核多体系物理学

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- 1. 有限温度･密度における場の理論入門
- (a) 経路積分･松原和･自由エネルギー
- (b)QCD におけるカイラル相転移、南部 - ヨナラシニョ模型、強結合格子 QCD
- 2. 状態方程式を記述する理論模型
- (a) 核力と位相差、有効相互作用
- (b) 核物質の状態方程式、平均場理論
- 3. 原子核反応理論
- (a) 核子 - 核子散乱、ハドロン - 原子核反応
- (b) 流体力学、輸送理論
- (c) ハイパー核･中間子核生成反応の概観と直接反応

シラバス

- [授業の概要・目的]
  - 核子・ハドロン・クォークからなる多体系の性質を量子色力学(QCD)、状態方程式、および核反応論の観点から議論する。強い相互作用の基本理論であるQCDの基本的性質、核物質の状態方程式を記述するために必要となる核多体理論(平均場理論、有効相互作用、有限温度・密度での場の理論、強結合格子QCD)、ハイパー核生成反応や重イオン反応を理解する上で必要とされる原子核反応理論(直接反応、輸送模型等)、等の理論の枠組について解説すると共に、これらについての最近の研究成果についても紹介する。
- [到達目標] 次の事項を習得する。
  - 有限温度・密度の場の理論、散乱理論、有効相互作用理論の基本を理解する。
  - 核力から有効相互作用、あるいは場の理論から状態方程式につながる理論体系を理解する。
  - 簡単な平均場理論・直接反応理論の範囲内で状態方程式・反応スペクトルを計算するための手法

Contents

- 核子･ハドロン･クォーク物質の相互作用と状態方程式について以下の内容で講義する。
  - 1. 有限温度･密度における場の理論入門
    - (a) 経路積分･松原和･自由エネルギー
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    - (c) ハイパー核･中間子核生成反応の概観と直接反応
- 講義回数は 1(全体の概要講義 (初回) を含めて 6 回), 2(6 回), 3(4 回) を予定している。

核力と位相差
(white board にて)

有効相互作用
(一部 white board にて)

Hartree-Fock Theory

- 平均場理論＝多体問題の基本

$$\delta \frac{\langle \Phi | H | \Phi \rangle}{\langle \Phi | \Phi \rangle} = 0, \qquad |\Phi\rangle = \det\{\phi_1 \cdots \phi_N\}: \text{Slater determinant}$$

- 電子系ではエネルギーをほぼ再現
- 原子核ではナイーブな HF は破綻
  - 短距離での斥力コア → エネルギー ＝∞
  - 2 体相関が決定的

$$\rho_2(\mathbf{r}_1, \mathbf{r}_2) = 0, \quad \text{for } |\mathbf{r}_1 - \mathbf{r}_2| < c$$

→ Brueckner 理論 (G-matrix)

原子 ·分子など、電子系

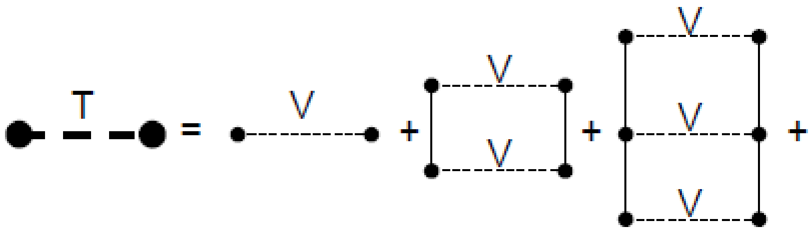
	HF	Exp
He	−2.86	−2.90
Li	−7.43	−7.48
Ne	−128.55	−128.94
Ar	−526.82	−527.60

原子単位 (27.2 eV)

Brueckner Theory

- **Lippmann-Schwinger Eq.**

$$T = V + VG_0 T$$



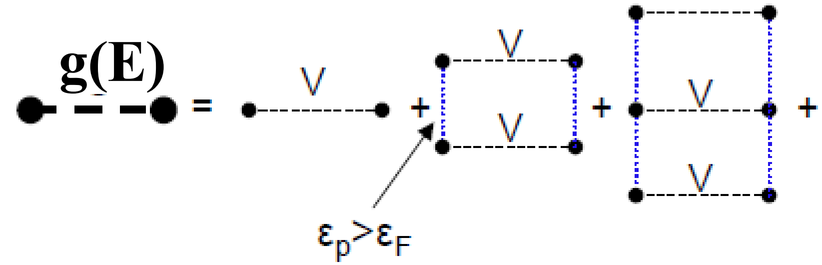
  - **V が singular でも T は有限**

$$V\left|\Psi_{\mathbf{k}}^{(+)}\right\rangle = T\left|\mathbf{k}\right\rangle$$

- **原子核中での 2 体散乱**
  → パウリ原理

$$g(E) = V + V\frac{Q}{E - H_0}g(E)$$

$$Q = 1 - \sum_{i,j<F}\left|ij\right\rangle\left\langle ij\right|$$



  - **原子核中では中間状態で**
    フェルミエネルギー以上の状態のみ
    伝播可能

$$V\left|\Psi\right\rangle = g(E)\left|\Phi\right\rangle$$

2 体相関を含む複雑な状態

2 体相関の無い状態 (E.g. Slater det.)

- **核内での散乱行列 =g-matrix**

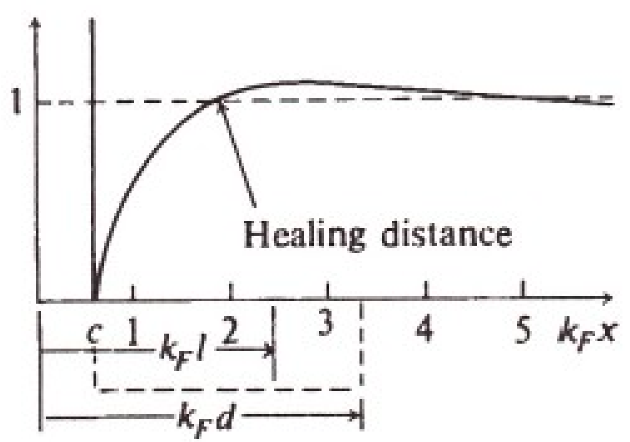
Healing distance

- (波動関数についての)
  Bethe-Goldstone 方程式

$$g_{12} = v_{12} + v_{12} \frac{Q_{12}}{E - (t_1 + t_2 + U_1 + U_2)} g_{12}$$

$$\rightarrow \left[E - (t_1 + t_2 + U_1 + U_2) \right] \psi_{12} = Q_{12} v_{12} \psi_{12}$$

  - BG 方程式の解は、$k_F\, l \sim 1.9$ 程度の距離で通常の平面波にほぼ一致する。
    (Healing distance)
    → 独立粒子描像



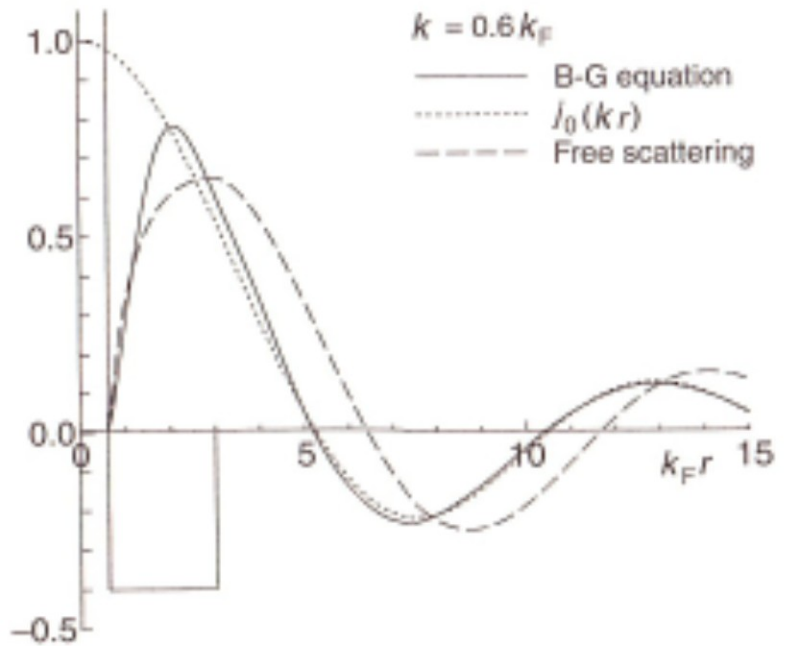
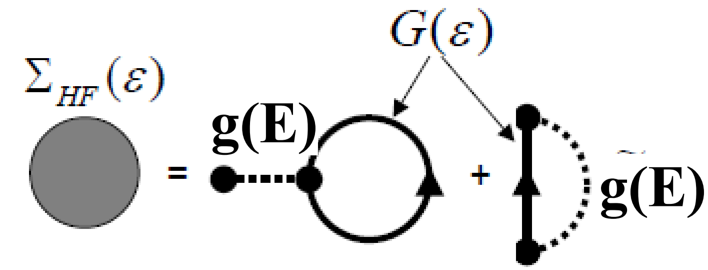


図 2.17 $k = 0.6\, k_F$ の場合の Bethe-Goldstone 方程式の解 (実線) と，自由空間内の 2 粒子散乱 (破線) および自由粒子の相対波動関数 (点線) の比較

$k_F = 1.27\ \mathrm{fm}^{-1}$，芯半径は $k_F r_c = 0.62$，井戸型ポテンシャルの半径は $k_F r_a = 3.0$，有効質量は $M^*/M = 0.6$ ととられている．

Brueckner-Hartree-Fock theory

- **g-matrix を 2 体相互作用とする HF**
  = Brueckner-Hartree-Fock

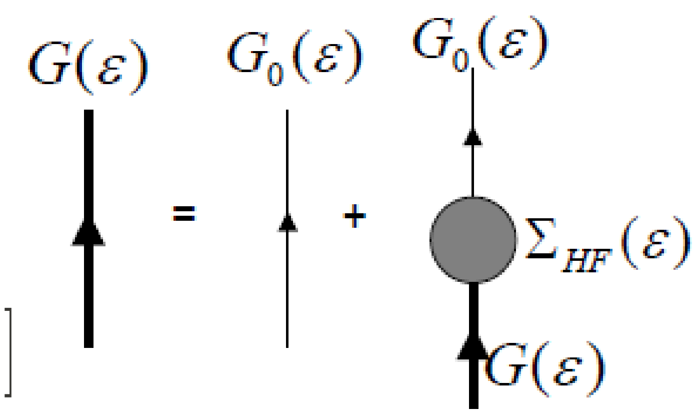


$$H = H_0 + V\,,\quad V = \frac{1}{2}\sum_{i \neq j} V_{ij}$$

$$H_0 = \sum_i \left[-\frac{\hbar^2}{2\mathrm{m}} \nabla_i^2 + U_i \right]$$

$$g(E) = V + V \frac{Q}{E - H_0} g(E)$$

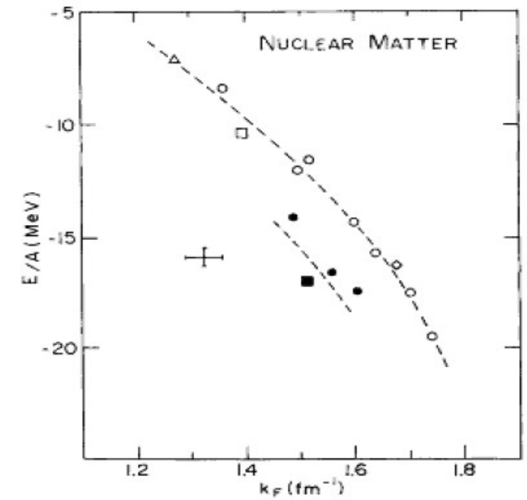
$$U_i(\varepsilon_i) = \sum_j \left[g_{ij,ij}(\varepsilon_i + \varepsilon_j) - g_{ij,ji}(\varepsilon_i + \varepsilon_j) \right]$$



$$E_{\mathrm{BHF}} = \sum_i^{\mathrm{occ.}} \langle i | -\frac{\hbar^2}{2\mathrm{m}} \nabla^2 | i \rangle + \frac{1}{2} \sum_{i \neq j}^{\mathrm{occ.}} \langle ij | g(\varepsilon_i + \varepsilon_j) | ij - ji \rangle$$

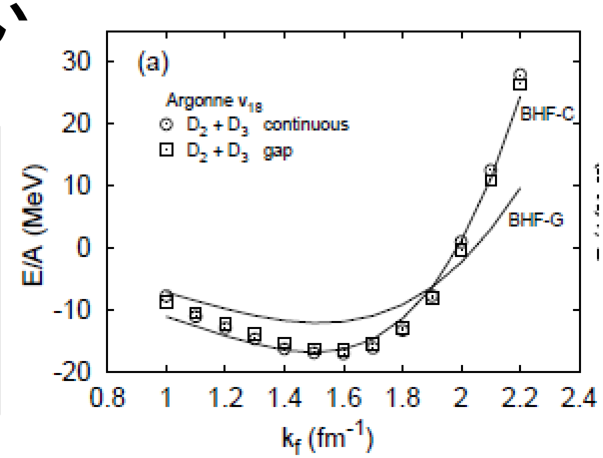
- **Self-consistent treatment**
  U → g-matrix & φ (s.p.w.f) → U

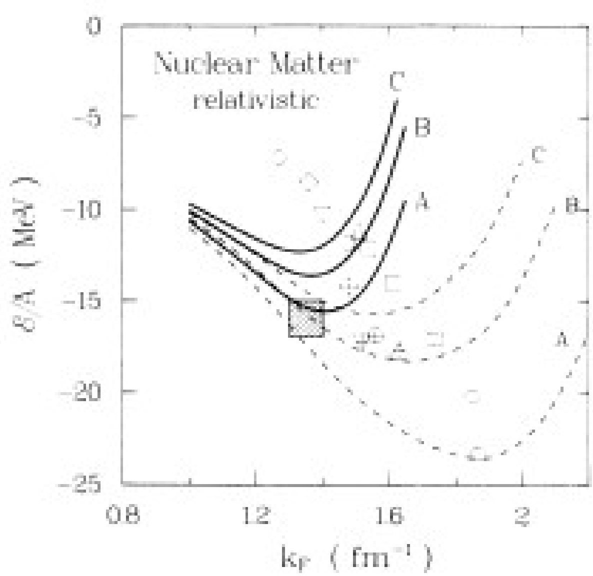
Brueckner-Hartree-Fock theory (cont.)



- **成功点**
  - **核物質の飽和性を定性的に説明**
  - **殻模型(独立粒子描像)の基礎を与える**
  - **有効核力の状態依存性を説明**
- **問題点**
  - **飽和点 (飽和密度、飽和エネルギー) の定量的理解 (Coester line)→ Relativity or 3 体力**
  - **展開の高次項→ Continuum choice では 3 体クラスター効果は小さい**
  - **スピン軌道力が足りない**

問題は残っているが、
現実的核力から出発して
多体問題に適用する
有効な手法





Why do we study Nuclear Matter EOS ?

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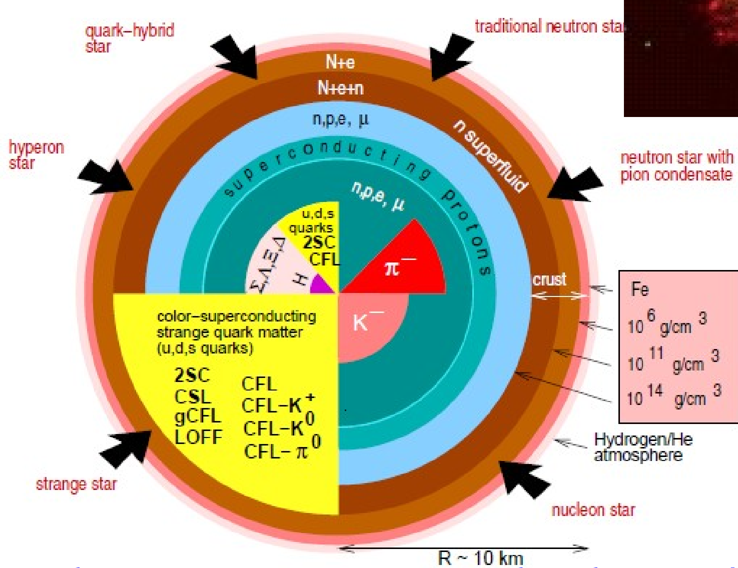
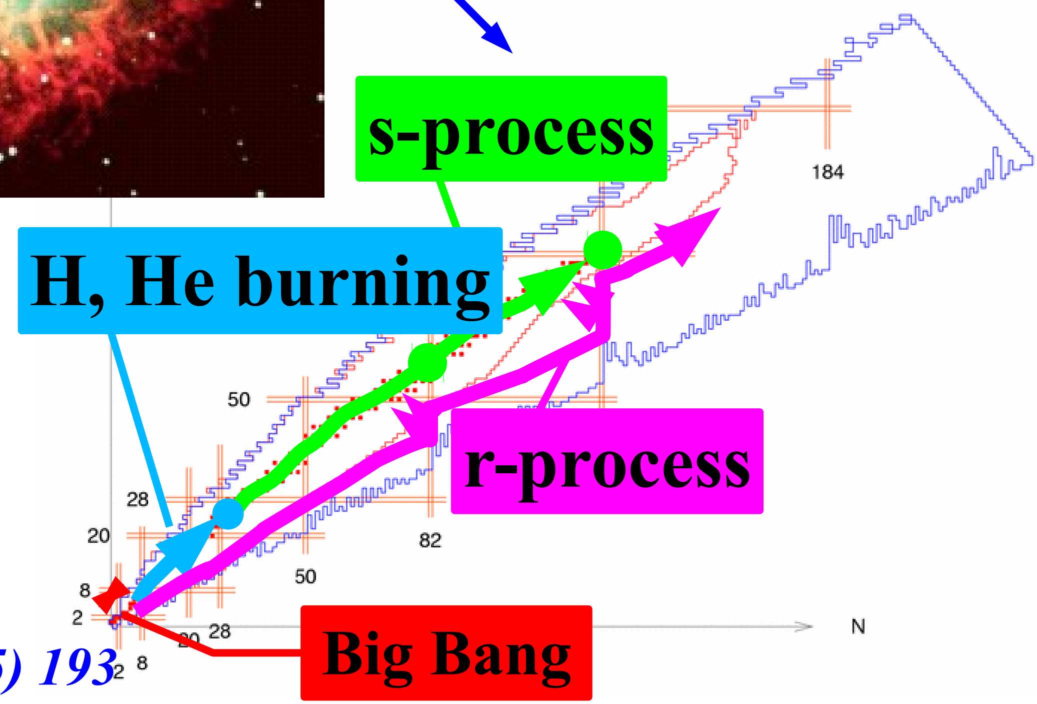
- **Answer 2: Since nuclear matter EOS is decisive in compact astrophysical objects such as neutron stars, supernovae, and black hole formation, EOS is important to understand where atomic elements are made.**

Supernova

Neutron Star



Nucleosynthesis



F. Weber, Prog. Part. Nucl. Phys. 54 (2005) 193

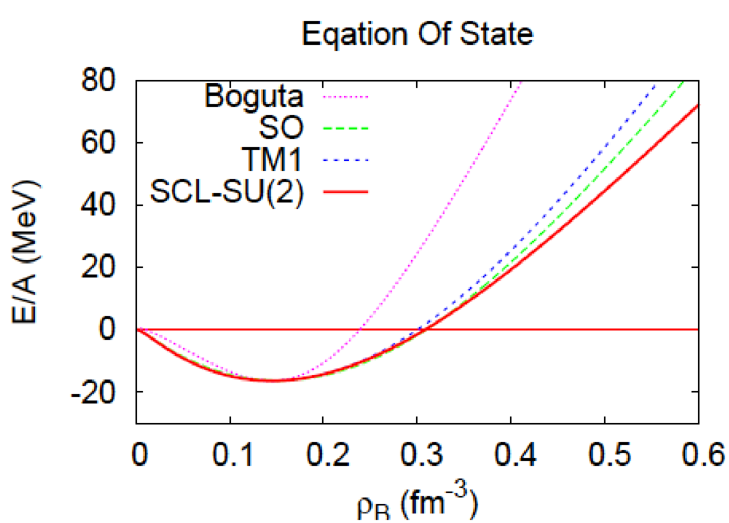
Why do we study Nuclear Matter EOS ?

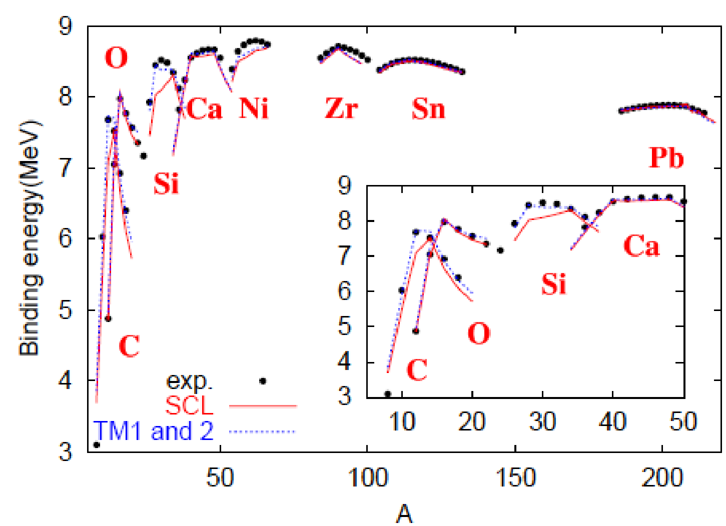
- **Answer 1: Since bulk nuclear properties are mainly determined by nuclear matter EOS, it is important for nuclear physics.**
  - **Nuclear Radius → Saturation of Density**

    $R_A = r_0 A^{1/3}$ (r_0=1.2 fm)
  - **Nuclear Binding Energy (Bethe-Weizsacker Formula)**

$$B(A,Z) = a_{vol} A - a_{surf} A^{2/3} - a_{Coulomb} \frac{Z^2}{A^{1/3}} - a_{sym} \frac{(N-Z)^2}{A} + a_{pair} \delta(A,Z) A^{-3/4}$$

Nuclear Matter





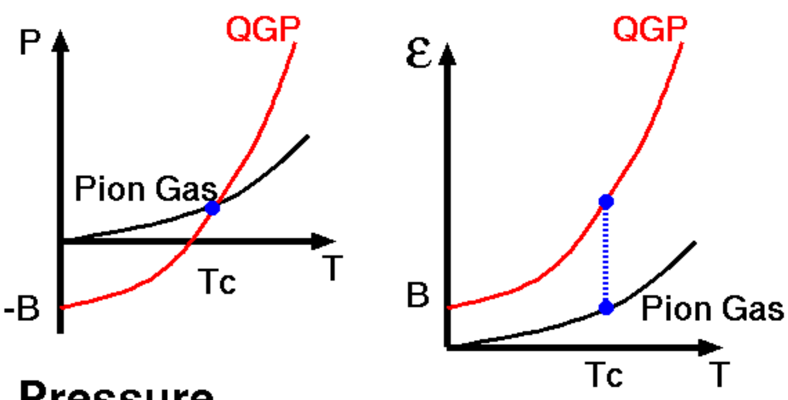
Why do we study Nuclear Matter EOS ?

- **Answer 3: Since the EOS should have singularity (or at least sudden change) at phase boundary, it would be possible to catch the signal of phase transition in nuclear collisions.**

- **Pressure and Energy Density of Free Massless Gas**

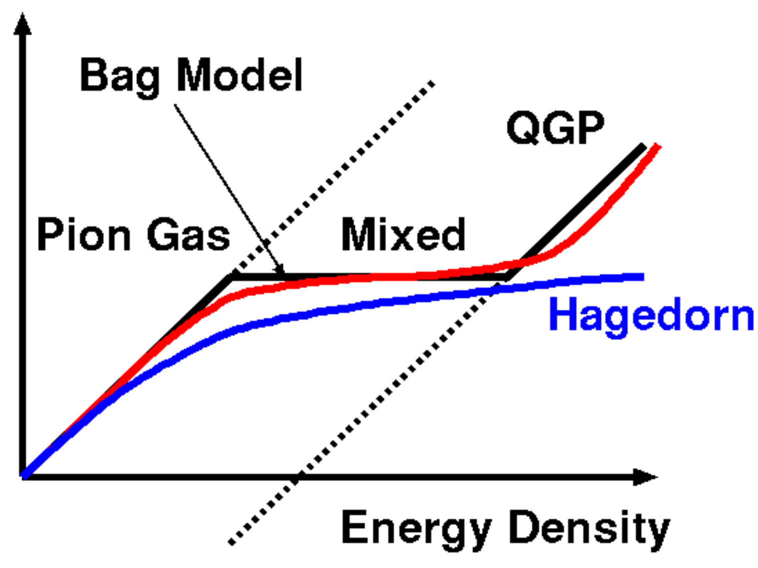
$$P=\frac{\pi^2}{90}N_B T^4 \quad , \quad \epsilon=\frac{\pi^2}{30}N_B T^4$$

N_B = Bosonic DOF (7/8 for Fermions)



- **Hadron Gas ~ 3 pions (N_B=3)**

$$P_\pi=\frac{\pi^2}{30}T^4 \quad , \quad \epsilon_\pi=\frac{\pi^2}{10}T^4$$



- **QGP N_B=16(gluon)+24 x 7/8 (quarks) and Bag Pressure**

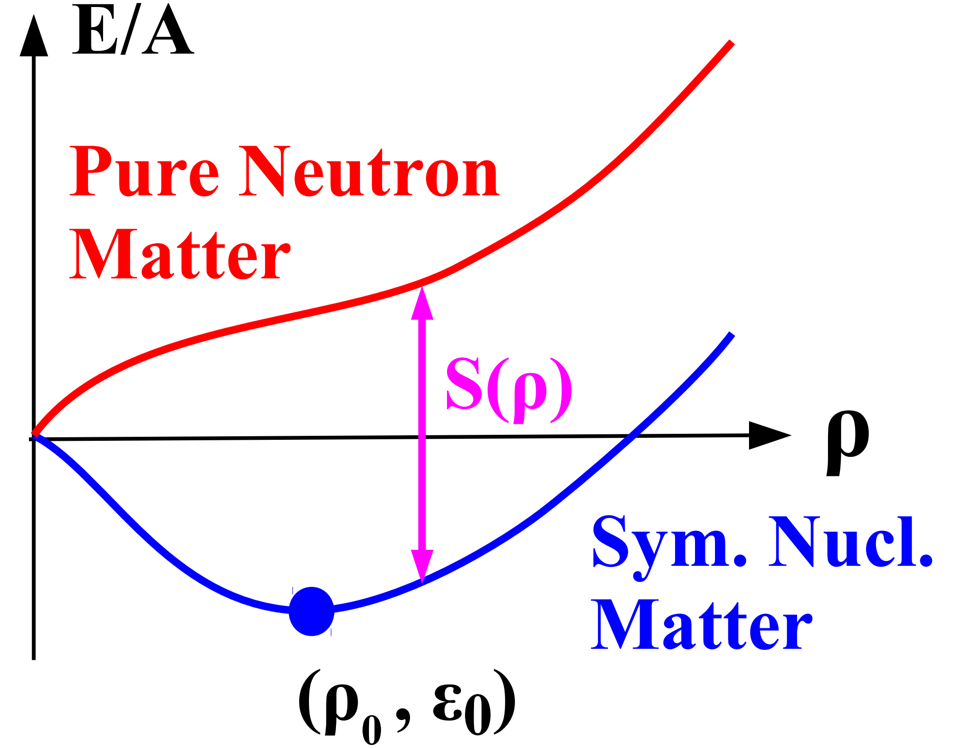
$$P_{QGP}=\frac{37\pi^2}{90}T^4-B \quad \epsilon_{QGP}=\frac{37\pi^2}{30}T^4+B$$

Nuclear Matter EOS

- **Energy per nucleon in nuclear matter**

$$E/A(\rho,\delta)=E_{\mathrm{SNM}}(\rho)+S(\rho)\delta^2 \quad , \quad \delta=(N-Z)/A$$

- **Saturation point (ρ_0, ε_0)**
  - **$\rho_0 \sim 0.15\ \mathrm{fm}^{-3}$**
  - **$\varepsilon_0 = -a_v \sim -16$ MeV**
  - **(nuclear radius and mass)**
- **Symmetry energy**
  - **$S(\rho) = E_{PNM}(\rho) - E_{SNM}(\rho)$**
  - **$= E(\rho, \delta=1) - E(\rho, \delta=0)$**
  - **$S_0 = S(\rho_0) \sim 30$ MeV**
  - **(mass formula)**



Nuclear Matter EOS can be, in principle, determined by terrestrial (laboratory) nuclear physics experiments !

Nuclear Matter EOS

- **Additional two important parameters: K and L**
- **Pressure is given by the derivative of E via ρ**

$$P = \rho^2 \left(\partial(E/A)/\partial\rho\right)$$

At ρ_0, L determines P

$$P = \rho_0 L/3 \quad (\text{at} \quad \rho = \rho_0)$$

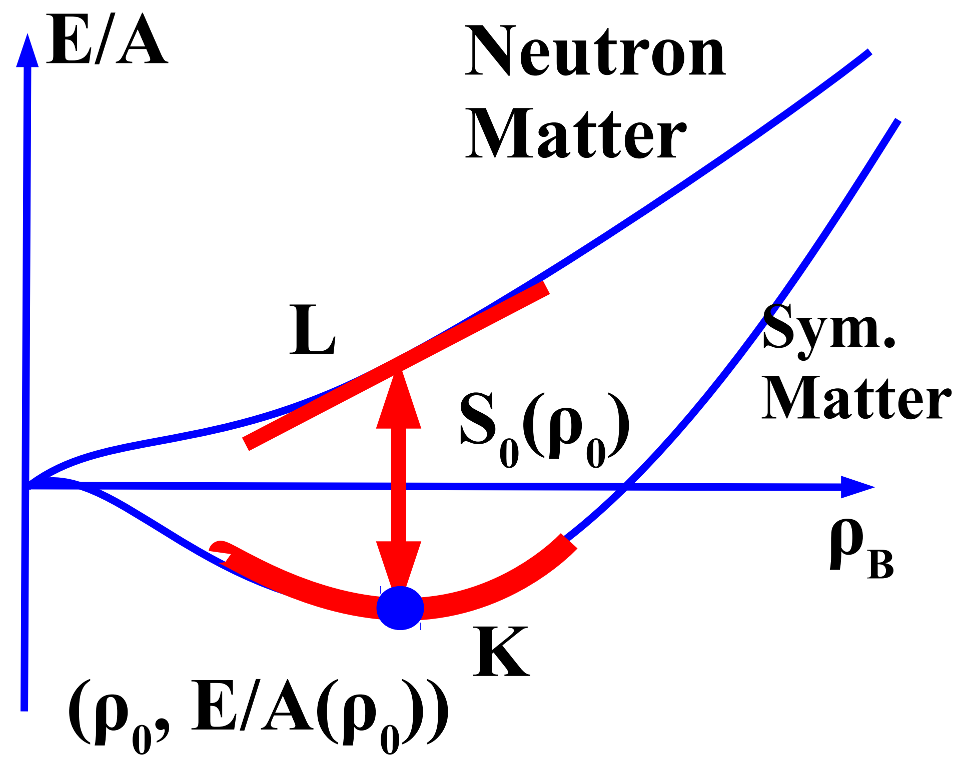
$$E/A(\rho,\delta) = E_{SNM}(\rho) + S(\rho)\delta^2 + O(\delta^4)$$

Symmetric Nuclear Matter

$$E_{SNM}(\rho) = E_{SNM}(\rho_0) + \frac{K(\rho-\rho_0)^2}{18\rho_0^2} + O((\rho-\rho_0)^3)$$

Symmetry Energy $(\delta = (N-Z)/A = 1 - 2Y_p)$

$$S(\rho) = S_0 + \frac{L(\rho-\rho_0)}{3\rho_0} + \frac{K_{sym}(\rho-\rho_0)^2}{18\rho_0^2} + O((\rho-\rho_0)^3)$$

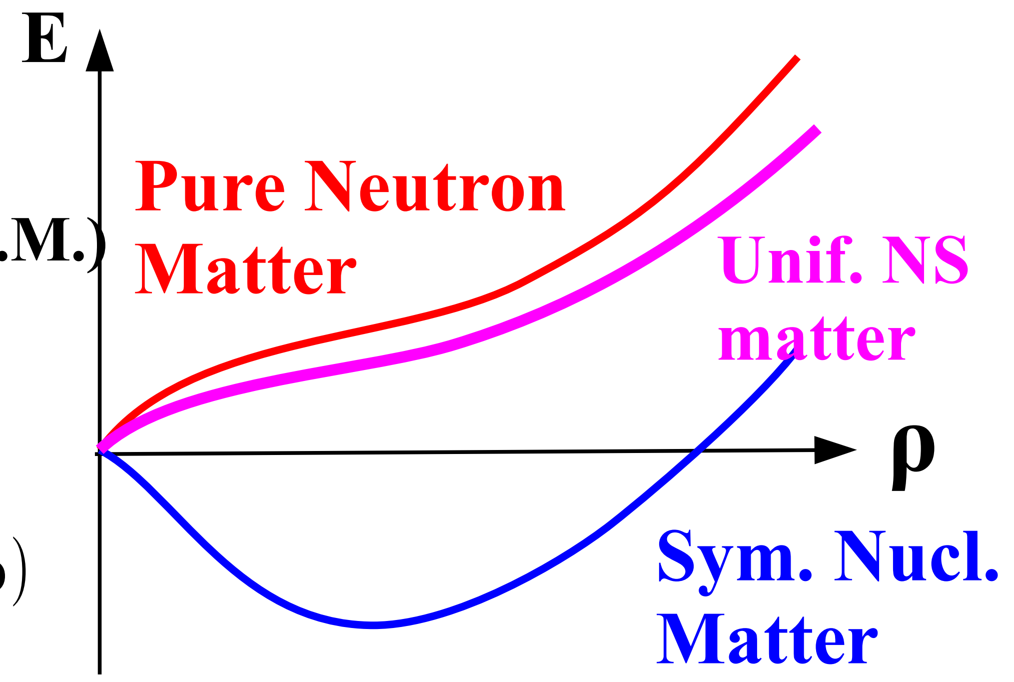


Neutron Star Matter EOS

- **What happens in low-density uniform neutron star matter ?**
  - **Constituents = proton, neutron and electron**
  - **Charge neutrality → # of electons= # of protons ($\rho_e=\rho_p=\rho(1-\delta)/2$)**

$$E_{\rm NSM}(\rho)=E_{\rm NM}(\rho,\delta)+E_e(\rho_e=\rho_p)$$
$$=E_{\rm SNM}(\rho)+S(\rho)\delta^2+\frac{\Delta M}{2}\delta+\frac{3}{8}\hbar k_F(1-\delta)^{4/3}$$

(electron mass neglected, neutron-proton mass diff. incl. k_F= Fermi wave num. in Sym. N.M.)



  - **δ is optimized to minimize energy per nucleon**

$$E_{\rm NSM}(\rho)\leq E_{\rm NM}(\rho,\delta=1)=E_{\rm PNM}(\rho)$$

対称エネルギーの起源

- **Fermi Gas model での核子あたりの運動エネルギー**

$$E_{\mathrm{sym},K} = \frac{Z}{A}\frac{3}{5}\frac{\hbar^2 k_{\mathrm{F}p}^2}{2m} + \frac{N}{A}\frac{3}{5}\frac{\hbar^2 k_{\mathrm{F}n}^2}{2m} = \frac{3}{5}E_{\mathrm{F}}\frac{1}{2}\left[(1-\delta)^{5/3}+(1+\delta)^{5/3}\right]$$

$$\simeq \frac{3}{5}E_{\mathrm{F}} + \frac{1}{3}E_{\mathrm{F}}\delta^2 + \mathcal{O}(\delta^4)$$

a_{sym} (FG)=E_F/3 ~ 11 MeV となり、質量公式の a_{sym} ~ 23 MeV (surface を考えると a_{sym} (vol) ~ 30 MeV) と比べて半分程度。残りは相互作用。

- **残りの半分の対称エネルギーを RMF で評価してみましょう。**

$$\Delta E_{\mathrm{sym},\rho} = \frac{1}{2}\frac{m_\rho^2 R^2}{\rho_B} = \frac{1}{2}\frac{g_\rho^2}{m_\rho^2}\rho_B\delta^2 = \Delta a_{\mathrm{sym}}\delta^2 \quad \left(R = \frac{g_\rho(\rho_n - \rho_p)}{m_\rho^2} = \frac{g_\rho \rho_B \delta}{m_\rho^2} \right)$$

$$g_\rho^2 = \frac{2 m_\rho^2 \Delta a_{\mathrm{sym}}}{\rho_B} \simeq (4.3)^2 \quad (a_{\mathrm{sym}} = 30\,\mathrm{MeV})$$

← **RMF par. より少し小さめ**

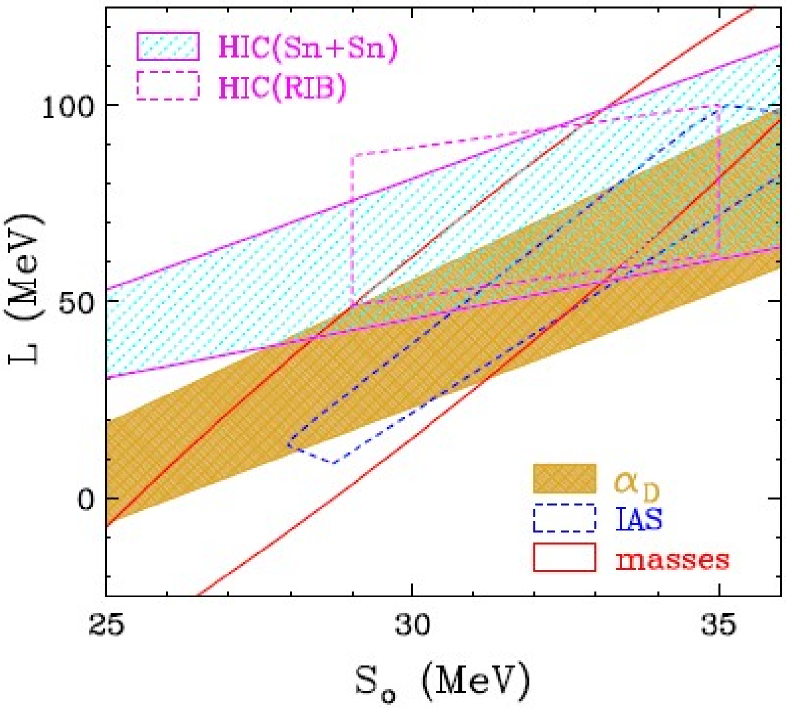
$$L \simeq E_F + 3\Delta a_{\mathrm{sym}} \simeq 90\,\mathrm{MeV}$$

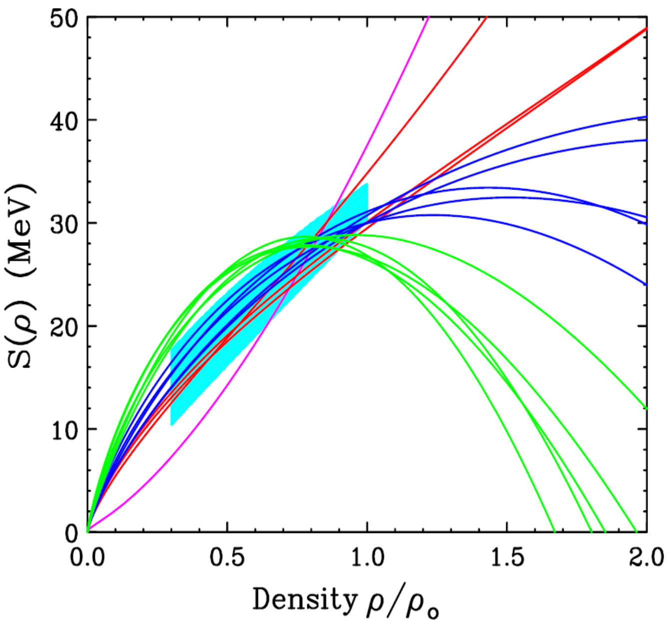
← **Optimal value より少し大きい**

Symmetry Energy

- **Symmetry Energy has been extracted from various observations.**
  - **Mass formula, Isobaric Analog State, Pygmy Dipole Resonance, Isospin Diffusion, Neutron Skin thickness, Dipole Polarizability, Asteroseismology**

Recent recommended value
S_0 = 30-35 MeV, L = 40-90 MeV
Is it enough for NS radii ?



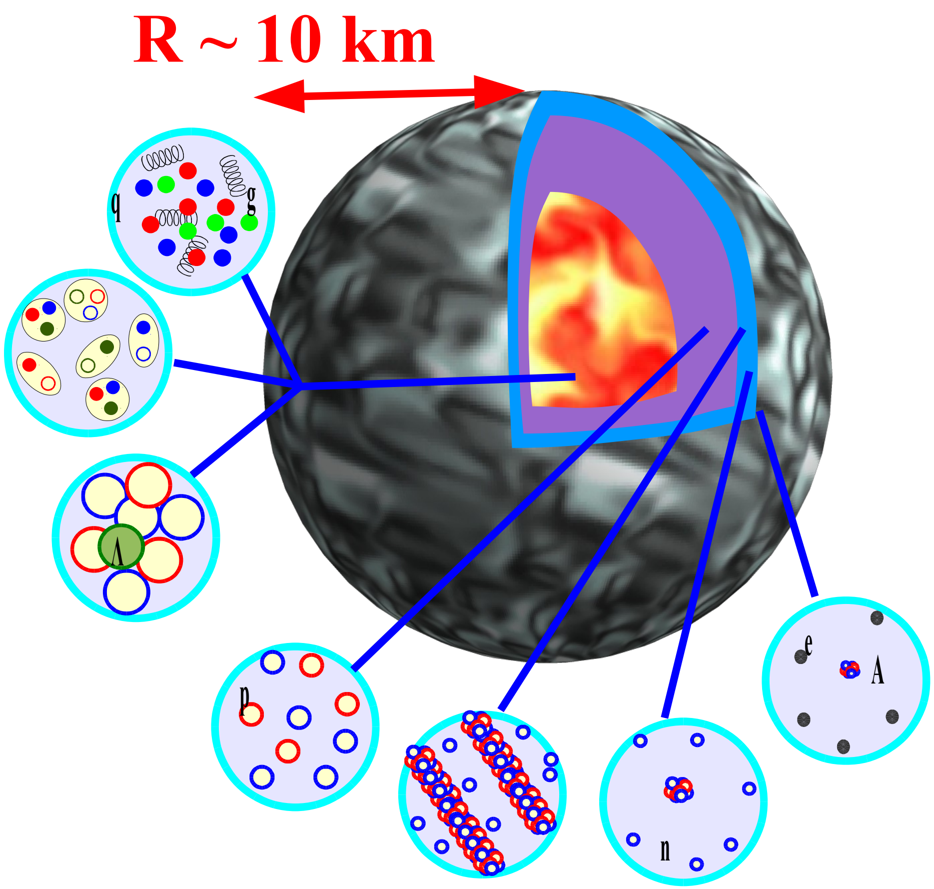


M.B.Tsang et al. (NuSYM2011), PRC 86 ('12)015803.

C.J.Horowitz, E.F.Brown, Y.Kim, W.G.Lynch, R.Michaels, A. Ono, J. Piekarewicz, M. B. Tsang, H.H.Wolter (NuSYM13), JPG41('14) 093001

Neutron Star

Star supported by nuclear force



$M \sim 1.4\, M_\odot$, $\rho_c \sim (3\text{-}10)\, \rho_0$

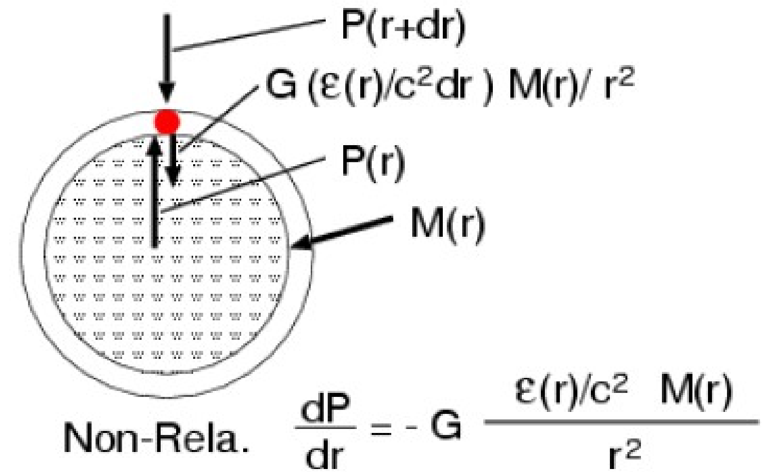
Wide density range → various constituents
NS = high-energy astrophysical objects
and laboratories of dense matter.

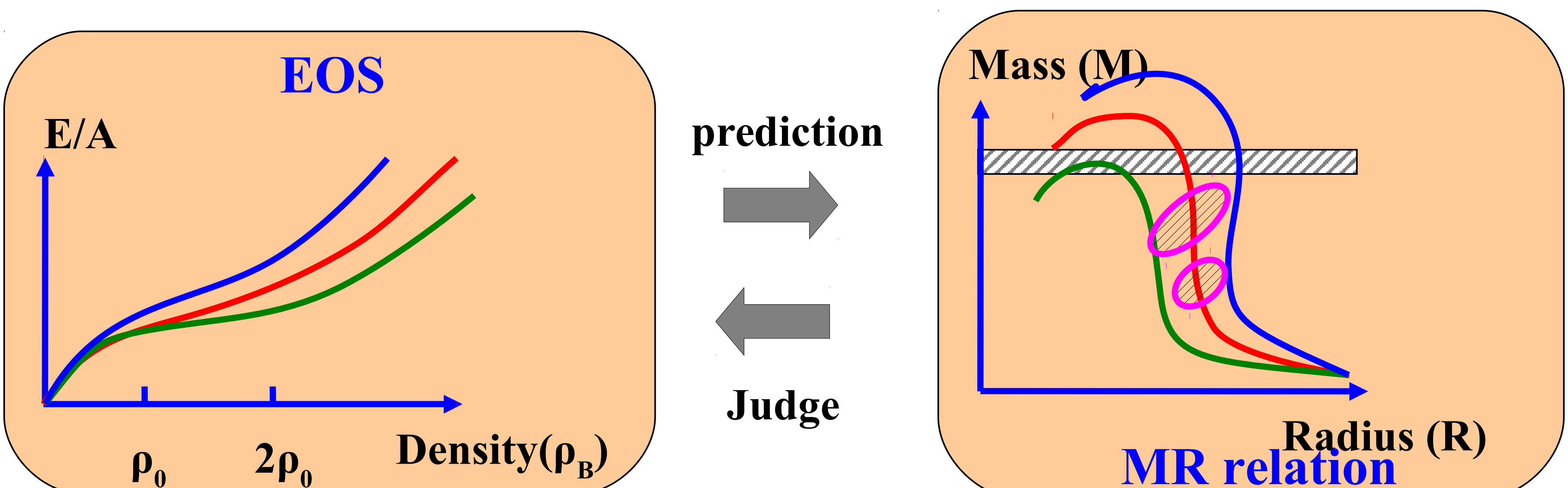
M-R curve and EOS

- **M-R curve and NS matter EOS has 1 to 1 correspondence**
  - **TOV(Tolman-Oppenheimer-Volkoff) equation =GR Hydrostatic Eq.**

$$\frac{dP}{dr} = -G\frac{(\varepsilon/c^2 + P/c^2)(M + 4\pi r^3 P/c^2)}{r^2(1-2GM/rc^2)}$$

$$\frac{dM}{dr} = 4\pi r^2 \varepsilon/c^2, \quad P = P(\varepsilon) \quad (\text{EOS})$$





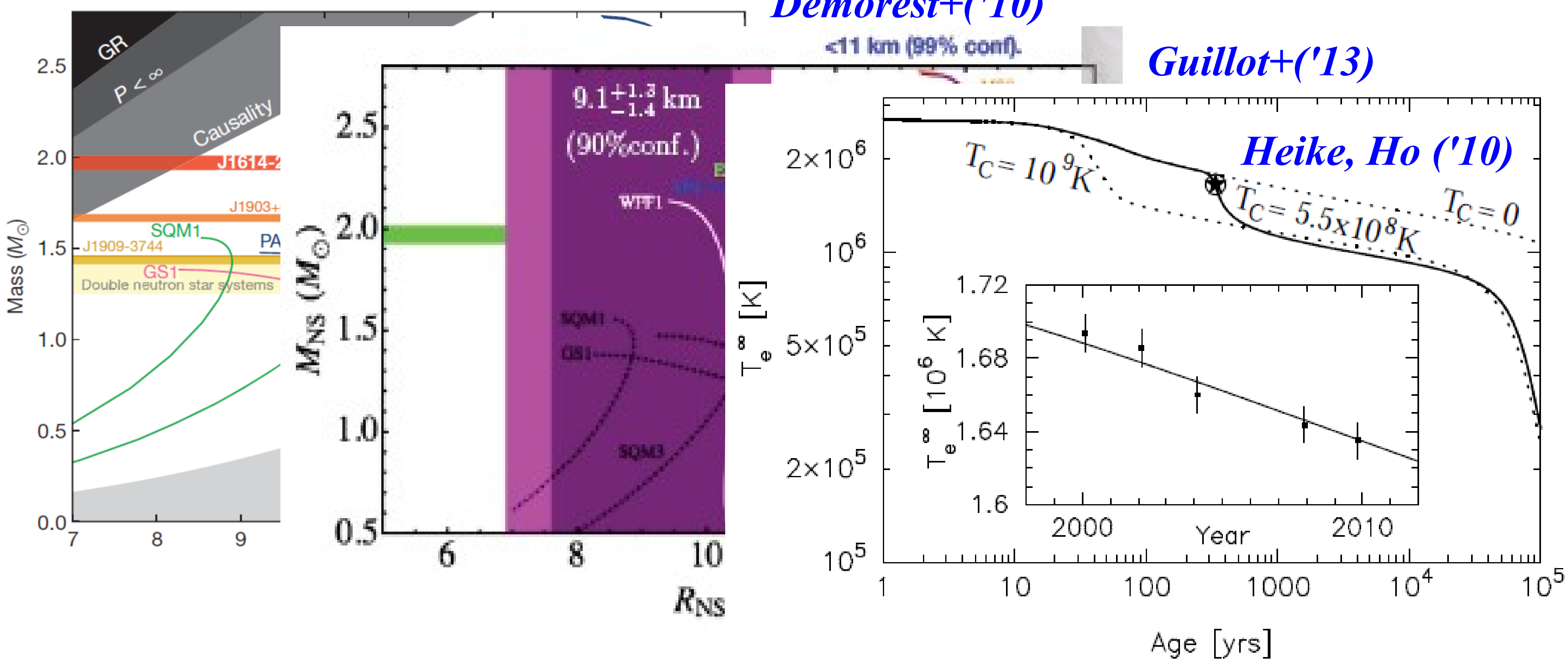
Current Big Puzzles in NS Physics

- **Massive NS puzzle (2 $M_\odot$ NS ?)**
- **Compact NS puzzle (9-10 km NS ?)**
- **Rapid NS cooling mystery (CasA cools too fast ?), Origin of Strong Mag. Field, …..**

Demorest+('10)

Guillot+('13)

Heike, Ho ('10)



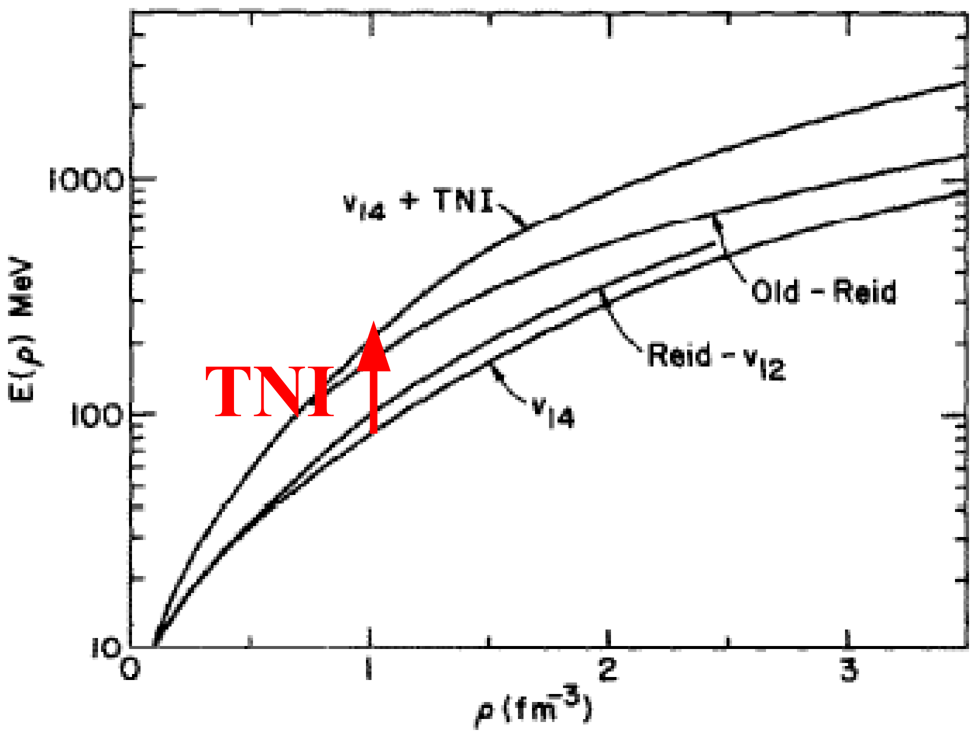
Nuclear Matter EOS Theories

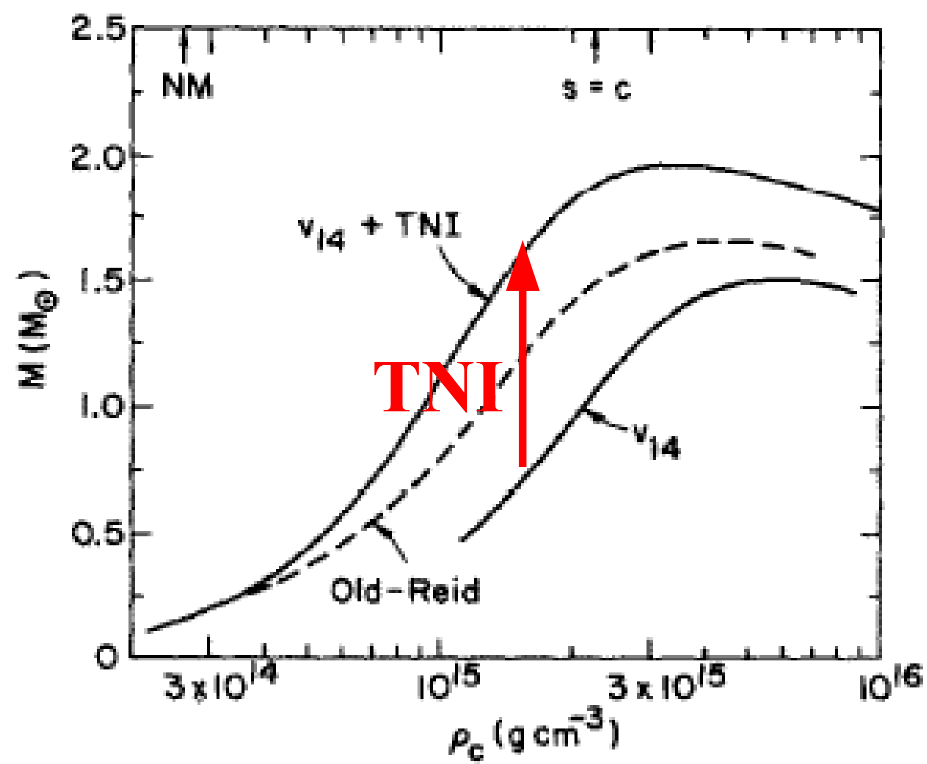
Theories/Models for Nuclear Matter EOS

- **Ab initio Approaches**
  - **LQCD, GFMC, Variational, BHF, DBHF, G-matrix, ...**
- **Mean Field from Effective Interactions ~ Nuclear Density Fuctionals**
  - **Skyrme Hartree-Fock(-Bogoliubov)**
    - **Non.-Rel.,Zero Range, Two-body + Three-body (or ρ-dep. two-body)**
    - **In HFB, Nuclear Mass is very well explained (Total B.E. ΔE ~ 0.6 MeV)**
    - **Causality is violated at very high densities.**
  - **Relativistic Mean Field**
    - **Relativistic, Meson-Baryon coupling, Meson self-energies**
    - **Successful in describing pA scatering (Dirac Phenomenology)**

Variational Calculations (1)

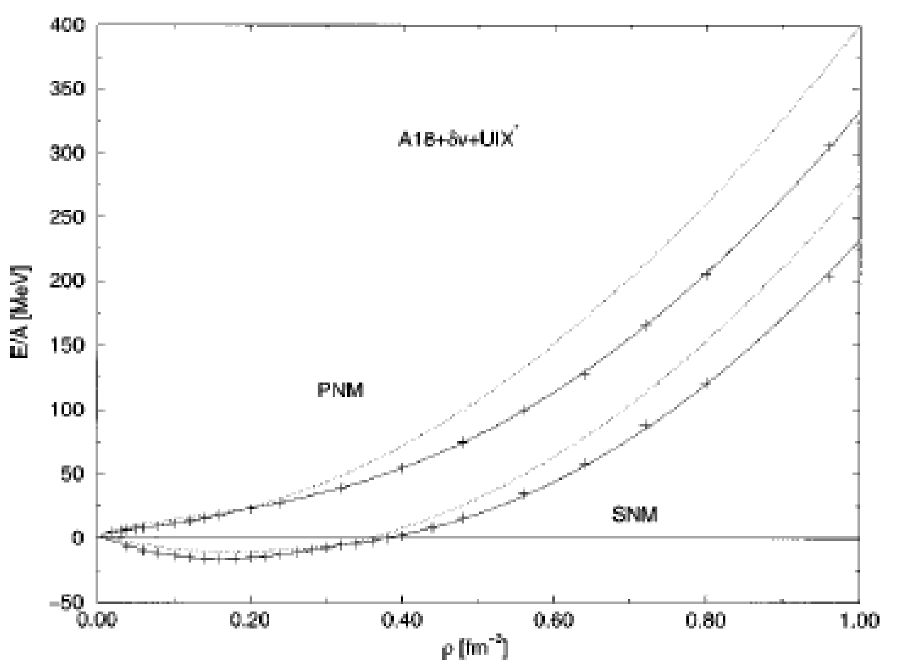
- **Variational Calculation starting from bare nuclear force**
  B. Friedman, V.R. Pandharipande, NPA361('81)502
  - **Argonne v14 + TNI (TNR+TNA)**
    (TNI/TNR/TNA: three-nucleon int./repulsion/attraction)

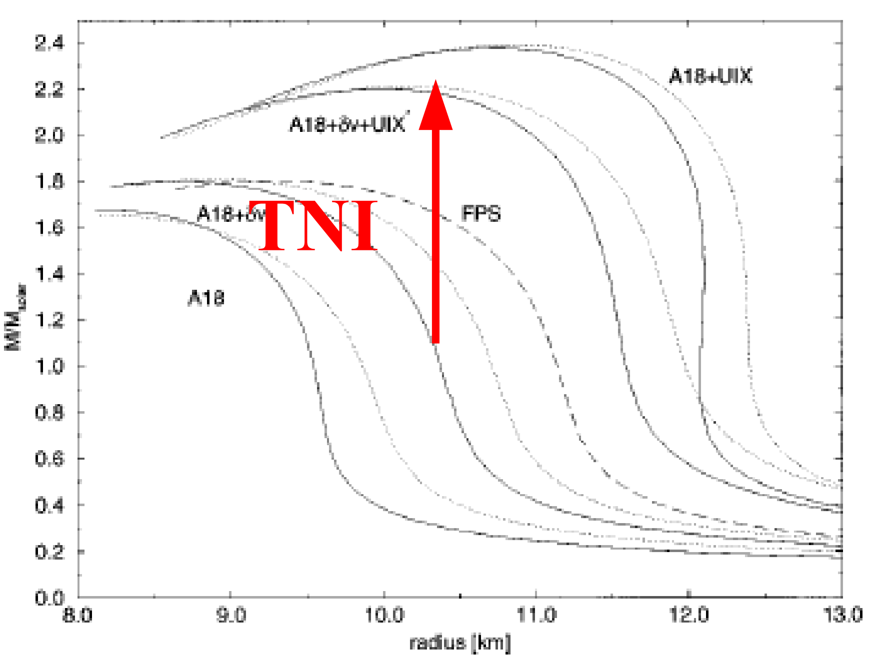




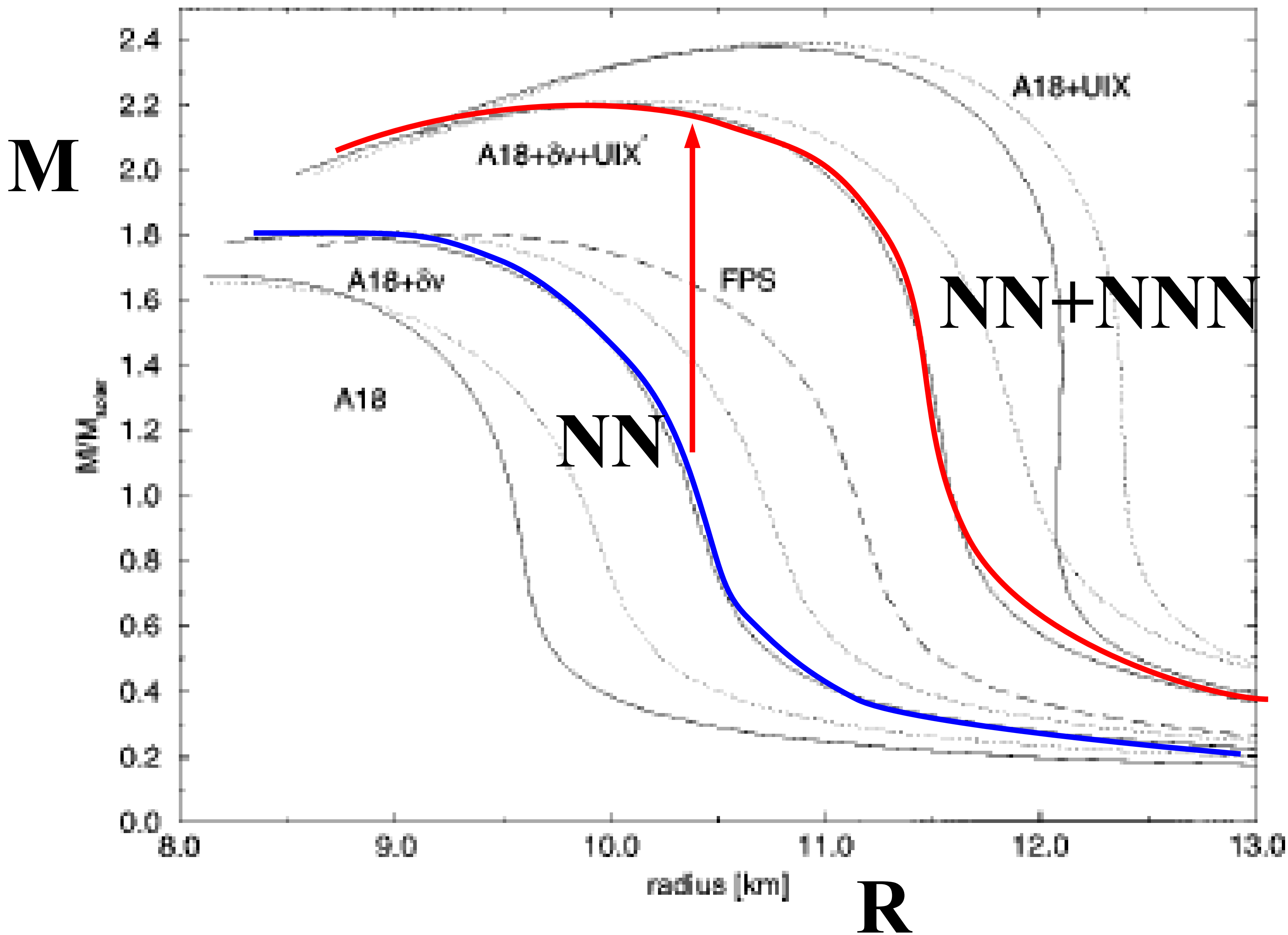
Variational Calculation (2)

- **Variational chain summation method**
  A. Akmal, V.R.Pandharipande, D.G. Ravenhall, PRC58('98)1804
  - **v18, relativistic correction, TNI**
  - **Existence of neutral pion condensation at $\rho_B > 0.2\ fm^{-3}$**



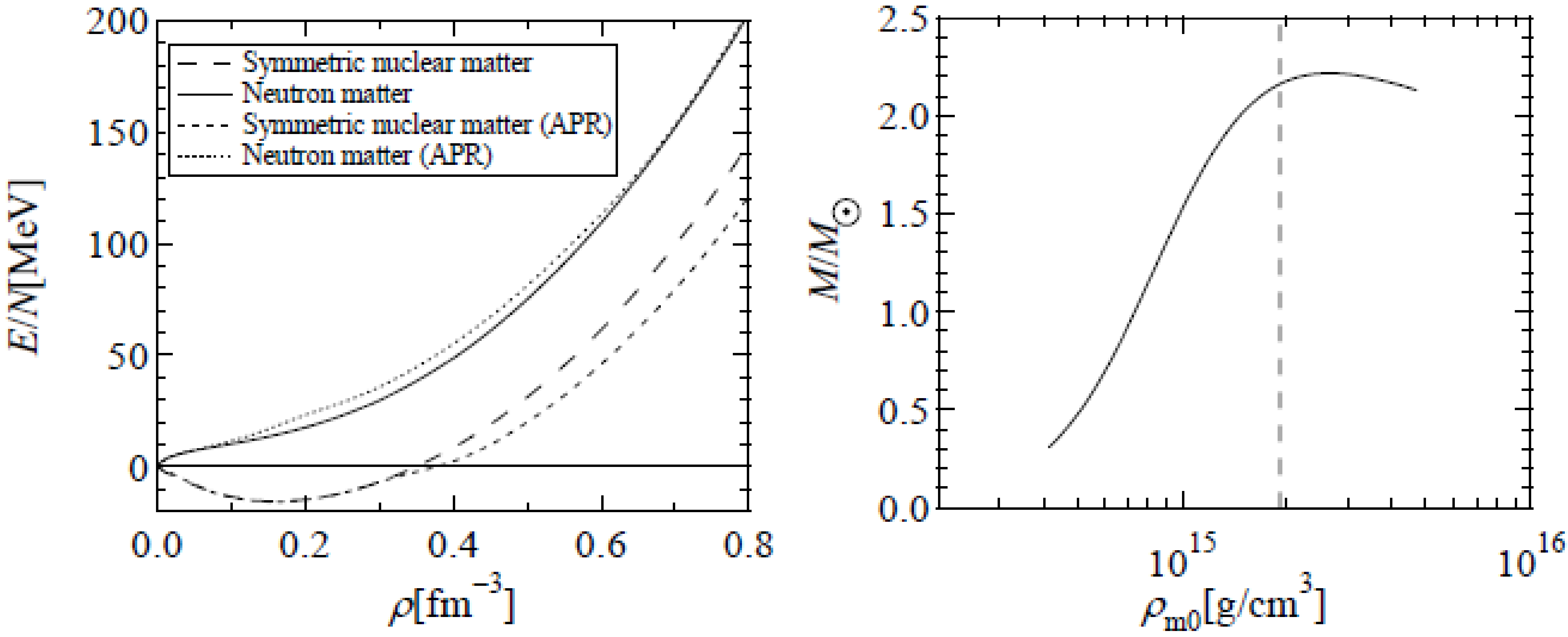


APR

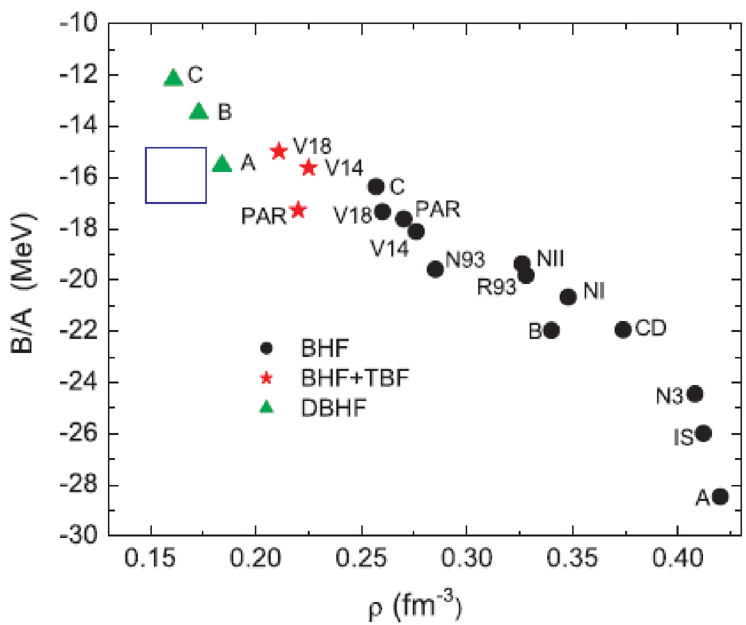

M
R
M/M_solar
radius [km]
A18+UIX
A18+δv+UIX*
A18+δv
FPS
A18
NN
NN+NNN
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
8.0
9.0
10.0
11.0
12.0
13.0

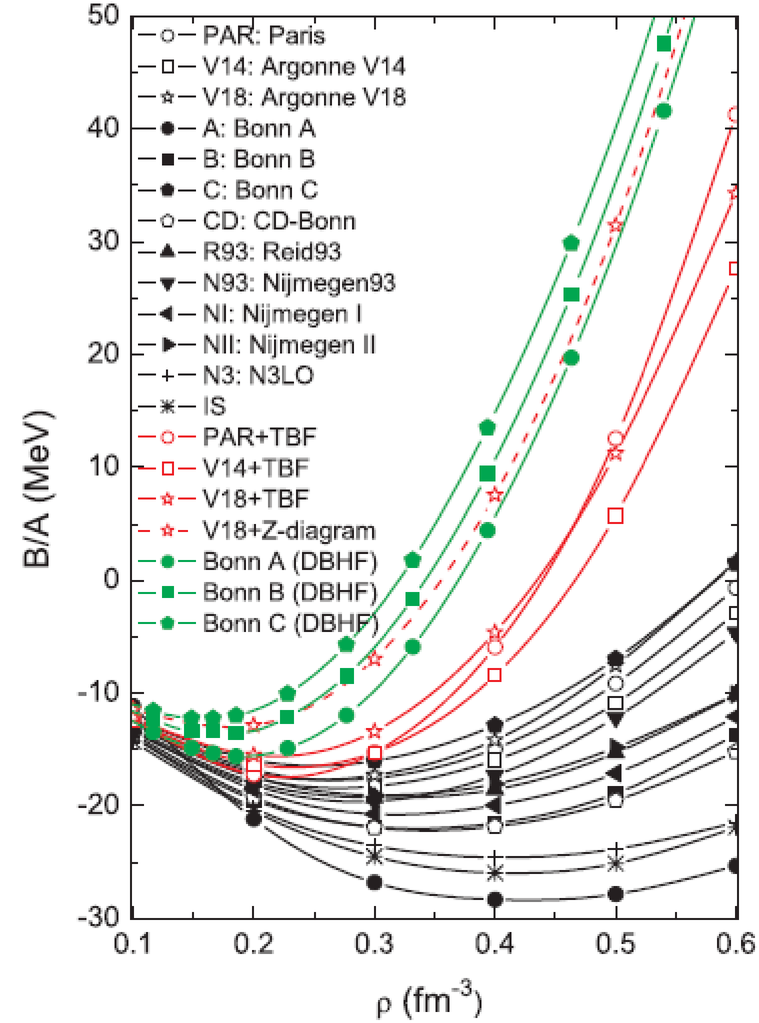
Variational Calculation (3)

- **Variational Calculation using v18+UIX**
  H. Kanzawa, K. Oyamatsu, K. Sumiyoshi, M. Takano, NPA791 ('07) 232
  - **Similar to APR, but healing-distance condition is required.**
    → no π^0 condensation



- **Self-consistent treatment of**
  Effective interaction (G-matrix) in the Bruckner Theory
  and Single particle energy from G-matrix
- **Need 3-body force to reproduce saturation point.**
  → FY type 2 π exchange
  + phen. or Z-diagram

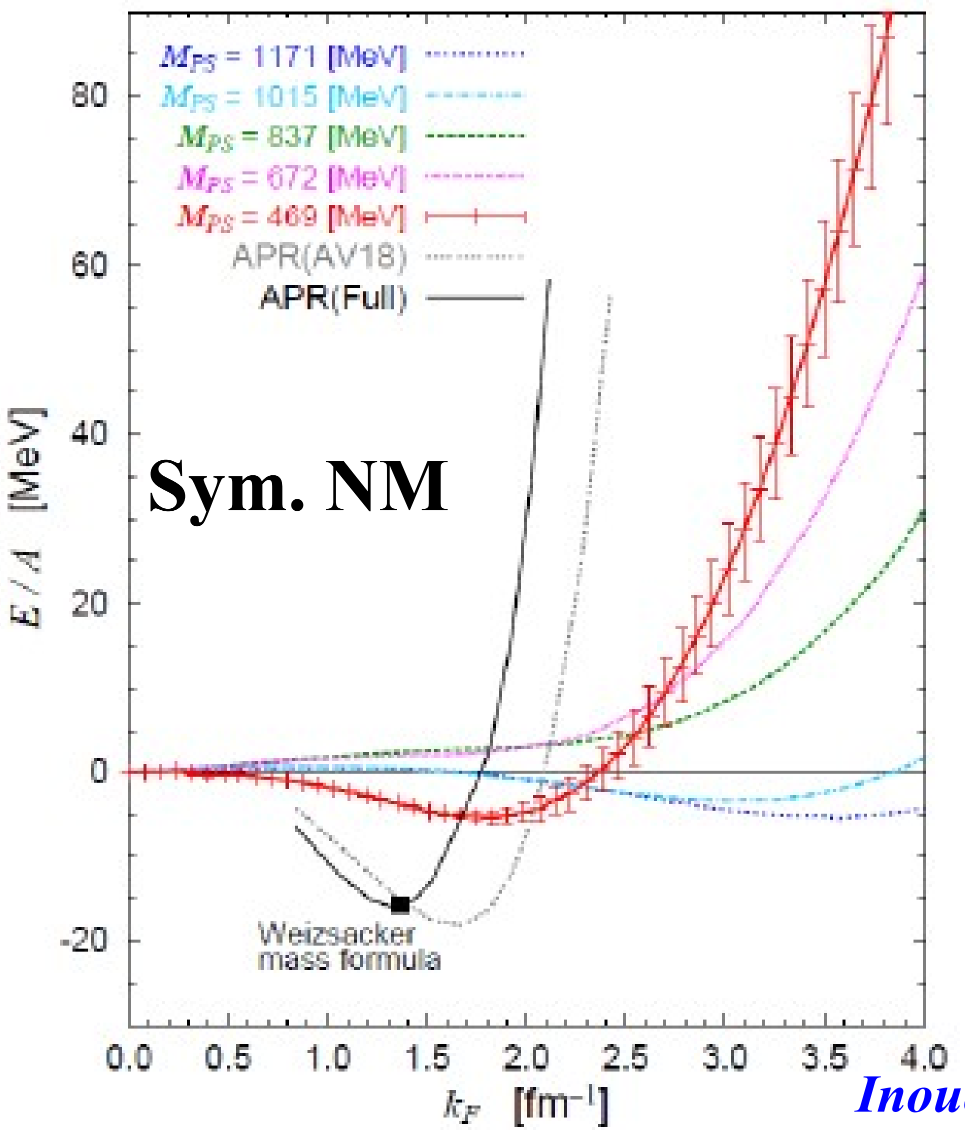


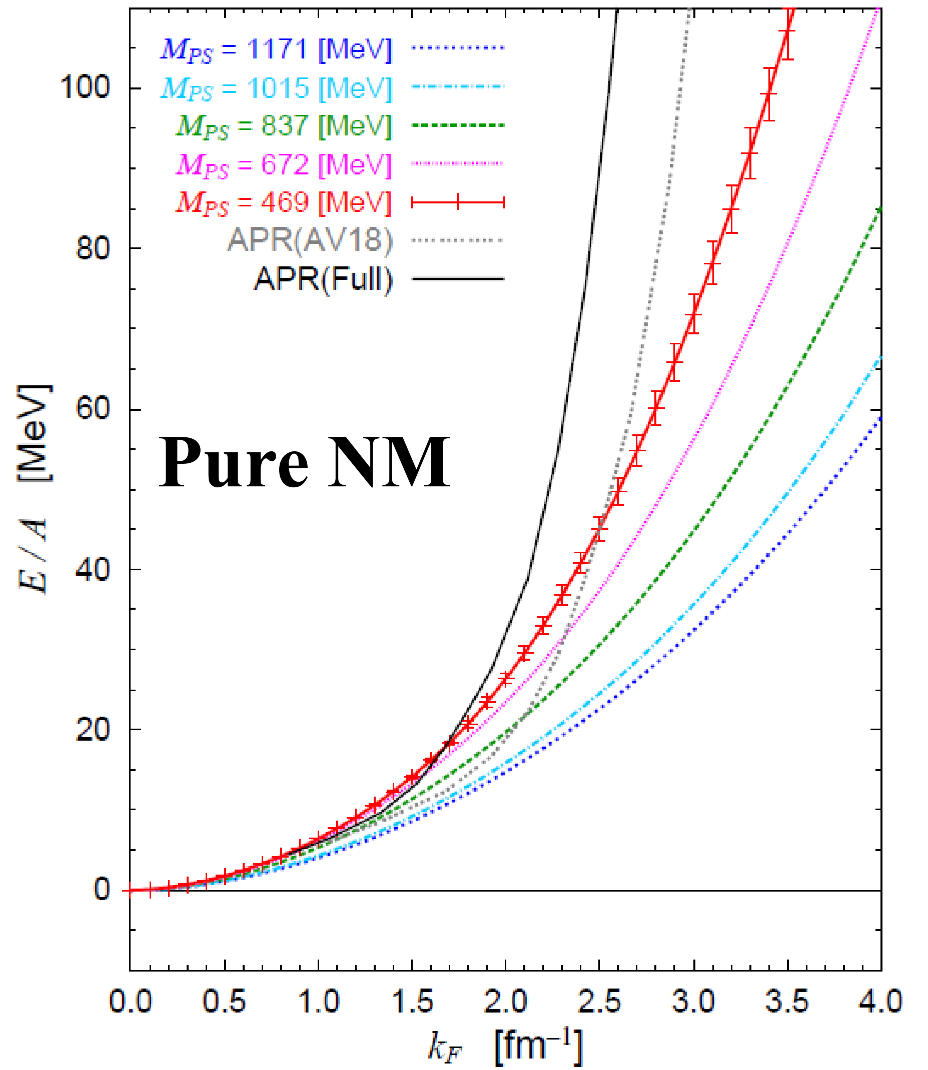


Z.H.Li, U. Lombardo, H.-J. Schulze, W. Zuo, L. W. Chen, H. R. Ma, PRC74('06)047304.

EOS from lattice NN force

- 格子 QCD 核力を用いた高密度状態方程式 (LQCD+BHF)
  NN force: $^1S_0$, $^3S_1$, $^3D_1$ only





Inoue et al. (HAL QCD Coll.), PRL111 ('13)112503

A simple model

Simple parametrized EOS

- **Skyrme int. motivated parameterization**

$$E_{\rm SNM} = \frac{3}{5}E_F(\rho) + \frac{\alpha}{2}\left(\frac{\rho}{\rho_0}\right) + \frac{\beta}{2+\gamma}\left(\frac{\rho}{\rho_0}\right)^{1+\gamma}$$

$$\alpha = \frac{2}{\gamma}\left(E_0(1+\gamma) - \frac{E_F(\rho_0)(1+3\gamma)}{5}\right) \ , \quad \beta = \frac{2+\gamma}{\gamma}\left[-E_0 + \frac{1}{5}E_F(\rho_0)\right] \ .$$

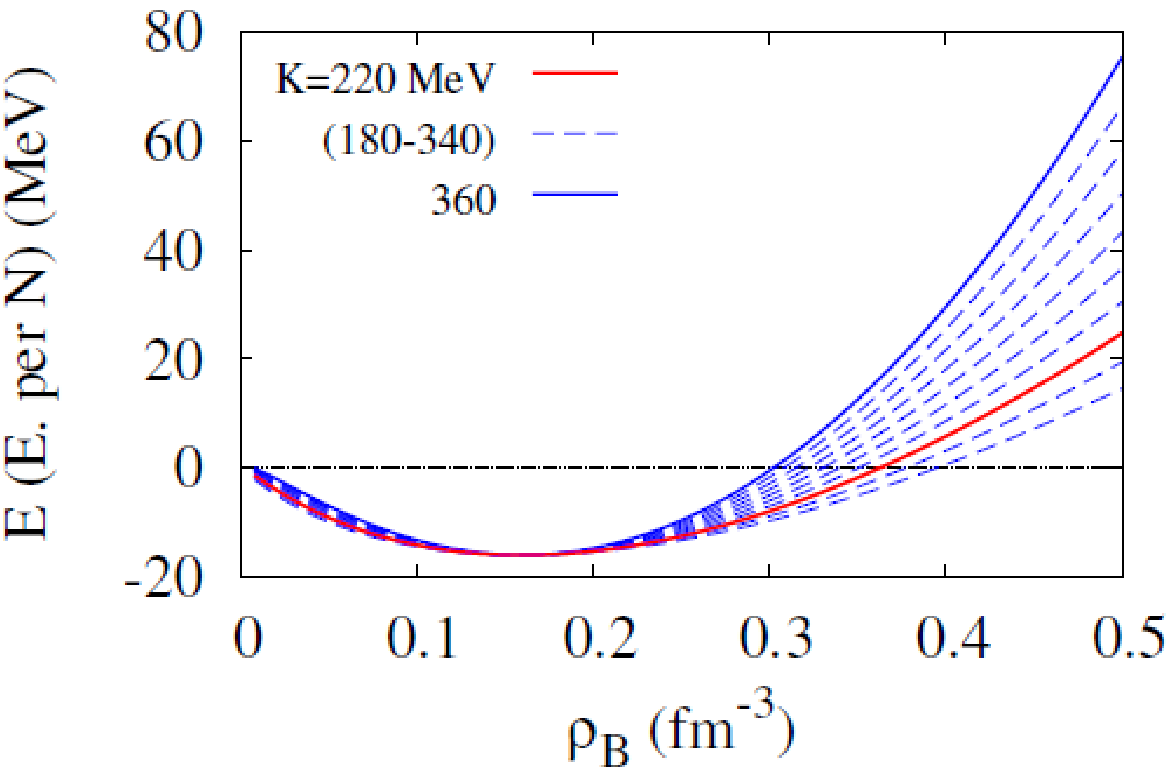
$$K = \frac{1+3\gamma}{5}E_F(\rho_0) - 3E_0(1+\gamma) \ .$$

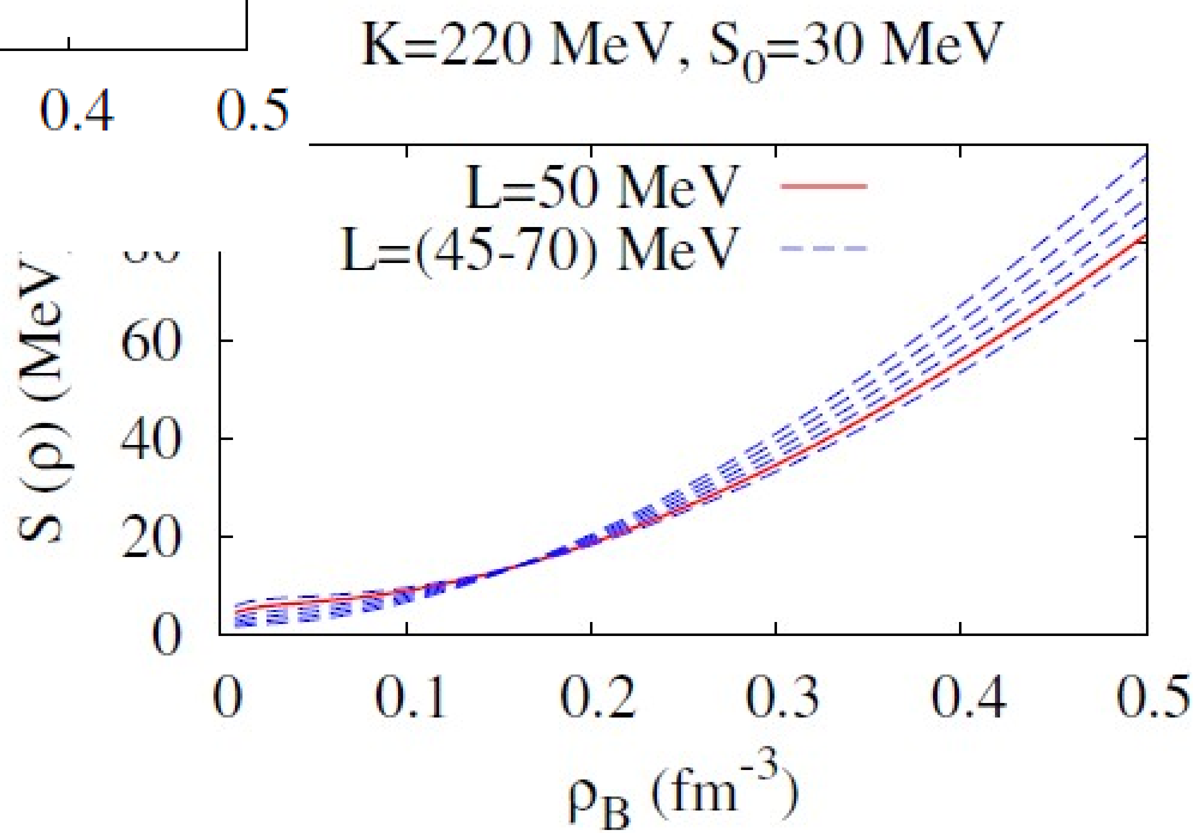
- **Symmetry energy parameterization**

$$S(\rho) = \frac{1}{3}E_F(\rho) + \left[S_0 - \frac{1}{3}E_F(\rho_0)\right]\left(\frac{\rho}{\rho_0}\right)^{\gamma_{\rm sym}}$$

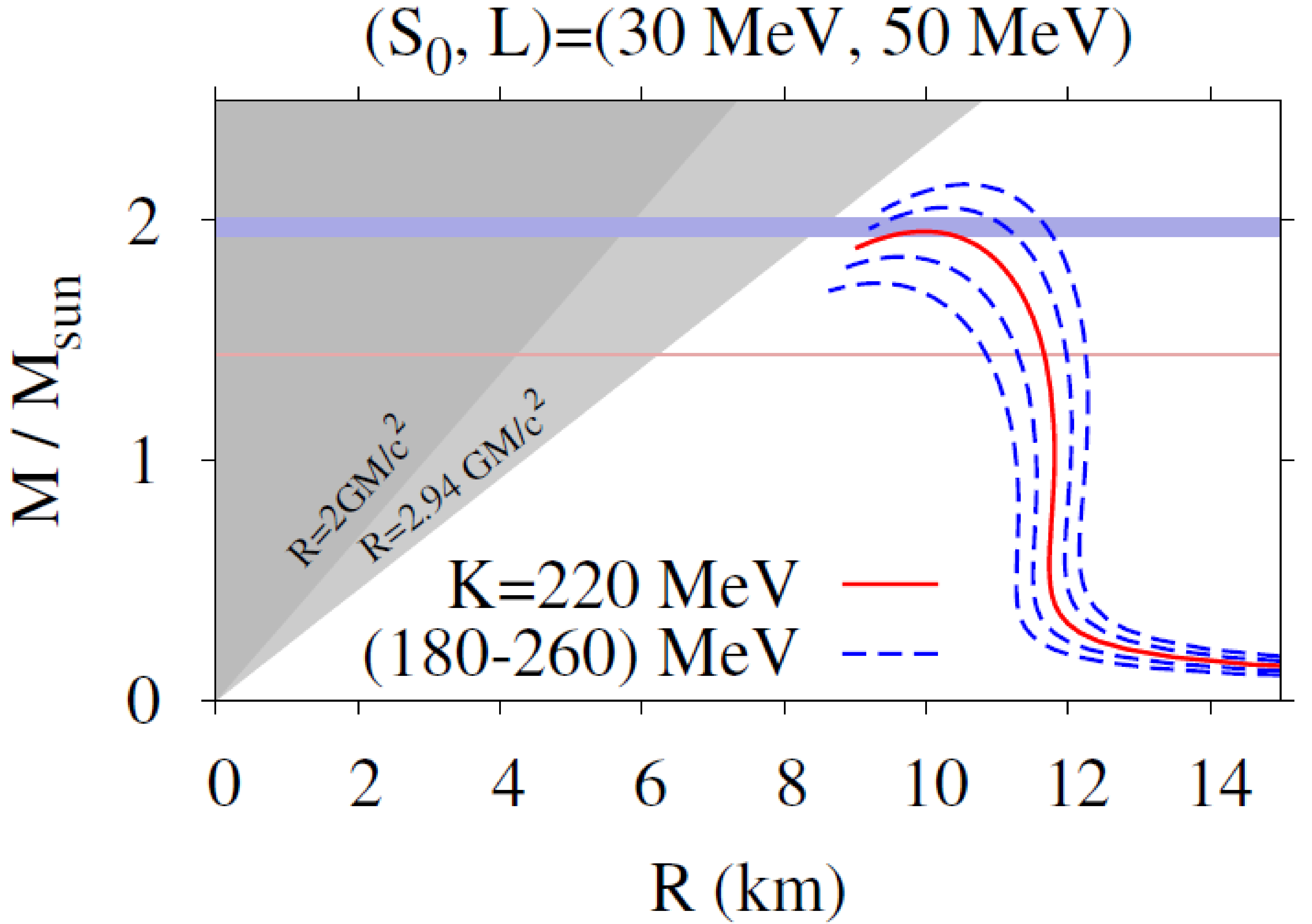
$$\gamma_{\rm sym} = \frac{L - \frac{2}{3}E_F(\rho_0)}{3S_0 - E_F(\rho_0)}$$

Simple parametrized EOS

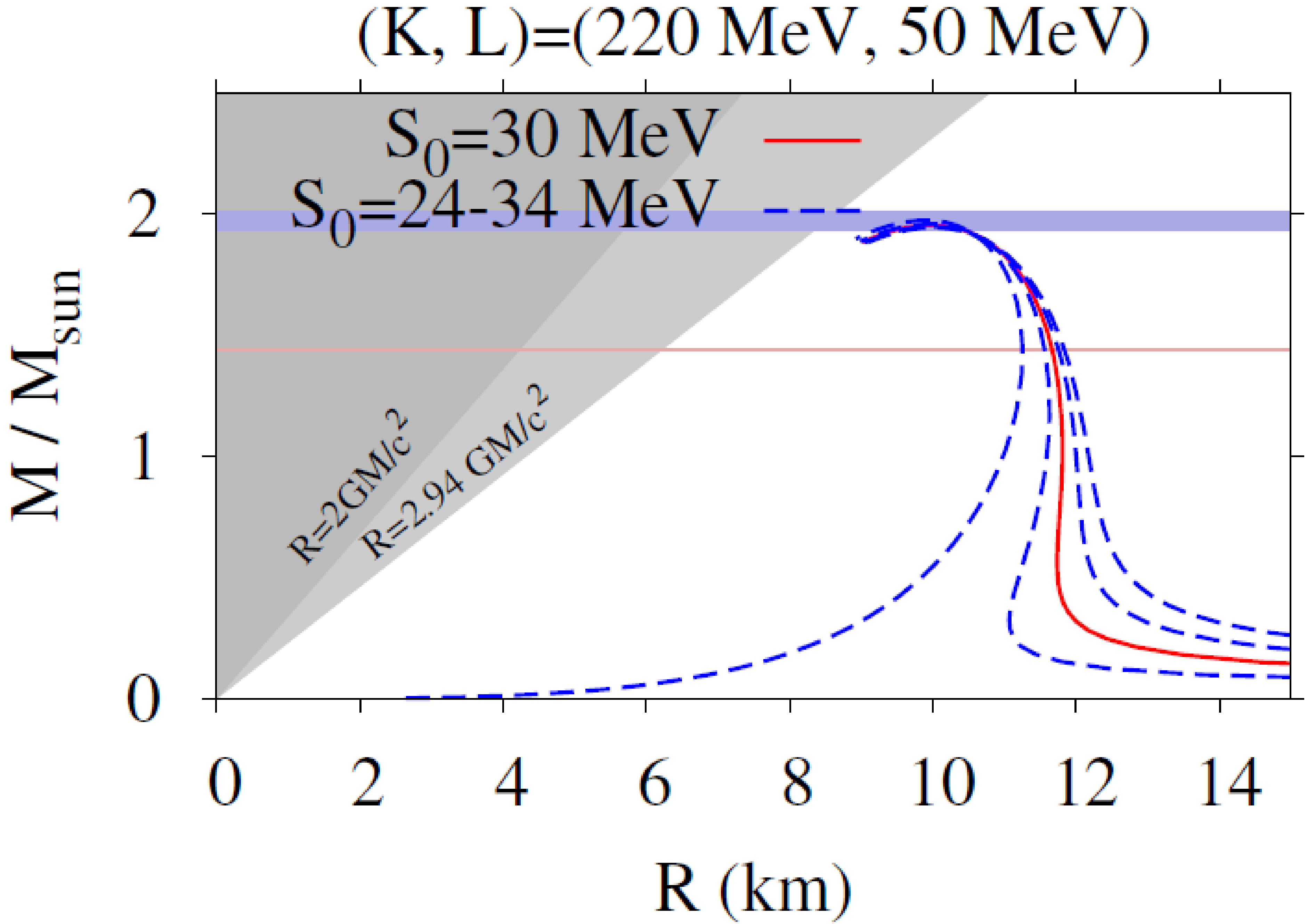




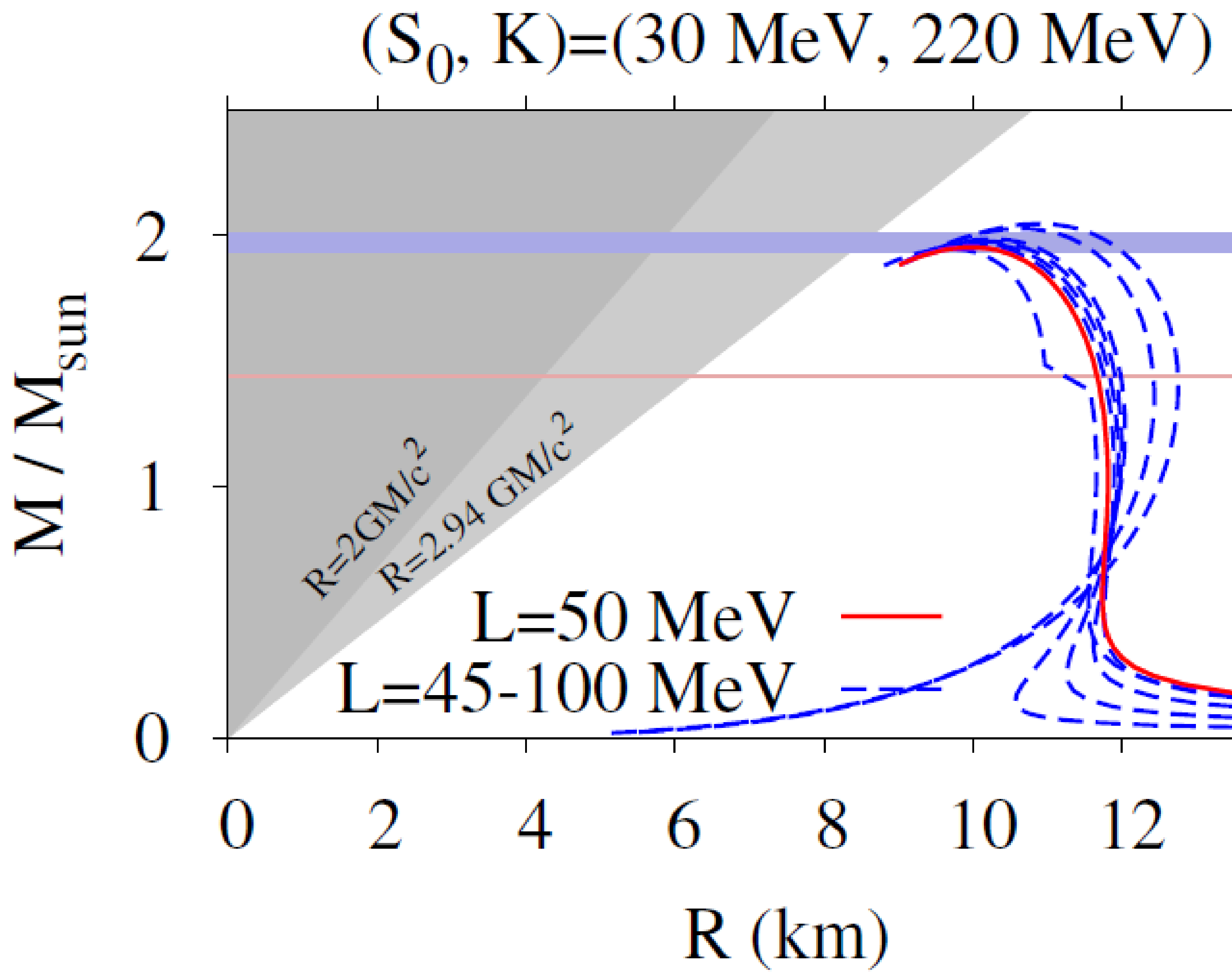
Simple parametrized EOS



Simple parametrized EOS



Simple parametrized EOS



Relativistic Mean Field

Relativistic Mean Field (1)

- **Effective Lagrangian of Baryons and Mesons + Mean Field App.**

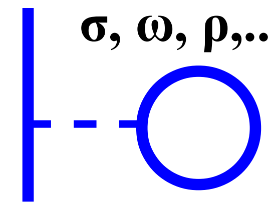
  B.D.Serot, J.D.Walecka, Adv.Nucl.Phys.16 ('86), 1

$$L = L_B^{\text{free}} + L_M^{\text{free}} + L_{BM} + L_M^{\text{Int}}$$

$$L_M^{\text{Int}} = -U_\sigma(\sigma) + \frac{1}{4} c_\omega (\omega_\mu \omega^\mu)^2 + \cdots$$

$$L_{BM} = -\sum_{B,S} g_{BS} \bar{\psi}_B \varphi_S \psi_B - \sum_{B,V} g_{BV} \bar{\psi}_B \gamma^\mu V_\mu \psi_B$$

$$L_B^{\text{free}} = \bar{\psi}_B (i \gamma^\mu \partial_\mu - M_B) \psi_B \ , \quad L_M^{\text{free}} = \sum_S \left[\frac{1}{2} \partial^\mu \phi_S \partial_\mu \phi_S - \frac{1}{2} m_S^2 \phi_S^2 \right] + \sum_V \left[-\frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \frac{1}{2} m_v^2 V_\mu V^\mu \right]$$



- **Baryons and Mesons: B=N, Λ, Σ, Ξ, ...,   S= σ, ς, ...,   V= ω, ρ, φ, …**
- **Based on Dirac phenomenology & Dirac Bruckner-Hatree-Fock**

  E.D. Cooper, S. Hama, B.C. Clark, R.L. Mercer, PRC47('93),297
  R. Brockmann, R. Machleidt, PRC42('90),1965

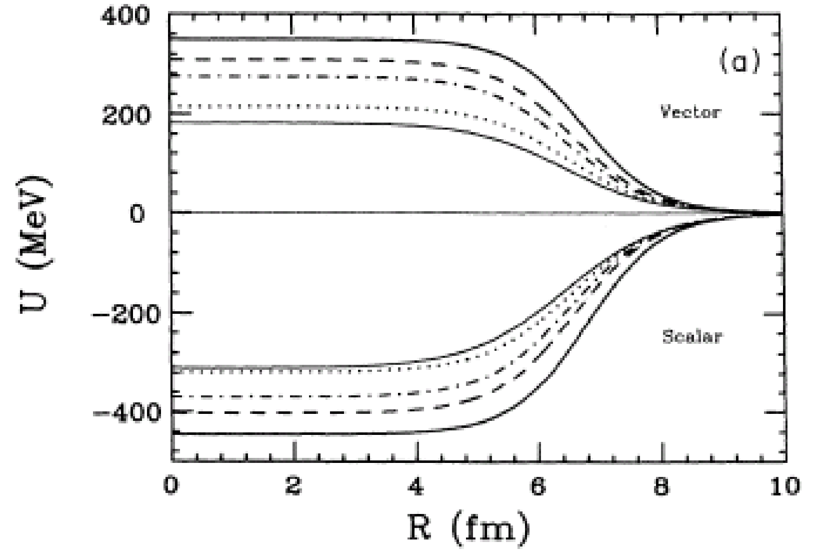
- **Large scalar (att.) and vector (repl.) → Large spin-orbit pot.**
  Relativistic Kinematics → Effective 3-body repulsion
- **Non-linear terms of mesons → Bare 3-body and 4-body force**

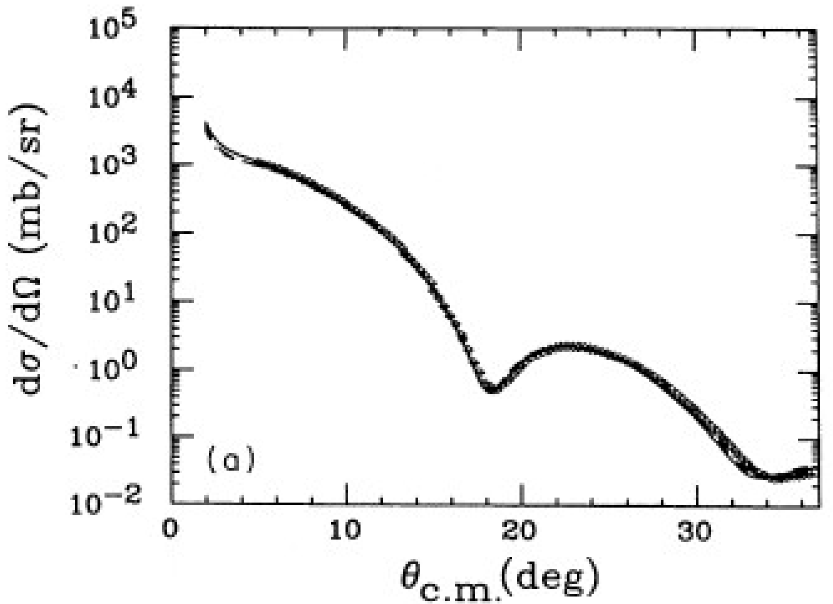
  Boguta, Bodmer ('77), NL1:Reinhardt, Rufa, Maruhn, Greiner, Friedrich ('86), NL3: Lalazissis, Konig, Ring ('97),TM1 and TM2: Sugahara, Toki ('94), Brockmann, Toki ('92)

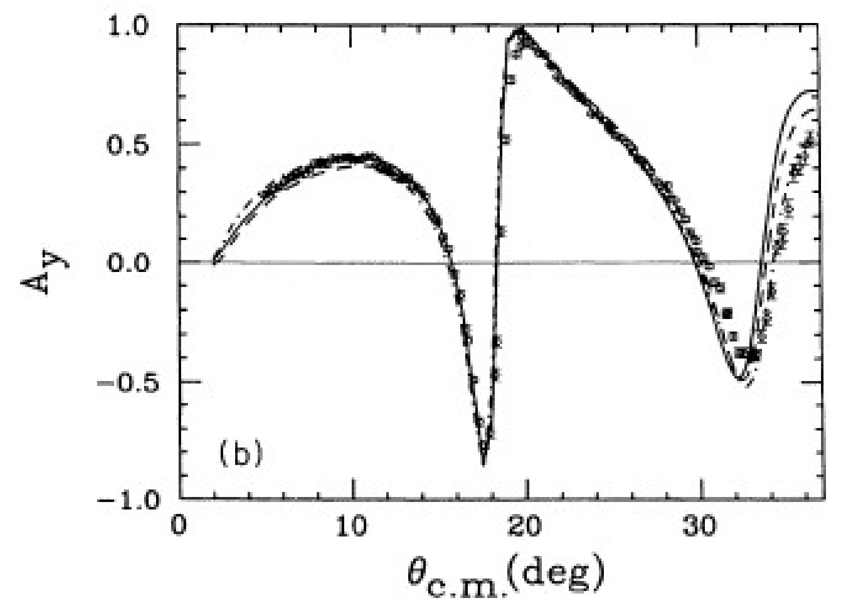
Dirac Phenomenology

E.D. Cooper, S. Hama, B.C. Clark, R.L. Mercer, PRC47('93),297

- **Dirac Eq. with**
  Scalar + Vector pA potential
  (-400 MeV + 350 MeV)
  → Cross Section, Spin Observables



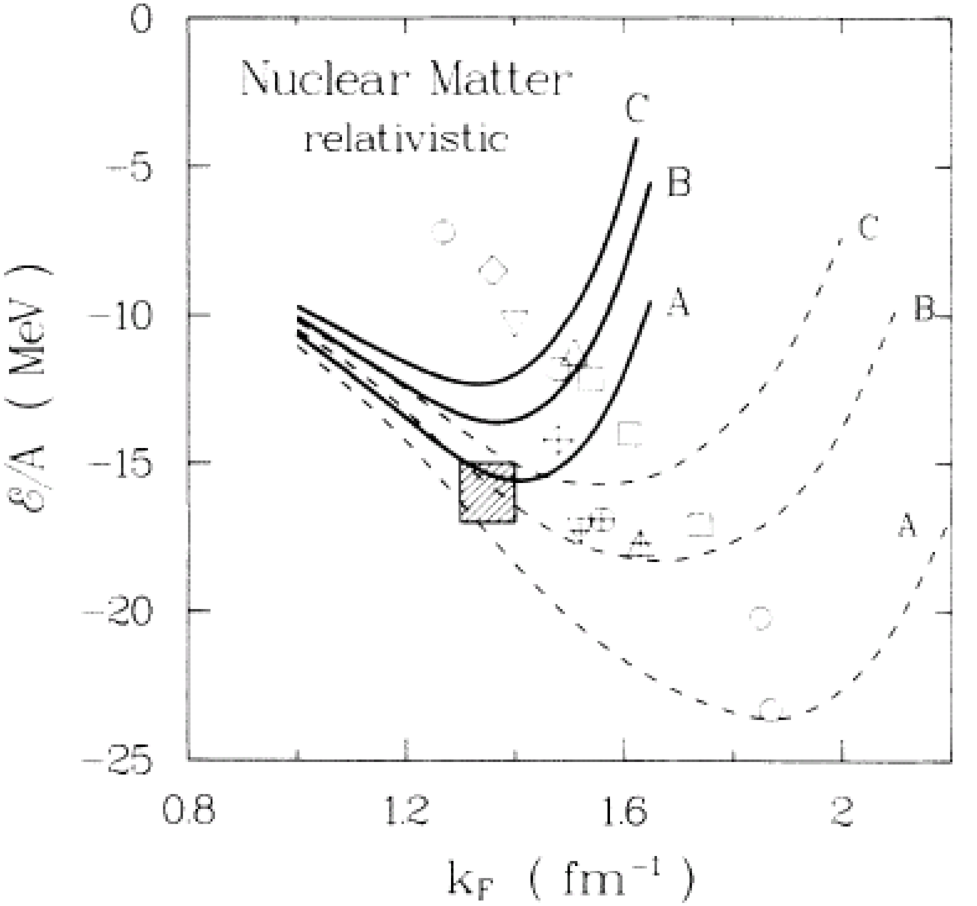




EOS in Dirac-Brueckner-Hartree-Fock

R. Brockmann, R. Machleidt, PRC42('90),1965

- **Non Relativistic Brueckner Calculation**
  → Nuclear Saturation Point cannot be reproduced (Coester Line)
- **Relativistic Approach (DBHF)**
  → Relativity gives additional repulsion, leading to successful description of the saturation point.



Relativistic Mean Field (2)

- **Mean Field treatment of meson field operator**
  = Meson ield operator is replaced with its expectation value
  $$\varphi(r) \rightarrow \langle\varphi(r)\rangle$$
  Ignoring fluctuations compared with the expectation value may be a good approximation at strong condensate.
- **Which Hadrons should be included in RMF ?**
  - **Baryons (1/2+)** p, n, Λ, Σ, Ξ, Δ, ...
  - **Scalar Mesons (0+)** σ(600), f_0(980), a_0(980), ...
  - **Vector Mesons (1-)** ω(783), ρ(770), ϕ(1020), ...
  - **Pseuso Scalar (0-)** π , K, η, η', ...
  - **Axial Vector (1+)** a_1, ...

  We require that the meson field can have uniform expectation values in nuclear matter.
  → Scalar and Time-Component of Vector Mesons (σ , ω, ρ, ...)

$\sigma\omega$ Model (1)

Serot, Walecka, Adv.Nucl.Phys.16 (1986),1

- **Consider only σ and ω mesons**
- **Lagrangian**

$$L = \bar{\psi}\left(i\gamma^\mu \partial_\mu - M + g_s \sigma - g_v \omega\right)\psi + \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_s^2 \sigma^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_v^2 \omega_\mu \omega^\mu$$

$$\left(F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu\right)$$

- **Equation of Motion**
  - **Euler-Lagrange Equation**

$$\frac{\partial}{\partial x^\mu}\left[\frac{\partial L}{\partial(\partial_\mu \phi_i)}\right] - \frac{\partial L}{\partial \phi_i} = 0$$

$$\sigma : \left[\partial_\mu \partial^\mu + m_s^2\right]\sigma = g_s \bar{\psi}\psi$$

$$\omega : \partial_\mu F^{\mu\nu} + m_v^2 \omega^\nu = g_v \bar{\psi}\gamma^\nu \psi \quad \rightarrow \quad \left[\partial_\mu \partial^\mu + m_v^2\right]\omega^\nu = g_v \bar{\psi}\gamma^\nu \psi$$

$$\psi : \left[\gamma^\mu\left(i\partial_\mu - g_v V_\mu\right) - \left(M - g_s \sigma\right)\right]\psi = 0$$

EOM of ω (for beginners)

- **Euler-Lagrange Eq.**

$$\partial_\mu F^{\mu\nu} + m_v^2 \omega^\nu = g_v \bar{\Psi} \gamma^\nu \psi$$

- **Divergence of LHS and RHS**

$$\partial_\nu \partial_\mu F^{\mu\nu} + m_v^2 (\partial_\nu \omega^\nu) = m_v^2 (\partial_\nu \omega^\nu) = g_v (\partial_\nu \bar{\Psi} \gamma^\nu \psi) = 0$$

LHS: derivatives are sym. and $F_{\mu\nu}$ is anti-sym.
RHS: Baryon Current = Conserved Current

- **Put it in the Euler-Lagrange Eq.**

$$\partial_\mu F^{\mu\nu} = \partial_\mu (\partial^\mu \omega^\nu - \partial^\nu \omega^\mu) = \partial_\mu \partial^\mu \omega^\nu - \partial^\nu (\partial_\mu \omega^\mu) = \partial_\mu \partial^\mu \omega^\nu$$

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Schroedinger Eq. for Upper Component (1)

- **Dirac Equation for Nucleons**

$$\left(i\gamma\partial-\gamma^0 U_v - M - U_s\right)\psi=0 \quad ,$$
$$U_v = g_\omega \omega \quad , \quad U_s = -g_\sigma \sigma$$

- **Decompose 4 spinor into Upper and Lower Components**

$$\begin{pmatrix} E-U_v-M-U_s & i\sigma\cdot\nabla \\ -i\sigma\cdot\nabla & -E+U_v-M-U_s \end{pmatrix}\begin{pmatrix} f \\ g \end{pmatrix}=0$$

$$g=\frac{-i}{E+M+U_s-U_V}(\sigma\cdot\nabla)f$$
$$\left(E-M-U_v-U_s\right)f=-i(\sigma\cdot\nabla)g$$

Schroedinger Eq. for Upper Component (2)

- **Erase Lower Component (assuming spherical sym.)**

$$-i(\sigma\cdot\nabla)g = -(\sigma\cdot\nabla)\frac{1}{X}(\sigma\cdot\nabla)f = -\frac{1}{X}\nabla^2 f - \frac{1}{r}\left[\frac{d}{dr}\frac{1}{X}\right](\sigma\cdot r)(\sigma\cdot\nabla)f$$

$$= -\nabla\frac{1}{X}\nabla f + \frac{1}{r}\left[\frac{d}{dr}\frac{1}{X}\right](\sigma\cdot l)f$$

$$(\sigma\cdot r)(\sigma\cdot\nabla) = (r\cdot\nabla) + i\sigma\cdot(r\times\nabla) = r\cdot\nabla - \sigma\cdot l$$

- **"Schroedinger-like" Eq. for Upper Component**

$$-\nabla\frac{1}{E+M+U_s-U_v}\nabla f + \left(U_s + U_v + U_{LS}(\sigma\cdot l)\right)f = (E-M)f$$

$$U_{LS} = \frac{1}{r}\left[\frac{d}{dr}\frac{1}{E+M+U_s-U_v}\right] < 0 \quad \text{on surface}$$

(U_s,U_v)~ (-350 MeV, 280 MeV)

→Small Central(U_s+U_v), Large LS (U_s-U_v)

Various Ways to Evaluate Non.-Rel. Potential

- **From Single Particle Energy**

$$\left(\gamma^0(E-U_v)+i\gamma\cdot\nabla-(M+U_s)\right)\psi=0 \;\rightarrow\; (E-U_v)^2=p^2+(M+U_s)^2$$

$$\rightarrow E=\sqrt{p^2+(M+U_s)^2}+U_v \approx E_p+\frac{M}{E_p}U_s+U_v+\frac{p^2}{2E_p^3}U_s^2$$

$$\left(E_p=\sqrt{p^2+M^2}\right)$$

- **Schroedinger Equivalent Potential (Uniform matter)**

$$-\frac{\nabla^2}{2\mathrm{M}}f+\left[U_s+\frac{E}{M}U_v+\frac{U_s^2-U_v^2}{2\mathrm{M}}\right]f=\frac{E+M}{2\mathrm{M}}(E-M)f$$

$$U_{\mathrm{SEP}}\approx U_s+\frac{E}{M}U_v$$

Anyway, slow baryons feel Non.-Rel. Potential,

$$U\approx U_s+U_v=-g_s\sigma+g_v\omega$$

Nuclear Matter in σω Model

Serot, Walecka, Adv.Nucl.Phys.16 (1986),1

- **Uniform Nuclear Matter**

$$E/V = \gamma_N \int^{P_F} \frac{d^3p}{(2\pi)^2} E^* + \frac{1}{2} m_s^2 \sigma^2 - \frac{1}{2} m_v^2 \omega^2 + g_v \rho_B \omega$$

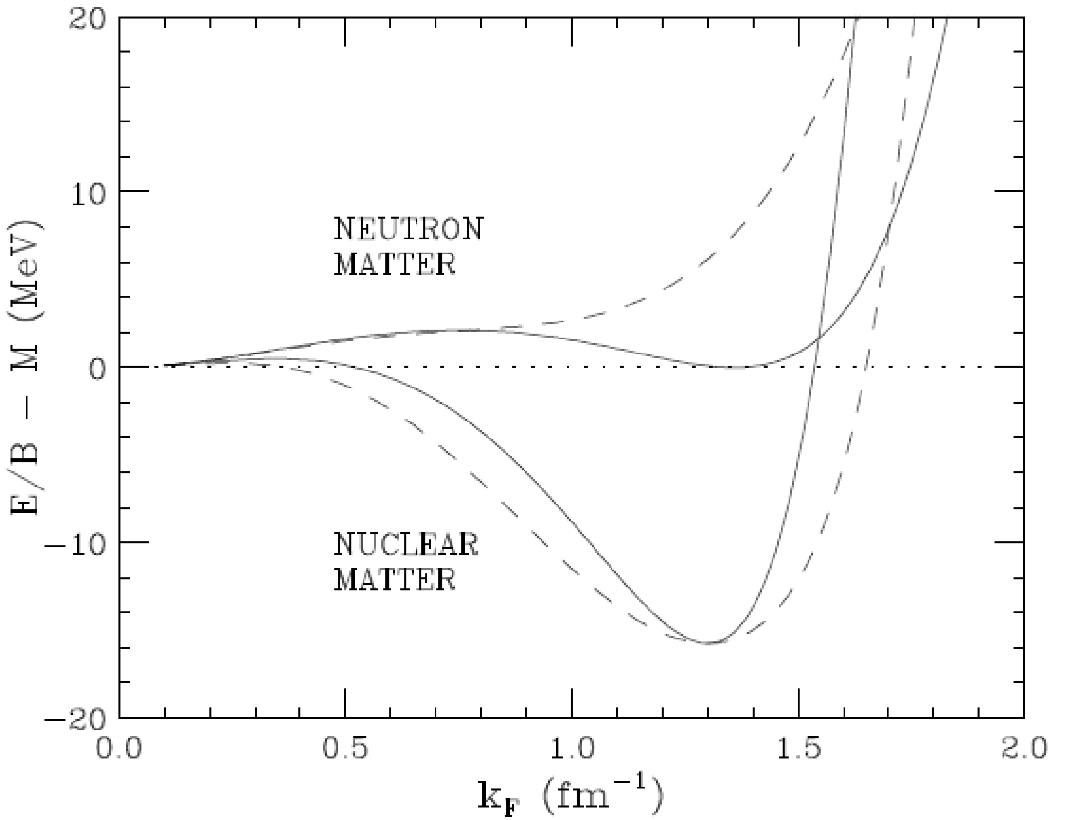
$$\left(M^* = M + U_s = M - g_s \sigma\,,\quad E^* = \sqrt{p^2 + M^{*2}}\right)$$

$$\sigma = \frac{g_s}{m_s^2} \rho_s = \frac{g_s}{m_s^2} \int^{P_F} \frac{\gamma_N \, d^3p}{(2\pi)^2} \frac{M^*}{E^*}$$

$$\omega = \frac{g_v}{m_v^2} \rho_B = \gamma_N \frac{g_v}{m_v^2} \int^{P_F} \frac{d^3p}{(2\pi)^3}$$

γ_N = Nucleon degeneracy

(=4 in sym. nuclear matte



Problem: EOS is too stiff
K ~ (500-600) MeV !
→ How can we avoid it ?

RMF with Non-Linear Meson Int. Terms

Boguta, Bodmer ('77), NL1:Reinhardt, Rufa, Maruhn, Greiner, Friedrich ('86), NL3: Lalazissis, Konig, Ring ('97),TM1 and TM2: Sugahara, Toki ('94), Brockmann, Toki ('92)

- **Too stiff EOS in the simplest RMF (σω model) is improved by introducing non-linear terms (σ^4, ω^4)**
  - **Fit B.E. of Stable as well as Unstable (n-rich) Nuclei**
  - **Three Mesons (σ,ω,ρ) are included**
  - **Meson Self-Energy Term (σ,ω)**

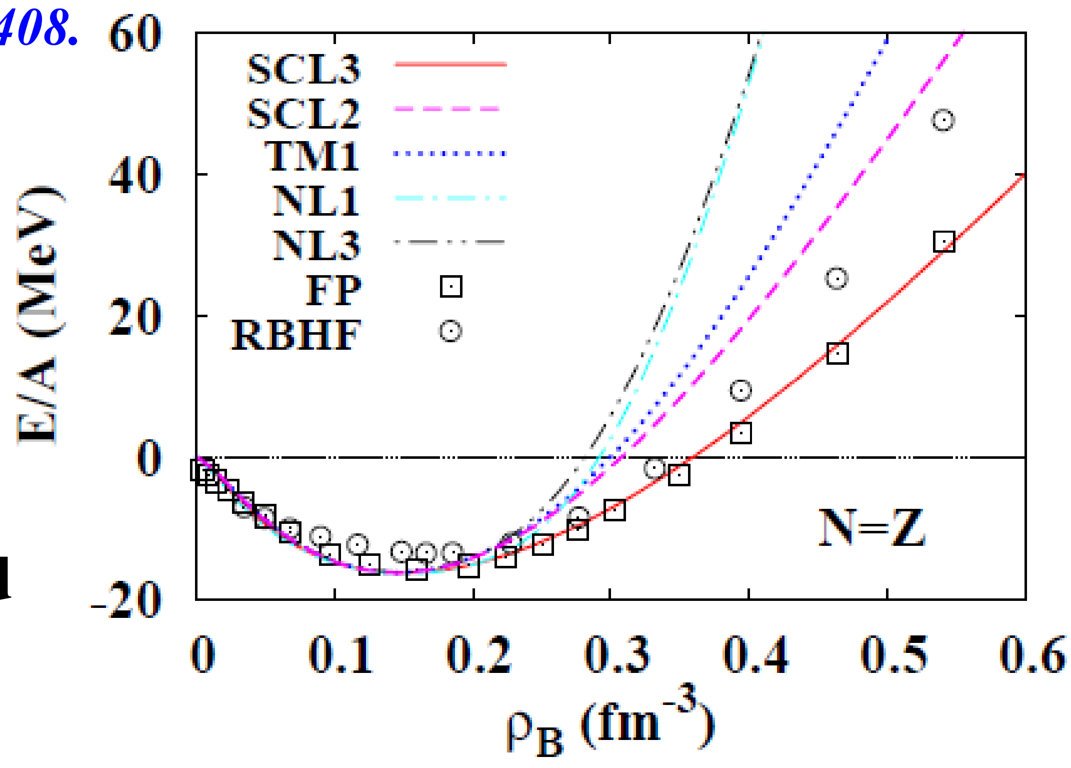
$$
\begin{aligned}
\mathcal{L} =& \bar{\psi}_N \left(i\partial\!\!\!/ - M - g_\sigma \sigma - g_\omega \omega\!\!\!/ - g_\rho \tau^a \rho\!\!\!/^a \right) \psi_N \\
&+ \frac{1}{2}\partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 \\
&- \frac{1}{4} W^{\mu\nu} W_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4} R^{a\mu\nu} R^a_{\mu\nu} + \frac{1}{2} m_\rho^2 \rho^{a\mu} \rho^a_\mu + \frac{1}{4} c_3 \left(\omega_\mu \omega^\mu \right)^2 \\
&+ \bar{\psi}_e \left(i\partial\!\!\!/ - m_e \right) \psi_e + \bar{\psi}_\nu i\partial\!\!\!/ \psi_\nu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \ ,
\end{aligned}
$$

$$
\begin{aligned}
W_{\mu\nu} &= \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \ , \\
R^a_{\mu\nu} &= \partial_\mu \rho^a_\nu - \partial_\nu \rho^a_\mu + g_\rho \epsilon^{abc} \rho^{b\mu} \rho^{c\nu} \ , \\
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \ .
\end{aligned}
$$

RMF models with Non-Linear Meson Int. Terms

- **Variety of the RMF models**
  → MB couplings, meson masses, meson self-energies
  - **σN, ωN, ρN couplings are well determined**
    → almost no model deps. in Sym. N.M. at low ρ
  - **ω^4 term is introduced to simulate DBHF results of vector pot.**
    TM1&2: Y. Sugahara, H. Toki, NPA579('94)557;
    R. Brockmann, H. Toki, PRL68('92)3408.
  - **σ^3 and σ^4 terms are introduced to soften EOS at ρ_0.**
    J. Boguta, A.R.Bodmer NPA292('77)413,
    NL1:P.-G.Reinhardt, M.Rufa, J.Maruhn, W.Greiner, J.Friedrich, ZPA323('86)13.
    NL3: G.A.Lalazissis, J.Konig, P.Ring, PRC55('97)540.

    → Large differences are found at high ρ



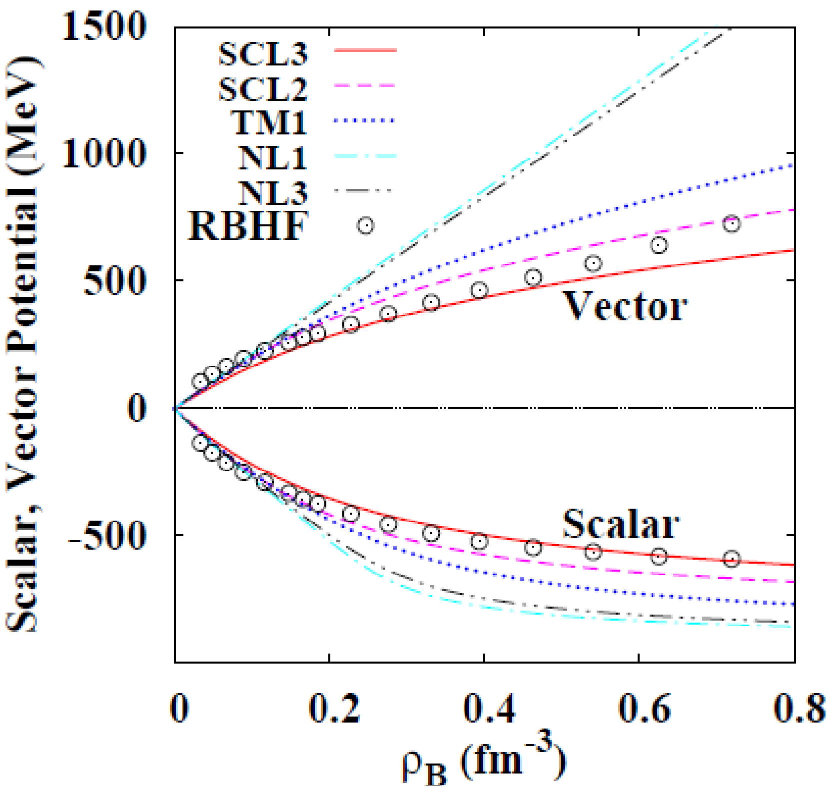
K. Tsubakihara, H. Maekawa, H. Matsumiya, AO,PRC81('10)065206.

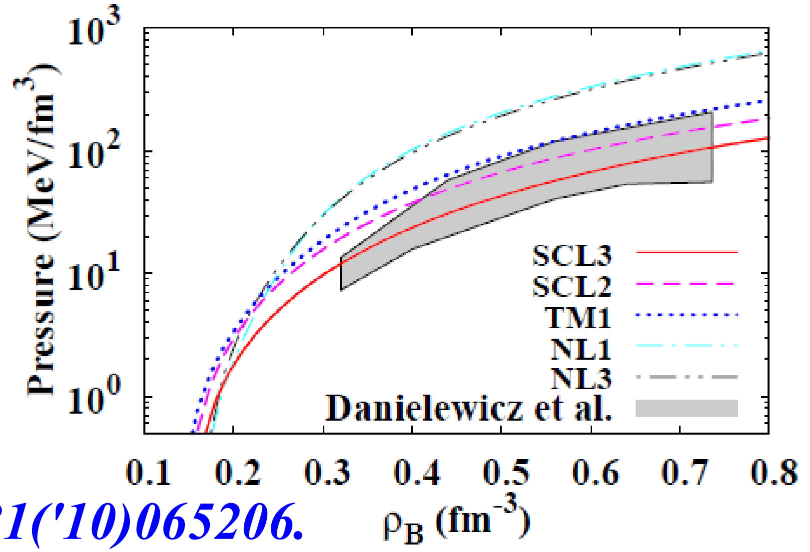
Vector potential in RMF

- **Vector potential from ω dominates at high density !**

$$U_v(\rho_B) = g_\omega \omega \sim \frac{g_\omega^2}{m_\omega^2} \rho_B$$

  - **Dirac-Bruckner-Hartree-Fock shows suppessed vector potential at high ρ_B.**
    R. Brockmann, R. Machleidt, PRC42('90)1965.
  - **Collective flow in heavy-ion collisions suggests pressure at high ρ_B.**
    P. Danielewicz, R. Lacey, W. G. Lynch, Science298('02)1592.
  - **Self-interaction of ω ~ $c_\omega(\omega_\mu\omega^\mu)^2$**
    → DBHF results & Heavy-ion data

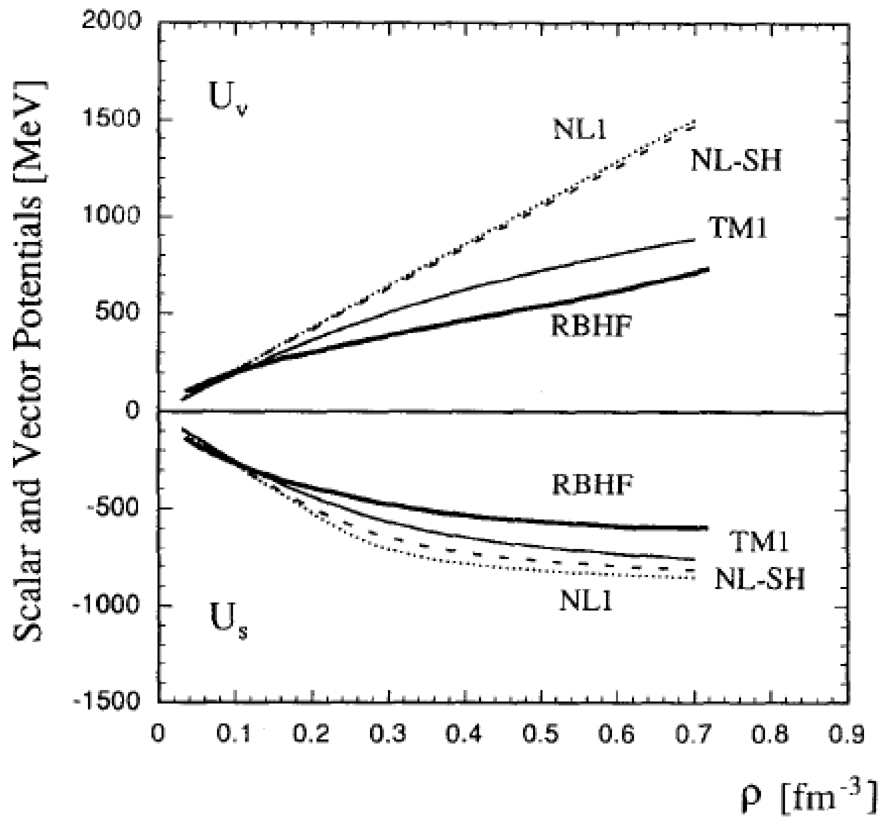


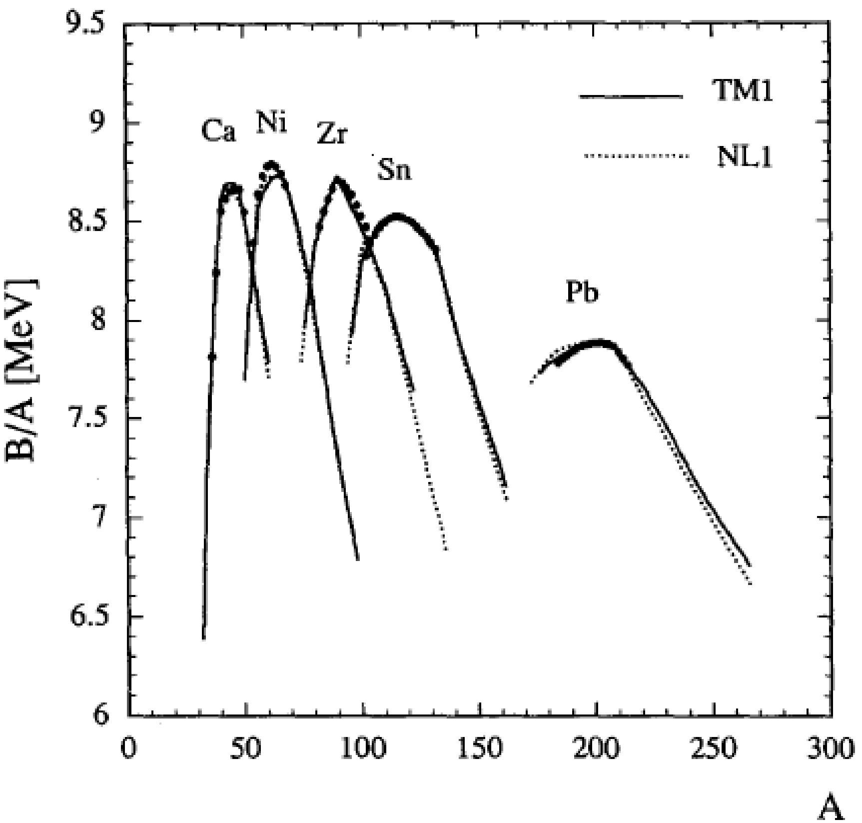


K. Tsubakihara, H. Maekawa, H. Matsumiya, AO,PRC81('10)065206.

TM1

- **TM1 Sugahara, Toki ('94)**
  - **Fit vector potential in RBHF by introducing ω^4 term.**
  - **Fit binding energies of neutron-rich nuclei**

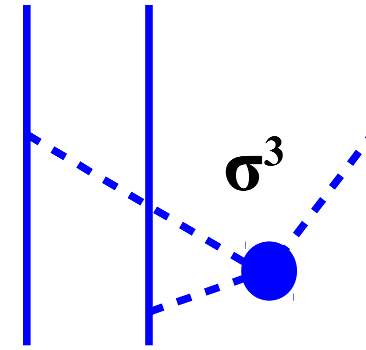


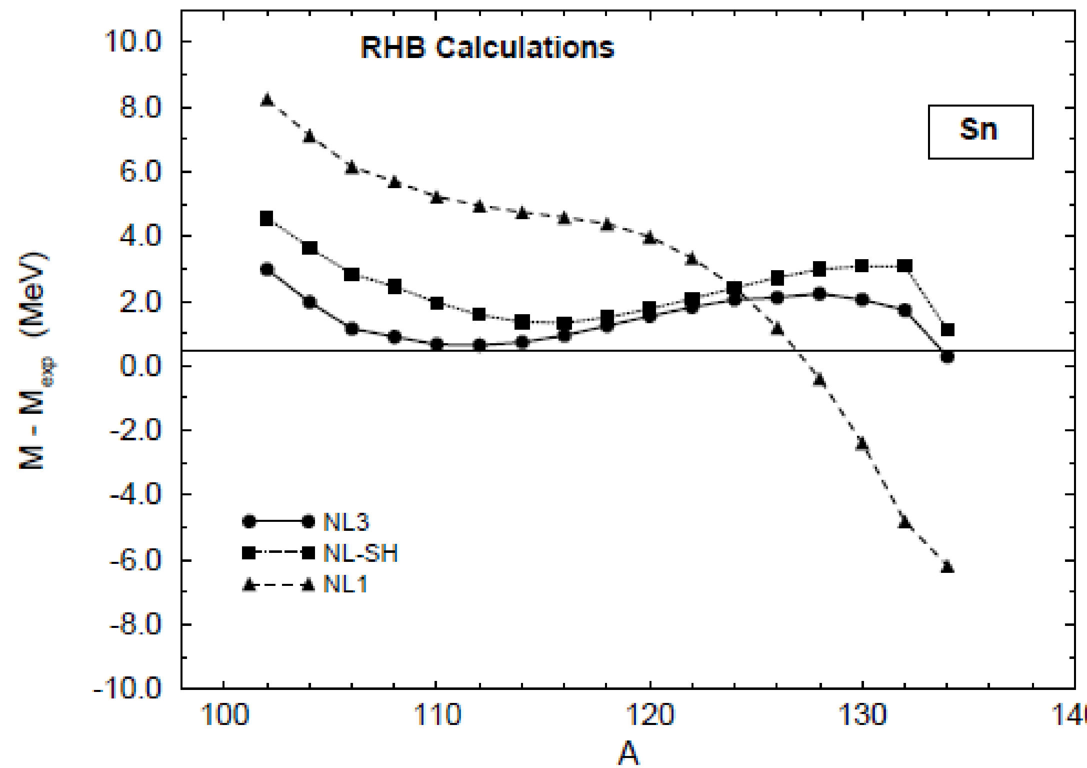


TM1: Sugahara, Toki ('94)

High Quality RMF models

- いくつかの RMF パラメータによる計算は、「質量公式」に迫る精度で原子核質量を記述!
  → High Quality RMF models.
  TM, NL1, NL3, ….
  - 全質量で 1-2 MeV の誤差 (NL3)
  - Linear coupling (σN, ωN, ρN), self-energy in σ, ω
  - 場合によっては結合定数の密度依存性を導入。





NL3: Lalazissis, Konig, Ring, PRC55 ('97)540

RMF with Non-Linear Meson Int. Terms

- **Are the Lagrangian parameters are well determined ?**

$$\mathcal{L} = \mathcal{L}_{\rm free}(\psi, \sigma, \omega, \rho, ...) + \bar{\psi}\left[g_\sigma \sigma - g_\omega \gamma^0 \omega - g_\rho \tau_z \gamma^0 \rho\right]\psi + c_\omega \omega^4/4 - V_\sigma(\sigma)\ , \qquad (3)$$

$$V_\sigma = \begin{cases} \frac{1}{3} g_3 \sigma^3 + \frac{1}{4} g_4 \sigma^4 & \text{(NL1, NL3, TM1)} \\ -a_\sigma f_{\rm SCL}(\sigma/f_\pi) & \text{(SCL)} \end{cases}, \qquad (4)$$

  - **Linear terms, Meson-Nucleon Coupling → Well determined**
  - **Negative Coef. of $\sigma^4 < 0$ in some of RMF models→ Vacuum is unstable**

TABLE II: RMF parameters

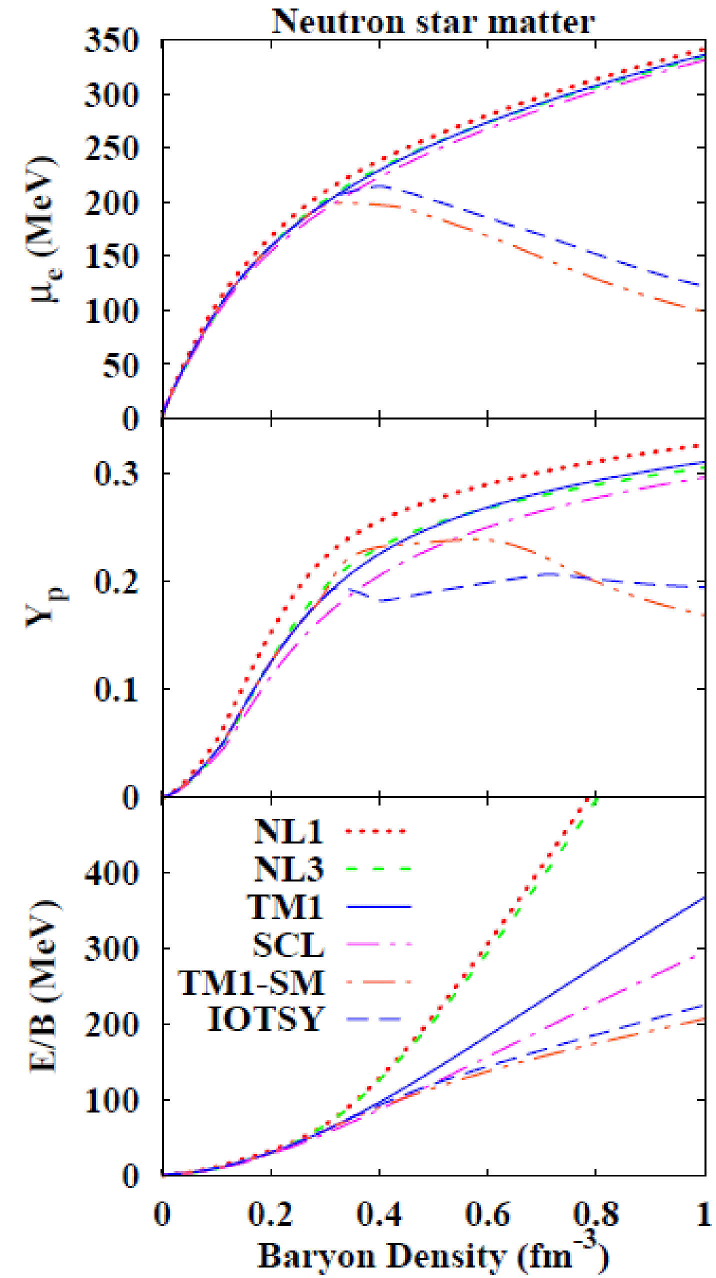
	$g_{\sigma N}$	$g_{\omega N}$	$g_{\rho N}$	g_3(MeV)	g_4	c_ω	m_σ(MeV)	m_ω(MeV)	m_ρ(MeV)
NL1[18]	10.138	13.285	4.976	2401.9	-36.265	0	492.25	795.359	763
NL3[19]	10.217	12.868	4.474	2058.35	-28.885	0	508.194	782.501	763
TM1[6]	10.0289	12.6139	4.6322	1426.466	0.6183	71.3075	511.198	783	770
SCL[20](*1)	10.08	13.02	4.40	1255.88	13.504	200	502.63	783	770

(*1): g_3 and g_4 are from the expansion of $f_{\rm SCL}$.

AO, Jido, Sekihara, Tsubakihara (2009)

Neutron Star Matter EOS

- **Difference in non-linear meson terms generate different predictions of EOS at high densities**



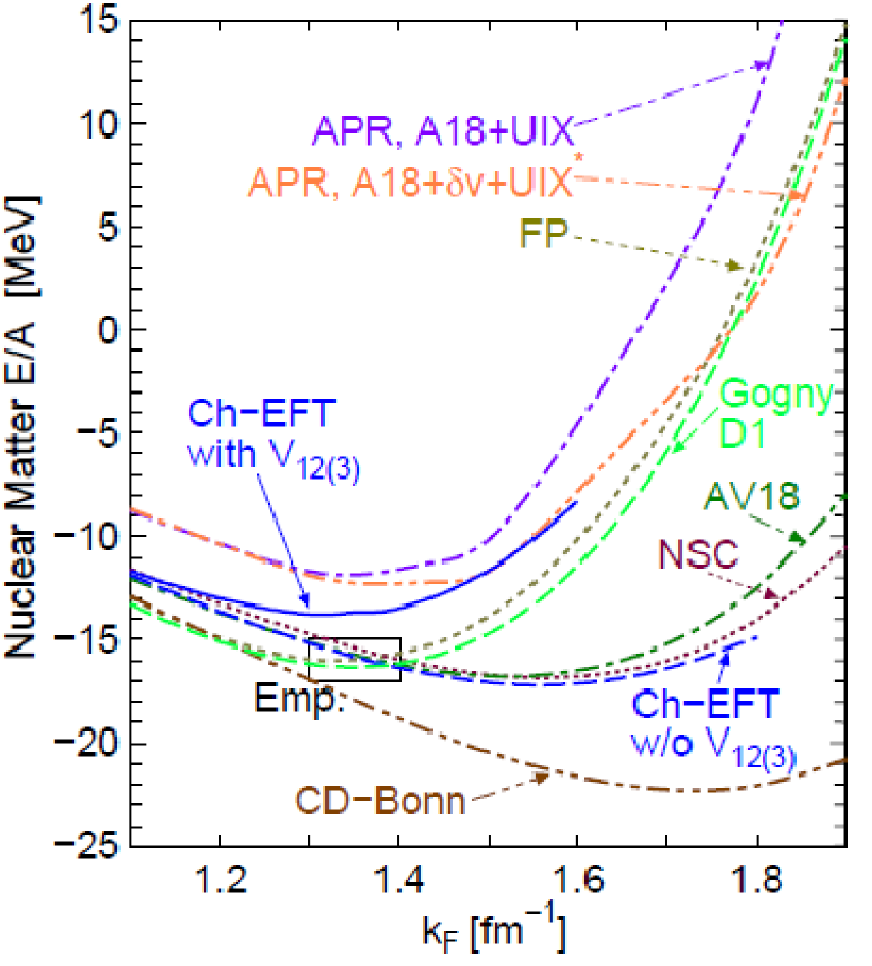
How can we fix non-linear terms ?

AO, Jido, Sekihara, Tsubakihara, Phys. Rev. C 80 (2009), 038202.

Ch-EFT EOS

- **Phen. models need inputs from**
  Experimental Data and/or Microscopic (Ab initio) Calc.
- **Recent Ch-EFT EOS is promising !**
  NN (N3LO)+3NF(N2LO)

M.Kohno ('13)

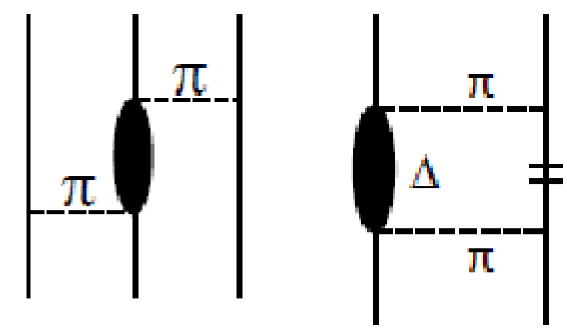


M. Kohno, PRC 88 ('13) 064005

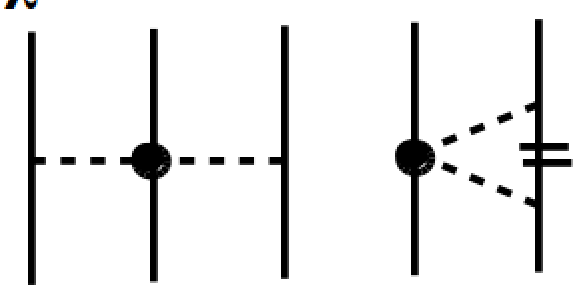
"Universal" mechanism of "Three-body" repulsion

- **"Universal" 3-body repulsion is necessary to support NS. Nishizaki, Takatsuka, Yamamoto ('02)**
- **Mechanism of "Universal" Three-Baryon Repulsion.**
  - **"σ"-exchange ~ two pion exch. w/ res.**
  - **Large attraction from two pion exchange is suppressed by the Pauli blocking in the intermediate stage. Kohno ('13)**

Physical Picture



χ EFT



"Universal" TBR
- ***Coupling to Res. (hidden DOF)***
- ***Reduced "σ" exch. pot. ?***

Short Summary

- Nuclear Matter EOS is important in many subjects of physics.
  - Bulk nuclear properties (B.E., radius)
  - Dense Matter in Compact Astrophysical Objects
  - High-Energy Heavy-Ion Collisions
- Relativistic Mean Field models
  - Simple description of nucleon scalar and vector potentials in terms of meson fields.
  - With non-linear meson interaction terms, nuclear binding energies (and radii) are well explained.
  - Ambiguities of non-linear couplings bring large differences of EOS at high densities, especially in asymmetric nuclear matter.
- It is promising to utilize the results of G-matrix based on Chiral EFT (2 and 3 nucleon force), which reproduces the saturation density in an "ab initio" manner.

レポート問題

- 以下の問題を 2 問以上 (核理論研究室の大学院生)、または 1 問以上 (他研究室の大学院生) 解き、レポートとして提出せよ。〆切は 1 月末。提出先は大西居室 (基研 , K407) 。
  - ボソン化した NJL 模型の作用から出発して、ゼロ温度（T=0）での有効ポテンシャルを求めよ。
    余裕があれば、有限温度・有限密度 (有限化学ポテンシャル) での有効ポテンシャルを構成子クォーク質量で 2 次まで展開し、カイラル極限で 2 次相転移線が $T^2+\mu_B^2/3\pi^2 = T_c^2$ で与えられることを示せ。
  - リンク積分を利用して、 Wilson ループの期待値を強結合領域で求めよ。
    余裕があれば、強結合極限での結果に加えて、 $1/g^2$ 補正がどのように与えられるか評価せよ。
  - 相対論的平均場理論 (σω 模型) において、核子の four spinor の上 2 成分が満たす方程式を導き、スピン軌道力の表式を与えよ。また、エネルギー密度の表式を求めよ。
    余裕があれば、核物質の飽和点を満たすように σN, ωN の結合定数を与えよ。飽和点は ρ_0=0.15 fm^{-3}, E/A=-16 MeV とする。
    (後半は多少の数値計算が必要である。)
  - 斥力芯ポテンシャルのみがある場合に、適当な条件のもとで Bethe-Goldstone 方程式を解き、 healing が起こることを確かめよ。