

核物質の状態方程式

質量公式と状態方程式

- $A \rightarrow \infty$ における核子あたりのエネルギー
(クーロンエネルギーは無視)

$$E = \lim_{A \rightarrow \infty} \frac{-B(A, Z)}{A} = \lim_{A \rightarrow \infty} \left[-a_v + a_s A^{-1/3} + a_a \frac{(N - Z)^2}{A^2} - a_p \frac{\delta_p}{A^{\gamma-1}} \right]$$
$$= -a_v + a_a \delta^2$$

- 密度と非対称度の関数と考えると、
核子あたりのエネルギーが最小となる密度が実現する

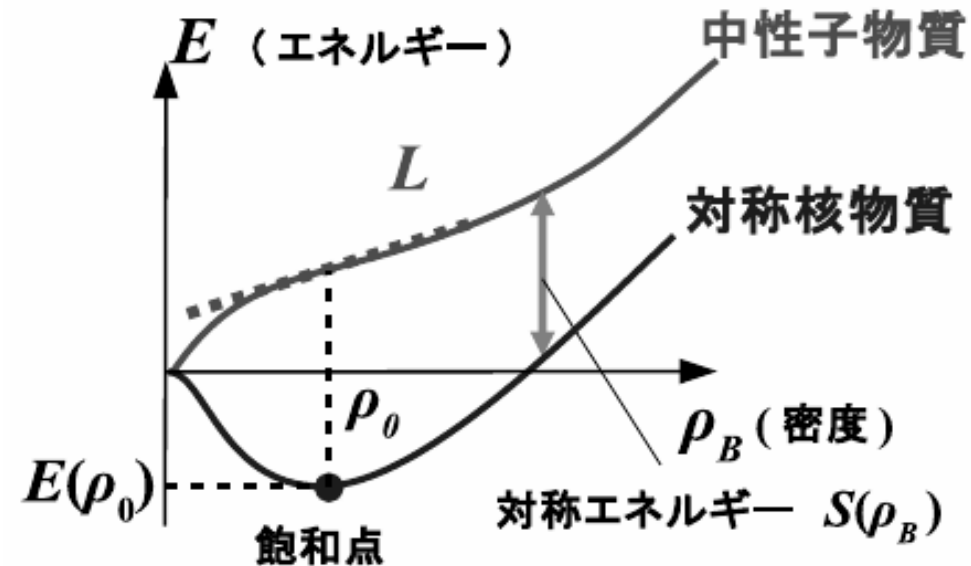
$$E = E(\rho_B, \delta)$$

→ 核物質の飽和性

- 飽和点

$$(\rho_0, E_0) \simeq (0.16 \text{ fm}^{-3}, -16 \text{ MeV})$$

状態方程式 (EOS)



対称エネルギー

■ 非対称核物質 ($N \neq Z$) のエネルギー

$$E(\rho_B, \delta) = E(\rho_B, \delta = 0) + S(\rho_B)\delta^2$$

■ 対称エネルギー

$$S(\rho_B) = E(\text{中性子物質}) - E(\text{対称核物質})$$

■ 飽和密度でのパラメータ

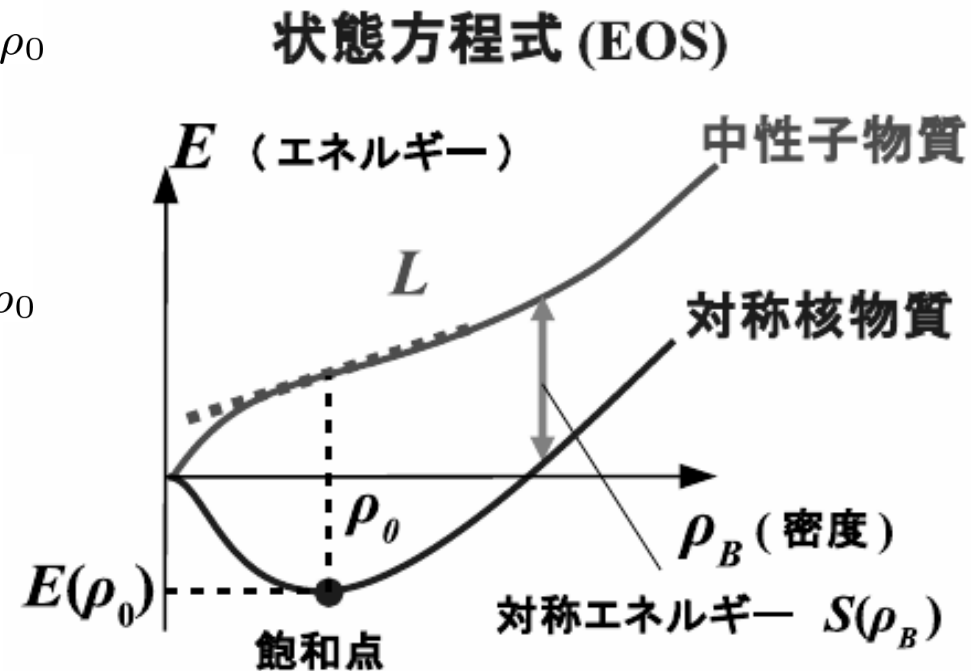
- 非圧縮率 $K \equiv 9\rho_0^2 \left. \frac{\partial^2 E(\rho_B)}{\partial \rho_B^2} \right|_{\rho_B=\rho_0}$

- 対称エネルギーの値と微分

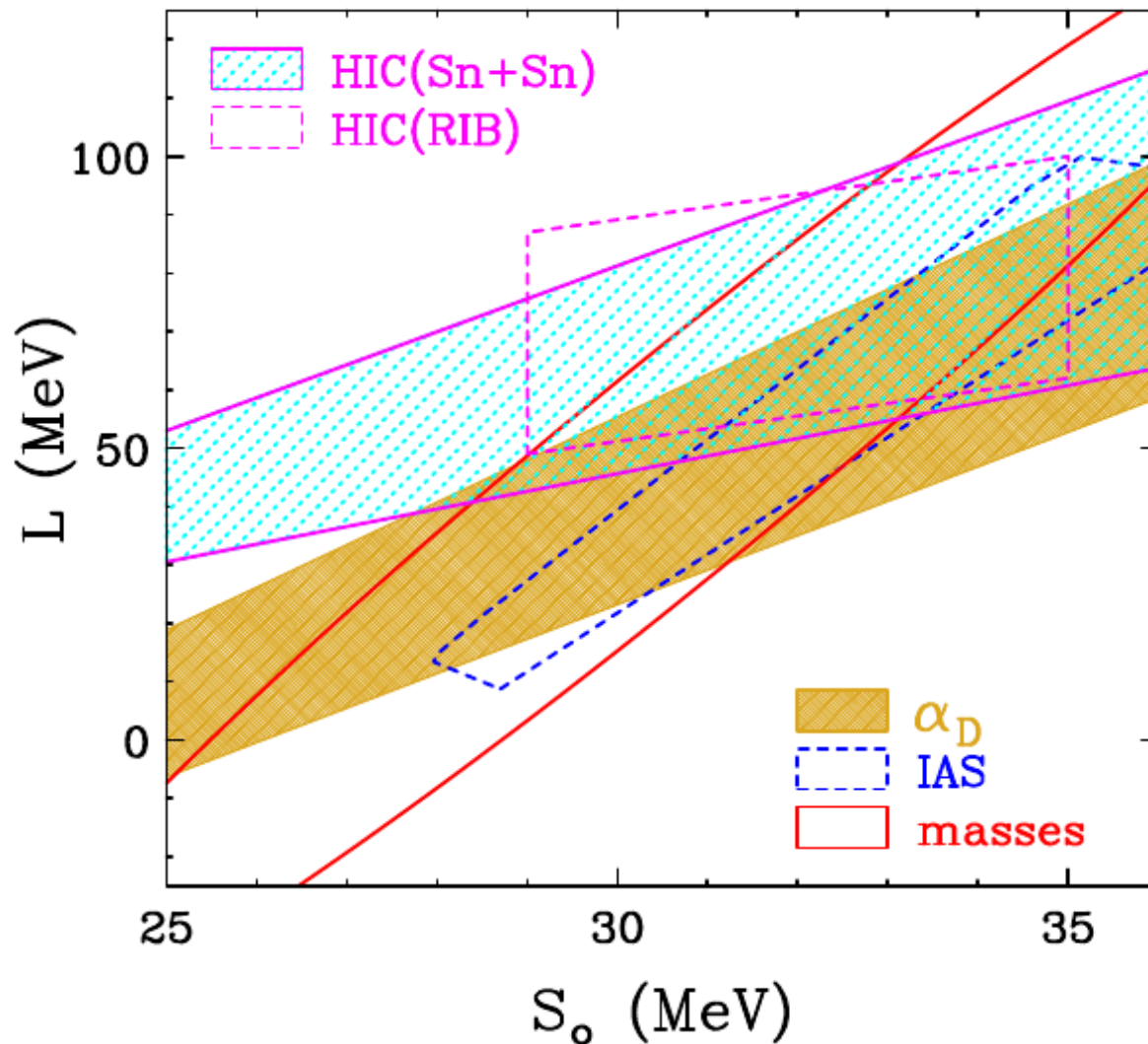
$$S_0 \equiv S(\rho_0), \quad L \equiv 3\rho_0 \left. \frac{dS(\rho_B)}{d\rho_B} \right|_{\rho_B=\rho_0}$$

$$E(\rho_B, \delta) \simeq E_0 + S_0 \delta^2 + \frac{L}{3} x \delta^2 + \frac{K}{18} x^2$$

$$(x = (\rho_B - \rho_0)/\rho_0)$$



対称エネルギーの実験的制限



$S_0 = (30-35) \text{ MeV}$
 $L = (50-90) \text{ MeV}$

様々な実験により対称エネルギーパラメータを制限！

C.J. Horowitz, E.F. Brown, Y. Kim, W.G. Lynch, R. Michaels, A. Ono, J. Piekarewicz, M.B. Tsang, H.H. Wolter, J. Phys. G 41 (2014) 093001.

A. Ohnishi

現象論的な核物質状態方程式

■ 相互作用エネルギー

$$V_{2B} = \frac{1}{2} \int d^3r d^3r' \rho_B(\mathbf{r}) v(\mathbf{r} - \mathbf{r}') \rho_B(\mathbf{r}') \rightarrow A \times \frac{\alpha}{2} \left(\frac{\rho_B}{\rho_0} \right)$$

$$V_{3B} = \frac{1}{3} \int d^3r d^3r' d^3r'' v(\mathbf{r}, \mathbf{r}', \mathbf{r}'') \rho_B(\mathbf{r}) \rho_B(\mathbf{r}') \rho_B(\mathbf{r}'') \rightarrow A \times \frac{\beta}{3} \left(\frac{\rho_B}{\rho_0} \right)^2$$

(一様密度、ゼロレンジの2体力・3体力)

■ 現象論的な状態方程式

● 対称核物質

$$E(\rho_B) = \frac{3}{5} E_F(\rho_B) + \frac{\alpha}{2} \left(\frac{\rho_B}{\rho_0} \right) + \frac{\beta}{2 + \gamma} \left(\frac{\rho_B}{\rho_0} \right)^{1+\gamma}$$

● 対称エネルギー

$$S(\rho_B) = \frac{1}{3} E_F(\rho_B) + \alpha_{\text{sym}} \left(\frac{\rho_B}{\rho_0} \right) + \beta_{\text{sym}} \left(\frac{\rho_B}{\rho_0} \right)^{\gamma_{\text{sym}}}$$

Simple parametrized EOS

■ Skyrme int. motivated parameterization

$$E_{\text{SNM}} = \frac{3}{5} E_F(\rho) + \frac{\alpha}{2} \left(\frac{\rho}{\rho_0} \right) + \frac{\beta}{2 + \gamma} \left(\frac{\rho}{\rho_0} \right)^{1+\gamma}$$

$$\alpha = \frac{2}{\gamma} \left(E_0(1 + \gamma) - \frac{E_F(\rho_0)(1 + 3\gamma)}{5} \right), \quad \beta = \frac{2 + \gamma}{\gamma} \left[-E_0 + \frac{1}{5} E_F(\rho_0) \right].$$

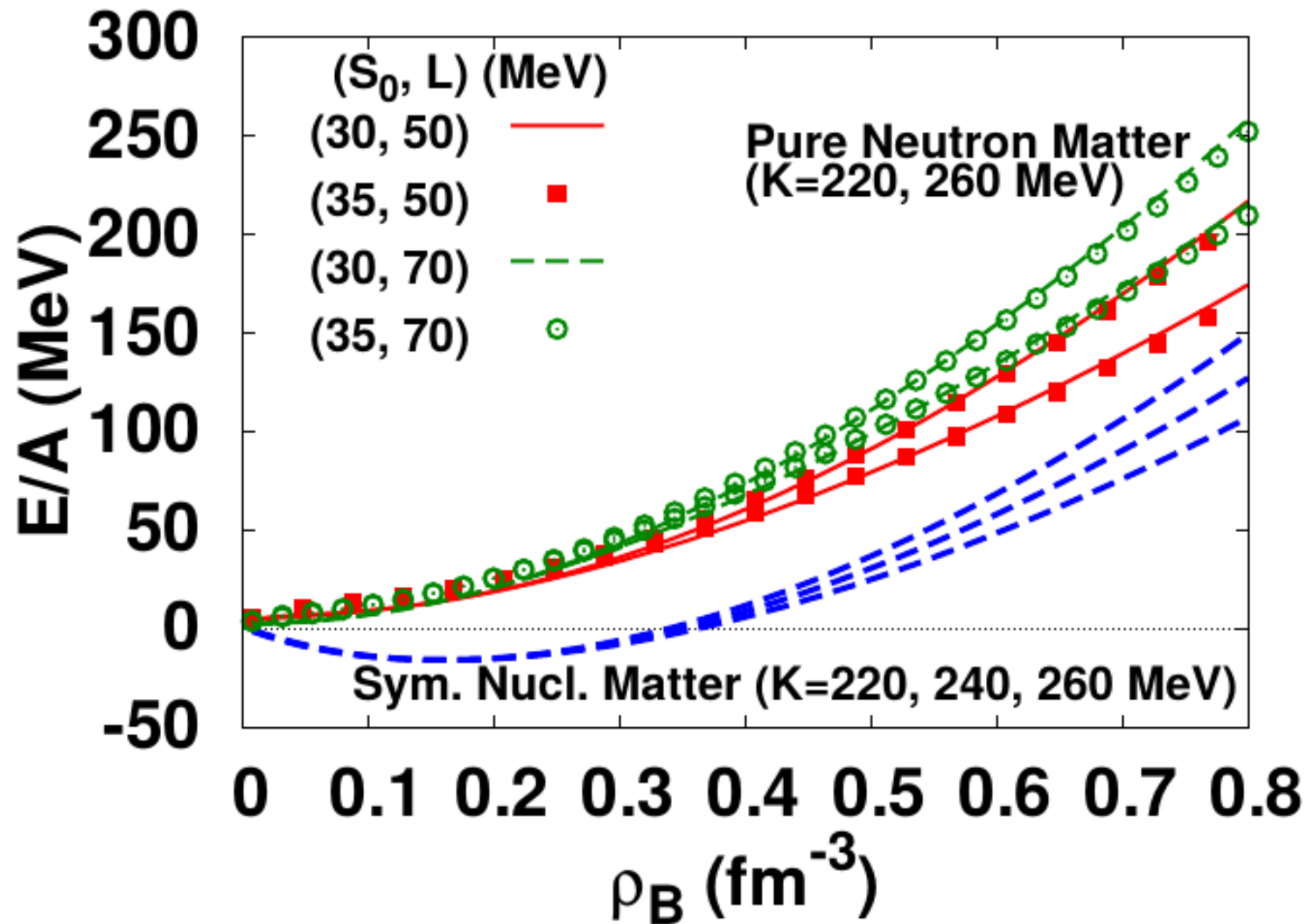
$$K = \frac{3(1 + 3\gamma)}{5} E_F(\rho_0) - 9E_0(1 + \gamma).$$

■ Symmetry energy parameterization

$$S(\rho) = \frac{1}{3} E_F(\rho) + \left[S_0 - \frac{1}{3} E_F(\rho_0) \right] \left(\frac{\rho}{\rho_0} \right)^{\gamma_{\text{sym}}}$$

$$\gamma_{\text{sym}} = \frac{L - \frac{2}{3} E_F(\rho_0)}{3S_0 - E_F(\rho_0)}$$

現象論的な核物質状態方程式

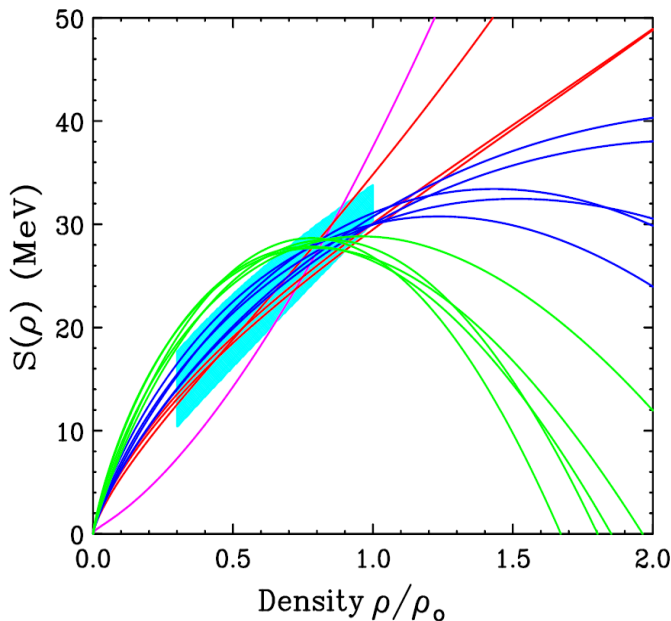


飽和密度近辺での不定性は少ないが、
中性子物質・高密度では大きな不定性

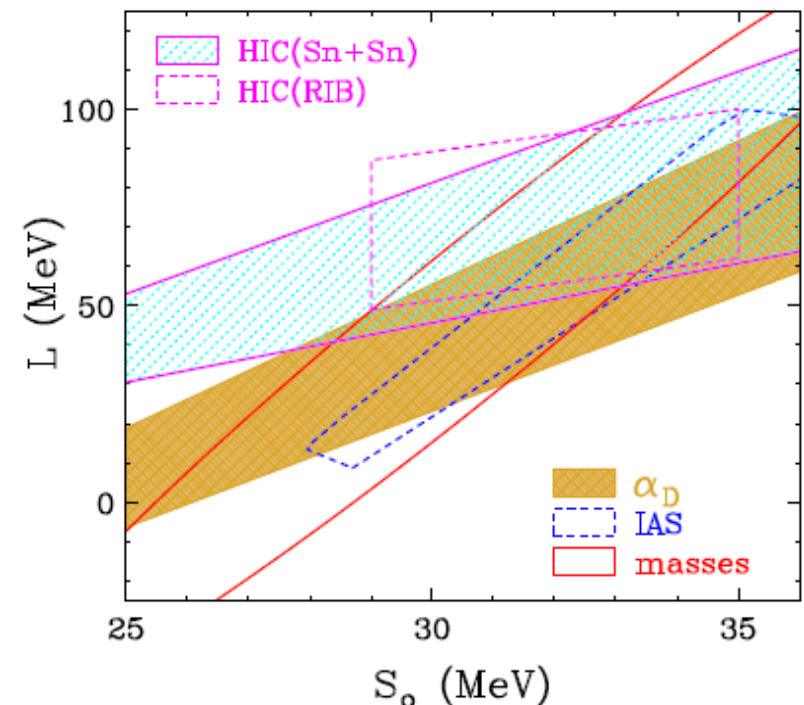
Symmetry Energy

- Symmetry Energy has been extracted from various observations.
 - Mass formula, Isobaric Analog State, Pygmy Dipole Resonance, Isospin Diffusion, Neutron Skin thickness, Dipole Polarizability, Asteroseismology

Recent recommended value
 $S_0 = 30\text{-}35 \text{ MeV}$, $L = 40\text{-}90 \text{ MeV}$
Is it enough for NS radii ?



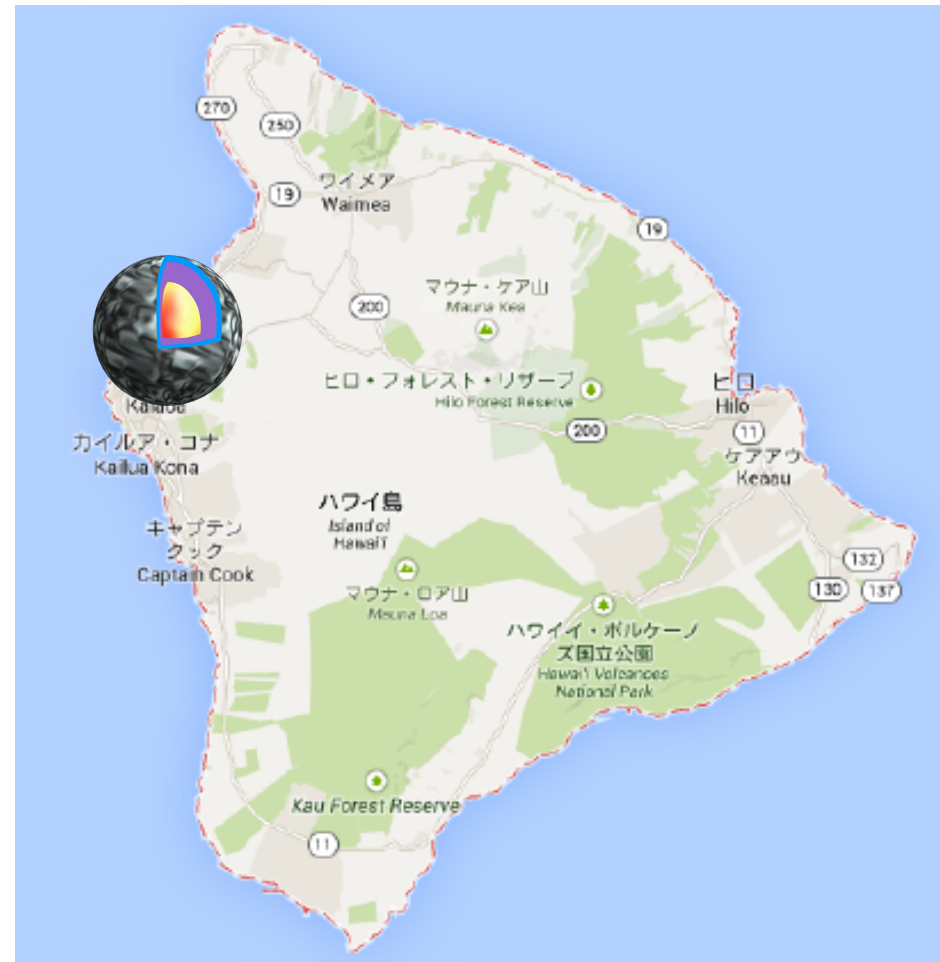
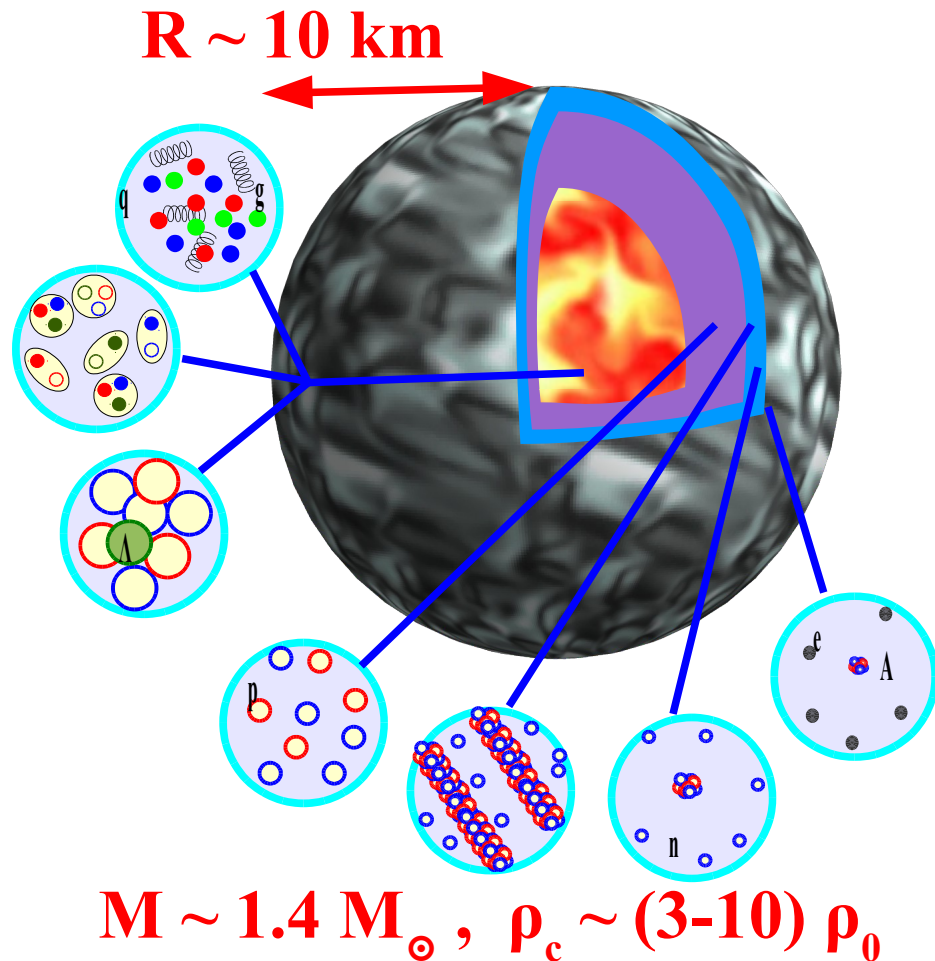
M.B. Tsang et al.
(NuSYM2011),
PRC 86 ('12)015803.



C.J. Horowitz, E.F. Brown, Y. Kim,
W.G. Lynch, R. Michaels, A. Ono, J.
Piekarewicz, M. B. Tsang, H.H. Wolter
(NuSYM13), JPG41('14) 093001

Neutron Star

Star supported by nuclear force



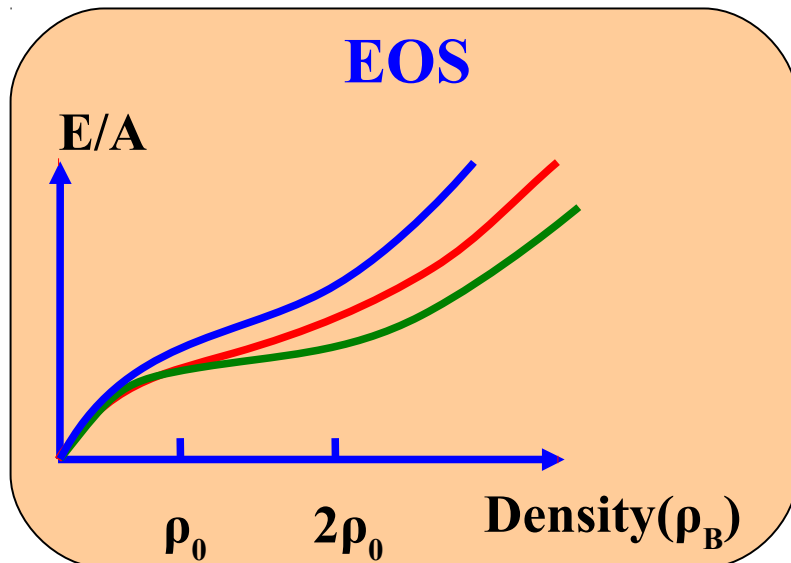
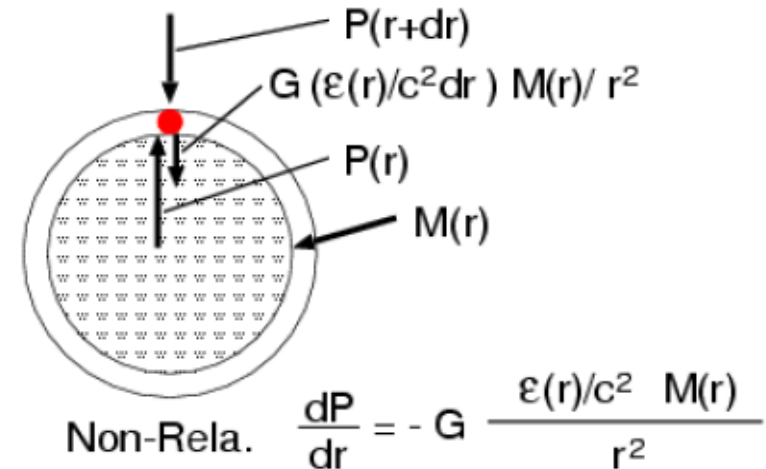
Wide density range $\rightarrow$ various constituents
NS = high-energy astrophysical objects
and laboratories of dense matter.

M-R curve and EOS

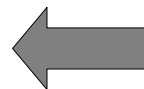
- M-R curve and NS matter EOS has 1 to 1 correspondence
 - TOV(Tolman-Oppenheimer-Volkoff) equation
=GR Hydrostatic Eq.

$$\frac{dP}{dr} = -G \frac{(\varepsilon/c^2 + P/c^2)(M + 4\pi r^3 P/c^2)}{r^2(1 - 2GM/rc^2)}$$

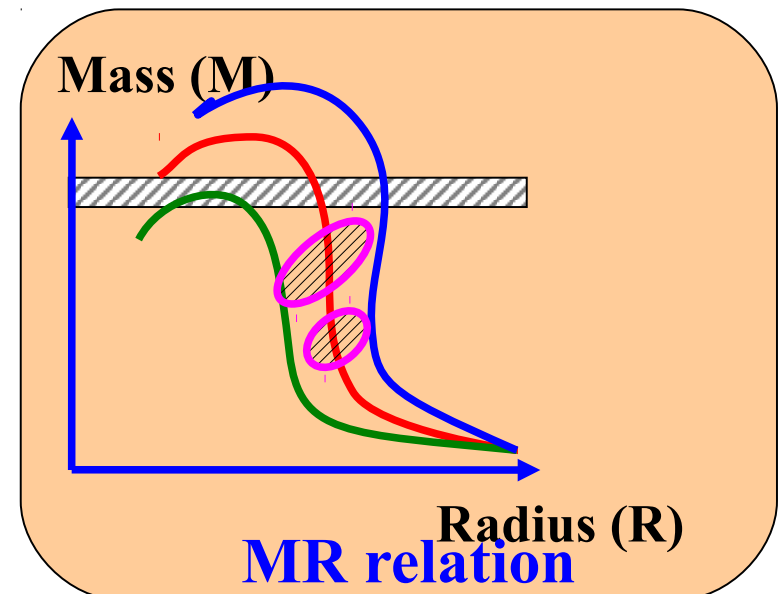
$$\frac{dM}{dr} = 4\pi r^2 \varepsilon/c^2, \quad P = P(\varepsilon) \quad (\text{EOS})$$



prediction



Judge



Neutron Star Matter EOS

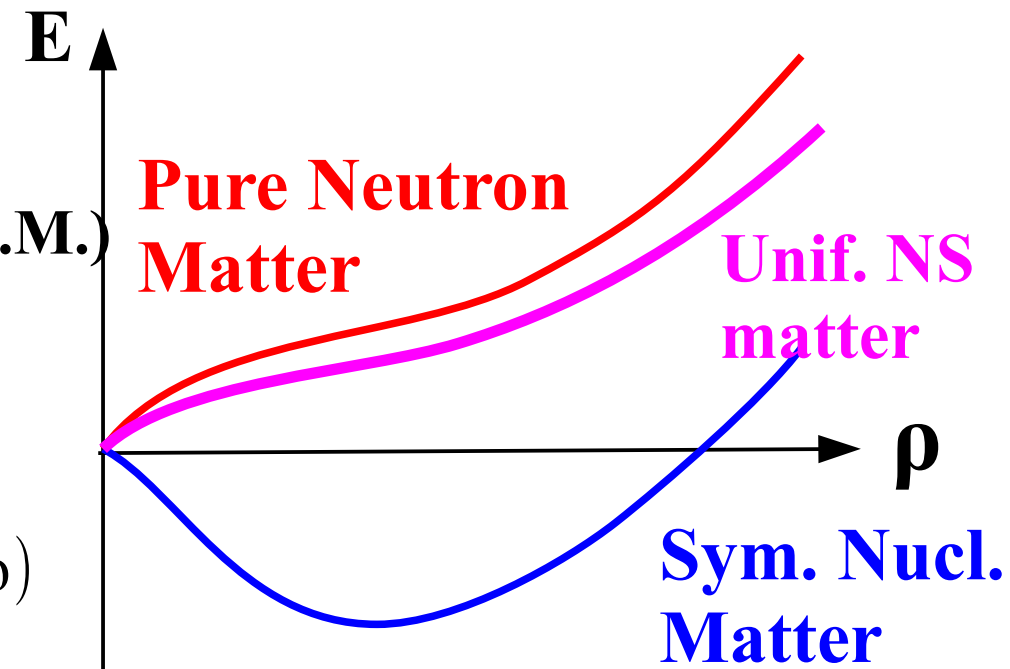
- What happens in low-density uniform neutron star matter ?
 - Constituents = proton, neutron and electron
 - Charge neutrality $\rightarrow$ # of electrons = # of protons ($\rho_e = \rho_p = \rho(1 - \delta)/2$)

$$\begin{aligned}
 E_{\text{NSM}}(\rho) &= E_{\text{NM}}(\rho, \delta) + E_e(\rho_e = \rho_p) \\
 &= E_{\text{SNM}}(\rho) + S(\rho)\delta^2 + \frac{\Delta M}{2}\delta + \frac{3}{8}\hbar k_F(1 - \delta)^{4/3}
 \end{aligned}$$

(electron mass neglected,
neutron-proton mass diff. incl.
 k_F = Fermi wave num. in Sym. N.M.)

- δ is optimized to minimize energy per nucleon

$$E_{\text{NSM}}(\rho) \leq E_{\text{NM}}(\rho, \delta = 1) = E_{\text{PNM}}(\rho)$$



状態方程式を記述する理論の枠組み

Theories/Models for Nuclear Matter EOS

■ Ab initio Approaches

- LQCD, GFMC, Variational, BHF, DBHF, G-matrix, ...

■ Mean Field from Effective Interactions ~ Nuclear Density Functionals

● Skyrme Hartree-Fock(-Bogoliubov)

- ◆ Non.-Rel., Zero Range, Two-body + Three-body (or ρ -dep. two-body)
- ◆ In HFB, Nuclear Mass is very well explained (Total B.E. $\Delta E \sim 0.6$ MeV)
- ◆ Causality is violated at very high densities.

● Relativistic Mean Field

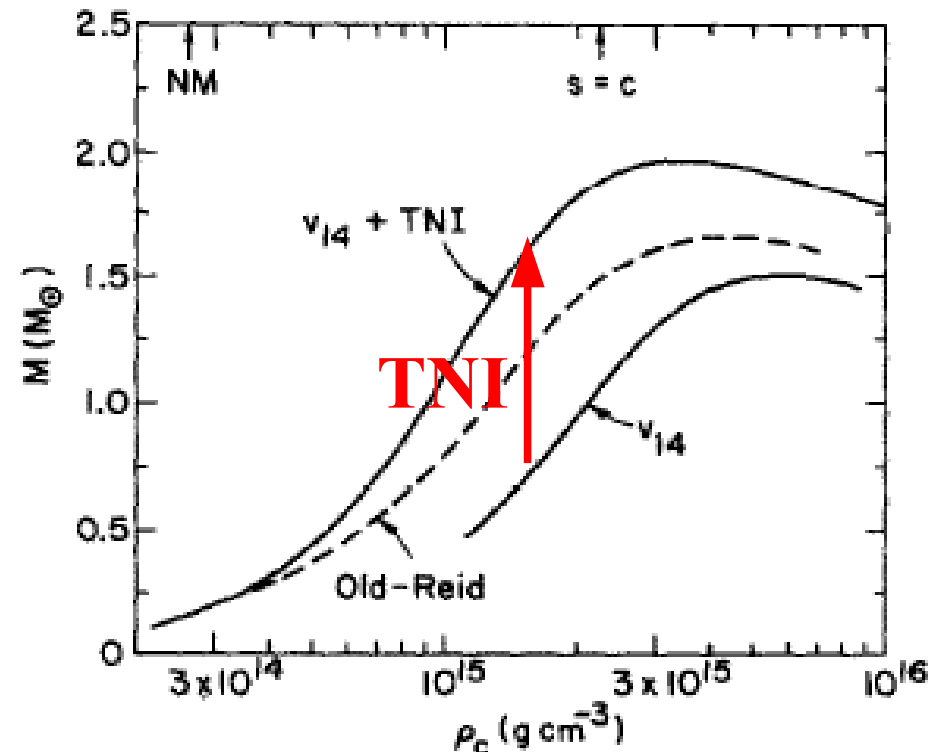
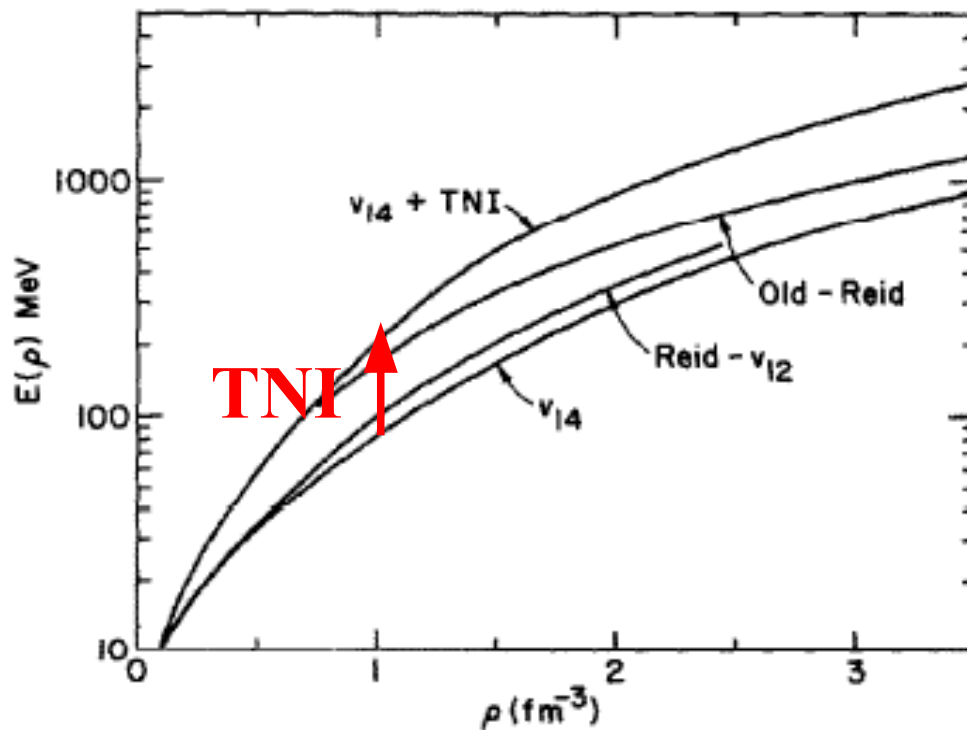
- ◆ Relativistic, Meson-Baryon coupling, Meson self-energies
- ◆ Successful in describing pA scattering (Dirac Phenomenology)

Variational Calculations (1)

■ Variational Calculation starting from bare nuclear force

B. Friedman, V.R. Pandharipande, NPA361('81)502

- Argonne v14 + TNI (TNR+TNA)
(TNI/TNR/TNA: three-nucleon int./repulsion/attraction)

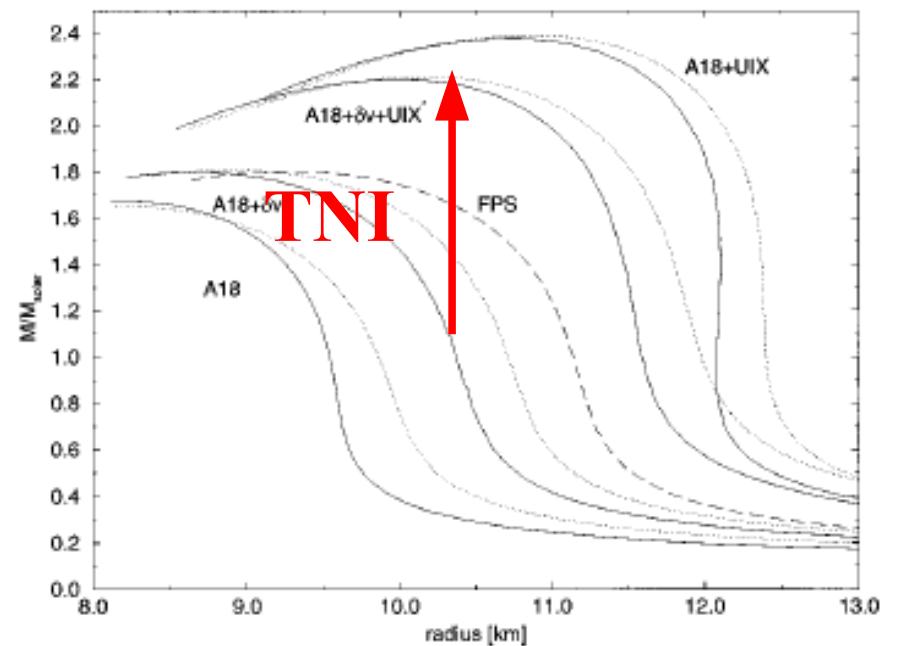
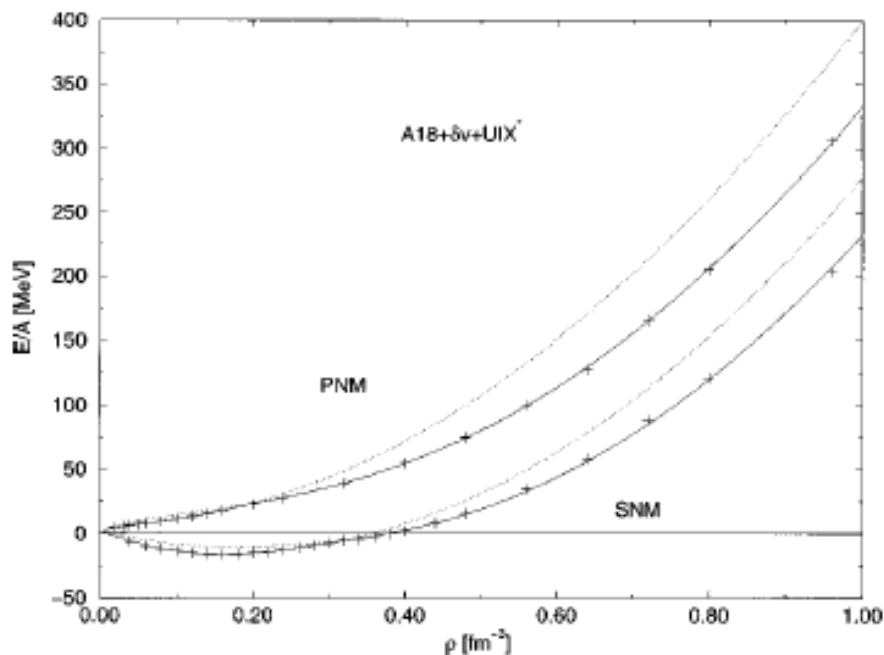


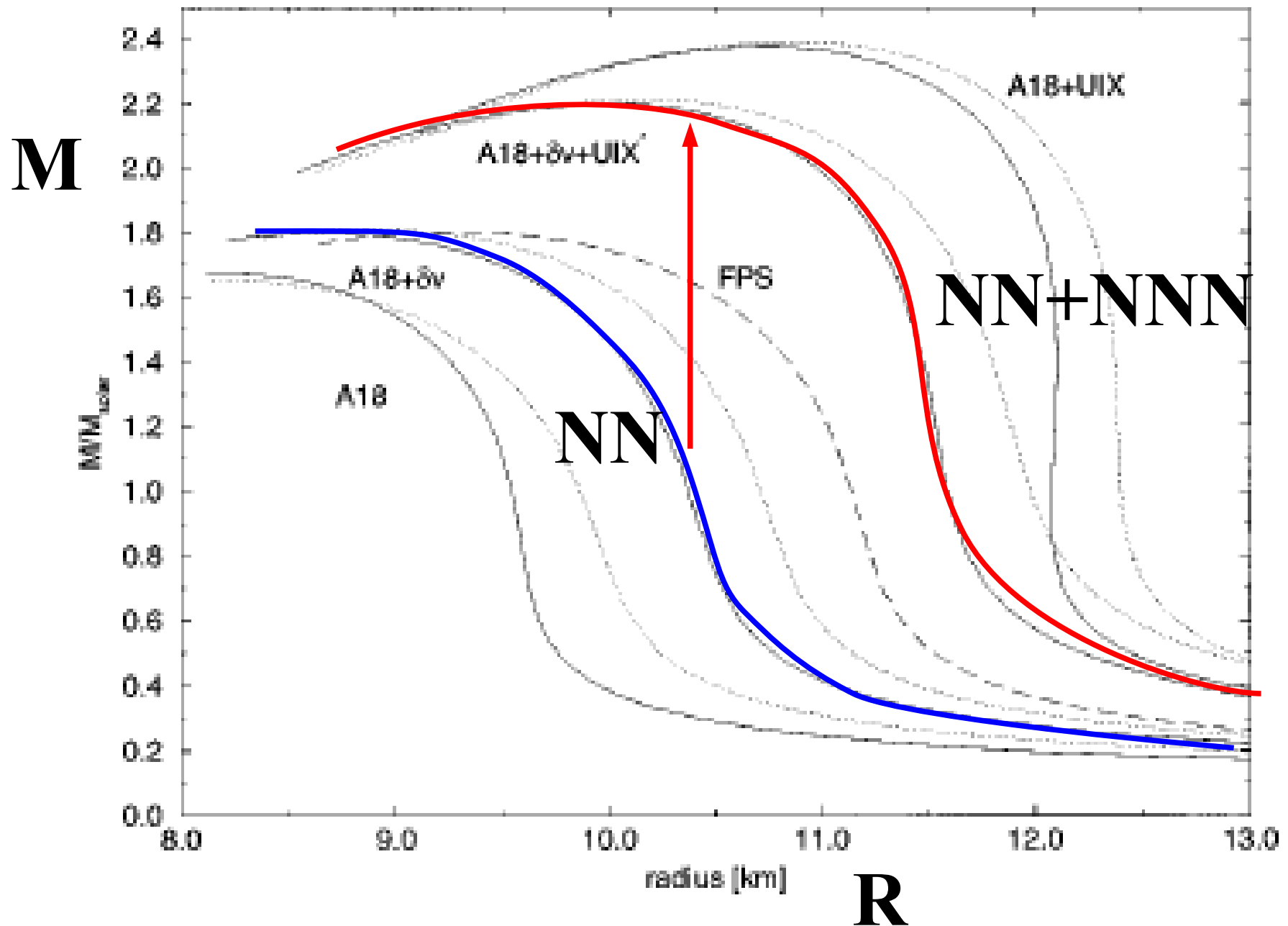
Variational Calculation (2)

■ Variational chain summation method

A. Akmal, V.R.Pandharipande, D.G. Ravenhall, PRC58('98)1804

- v18, relativistic correction, TNI
- Existence of neutral pion condensation at $\rho_B > 0.2 \text{ fm}^{-3}$



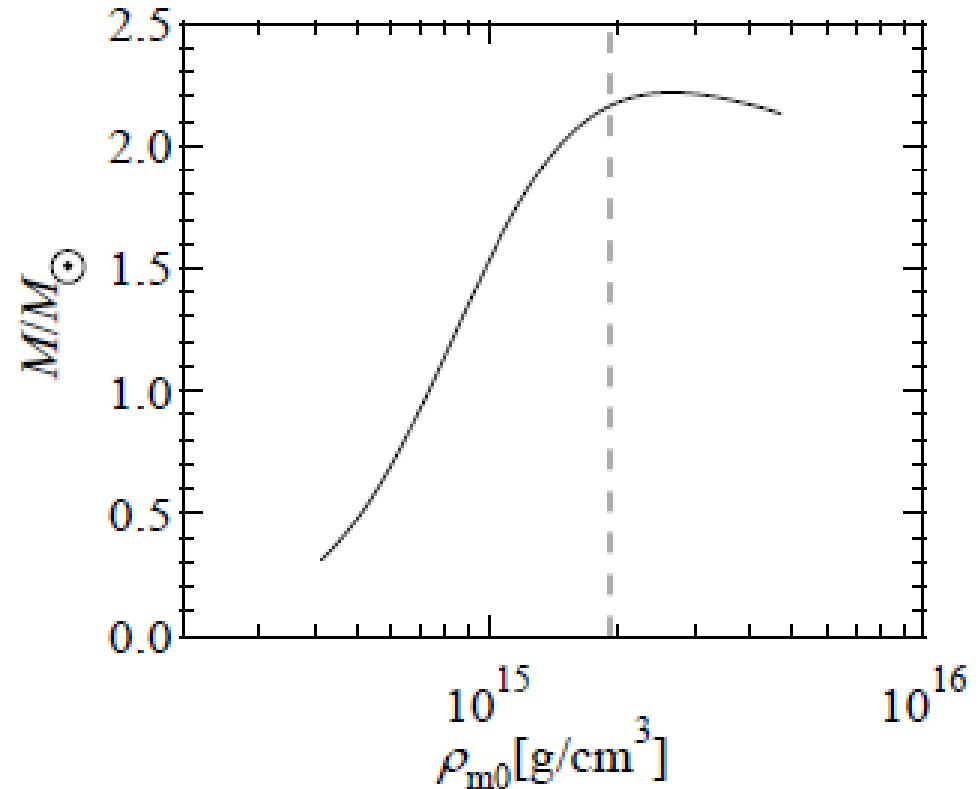
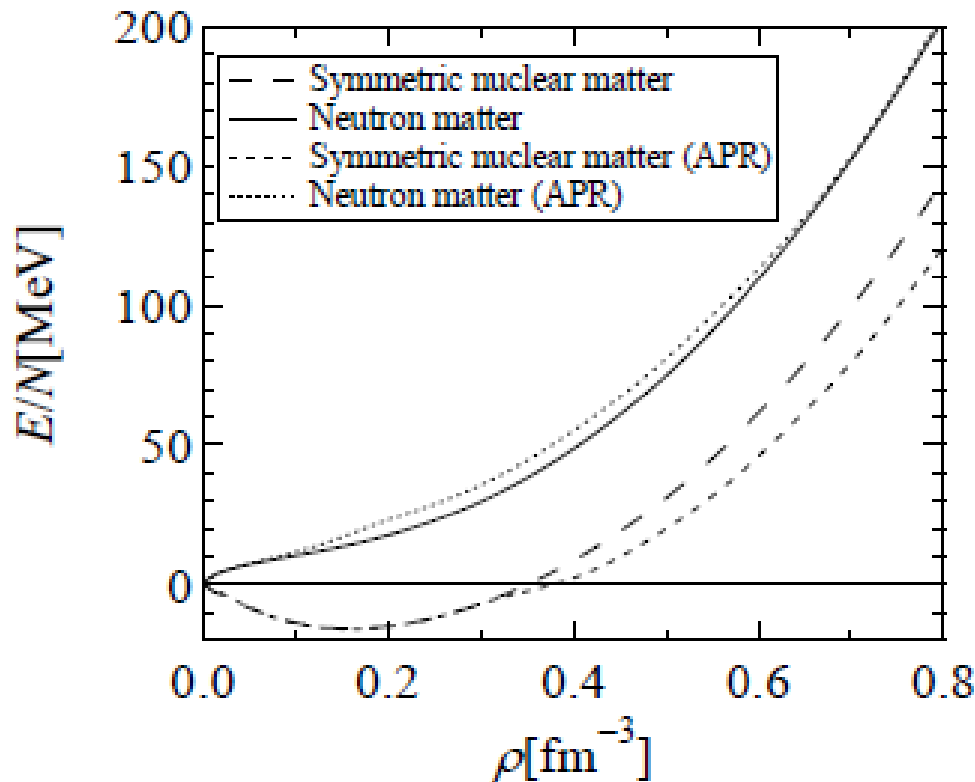


Variational Calculation (3)

■ Variational Calculation using v18+UIX

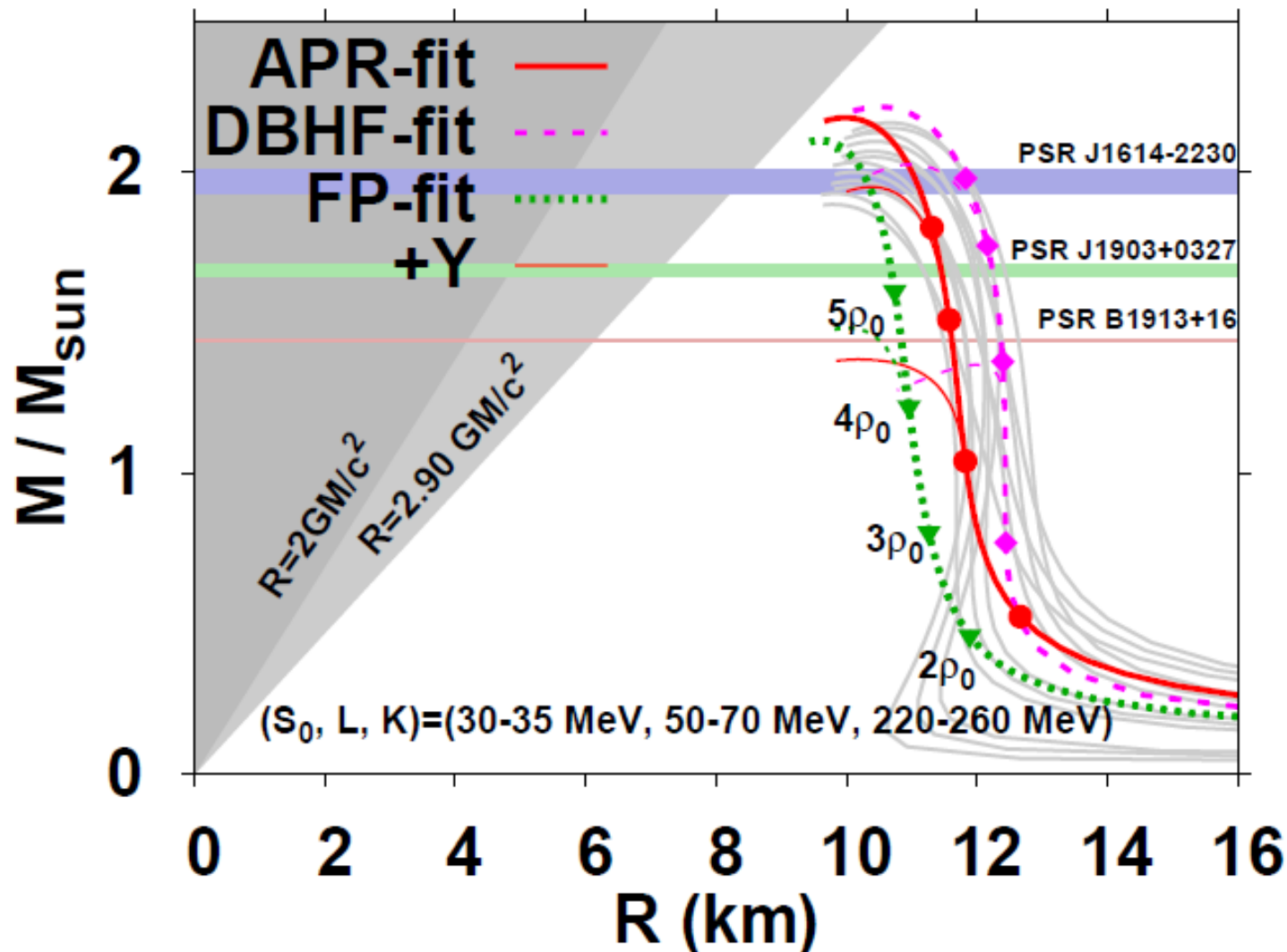
H. Kanzawa, K. Oyamatsu, K. Sumiyoshi, M. Takano, NPA791 ('07) 232

- Similar to APR, but healing-distance condition is required.
→ no π^0 condensation



Ab initio & 現象論的EOSでのMR曲線

- 現象論的状态方程式から推測されるMR曲線(灰色)
 - 半径 $R=(11-13)$ km ($M=1.4 M_{\odot}$)、最大質量 $M_{\max}=(1.9-2.2) M_{\odot}$ (核子のみの場合)



Hartree-Fock Theory

■ 平均場理論 = 多体問題の基本

$$\delta \frac{\langle \Phi | H | \Phi \rangle}{\langle \Phi | \Phi \rangle} = 0, \quad |\Phi\rangle = \det\{\phi_1 \cdots \phi_N\}: \text{Slater determinant}$$

■ 電子系ではエネルギーをほぼ再現

■ 原子核ではナイーブな HF は破綻

- 短距離での斥力コア → エネルギー $= \infty$
- 2体相関が決定的

$$\rho_2(\mathbf{r}_1, \mathbf{r}_2) = 0, \quad \text{for } |\mathbf{r}_1 - \mathbf{r}_2| < c$$

→ Brueckner 理論 (G-matrix)

原子・分子など、電子系

	HF	Exp
He	-2.86	-2.90
Li	-7.43	-7.48
Ne	-128.55	-128.94
Ar	-526.82	-527.60

原子単位 (27.2 eV)

Brueckner Theory

■ Lippmann-Schwinger Eq.

$$T = V + VG_0 T$$

- V が singular でも T は有限

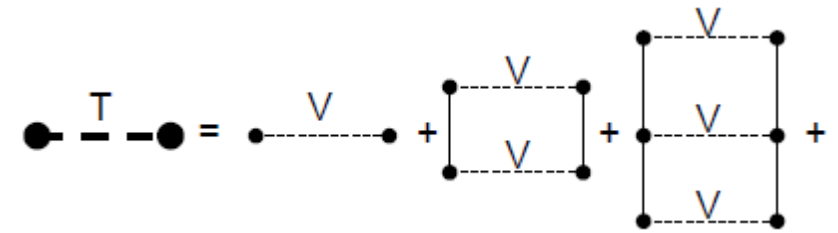
■ 原子核中での 2 体散乱 → パウリ原理

$$g(E) = V + V \frac{Q}{E - H_0} g(E)$$

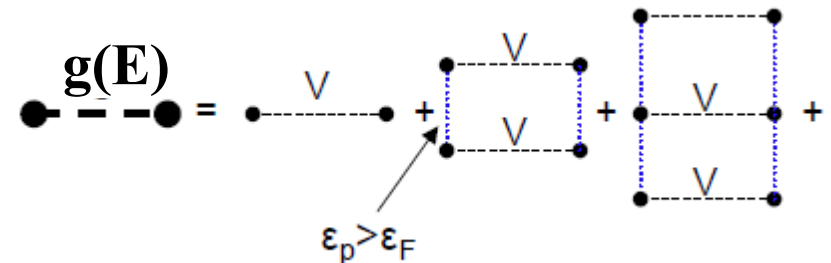
$$Q = 1 - \sum_{i,j < F} |ij\rangle \langle ij|$$

- 原子核中では中間状態で
フェルミエネルギー以上の状態のみ
伝播可能

■ 核内での散乱行列 = g-matrix



$$V |\Psi_k^{(+)}\rangle = T |\mathbf{k}\rangle$$



$$V |\Psi\rangle = g(E) |\Phi\rangle$$

2 体相関を含む
複雑な状態

2 体相関の
無い状態
(E.g. Slater det.)

Healing distance

■ (波動関数についての) Bethe-Goldstone 方程式

$$g_{12} = v_{12} + v_{12} \frac{Q_{12}}{E - (t_1 + t_2 + U_1 + U_2)} g_{12}$$

$$\rightarrow [E - (t_1 + t_2 + U_1 + U_2)] \psi_{12} = Q_{12} v_{12} \psi_{12}$$

- BG 方程式の解は、 $k_F l \sim 1.9$ 程度の距離で通常の平面波にほぼ一致する (Healing distance)
→ 独立粒子描像

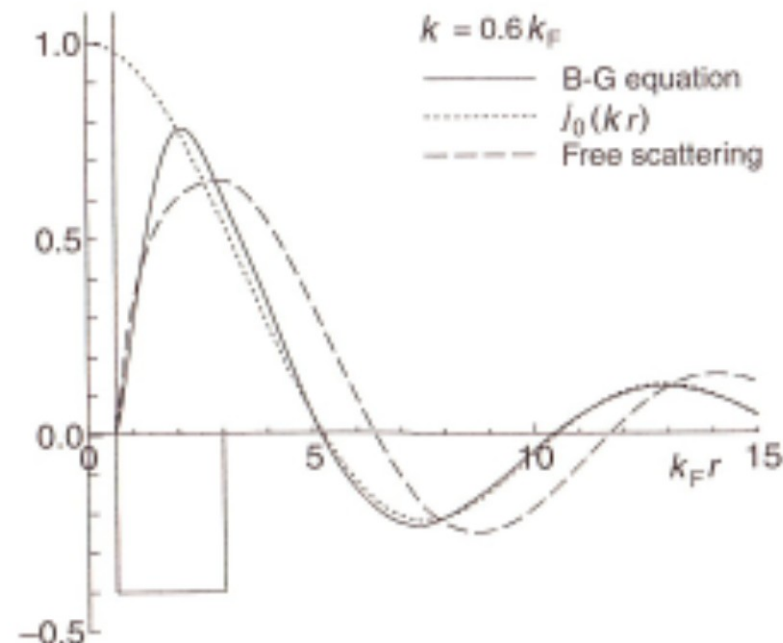
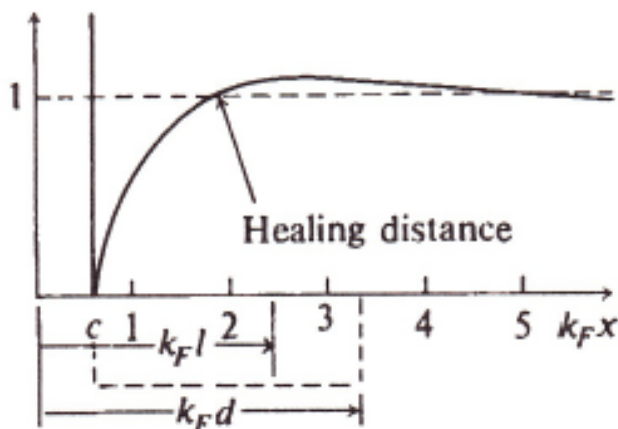


図 2.17 $k = 0.6 k_F$ の場合の Bethe-Goldstone 方程式の解 (実線) と、自由空間内の 2 粒子散乱 (破線) および自由粒子の相対波動関数 (点線) の比較

$k_F = 1.27 \text{ fm}^{-1}$, 芯半径は $k_F r_c = 0.62$, 井戸型ポテンシャルの半径は $k_F r_0 = 3.0$, 有効質量は $M^*/M = 0.6$ ととられている。

Brueckner-Hartree-Fock theory

- g-matrix を 2 体相互作用とする HF
= Brueckner-Hartree-Fock

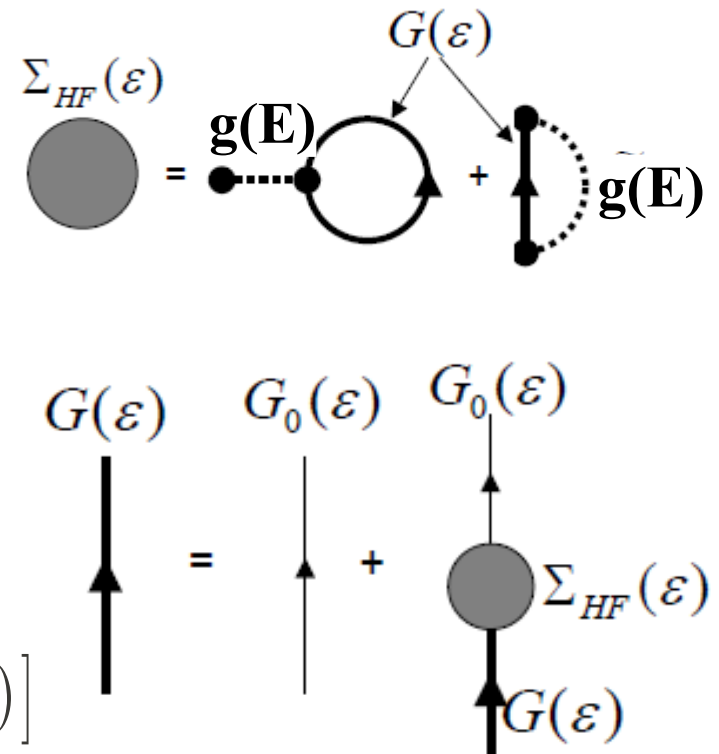
$$H = H_0 + V, \quad V = \frac{1}{2} \sum_{i \neq j} V_{ij}$$

$$H_0 = \sum_i \left[-\frac{\hbar^2}{2m} \nabla_i^2 + U_i \right]$$

$$g(E) = V + V \frac{Q}{E - H_0} g(E)$$

$$U_i(\varepsilon_i) = \sum_j \left[g_{ij,ij}(\varepsilon_i + \varepsilon_j) - g_{ij,ji}(\varepsilon_i + \varepsilon_j) \right]$$

$$E_{\text{BHF}} = \sum_i^{\text{occ.}} \langle i | -\frac{\hbar^2}{2m} \nabla^2 | i \rangle + \frac{1}{2} \sum_{i \neq j}^{\text{occ.}} \langle ij | g(\varepsilon_i + \varepsilon_j) | ij - ji \rangle$$



- Self-consistent treatment

$U \rightarrow \text{g-matrix} \ \& \ \varphi \text{ (s.p.w.f)} \rightarrow U$

Brueckner-Hartree-Fock theory (cont.)

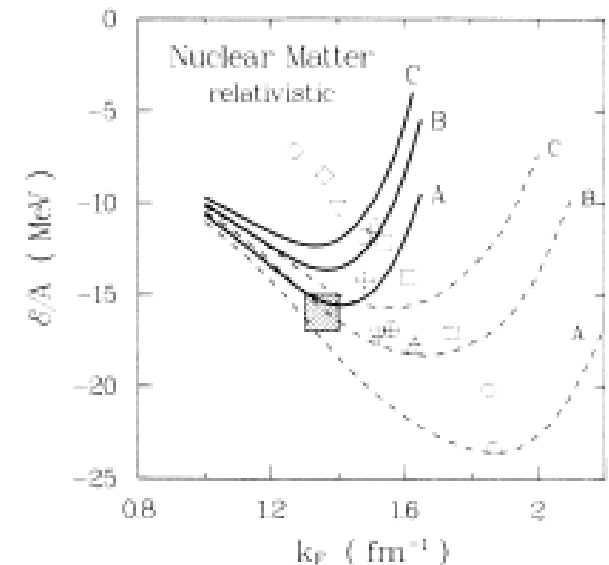
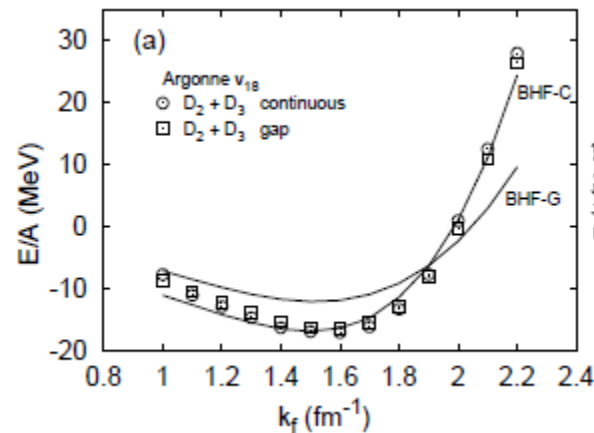
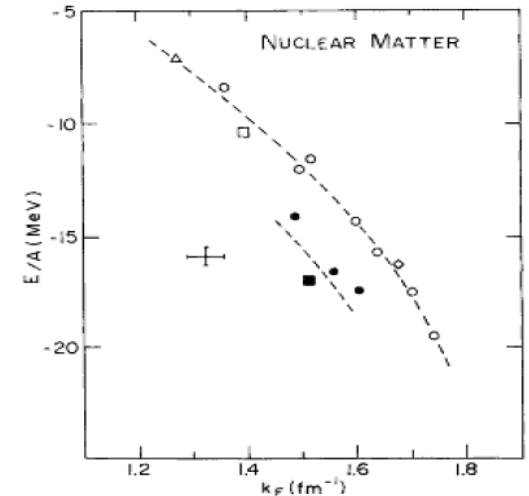
■ 成功点

- 核物質の飽和性を定性的に説明
- 殻模型(独立粒子描像)の基礎を与える
- 有効核力の状態依存性を説明

■ 問題点

- 飽和点(飽和密度、飽和エネルギー)の定量的理解(Coester line)→ Relativity or 3体力
- 展開の高次項→ Continuum choice では3体クラスター効果は小さい
- スピン軌道力が足りない

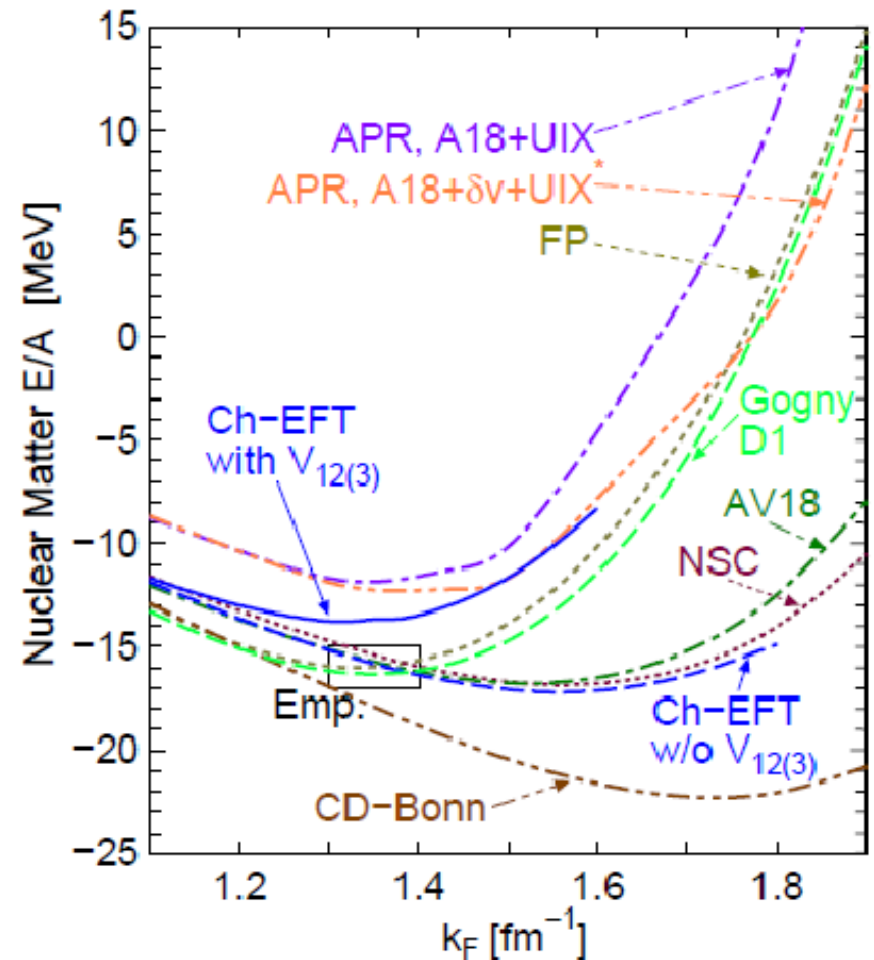
問題は残っているが、
現実的核力から出発して
多体問題に適用する
有効な手法



Ch-EFT EOS

- Phen. models need inputs from
Experimental Data and/or Microscopic (Ab initio) Calc.
- Recent Ch-EFT EOS is promising !
NN (N3LO)+3NF(N2LO)

M.Kohno ('13)

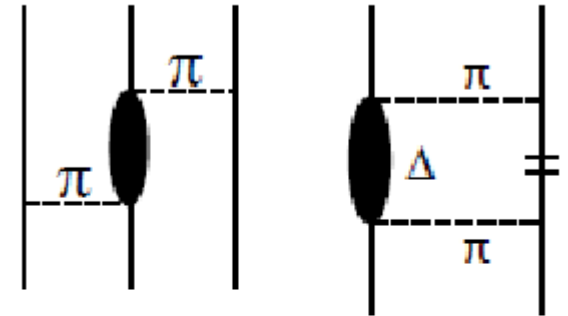


M. Kohno, PRC 88 ('13) 064005

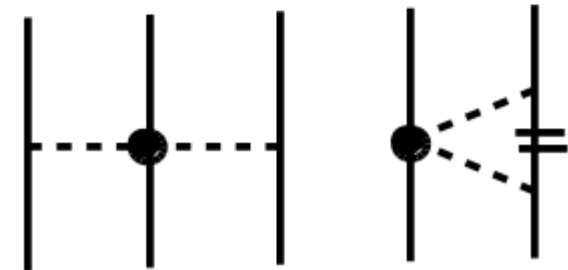
“Universal” mechanism of “Three-body” repulsion

- “Universal” 3-body repulsion is necessary to support NS.
Nishizaki, Takatsuka, Yamamoto (‘02)
- Mechanism of “Universal” Three-Baryon Repulsion.
 - “ σ ”-exchange $\sim$ two pion exch. w/ res.
 - Large attraction from two pion exchange is suppressed by the Pauli blocking in the intermediate stage.
Kohn ('13)

Physical Picture



χ EFT

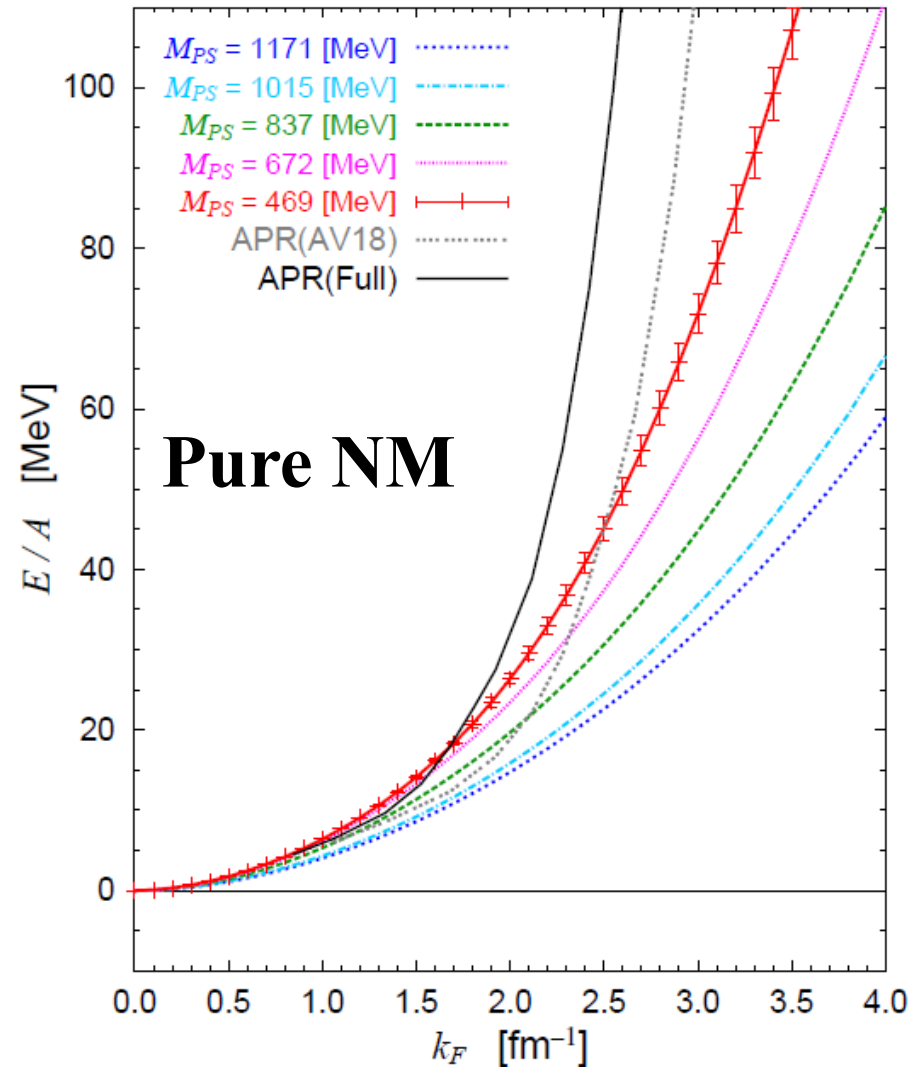
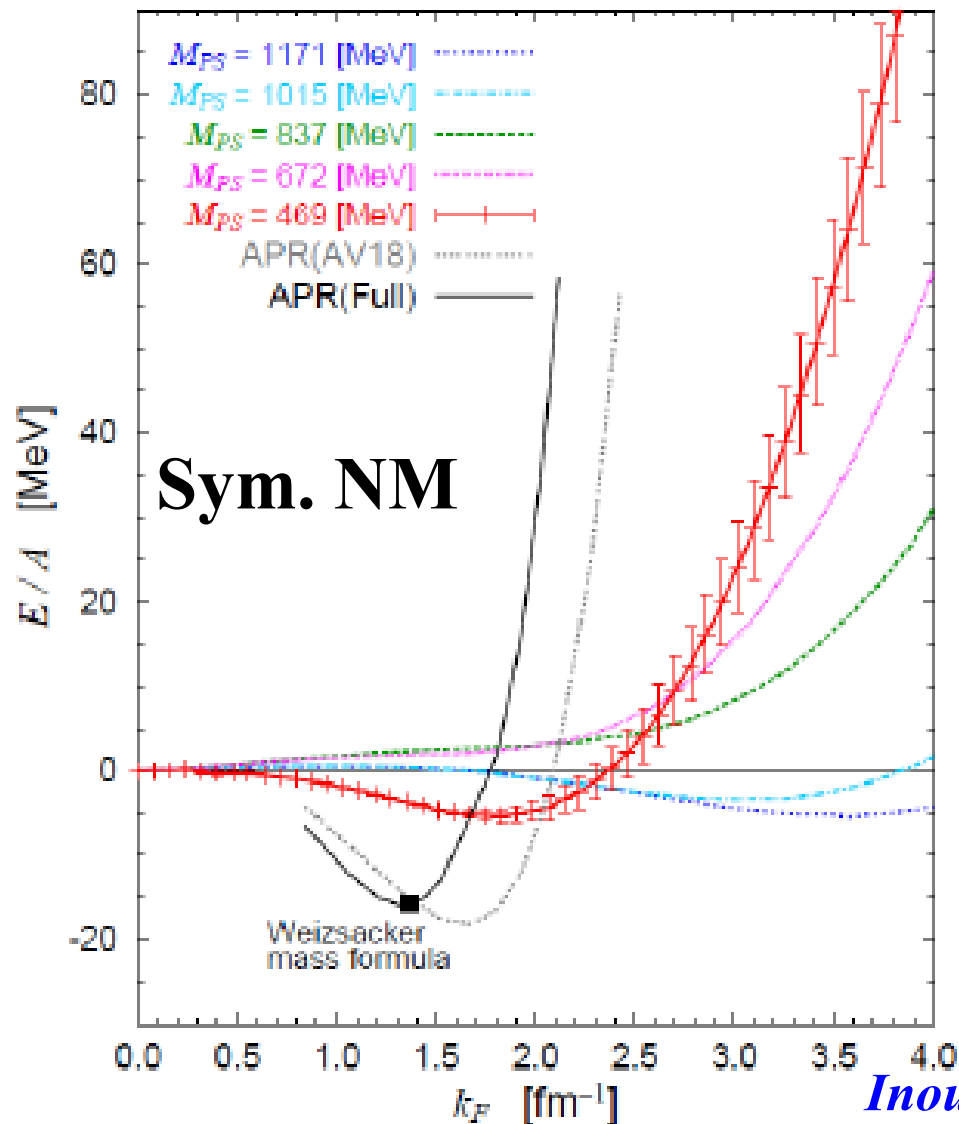


“Universal” TBR

- *Coupling to Res. (hidden DOF)*
- *Reduced “ σ ” exch. pot. ?*

EOS from lattice NN force

- 格子 QCD 核力を用いた高密度状態方程式 (LQCD+BHF)
NN force: 1S_0 , 3S_1 , 3D_1 only



Inoue et al. (HAL QCD Coll.), PRL111 ('13)112503

Relativistic Mean Field

Relativistic Mean Field (1)

Effective Lagrangian of Baryons and Mesons + Mean Field App.

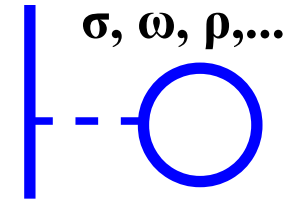
B.D.Serot, J.D.Walecka, Adv.Nucl.Phys.16 ('86), 1

$$L = L_B^{\text{free}} + L_M^{\text{free}} + L_{BM} + L_M^{\text{Int}}$$

$$L_M^{\text{Int}} = -U_\sigma(\sigma) + \frac{1}{4}c_\omega(\omega_\mu\omega^\mu)^2 + \dots$$

$$L_{BM} = -\sum_{B,S} g_{BS} \bar{\Psi}_B \varphi_S \Psi_B - \sum_{B,V} g_{BV} \bar{\Psi}_B \gamma^\mu V_\mu \Psi_B$$

$$L_B^{\text{free}} = \bar{\Psi}_B (i \gamma^\mu \partial_\mu - M_B) \Psi_B, \quad L_M^{\text{free}} = \sum_S \left[\frac{1}{2} \partial^\mu \varphi_S \partial_\mu \varphi_S - \frac{1}{2} m_S^2 \varphi_S^2 \right] + \sum_V \left[-\frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \frac{1}{2} m_V^2 V_\mu V^\mu \right]$$



• **Baryons and Mesons:** B=N, Λ , Σ , Ξ , ..., S= σ , ζ , ..., V= ω , ρ , φ , ...

• **Based on Dirac phenomenology & Dirac Bruckner-Hatree-Fock**

E.D. Cooper, S. Hama, B.C. Clark, R.L. Mercer, PRC47('93),297

R. Brockmann, R. Machleidt, PRC42('90),1965

• **Large scalar (att.) and vector (repl.) → Large spin-orbit pot.**

Relativistic Kinematics → Effective 3-body repulsion

• **Non-linear terms of mesons → Bare 3-body and 4-body force**

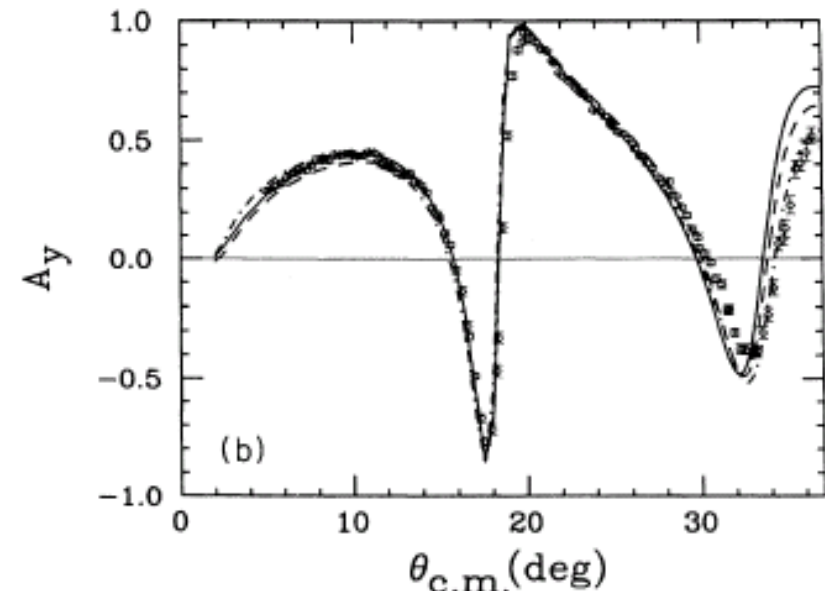
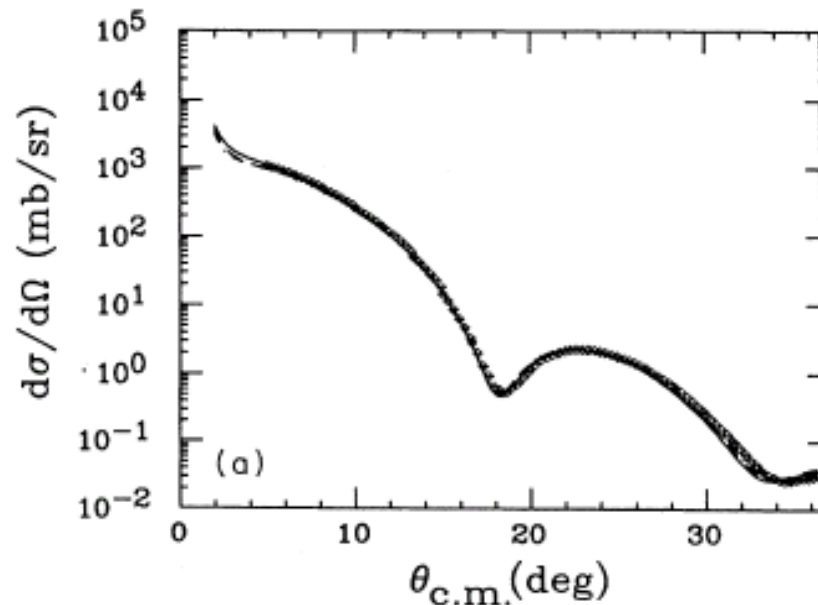
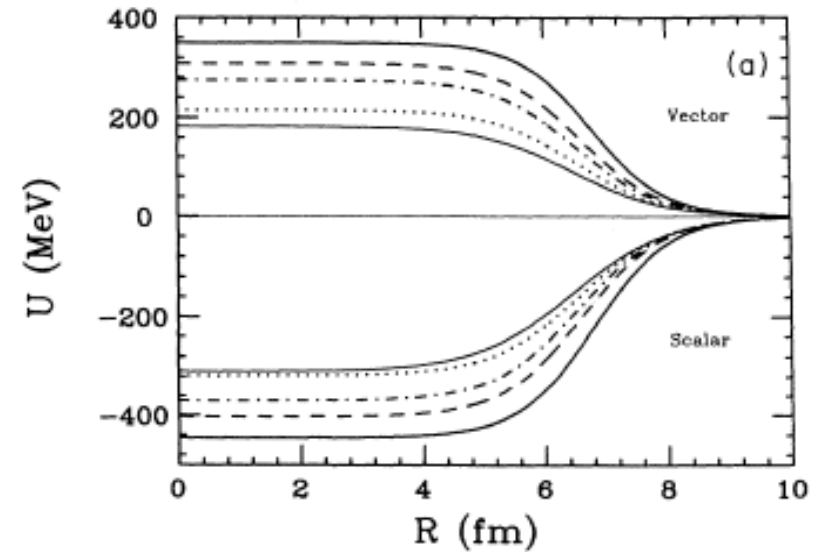
Boguta, Bodmer ('77), NL1:Reinhardt, Rufa, Maruhn, Greiner, Friedrich ('86), NL3:

Lalazissis, Konig, Ring ('97), TM1 and TM2: Sugahara, Toki ('94), Brockmann, Toki ('92)

Dirac Phenomenology

E.D. Cooper, S. Hama, B.C. Clark, R.L. Mercer, PRC47('93),297

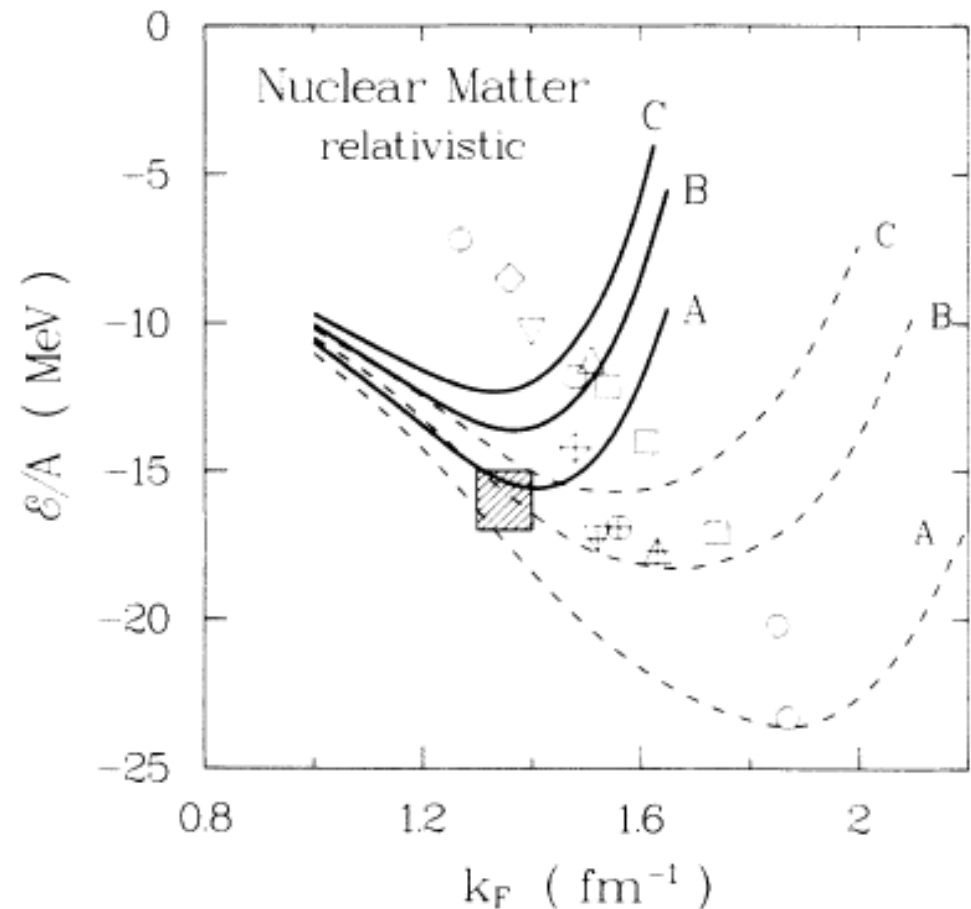
- Dirac Eq. with
Scalar + Vector pA potential
(-400 MeV + 350 MeV)
→ Cross Section, Spin Observable



EOS in Dirac-Brueckner-Hartree-Fock

R. Brockmann, R. Machleidt, PRC42('90),1965

- **Non Relativistic Brueckner Calculation**
→ **Nuclear Saturation Point cannot be reproduced (Coester Line)**
- **Relativistic Approach (DBHF)**
→ **Relativity gives additional repulsion, leading to successful description of the saturation point.**



Relativistic Mean Field (2)

- **Mean Field treatment of meson field operator**
= Meson field operator is replaced with its expectation value
 $\varphi(r) \rightarrow \langle \varphi(r) \rangle$

Ignoring fluctuations compared with the expectation value may be a good approximation at strong condensate.

- **Which Hadrons should be included in RMF ?**
 - **Baryons (1/2+)** $p, n, \Lambda, \Sigma, \Xi, \Delta, \dots$
 - **Scalar Mesons (0+)** $\sigma(600), f_0(980), a_0(980), \dots$
 - **Vector Mesons (1-)** $\omega(783), \rho(770), \phi(1020), \dots$
 - **Pseudo Scalar (0-)** $\pi, K, \eta, \eta', \dots$
 - **Axial Vector (1+)** $a_1, \dots$

We require that the meson field can have uniform expectation values in nuclear matter.

→ Scalar and Time-Component of Vector Mesons ($\sigma, \omega, \rho, \dots$)

$\sigma\omega$ Model (1)

Serot, Walecka, Adv.Nucl.Phys.16 (1986),1

■ Consider only σ and ω mesons

■ Lagrangian

$$L = \bar{\psi} (i \gamma^\mu \partial_\mu - M + g_s \sigma - g_v \gamma^\mu \omega_\mu) \psi \\ + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_s^2 \sigma^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_v^2 \omega_\mu \omega^\mu \\ (F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu)$$

■ Equation of Motion

● Euler-Lagrange Equation

$$\frac{\partial}{\partial x^\mu} \left[\frac{\partial L}{\partial (\partial_\mu \phi_i)} \right] - \frac{\partial L}{\partial \phi_i} = 0$$

$$\sigma : \left[\partial_\mu \partial^\mu + m_s^2 \right] \sigma = g_s \bar{\psi} \psi$$

$$\omega : \partial_\mu F^{\mu\nu} + m_v^2 \omega^\nu = g_v \bar{\psi} \gamma^\nu \psi \quad \rightarrow \quad \left[\partial_\mu \partial^\mu + m_v^2 \right] \omega^\nu = g_v \bar{\psi} \gamma^\nu \psi$$

$$\psi : \left[\gamma^\mu (i \partial_\mu - g_v V_\mu) - (M - g_s \sigma) \right] \psi = 0$$

EOM of ω (for beginners)

■ Euler-Lagrange Eq.

$$\partial_\mu F^{\mu\nu} + m_\nu^2 \omega^\nu = g_\nu \bar{\psi} \gamma^\nu \psi$$

■ Divergence of LHS and RHS

$$\partial_\nu \partial_\mu F^{\mu\nu} + m_\nu^2 (\partial_\nu \omega^\nu) = m_\nu^2 (\partial_\nu \omega^\nu) = g_\nu (\partial_\nu \bar{\psi} \gamma^\nu \psi) = 0$$

LHS: derivatives are sym. and $F_{\mu\nu}$ is anti-sym.

RHS: Baryon Current = Conserved Current

■ Put it in the Euler-Lagrange Eq.

$$\partial_\mu F^{\mu\nu} = \partial_\mu (\partial^\mu \omega^\nu - \partial^\nu \omega^\mu) = \partial_\mu \partial^\mu \omega^\nu - \partial^\nu (\partial_\mu \omega^\mu) = \partial_\mu \partial^\mu \omega^\nu$$

Schroedinger Eq. for Upper Component (1)

■ Dirac Equation for Nucleons

$$\left(i\gamma\partial - \gamma^0 U_v - M - U_s \right) \psi = 0 \quad ,$$
$$U_v = g_\omega \omega \quad , \quad U_s = -g_\sigma \sigma$$

■ Decompose 4 spinor into Upper and Lower Components

$$\begin{pmatrix} E - U_v - M - U_s & i\sigma \cdot \nabla \\ -i\sigma \cdot \nabla & -E + U_v - M - U_s \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix} = 0$$

$$g = \frac{-i}{E + M + U_s - U_v} (\sigma \cdot \nabla) f$$

$$(E - M - U_v - U_s) f = -i(\sigma \cdot \nabla) g$$

Schroedinger Eq. for Upper Component (2)

■ Erase Lower Component (assuming spherical sym.)

$$\begin{aligned}
 -i(\sigma \cdot \nabla) g &= -(\sigma \cdot \nabla) \frac{1}{X} (\sigma \cdot \nabla) f = -\frac{1}{X} \nabla^2 f - \frac{1}{r} \left[\frac{d}{dr} \frac{1}{X} \right] (\sigma \cdot r) (\sigma \cdot \nabla) f \\
 &= -\nabla \frac{1}{X} \nabla f + \frac{1}{r} \left[\frac{d}{dr} \frac{1}{X} \right] (\sigma \cdot l) f
 \end{aligned}$$

$$(\sigma \cdot r)(\sigma \cdot \nabla) = (r \cdot \nabla) + i \sigma \cdot (r \times \nabla) = r \cdot \nabla - \sigma \cdot l$$

■ “Schroedinger-like” Eq. for Upper Component

$$-\nabla \frac{1}{E + M + U_s - U_v} \nabla f + \left(U_s + U_v + U_{LS}(\sigma \cdot l) \right) f = (E - M) f$$

$$U_{LS} = \frac{1}{r} \left[\frac{d}{dr} \frac{1}{E + M + U_s - U_v} \right] < 0 \quad \text{on surface}$$

$$(U_s, U_v) \sim (-350 \text{ MeV}, 280 \text{ MeV})$$

→ Small Central ($U_s + U_v$), Large LS ($U_s - U_v$)

Various Ways to Evaluate Non.-Rel. Potential

■ From Single Particle Energy

$$\begin{aligned} & \left(\gamma^0 (E - U_v) + i \gamma \cdot \nabla - (M + U_s) \right) \psi = 0 \rightarrow (E - U_v)^2 = p^2 + (M + U_s)^2 \\ & \rightarrow E = \sqrt{p^2 + (M + U_s)^2} + U_v \approx E_p + \frac{M}{E_p} U_s + U_v + \frac{p^2}{2 E_p^3} U_s^2 \\ & (E_p = \sqrt{p^2 + M^2}) \end{aligned}$$

■ Schroedinger Equivalent Potential (Uniform matter)

$$\begin{aligned} -\frac{\nabla^2}{2M} f + \left[U_s + \frac{E}{M} U_v + \frac{U_s^2 - U_v^2}{2M} \right] f &= \frac{E + M}{2M} (E - M) f \\ U_{\text{SEP}} &\approx U_s + \frac{E}{M} U_v \end{aligned}$$

Anyway, slow baryons feel Non.-Rel. Potential,

$$U \approx U_s + U_v = -g_s \sigma + g_v \omega$$

Nuclear Matter in $\sigma\omega$ Model

Serot, Walecka, *Adv.Nucl.Phys.*16 (1986),1

■ Uniform Nuclear Matter

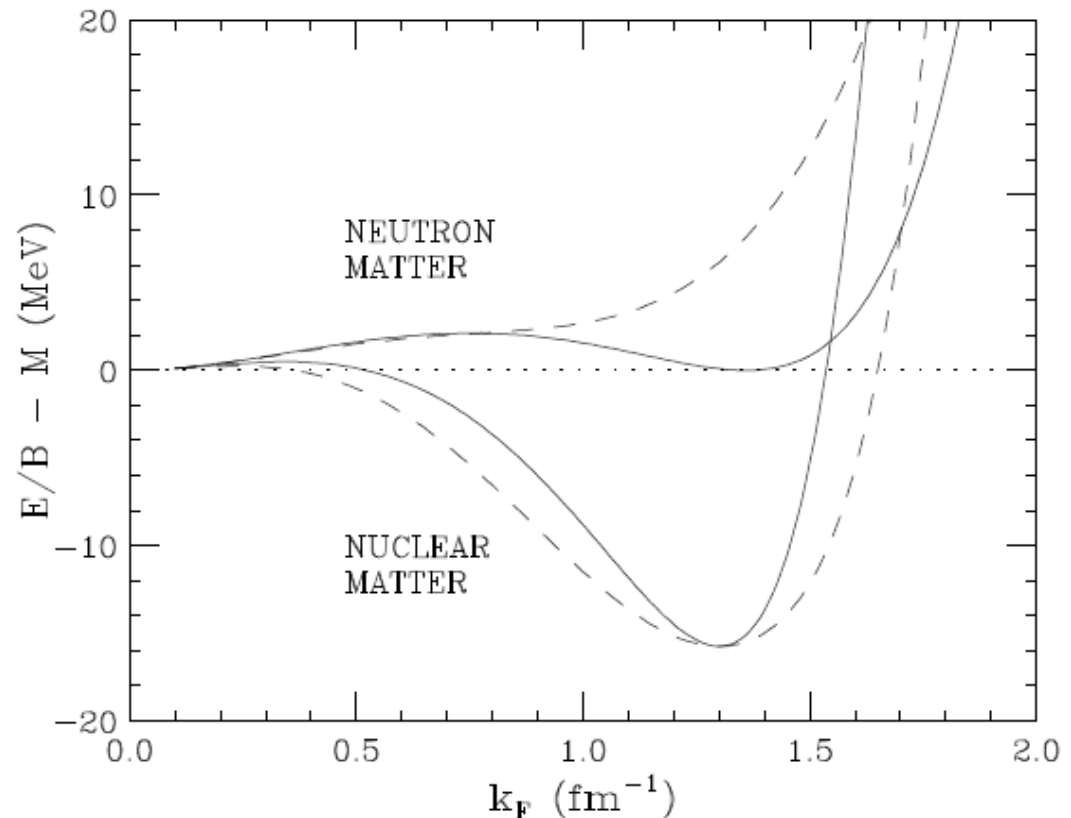
$$E/V = \gamma_N \int^{P_F} \frac{d^3 p}{(2\pi)^2} E^* + \frac{1}{2} m_s^2 \sigma^2 - \frac{1}{2} m_v^2 \omega^2 + g_v \rho_B \omega$$

$$\sigma = \frac{g_s}{m_s^2} \rho_s = \frac{g_s}{m_s^2} \int^{P_F} \frac{\gamma_N d^3 p}{(2\pi)^2} \frac{M^*}{E^*} \quad (M^* = M + U_s = M - g_s \sigma, \quad E^* = \sqrt{p^2 + M^{*2}})$$

$$\omega = \frac{g_v}{m_v^2} \rho_B = \gamma_N \frac{g_v}{m_v^2} \int^{P_F} \frac{d^3 p}{(2\pi)^3}$$

γ_N = Nucleon degeneracy
(=4 in sym. nuclear matter)

Problem: EOS is too stiff
 $K \sim (500-600) \text{ MeV}!$
 $\rightarrow$ How can we avoid it ?



RMF with Non-Linear Meson Int. Terms

*Boguta, Bodmer ('77), NL1:Reinhardt, Rufa, Maruhn, Greiner, Friedrich ('86),
NL3: Lalazissis, Konig, Ring ('97), TM1 and TM2: Sugahara, Toki ('94), Brockmann, Toki ('92)*

- Too stiff EOS in the simplest RMF ($\sigma\omega$ model) is improved by introducing non-linear terms (σ^4, ω^4)
- Fit B.E. of Stable as well as Unstable (n-rich) Nuclei
- Three Mesons (σ, ω, ρ) are included
- Meson Self-Energy Term (σ, ω)

$$\begin{aligned}
 \mathcal{L} = & \bar{\psi}_N (i\partial - M - g_\sigma \sigma - g_\omega \not{\omega} - g_\rho \tau^a \not{\rho}^a) \psi_N \\
 & + \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 \\
 & - \frac{1}{4} W^{\mu\nu} W_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4} R^{a\mu\nu} R_{\mu\nu}^a + \frac{1}{2} m_\rho^2 \rho^{a\mu} \rho_\mu^a + \frac{1}{4} c_3 (\omega_\mu \omega^\mu)^2 \\
 & + \bar{\psi}_e (i\partial - m_e) \psi_e + \bar{\psi}_\nu i\partial \psi_\nu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} , \\
 W_{\mu\nu} = & \partial_\mu \omega_\nu - \partial_\nu \omega_\mu , \\
 R_{\mu\nu}^a = & \partial_\mu \rho_\nu^a - \partial_\nu \rho_\mu^a + g_\rho \epsilon^{abc} \rho^{b\mu} \rho^{c\nu} , \\
 F_{\mu\nu} = & \partial_\mu A_\nu - \partial_\nu A_\mu .
 \end{aligned}$$

RMF models with Non-Linear Meson Int. Terms

■ Variety of the RMF models

→ MB couplings, meson masses, meson self-energies

- σN , ωN , ρN couplings are well determined

→ almost no model deps. in Sym. N.M. at low ρ

- ω^4 term is introduced to simulate DBHF results of vector pot.

TM1&2: Y. Sugahara, H. Toki, NPA579('94)557;

R. Brockmann, H. Toki, PRL68('92)3408.

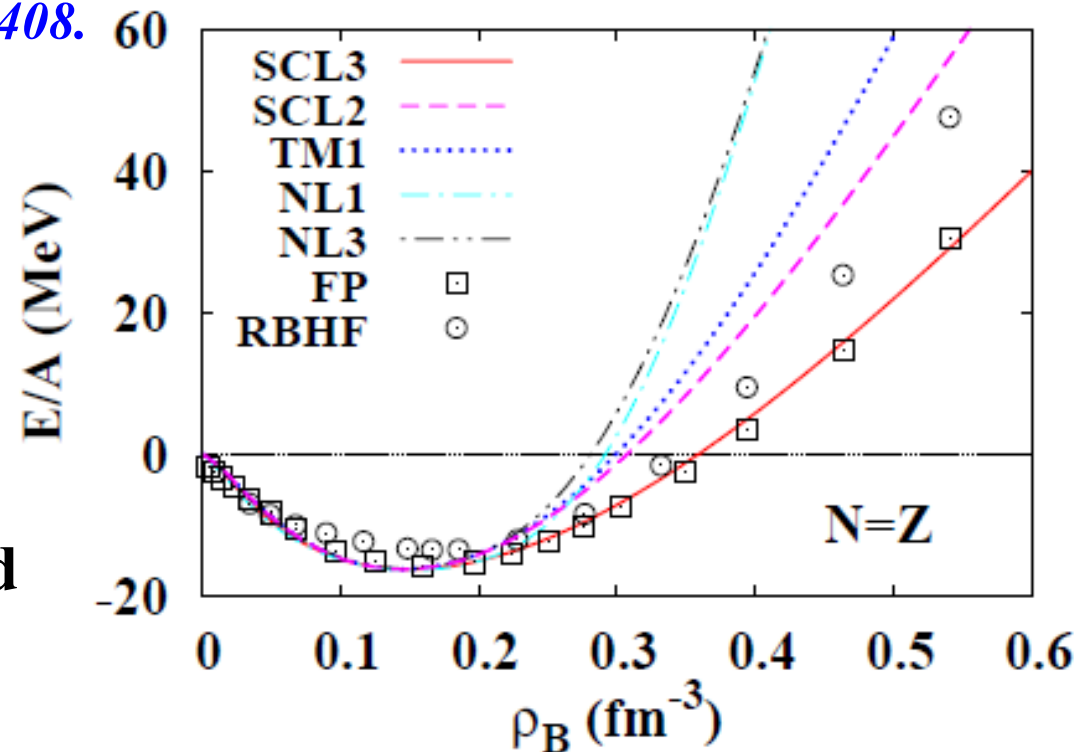
- σ^3 and σ^4 terms are introduced to soften EOS at ρ_0 .

J. Boguta, A.R. Bodmer NPA292('77)413,

NL1: P.-G. Reinhardt, M. Rufa, J. Maruhn, W. Greiner, J. Friedrich, ZPA323('86)13.

NL3: G.A. Lalazissis, J. Konig, P. Ring, PRC55('97)540.

→ Large differences are found at high ρ



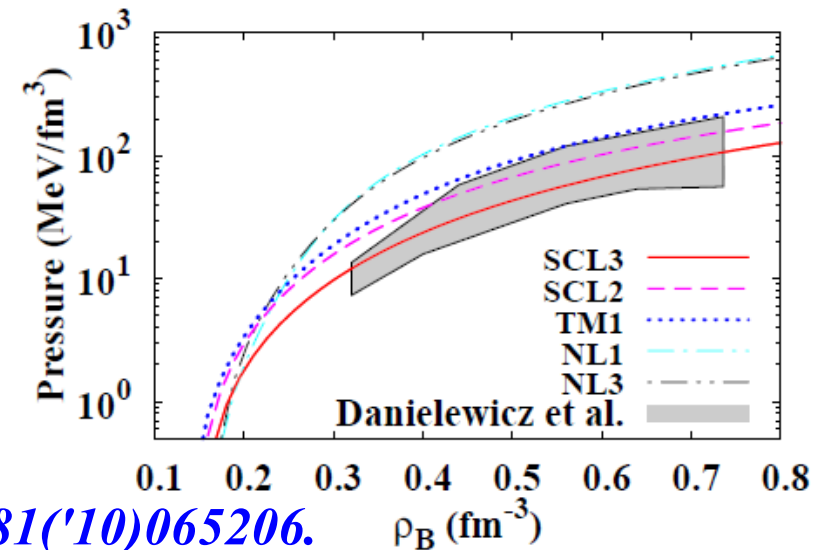
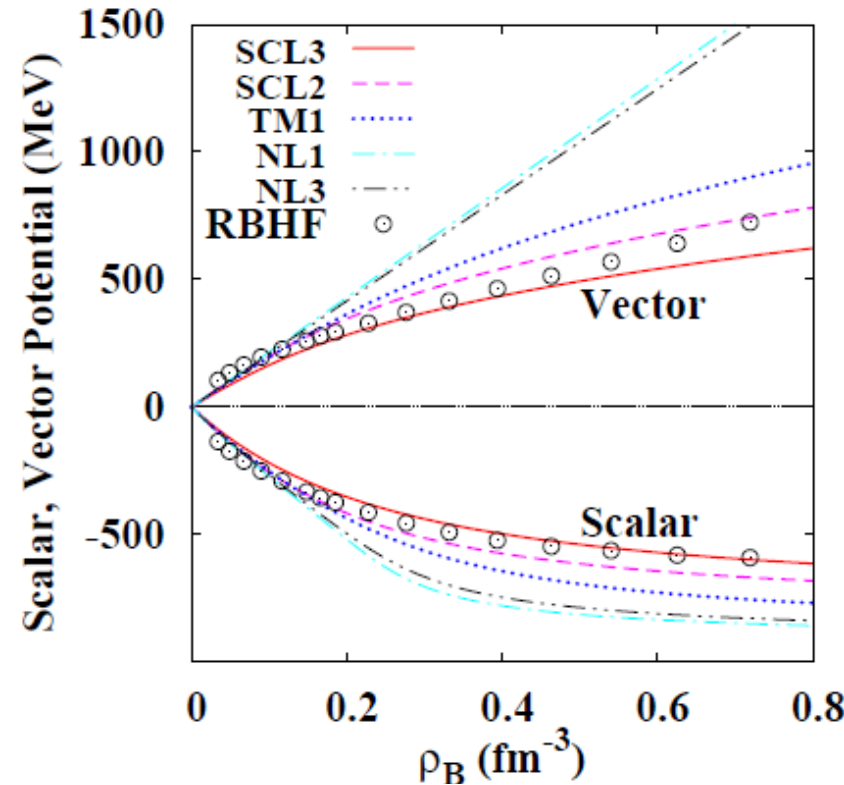
K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.

Vector potential in RMF

- Vector potential from ω dominates at high density !

$$U_v(\rho_B) = g_\omega \omega \sim \frac{g_\omega^2}{m_\omega^2} \rho_B$$

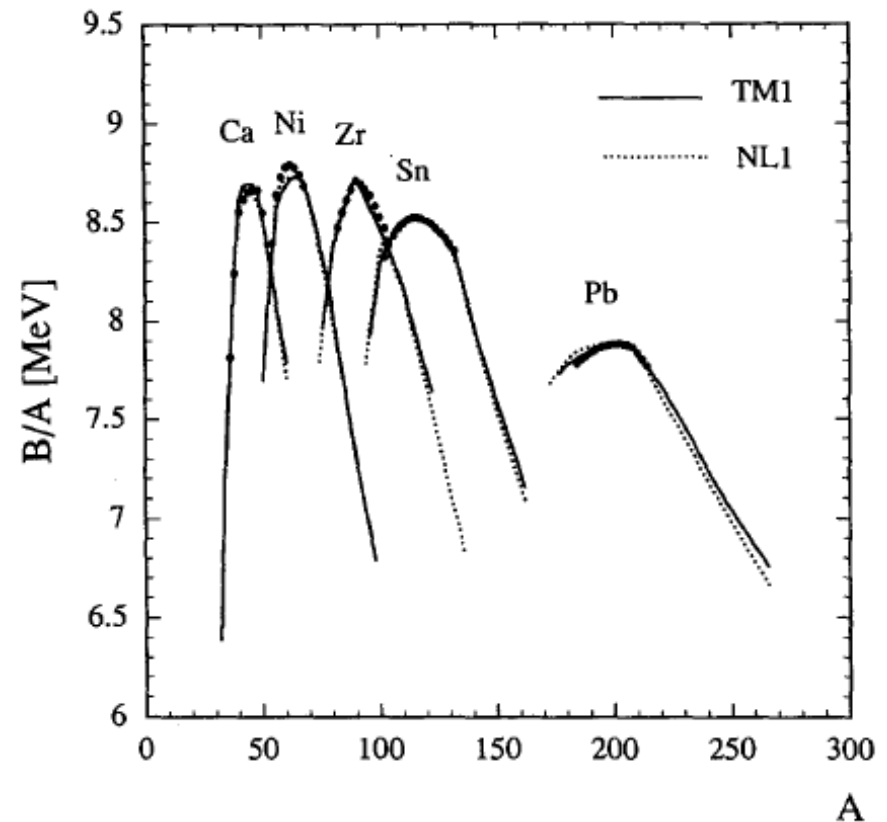
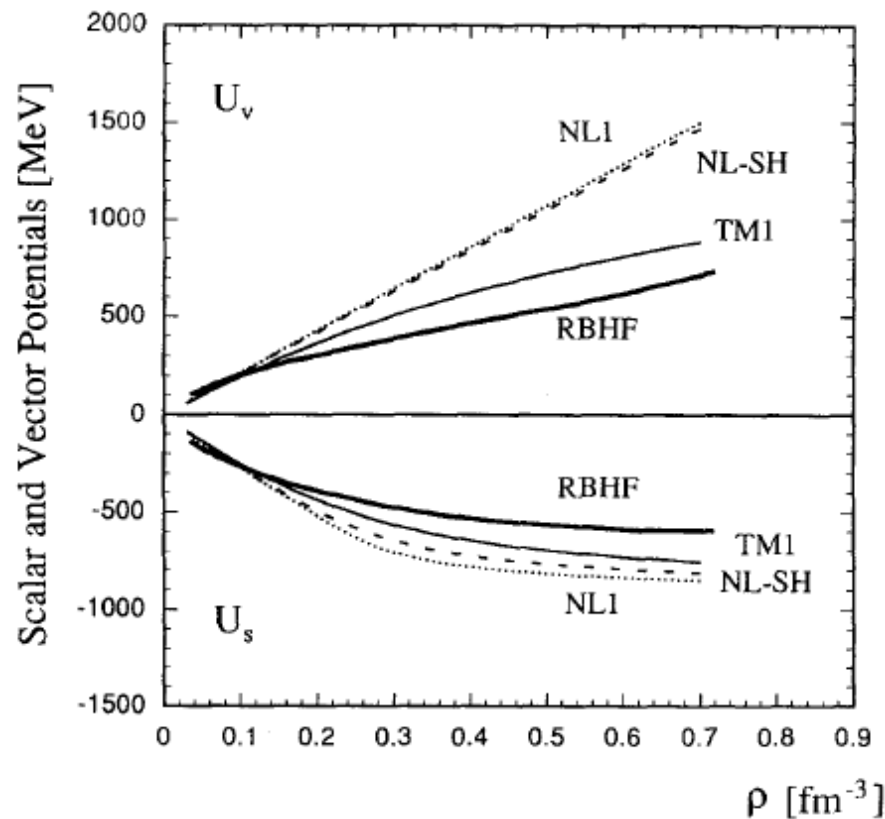
- Dirac-Bruckner-Hartree-Fock shows suppressed vector potential at high ρ_B .
R. Brockmann, R. Machleidt, PRC42('90)1965.
- Collective flow in heavy-ion collisions suggests pressure at high ρ_B .
P. Danielewicz, R. Lacey, W. G. Lynch, Science298('02)1592.
- Self-interaction of $\omega \sim c_\omega (\omega_\mu \omega^\mu)^2$
→ DBHF results & Heavy-ion data



K. Tsubakihara, H. Maekawa, H. Matsumiya, AO, PRC81('10)065206.

■ TM1 Sugahara, Toki ('94)

- Fit vector potential in RBHF by introducing ω^4 term.
- Fit binding energies of neutron-rich nuclei



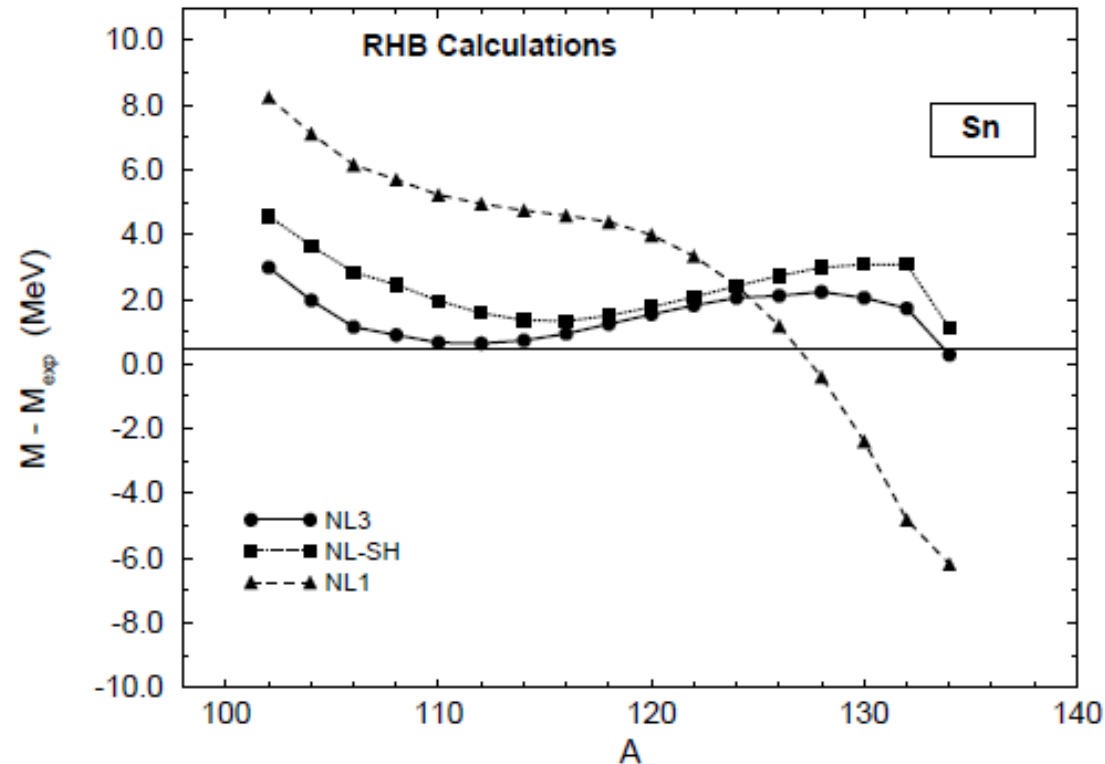
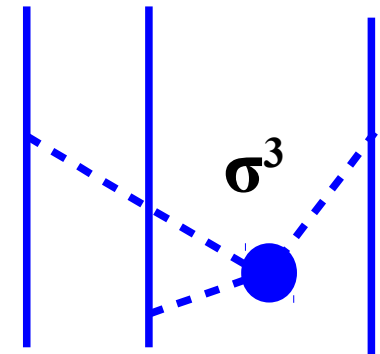
TM1: Sugahara, Toki ('94)

High Quality RMF models

- いくつかの RMF パラメータによる計算は、「質量公式」に迫る精度で原子核質量を記述！
→ High Quality RMF models.

TM, NL1, NL3,

- 全質量で 1-2 MeV の誤差 (NL3)
- Linear coupling ($\sigma N, \omega N, \rho N$), self-energy in σ, ω
- 場合によっては結合定数の密度依存性を導入。



NL3: Lalazissis, Konig, Ring, PRC55 ('97)540

RMF with Non-Linear Meson Int. Terms

- Are the Lagrangian parameters are well determined ?

$$\mathcal{L} = \mathcal{L}_{\text{free}}(\psi, \sigma, \omega, \rho, \dots) + \bar{\psi} [g_{\sigma}\sigma - g_{\omega}\gamma^0\omega - g_{\rho}\tau_z\gamma^0\rho] \psi + c_{\omega}\omega^4/4 - V_{\sigma}(\sigma), \quad (3)$$

$$V_{\sigma} = \begin{cases} \frac{1}{3}g_3\sigma^3 + \frac{1}{4}g_4\sigma^4 & (\text{NL1, NL3, TM1}) \\ -a_{\sigma}f_{\text{SCL}}(\sigma/f_{\pi}) & (\text{SCL}) \end{cases}, \quad (4)$$

- Linear terms, Meson-Nucleon Coupling → Well determined
- Negative Coef. of $\sigma^4 < 0$ in some of RMF models → Vacuum is unstable

TABLE II: RMF parameters

	$g_{\sigma N}$	$g_{\omega N}$	$g_{\rho N}$	$g_3(\text{MeV})$	g_4	c_{ω}	$m_{\sigma}(\text{MeV})$	$m_{\omega}(\text{MeV})$	$m_{\rho}(\text{MeV})$
NL1[18]	10.138	13.285	4.976	2401.9	-36.265	0	492.25	795.359	763
NL3[19]	10.217	12.868	4.474	2058.35	-28.885	0	508.194	782.501	763
TM1[6]	10.0289	12.6139	4.6322	1426.466	0.6183	71.3075	511.198	783	770
SCL[20](*)	10.08	13.02	4.40	1255.88	13.504	200	502.63	783	770

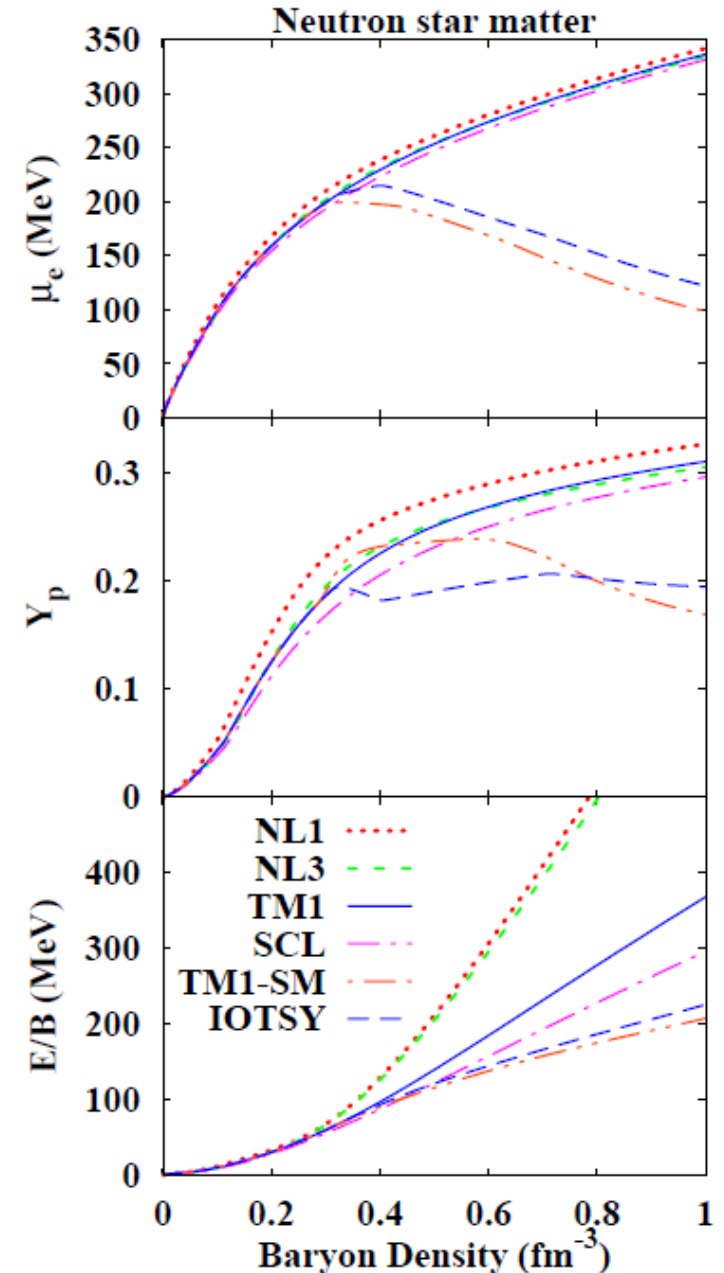
(*)1): g_3 and g_4 are from the expansion of f_{SCL} .

AO, Jido, Sekihara, Tsubakihara (2009)

Neutron Star Matter EOS

- Difference in non-linear meson terms generate different predictions of EOS at high densities

How can we fix non-linear terms ?



AO, Jido, Sekihara, Tsubakihara, Phys. Rev. C 80 (2009), 038202.

Summary

- **Nuclear Matter EOS is important in many subjects of physics.**
 - Bulk nuclear properties (B.E., radius)
 - Dense Matter in Compact Astrophysical Objects
 - High-Energy Heavy-Ion Collisions
- **Relativistic Mean Field models**
 - Simple description of nucleon scalar and vector potentials in terms of meson fields.
 - With non-linear meson interaction terms, nuclear binding energies (and radii) are well explained.
 - Ambiguities of non-linear couplings bring large differences of EOS at high densities, especially in asymmetric nuclear matter.
- **It is promising to utilize the results of G-matrix based on Chiral EFT (2 and 3 nucleon force), which reproduces the saturation density in an “ab initio” manner.**