核物質の状態方程式

質量公式と状態方程式

- $A \to \infty$ における核子あたりのエネルギー
  (クーロンエネルギーは無視)

$$E = \lim_{A\to\infty} \frac{-B(A,Z)}{A} = \lim_{A\to\infty}\left[-a_v + a_s A^{-1/3} + a_a \frac{(N-Z)^2}{A^2} - a_p \frac{\delta_p}{A^{\gamma-1}}\right]$$

$$= -a_v + a_a \delta^2$$

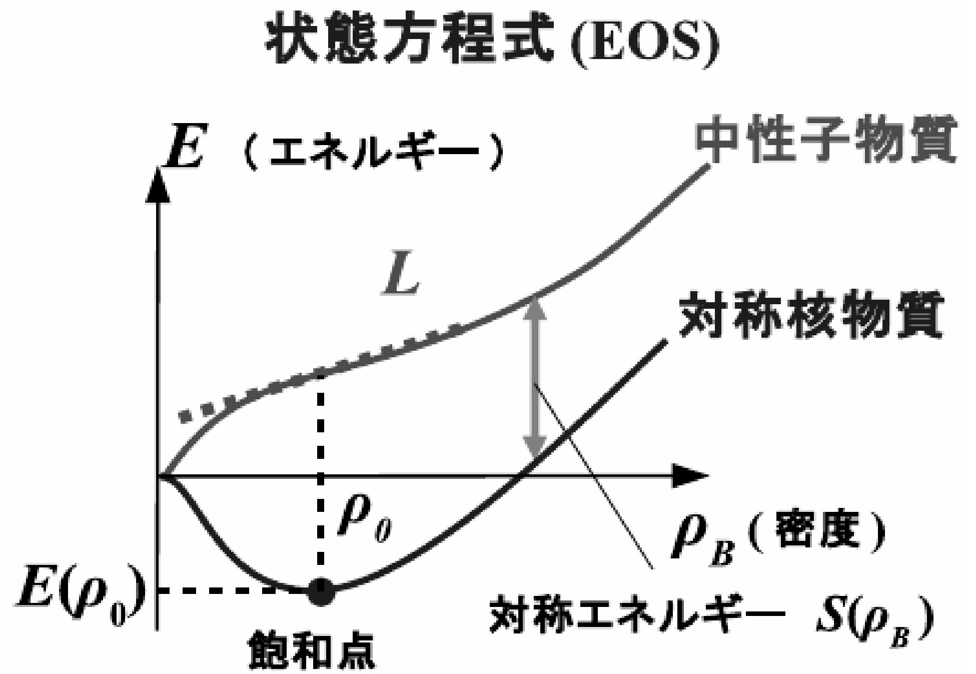
- 密度と非対称度の関数と考えると、
  核子あたりのエネルギーが最小となる密度が実現する

$$E = E(\rho_B, \delta)$$

→ 核物質の飽和性

- 飽和点

$$(\rho_0, E_0) \simeq (0.16\ \mathrm{fm}^{-3}, -16\ \mathrm{MeV})$$



対称エネルギー

- **非対称核物質 (N ≠ Z) のエネルギー**

$$E(\rho_B,\delta) = E(\rho_B,\delta=0) + S(\rho_B)\delta^2$$

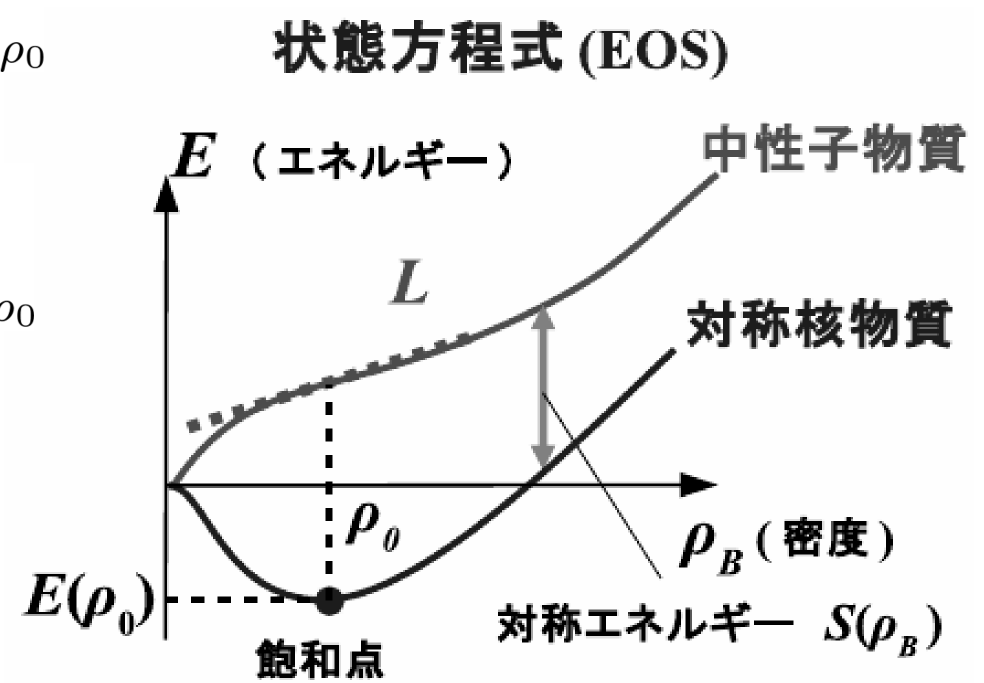
- **対称エネルギー**
  $S(\rho_B)$ = E(中性子物質)- E(対称核物質)

- **飽和密度でのパラメータ**
  - **非圧縮率** $$K \equiv 9\,\rho_0^2 \frac{\partial^2 E(\rho_B)}{\partial \rho_B^2}\bigg|_{\rho_B=\rho_0}$$
  - **対称エネルギーの値と微分**

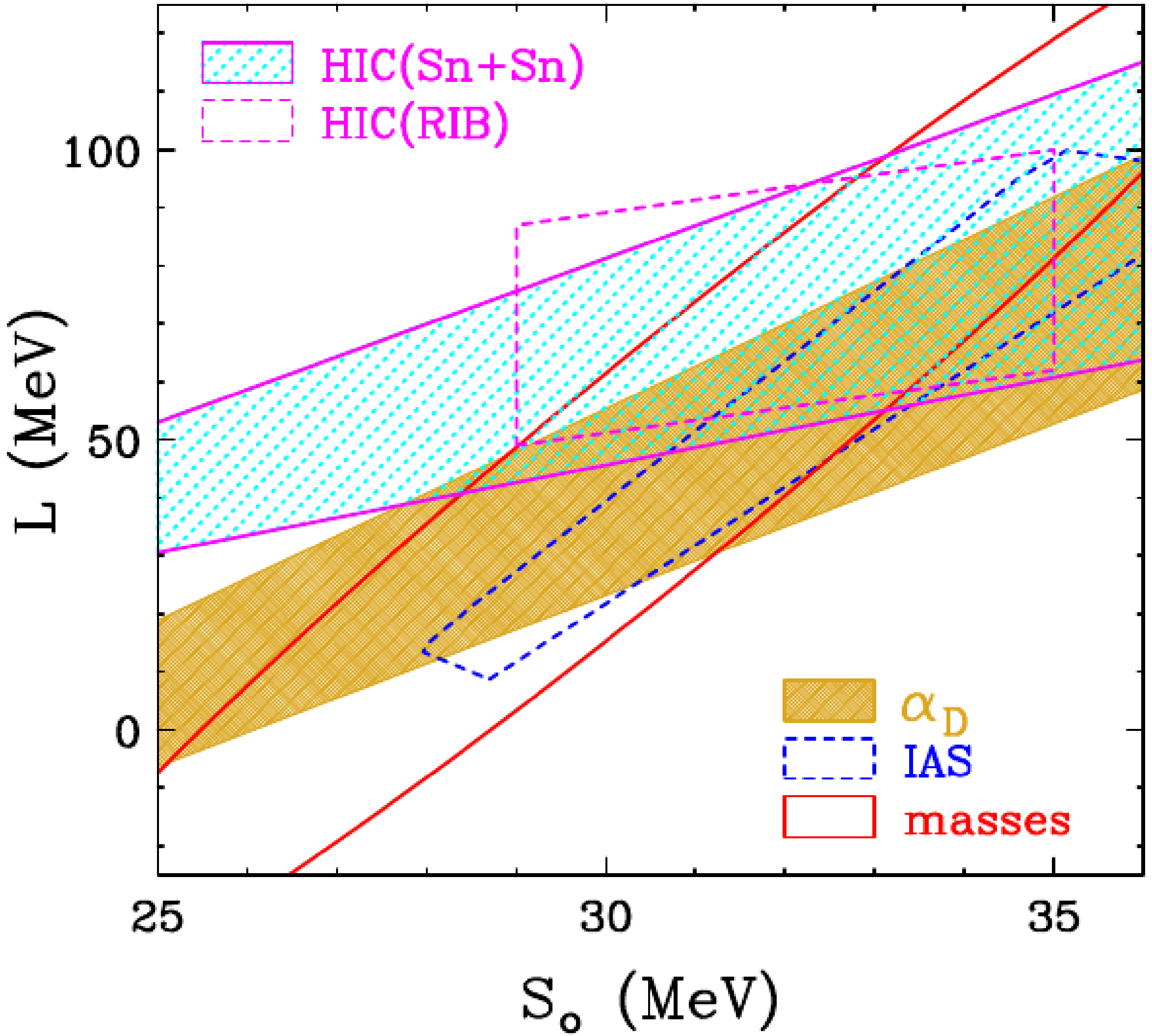
$$S_0 \equiv S(\rho_0)\ , \quad L \equiv 3\rho_0 \frac{dS(\rho_B)}{d\rho_B}\bigg|_{\rho_B=\rho_0}$$

$$E(\rho_B,\delta) \simeq E_0 + S_0\,\delta^2 + \frac{L}{3}\,x\,\delta^2 + \frac{K}{18}\,x^2$$

$$(x = (\rho_B - \rho_0)/\rho_0)$$



対称エネルギーの実験的制限



S_0=(30-35) MeV

L = (50-90) MeV

様々な実験により対称エネルギーパラメータを制限！

C.J. Horowitz, E.F. Brown, Y. Kim, W.G. Lynch, R. Michaels, A. Ono, J. Piekarewicz, M.B. Tsang, H.H. Wolter, J. Phys. G 41 (2014) 093001.

現象論的な核物質状態方程式

- **相互作用エネルギー**

$$V_{2B} = \frac{1}{2}\int d^3r\, d^3r' \rho_\mathrm{B}(\mathbf{r}) v(\mathbf{r}-\mathbf{r}')\rho_\mathrm{B}(\mathbf{r}') \to A\times\frac{\alpha}{2}\left(\frac{\rho_\mathrm{B}}{\rho_0}\right)$$

$$V_{3B} = \frac{1}{3}\int d^3r\, d^3r'\, d^3r''\, v(\mathbf{r},\mathbf{r}',\mathbf{r}'')\rho_\mathrm{B}(\mathbf{r})\rho_\mathrm{B}(\mathbf{r}')\rho_\mathrm{B}(\mathbf{r}'') \to A\times\frac{\beta}{3}\left(\frac{\rho_\mathrm{B}}{\rho_0}\right)^2$$

（一様密度、ゼロレンジの2体力･3体力）

- **現象論的な状態方程式**
  - **対称核物質**

$$E(\rho_\mathrm{B}) = \frac{3}{5}E_F(\rho_\mathrm{B}) + \frac{\alpha}{2}\left(\frac{\rho_\mathrm{B}}{\rho_0}\right) + \frac{\beta}{2+\gamma}\left(\frac{\rho_\mathrm{B}}{\rho_0}\right)^{1+\gamma}$$

  - **対称エネルギー**

$$S(\rho_\mathrm{B}) = \frac{1}{3}E_F(\rho_\mathrm{B}) + \alpha_\mathrm{sym}\left(\frac{\rho_\mathrm{B}}{\rho_0}\right) + \beta_\mathrm{sym}\left(\frac{\rho_\mathrm{B}}{\rho_0}\right)^{\gamma_\mathrm{sym}}$$

Simple parametrized EOS

- **Skyrme int. motivated parameterization**

$$E_{\rm SNM} = \frac{3}{5}E_F(\rho) + \frac{\alpha}{2}\left(\frac{\rho}{\rho_0}\right) + \frac{\beta}{2+\gamma}\left(\frac{\rho}{\rho_0}\right)^{1+\gamma}$$

$$\alpha = \frac{2}{\gamma}\left(E_0(1+\gamma) - \frac{E_F(\rho_0)(1+3\gamma)}{5}\right) \ , \quad \beta = \frac{2+\gamma}{\gamma}\left[-E_0 + \frac{1}{5}E_F(\rho_0)\right] \ .$$

$$K = \frac{3(1+3\gamma)}{5}E_F(\rho_0) - 9E_0(1+\gamma) \ .$$

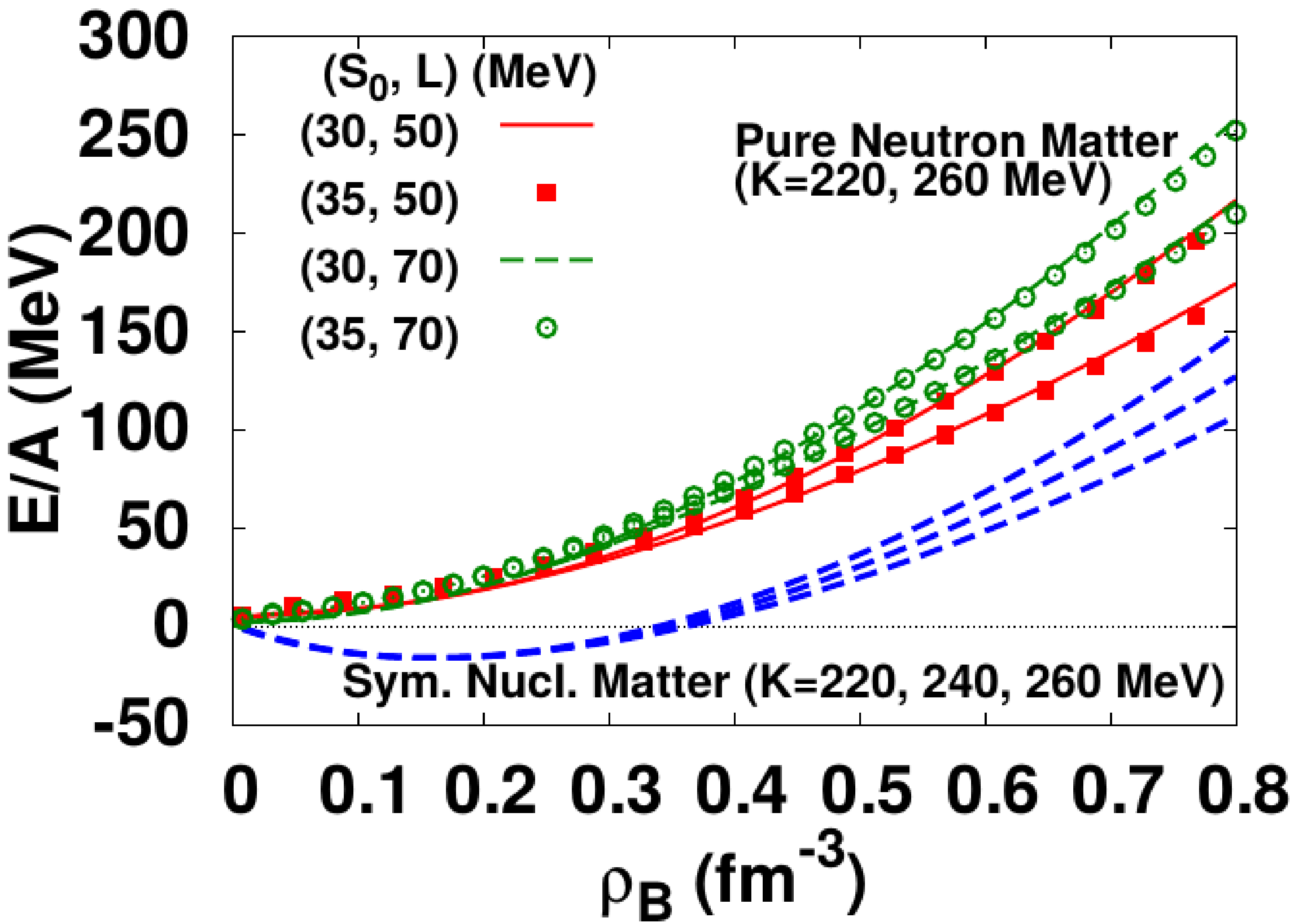
(corrected on Dec.18, 2016)

- **Symmetry energy parameterization**

$$S(\rho) = \frac{1}{3}E_F(\rho) + \left[S_0 - \frac{1}{3}E_F(\rho_0)\right]\left(\frac{\rho}{\rho_0}\right)^{\gamma_{\rm sym}}$$

$$\gamma_{\rm sym} = \frac{L - \frac{2}{3}E_F(\rho_0)}{3S_0 - E_F(\rho_0)}$$

現象論的な核物質状態方程式

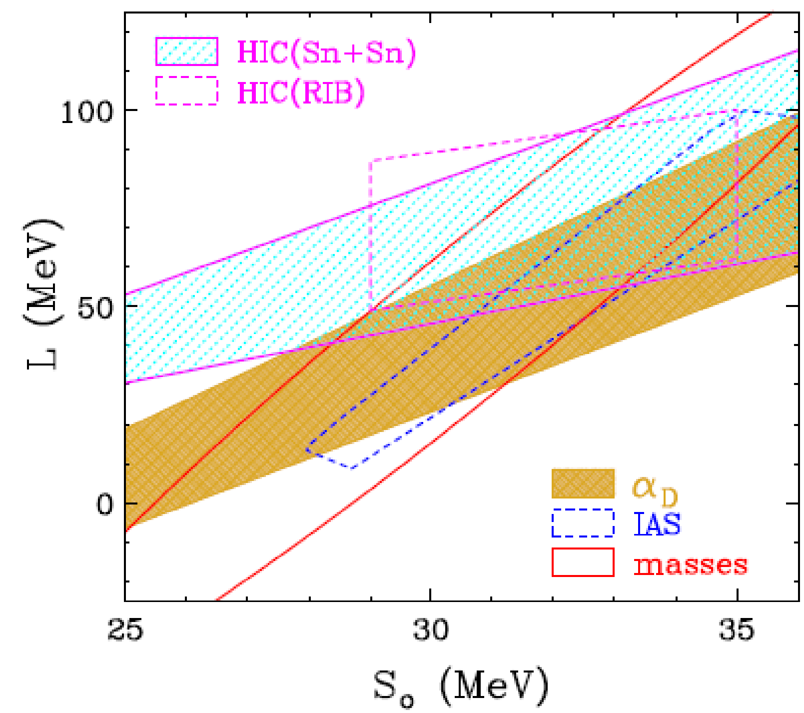


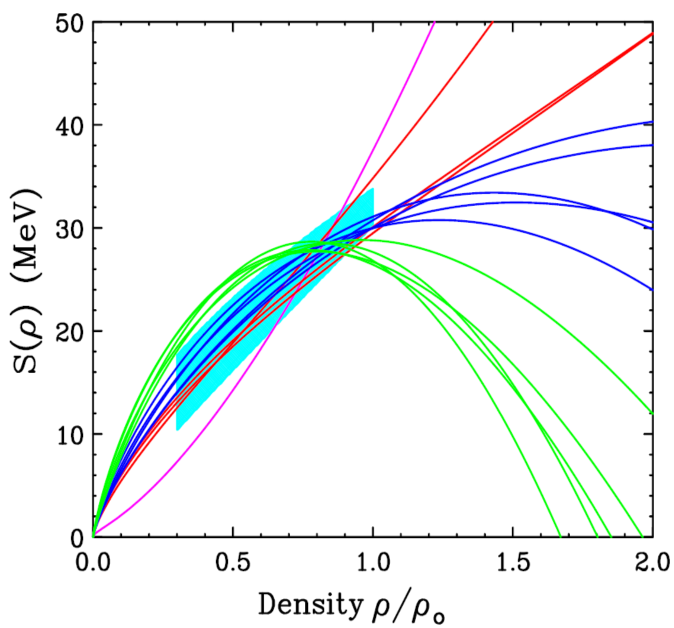
飽和密度近辺での不定性は少ないが、
中性子物質・高密度では大きな不定性

Symmetry Energy

- **Symmetry Energy has been extracted from various observations.**
  - **Mass formula, Isobaric Analog State, Pygmy Dipole Resonance, Isospin Diffusion, Neutron Skin thickness, Dipole Polarizability, Asteroseismology**

Recent recommended value
S_0 = 30-35 MeV, L = 40-90 MeV
Is it enough for NS radii ?





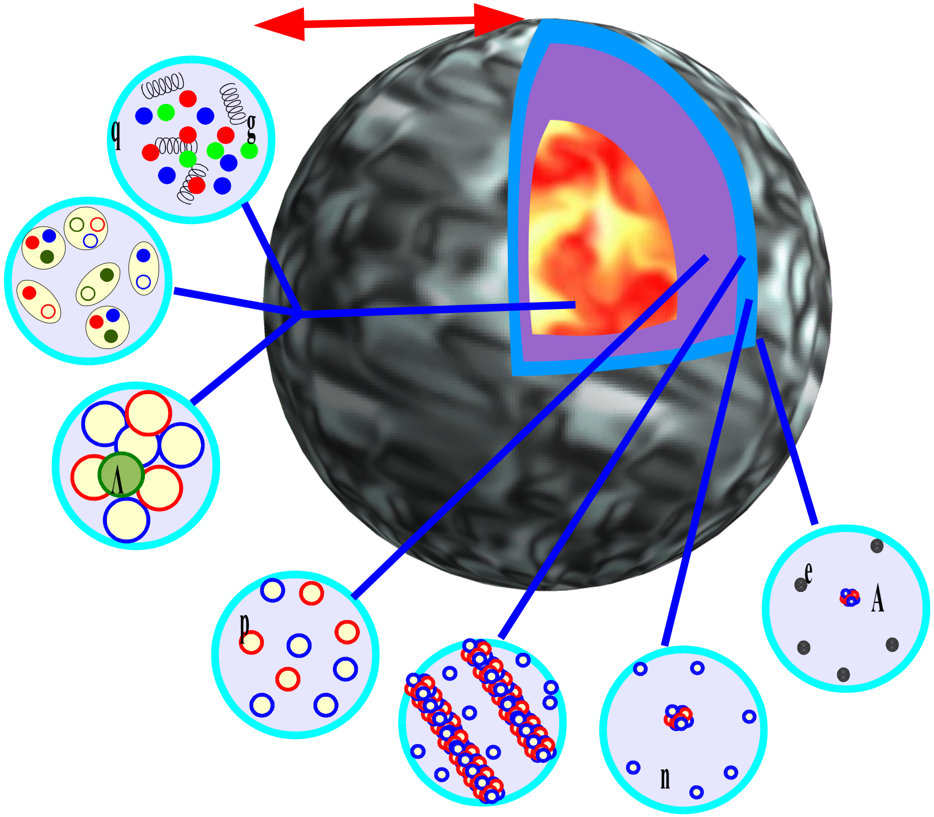
M.B.Tsang et al. (NuSYM2011), PRC 86 ('12)015803.

C.J.Horowitz, E.F.Brown, Y.Kim, W.G.Lynch, R.Michaels, A. Ono, J. Piekarewicz, M. B. Tsang, H.H.Wolter (NuSYM13), JPG41('14) 093001

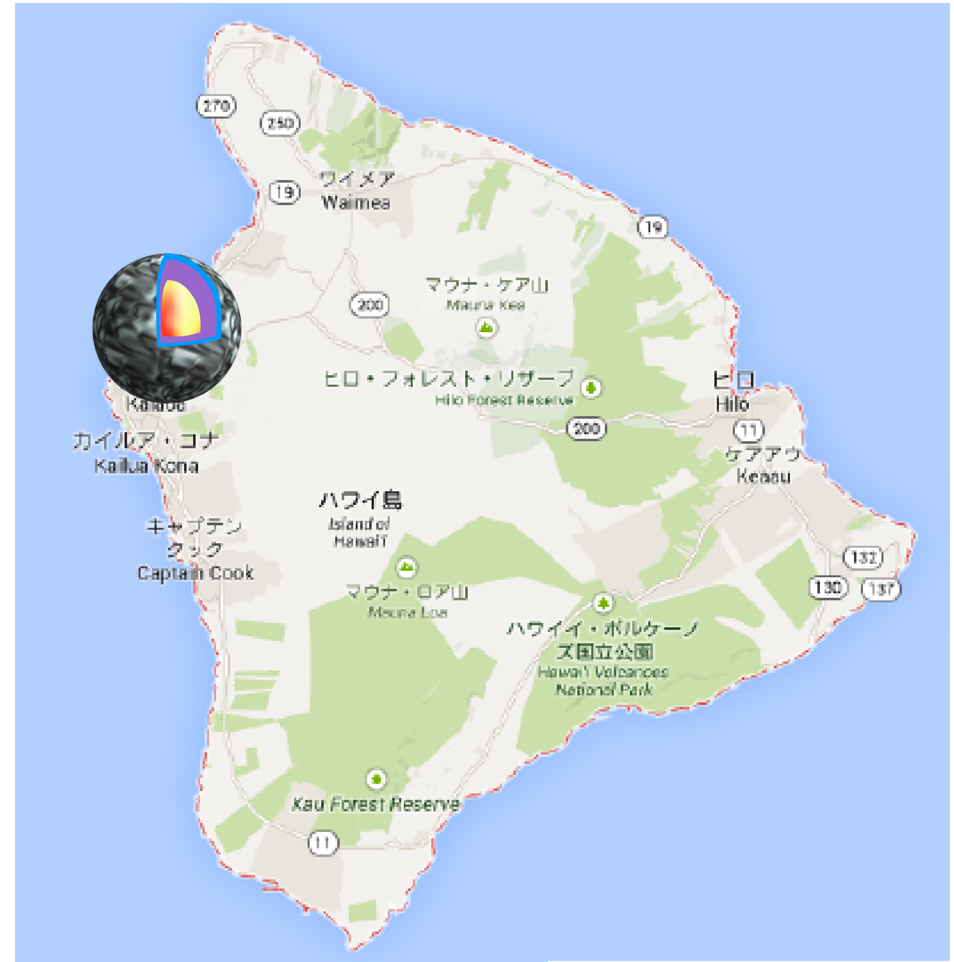
Neutron Star

Star supported by nuclear force

R ~ 10 km



M ~ 1.4 $M_\odot$, ρ_c ~ (3-10) ρ_0



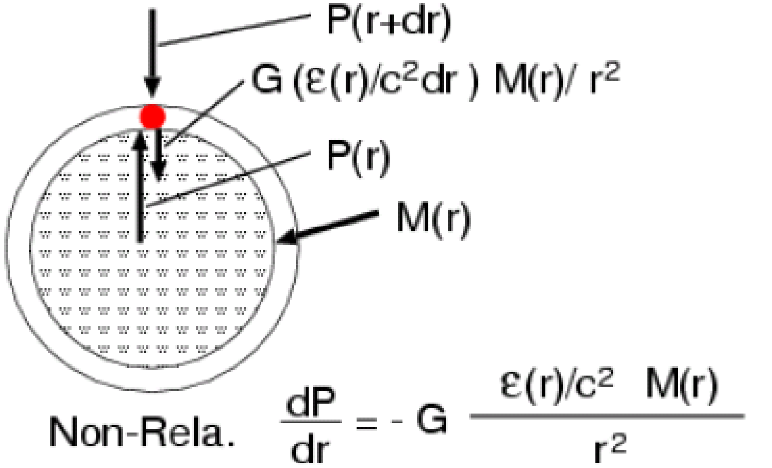
Wide density range → various constituents
NS = high-energy astrophysical objects
and laboratories of dense matter.

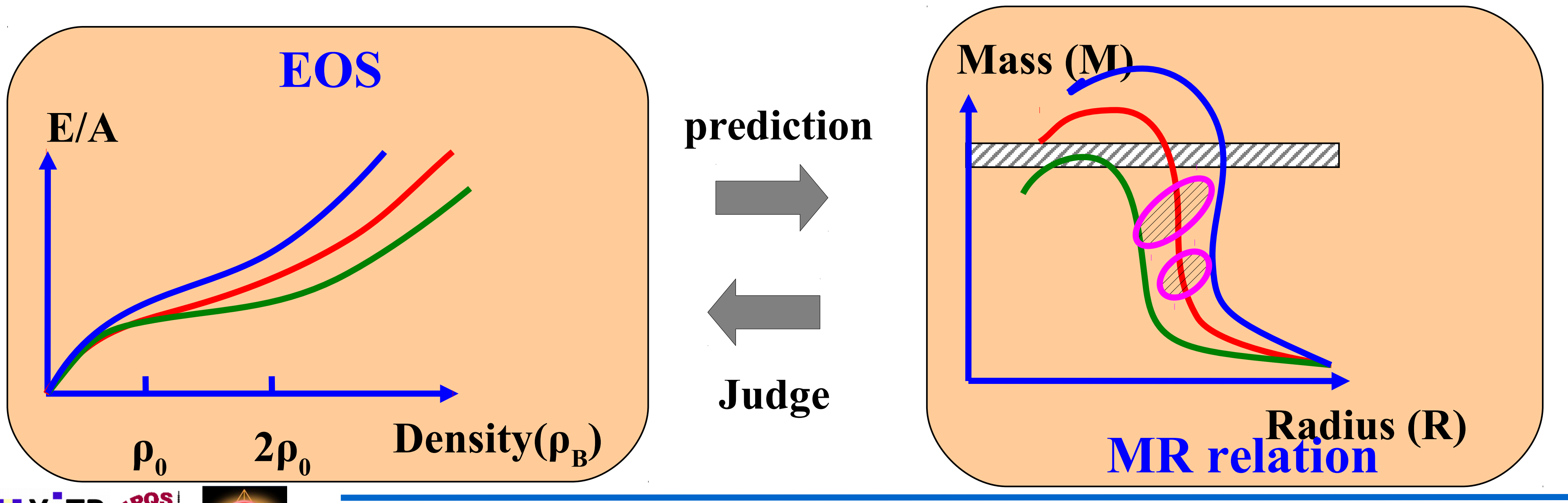
M-R curve and EOS

- **M-R curve and NS matter EOS has 1 to 1 correspondence**
  - **TOV(Tolman-Oppenheimer-Volkoff) equation =GR Hydrostatic Eq.**

$$\frac{dP}{dr} = -G \frac{(\varepsilon/c^2 + P/c^2)(M + 4\pi r^3 P/c^2)}{r^2(1 - 2GM/rc^2)}$$

$$\frac{dM}{dr} = 4\pi r^2 \varepsilon/c^2, \quad P = P(\varepsilon) \quad (\mathrm{EOS})$$



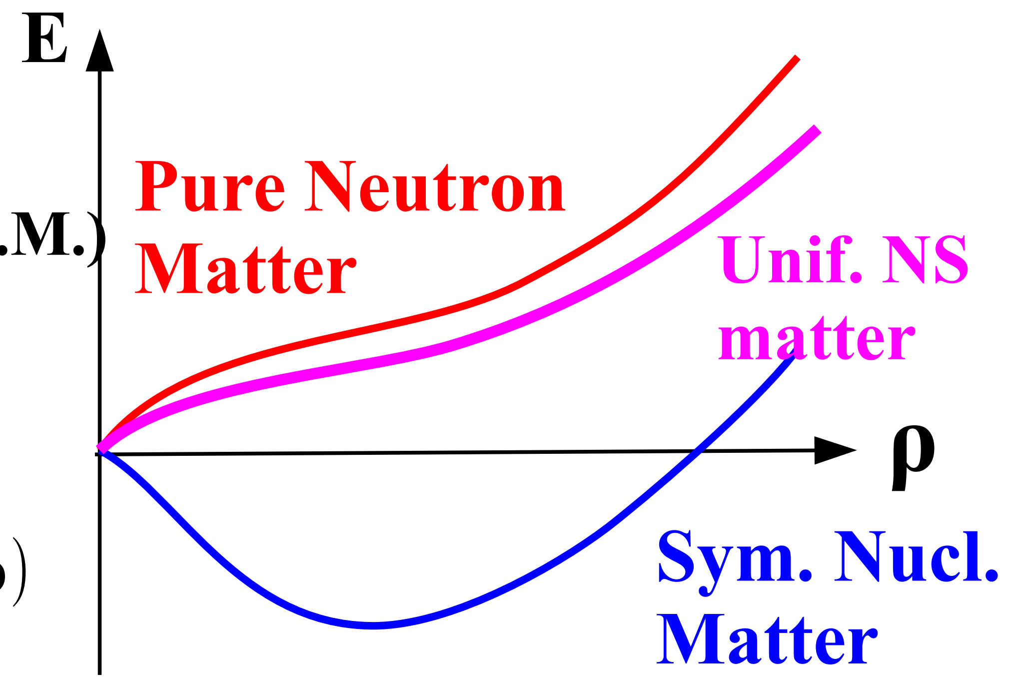


Neutron Star Matter EOS

- **What happens in low-density uniform neutron star matter ?**
  - **Constituents = proton, neutron and electron**
  - **Charge neutrality → # of electons= # of protons ($\rho_e = \rho_p = \rho(1-\delta)/2$)**

$$E_{\rm NSM}(\rho) = E_{\rm NM}(\rho,\delta) + E_e(\rho_e = \rho_p)$$
$$= E_{\rm SNM}(\rho) + S(\rho)\delta^2 + \frac{\Delta M}{2}\delta + \frac{3}{8}\hbar k_F (1-\delta)^{4/3}$$

(electron mass neglected,
neutron-proton mass diff. incl.
k_F= Fermi wave num. in Sym. N.M.)



  - **δ is optimized to minimize energy per nucleon**

$$E_{\rm NSM}(\rho) \le E_{\rm NM}(\rho,\delta=1) = E_{\rm PNM}(\rho)$$

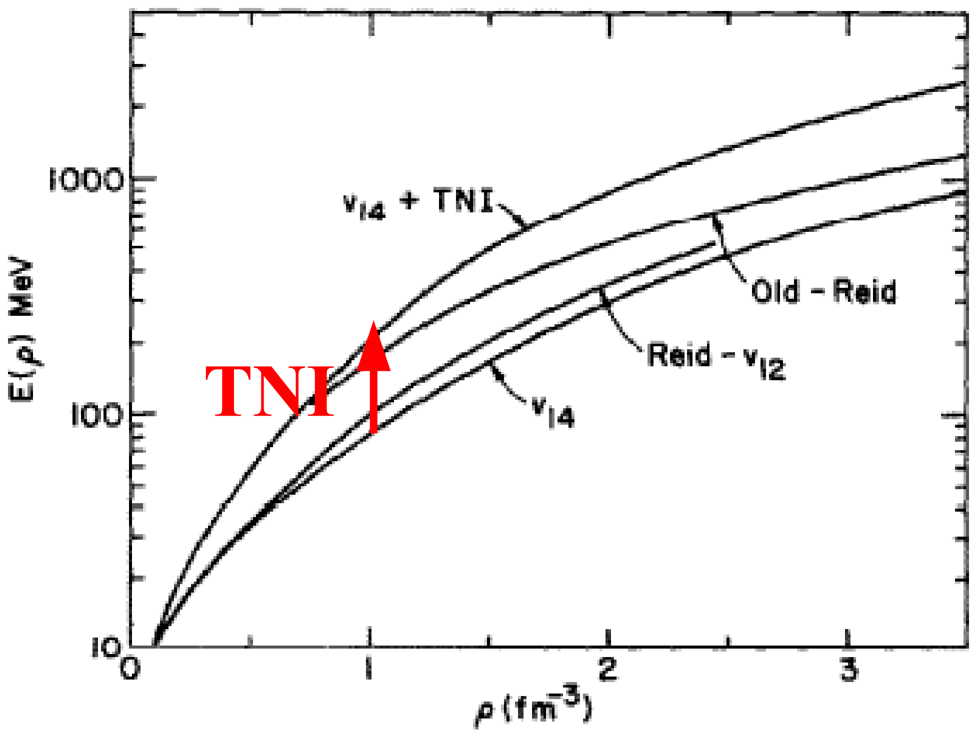
状態方程式を記述する理論の枠組み

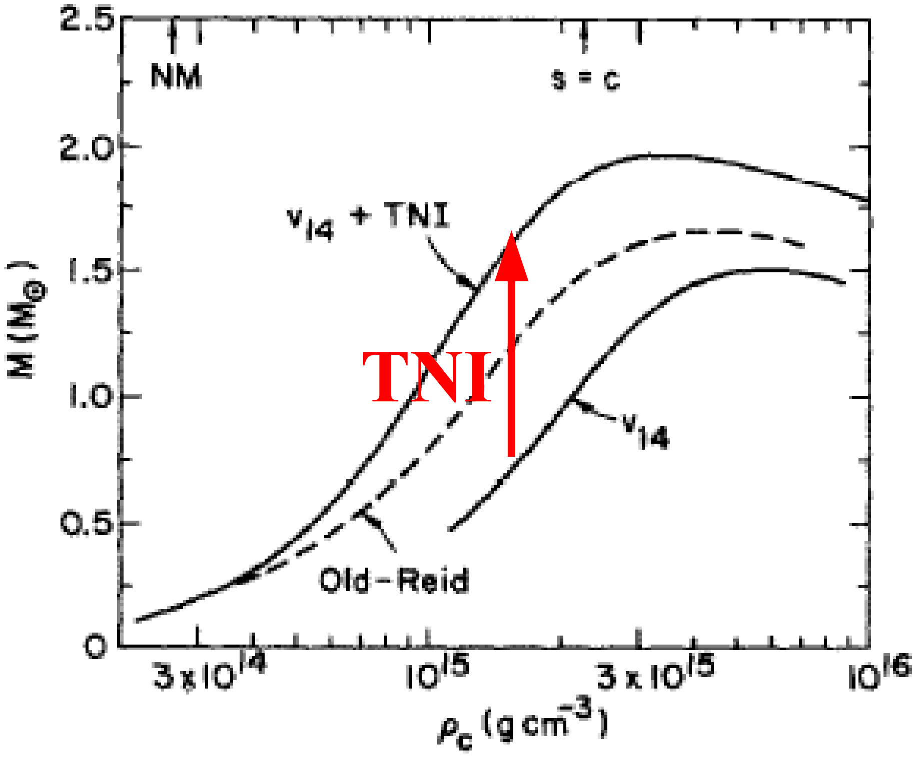
Theories/Models for Nuclear Matter EOS

- Ab initio Approaches
  - LQCD, GFMC, Variational, BHF, DBHF, G-matrix, ...
- Mean Field from Effective Interactions ~ Nuclear Density Fuctionals
  - Skyrme Hartree-Fock(-Bogoliubov)
    - Non.-Rel.,Zero Range, Two-body + Three-body (or ρ-dep. two-body)
    - In HFB, Nuclear Mass is very well explained (Total B.E. $\Delta E \sim 0.6$ MeV)
    - Causality is violated at very high densities.
  - Relativistic Mean Field
    - Relativistic, Meson-Baryon coupling, Meson self-energies
    - Successful in describing pA scatering (Dirac Phenomenology)

Variational Calculations (1)

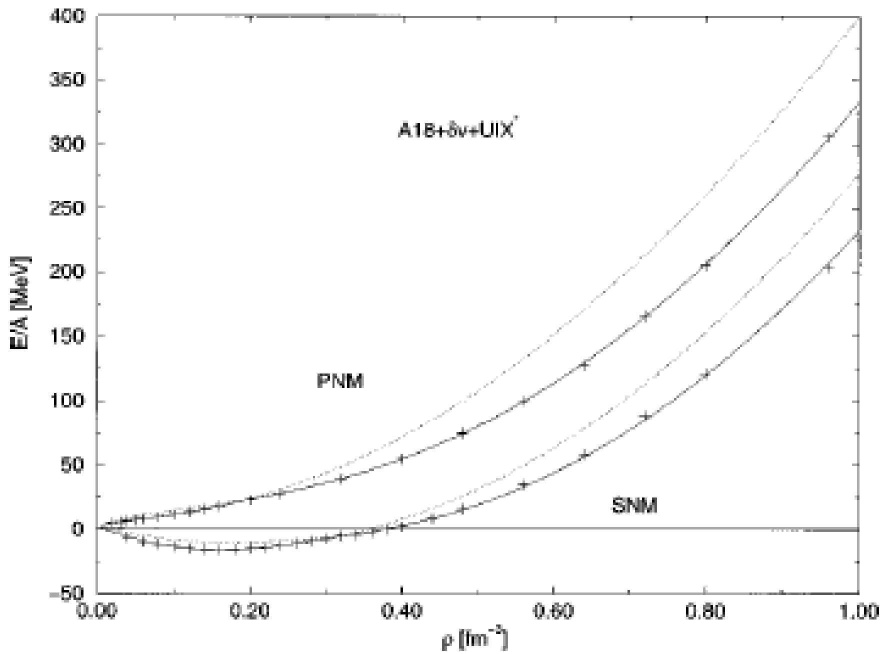
- **Variational Calculation starting from bare nuclear force**
  B. Friedman, V.R. Pandharipande, NPA361('81)502
  - **Argonne v14 + TNI (TNR+TNA)**
    (TNI/TNR/TNA: three-nucleon int./repulsion/attraction)

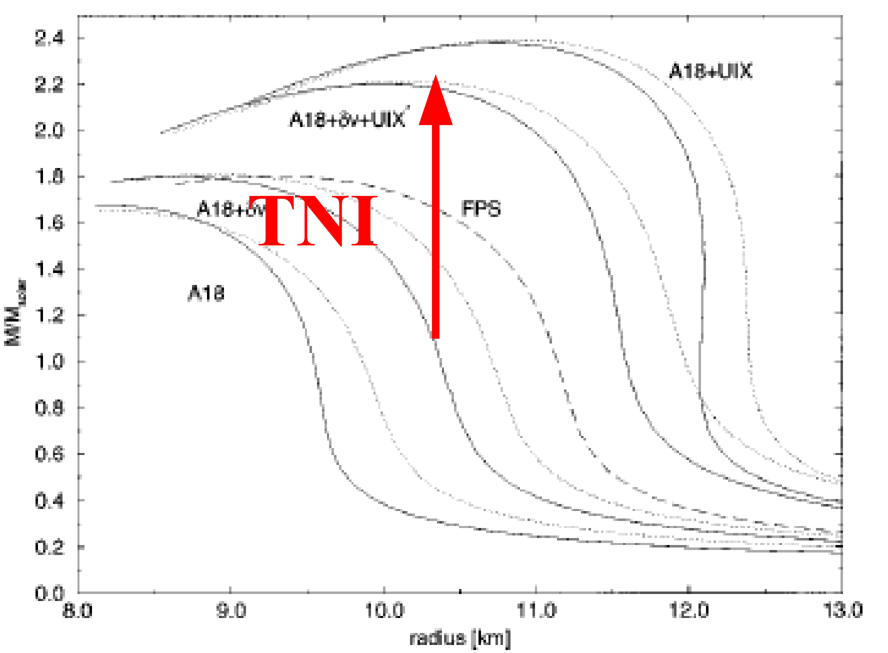




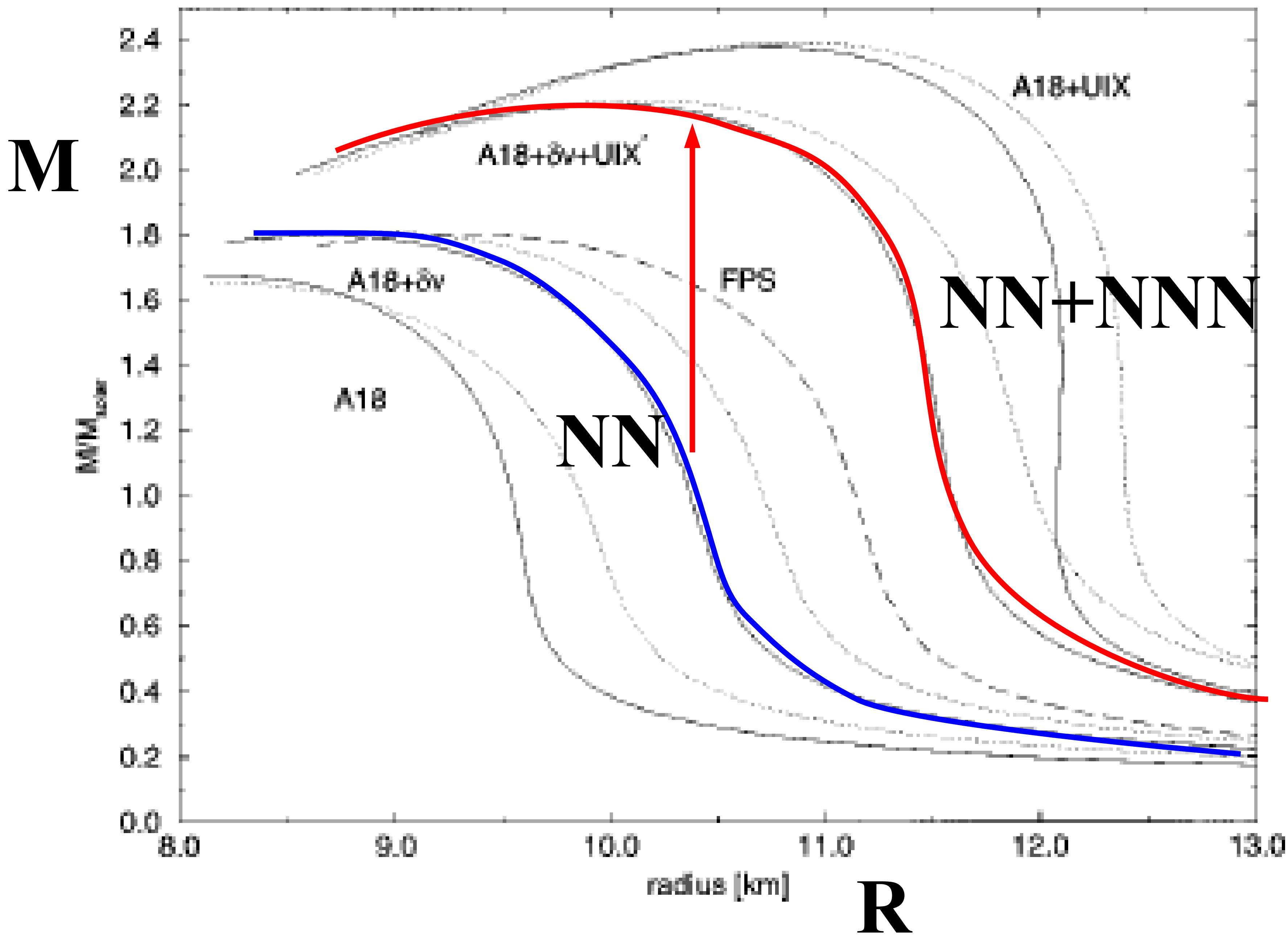
Variational Calculation (2)

- **Variational chain summation method**
  A. Akmal, V.R.Pandharipande, D.G. Ravenhall, PRC58('98)1804
  - **v18, relativistic correction, TNI**
  - **Existence of neutral pion condensation at $\rho_B > 0.2\ \mathrm{fm}^{-3}$**

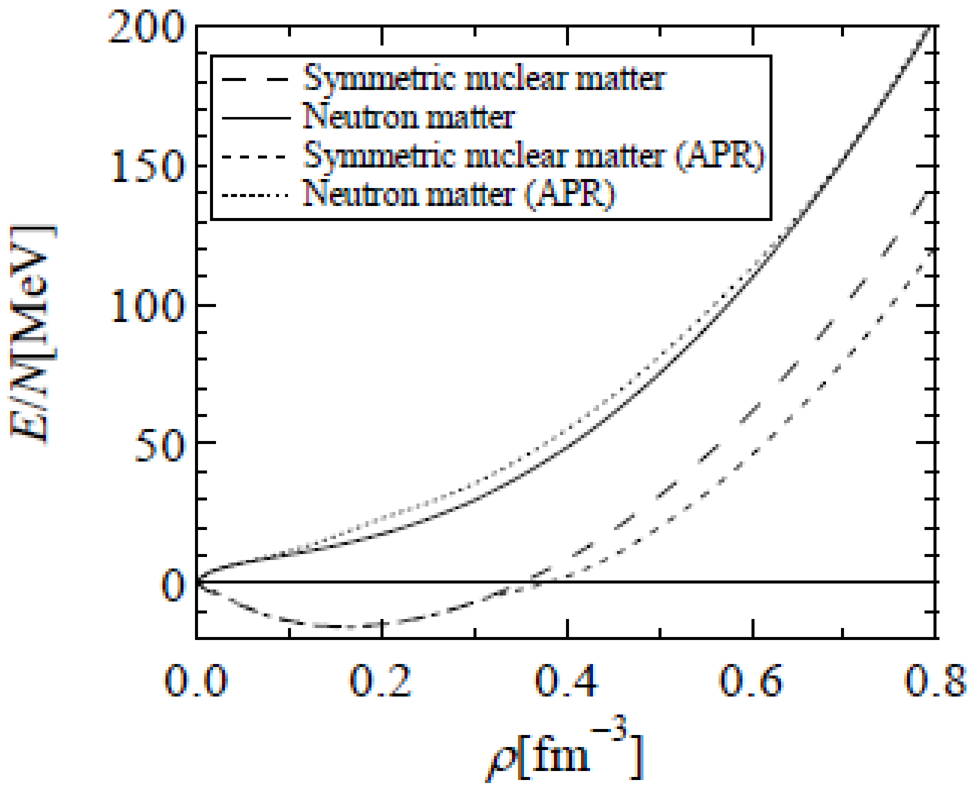


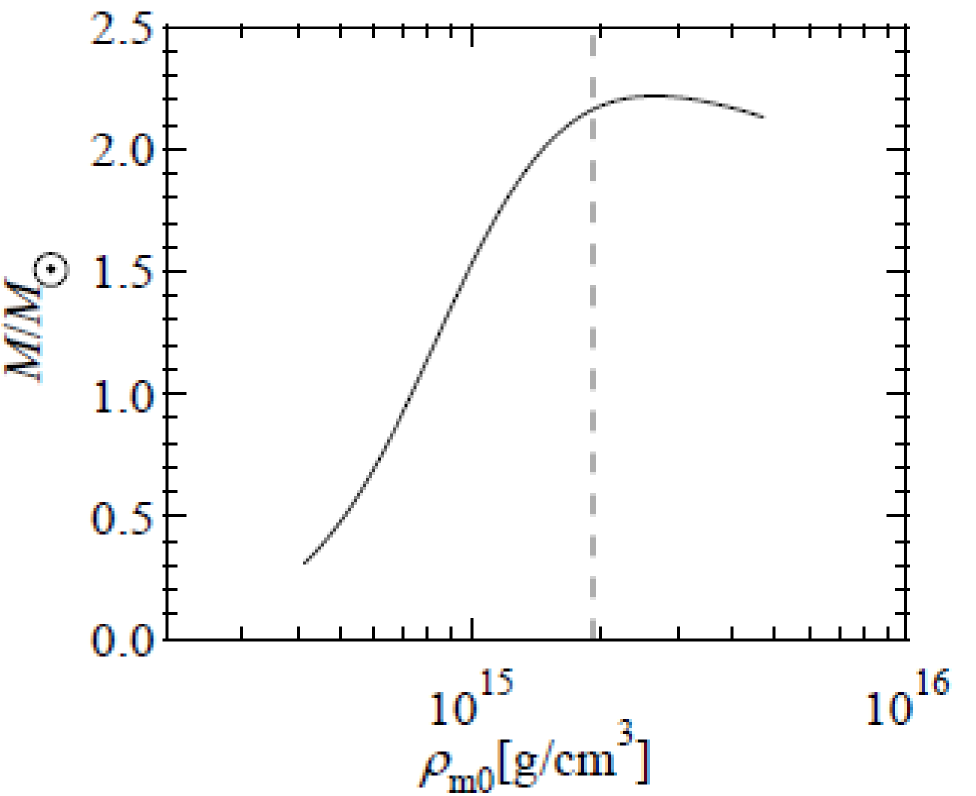


APR


M
R
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
8.0
9.0
10.0
11.0
12.0
13.0
radius [km]
A18+UIX
A18+δv+UIX*
A18+δv
FPS
A18
NN
NN+NNN

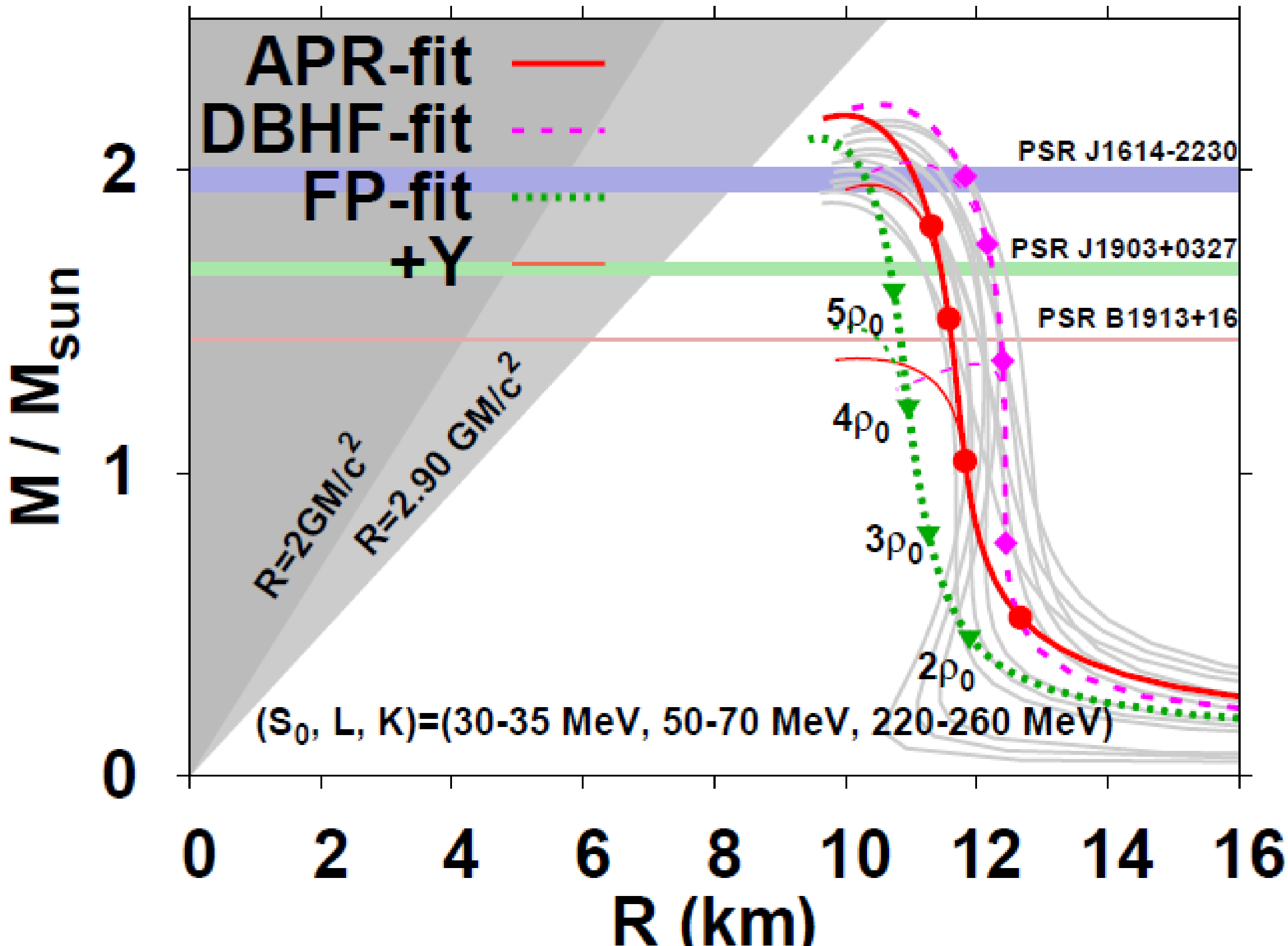
- **Variational Calculation using v18+UIX**
  H. Kanzawa, K. Oyamatsu, K. Sumiyoshi, M. Takano, NPA791 ('07) 232
  - **Similar to APR, but healing-distance condition is required.**
    → no π^0 condensation





Ab initio & 現象論的 EOS での MR 曲線

- 現象論的状態方程式から推測される MR 曲線（灰色）
  - 半径　R=(11-13) km (M= 1.4 $M_\odot$) 、最大質量　M_{max} = (1.9-2.2) $M_\odot$ (核子のみの場合)



Hartree-Fock Theory

- **平均場理論 = 多体問題の基本**

$$\delta \frac{\langle \Phi | H | \Phi \rangle}{\langle \Phi | \Phi \rangle} = 0, \qquad |\Phi\rangle = \det\{\phi_1 \cdots \phi_N\}: \text{Slater determinant}$$

- **電子系ではエネルギーをほぼ再現**
- **原子核ではナイーブな HF は破綻**
  - **短距離での斥力コア → エネルギー = ∞**
  - **2 体相関が決定的**

$$\rho_2(\mathbf{r}_1, \mathbf{r}_2) = 0, \quad \text{for } |\mathbf{r}_1 - \mathbf{r}_2| < c$$

→ Brueckner 理論 (G-matrix)

原子 ·分子など、電子系

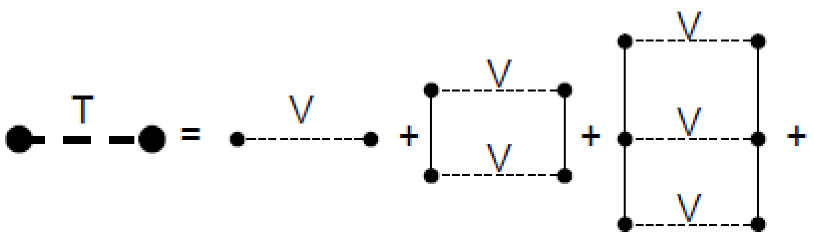
	HF	Exp
He	−2.86	−2.90
Li	−7.43	−7.48
Ne	−128.55	−128.94
Ar	−526.82	−527.60

原子単位 (27.2 eV)

Brueckner Theory

- **Lippmann-Schwinger Eq.**

$$T = V + VG_0 T$$



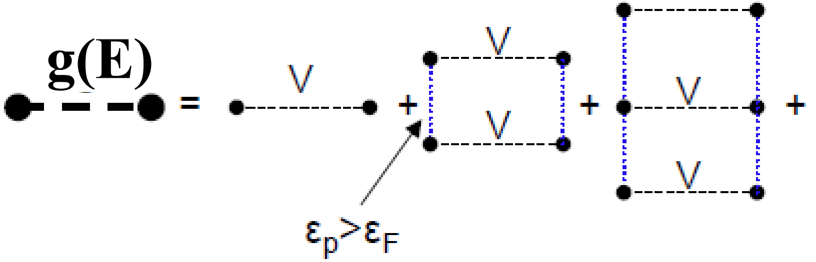
  - **V が singular でも T は有限**

- **原子核中での 2 体散乱**
  → パウリ原理

$$V\left|\Psi_{\mathbf{k}}^{(+)}\right\rangle = T\left|\mathbf{k}\right\rangle$$

$$g(E) = V + V\frac{Q}{E - H_0}g(E)$$

$$Q = 1 - \sum_{i,j<F}\left|ij\right\rangle\left\langle ij\right|$$



  - **原子核中では中間状態で**
    フェルミエネルギー以上の状態のみ
    伝播可能

$$V\left|\Psi\right\rangle = g(E)\left|\Phi\right\rangle$$

2 体相関を含む複雑な状態 ↑ (for $|\Psi\rangle$)

2 体相関の無い状態 (E.g. Slater det.) ↑ (for $|\Phi\rangle$)

- **核内での散乱行列 =g-matrix**

Healing distance

- **(波動関数についての)**
  Bethe-Goldstone 方程式

$$g_{12} = v_{12} + v_{12} \frac{Q_{12}}{E - (t_1 + t_2 + U_1 + U_2)} g_{12}$$

$$\rightarrow \left[E - (t_1 + t_2 + U_1 + U_2) \right] \psi_{12} = Q_{12} v_{12} \psi_{12}$$

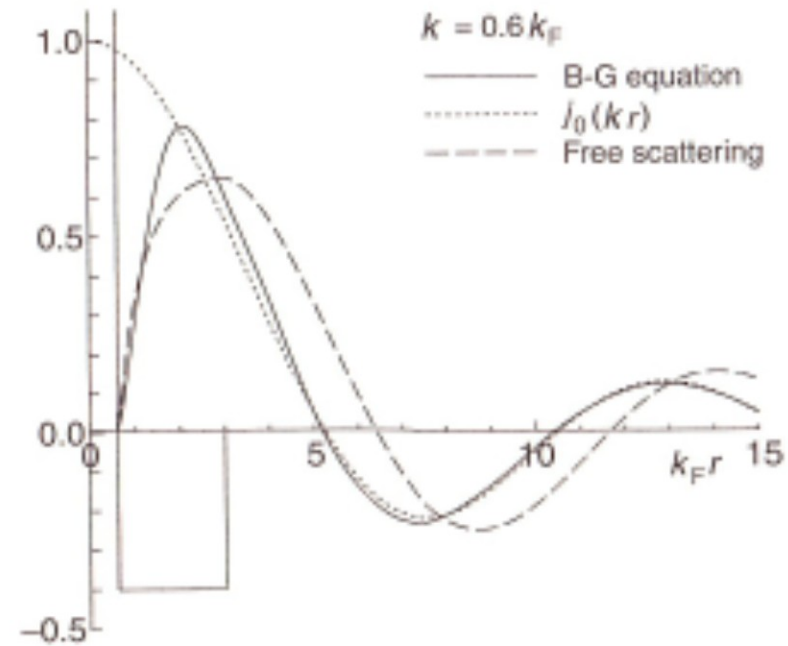
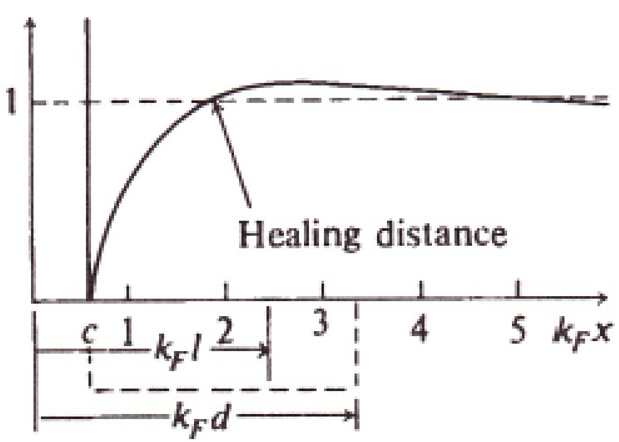


図 2.17 $k = 0.6\,k_F$ の場合の Bethe-Goldstone 方程式の解 (実線) と，自由空間内の 2 粒子散乱 (破線) および自由粒子の相対波動関数 (点線) の比較

$k_F = 1.27\ \mathrm{fm}^{-1}$, 芯半径は $k_F r_c = 0.62$, 井戸型ポテンシャルの半径は $k_F r_a = 3.0$, 有効質量は $M^*/M = 0.6$ ととられている.

- **BG 方程式の解は、$k_F\, l \sim 1.9$ 程度の距離で通常の平面波にほぼ一致する (Healing distance)**
  → 独立粒子描像



Brueckner-Hartree-Fock theory

- **g-matrix を 2 体相互作用とする HF**
  = Brueckner-Hartree-Fock

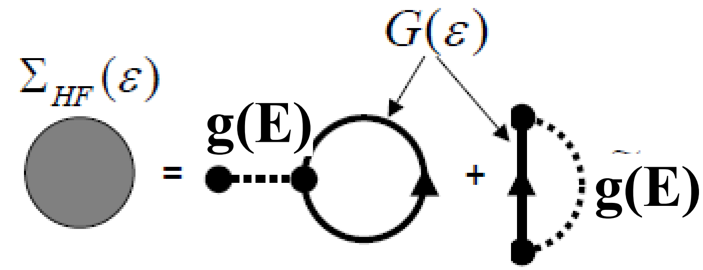
$$H = H_0 + V\,, \quad V = \frac{1}{2}\sum_{i\neq j} V_{ij}$$

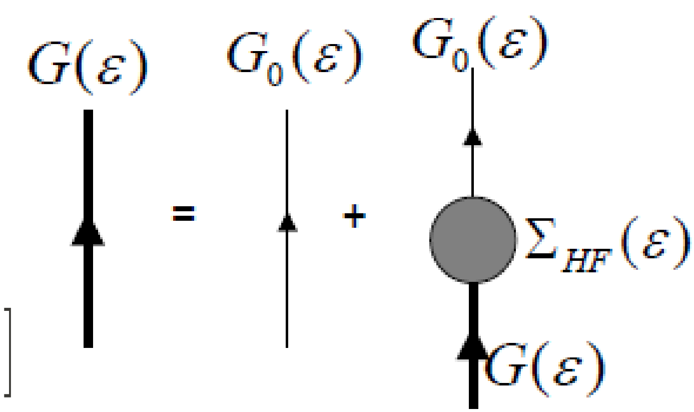
$$H_0 = \sum_i \left[-\frac{\hbar^2}{2\mathrm{m}} \nabla_i^2 + U_i \right]$$

$$g(E) = V + V \frac{Q}{E - H_0} g(E)$$

$$U_i(\varepsilon_i) = \sum_j \left[g_{ij,ij}(\varepsilon_i + \varepsilon_j) - g_{ij,ji}(\varepsilon_i + \varepsilon_j) \right]$$

$$E_{\mathrm{BHF}} = \sum_i^{\mathrm{occ.}} \langle i | -\frac{\hbar^2}{2\mathrm{m}} \nabla^2 | i \rangle + \frac{1}{2} \sum_{i\neq j}^{\mathrm{occ.}} \langle ij | g(\varepsilon_i + \varepsilon_j) | ij - ji \rangle$$

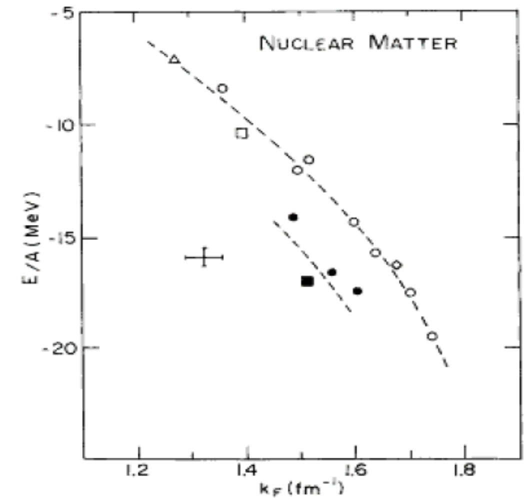




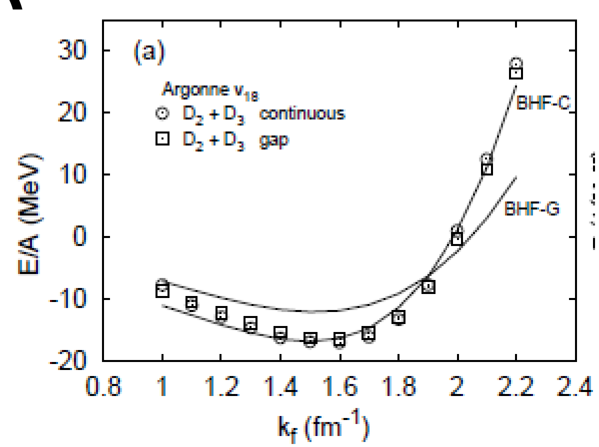
- **Self-consistent treatment**
  U → g-matrix & φ (s.p.w.f) → U

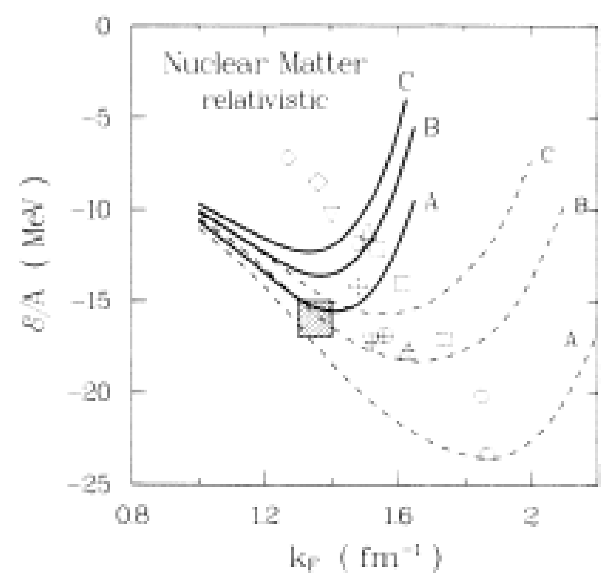
Brueckner-Hartree-Fock theory (cont.)

- **成功点**
  - 核物質の飽和性を定性的に説明
  - 殻模型（独立粒子描像）の基礎を与える
  - 有効核力の状態依存性を説明
- **問題点**
  - 飽和点 (飽和密度、飽和エネルギー) の
    定量的理解 (Coester line)→ Relativity or 3 体力
  - 展開の高次項→ Continuum choice では
    3 体クラスター効果は小さい
  - スピン軌道力が足りない



問題は残っているが、
現実的核力から出発して
多体問題に適用する
有効な手法

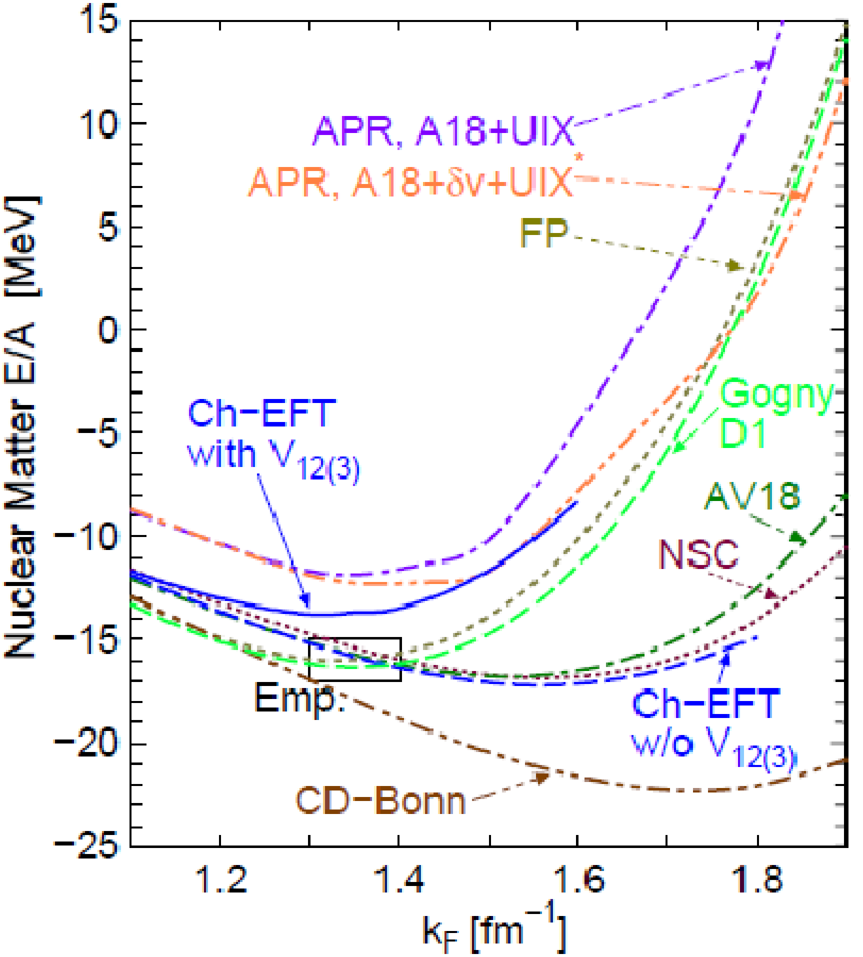




Ch-EFT EOS

- **Phen. models need inputs from**
  Experimental Data and/or Microscopic (Ab initio) Calc.
- **Recent Ch-EFT EOS is promising !**
  NN (N3LO)+3NF(N2LO)

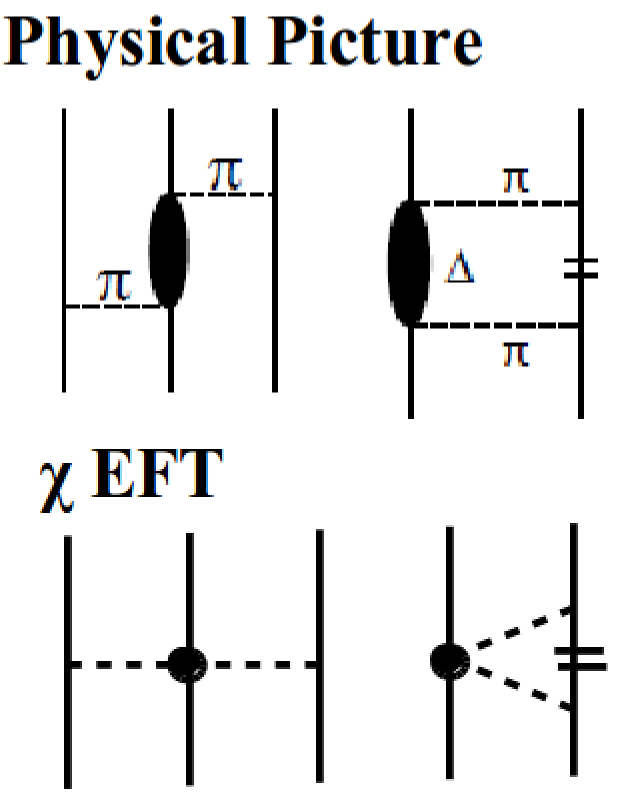
M.Kohno ('13)



M. Kohno, PRC 88 ('13) 064005

"Universal" mechanism of "Three-body" repulsion

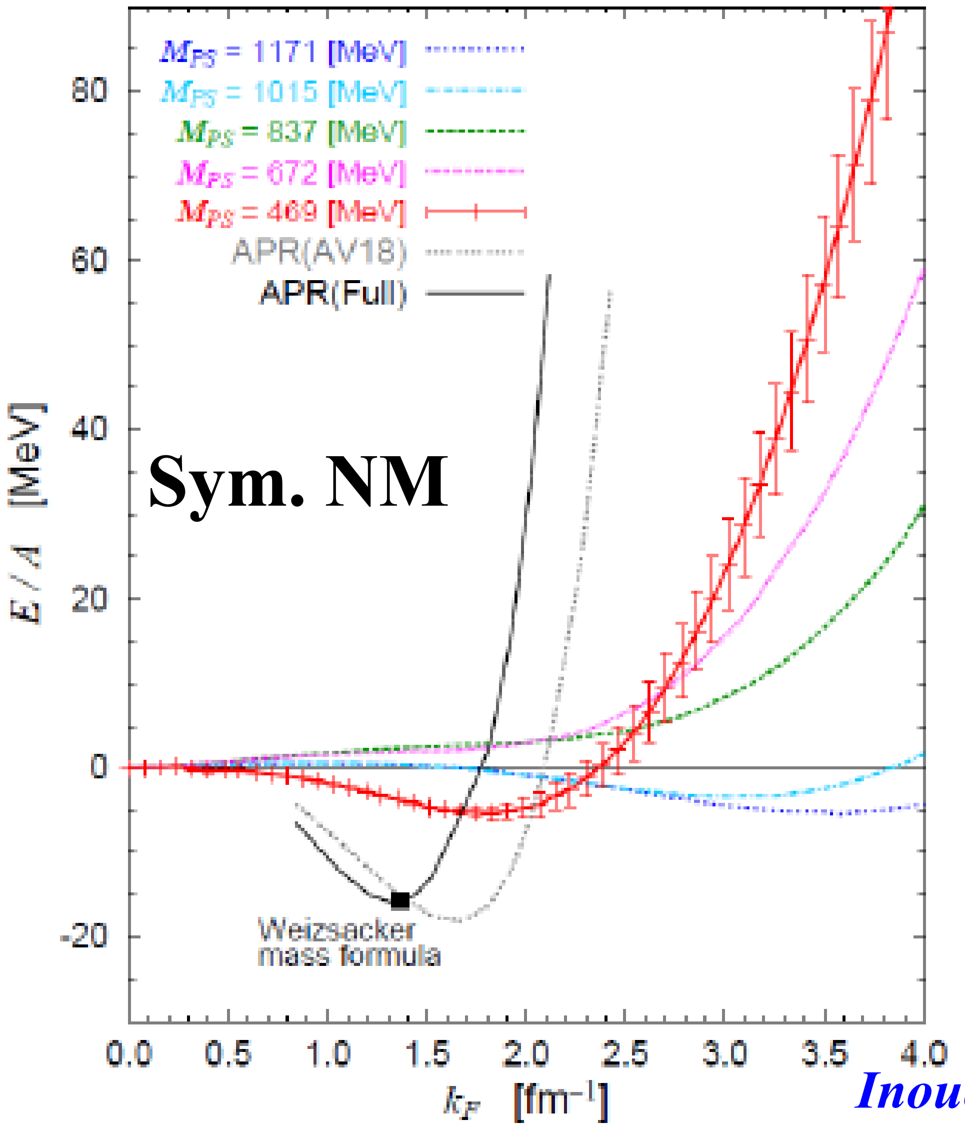
- **"Universal" 3-body repulsion is necessary to support NS. Nishizaki, Takatsuka, Yamamoto ('02)**
- **Mechanism of "Universal" Three-Baryon Repulsion.**
  - **"σ"-exchange ~ two pion exch. w/ res.**
  - **Large attraction from two pion exchange is suppressed by the Pauli blocking in the intermediate stage. Kohno ('13)**

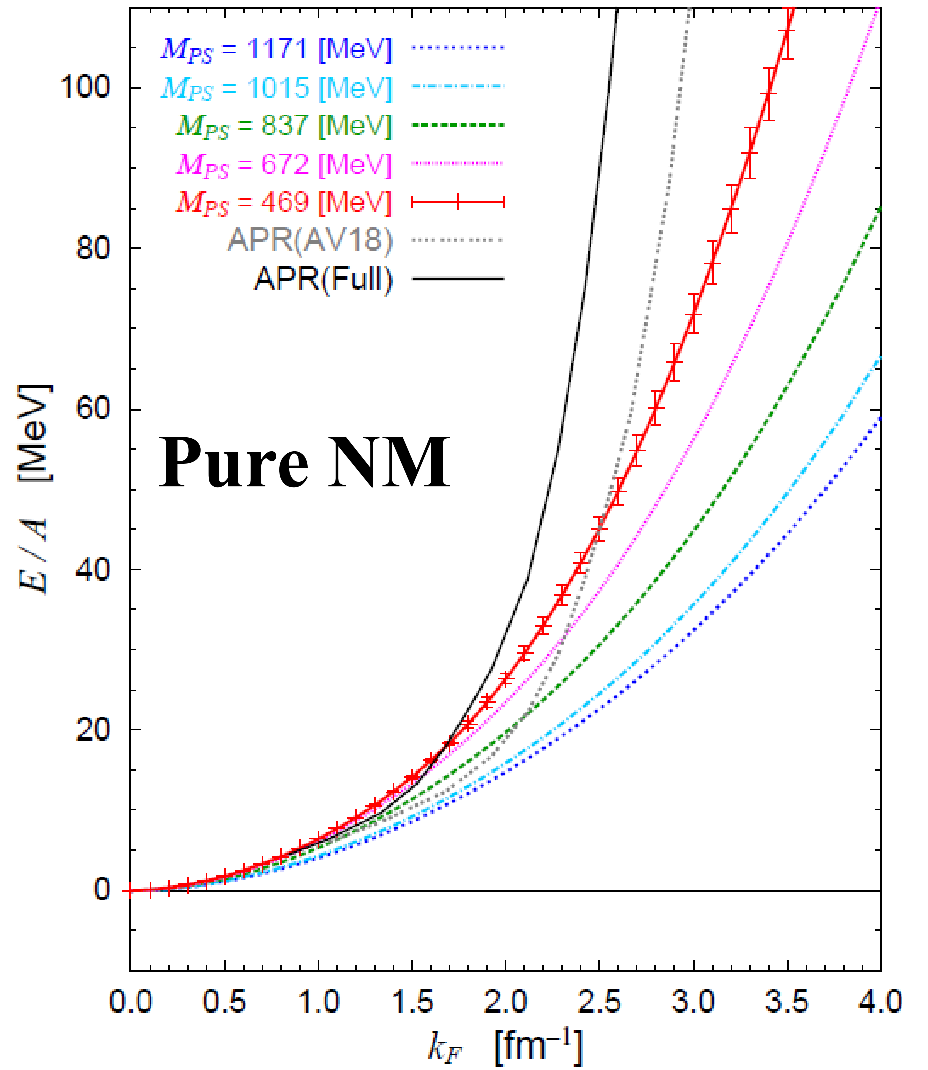


"Universal" TBR
- ***Coupling to Res. (hidden DOF)***
- ***Reduced "σ" exch. pot. ?***

EOS from lattice NN force

- **格子 QCD 核力を用いた高密度状態方程式 (LQCD+BHF)**
  NN force: $^1S_0$, $^3S_1$, $^3D_1$ only





Inoue et al. (HAL QCD Coll.), PRL111 ('13)112503

Relativistic Mean Field

Relativistic Mean Field (1)

- **Effective Lagrangian of Baryons and Mesons + Mean Field App.**

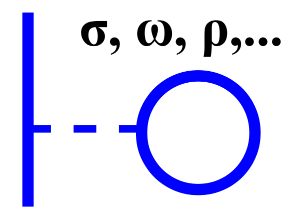
  B.D.Serot, J.D.Walecka, Adv.Nucl.Phys.16 ('86), 1

$$L = L_B^{\mathrm{free}} + L_M^{\mathrm{free}} + L_{BM} + L_M^{\mathrm{Int}}$$

$$L_M^{\mathrm{Int}} = -U_\sigma(\sigma) + \frac{1}{4} c_\omega (\omega_\mu \omega^\mu)^2 + \cdots$$

$$L_{BM} = -\sum_{B,S} g_{BS} \bar{\psi}_B \varphi_S \psi_B - \sum_{B,V} g_{BV} \bar{\psi}_B \gamma^\mu V_\mu \psi_B$$

$$L_B^{\mathrm{free}} = \bar{\psi}_B (i\gamma^\mu \partial_\mu - M_B) \psi_B \ , \quad L_M^{\mathrm{free}} = \sum_S \left[\frac{1}{2} \partial^\mu \varphi_S \partial_\mu \varphi_S - \frac{1}{2} m_S^2 \varphi_S^2\right] + \sum_V \left[-\frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \frac{1}{2} m_v^2 V_\mu V^\mu\right]$$



- **Baryons and Mesons: B=N, Λ, Σ, Ξ, ...,   S= σ, ς, ...,   V= ω, ρ, φ, …**
- **Based on Dirac phenomenology & Dirac Bruckner-Hatree-Fock**

  E.D. Cooper, S. Hama, B.C. Clark, R.L. Mercer, PRC47('93),297
  R. Brockmann, R. Machleidt, PRC42('90),1965

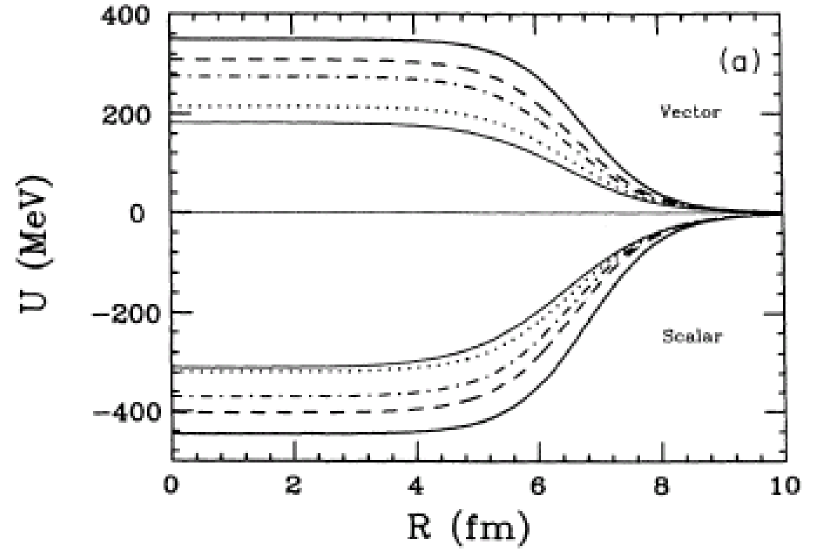
- **Large scalar (att.) and vector (repl.) → Large spin-orbit pot.**
  Relativistic Kinematics → Effective 3-body repulsion
- **Non-linear terms of mesons → Bare 3-body and 4-body force**

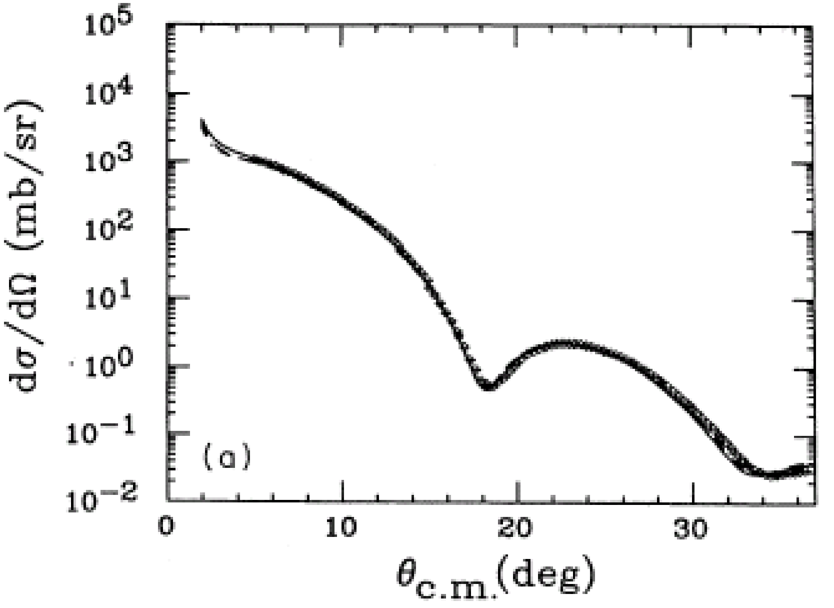
  Boguta, Bodmer ('77), NL1:Reinhardt, Rufa, Maruhn, Greiner, Friedrich ('86), NL3: Lalazissis, Konig, Ring ('97),TM1 and TM2: Sugahara, Toki ('94), Brockmann, Toki ('92)

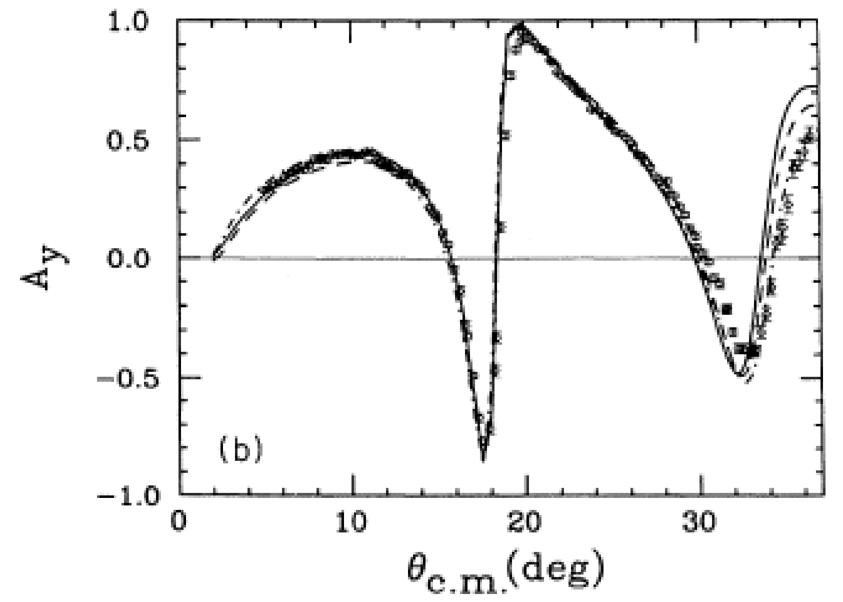
Dirac Phenomenology

E.D. Cooper, S. Hama, B.C. Clark, R.L. Mercer, PRC47('93),297

- **Dirac Eq. with Scalar + Vector pA potential (-400 MeV + 350 MeV) → Cross Section, Spin Observables**



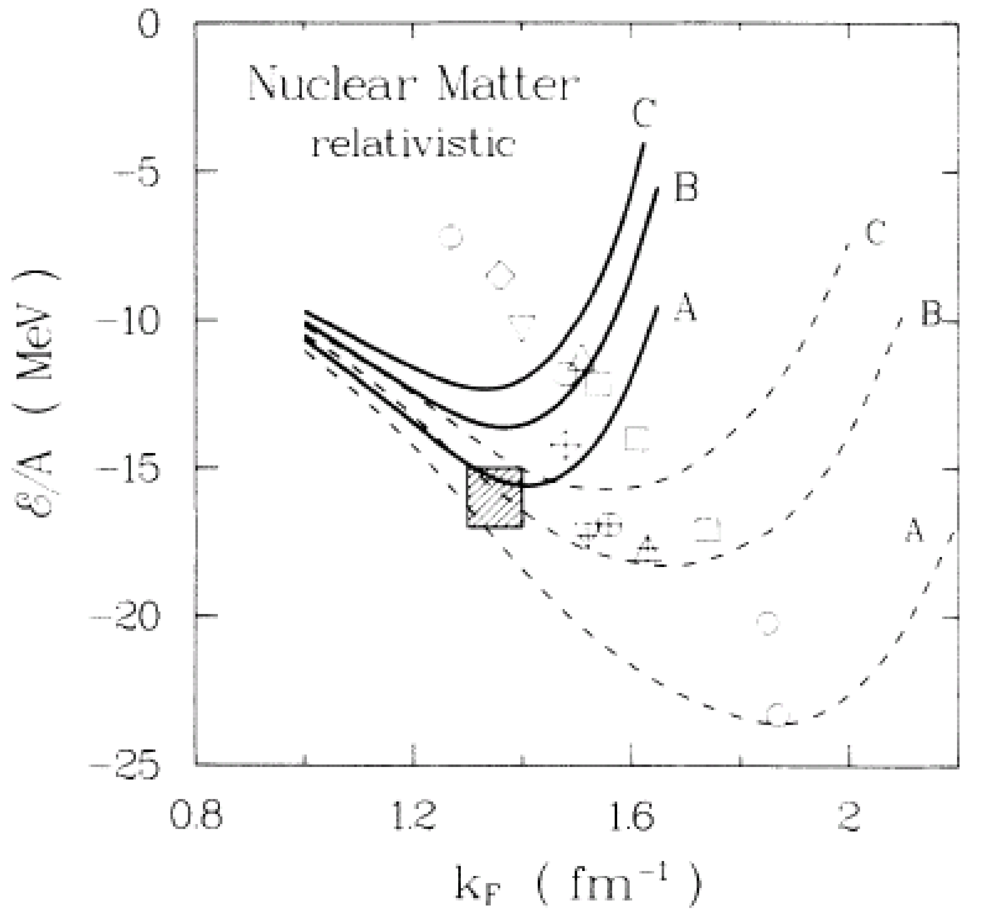




EOS in Dirac-Brueckner-Hartree-Fock

R. Brockmann, R. Machleidt, PRC42('90),1965

- **Non Relativistic Brueckner Calculation**
  → Nuclear Saturation Point cannot be reproduced (Coester Line)
- **Relativistic Approach (DBHF)**
  → Relativity gives additional repulsion, leading to successful description of the saturation point.



Relativistic Mean Field (2)

- **Mean Field treatment of meson field operator**
  = Meson ield operator is replaced with its expectation value
  $\varphi(r) \to \langle\varphi(r)\rangle$
  Ignoring fluctuations compared with the expectation value may be a good approximation at strong condensate.
- **Which Hadrons should be included in RMF ?**
  - **Baryons (1/2+)** **p, n, Λ, Σ, Ξ, Δ, ...**
  - **Scalar Mesons (0+)** **σ(600), f_0(980), a_0(980), ...**
  - **Vector Mesons (1-)** **ω(783), ρ(770), ϕ(1020), ...**
  - **Pseuso Scalar (0-)** **π , K, η, η', ...**
  - **Axial Vector (1+)** **a_1, ...**

  We require that the meson field can have uniform expectation values in nuclear matter.
  → Scalar and Time-Component of Vector Mesons (σ , ω, ρ, ...)

σω Model (1)

Serot, Walecka, Adv.Nucl.Phys.16 (1986),1

- **Consider only σ and ω mesons**
- **Lagrangian**

$$L=\bar{\psi}\left(i\gamma^{\mu}\partial_{\mu}-M+g_s\sigma-g_v\gamma^{\mu}\omega_{\mu}\right)\psi$$
$$+\frac{1}{2}\partial_{\mu}\sigma\partial^{\mu}\sigma-\frac{1}{2}m_s^2\sigma^2-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}m_v^2\omega_{\mu}\omega^{\mu}$$
$$\left(F_{\mu\nu}=\partial_{\mu}\omega_{\nu}-\partial_{\nu}\omega_{\mu}\right)$$

- **Equation of Motion**
  - **Euler-Lagrange Equation**

$$\frac{\partial}{\partial x^{\mu}}\left[\frac{\partial L}{\partial(\partial_{\mu}\phi_i)}\right]-\frac{\partial L}{\partial\phi_i}=0$$

$$\sigma:\left[\partial_{\mu}\partial^{\mu}+m_s^2\right]\sigma=g_s\bar{\psi}\psi$$

$$\omega:\partial_{\mu}F^{\mu\nu}+m_v^2\omega^{\nu}=g_v\bar{\psi}\gamma^{\nu}\psi \quad\rightarrow\quad \left[\partial_{\mu}\partial^{\mu}+m_v^2\right]\omega^{\nu}=g_v\bar{\psi}\gamma^{\nu}\psi$$

$$\psi:\left[\gamma^{\mu}\left(i\partial_{\mu}-g_vV_{\mu}\right)-\left(M-g_s\sigma\right)\right]\psi=0$$

EOM of ω (for beginners)

- **Euler-Lagrange Eq.**

$$\partial_\mu F^{\mu\nu} + m_v^2 \omega^\nu = g_v \bar{\psi} \gamma^\nu \psi$$

- **Divergence of LHS and RHS**

$$\partial_\nu \partial_\mu F^{\mu\nu} + m_v^2 (\partial_\nu \omega^\nu) = m_v^2 (\partial_\nu \omega^\nu) = g_v (\partial_\nu \bar{\psi} \gamma^\nu \psi) = 0$$

LHS: derivatives are sym. and $F_{\mu\nu}$ is anti-sym.

RHS: Baryon Current = Conserved Current

- **Put it in the Euler-Lagrange Eq.**

$$\partial_\mu F^{\mu\nu} = \partial_\mu (\partial^\mu \omega^\nu - \partial^\nu \omega^\mu) = \partial_\mu \partial^\mu \omega^\nu - \partial^\nu (\partial_\mu \omega^\mu) = \partial_\mu \partial^\mu \omega^\nu$$

Schroedinger Eq. for Upper Component (1)

- **Dirac Equation for Nucleons**

$$\left(i\gamma\partial-\gamma^0 U_V - M - U_S\right)\psi=0 \quad ,$$
$$U_V = g_\omega \omega \quad , \quad U_S = -g_\sigma \sigma$$

- **Decompose 4 spinor into Upper and Lower Components**

$$\begin{pmatrix} E-U_V-M-U_S & i\sigma\cdot\nabla \\ -i\sigma\cdot\nabla & -E+U_V-M-U_S \end{pmatrix}\begin{pmatrix} f \\ g \end{pmatrix}=0$$

$$g=\frac{-i}{E+M+U_s-U_V}(\sigma\cdot\nabla)f$$
$$(E-M-U_v-U_s)f=-i(\sigma\cdot\nabla)g$$

Schroedinger Eq. for Upper Component (2)

- **Erase Lower Component (assuming spherical sym.)**

$$-i(\sigma\cdot\nabla)g=-(\sigma\cdot\nabla)\frac{1}{X}(\sigma\cdot\nabla)f=-\frac{1}{X}\nabla^2 f-\frac{1}{r}\left[\frac{d}{dr}\frac{1}{X}\right](\sigma\cdot r)(\sigma\cdot\nabla)f$$

$$=-\nabla\frac{1}{X}\nabla f+\frac{1}{r}\left[\frac{d}{dr}\frac{1}{X}\right](\sigma\cdot l)f$$

$$(\sigma\cdot r)(\sigma\cdot\nabla)=(r\cdot\nabla)+i\sigma\cdot(r\times\nabla)=r\cdot\nabla-\sigma\cdot l$$

- **"Schroedinger-like" Eq. for Upper Component**

$$-\nabla\frac{1}{E+M+U_s-U_v}\nabla f+\left(U_s+U_v+U_{LS}(\sigma\cdot l)\right)f=(E-M)f$$

$$U_{LS}=\frac{1}{r}\left[\frac{d}{dr}\frac{1}{E+M+U_s-U_v}\right]<0 \quad \text{on surface}$$

(U_s,U_v)~ (-350 MeV, 280 MeV)

→Small Central(U_s+U_v), Large LS (U_s-U_v)

Various Ways to Evaluate Non.-Rel. Potential

- **From Single Particle Energy**

$$\left(\gamma^0(E-U_v)+i\gamma\cdot\nabla-(M+U_s)\right)\psi=0 \;\rightarrow\; (E-U_v)^2=p^2+(M+U_s)^2$$

$$\rightarrow E=\sqrt{p^2+(M+U_s)^2}+U_v\approx E_p+\frac{M}{E_p}U_s+U_v+\frac{p^2}{2E_p^3}U_s^2$$

$$(E_p=\sqrt{p^2+M^2})$$

- **Schroedinger Equivalent Potential (Uniform matter)**

$$-\frac{\nabla^2}{2\mathrm{M}}f+\left[U_s+\frac{E}{M}U_v+\frac{U_s^2-U_v^2}{2\mathrm{M}}\right]f=\frac{E+M}{2\mathrm{M}}(E-M)f$$

$$U_{\mathrm{SEP}}\approx U_s+\frac{E}{M}U_v$$

Anyway, slow baryons feel Non.-Rel. Potential,

$$U\approx U_s+U_v=-g_s\sigma+g_v\omega$$

YUKAWA INSTITUTE FOR THEORETICAL PHYSICS YIPQS YITP Kyoto

Nuclear Matter in σω Model

Serot, Walecka, Adv.Nucl.Phys.16 (1986),1

- **Uniform Nuclear Matter**

$$E/V = \gamma_N \int^{P_F} \frac{d^3p}{(2\pi)^2} E^* + \frac{1}{2} m_s^2 \sigma^2 - \frac{1}{2} m_v^2 \omega^2 + g_v \rho_B \omega$$

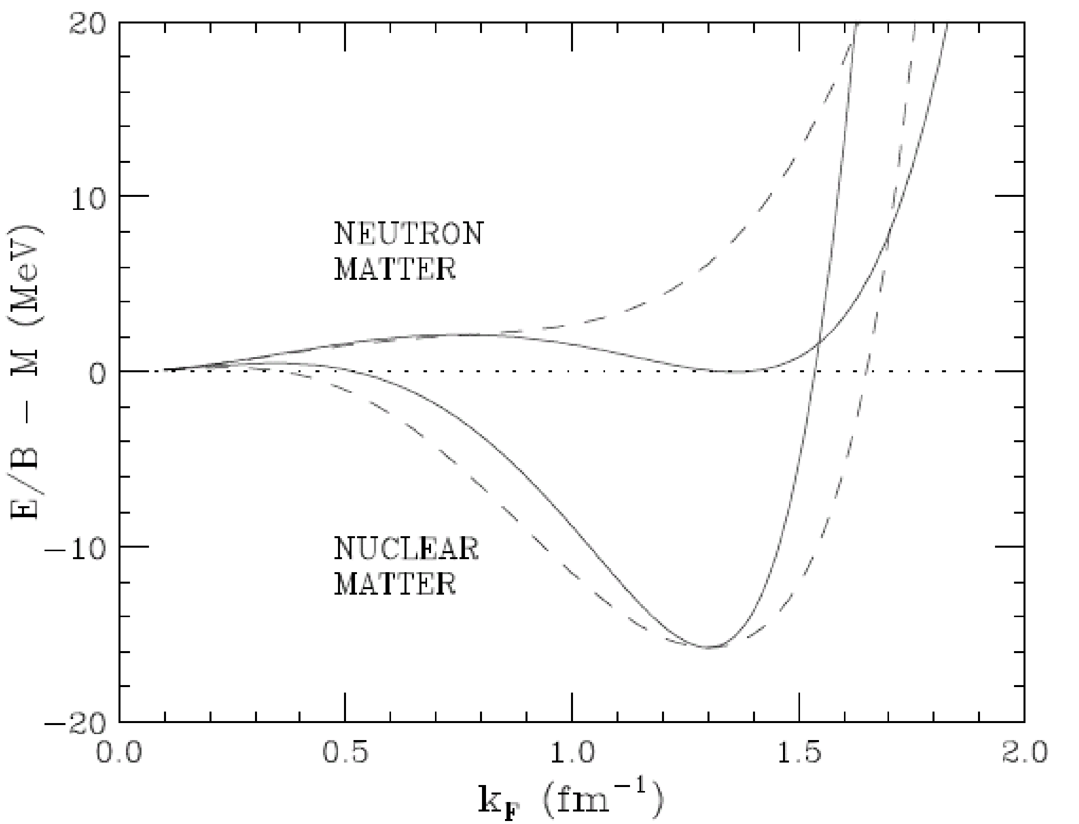
$$\left(M^* = M + U_s = M - g_s \sigma\,, \quad E^* = \sqrt{p^2 + M^{*2}} \right)$$

$$\sigma = \frac{g_s}{m_s^2} \rho_s = \frac{g_s}{m_s^2} \int^{P_F} \frac{\gamma_N \, d^3p}{(2\pi)^2} \frac{M^*}{E^*}$$

$$\omega = \frac{g_v}{m_v^2} \rho_B = \gamma_N \frac{g_v}{m_v^2} \int^{P_F} \frac{d^3p}{(2\pi)^3}$$

γ_N = Nucleon degeneracy
(=4 in sym. nuclear matte

Problem: EOS is too stiff
K ~ (500-600) MeV !
→ How can we avoid it ?



RMF with Non-Linear Meson Int. Terms

Boguta, Bodmer ('77), NL1:Reinhardt, Rufa, Maruhn, Greiner, Friedrich ('86),
NL3: Lalazissis, Konig, Ring ('97),TM1 and TM2: Sugahara, Toki ('94), Brockmann, Toki ('92)

- **Too stiff EOS in the simplest RMF (σω model) is improved by introducing non-linear terms (σ^4, ω^4)**
  - **Fit B.E. of Stable as well as Unstable (n-rich) Nuclei**
  - **Three Mesons (σ,ω,ρ) are included**
  - **Meson Self-Energy Term (σ,ω)**

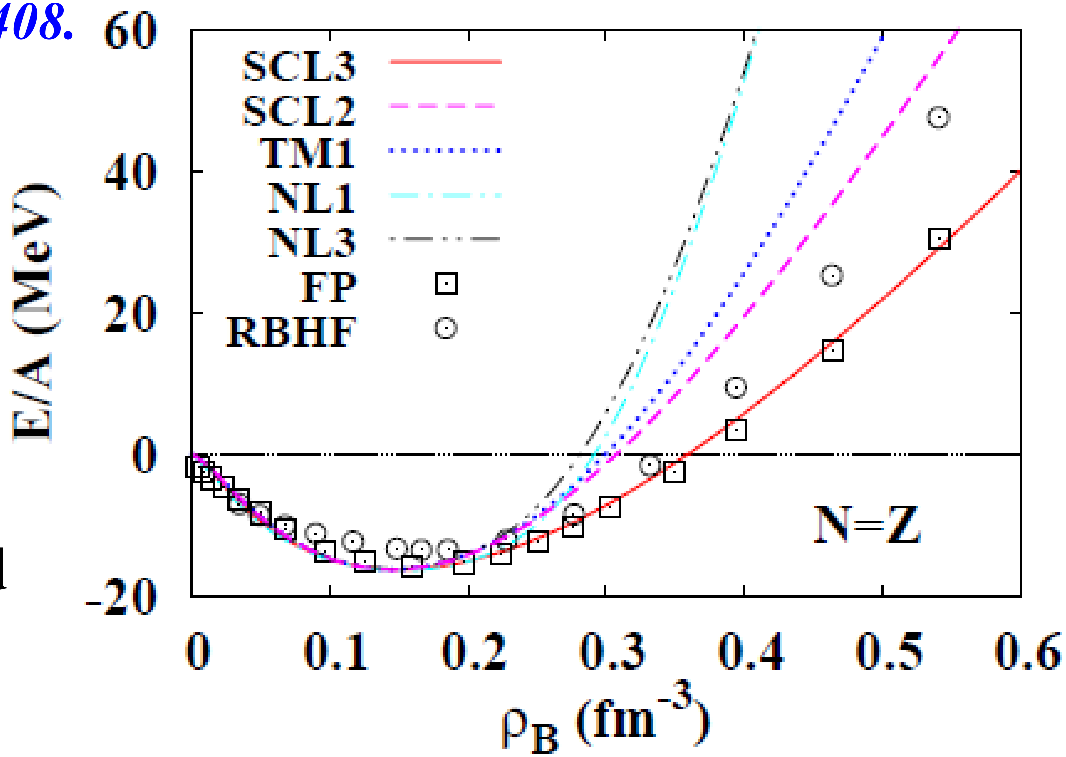
$$
\begin{aligned}
\mathcal{L} = & \bar{\psi}_N \left(i\partial\!\!\!/ - M - g_\sigma \sigma - g_\omega \omega\!\!\!/ - g_\rho \tau^a \rho\!\!\!/^a \right) \psi_N \\
& + \frac{1}{2}\partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 \\
& - \frac{1}{4} W^{\mu\nu} W_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4} R^{a\mu\nu} R^a_{\mu\nu} + \frac{1}{2} m_\rho^2 \rho^{a\mu} \rho^a_\mu + \frac{1}{4} c_3 \left(\omega_\mu \omega^\mu \right)^2 \\
& + \bar{\psi}_e \left(i\partial\!\!\!/ - m_e \right) \psi_e + \bar{\psi}_\nu i\partial\!\!\!/ \psi_\nu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \ ,
\end{aligned}
$$

$$
\begin{aligned}
W_{\mu\nu} &= \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \ , \\
R^a_{\mu\nu} &= \partial_\mu \rho^a_\nu - \partial_\nu \rho^a_\mu + g_\rho \epsilon^{abc} \rho^{b\mu} \rho^{c\nu} \ , \\
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu \ .
\end{aligned}
$$

RMF models with Non-Linear Meson Int. Terms

- Variety of the RMF models
  → MB couplings, meson masses, meson self-energies
  - σN, ωN, ρN couplings are well determined
    → almost no model deps. in Sym. N.M. at low ρ
  - ω^4 term is introduced to simulate DBHF results of vector pot.
    TM1&2: Y. Sugahara, H. Toki, NPA579('94)557;
    R. Brockmann, H. Toki, PRL68('92)3408.
  - σ^3 and σ^4 terms are introduced to soften EOS at ρ_0.
    J. Boguta, A.R.Bodmer NPA292('77)413,
    NL1:P.-G.Reinhardt, M.Rufa, J.Maruhn, W.Greiner, J.Friedrich, ZPA323('86)13.
    NL3: G.A.Lalazissis, J.Konig, P.Ring, PRC55('97)540.

    → Large differences are found at high ρ



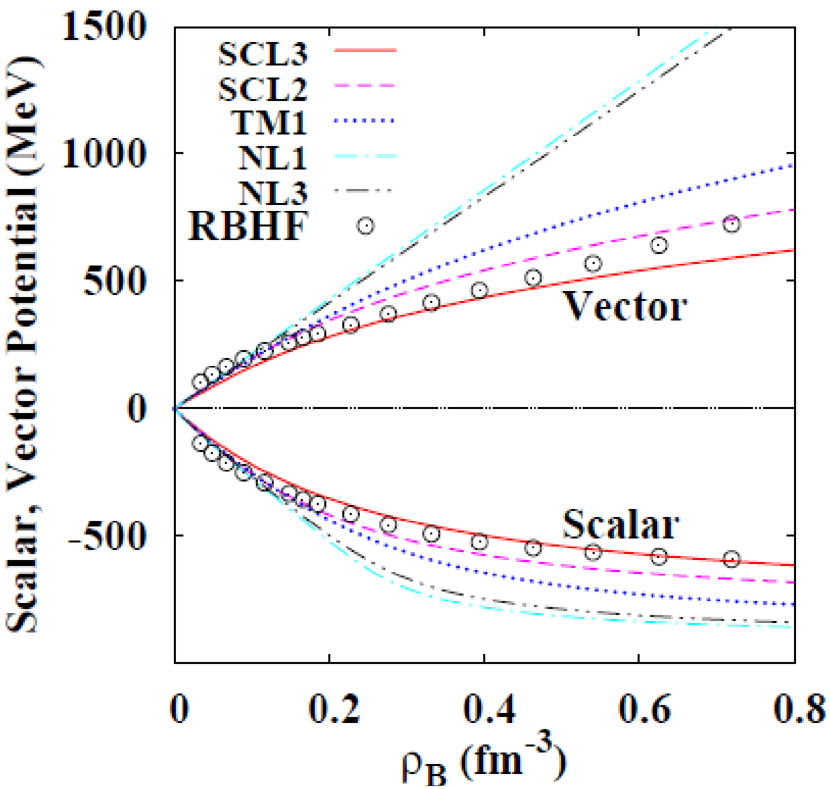
K. Tsubakihara, H. Maekawa, H. Matsumiya, AO,PRC81('10)065206.

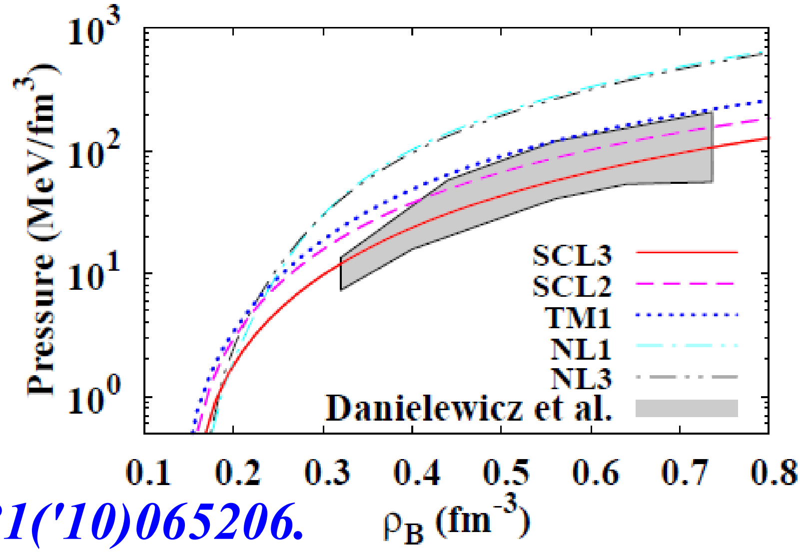
Vector potential in RMF

- **Vector potential from ω dominates at high density !**

$$U_v(\rho_B) = g_\omega \omega \sim \frac{g_\omega^2}{m_\omega^2} \rho_B$$

  - **Dirac-Bruckner-Hartree-Fock shows suppressed vector potential at high ρ_B.**
    R. Brockmann, R. Machleidt, PRC42('90)1965.
  - **Collective flow in heavy-ion collisions suggests pressure at high ρ_B.**
    P. Danielewicz, R. Lacey, W. G. Lynch, Science298('02)1592.
  - **Self-interaction of ω ~ $c_\omega(\omega_\mu \omega^\mu)^2$**
    → DBHF results & Heavy-ion data

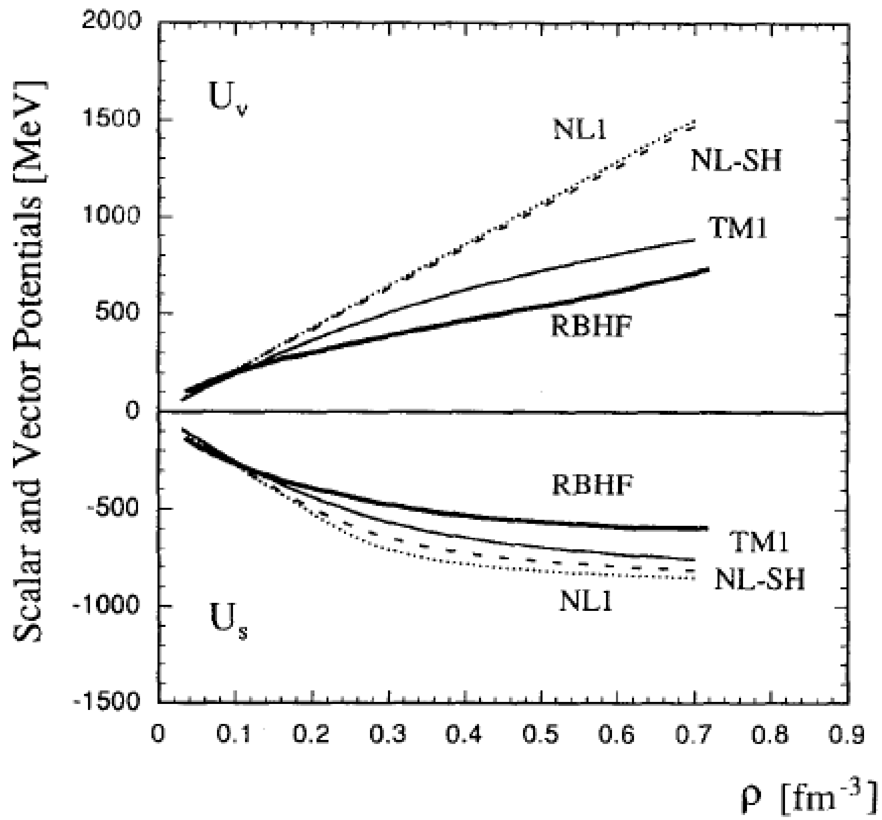


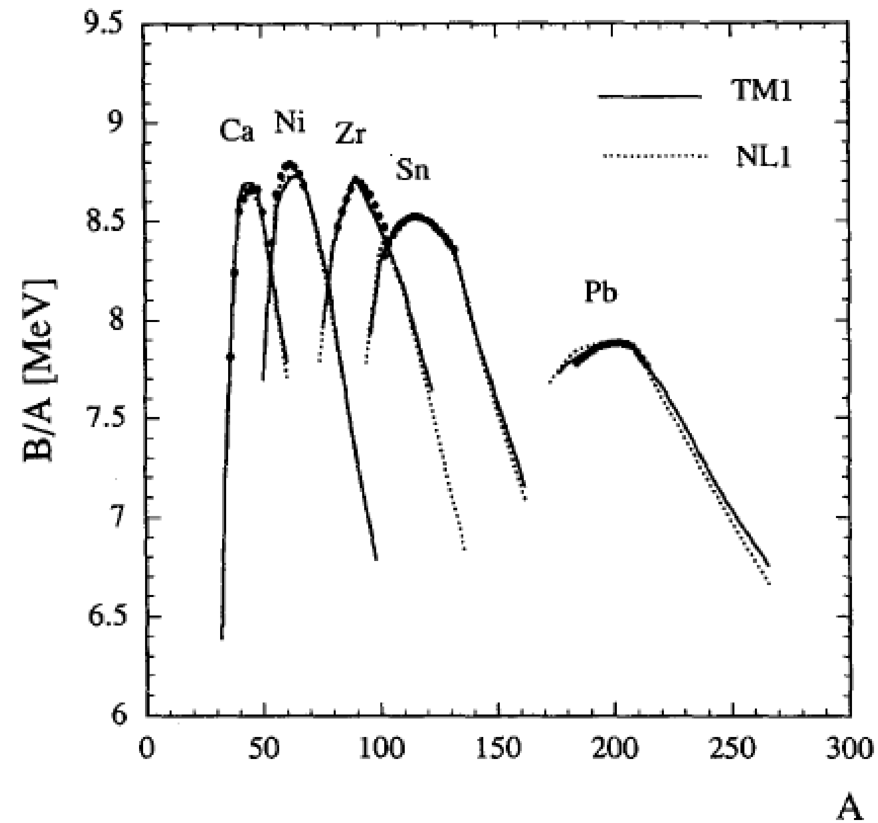


K. Tsubakihara, H. Maekawa, H. Matsumiya, AO,PRC81('10)065206.

TM1

- **TM1 Sugahara, Toki ('94)**
  - **Fit vector potential in RBHF by introducing ω^4 term.**
  - **Fit binding energies of neutron-rich nuclei**

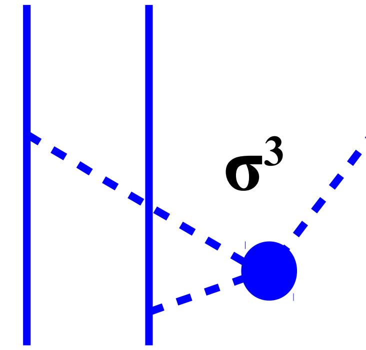


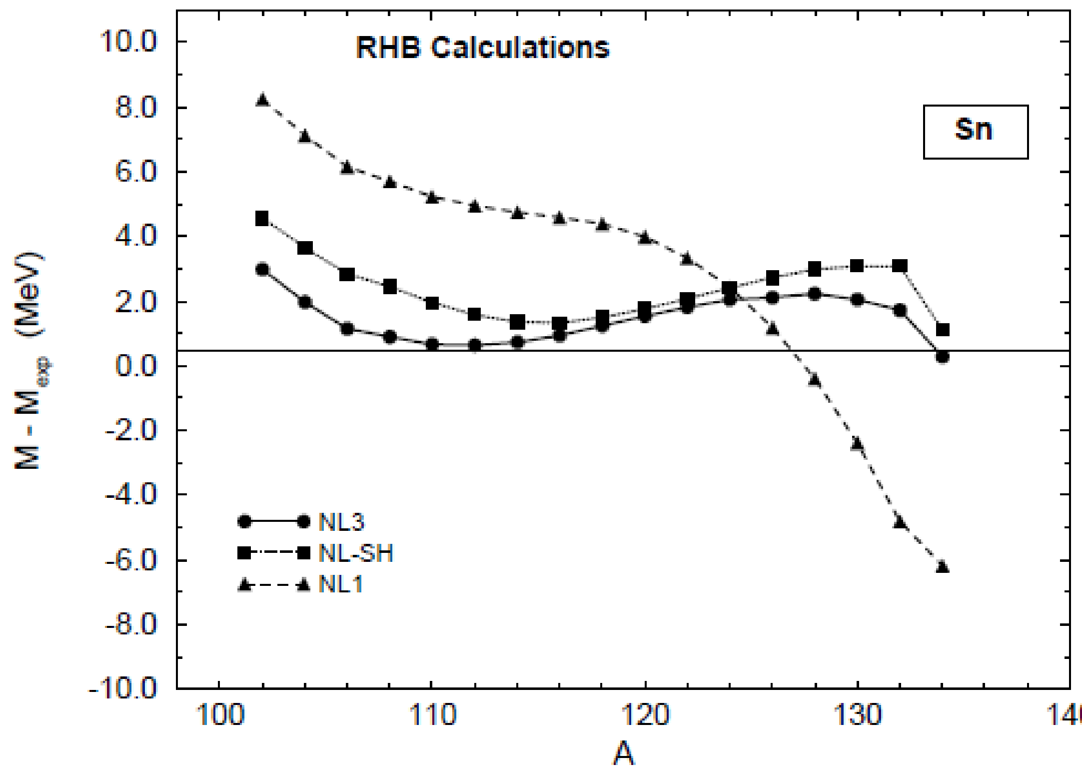


TM1: Sugahara, Toki ('94)

High Quality RMF models

- いくつかの RMF パラメータによる計算は、
「質量公式」に迫る精度で原子核質量を記述！
→ High Quality RMF models.
TM, NL1, NL3, ….
  - 全質量で 1-2 MeV の誤差 (NL3)
  - Linear coupling (σN, ωN, ρN), self-energy in σ, ω
  - 場合によっては結合定数の密度依存性を導入。





NL3: Lalazissis, Konig, Ring, PRC55 ('97)540

RMF with Non-Linear Meson Int. Terms

- **Are the Lagrangian parameters are well determined ?**

$$\begin{aligned}\mathcal{L} =& \mathcal{L}_{\rm free}(\psi,\sigma,\omega,\rho,...) + \bar{\psi}\left[g_\sigma\sigma - g_\omega\gamma^0\omega - g_\rho\tau_z\gamma^0\rho\right]\psi \\ &+ c_\omega\omega^4/4 - V_\sigma(\sigma)\ , \qquad (3)\end{aligned}$$

$$V_\sigma = \begin{cases} \frac{1}{3}g_3\sigma^3 + \frac{1}{4}g_4\sigma^4 & \text{(NL1, NL3, TM1)} \\ -a_\sigma f_{\rm SCL}(\sigma/f_\pi) & \text{(SCL)} \end{cases}\ , \qquad (4)$$

  - **Linear terms, Meson-Nucleon Coupling → Well determined**
  - **Negative Coef. of $\sigma^4 < 0$ in some of RMF models→ Vacuum is unstable**

TABLE II: RMF parameters

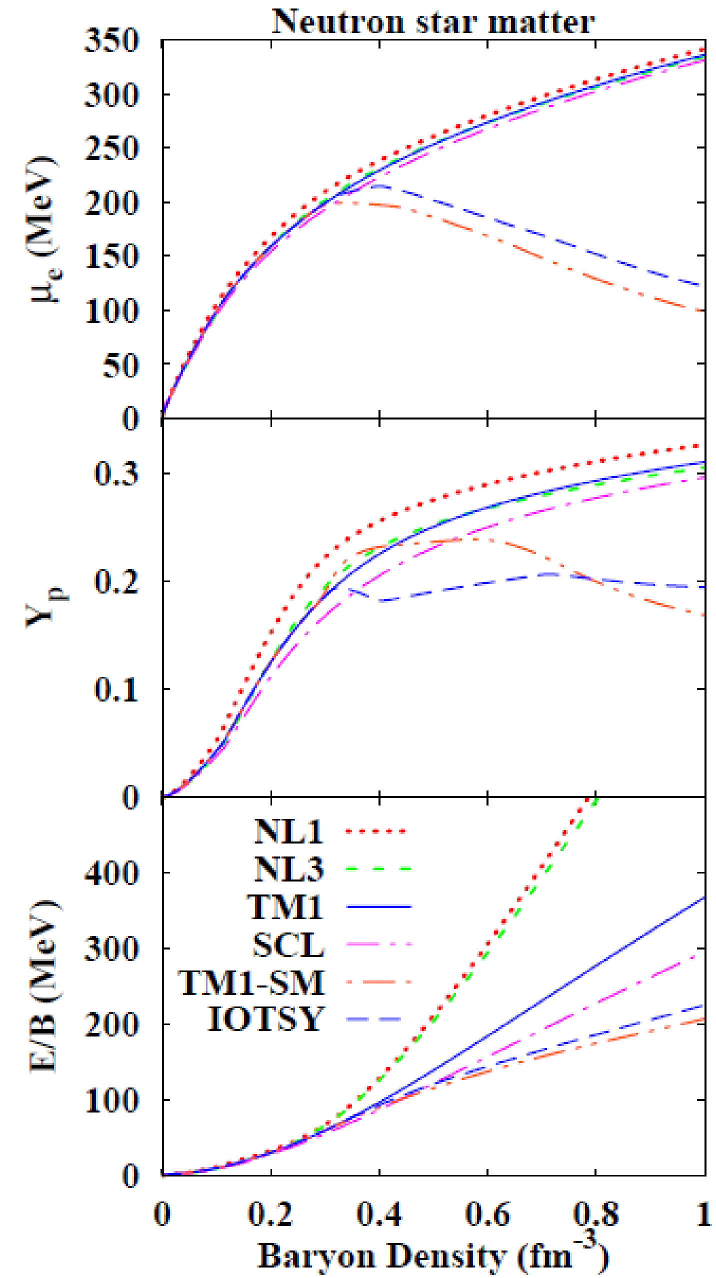
	$g_{\sigma N}$	$g_{\omega N}$	$g_{\rho N}$	g_3(MeV)	g_4	c_ω	m_σ(MeV)	m_ω(MeV)	m_ρ(MeV)
NL1[18]	10.138	13.285	4.976	2401.9	-36.265	0	492.25	795.359	763
NL3[19]	10.217	12.868	4.474	2058.35	-28.885	0	508.194	782.501	763
TM1[6]	10.0289	12.6139	4.6322	1426.466	0.6183	71.3075	511.198	783	770
SCL[20](*1)	10.08	13.02	4.40	1255.88	13.504	200	502.63	783	770

(*1): g_3 and g_4 are from the expansion of $f_{\rm SCL}$.

AO, Jido, Sekihara, Tsubakihara (2009)

Neutron Star Matter EOS

- **Difference in non-linear meson terms generate different predictions of EOS at high densities**



How can we fix non-linear terms ?

AO, Jido, Sekihara, Tsubakihara, Phys. Rev. C 80 (2009), 038202.

Short Summary

- **Nuclear Matter EOS is important in many subjects of physics.**
  - **Bulk nuclear properties (B.E., radius)**
  - **Dense Matter in Compact Astrophysical Objects**
  - **High-Energy Heavy-Ion Collisions**
- **Relativistic Mean Field models**
  - **Simple description of nucleon scalar and vector potentials in terms of meson fields.**
  - **With non-linear meson interaction terms, nuclear binding energies (and radii) are well explained.**
  - **Ambiguities of non-linear couplings bring large differences of EOS at high densities, especially in asymmetric nuclear matter.**
- **It is promising to utilize the results of G-matrix based on Chiral EFT (2 and 3 nucleon force), which reproduces the saturation density in an "ab initio" manner.**

Thank you !