



Impulse Approximation (1)

- 短時間の反応であれば 入射粒子と核の反応 ~ Σ (入射粒子と核内核子の反応) → Impulse 近似
- Lippmann-Schwinger 方程式 (potential 問題)

$$\Psi^{(+)} = \Phi + \hat{G}_0 \hat{V} \Psi^{(+)} \qquad n$$

$$\hat{H} = \hat{K} + \hat{V}, \quad \Phi = \exp(i \, \vec{k}_i \cdot \vec{r}), \quad \hat{G}_0 = (E - \hat{K} + i \, \varepsilon)^{-1}$$

Green's function

$$G_0(\vec{r},\vec{r}') = -\frac{2m}{\hbar^2} \frac{1}{4\pi} \frac{e^{ikr}}{r} \rightarrow f(\theta) = -\frac{2m}{\hbar^2} \frac{1}{4\pi} \langle \vec{k_f} | \hat{V} | \Psi^{(+)} \rangle$$

T-matrix (transition matrix)

$$\langle \vec{k}_{f} | \hat{T} | \vec{k}_{i} \rangle \equiv \langle \vec{k}_{f} | \hat{V} | \Psi^{(+)} \rangle \rightarrow \hat{T} = \hat{V} + \hat{V} \hat{G}_{0} \hat{T}$$



Impulse Approximation (2)



形式的にはポテンシャル問題と同じだが、 多体問題なのでそのままでは解けない。

■ 方針: *T* を核内核子との散乱振幅 τ_i で表し、 最後に *i* と散乱した振幅 T_i の和で *T* を求める。 $\tau_i = v_i + v_i G_0 \tau_i$, $T_i = v_i + v_i G_0 T$, $T = \sum_i T_i$ $\rightarrow T = \sum_i T_i = \sum_i \tau_i + \sum_i \tau_i G_0 \sum_{i \neq i} T_j$



Impulse Approximation (3)

- 近似的取扱い
 - 近似1:質量数が大きいとして、O(1/A) を無視する。 $\sum_{j \neq i} T_j \simeq T \rightarrow T = \tilde{T} + \tilde{T} G_0 T \quad (\tilde{T} = \sum_i \tau_i)$
 - 近似 2:核子波動関数は反対称化されているので、どの T_i も同じ。 (Kerman-McManus-Thaler の多重散乱理論) $\sum_{j \neq i} T_j \simeq \frac{A-1}{A} T \rightarrow T' = \tilde{T}' + \tilde{T}' G_0 T' \quad (\tilde{T}' = \frac{A-1}{A} \sum_i \tau_i, T' = \frac{A-1}{A} T)$
 - いずれの場合も、 $\Sigma \tau_i$ は光学ポテンシャルの役割を果たす
- Impulse approximation $T \simeq \sum_{i} \tau_{i} \simeq \sum_{i} t_{i}$
 - 1段階反応を仮定
 - 核内2体散乱振幅を自由空間の散乱振幅と同じと仮定





Fermi's Golden Rule

$$W = \frac{2\pi}{\hbar} |\langle f | \hat{V} | i \rangle|^{2} \rho_{E}$$

$$\rightarrow d\sigma = \frac{W}{v_{i}} = \frac{1}{v_{i}} (2\pi)^{4} \delta^{4} (p_{1} + p_{2} - p_{3} - p_{4}) |T_{fi}|^{2} \frac{d\vec{p}_{3}}{(2\pi)^{3}} \frac{d\vec{p}_{4}}{(2\pi)^{3}}$$

$$\rightarrow \frac{d^{2}\sigma}{dE_{3}d\Omega_{3}} = \frac{p_{3}E_{3}}{(2\pi)^{2}v_{1}} |T_{fi}|^{2} \delta (\omega - E_{1} + E_{3}) \quad (\omega = E_{4} - E_{2})$$

$$T_{fi} = \langle \chi^{(-)}(\vec{k}_{f}) f | \sum_{i} t_{i} | \chi^{(+)} i \rangle$$

$$1 (\pi) \qquad 3 (1)$$

χ:入射・出射粒子の w.f.、 i, f:核の始状態、終状態





断面積と有効核子数(2)

- ■素過程散乱振幅についての近似
 - 入射・出射粒子の運動量と入射エネルギーで表せる
 - 短距離力である

 $\langle \vec{r}_{2}\vec{r}_{4}|t|\vec{r}_{1}\vec{r}_{2}\rangle \simeq t(E,\vec{k}_{1},\vec{k}_{3})\delta(\vec{r}_{2}-\vec{r}_{4})\delta(\vec{r}_{1}-\vec{r}_{3})\delta(\vec{r}_{1}-\vec{r}_{2})\hat{O}$ $\frac{d^{2}\sigma}{d\Omega_{3}dE_{3}} = \frac{p_{3}E_{3}}{(2\pi)^{3}v_{1}}|t|^{2}N_{\text{eff}}\delta(\omega+E_{3}-E_{1})$ $\simeq \beta \left(\frac{d\sigma}{d\Omega}\right)_{\text{Lab.}}^{\text{elem.}}N_{\text{eff}}\delta(\omega+E_{3}-E_{1})$ $N_{\text{eff}} = \sum_{f} \left|\int d\vec{r}\chi_{3}^{(-)*}(\vec{r})\chi_{1}^{(+)}(\vec{r})\langle f|\sum\hat{O}_{j}\delta(\vec{r}-\vec{r}_{j})|i\rangle\right|^{2}$



Green's Function Method

■ 1 粒子過程を仮定し、終状態の原子核の状態を見ないとする。 → 完全系を作るとすれば、δ 関数をグリーン関数に置き換え可能 $\frac{d^2\sigma}{dE_3d\Omega_3} = \frac{p_3E_3}{(2\pi)^2v_1} |T_{fi}|^2 \delta(\omega - E_1 + E_3) \quad (\omega = E_4 - E_2)$

$$\sum_{f} |\langle \chi_3 f | \hat{t} | \chi_1 i \rangle|^2 \delta (E_1 + E_2 - E_3 - E_4) = -\frac{|t|^2}{\pi} \operatorname{Im} \left[\sum_{\alpha} \langle F_{\alpha} | \frac{1}{E - \hat{H}_Y + i \varepsilon} | F_{\alpha} \rangle \right]$$

$$\frac{d^2\sigma}{dE_3 d\Omega_3} = -\frac{p_3 E_3}{(2\pi)^2 v_1} |t|^2 \frac{1}{\pi} \operatorname{Im}\left[\sum_{\alpha} \langle F_{\alpha} | G_{\alpha\alpha} (E - E_{\alpha}) | F_{\alpha} \rangle\right]$$

 $F_{\alpha}(\vec{r}) = \chi_3^{(-)*}(\vec{r})\chi_1(\vec{r})\psi_{\alpha}$



Deeply Bound pionic atom (1)



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(a)

π⁰ peak

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Deeply Bound pionic atom (2)

- 正確にスペクトルを再現するには、
 - 波動関数の歪曲 (Distorted Wave Impulse Approximation)
 - 始状態・終状態の波動関数の光学因子 (spectroscopic factor)
 が必要

$$\chi_{f}^{*}(\mathbf{r})\chi_{i}(\mathbf{r}) = \exp(i\mathbf{q}\cdot\mathbf{r})D(\mathbf{b},z)$$
$$D(\mathbf{b},z) = \exp\left[-\frac{1}{2}\left(\int_{-\infty}^{z}\sigma_{d}\rho(\mathbf{b},z')dz'\right) + \int_{z}^{\infty}\sigma_{3}_{\mathrm{He}}\rho(\mathbf{b},z')dz'\right)\right]$$

Star Ma



<u>K. Itahashi et al., PRC62('00)025202</u>

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Hyperons (Baryons with Strangeness)

Ground state baryon SU(3)_f octet ($J^{\pi}=1/2+$)

Baryon	M(Mev)	S	Comp.
n	940	0	udd
р	938	0	uud
Λ	1116	-1	(uds-dus)/√2
Σ^+	1189	-1	uus
Σ^0	1193	-1	(uds+dus)/√2
Σ^{-}	1197	-1	dds
Ξ^0	1315	-2	uss
Ξ^-	1321	-2	dss





 $SU(3)_{f}$ transformation

- Fundamental triplet $(u,d,s)^T = q \rightarrow q'=U q \ (U \in SU(3))$
- **Diquark** $\mathbf{D}_{i} = \varepsilon_{ijk} \mathbf{q}_{j} \mathbf{q}_{k} \rightarrow \mathbf{D'} = \mathbf{D} \mathbf{U}^{+}$
- **Baryon octet** $\mathbf{B}_{ij} = \mathbf{D}_j \mathbf{q}_i \rightarrow \mathbf{B'} = \mathbf{U}\mathbf{B}\mathbf{U}^+$

$$\begin{pmatrix} [ds]u & [su]u & [ud]u\\ [ds]d & [su]d & [ud]d\\ [ds]s & [su]s & [ud]s \end{pmatrix} = \begin{pmatrix} \frac{\Lambda}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ & p\\ \Sigma^- & \frac{\Lambda}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & n\\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$



 $SU(3)_{f}$ transformation

- Fundamental triplet $(u,d,s)^T = q \rightarrow q'=U q \ (U \in SU(3))$
- Anti-quark $\overline{\mathbf{q}} \to \overline{\mathbf{q}}' = \overline{\mathbf{q}} \mathbf{U}^+$
- Meson octet $M_{ij} = \overline{q}_j q_i \rightarrow M' = UMU^+$

$$\begin{pmatrix} \overline{u} \ u & \overline{d} \ u & \overline{s} \ u \\ \overline{u} \ d & \overline{d} \ d & \overline{s} \ d \\ \overline{u} \ s & \overline{d} \ s & \overline{s} \ s \end{pmatrix} = \begin{pmatrix} \frac{\eta}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} & K^0 \\ K^- & \overline{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix} = P$$

$$S = \begin{pmatrix} \frac{\sigma}{\sqrt{2}} + \frac{a_0}{\sqrt{2}} & a_0^+ & \kappa^+ \\ a_0^- & \frac{\sigma}{\sqrt{2}} - \frac{a_0}{\sqrt{2}} & \kappa^0 \\ \kappa^- & \bar{\kappa}^0 & \zeta \end{pmatrix} \qquad \qquad V = \begin{pmatrix} \frac{\omega}{\sqrt{2}} + \frac{\rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega}{\sqrt{2}} - \frac{\rho^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \varphi \end{pmatrix}$$



SU(3), invariant coupling

- Baryon-Meson coupling
 - $\mathcal{L}_{\rm BV} = \sqrt{2} \{ g_s \operatorname{tr} (M_v) \operatorname{tr} (\bar{B}B) + g_D \operatorname{tr} (\bar{B} \{M_v, B\}) + g_F \operatorname{tr} (\bar{B} [M_v, B]) \}$ $= \sqrt{2} \{ g_s \operatorname{tr} (M_v) \operatorname{tr} (\bar{B}B) + g_1 \operatorname{tr} (\bar{B}M_vB) + g_2 \operatorname{tr} (BBM_v) \}$
- Assumption
 - BM coupling is SU(3) invariant
 - N does not couple with s vector meson

$$g_{\omega\Lambda} = \frac{5}{6}g_{\omega N} - \frac{1}{2}g_{\rho N}, \ g_{\phi\Lambda} = \frac{\sqrt{2}}{6}(g_{\omega N} + 3g_{\rho N})$$

Further simplification: $g_{\rho N} = g_{\omega N}/3$ (quark counting)

$$g_{\omega N} = g_{\nu}, g_{\rho N} = g_{\nu}/3, g_{\omega \Lambda} = 2g_{\nu}/3, g_{\varphi \Lambda} = \sqrt{2}g_{\nu}/3$$



Hypernuclear formation

■ (K⁻, π), (π , K⁺), and (K⁻,K⁺) reactions on nuclei \rightarrow Hypernuclei

Reaction	Elementary Processes		
	Main Process	Other Processes	
(K^{-},π^{-})	$K^-n \to \pi^-\Lambda,$	$K^-n \to \pi^- \Sigma^0, \ K^-p \to \pi^- \Sigma^+$	
(K^-,π^+)	$K^- p \to \pi^+ \Sigma^-,$	$K^-pp \to \pi^+\Lambda n$ (n-rich hypernuclear formation)	
(π^+, K^+)	$\pi^+ n \to K^+ \Lambda,$	$\pi^+ n \to K^+ \Sigma^0, \ \pi^+ p \to K^+ \Sigma^+$	
(π^{-}, K^{+})	$\pi^- p \to K^+ \Sigma^-,$	$\pi^- pp \to K^+ \Lambda n$ (n-rich hypernuclear formation)	
(K^{-}, K^{+})	$K^- p \to K^+ \Xi^-,$	$K^- pp \to K^+ \Lambda \Lambda$	





Hypernuclear formation





Single particles states of A in nuclei

- Single particle potential depth of Λ is around -30 MeV
 - s, p, d, f, ... states are clearly seen

•
$$A_{core}^{-2/3} \propto R^{-2} \propto K.E.$$
 of Λ





Hypernuclear production (discrete state)

- Substitutional reaction
 - Magic momentum 近辺では q~0
 → 核子軌道にハイペロンが入る状態が有利

H. Bando, T. Motoba, J. Zofca, Int. J. Mod. Phys. A 5 (1990), 4021-4198.







the reaction $aN \rightarrow Yb$ at $\theta_{b,L} = 0^{\circ}$.

A hypernuclear formation

- **(** π^+ , K⁺) reactions on nuclei
 - $q \sim k_F \rightarrow various s.p.$ states of Λ are populated



Hasegawa et al.(1996)



Quasi-Free A **Production** (1)

- 有効核子数法 → 十分に離散的な束縛状態で成功
 - → 連続状態では?
- **Green's Function Method** O.Morimatsu, K. Yazaki, NPA483('88)493
 - 束縛状態と連続状態を同時に記述可能

Harada, Hirabayashi ('04)

CROSS

SECTION (µb/sr MeV)[CAL.

12



350

300

Quasi-Free A Production

- Kinematical factor による t-matrix の表現
 - 狙うエネルギー領域が狭い場合には有効
 - → 広いエネルギー領域をカバーする時には最適化運動量を用いる



T. Harada, Y. Hirabayashi / Nuclear Physics A 744 (2004) 323–343



S Potential in Nuclear Matter

- **U**_{Λ}(ρ_0) ~ 30 MeV: Well known from single particle energies
 - Tsubakihara, Maekawa, AO, Naïve expectation EPJA33('07),295. = Quark Number (ud number) Scaling S_{Λ} from $A+1_{\Lambda}Z$ Chiral SU(3) 25 $U_{\Lambda} \sim 2/3 U_{N} \rightarrow U_{\Sigma} \sim 2/3 U_{N} \sim -30 \text{ MeV}$ 20(MeV) 15 10 Problems with Σ • Only one bound state ⁴, He (Too light !) 0 \rightarrow Continuum (Quasi-Free) Spectroscopy -5 0.05 0.1 0.15 0.2 0.250.3 -2/3 is necessary QF Peak Acore $\omega = q^2/2M_Y^* + \Delta M + U(Y) - U(N)$ $\partial dE(X)/d\Omega$ U(Y)p π S N **30 MeV** E(Y)**Threshold**



Quasi-Free *S Production*

KEK data of \Sigma- production on nuclear target

H. Noumi, et al., Phys. Rev. Lett. 89 (2002) 072301;
H. Noumi, et al., Phys. Rev. Lett. 90 (2003) 049902, Erratum.

- Naïve analysis suggest Re U ~ 90 MeV
- Green's Function Method
 +OFA t-matrix (DWIA)
 - QF data is consistent with Atom data, but sensitivity is

small.





T. Harada, Y. Hirabayashi / Nuclear Physics A 759 (2005) 143-169

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Ξ hypernuclear formation

- Missing mass spectroscopy BNL E885 ¹²C(K⁻,K⁺) *Fukuda et al. PRC58('98),1306; Khaustov et al. PRC61('00), 054603.*
 - No clear bound states found
- Twin hypernuclear formation *Aoki et al. PLB355('95),45.*
- Potential depth U_± ~ -14 MeV





"Stars" of Hyperon Potentials (A la Michelin)

- Bound State Spectroscopy + Continuum Spectroscopy
 U_Σ(ρ₀) > +15 MeV ²³ ²³ ²³
 - Continuum (Quasi-Free) spectroscopy with Optimal Fermi Averaging t-matrix



- Atomic shift data (attractive at surface) should be respected.
- First example of quark Pauli blocking effects in potential ?
- $U_{\Xi}(\rho_0) \sim -14 \text{ MeV}$
 - No confirmed bound state, No atomic data, High mom. transf., → Small Potential Deps.
 - Continuum low-res. spectrum shape $\rightarrow -14$ MeV
 - Spin-Isospin deps. (π exch.) → Deformation
 → Spectrum shape may be modified.



Strangeness Nuclear Physics

Before Oct.2010,

 $U_{\Lambda}(\rho_0) \sim -30 \text{ MeV}, U_{\Sigma}(\rho_0) > +20 \text{ MeV}, U_{\Xi}(\rho_0) \sim -14 \text{ MeV}$

Harada, Hirabayashi ('05), Noumi et al. ('02), Fukuda et al. PRC58('98),1306; Khaustov et al. PRC61('00), 054603; Aoki et al. PLB355('95),45.

 \rightarrow Maximum mass of NS \sim 1.6 $M_{_{\odot}}$



Ishizuka, AO, Tsubakihara, Sumiyoshi, Yamada ('08)



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Hyperon Puzzle



What did we miss ?

- Hyperon potential in nuclear matter ?
 - $U_{\Lambda}(\rho_0) \sim -30$ MeV, $U_{\Sigma}(\rho_0) > +20$ MeV, $U_{\Xi}(\rho_0) \sim -14$ MeV
- Hyperon-Hyperon potential ?
 - If vacuum ΛΛ potential is much more attractive than Nagara event implies, ΛΛΝ potential must be very repulsive.
- Kaon potential in nuclear matter ?
- Three-baryon (3B) interaction ?
- Quark matter core ?
- Modified gravity ?



Σ or Ξ potential in nuclei ?

- New analysis of Σ production reaction: ⁶Li (π⁻, K⁺) Σ⁻⁵He (Honda, Harada)
 → U_Σ ~ +30 MeV (consistent)
- New Ξ hypernuclei → B.E. = 9 MeV & 1 MeV (Takahashi (A01), Nakazawa, Kanatsuki, Yamamoto) → Deeper than previous estimate !





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T. Harada

<u>Repulsion and absorption of the Σ-</u> <u>nucleus potential</u> <u>in the ⁶Li(π =,K[±]) reactions</u>

Dependence of the calculated average spectra for the ${}^{6}Li(\pi^{\pm}, K^{\pm})$ reaction



$$p_{\pi-}=1.2 \text{ GeV/c}$$
 WS potential

The shape and magnitude of the spectrum are sensitive to the strengths of (V_{Σ}, W_{Σ}) .

$$V_{\Sigma}, W_{\Sigma}) = (+30, -15) \text{ MeV}$$





Remarks

- The optimal Fermi-averaged amplitudes of $f_{\pi-p\to K+\Sigma-}$ in our DWIA calculations are essential to describe the energy and angular dependence of the data of the ⁶Li(π^- , K⁺) reaction at 1.2 GeV/c.
- The calculated spectrum indicates the repulsive and absorptive components of the Σ^{-5} He potential.



The repulsive Σ -nucleus potential for Σ^{-5} He with (V_{Σ} , W_{Σ})= (+30 MeV, -15 MeV) can fully reproduce the data of the ⁶Li(π^{-} , K⁺) reaction at 1.2 GeV/c.

Anti-Kaon potential in Nuclear Matter ?

- K-pp binding energy (Takahashi (A02), Outa, Dote)
 - E15: One state at B.E.~ (15-30) MeV, Strength at B.E. ~ 100 MeV E27: B.E.~100 MeV ?
 - Dote: Higher pole B.E.~ 27 MeV, Lower pole B.E.~ 79 MeV (?) Akaishi: B.E. ~ 100 MeV (DISTO, FINUDA) S.Ohnishi: Saturating B.E. in heavier kaonic nuclei



AA potential ?

■ Nagara fit $\rightarrow a_0(\Lambda\Lambda) = -0.575$ fm or -0.77 fm

Hiyama, Kamimura, Motoba, Yamada, Yamamoto ('02), Filikhin, Gal ('02)

New approach: $\Lambda\Lambda$ correlation from HIC (Morita) \rightarrow -1.25 fm < $a_0(\Lambda\Lambda)$ < 0 (Consistent with Nagara)

Exp: Adamczyk et al. (STAR Collaboration), PRL 114 ('15) 022301. Theor.:Morita et al., T.Furumoto, AO, PRC91('15)024916.





Remaining possibilities

- Three-baryon (3B) interaction ?
 - "Universal" 3B repulsion Nishizaki, Takatsuka, Yamamoto ('02), Tamagaki ('08), Yamamoto, Furumoto, Yasutake, Rijken ('13)
 - Repulsive ANN potential (or density dep. AN pot.) Lonardoni, Lovato, Gandolfi, Pederiva ('15), Togashi, Hiyama, Yamamoto, Takano ('16), Tsubakihara, Harada, AO ('16)
 - Medium modification of baryons (Quark Meson Coupling model) J.Rikovska-Stone, P.A.M.Guichon, H.H.Matevosyan, A.W.Thomas ('07), Miyatsu, Yamamuro, Nakazato ('13)

Quark matter NS core ?

First order phase transition

L. Bonanno, A. Sedrakian, Astron. Astrophys. 539 (2012) A16; M. Bejger, D. Blaschke, P. Haensel, J. L. Zdunik, M. Fortin, arXiv:1608.07049.

- Crossover transition to quark matter Masuda, Hatsuda, Takatsuka ('12)
- Modified Gravity Astashenok et al. ('14), M.-K. Cheoun's talk



Hyperon Puzzle



Lonardoni, Lovato, Gandolfi, Pederiva ('15),



QMC, Miyatsu, Yamamuro, Nakazato ('13)





Yamamoto, Furumoto, Yasutake, Rijken ('13)









- ハドロン 原子核反応を記述する、現時点で「最良」の方法
 - 束縛状態の生成
 → 正確な核構造計算
 (配位混合を取り入れた Shell 模型計算、クラスター計算、....)
 + 歪曲波インパルス近似 (DWIA)
 (spectroscopic factor と光学ポテンシャルによる
 入射波・出射波の歪曲を考慮)
 - 連続状態のスペクトル
 - → 正しい境界条件を与える Green's Function Method を 用いた DWIA
- Hyperon Puzzle
 - $U_{\Lambda}(\rho_0) \sim -30$ MeV, $U_{\Sigma}(\rho_0) > +20$ MeV, $U_{\Xi}(\rho_0) \sim -14$ MeV
 - Hyperon, Kaon の混在を考慮した EOS では、2M_☉を支えられない。
 - 新たなデータも Hyperon Puzzle 解決とならない。
 - 3B repulsion, Quark Matter or Modified gravity

