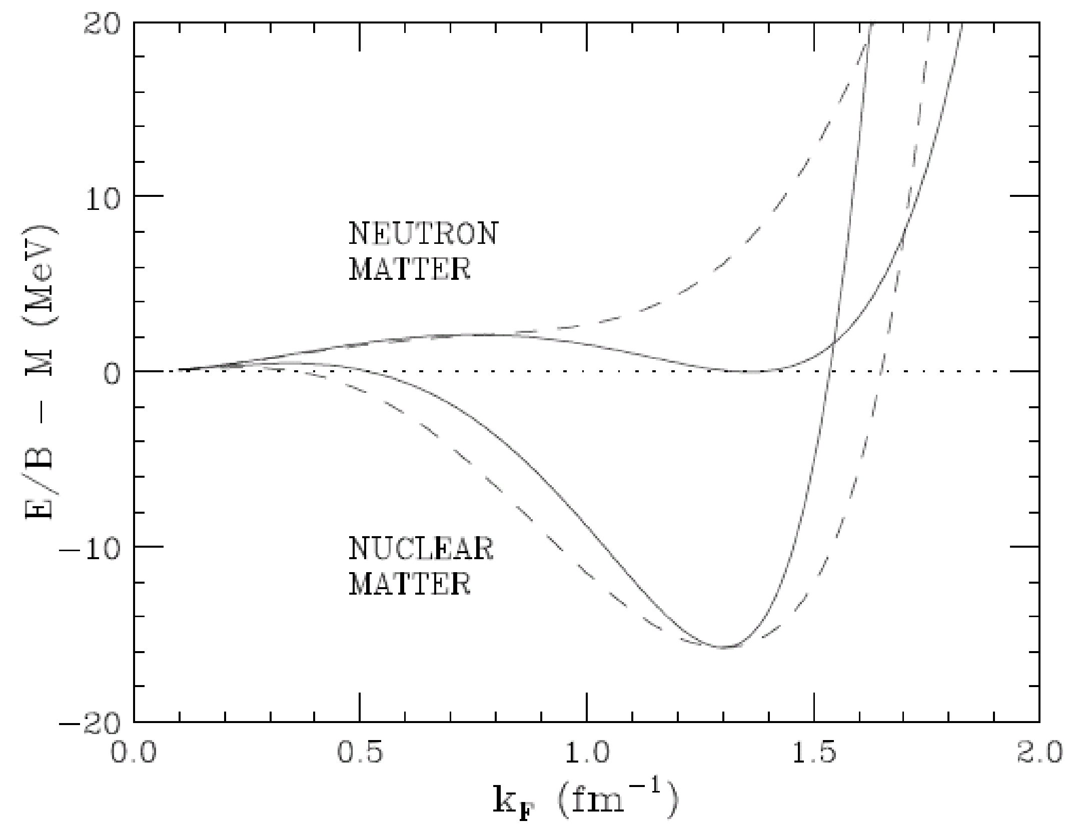
$$E/V = \gamma_N \int^{P_F} \frac{d^3p}{(2\pi)^2} E^* + \frac{1}{2} m_s^2 \sigma^2 - \frac{1}{2} m_v^2 \omega^2 + g_v \rho_B \omega$$

$$\sigma = \frac{g_s}{m_s^2} \rho_s = \frac{g_s}{m_s^2} \int^{P_F} \frac{\gamma_N \; d^3p}{(2\pi)^2} \frac{M^*}{E^*} \qquad \left(M^* = M + U_s = M - g_s\sigma\,, \quad E^* = \sqrt{p^2 + M^{*2}}\right)$$

$$\omega = \frac{g_v}{m_v^2} \rho_B = \gamma_N \frac{g_v}{m_v^2} \int^{P_F} \frac{d^3p}{(2\pi)^3}$$

γ_N = Nucleon degeneracy
(=4 in sym. nuclear matter)

Problem: EOS is too stiff
K ~ (500-600) MeV !
→ How can we avoid it ?



RMF with Non-Linear Meson Int. Terms

Boguta, Bodmer ('77), NL1:Reinhardt, Rufa, Maruhn, Greiner, Friedrich ('86), NL3: Lalazissis, Konig, Ring ('97),TM1 and TM2: Sugahara, Toki ('94), Brockmann, Toki ('92)

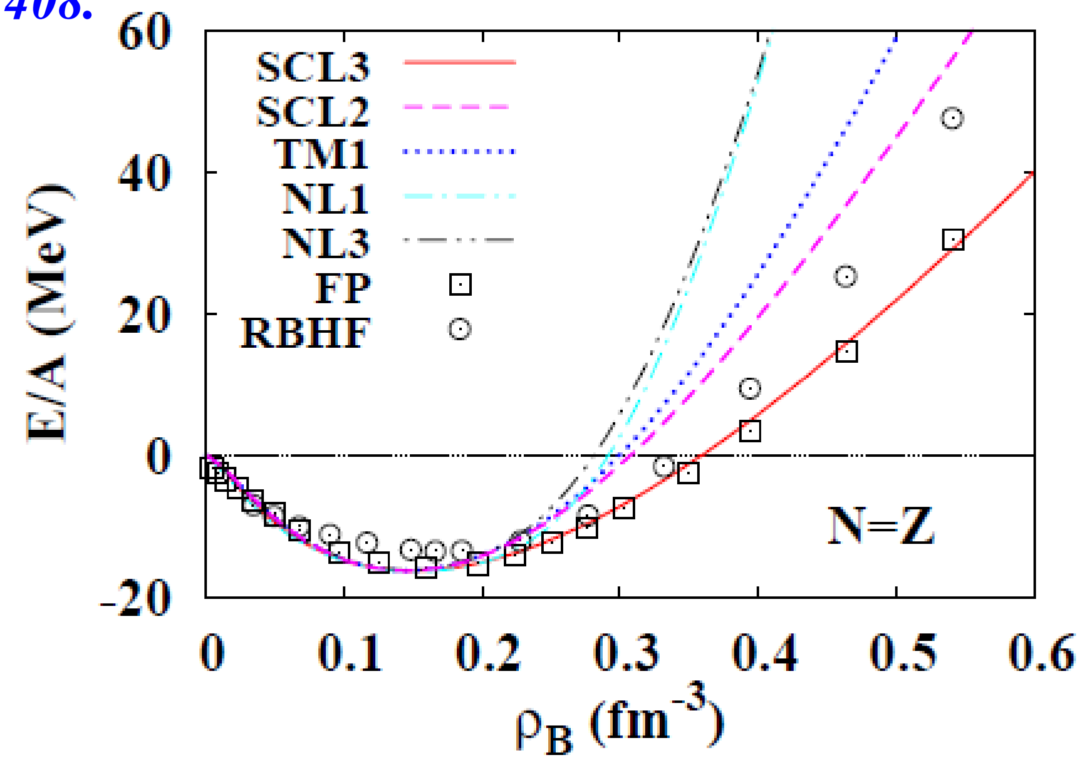
- **Too stiff EOS in the simplest RMF (σω model) is improved by introducing non-linear terms (σ^4, ω^4)**
  - **Fit B.E. of Stable as well as Unstable (n-rich) Nuclei**
  - **Three Mesons (σ,ω,ρ) are included**
  - **Meson Self-Energy Term (σ,ω)**

$$
\begin{aligned}
\mathcal{L} =& \bar{\psi}_N \left(i\partial\!\!\!/ - M - g_\sigma \sigma - g_\omega \omega\!\!\!/ - g_\rho \tau^a \rho\!\!\!/^a\right) \psi_N \\
&+ \frac{1}{2}\partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 \boxed{- \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4} \\
&- \frac{1}{4} W^{\mu\nu} W_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4} R^{a\mu\nu} R^a_{\mu\nu} + \frac{1}{2} m_\rho^2 \rho^{a\mu} \rho^a_\mu + \boxed{\frac{1}{4} c_3 \left(\omega_\mu \omega^\mu\right)^2} \\
&+ \bar{\psi}_e \left(i\partial\!\!\!/ - m_e\right) \psi_e + \bar{\psi}_\nu i\partial\!\!\!/ \psi_\nu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \ , \\
W_{\mu\nu} =& \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \ , \\
R^a_{\mu\nu} =& \partial_\mu \rho^a_\nu - \partial_\nu \rho^a_\mu + g_\rho \epsilon^{abc} \rho^{b\mu} \rho^{c\nu} \ , \\
F_{\mu\nu} =& \partial_\mu A_\nu - \partial_\nu A_\mu \ .
\end{aligned}
$$

RMF models with Non-Linear Meson Int. Terms

- Variety of the RMF models
  → MB couplings, meson masses, meson self-energies
  - σN, ωN, ρN couplings are well determined
    → almost no model deps. in Sym. N.M. at low ρ
  - ω^4 term is introduced to simulate DBHF results of vector pot.
    TM1&2: Y. Sugahara, H. Toki, NPA579('94)557;
    R. Brockmann, H. Toki, PRL68('92)3408.
  - σ^3 and σ^4 terms are introduced to soften EOS at ρ_0.
    J. Boguta, A.R.Bodmer NPA292('77)413,
    NL1:P.-G.Reinhardt, M.Rufa, J.Maruhn, W.Greiner, J.Friedrich, ZPA323('86)13.
    NL3: G.A.Lalazissis, J.Konig, P.Ring, PRC55('97)540.

    → Large differences are found at high ρ



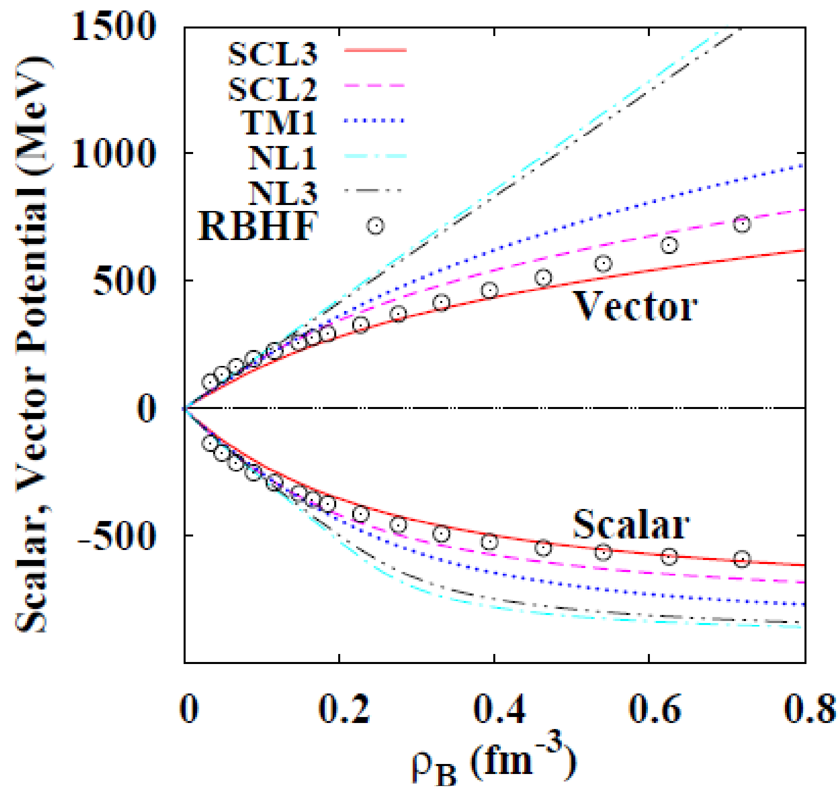
K. Tsubakihara, H. Maekawa, H. Matsumiya, AO,PRC81('10)065206.

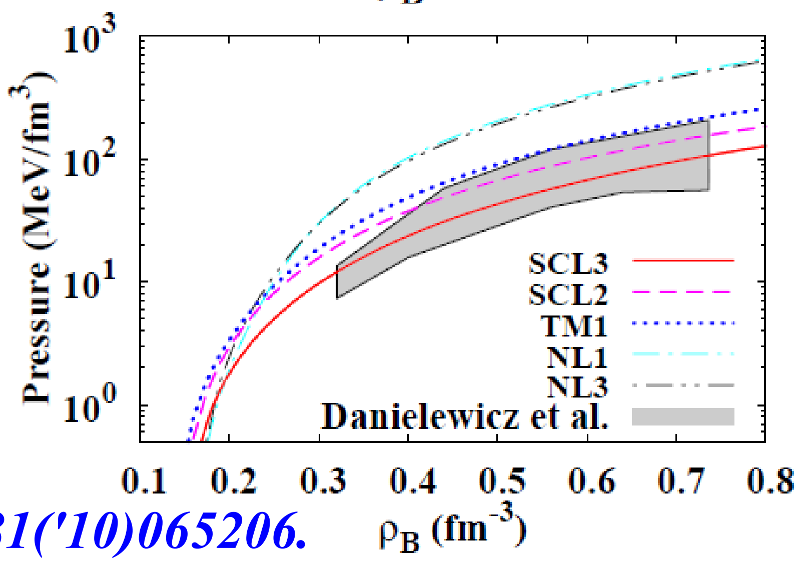
Vector potential in RMF

- **Vector potential from ω dominates at high density !**

$$U_v(\rho_B) = g_\omega \omega \sim \frac{g_\omega^2}{m_\omega^2}\rho_B$$

  - **Dirac-Bruckner-Hartree-Fock shows suppressed vector potential at high ρ_B.**
    R. Brockmann, R. Machleidt, PRC42('90)1965.
  - **Collective flow in heavy-ion collisions suggests pressure at high ρ_B.**
    P. Danielewicz, R. Lacey, W. G. Lynch, Science298('02)1592.
  - **Self-interaction of ω ~ $c_\omega(\omega_\mu\omega^\mu)^2$**
    → DBHF results & Heavy-ion data

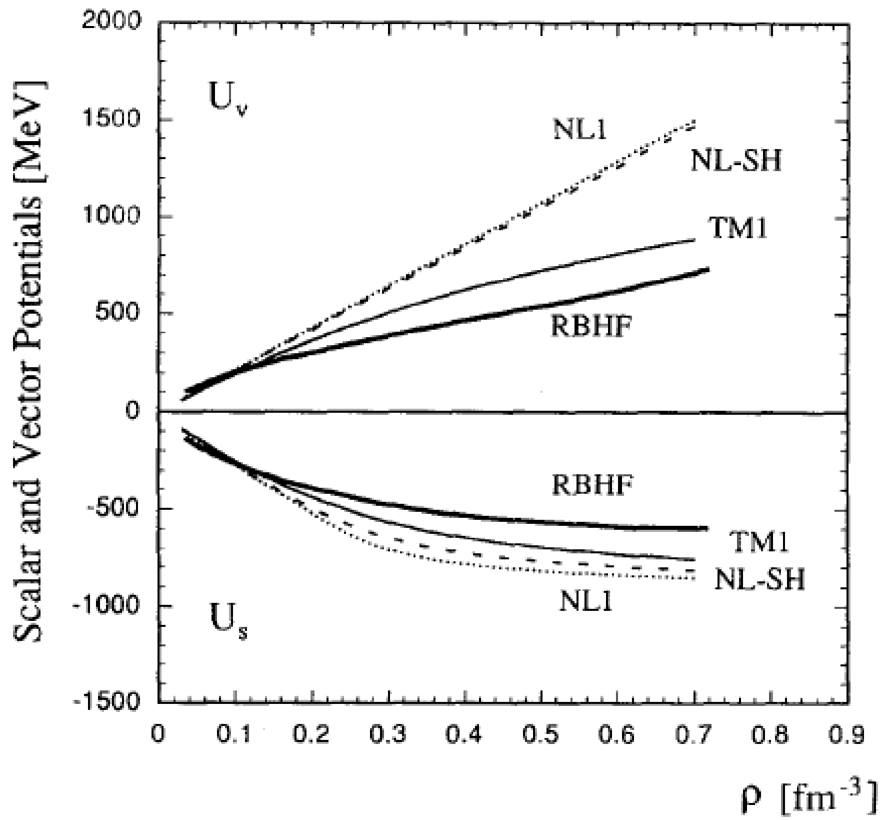


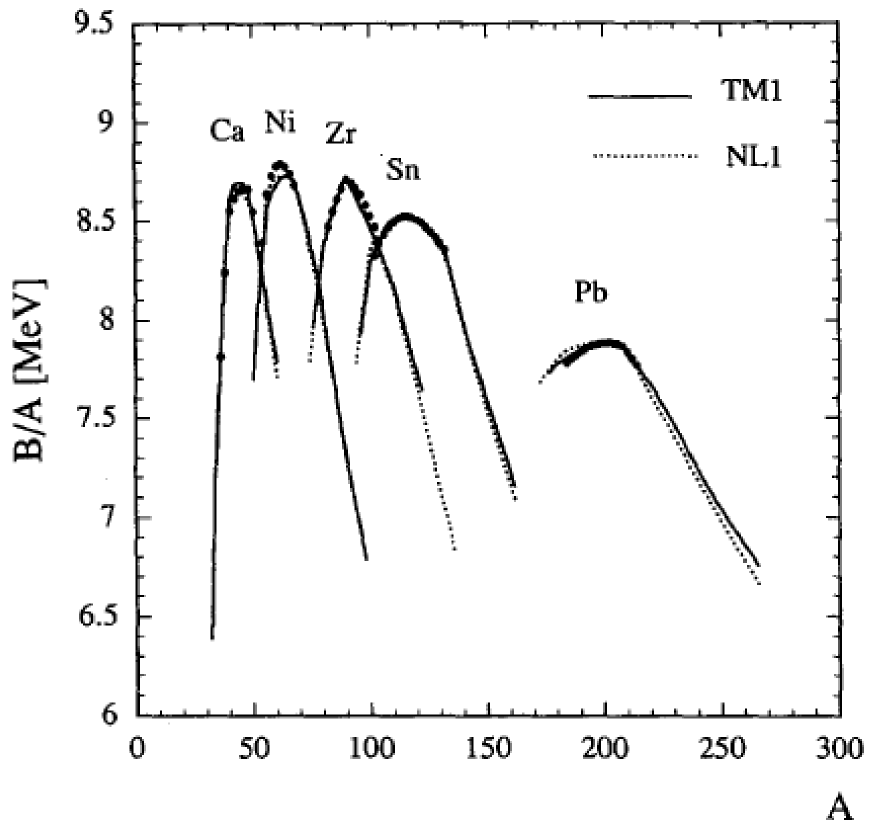


K. Tsubakihara, H. Maekawa, H. Matsumiya, AO,PRC81('10)065206.

TM1

- **TM1 Sugahara, Toki ('94)**
  - **Fit vector potential in RBHF by introducing ω⁴ term.**
  - **Fit binding energies of neutron-rich nuclei**

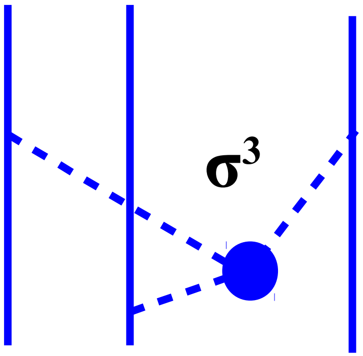




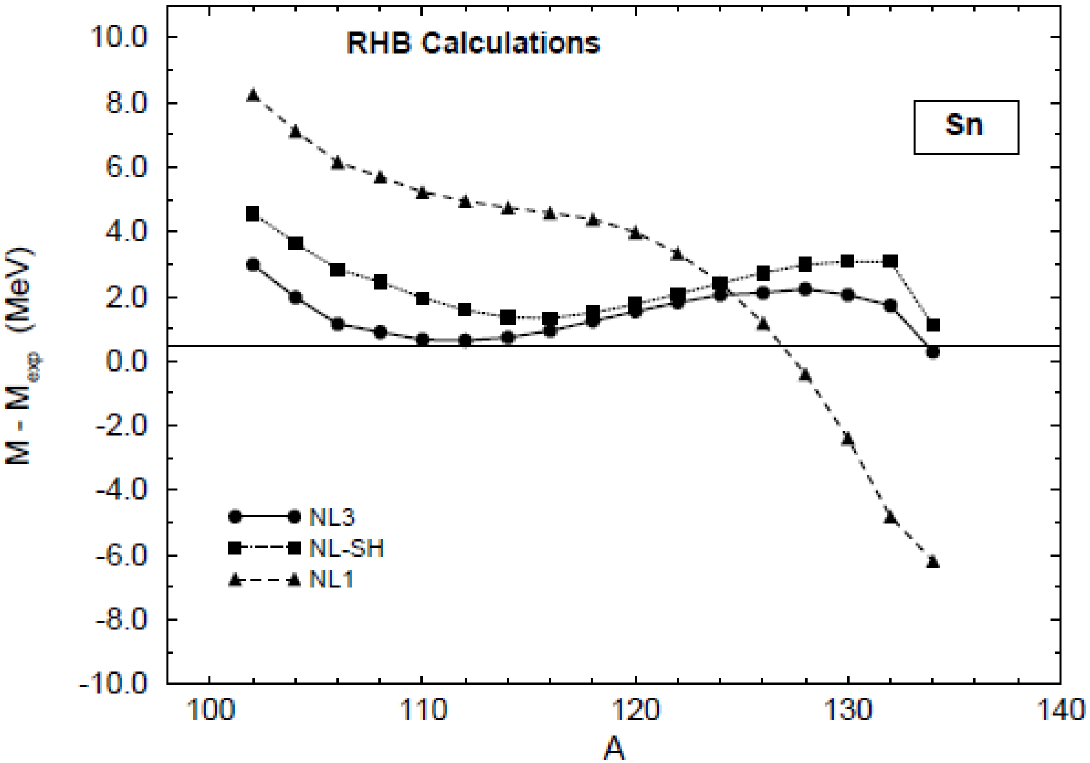
TM1: Sugahara, Toki ('94)

High Quality RMF models

- **いくつかの RMF パラメータによる計算は、「質量公式」に迫る精度で原子核質量を記述！**
  → High Quality RMF models.
  TM, NL1, NL3, ….



  - **全質量で 1-2 MeV の誤差 (NL3)**
  - **Linear coupling (σN, ωN, ρN), self-energy in σ, ω**
  - **場合によっては結合定数の密度依存性を導入。**



NL3: Lalazissis, Konig, Ring, PRC55 ('97)540

RMF with Non-Linear Meson Int. Terms

- **Are the Lagrangian parameters are well determined ?**

$$\mathcal{L} = \mathcal{L}_{\text{free}}(\psi, \sigma, \omega, \rho, \ldots) + \bar{\psi}\left[g_\sigma \sigma - g_\omega \gamma^0 \omega - g_\rho \tau_z \gamma^0 \rho\right]\psi + c_\omega \omega^4/4 - V_\sigma(\sigma) \, , \quad (3)$$

$$V_\sigma = \begin{cases} \frac{1}{3} g_3 \sigma^3 + \frac{1}{4} g_4 \sigma^4 & \text{(NL1, NL3, TM1)} \\ -a_\sigma f_{\text{SCL}}(\sigma/f_\pi) & \text{(SCL)} \end{cases} \, , \quad (4)$$

  - **Linear terms, Meson-Nucleon Coupling → Well determined**
  - **Negative Coef. of $\sigma^4 < 0$ in some of RMF models→ Vacuum is unstable**
  - **Meson interaction terms → Different in RMF parameterization**

TABLE II: RMF parameters

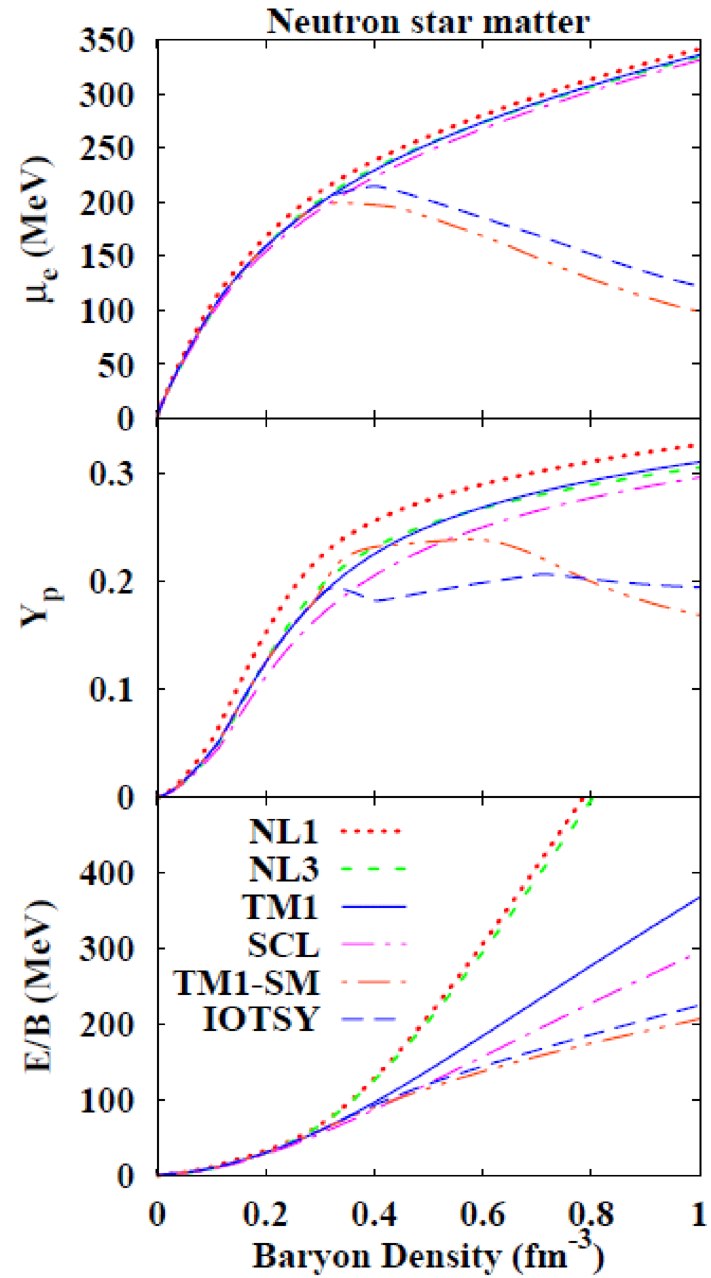
	$g_{\sigma N}$	$g_{\omega N}$	$g_{\rho N}$	g_3(MeV)	g_4	c_ω	m_σ(MeV)	m_ω(MeV)	m_ρ(MeV)
NL1[18]	10.138	13.285	4.976	2401.9	-36.265	0	492.25	795.359	763
NL3[19]	10.217	12.868	4.474	2058.35	-28.885	0	508.194	782.501	763
TM1[6]	10.0289	12.6139	4.6322	1426.466	0.6183	71.3075	511.198	783	770
SCL[20](*1)	10.08	13.02	4.40	1255.88	13.504	200	502.63	783	770

(*1): g_3 and g_4 are from the expansion of f_{SCL}.

AO, Jido, Sekihara, Tsubakihara (2009)

Neutron Star Matter EOS

- **Difference in non-linear meson terms generate different predictions of EOS at high densities**



How can we fix non-linear terms ?

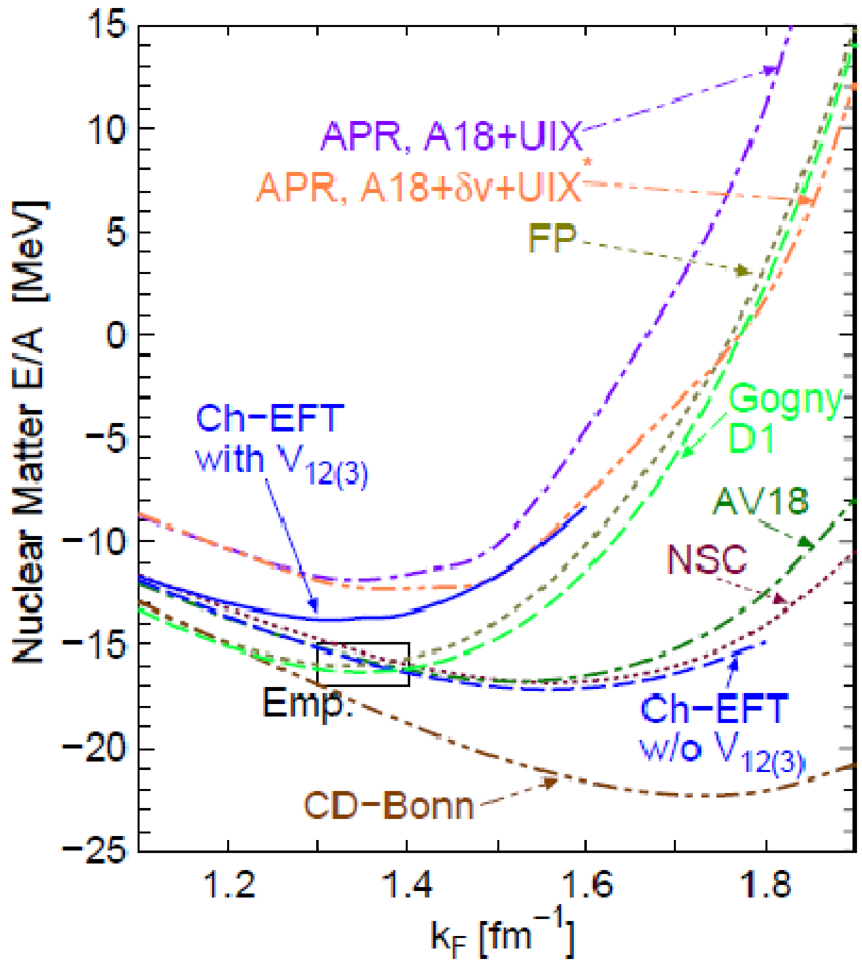
AO, Jido, Sekihara, Tsubakihara, Phys. Rev. C 80 (2009), 038202.

YITP YUKAWA INSTITUTE FOR THEORETICAL PHYSICS
YIPQS YITP Kyoto
Neutron Star Matter

Ch-EFT EOS

- **Phen. models need inputs from**
  Experimental Data and/or Microscopic (Ab initio) Calc.
- **Recent Ch-EFT EOS is promising !**
  NN (N3LO)+3NF(N2LO)

M.Kohno ('13)

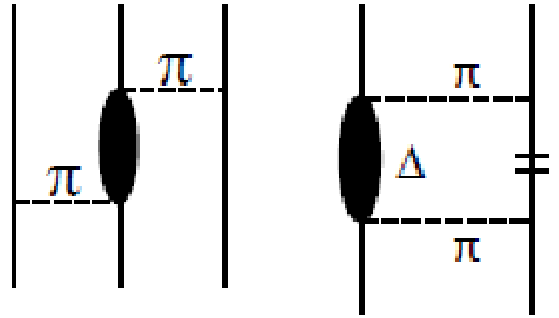


M. Kohno, PRC 88 ('13) 064005

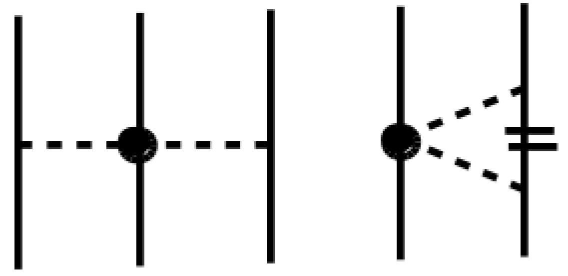
"Universal" mechanism of "Three-body" repulsion

- **"Universal" 3-body repulsion is necessary to support NS.**
  Nishizaki, Takatsuka, Yamamoto ('02)
- **Mechanism of "Universal" Three-Baryon Repulsion.**
  - **"σ"-exchange ~ two pion exch. w/ res.**
  - **Large attraction from two pion exchange is suppressed by the Pauli blocking in the intermediate stage.**
    Kohno ('13)

Physical Picture



χ EFT



"Universal" TBR
- ***Coupling to Res. (hidden DOF)***
- ***Reduced "σ" exch. pot. ?***

Summary of Lecture 2.3

- **Nuclear Matter EOS is important in many subjects of physics.**
  - **Bulk nuclear properties (B.E., radius)**
  - **Dense Matter in Compact Astrophysical Objects**
  - **High-Energy Heavy-Ion Collisions**
- **Relativistic Mean Field models**
  - **Simple description of nucleon scalar and vector potentials in terms of meson fields.**
  - **With non-linear meson interaction terms, nuclear binding energies (and radii) are well explained.**
  - **Ambiguities of non-linear couplings bring large differences of EOS at high densities, especially in asymmetric nuclear matter.**
- **It is promising to utilize the results of G-matrix based on Chiral EFT (2 and 3 nucleon force), which reproduces the saturation density in an "ab initio" manner.**

Thank you !

Field Theory at Finite T & ρ
– Short Course –

経路積分

- **量子力学での経路積分 (Path integral)**
  - 時刻 t_i で位置 q_i にいた粒子が時刻 t_f で位置 q_f に到着する振幅

$$S_{fi}=\langle q_f, t_f|\exp[-i\hat{H}(t_f-t_i)]|q_i, t_i\rangle=\int Dq\exp(iS[q])$$

$$S[q]=\int_{q(t_i)=q_i, q(t_f)=q_f} dt\, L(q,\dot{q})$$

経路 $q(t)$ についての和 → 経路積分

  - 特徴
    - 演算子の代わりに通常の数 (c- 数) で表せる。
    - 作用 S の構成時に正準交換関係を用いることにより「量子論」の性質を取り込む。

- **場の理論＝各点での場の振幅 φ(x,t) を座標とする量子力学**

$$S_{fi}=\langle \Psi_f|\exp[-i\hat{H}(t_f-t_i)]|\Psi_i\rangle=\int D\phi\exp(iS[\phi])$$

$$S[\phi]=\int_{\Psi(t_i)=\Psi_i, \Psi(t_f)=\Psi_f} d^4x\, L(\phi,\partial_\mu\phi)$$

分配関数とユークリッド化

分配関数

$$Z=\sum_n \exp(-E_n/T)=\sum_n \langle n|\exp[-\hat{H}/T]|n\rangle$$

$$=\sum_n \langle n|\exp[-i\hat{H}(t_f-t_i)]|n\rangle_{t_f-t_i=-i/T}=\int D\phi \exp(-S_E[\phi])$$

$$S_E[\phi]=\int_0^\beta d\tau\, d^3x\, L_E(\phi,\partial_i\phi,\partial_\tau\phi)\,\big|_{\phi(x,\beta)=\phi(x,0)}$$

$$L_E(\phi,\partial_i\phi,\partial_\tau\phi)=-L(\phi,\partial_i\phi,i\partial_t\phi)$$

$$t=-i\tau,\quad \partial_\tau=-i\partial_t,\ \beta=1/T$$

$$iS=i\int_0^{-i\beta} dt\int d^3x\, L=\int_0^\beta d\tau\, d^3x\, L=-\int_0^\beta d\tau\, d^3x\, L_E$$

- 統計力学の分配関数は虚時間発展の振幅の和である。
- 全ての状態について和 → $\tau=0, \beta$ で周期境界条件をつけて任意の $\varphi(x,t)$ について足し合わせる。

Example: Scalar Field

- **Lagrangian density**

$$L=\frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi-\frac{1}{2}m^2\phi^2-U(\phi)$$

Euler-Lagrange equation (principle of least action)

$$\partial_\mu\left[\frac{\partial L}{\partial(\partial_\mu\phi)}\right]-\frac{\partial L}{\partial\phi}=0\ \rightarrow\ \partial_\mu\partial^\mu\phi+m^2\phi+\frac{\partial U}{\partial\phi}=0\,(\text{Klein-Gordon eq.})$$

- **Euclidean Lagrangian**
  - **Euclid 化のルール** $t = -i\tau$, $x_4=\tau$, $g_{\mu\nu}=(1,1,1,1)$, $L_E=-L$

$$L_E=\frac{1}{2}\partial_\mu\phi\,\partial_\mu\phi+\frac{1}{2}m^2\phi^2+U(\phi)$$

→ 相互作用がない場合に実際に経路積分してみましょう。

Partition Func. of Free Scalar Field

- 自由スカラー場の分配関数
  - 有限のサイズの箱 (体積 V) の中で自由スカラー場 (U=0)
  - フーリエ変換

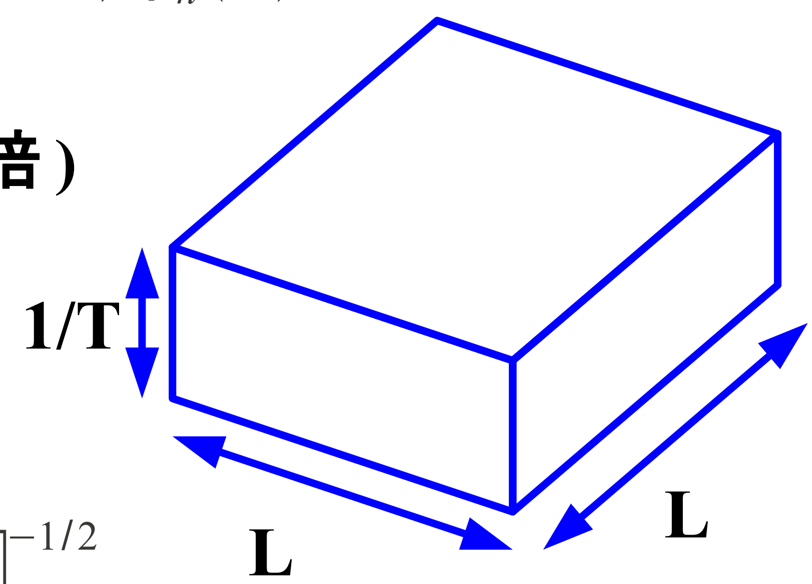
$$\phi(\tau, \boldsymbol{x}) = \frac{1}{\sqrt{V/T}} \sum_{n,\boldsymbol{k}} \exp(-i\omega_n \tau + i\boldsymbol{k}\cdot\boldsymbol{x}) \phi_n(\boldsymbol{k})$$

Periodic boudnary condition $\omega_n = 2\pi n T, \quad k_i = 2\pi n_i / L$

Euclidean action $S_E = \frac{1}{2} \sum_{n,\boldsymbol{k}} (\omega_n^2 + \boldsymbol{k}^2 + m^2) \phi_n^2(\boldsymbol{k})$

  - フーリエ変換はユニタリー変換だから、
    積分の測度は変わらない。(高々定数倍)

$$D\phi = N \prod_{n,\boldsymbol{k}} d\phi_n(\boldsymbol{k})$$



  - ガウス積分 → 分配関数

$$Z = \int D\phi\, e^{-S_E} = N \prod_{n,\boldsymbol{k}} \sqrt{2\pi} \left[\omega_n^2 + \boldsymbol{k}^2 + m^2\right]^{-1/2}$$

Partition Func. of Free Scalar Field (cont.)

- 自由エネルギー

$$\Omega = -T\log Z = \frac{1}{2}\sum_{\boldsymbol{k}}\left[T\sum_n \log(\omega_n^2 + \underbrace{\boldsymbol{k}^2 + m^2}_{E_k^2})\right] + \text{const.}$$

$$= \frac{1}{2}\sum_{\boldsymbol{k}} I(E_{\boldsymbol{k}}, T) + \text{const.}$$

- **松原和 (Matsubara Frequency summation)**

$$\sum_n \frac{1}{a^2 + \bar{n}^2} = \frac{\pi}{2a} \times \begin{cases} \coth(\pi a/2) & (\bar{n} = 2\mathrm{n}) \\ \tanh(\pi a/2) & (\bar{n} = 2\mathrm{n}+1) \end{cases}$$

$$\frac{\partial I(E_k, T)}{\partial E_k} = \sum_n \frac{2TE_k}{\omega_n^2 + E_k^2} = \cdots = \frac{1 + \exp(-E_k/T)}{1 - \exp(-E_k/T)}$$

$$I(E_k, T) = E_{\boldsymbol{k}} + 2T\log\left(1 - \exp(-E_k/T)\right) + \text{const.}$$

Partition Func. of Free Scalar Field (cont.)

- 自由エネルギー (グランド･ポテンシャル)

$$\Omega = \sum_{\boldsymbol{k}} \left\{ \frac{E_{\boldsymbol{k}}}{2} + T \log\left(1 - e^{-E_{\boldsymbol{k}}/T}\right) \right\} + \text{const.}$$

$$= V \int \frac{d^3 k}{(2\pi)^3} \left[\frac{E_{\boldsymbol{k}}}{2} + T \log\left(1 - e^{-E_{\boldsymbol{k}}/T}\right) \right]$$

ゼロ点エネルギー ($\hbar\omega/2$)　　熱的励起

ゼロ点エネルギー部分を無視して部分積分すると、
通常の圧力を得る。

$$P = -\Omega / V = \int \frac{d^3 k}{(2\pi)^3} \frac{\boldsymbol{k}\cdot\boldsymbol{v}}{3} \frac{e^{-E_{\boldsymbol{k}}/T}}{1 - e^{-E_{\boldsymbol{k}}/T}} \qquad \left(\boldsymbol{v} = \frac{\partial E_{\boldsymbol{k}}}{\partial \boldsymbol{k}} \right)$$

場の理論 → *Euclid* 化 + *Imag. Time* → 統計力学

Matsubara Frequency Summation

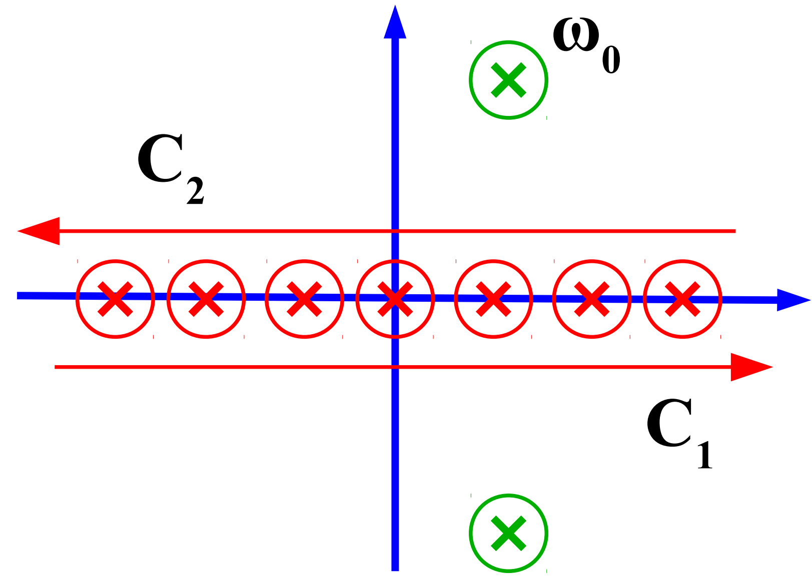
- **Contour integral technique**

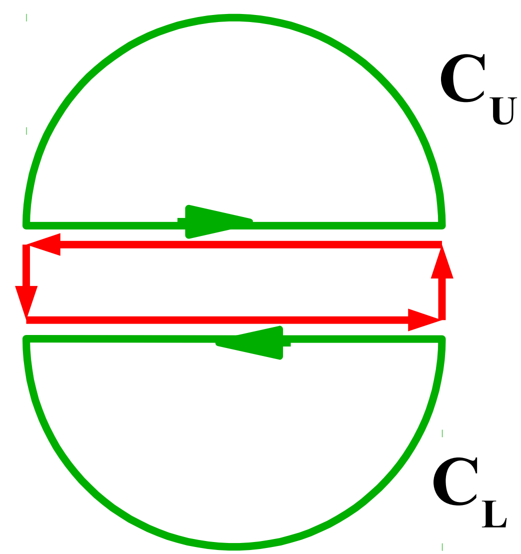
$$S = T \sum_n g(\omega_n = 2\pi n T, \pi(2n+1)T)$$

$$= \pm \int_{C_1+C_2} \frac{dz}{2\pi} \frac{g(z)}{e^{i\beta z} \mp 1} = \mp i \sum_{\omega_0} \frac{\mathrm{Res}\, g(\omega_0)}{e^{i\beta\omega_0} \mp 1}$$

(g: meromorphic (有理型),
no pole on real axis,
decreases faster than 1/ω at ω → ∞)

- **Applicable to more general cases !**
- **Anti-periodic condition → Fermi-Dirac dist.**





$C_1+C_2+C_U+C_L=0$

- **Example: $g(\omega)=1/(\omega^2+E^2)$**
  → $\omega_0 = \pm iE$, Res $g = \pm 1/2iE$

$$S = \frac{1}{2E} \frac{e^{\beta E} \pm 1}{e^{\beta E} \mp 1}$$

Fermion

- **Lagrangian**

$$L = \bar{N}\left(i\gamma^{\mu}\partial_{\mu} - m\right)N$$

- **Euclidean**

$$(x_{\mu})_E = (\tau = it, \boldsymbol{x}), \quad (\gamma_{\mu})_E = (\gamma_4 = i\gamma^0, \boldsymbol{\gamma})$$

$$L_E = \bar{N}\left(-i\gamma_{\mu}\partial_{\mu} + m\right)N$$

- **Grassman number**
経路積分において、フェルミオンは反可換な Grassmann 数

$$\int d\chi \cdot 1 = \text{anti-comm. constant} = 0 \quad , \quad \int d\chi \cdot \chi = \text{comm. constant} \equiv 1$$

$$\int d\chi\, d\bar{\chi} \exp\left[\bar{\chi} A \chi\right] = \int d\chi\, d\bar{\chi} \frac{1}{N!}(\bar{\chi} A \chi)^N = \cdots = \det A$$

$$= \exp\left[-(-\log\det A)\right]$$

Bi-linear Fermion action leads to -log(det A) effective action

- **Example: Relativistic Mean Field (RMF)**

$$L = \bar{\psi}(i\gamma^\mu \partial_\mu - m - \Sigma)\psi + L_{\text{meson}}(\Phi) \quad (\Phi = \sigma, \omega, \rho)$$
$$\Sigma = g_\sigma \sigma + \gamma^0 (g_\omega \omega^0 + g_\rho \rho^0 \boldsymbol{\tau})$$

- **Euclid 化 + 化学ポテンシャルの導入**

$$\begin{aligned} Z &= \int D\psi D\bar{\psi} D\Phi \ \exp\left[-\int d^4x (L - \mu\psi^+\psi)\right] \\ &= \int D\psi D\bar{\psi} D\Phi \ \exp\left[-\int d^4x \{\bar{\psi} D\psi + L_{\text{meson}}(\Phi)\}\right] \\ &= \int D\Phi \exp\left[-S_{\text{eff}}(\Phi; T, \mu)\right] \end{aligned}$$
$$D = -i\gamma\partial - \mu\gamma^0 + m + \Sigma$$

- **有効作用**

$$S_{\text{eff}} = S_{\text{eff}}^{(F)} + S_{\text{meson}} = -\sum_{n,\boldsymbol{k}} \log\det D_{n,\boldsymbol{k}} + \int d^4x \, L_{\text{meson}}$$

RMF (cont.)

- 一様な場を仮定 → Fourier 変換により D をブロック対角化

$$D_{n,\boldsymbol{k}}=\gamma^0\left(-i\omega_n-\left(\mu-V^0\right)\right)+\boldsymbol{\gamma}\cdot\boldsymbol{k}+M+g_\sigma\sigma$$

$$\rightarrow \det D=\left[\left(\omega_n+i\mu^*\right)^2+E^{*2}\right]^2$$

$$\mu^*=\mu-g_\omega\omega^0-g_\rho\rho^0\boldsymbol{\tau},\quad E^*=\sqrt{\boldsymbol{k}^2+M^{*2}},\quad M^*=m+g_\sigma\sigma$$

- 松原振動数和を実行

$$F_{\text{eff}}^{(F)}=-\frac{d_f}{2}\int\frac{d^3k}{(2\pi)^3}\left[E^*+T\log\left(1+e^{-(E^*-\mu^*)/T}\right)+T\log\left(1+e^{-(E^*+\mu^*)/T}\right)\right]$$

ゼロ点　　粒子 (核子)　　反粒子 (反核子)

- 温度 0 の場合

$$F_{\text{eff}}^{(F)}=-\frac{d_f}{2}\int^{\Lambda}\frac{d^3k}{(2\pi)^3}E^*+d_N\int^{k_F}\frac{d^3k}{(2\pi)^3}E^*-\mu^*\rho_B\quad\left(d_N=d_f/2\right)$$

ゼロ点エネルギーは核子のループから現れる
(RMF では通常無視)

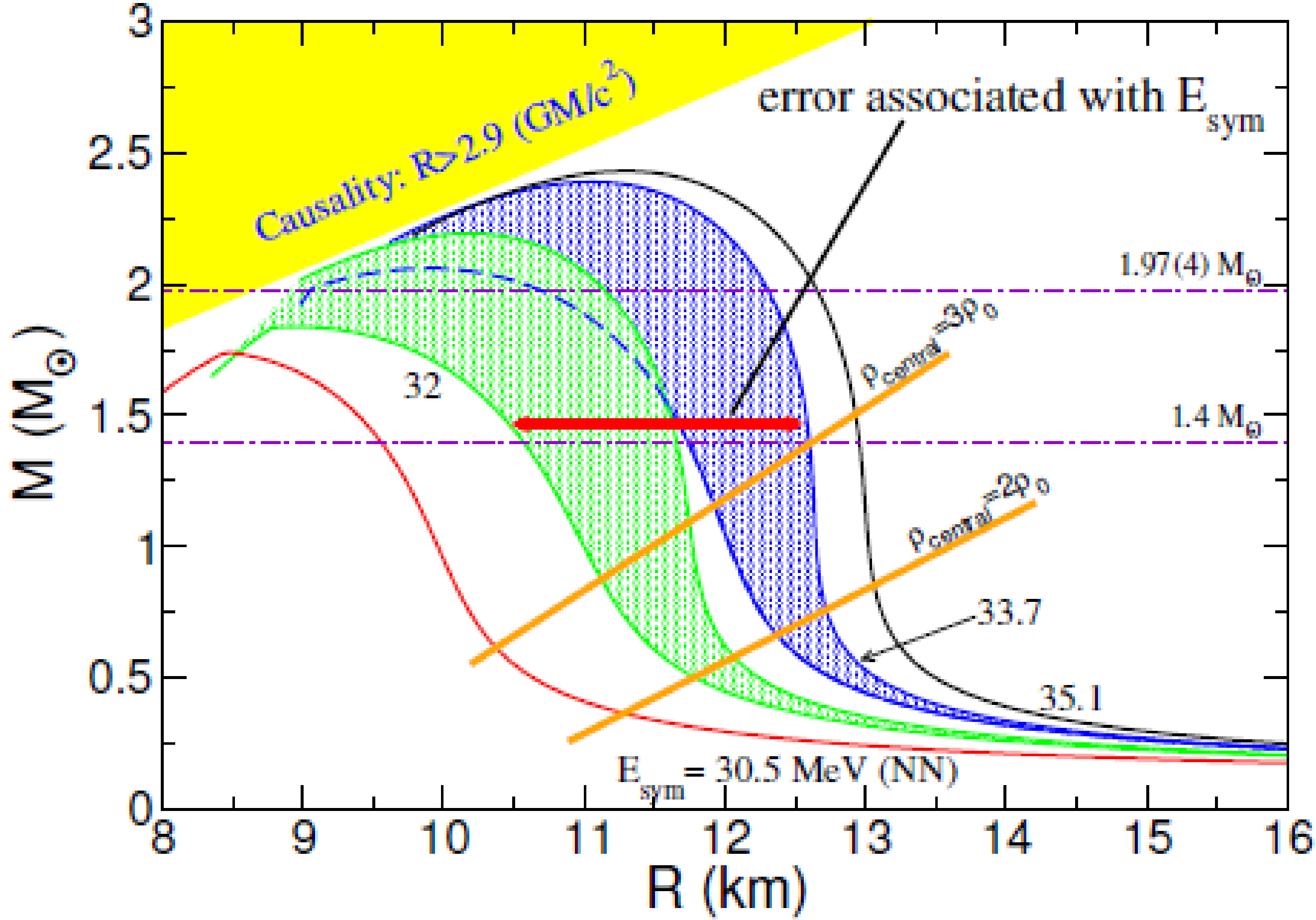
YUKAWA INSTITUTE FOR THEORETICAL PHYSICS　YIPQS　YITP Kyoto

有限温度の場の理論 Short Summary

- 量子力学・場の理論
  - 経路積分によって通常の数 (c- 数) による記述が可能。
  - ただし、全ての経路の足し合わせが必要。
- 統計力学・分配関数
  - 虚時間による定式化により、分配関数が経路積分で表現できる。
  - ユークリッド化することにより、時間と空間を同様に扱える。
    ただし、(虚) 時間 τ ($=it$) の範囲には制限がつく。($0 \leq \tau \leq \beta$)
  - 分配関数はすべての状態での期待値の和
    → $\tau=0, \beta$ で周期境界条件 → 松原振動数 $\omega_n = 2\pi n T$
    (フェルミオンの場合には反周期境界条件、$\omega_n = 2\pi (n+1/2) T$)
- 自由スカラー場の分配関数
  - 松原振動数についての和 → ゼロ点のエネルギー + 熱的部分
- 勉強すべきこと
  - グリーン関数、摂動論、Hard Thermal Loop 、Debye 遮蔽、...

Symmetry Energy affects MR Relation of NS

- **Nuclear pressure at ρ_0 comes ONLY from Esym, then Esym dominates pressure around ρ_0 !**
- **5 MeV Difference in Esym results in (3-4) km difference in R_{NS} prediction.**



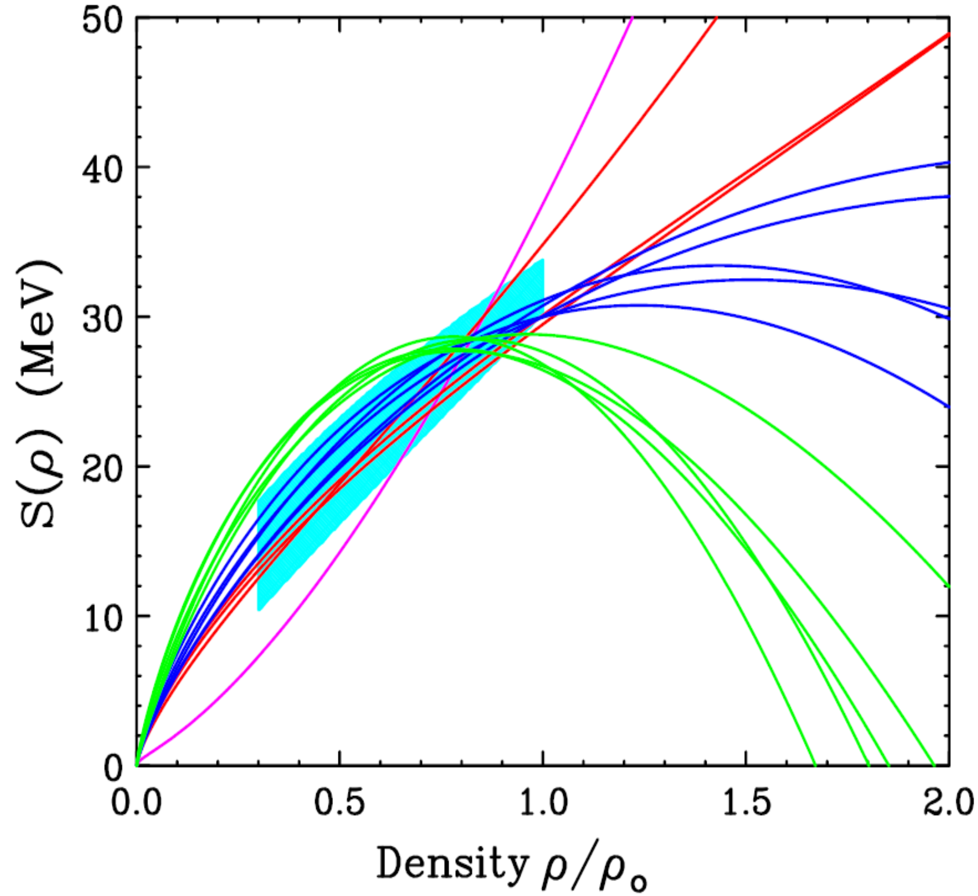
Gandolfi, Carlson, Reddy, PRC 032801, 85 (2012).

Symmetry Energy

- **Summary of Nuclear Symmetry Energy workshop NuSym11 http://www.smith.edu/nusym11**

  $$E_{sym}(\rho_0) = 31\text{-}34 \text{ MeV}, L = 50\text{-}110 \text{ MeV}$$

  extracted from various observations.
  - **Mass formula** *Moller ('10)*
  - **Isobaric Analog State** *Danielewicz, Lee ('11)*
  - **Pygmy Dipole Resonance** *Carbone+ ('10)*
  - **Isospin Diffusion** *Tsang et al. ('04)*
  - **Neutron Skin thickness** *J.Zenihiro+ ('10)*
- これらの多くは ρ_0 以下の密度での Esym に敏感。



M. B. Tsang et al., Phys. Rev. C 86 (2012) 015803.