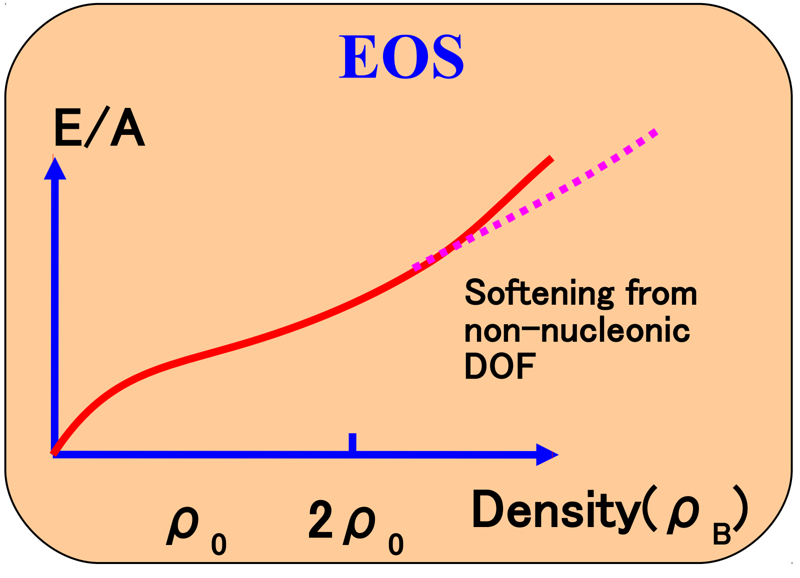
Nuclear Symmetry Energy

M-R Relation and EOS

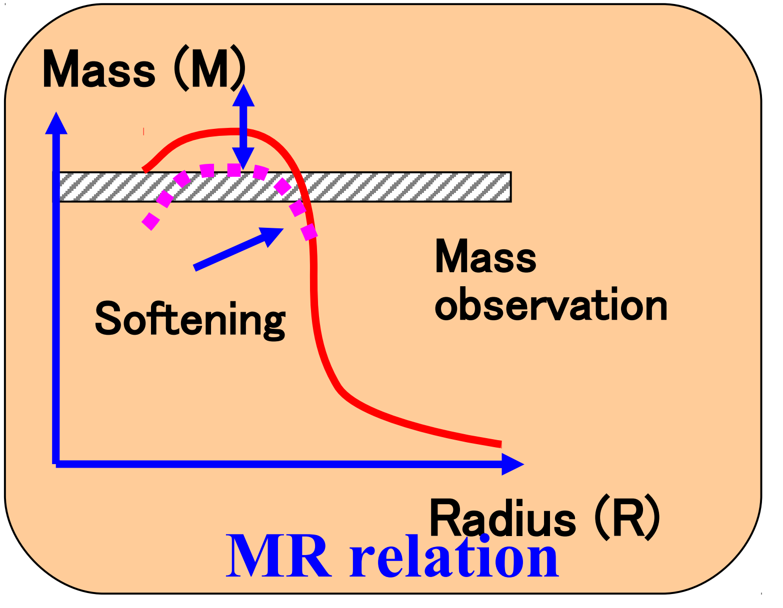
- **Solving TOV eq.**
  starting from the "initial" condition, ε(r=0) = ε_c = given
  until the "boundary" condition P(r)=0 is satisfied.
  → M and R are the functions of ε(r=0)
  and functionals of EOS, P=P(ε).

$$M = M(\varepsilon_c)[P(\varepsilon)] \quad , \quad R = R(\varepsilon_c)[P(\varepsilon)]$$

→ M-R curve and NS matter EOS : 1 to 1 correspondence



TOV Eq.



Nuclear Mass

- **Bethe-Weizsacker mass formula**
  Nuclear binding energy is roughly given by Liquid drop.
  Nuclear size measurement → $R = r_0 A^{1/3}$

$$B(A,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} + a_p \frac{\delta_p}{A^\gamma}$$

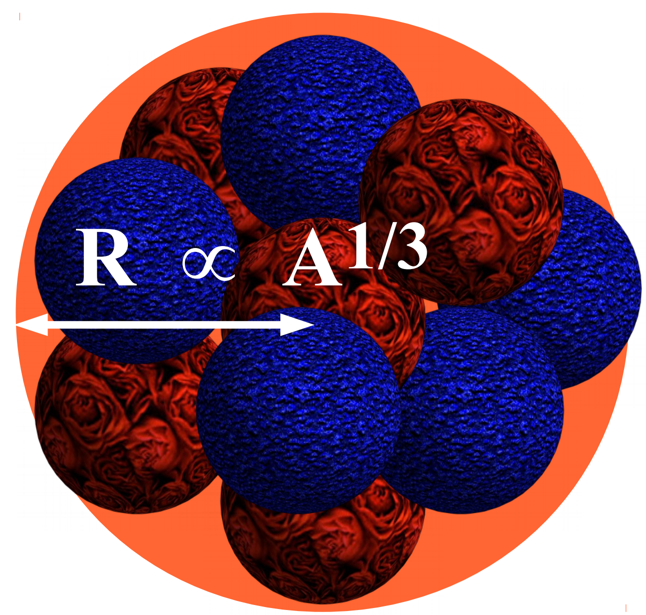
Volume **Surface** **Coulomb** **Symmetry** **Paring**

$A \propto \frac{4\pi}{3} R^3$ $\quad A^{2/3} \propto 4\pi R^2$ $\quad \propto \frac{Q^2}{R}$

- **Ignore Coulomb, consider A → ∞,**

$$B/A = a_v(\rho) - a_a(\rho)\delta^2 \quad , \quad \delta = (N-Z)/A$$

$$a_v \simeq 16\,\mathrm{MeV} \quad , \quad a_a \simeq 30\,\mathrm{MeV}$$



Coef. may depend on the number density ρ
→ Nuclear Matter EOS

Nuclear Matter EOS

- **Energy per nucleon in nuclear matter**

$$E(\rho,\delta)=E_{\rm SNM}(\rho)+E_{\rm Sym}(\rho)\delta^2 \quad , \quad \delta=(N-Z)/A$$

- **Saturation point (ρ_0, E_0)**

  $\rho_0 \sim 0.15\ \mathrm{fm}^{-3}$

  $E_0 = -a_v \sim -16$ MeV

  (nuclear radius and mass)

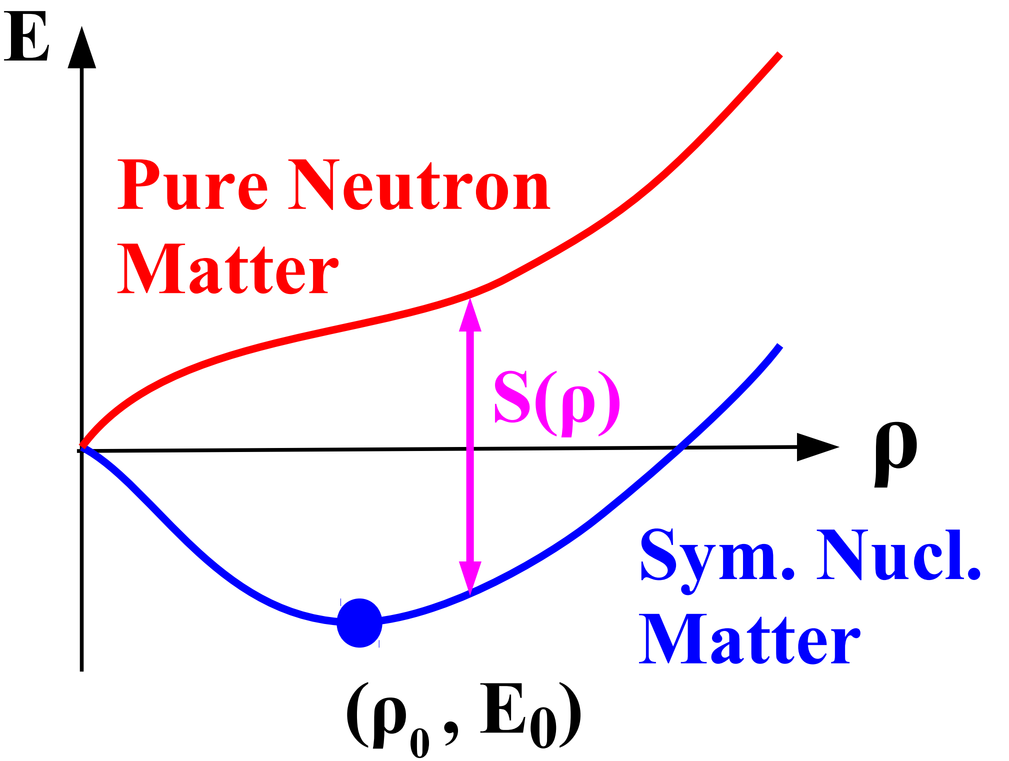
- **Symmetry energy**

  $S(\rho) = E_{PNM}(\rho) - E_{SNM}(\rho)$

  $= E(\rho, \delta=1) - E(\rho, \delta=0)$

  $S_0 = S(\rho_0) \sim 30$ MeV

  (mass formula)



Nuclear Matter EOS can be, in principle, determined by terrestrial (laboratory) nuclear physics experiments !

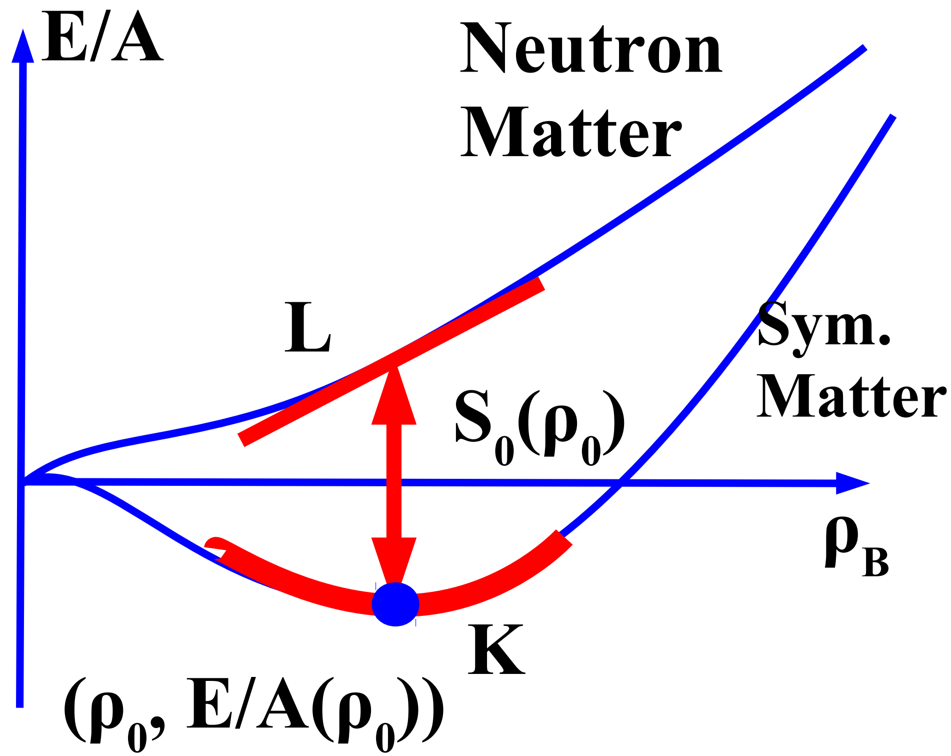
Nuclear Matter EOS

- **Additional two important parameters: K and L**
- **Pressure is given by the derivative of E via ρ**

$$P = \rho^2 (\partial E / \partial \rho)$$

At ρ_0, L determines P

$$P = \rho_0 L / 3 \ (\text{at} \quad \rho = \rho_0)$$



$$E/A(\rho, \delta) = \varepsilon(\rho) + E_{\text{sym}}(\rho)\delta^2 + O(\delta^4)$$

Symmetric Nuclear Matter

$$\varepsilon(\rho) = \varepsilon(\rho_0) + \frac{K(\rho - \rho_0)^2}{18\rho_0^2} + O((\rho - \rho_0)^3)$$

Symmetry Energy $(\delta = (N - Z)/A = 1 - 2Y_p)$

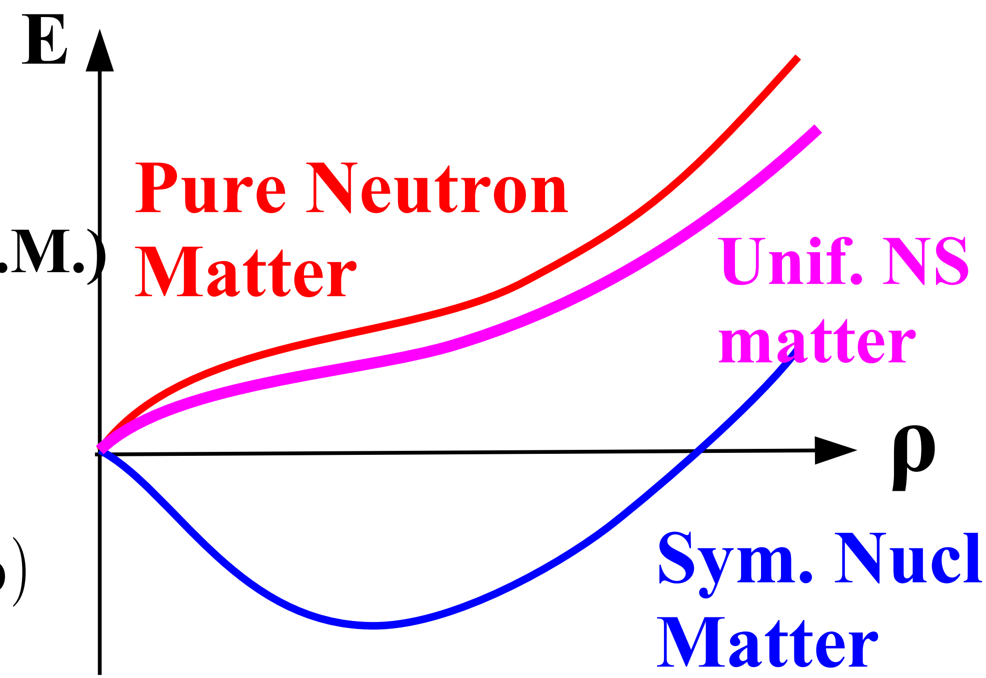
$$E_{\text{sym}}(\rho) = S_0 + \frac{L(\rho - \rho_0)}{3\rho_0} + \frac{K_{\text{sym}}(\rho - \rho_0)^2}{18\rho_0^2} + O((\rho - \rho_0)^3)$$

Neutron Star Matter EOS

- **What happens in low-density uniform neutron star matter ?**
  - **Constituents = proton, neutron and electron**
  - **Charge neutrality → # of electons= # of protons ($\rho_e = \rho_p = \rho(1-\delta)/2$)**

$$E_{\rm NSM}(\rho) = E_{\rm NM}(\rho, \delta) + E_e(\rho_e = \rho_p)$$
$$= E_{\rm SNM}(\rho) + S(\rho)\delta^2 + \frac{\Delta M}{2}\delta + \frac{3}{8}\hbar k_F (1-\delta)^{4/3}$$

**(electron mass neglected,
neutron-proton mass diff. incl.
k_F= Fermi wave num. in Sym. N.M.)**



  - **δ is optimized to minimize energy per nucleon**

$$E_{\rm NSM}(\rho) \le E_{\rm NM}(\rho, \delta=1) = E_{\rm PNM}(\rho)$$

対称エネルギーの起源

- Fermi Gas model での核子あたりの運動エネルギー

$$E_{\mathrm{sym},K} = \frac{Z}{A}\frac{3}{5}\frac{\hbar^2 k_{\mathrm{F}p}^2}{2m} + \frac{N}{A}\frac{3}{5}\frac{\hbar^2 k_{\mathrm{F}n}^2}{2m} = \frac{3}{5}E_{\mathrm{F}}\frac{1}{2}\left[(1-\delta)^{5/3}+(1+\delta)^{5/3}\right]$$

$$\simeq \frac{3}{5}E_{\mathrm{F}} + \frac{1}{3}E_{\mathrm{F}}\delta^2 + \mathcal{O}(\delta^4)$$

a_{sym} (FG)=$E_F/3 \sim 11$ MeV となり、質量公式の $a_{sym} \sim 23$ MeV (surface を考えると a_{sym} (vol) ~ 30 MeV) と比べて半分程度。残りは相互作用。

- 残りの半分の対称エネルギーを RMF で評価してみましょう。

$$\Delta E_{\mathrm{sym},\rho} = \frac{1}{2}\frac{m_\rho^2 R^2}{\rho_B} = \frac{1}{2}\frac{g_\rho^2}{m_\rho^2}\rho_B\delta^2 = \Delta a_{\mathrm{sym}}\delta^2 \quad \left(R = \frac{g_\rho(\rho_n-\rho_p)}{m_\rho^2} = \frac{g_\rho\rho_B\delta}{m_\rho^2}\right)$$

$$g_\rho^2 = \frac{2m_\rho^2\Delta a_{\mathrm{sym}}}{\rho_B} \simeq (4.3)^2 \quad (a_{\mathrm{sym}} = 30\,\mathrm{MeV})$$

← RMF par. より少し小さめ

$$L \simeq E_F + 3\Delta a_{\mathrm{sym}} \simeq 90\,\mathrm{MeV}$$

← Optimal value より少し大きい

Simple parametrized EOS

- **Skyrme int. motivated parameterization**

$$E_{\mathrm{SNM}} = \frac{3}{5}E_F(\rho) + \frac{\alpha}{2}\left(\frac{\rho}{\rho_0}\right) + \frac{\beta}{2+\gamma}\left(\frac{\rho}{\rho_0}\right)^{1+\gamma}$$

$$\alpha = \frac{2}{\gamma}\left(E_0(1+\gamma) - \frac{E_F(\rho_0)(1+3\gamma)}{5}\right) \ , \quad \beta = \frac{2+\gamma}{\gamma}\left[-E_0 + \frac{1}{5}E_F(\rho_0)\right] \ .$$

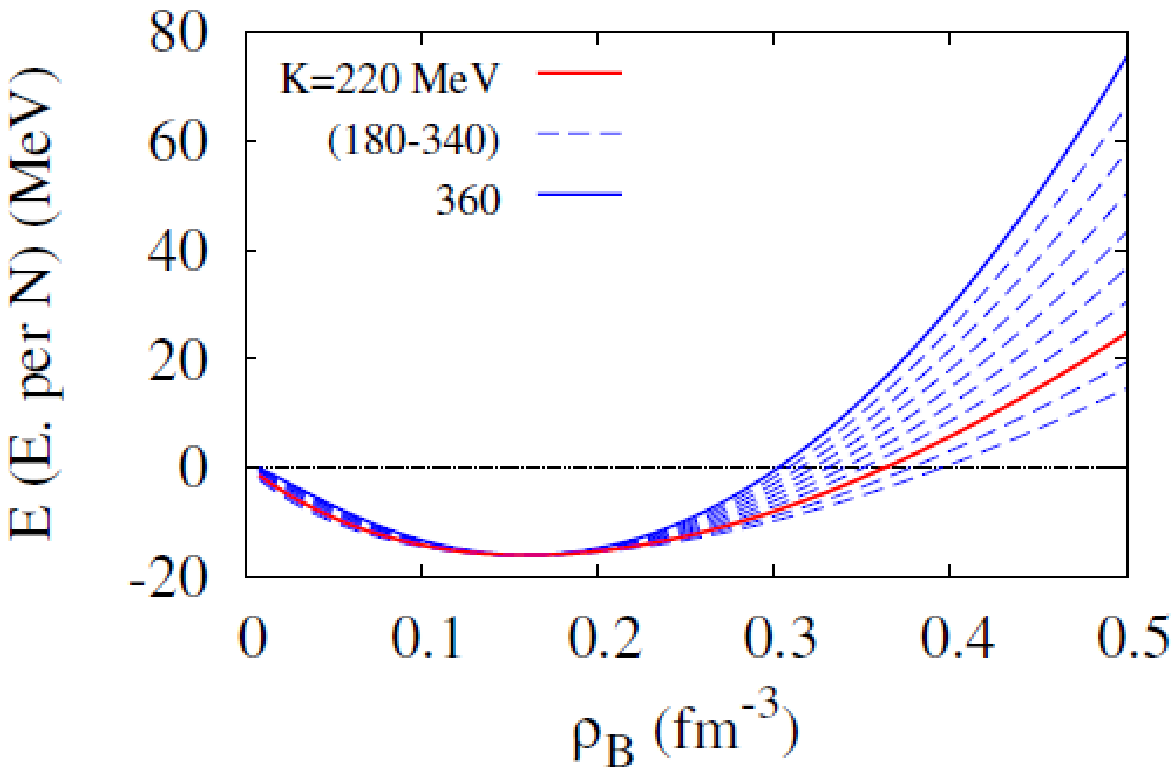
$$K = \frac{1+3\gamma}{5}E_F(\rho_0) - 3E_0(1+\gamma) \ .$$

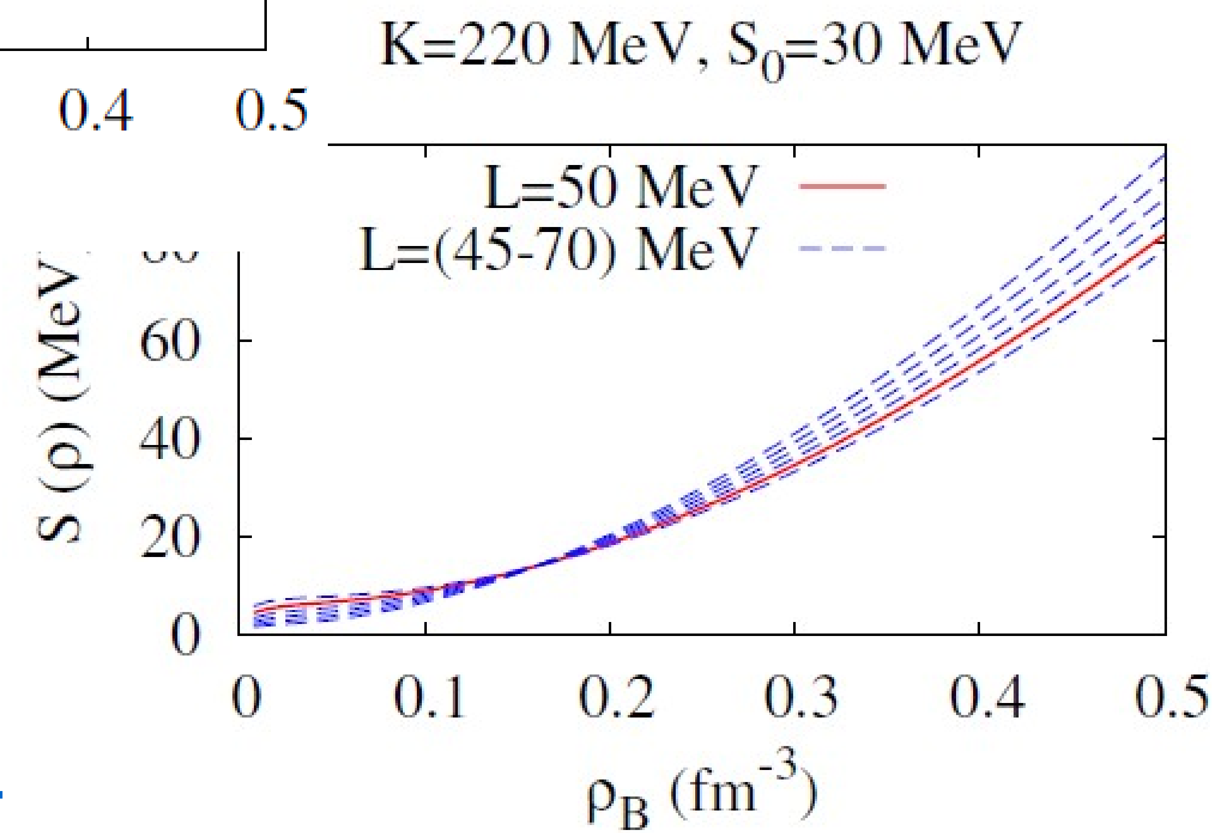
- **Symmetry energy parameterization**

$$S(\rho) = \frac{1}{3}E_F(\rho) + \left[S_0 - \frac{1}{3}E_F(\rho_0)\right]\left(\frac{\rho}{\rho_0}\right)^{\gamma_{\mathrm{sym}}}$$

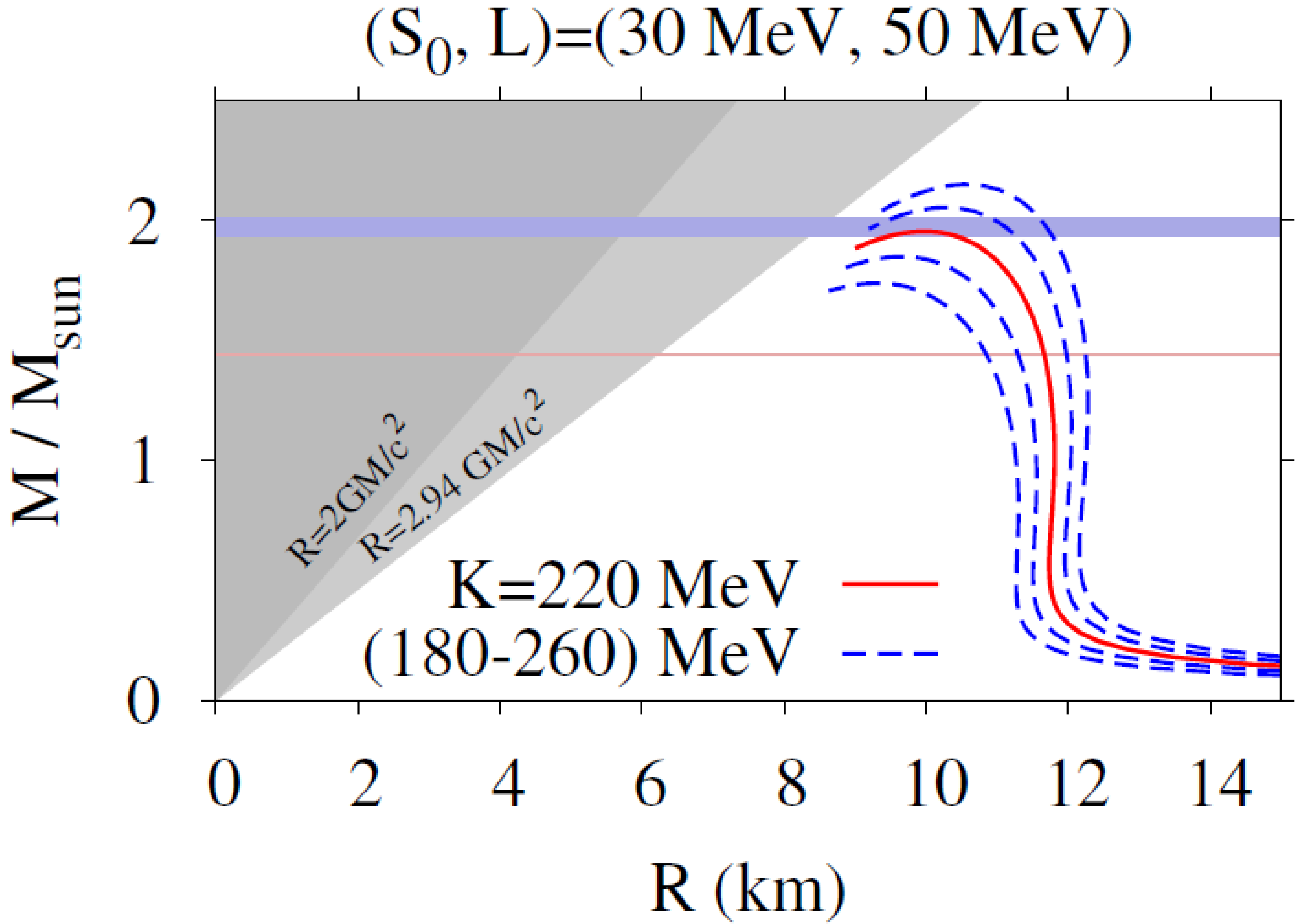
$$\gamma_{\mathrm{sym}} = \frac{L - \frac{2}{3}E_F(\rho_0)}{3S_0 - E_F(\rho_0)}$$

Simple parametrized EOS

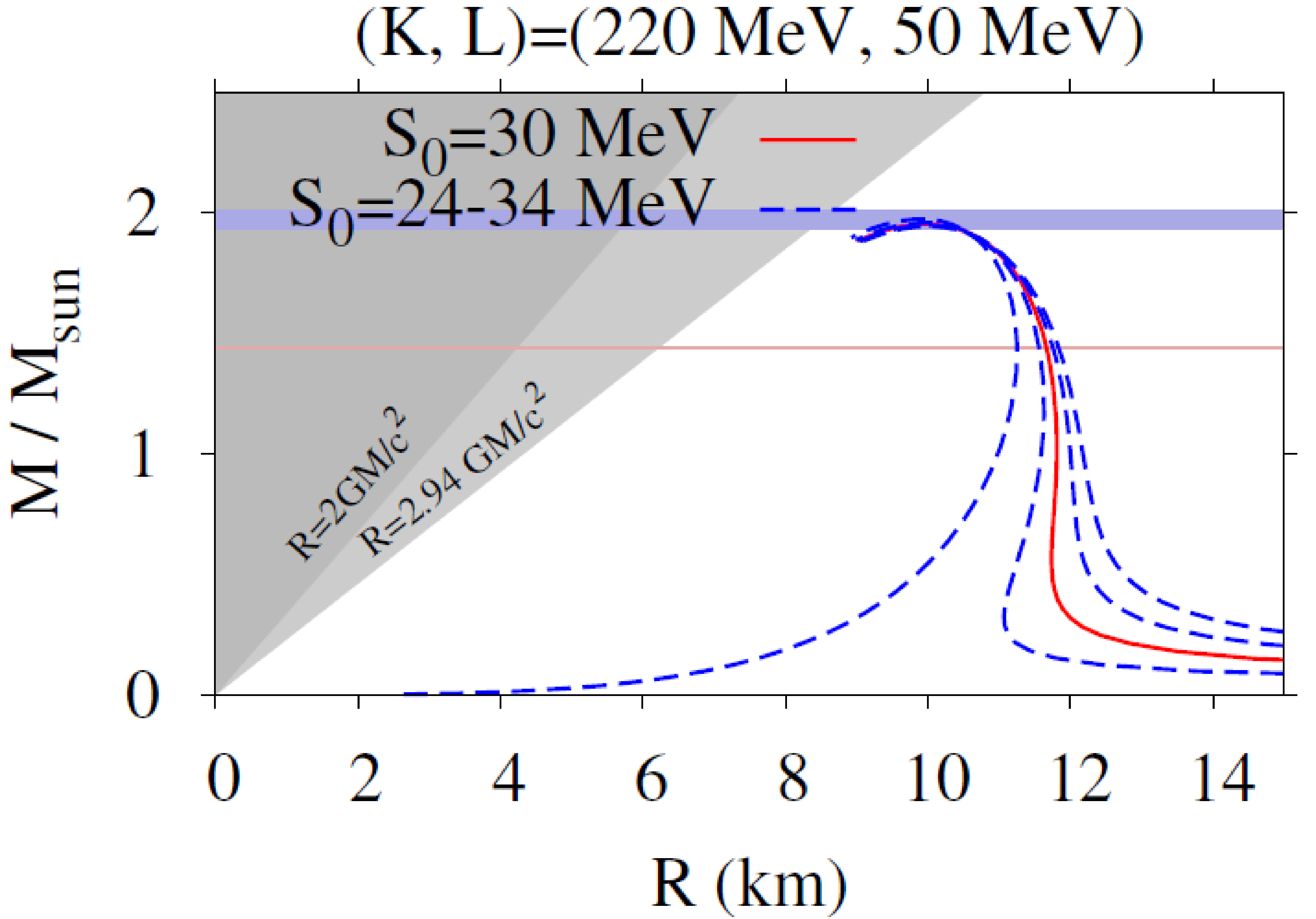




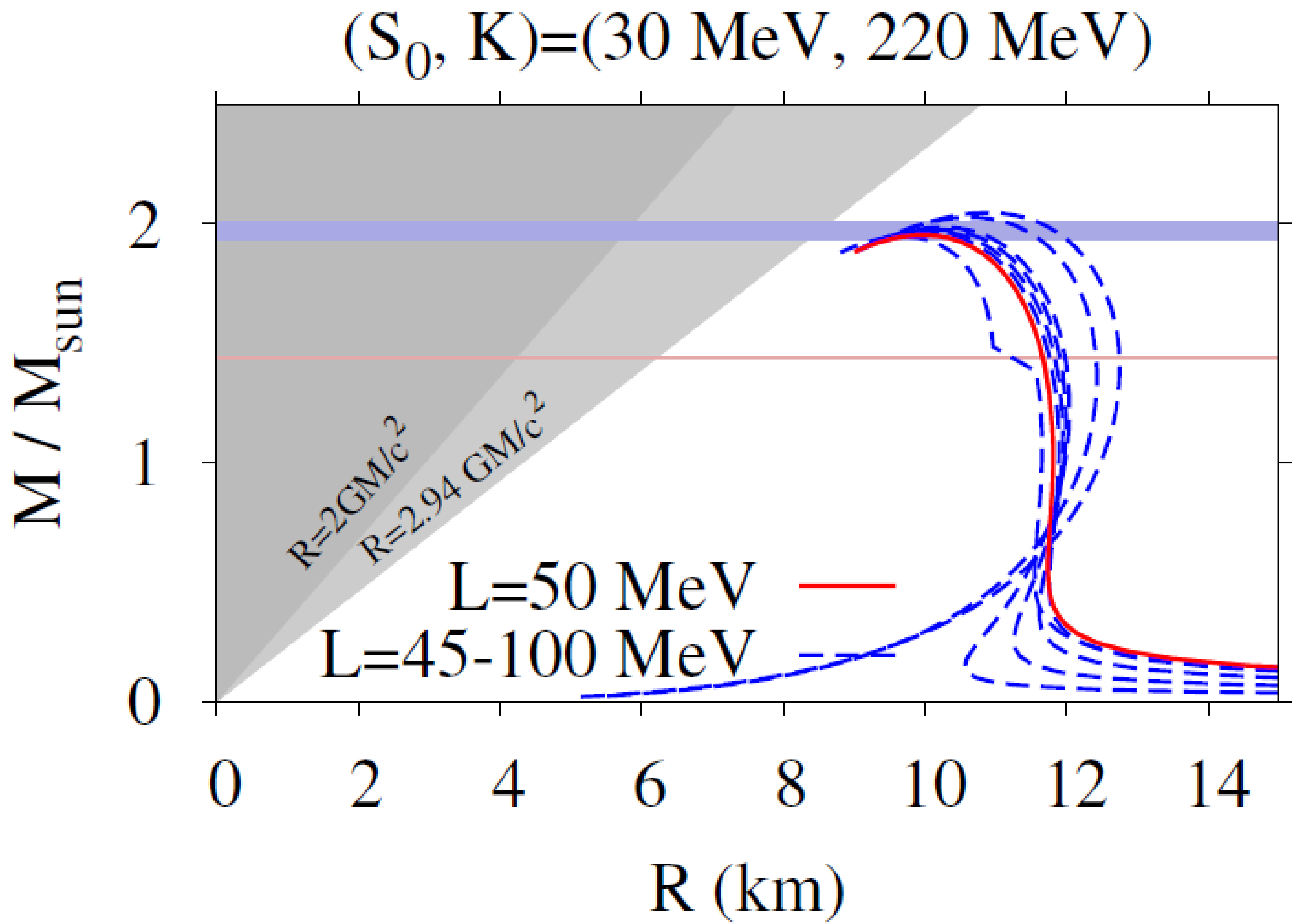
Simple parametrized EOS



Simple parametrized EOS

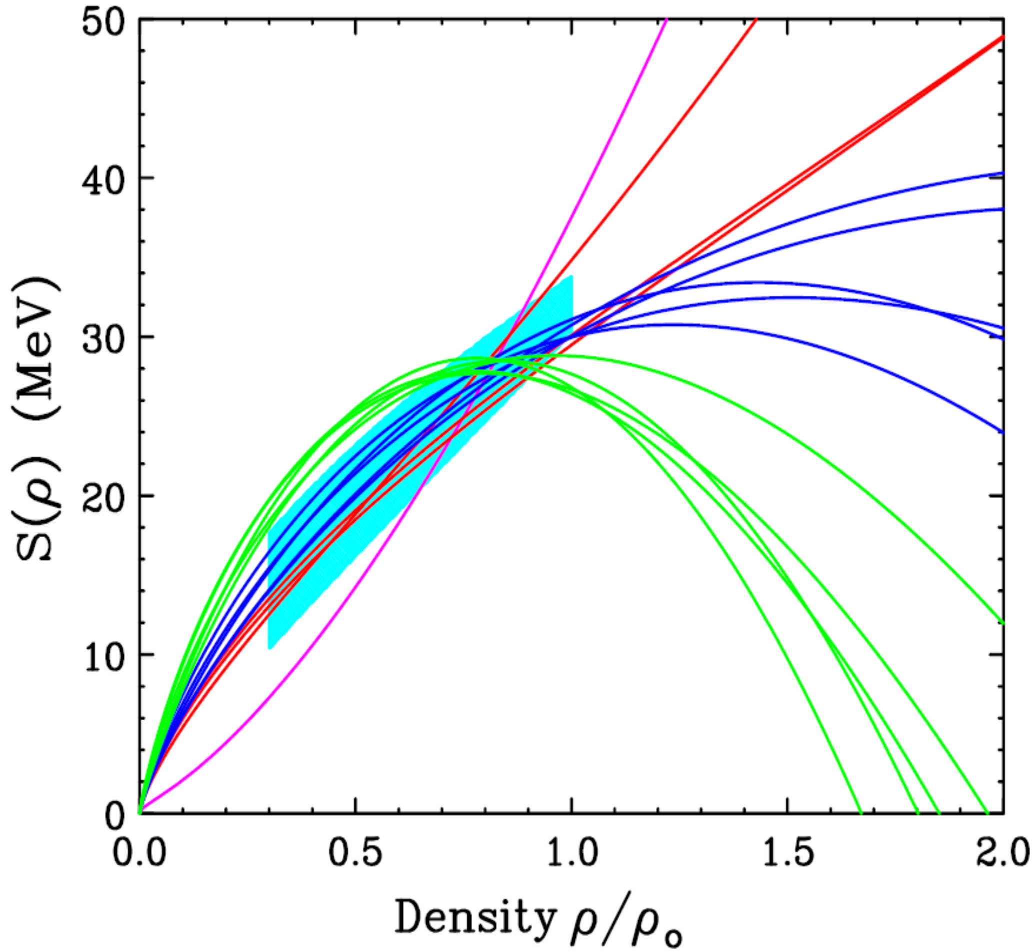


Simple parametrized EOS



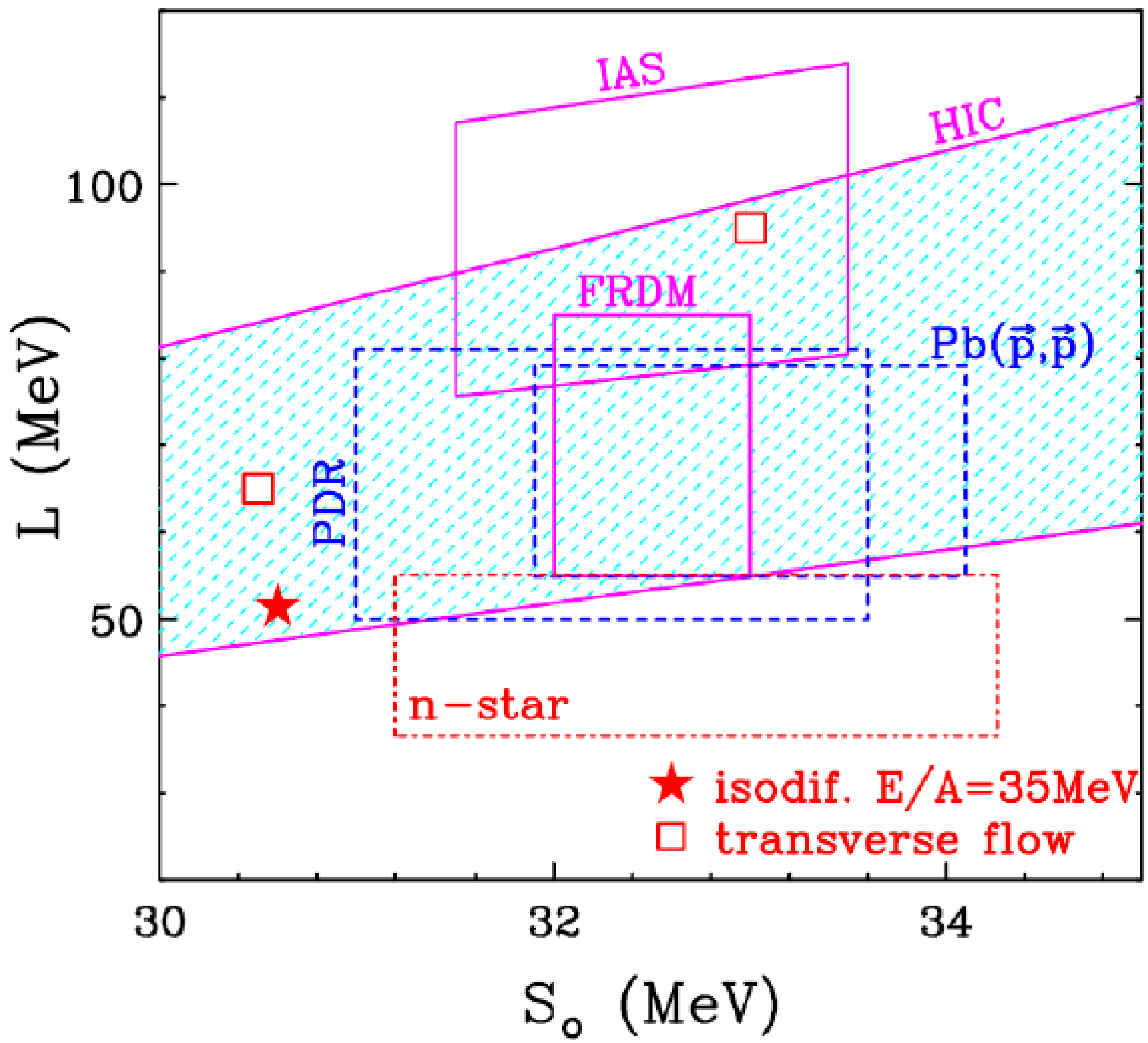
Symmetry Energy

- Summary of Nuclear Symmetry Energy workshop NuSym11 http://www.smith.edu/nusym11
  $E_{sym}(\rho_0)$ = 31-34 MeV, L = 50-110 MeV
  extracted from various observations.
  - Mass formula *Moller ('10)*
  - Isobaric Analog State *Danielewicz, Lee ('11)*
  - Pygmy Dipole Resonanc *Carbone+ ('10)*
  - Isospin Diffusion *Tsang et al. ('04)*
  - Neutron Skin thickness *J.Zenihiro+ ('10)*
- これらの多くは ρ_0 以下の密度での Esym に敏感。



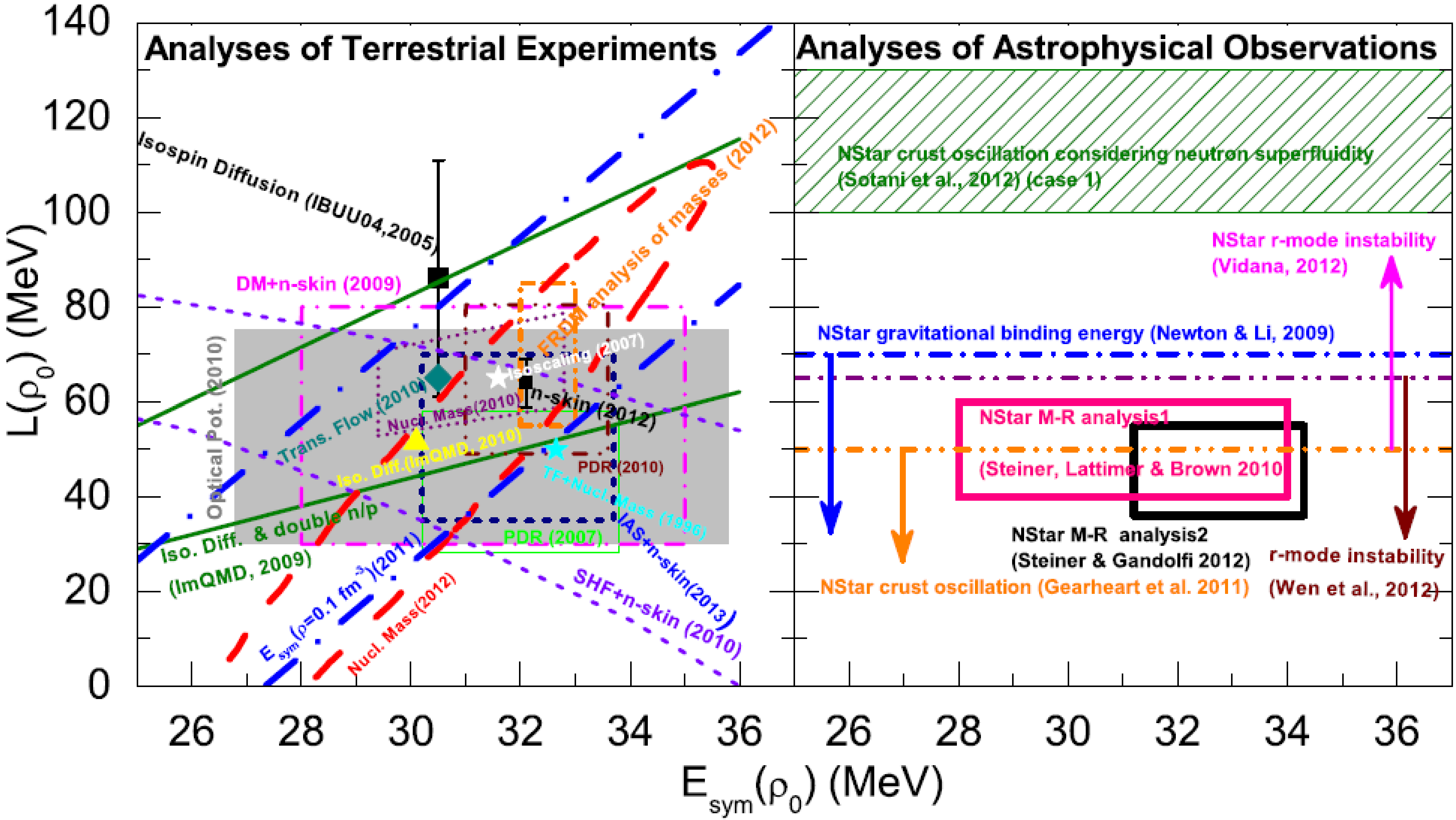
M. B. Tsang et al., Phys. Rev. C 86 (2012) 015803.

Nuclear Symmetry Energy (NuSYM 2011)



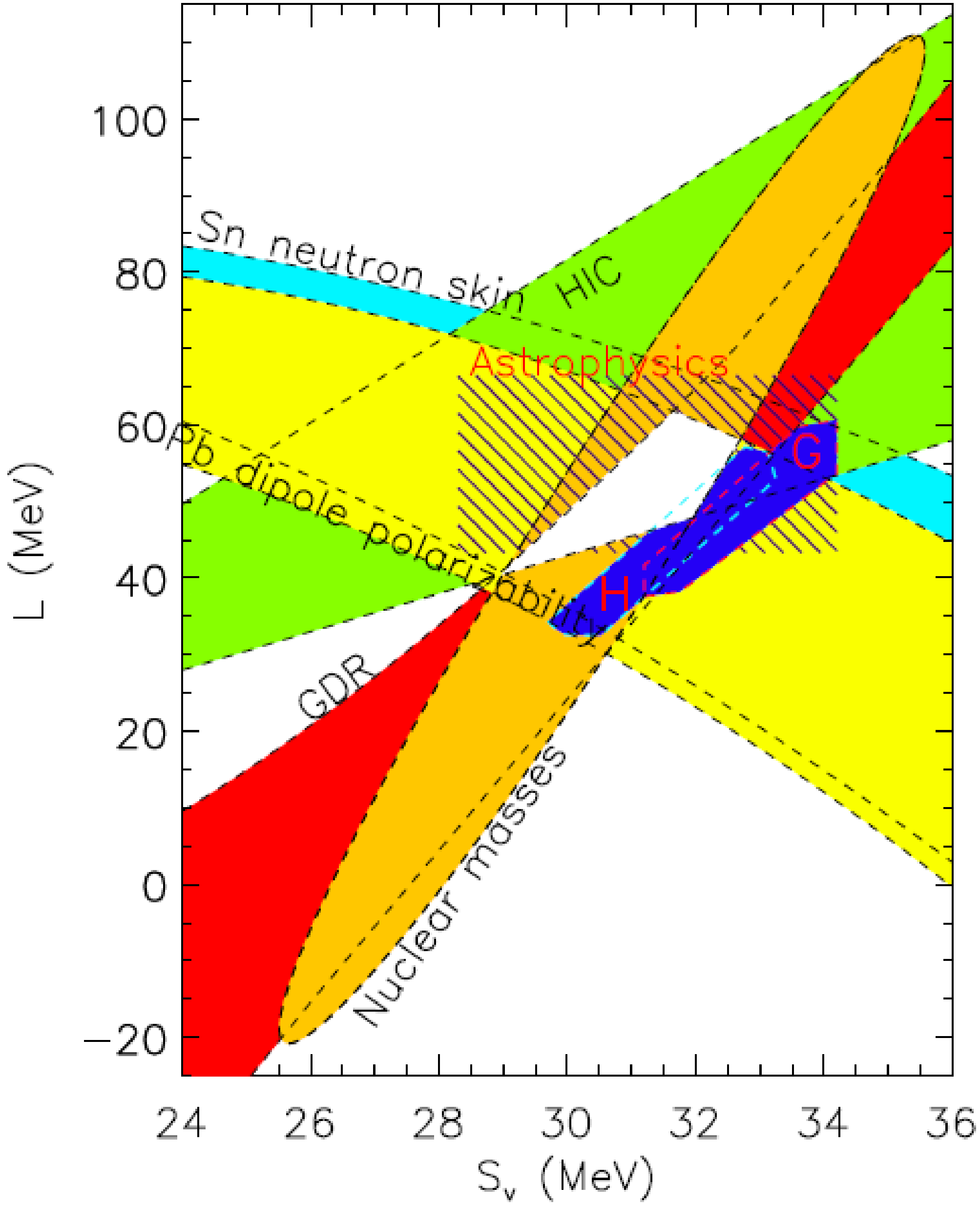
Tsang et al. ('12): NuSYM 2011

Nuclear Symmetry Energy (NuSYM 2013)



B. A. Li et al. ('13)

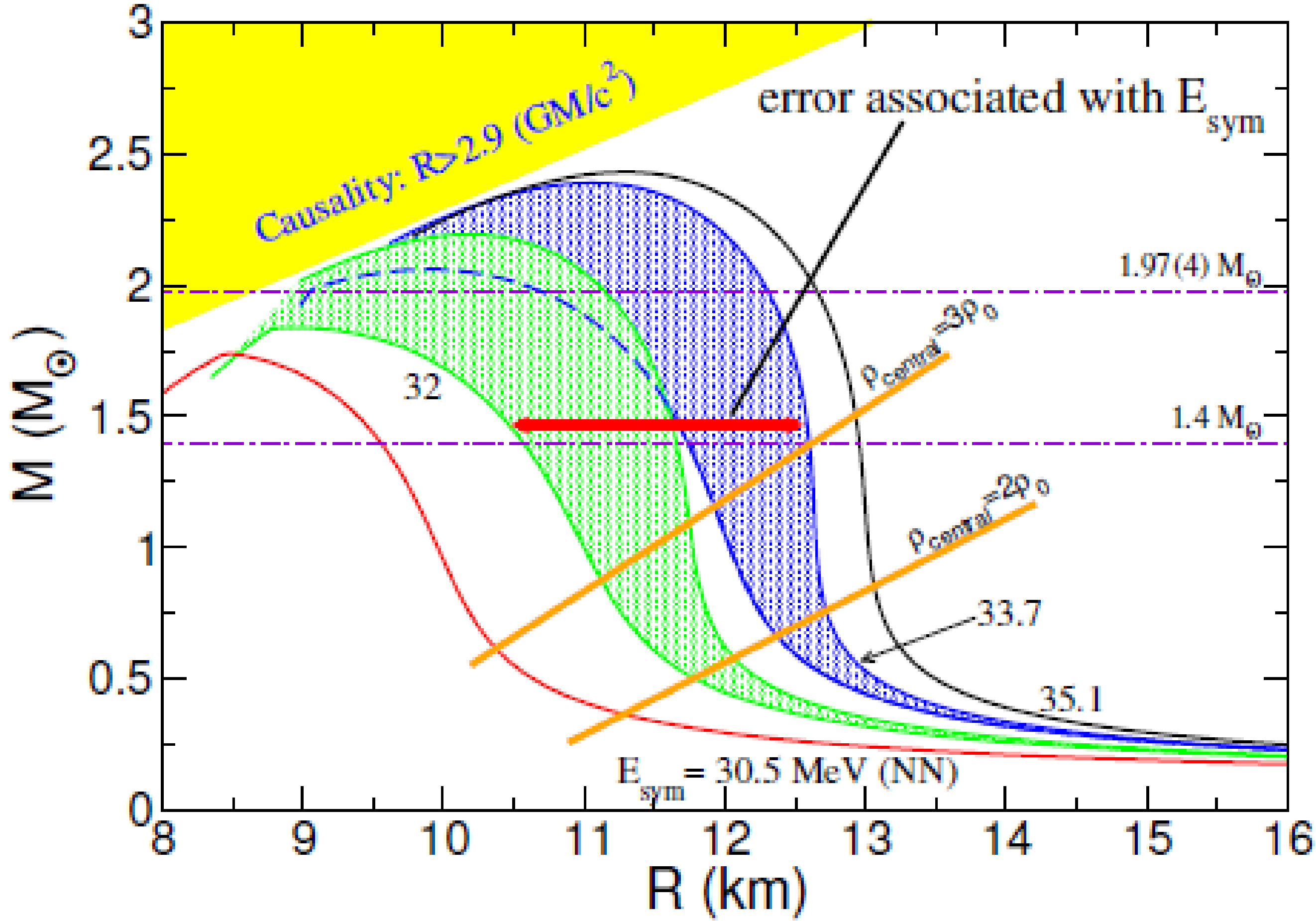
Nuclear Symmetry Energy (Lattmier-Lim, 2013)



Lattimer, Lim ('13)

Symmetry Energy affects MR Relation of NS

- **Nuclear pressure at ρ_0 comes ONLY from Esym, then Esym dominates pressure around ρ_0 !**
- **5 MeV Difference in Esym results in (3-4) km difference in R_{NS} prediction.**



Gandolfi, Carlson, Reddy, PRC 032801, 85 (2012).

Nuclear Mass

- **Larger symmetry energy → B.E. of n-rich nuclei become smaller.**
  - **Volume and surface symmetry energy**

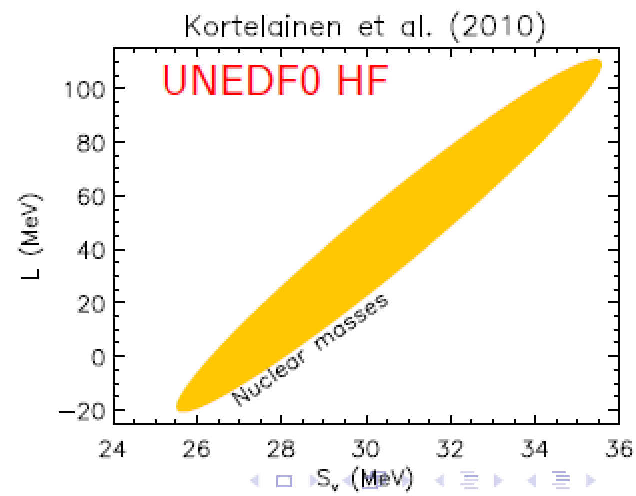
$$E_{\rm sym}(A) = a_a(A) = S_v - S_s A^{-1/3}$$
$$(\delta = (N-Z)/A = 1 - 2Y_p)$$

$$B(A,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} + a_p \frac{\delta_p}{A^\gamma}$$

  - **Finite Range Droplet Model**
    P.Moller, W.D.Myers, H.Sagawa, S.Yoshida, Phys. Rev. Lett. 108, 052501 (2012)

$$S_v = 32.5 \pm 0.5\,\text{MeV}, \; L = 70 \pm 15\,\text{MeV}$$

  - **Density Functional (UNEDF)**
    Kortelainen et al. (2010)



Lattimer, Lim ('13)

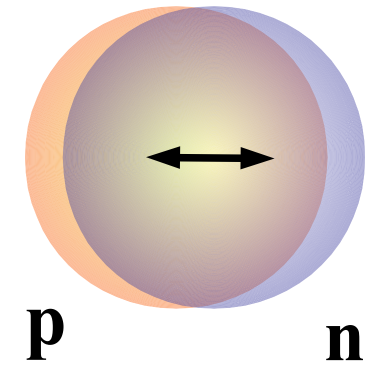
Well, relax (rough idea)

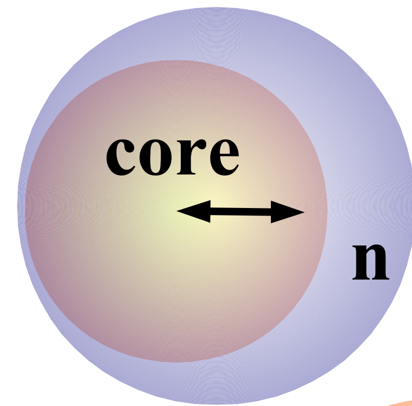
- **When L and Sv are linearly correlated as**
  $L = a\,S_v + b$,
  Symmetry energy is given as
  $E_{sym}(x) = Sv + L(x-1)/3 = Sv\,(1+a(x-1)/3) + \text{const. (indep. of Sv)}$
  $(x = \rho/\rho_0)$
  → That observable determines symmetry energy most effectively at $x = 1 - 3/a$.

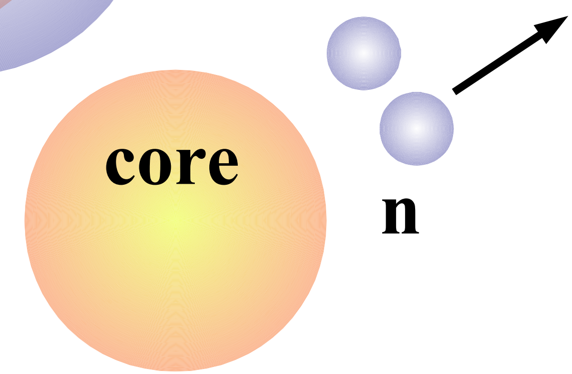
- **Nuclear mass: $a \sim 14$**
  原子核質量は $x \sim 1 - 3/14 = 0.78$ 近辺の対称エネルギーをよく決める

Pigmy Dipole Resonance

- **E1 response of nuclei**
  - **Giant Resonance: p and n oscillates collectively.**
    E* ~ 80 $A^{-1/3}$ MeV
  - **Pigmy Dipole Resonance:**
    Core oscillates in neutron skin/halo
    E* ~ (5-10) MeV
  - **Soft E1 excitation**
    When n wf is extended,
    direct dissociation σ also shows a peak.



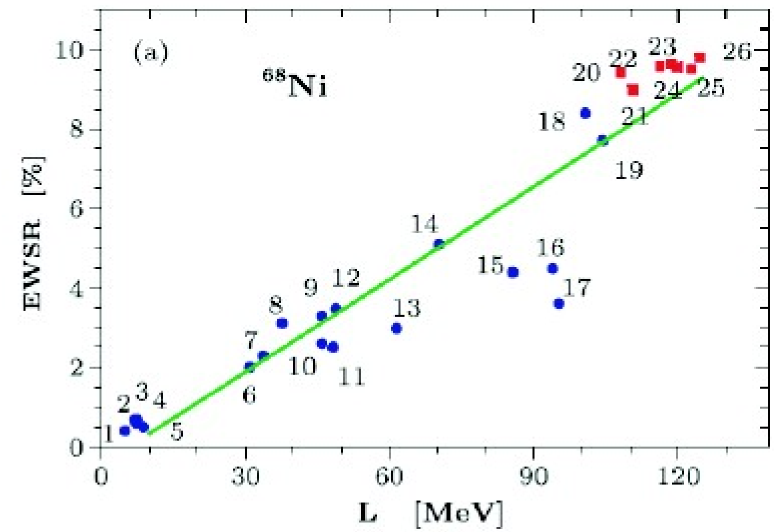


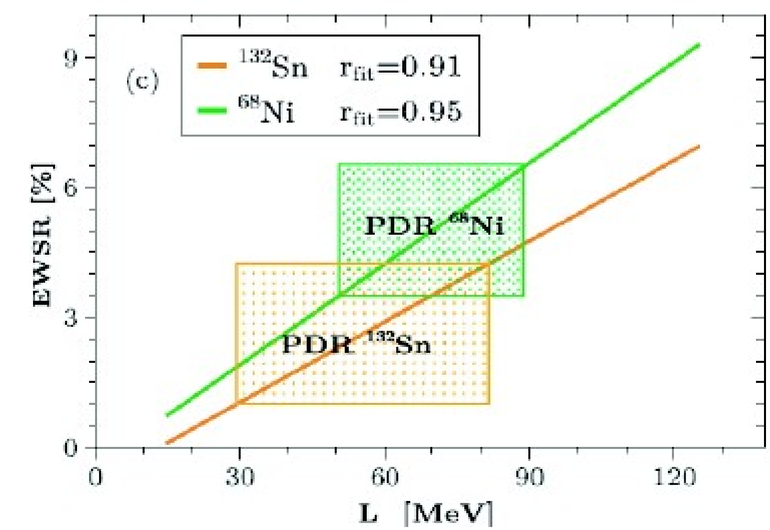


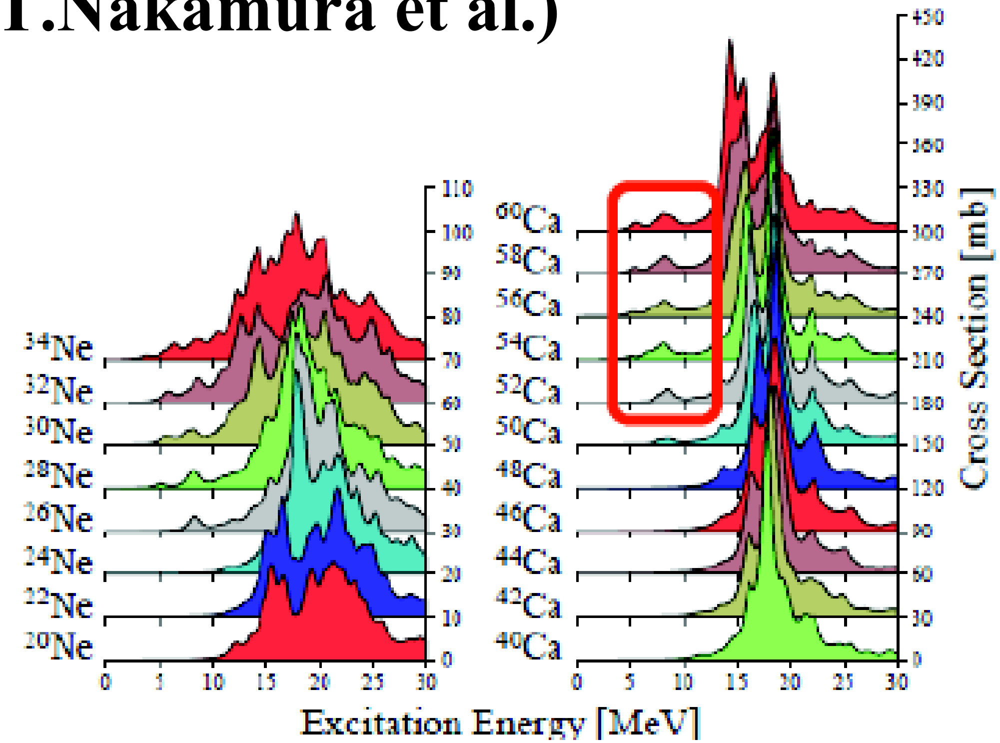
PDR would be sensitive to Esym

PDR of neutron skin nuclei

- **Energy Weighted Sum Rule value of PDR would has linear dep. on L**
- **PDR of very neutron rich nuclei will be measured at RIBF-SAMURAI (T.Nakamura et al.)**







T.Inakura, T.Nakatsukasa, K.Yabana Phys.Rev. C 84, 021302(R) (2011)

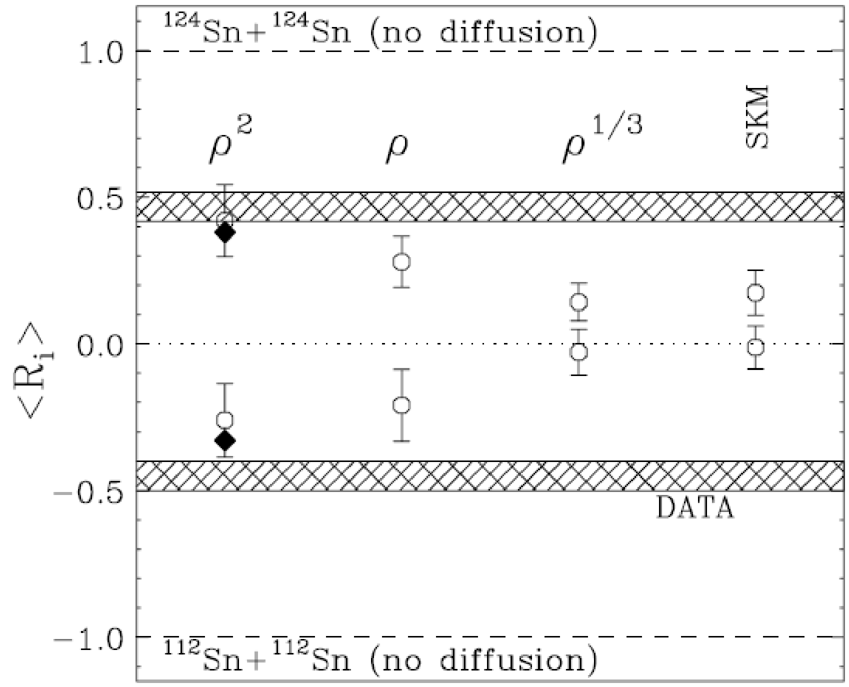
P. Adrich et al., PRL 95, 132501 (2005) .(GSI) 130,132Sn
O.Wieland et al., PRL 102, 092502 (2009) . (GSI) 68Ni

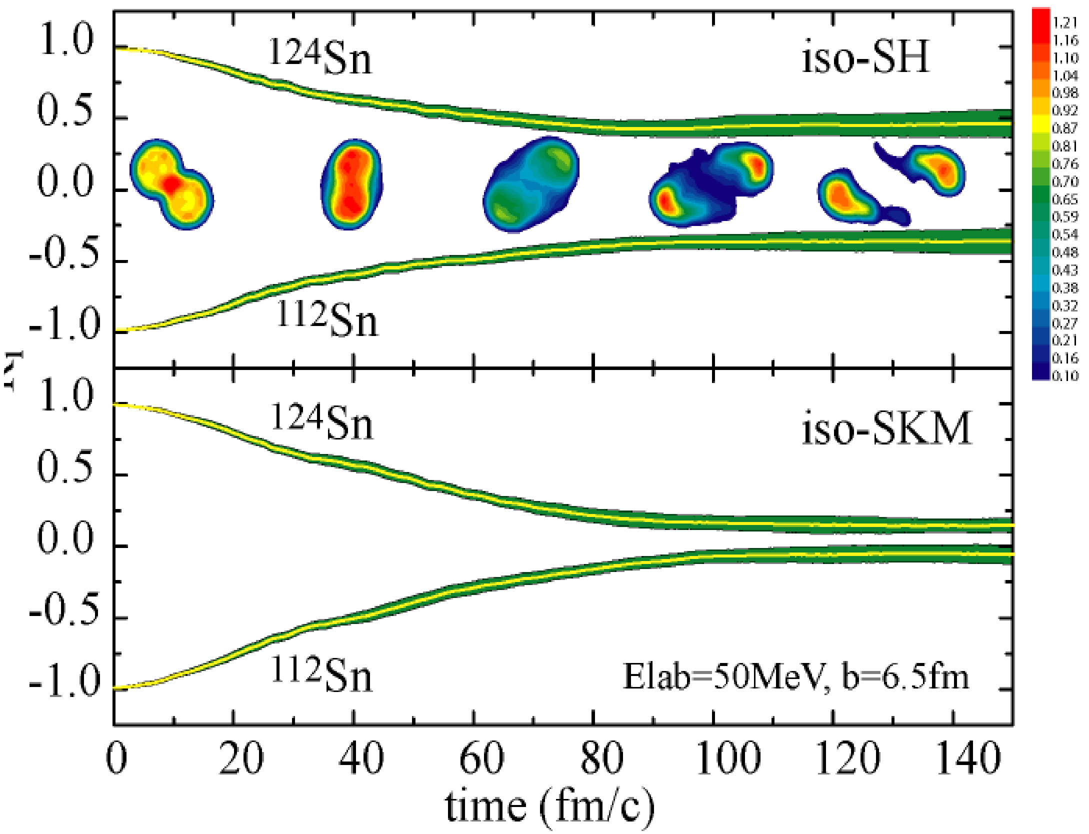
Isospin Diffusion

- **Collision of nuclei having different n/p ratio**
  → fragments with medium n/p ratio will be formed
  (Isospin diffusion)

- **Driving force of isospin diffusion**
  = Symmetry energy

$$R_i = \frac{2x - x_{124+124} - x_{112+112}}{x_{124+124} - x_{112+112}}$$

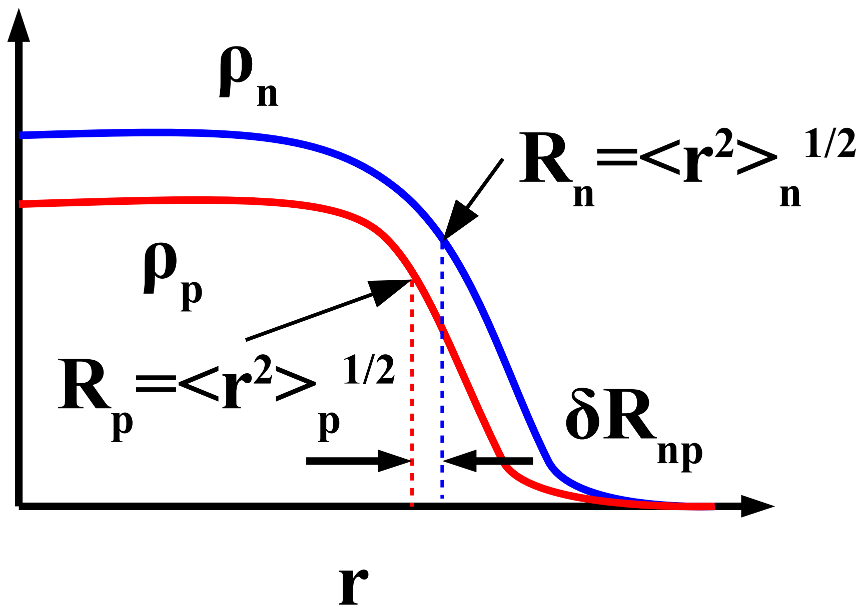




Tsang et al. ('04)

Skin Thickness & Dipole Polarizability

- **Skin Thickness δR_{np}**
  - **Larger L**
    - → **Small E_{sym} at low ρ**
    - **Large E_{sym} at high ρ**
    - → **Larger δR_{np}**



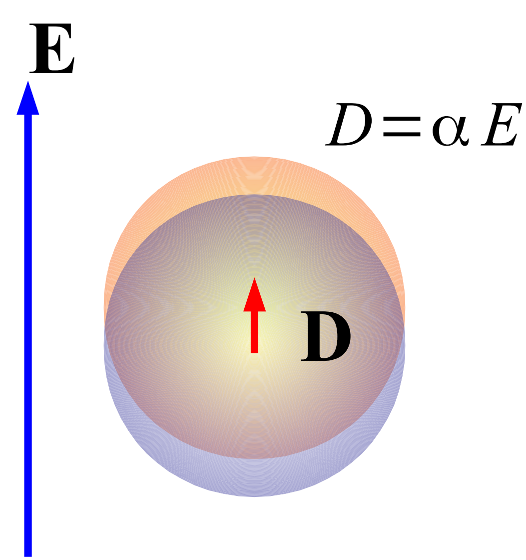
- **Electric Dipole Polarizability α**

$$H = H_0 - eE\sum_{i\in p} x_i = H_0 - E\hat{D}$$

$$|\psi\rangle = |0\rangle - \sum_{n>0} \frac{|n\rangle\langle n|V|0\rangle}{E_n - E_0} + O(E^2)$$

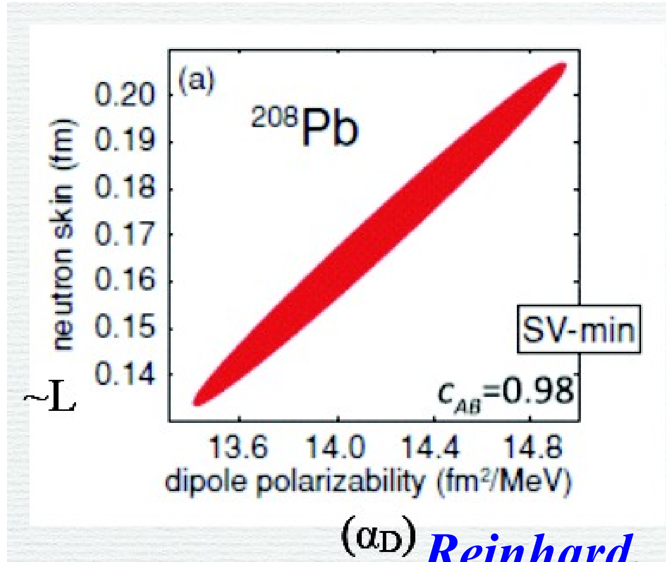
$$D = \langle\psi|\hat{D}|\psi\rangle = 2E\sum_{n>0}\frac{\langle 0|\hat{D}|n\rangle\langle n|\hat{D}|0\rangle}{E_n - E_0}$$

$$\alpha = \frac{8\pi}{9}\int\frac{dB(E1)}{\omega}$$

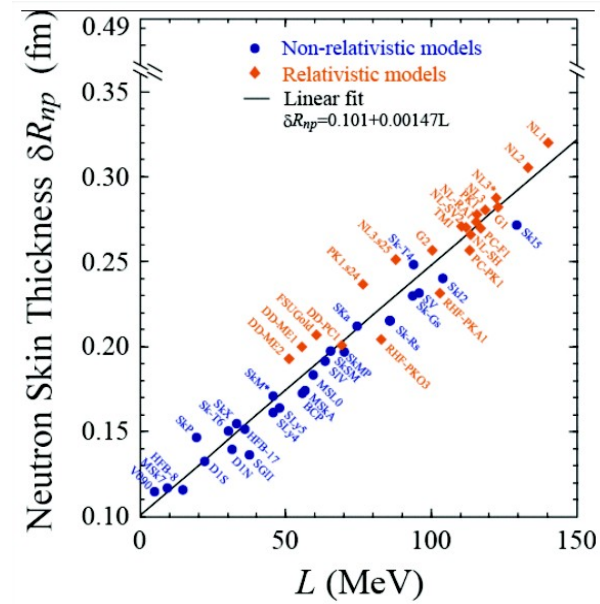


Skin Thickness & Dipole Polarizability

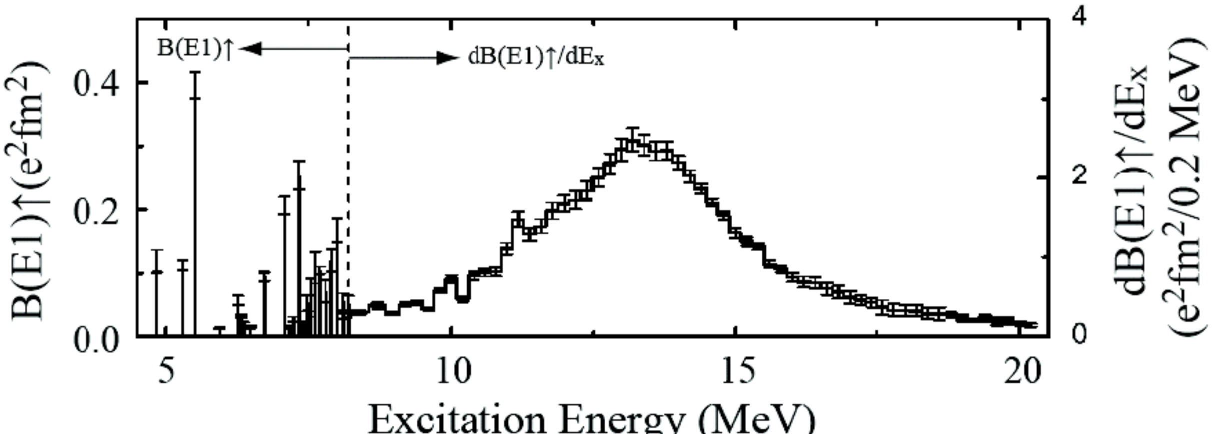
- **Strong corr. btw α and skin thickness (smaller restoring force → soft)**
- **Skin thickness is also correlated with L.**
- **Precise data from RCNP**



Reinhard, Nazarewicz ('10)

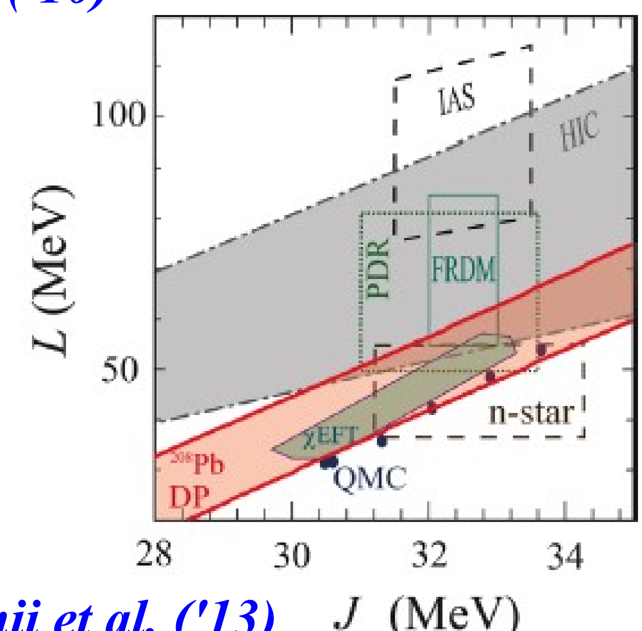


Roca-Maza et al. ('11)



E_x	0 – 10	10 – 20	20 – 130 MeV
α_D	2.7	16.2	1.2 fm³

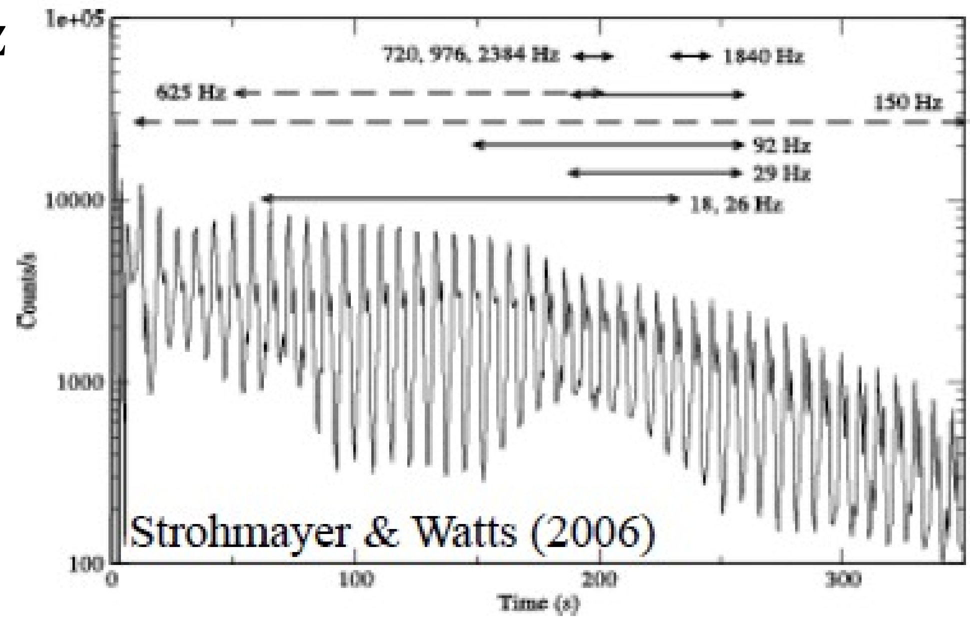
combined data — total 20.1±0.6 fm³



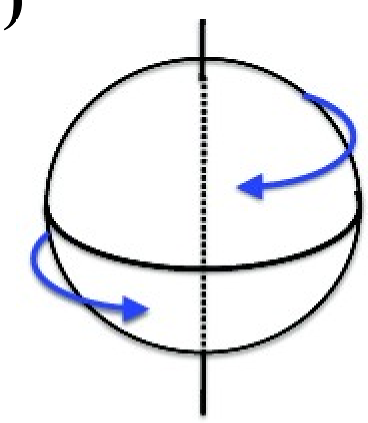
Tamii et al. ('13)

Quasi Periodic Oscillation of Neutron Stars

- **QPOs in afterglow of giant flares from soft-gamma repeaters (SGRs)** *(Barat+ 83, Israel+ 05, Strohmayer & Watts 05, Watts & Strohmayer 06)*
  - **SGR 0526-66 (5th/3/1979) : 43 Hz**
  - **SGR 1900+14 (27th/8/1998) :**
    28, 54, 84, 155 Hz
  - **SGR 1806-20 (27th/12/2004) :**
    18, 26, 30, 92.5, 150,
    626.5, 1837 Hz



- **Asteroseismology**
  - **From star quake to stellar properties (M, R, B, EOS …)**
  - **Low frequency (e.g. 28 Hz) requires long wave mode.**
    → Torsional oscillations of the crust

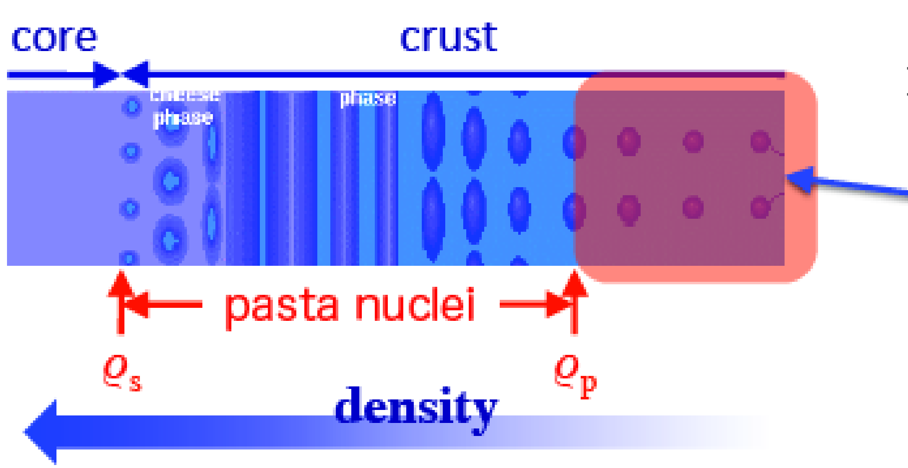


QPO and Symmetry Energy

- **Torsional oscillations (ねじれ振動)**
  - **incompressible (no density perturbations)**
  - **Frequency** $${}_{l}t_{0}=\sqrt{l(l+1)}\frac{v_s}{2\pi R}\ ,\quad v_s=\sqrt{\mu/\rho}$$

  μ: shear modulus, vs: shear velocity
  (Hansen & Cioff 1980)

- **Shear modulus of bcc lattice depends on nuclear charge**
  → Dependence on the symmetry energy



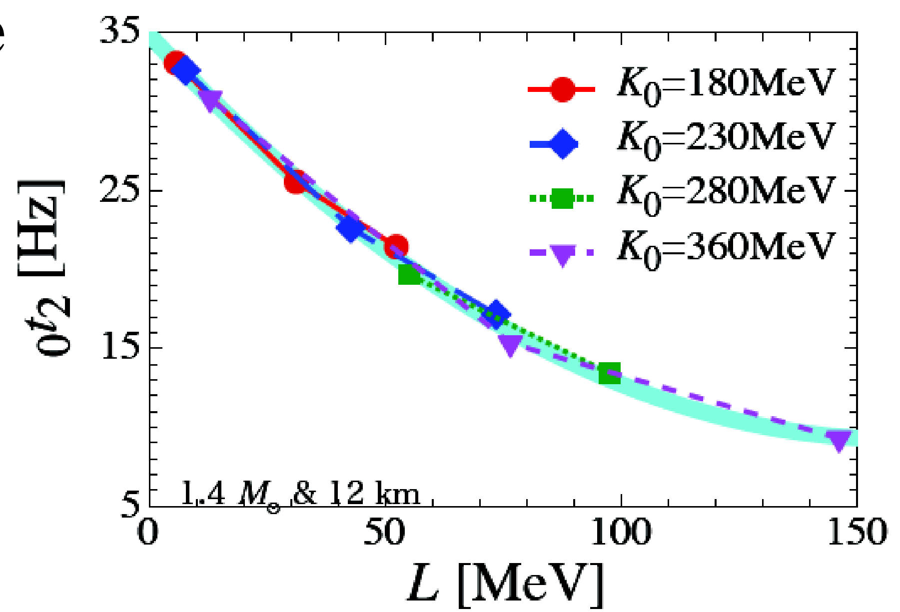
for bcc lattice (Strohmayer+ 1991)

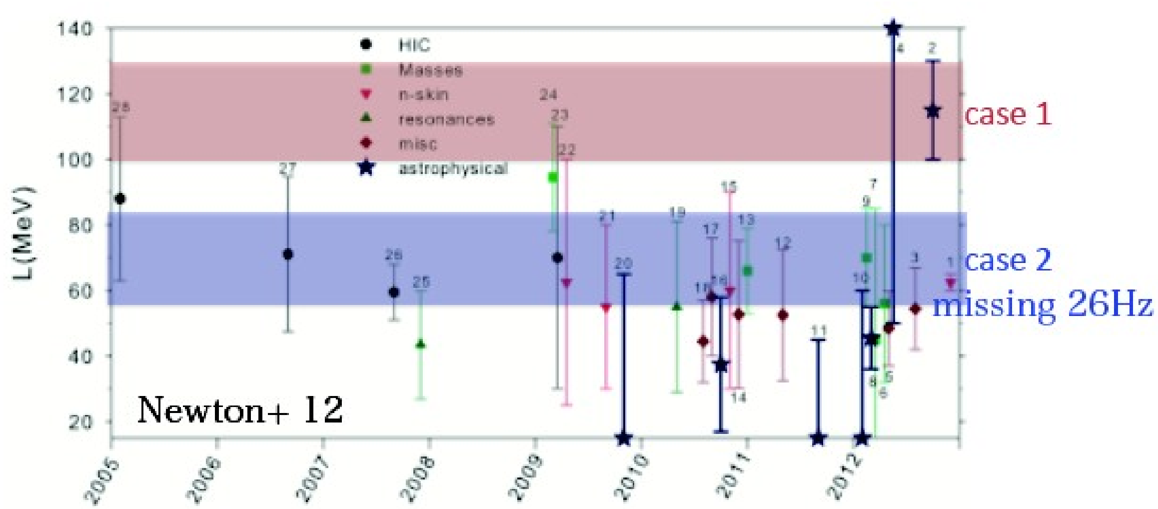
$$\mu = 0.1194\frac{n_i(Ze)^2}{a}$$

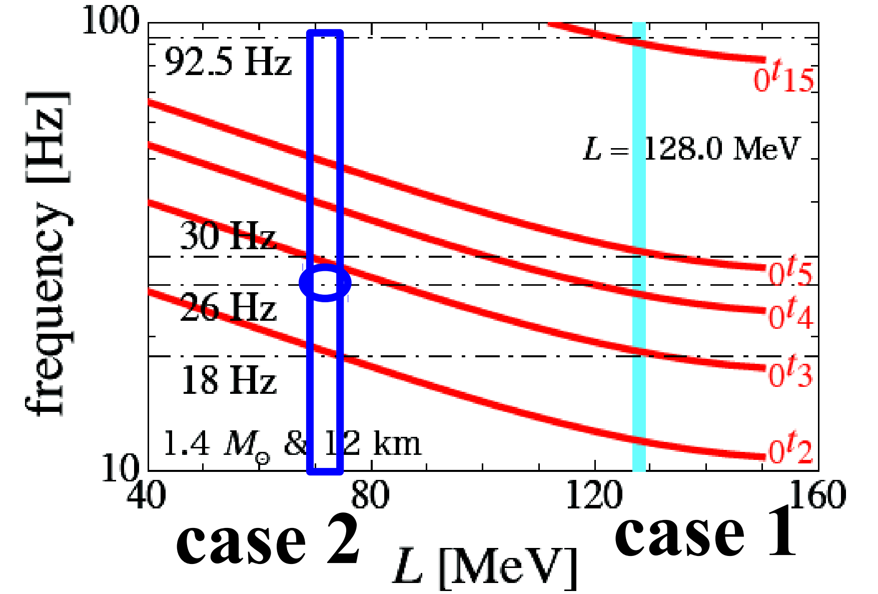
n_i : number density of quark droplet
Z : charge of quark droplet
a : Wigner-Seitz radius

QPO and Symmetry Energy

- **For a given set of (M,R), we can solve TOV equation from the surface.**
- **Torsional oscillation frequencies are calculated using EOSs with various (L,K).**
  Oyamatsu, Iida ('03,'07)
- **Compared with observed freq.**
  → constrains on L (small dep. on K)







Large L or missing 26 Hz

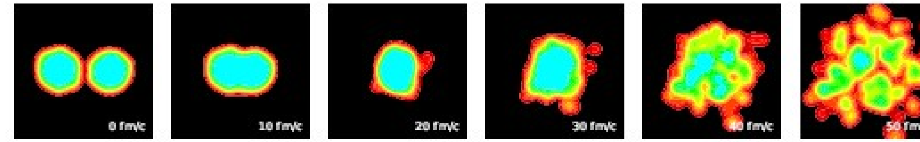
Sotani+ '13

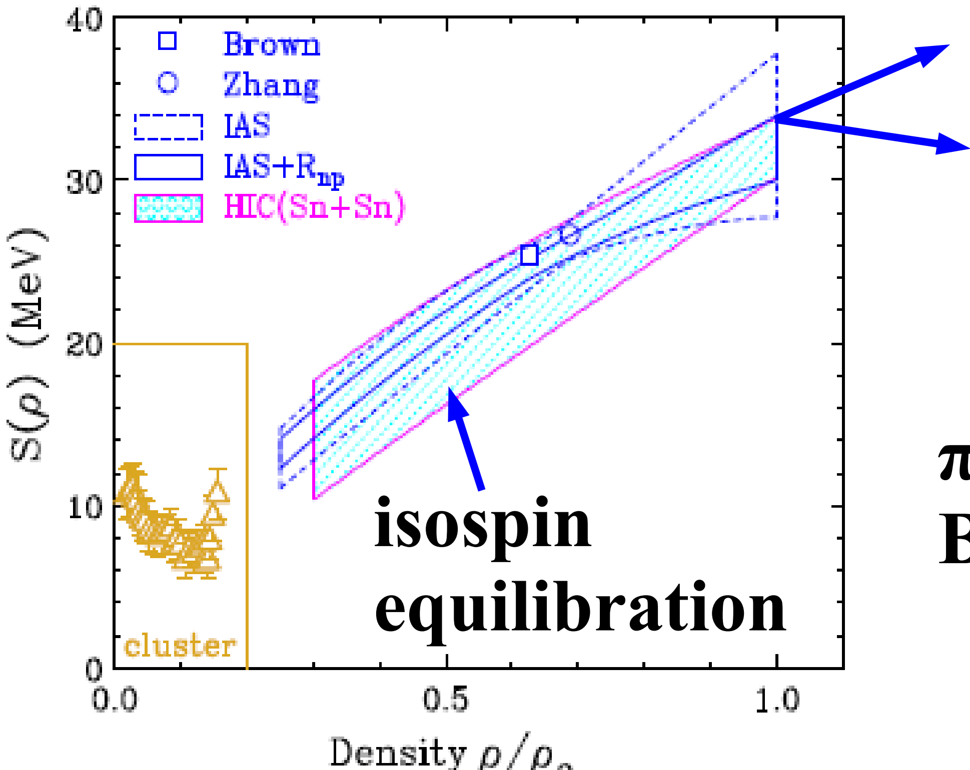
High Density Symmetry Energy

- Symmetry energy at $\rho=(2\text{-}4)\ \rho_0$ dominantly determines NS radius.

  → Central Heavy-Ion collisions at a few 100 MeV !

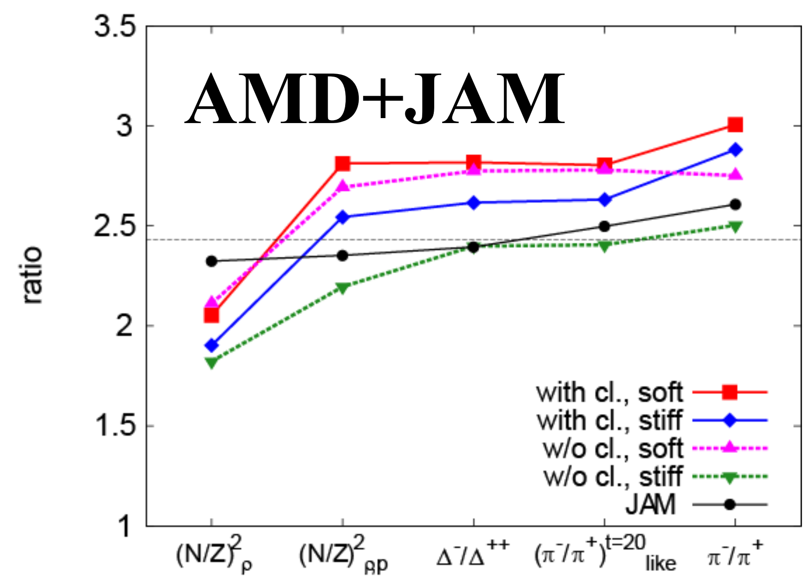
  (Li, Trautmann, Murakami, Ono)





Let's wait for SπRIT B01 results (Murakami) ! More theor. work needed.

π^-/π^+
B.A.Li

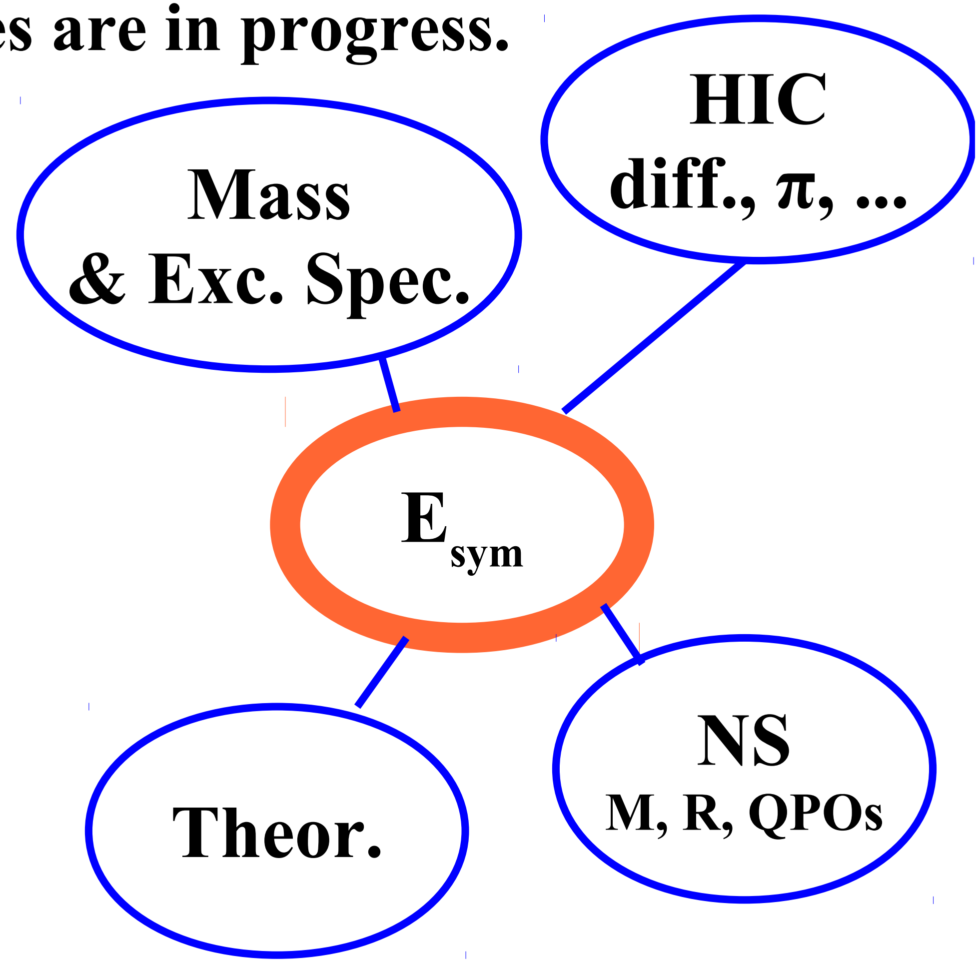


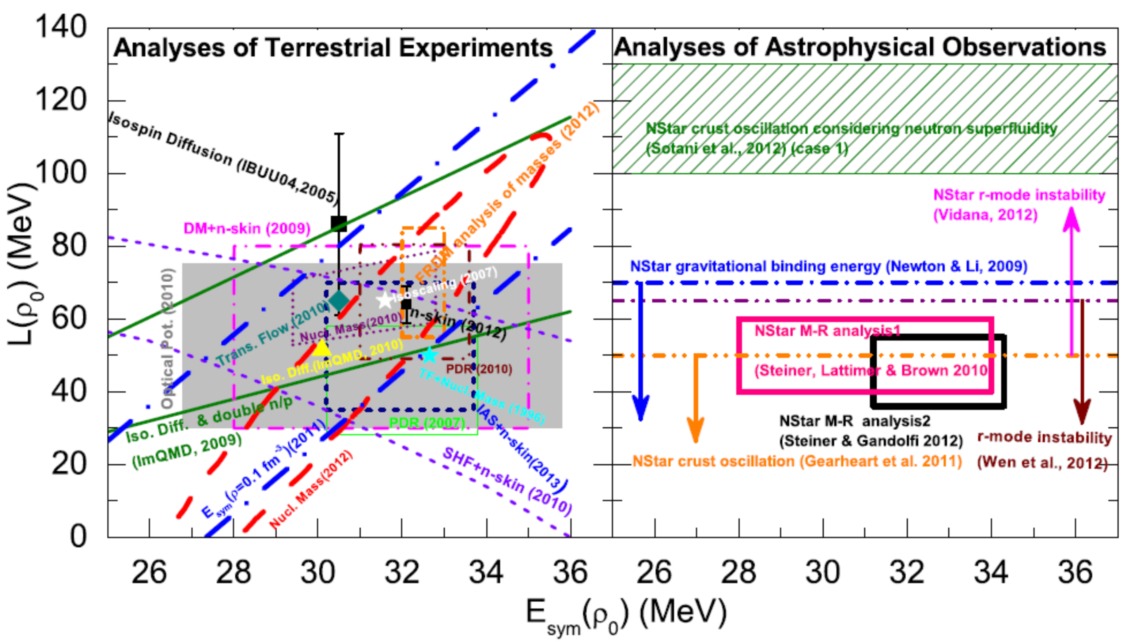
C.J.Horowitz, E.F.Brown, Y.Kim, W.G.Lynch, R.Michaels, A. Ono, J. Piekarewicz, M. B. Tsang, H.H.Wolter (NuSYM13), JPG41('14) 093001

Ikeno, Ono, Nara, AO ('16)

Summary

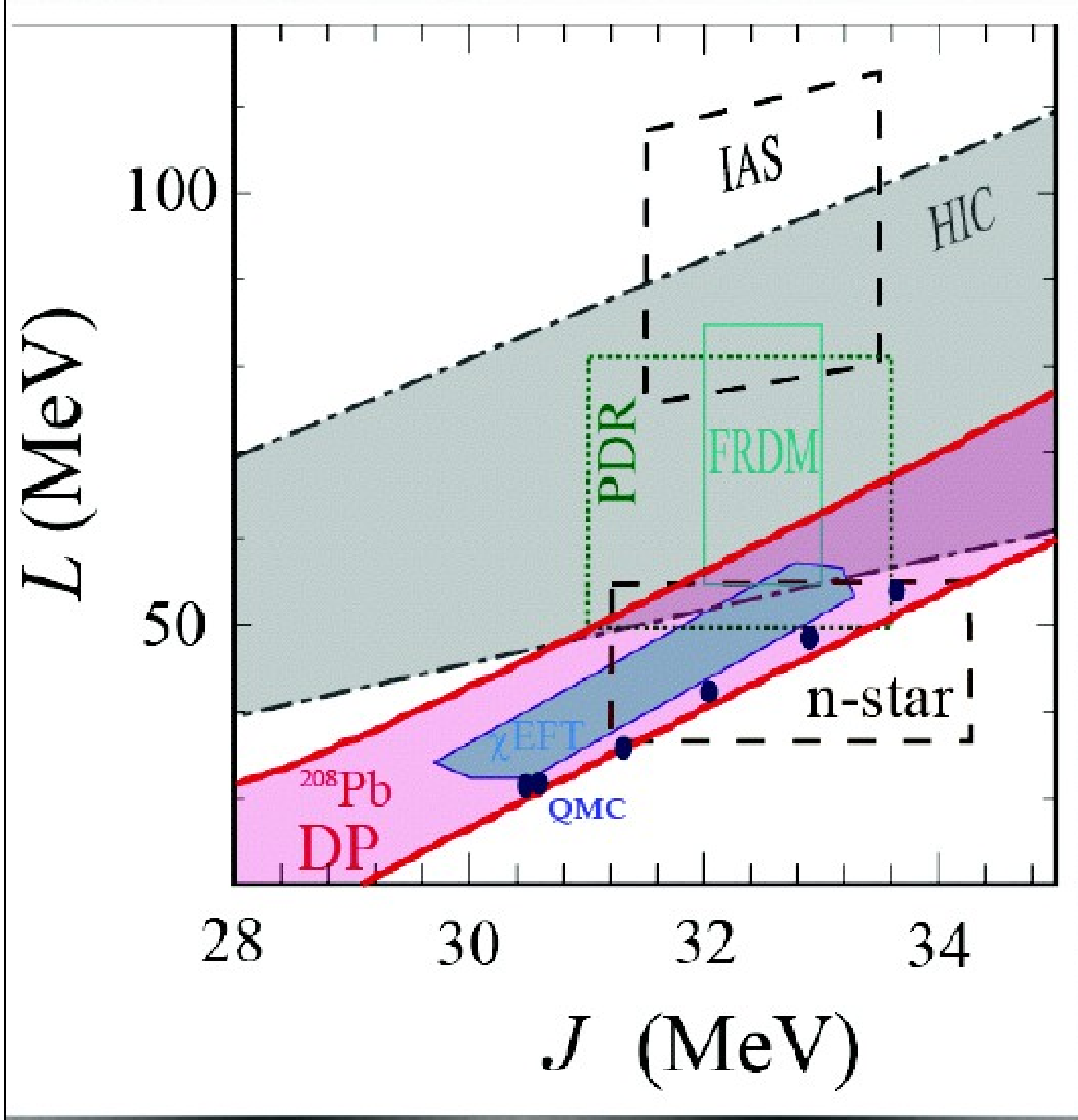
- **Symmetry energy is decisive in neutron star matter EOS, and is related to various properties of nuclei.**
- **Experimental & Theoretical studies are in progress.**
- **If you have a new idea to determine Esym, please propose as soon as possible.**





Thank you !

Constraints on J and L



AT *et al.*, to be published in EPJA.

M.B. Tsang *et al.*, PRC86, 015803 (2012).

I. Tews et al., PRL110, 032504 (2013)

DP: Dipole Polarizability (this work)
HIC: Heavy Ion Collision
PDR: Pygmy Dipole Resonance
IAS: Isobaric Analogue State
FRDM: Finite Range Droplet Model (nuclear mass analysis)
n-star: Neutron Star Observation (A.W. Steiner et al.,)
cEFT: Chiral Effective Field Theory
QMC: Quantum Monte-Carlo Calc.