Three Baryon Interaction in the Quark Cluster Model – 3B Interaction from Determinant Interaction of Quarks as an example –

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Seminar at Hokkaido U., Nov. 30, 2016

AO, K. Kashiwa, K. Morita, arXiv:1610.06306

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- **CV**
 - Born in Kobe ('64), B. Sc. '87, M. Sc. '89, D. Sc. '92 (Kyoto Univ.)
 - JSPS fellow @ RCNP ('92-'93), Hokkaido U., ('93-'08), YITP ('08-)





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- Introduction
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Hyperon Puzzle (or Hyperon Crisis)

Demorest et al., Nature 467 (2010) 1081 (Oct.28, 2010).



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Proposed Solutions

Hyperonic EOS cannot support massive NS (M ~ 2 M_o). Demorest et al. (2010), Antoniadis et al. (2013)

Proposed Solutions

- Hyperonic Three-Body Force (or density dep. coupling) Bednarek et al. ('12), Jiang et al. ('12); Long et al. ('12); Yamamoto et al. ('14); Lonardoni et al. ('15); Tsubakihara et al. ('13), T. Miyatsu et al. ('13), ...
- Crossover Transition to Quark Matter Bonanno et al.('12); Masuda et al. ('13); Bejger et al.('16), ...

Modified Gravity

Astashenok et al. ('14)

Three-nucleon interaction is known to be necessary. How can we determine YNN (+YYN, YYY) potential ?



Massive Neutron Stars with Hyperons



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Baryon-Baryon Force

- **Long-range (r>2 fm):** π exch.
- Intermediate (r ~ 1 fm): multi π exch., boson exch., ...
- Short range (r < 0.6 fm): vector boson exch., Pomeron exch., quark exclusion + one gluon exch.,

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V.G. Neudachin, Yu.F. Smirnov, R. Tamagaki,
PTP 58 ('77) 1072; M. Oka, K. Yazaki,
PTP ('81)572.
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Quark model description of 3B repulsion should be a promising approach !

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Fujiwara, Suzuki, Nakamoto ('07)

Three-Baryon force



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Kobayashi-Maskawa-'t Hooft (KMT) interaction

KMT interaction

Kobayashi, Maskawa ('70), 't Hooft ('76)

- $\mathcal{L}_{=}g_D (\det \Phi + \text{h.c.}), \quad \Phi_{ij} = \bar{q}_j(1 \gamma_5)q_i$
- Determinant interaction in flavor for three quarks (SU(3),)
- Responsible for U(1)_A anomaly η-η' mass diff.
 → g_D= -9.29 Hatsuda, Kunihiro ('94) -12.36 Rehberg, Klevanski, Hufner ('96)
- KMT interaction should generate 2B and 3B interaction when hyperons are involved.
- Repulsive in ΛΛ system
 → Pushes up H particle energy. *Takeuchi, Oka ('91)*





Kobayashi-Maskawa-'t Hooft (KMT) interaction



Does anomaly support massive NS ?



Quark Cluster model

Totally anti-symmetrized wave function of baryons

$$|\Psi\rangle = \mathcal{A}|\chi_{12}B_1B_2\rangle$$
$$|\Psi\rangle = \mathcal{A}|\chi_{123}B_1B_2B_3\rangle$$

Resonating Group Method

$$\int d\mathbf{r}' H(\mathbf{r}, \mathbf{r}') \chi(\mathbf{r}') = E \int d\mathbf{r}' N(\mathbf{r}, \mathbf{r}') \chi(\mathbf{r}')$$

$$\rightarrow -\frac{\hbar^2}{2\mu} \nabla^2 \chi^{(N)} + (V\chi^{(N)}) = E\chi^{(N)} \quad (\chi^{(N)} = \mathcal{N}^{1/2}\chi)$$

$$H(\mathbf{r}, \mathbf{r}') = \langle \mathbf{r} B_1 B_2 \dots | H | \mathcal{A}(\mathbf{r}' B_1 B_2 \dots) \rangle$$

$$N(\mathbf{r}, \mathbf{r}') = \langle \mathbf{r} B_1 B_2 \dots | \mathcal{A}(\mathbf{r}' B_1 B_2 \dots) \rangle$$

• When (wave length of χ) >> (baryon size), $V(\mathbf{r}) \simeq \Delta K + \langle V \mathcal{A} \rangle / \langle \mathcal{A} \rangle$



Norm Kernel

Antisymmetrizer makes the calculation complicated !

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Norm Kernel

- Single baryon w.f. $|\psi_{\mathcal{A}}\rangle = \mathcal{A}/\sqrt{3!} \times \varepsilon_{abc}/\sqrt{3!} \times [|\operatorname{Flavor}\rangle \otimes |\operatorname{Spin}\rangle \otimes |\operatorname{Spatial w.f.}\rangle]^{abc}$.
- Two barvon w.f.

$$|\psi_{\mathcal{A}}(n_{\uparrow}, n_{\downarrow})\rangle = \frac{1}{\sqrt{6!}} |\mathcal{A}[\psi(n_{\uparrow})\psi(n_{\downarrow})]\rangle$$

Norm

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$$\begin{split} \mathcal{N}_{\mathcal{A}} = & \langle \psi_{\mathcal{A}}(n_{\uparrow}, n_{\downarrow}) \mid \psi_{\mathcal{A}}(n_{\uparrow}, n_{\downarrow}) \rangle \\ = & \langle \psi(n_{\uparrow})\psi(n_{\downarrow}) \mid \mathcal{A}[\psi(n_{\uparrow})\psi(n_{\downarrow})] \rangle \\ = & \frac{1}{(3!)^2} \sum_{i,j,k,l} c_i^*(n_{\uparrow})c_j^*(n_{\downarrow})c_k(n_{\uparrow})c_l(n_{\downarrow}) \varepsilon_{abc} \varepsilon_{def} \varepsilon_{a'b'c'} \varepsilon_{d'e'f'} \\ & \times \langle \phi_i^{abc}(n_{\uparrow})\phi_j^{def}(n_{\downarrow}) \mid \mathcal{A}[\phi_k^{a'b'c'}(n_{\uparrow})\phi_l^{d'e'f'}(n_{\downarrow})] \rangle \\ = & \sum_{i,j,k,l} c_i^*(n_{\uparrow})c_j^*(n_{\downarrow})c_k(n_{\uparrow})c_l(n_{\downarrow}) \sum_P C_P(\phi_i\phi_j,\phi_k\phi_l) F_P(\phi_i\phi_j,\phi_k\phi_l) \end{split}$$

Anti-symmetrization

$$\mathcal{A}[1^{a}1^{b}1^{c}2^{d}2^{e}2^{f}] = 1^{a}1^{b}1^{c}2^{d}2^{e}2^{f} - 1^{a}1^{b}2^{d}1^{c}2^{e}2^{f} + 1^{a}2^{e}2^{d}1^{c}1^{b}2^{f} + \cdots$$

$$C_{P} = -\frac{1}{(3!)^{2}}\varepsilon_{abd}\varepsilon_{cef}\varepsilon_{abc}\varepsilon_{def} = -\frac{1}{36}2\delta_{dc}2\delta_{cd} = -\frac{1}{3}$$

$$F_{P}(\phi_{i}\phi_{j},\phi_{k}\phi_{l}) = \langle\phi_{i}(n_{\uparrow})\phi_{j}(n_{\downarrow}) \mid P[\phi_{k}(n_{\uparrow})\phi_{l}(n_{\downarrow})]\rangle_{\text{fss}} = 0 \text{ or } 1 \checkmark \text{Numerical}$$

$$\mathsf{Numerical}$$

B	$ $ Flavor \rangle	$ $ Spin \rangle
n_\uparrow	$ ddu \rangle / \sqrt{2}$	$ \uparrow\downarrow\uparrow+\downarrow\uparrow\uparrow-2\uparrow\uparrow\downarrow\rangle/\sqrt{6}$
p_\uparrow	$ uud\rangle/\sqrt{2}$	$ \uparrow\downarrow\uparrow+\downarrow\uparrow\uparrow-2\uparrow\uparrow\downarrow\rangle/\sqrt{6}$
Λ_{\uparrow}	$ uds \rangle$	$ \uparrow\downarrow\uparrow-\downarrow\uparrow\uparrow angle/\sqrt{2}$
Σ^{-}_{\uparrow}	$ dds \rangle / \sqrt{2}$	$ \uparrow\downarrow\uparrow+\downarrow\uparrow\uparrow-2\uparrow\uparrow\downarrow\rangle/\sqrt{6}$
$\Sigma^{\dot{0}}_{\uparrow}$	$ uds \rangle$	$ \uparrow\downarrow\uparrow+\downarrow\uparrow\uparrow-2\uparrow\uparrow\downarrow\rangle/\sqrt{6}$
Σ^+_\uparrow	$ uus\rangle/\sqrt{2}$	$ \uparrow\downarrow\uparrow+\downarrow\uparrow\uparrow-2\uparrow\uparrow\downarrow\rangle/\sqrt{6}$
Ξ↑	$ ssd\rangle/\sqrt{2}$	$ \uparrow\downarrow\uparrow+\downarrow\uparrow\uparrow-2\uparrow\uparrow\downarrow\rangle/\sqrt{6}$
Ξî	$ ssu\rangle/\sqrt{2}$	$ \uparrow\downarrow\uparrow+\downarrow\uparrow\uparrow-2\uparrow\uparrow\downarrow\rangle/\sqrt{6}$

Norm Kernel

					_	
Baryon(s)	$\mathcal{N}_{\mathcal{A}}$	$\mathcal{T}_{\mathcal{A}}$	\mathcal{T}	$\mathcal{T}_{nB}(n=2,3)$	-	
$(NN)_{(S,T)=(0,1),(1,0)}$	10/9	0	0	0	-	
$N_{\uparrow}\Lambda_{\uparrow}, N_{\downarrow}\Lambda_{\downarrow}$	1	20/3	20/3	20/3		
$N_{\uparrow}\Lambda_{\downarrow}, N_{\downarrow}\Lambda_{\uparrow}$	1	10/3	10/3	10/3		
$(\Lambda\Lambda)_{S=0}$	1	18/3	18/3	18/3		
$(NNN)_{(S,T)=(1/2,1/2)}$	100/81	0	0	0	-	
$n_{\uparrow}n_{\downarrow}\Lambda, p_{\uparrow}p_{\downarrow}\Lambda$	25/27	350/27	14	12/3		
$n_{\uparrow}p_{\uparrow}\Lambda_{\uparrow}, n_{\downarrow}p_{\downarrow}\Lambda_{\downarrow}$	25/27	750/27	30	50/3		
$n_{\uparrow}p_{\uparrow}\Lambda_{\downarrow}, n_{\downarrow}p_{\downarrow}\Lambda_{\uparrow}$	25/27	250/27	10	10/3		
$n_{\uparrow}p_{\downarrow}\Lambda, n_{\downarrow}p_{\uparrow}\Lambda$	25/27	425/27	17	21/3		
$N\Lambda_{\uparrow}\Lambda_{\downarrow}$	45/54	1035/54	23	21/3	_	
				1.4	- S=0, -1, -2, -3, -4	
	1.2	2B S=0, -1,				
				1		
Nat				Ēns		
INOL	very s	sman		N O.		
	- 0.8					
				0.4	-	
	0.2					
	0	4				
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Star Matter Channel AO, Kashiwa, Morita ('16)

-3,

3B

-4,

-5, -6

-2,

KMT matrix element

Reduction of KMT interaction to 3 quark pot.

$$\begin{split} V_{\text{KMT}} \simeq &- 2g_{\text{D}} \int d^3 x \, \varepsilon_{ijk} \, u^{\dagger}(\boldsymbol{x}) q_i(\boldsymbol{x}) \, d^{\dagger}(\boldsymbol{x}) q_j(\boldsymbol{x}) \, s^{\dagger}(\boldsymbol{x}) q_k(\boldsymbol{x}) \\ &= - 2g_{\text{D}} \varepsilon_{ijk} \sum_{\{\alpha,\beta,\gamma\}} \hat{T}_{\alpha}^{u,i} \, \hat{T}_{\beta}^{d,j} \, \hat{T}_{\gamma}^{s,k} \delta(\boldsymbol{x}_{\alpha} - \boldsymbol{x}_{\beta}) \delta(\boldsymbol{x}_{\beta} - \boldsymbol{x}_{\gamma}) \end{split}$$

Flavor exchanging operator

$$\begin{aligned} \hat{\mathcal{T}}^{\text{KMT}} &= \sum_{\{\alpha,\beta,\gamma\}} \varepsilon_{ijk} \, \hat{T}^{u,i}_{\alpha} \, \hat{T}^{d,j}_{\beta} \, \hat{T}^{s,k}_{\gamma} \\ \mathcal{T}_{\mathcal{A}} \equiv & \langle \psi_{\mathcal{A}} \mid \hat{\mathcal{T}}^{\text{KMT}} \mid \psi_{\mathcal{A}} \rangle \qquad \mathcal{T} = \mathcal{T}_{\mathcal{A}} / \mathcal{N}_{\mathcal{A}} \end{aligned}$$

Subtract the two-body part

$$\mathcal{T}_{3B}(n_{\uparrow}n_{\downarrow}\Lambda_{\uparrow}) = \mathcal{T}(n_{\uparrow}n_{\downarrow}\Lambda_{\uparrow}) - \mathcal{T}(n_{\uparrow}\Lambda_{\uparrow}) - \mathcal{T}(n_{\downarrow}\Lambda_{\uparrow})$$



KMT matrix element

$$\begin{split} \langle \phi \mid V_{\text{KMT}} \mid \phi' \rangle &= \sum_{\{\alpha,\beta,\gamma\}} \langle q_{\alpha}q_{\beta}q_{\gamma} \mid V_{\text{KMT}} \mid q'_{\alpha}q'_{\beta}q'_{\gamma} \rangle \prod_{i \neq \{\alpha,\beta,\gamma\}} \langle q_i \mid q'_i \rangle \text{ irrelevant quarks} \\ \textbf{product w.f.} &= -2g_{\text{D}} \langle \sigma \mid \sigma' \rangle \sum_{\{\alpha,\beta,\gamma\}} F_{\alpha\beta\gamma}^{\text{KMT}}(f,f') R_{\alpha\beta\gamma}^{\text{KMT}}(\varphi,\varphi') , \\ \langle \sigma \mid \sigma' \rangle &= \prod_{\alpha} \langle \sigma_{\alpha} \mid \sigma'_{\alpha} \rangle , \text{ flavor matching } \\ \langle \sigma \mid \sigma' \rangle &= \langle f \mid \varepsilon_{ijk} \hat{T}_{\alpha}^{u,i} \hat{T}_{\beta}^{d,j} \hat{T}_{\gamma}^{s,k} \mid f' \rangle \text{ (numerical)} \\ &= \delta_{u,f_{\alpha}} \delta_{d,f_{\beta}} \delta_{s,f_{\gamma}} \sum_{ijk} \varepsilon_{ijk} \underbrace{\delta_{i,f'_{\alpha}} \delta_{j,f'_{\beta}} \delta_{k,f'_{\gamma}}}_{\mu \neq \{\alpha,\beta\gamma\}} \prod_{\mu \neq \alpha,\beta\gamma} \delta_{f_{\mu},f'_{\mu}} , \\ R_{\alpha\beta\gamma}^{\text{KMT}}(\varphi,\varphi') &= \langle \varphi \mid \delta(x_{\alpha} - x_{\beta}) \delta(x_{\beta} - x_{\gamma}) \mid \varphi' \rangle \\ &= \int d^{3}x \varphi_{\alpha}^{*}(x) \varphi_{\beta}^{*}(x) \varphi_{\gamma}^{*}(x) \varphi'_{\alpha}(x) \varphi'_{\beta}(x) \varphi'_{\gamma}(x) \prod_{\mu \neq \alpha,\beta,\gamma} \langle \varphi_{\mu} \mid \varphi'_{\mu} \rangle . \end{split}$$



KMT matrix element

Baryon(s)	$\mathcal{N}_\mathcal{A}$	$\mathcal{T}_{\mathcal{A}}$	${\mathcal T}$	$\mathcal{T}_{nB}(n=2,3)$				
$(NN)_{(S,T)=(0,1),(1,0)}$	10/9	0	0	0				
$N_{\uparrow}\Lambda_{\uparrow}, N_{\downarrow}\Lambda_{\downarrow}$	1	20/3	20/3	20/3				
$N_{\uparrow}\Lambda_{\downarrow}, N_{\downarrow}\Lambda_{\uparrow}$	1	10/3	10/3	10/3	/	Big for	· npΛ	
$(\Lambda\Lambda)_{S=0}$	1	18/3	18/3	18/3		(S-2/2))	
$(NNN)_{(S,T)=(1/2,1/2)}$	100/81	0	0	0		(3-3/2)	
$n_{\uparrow}n_{\downarrow}\Lambda, p_{\uparrow}p_{\downarrow}\Lambda$	25/27	350/27	14	12/3				
$n_{\uparrow}p_{\uparrow}\Lambda_{\uparrow}, n_{\downarrow}p_{\downarrow}\Lambda_{\downarrow}$ 25/27 750/27		750/27	30	50/3				
$n_{\uparrow}p_{\uparrow}\Lambda_{\downarrow}, n_{\downarrow}p_{\downarrow}\Lambda_{\uparrow}$	25/27	250/27	10	10/3				
$n_{\uparrow}p_{\downarrow}\Lambda, n_{\downarrow}p_{\uparrow}\Lambda$	25/27	425/27	17	21/3				
$N\Lambda_{\uparrow}\Lambda_{\downarrow}$	45/54	1035/54	23	21/3	=0, -1, -2	2, -3,	-4,	-5, -6
						3B		
KMT matrix elements strongly depend on the channel			T _{1B,2B,3B}	20 2B 15 10 5 1B 18 18 18 18 18 10 10 10 10 10 10 10 10 10 10				
			Channel					



3B potential from KMT interaction

■ 3q int. → 3B potential

$$V_{3B}^{KMT} = -2 g_D T_{3B} \int d^3 x \varphi_{R_a}^*(x) \varphi_{R_b}^*(x) \varphi_{R_c}^*(x) \varphi_{R_d}(x) \varphi_{R_e}(x) \varphi_{R_f}(x)$$

$$V_{3B}^{KMT}(R_1, R_2, R_3) \simeq V_0 T_{3B} \exp \left[-\frac{2\nu}{3} (R_{12}^2 + R_{23}^2 + R_{31}^2) \right]$$

$$V_0 \equiv \frac{-2g_D}{(\sqrt{3}\pi b^2)^3} = \frac{-2g_D \Lambda^5}{(\sqrt{3}\pi b^2 \Lambda^2)^3} \Lambda = \begin{cases} 1.45 \text{ MeV} & (b = 0.6 \text{ fm}) \\ 2.29 \text{ MeV} & (b = 0.5562 \text{ fm}) \end{cases}.$$

Parameters are taken from *Hatsuda, Kunihiro ('94), Rehberg, Klevanski, Hufner ('96), Fujiwara, Suzuki, Nakamoto ('07), Oka, Yazaki ('81)*



3B potential from KMT interaction

KMT 3B Potential



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Lattice data: Doi et al. (HAL QCD) ('07)

KMT-3B Contribution to A potential



Density is assumed to be uniform. No correlation effects.

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3B potential from KMT: Repulsive enough ?



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Summary

- Quark model three-baryon (3B) potential may be a promising method to evaluate the 3B potential at short distances.
- Kobayashi-Maskawa-'t Hooft (KMT) interaction generates 3q potential among u,d,s quarks, and generates 3B potential only when hyperons are involved.
- Expectation value of the KMT interaction is evaluated in the cases where 3B are located at the same spatial point. Matrix elements strongly depend on the baryon trio.
- **3B** potential from KMT interaction is obtained.
 - It is comparable in strength to the lattice 3N potential.
 - More repulsive in npΛ than in nnΛ (Negative contribution to symmetry energy.)
- 3B pot. from KMT is not strong enough to solve the hyperon puzzle, but contributes to hyperon suppression.



Three-Baryon force



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Confinement Potential → *3B Potential* ?



Takahashi, Suganuma, Nemoto, Matsufuru ('02)



Thank you for your attention !



Massive Neutron Stars with Hyperons

Tsubakihara, Harada, AO, arXiv:1402.0979

- Ruled-out EOS with hyperons = GM3 Glendenning & Moszkowski (1991)
- \blacksquare We did NOTHING special and find 2 M_{\odot} NS can be supported.
 - "Typical" RMF for nucl. matter NL1, NL-SH, TM1 Reinhardt et al. ('86); Sharma, Nagarajan, Ring ('93); Sugahara, Toki ('94).
 - ss mesons are introduced
 - Hypernuclear data

 Λ, ΛΛ hypernuclei
 Σ atomic shifts
 SU(3) relation to isoscalar
 -vector couplings





What is necessary to solve the massive NS puzzle ?

There are many "model" solutions.

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- Ab initio calculation including three-baryon force (3BF)
 - Bare 2NF+Phen. 3NF(UIX, IL2-7) + many-body theory (verified in light nuclei).
 - Chiral EFT (2NF+3NF) + many-body theory



J. Carlson et al. ('14)

3BF including Hyperons

3BF incl. YNN, YYN and YYY should exist and contribute to EOS.

Nishizaki, Takatsuka, Yamamoto ('02)

- Chiral EFT, Multi-Pomeron exch., Quark Pauli, Lattice 3BF, SJ, .. Kohno('10); Heidenbauer+('13); Yamamoto+('14); Nakamoto, Suzuki; Doi+(HALQCD,'12); Tamagaki('08); ...
- Quant. MC study Lonardoni et al.('14)
- Quark Meson Coupling Miyatsu et al.; Thomas (HHIQCD)
- AAN K. Morita, T. Furumoto, AO, PRC91('15)024916

Caveat: Missing data

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Fitting "Ab initio" EOS via RMF





Symmetry Energy





Neutron Star Matter EOS

Neutron Star Matter EOS Lressure (MeV/fm³) 0² 10¹ 10 APR-fit DBHF-fit P (MeV/fm³) FP-fit NL1 HS(TMA) NL3 TM1 SFHx TM1 SFHo LS(220) 10⁰ SCL LS(180) HS(FSUGold 10² 10³ 10^{3} ϵ (MeV/fm³) Energy Density (MeV/fm³) A. W. Steiner, M. Hempel, T. Fischer, ApJ 774 (2013) 17 A.Ohnishi @ Hokkaido U., Nov. 11, 2016 32 utro Star Matter KAWA INSTITUTE FOR

NS matter in "ab initio"-fit + *A*

A potential in nuclear matter at $\rho_0 \sim -30$ MeV

- Scheme 1: $U_{\Lambda}(\rho) = \alpha U_{N}(\rho)$
- Scheme 2: $U_{\Lambda}(\rho) = 2/3 U^{n=2}N(\rho) + \beta U^{n>2}N(\rho)$





M-R curve of Neutron Stars



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