原子核基礎論B

京大基研　大西　明
Akira Ohnishi (YITP, Kyoto U.)

1. 核力・特に非中心力や 3 体力 (1 回)
2. 原子核構造を記述するための種々の模型の最近の進展 (2 回)
3. 最近の中性子過剰核の物理の最近の進展 (2 回)
4. 原子核構造における異なる状態の混合や競合 (2 回)

板垣

5. 高温・高密度核物質概観 (1 回) (高エネルギー重イオン衝突、コンパクト天体現象)
   → 前期の Sec. 3 と重なりが大きいのでスキップ
6. 有限温度・密度における場の理論入門 (2 回)
7. QCD 有効模型における相転移と相図 (2 回)
8. 有限温度・密度格子 QCD と符号問題 (1 回)
9. 高エネルギー重イオン衝突における輸送理論 (1 回)

大西

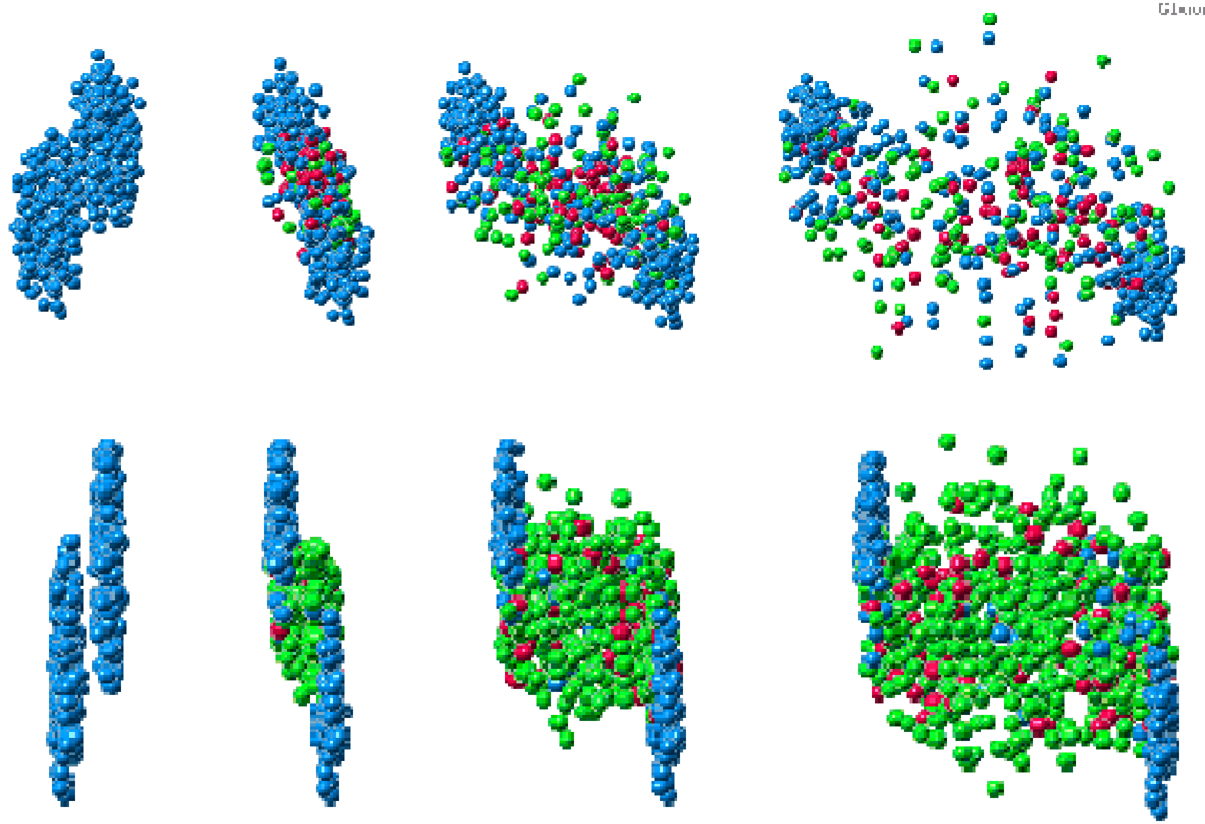
Nuclear Transport Models and Heavy-Ion Collision

Heavy-Ion Collisions at Einc ~ (1-100) A GeV

- **Study of Hot and Dense Hadronic Matter**
  → Particle Yield, Collective Dynamics (Flow), EOS, ...

AGS

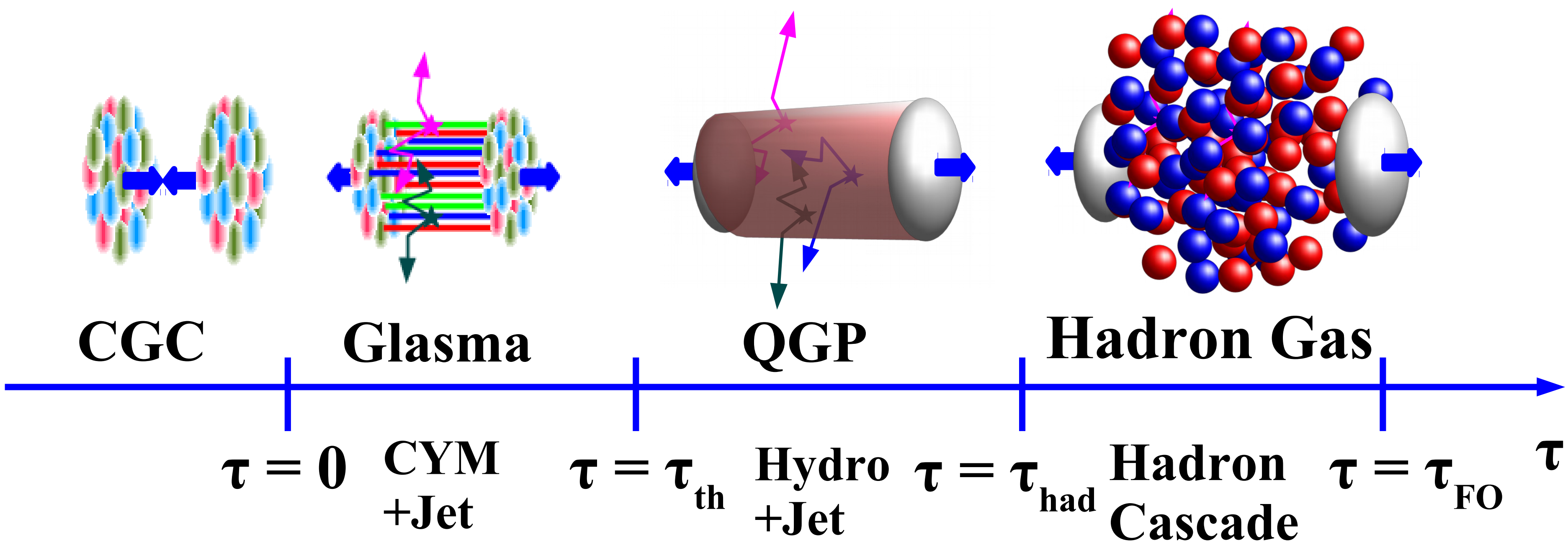
SPS



JAMming on the Web, linked from http://www.jcprg.org/

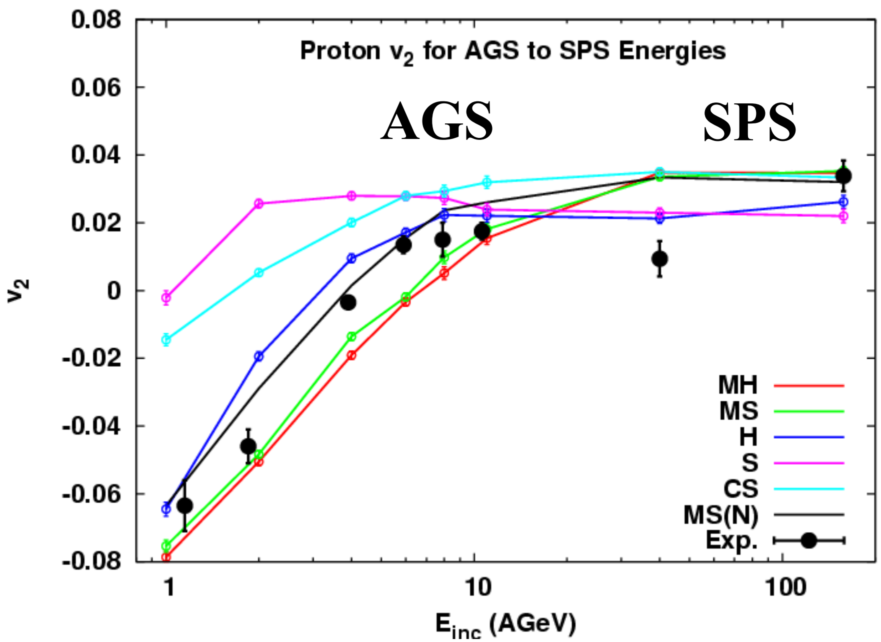
RHIC における2つの驚き (cont.)

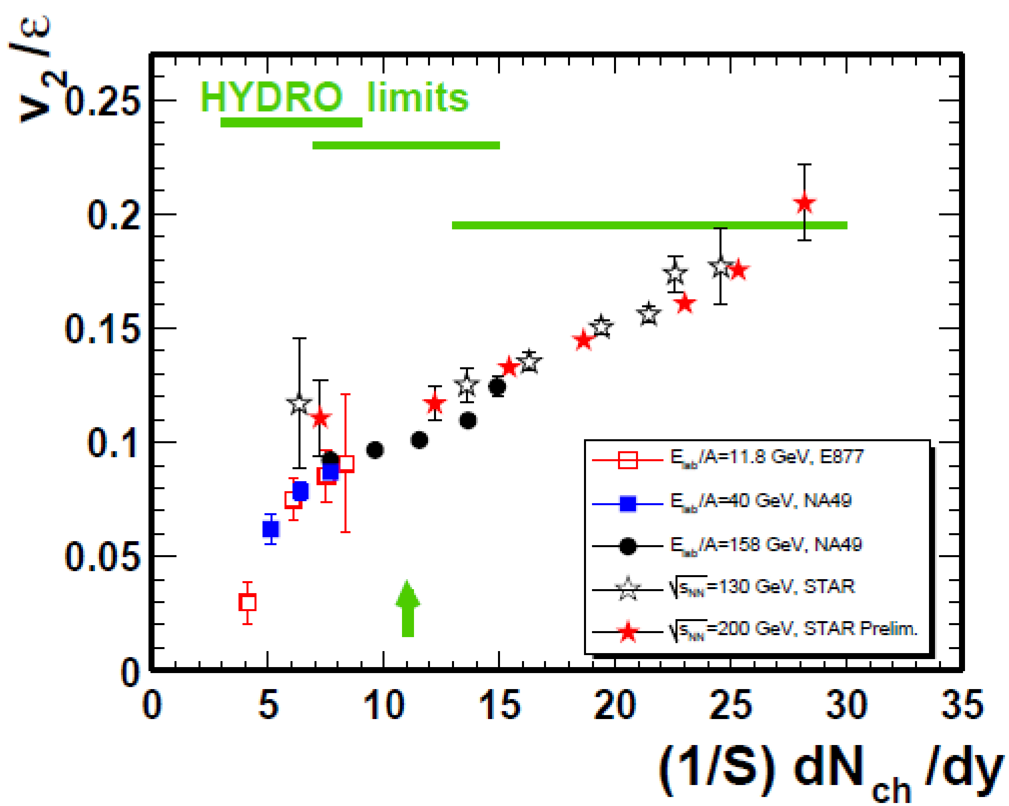
- **2つの驚き (2): 早い熱平衡化**
  - **摂動論的 QCD の予言 (2-5 fm/c) に比べて有意に早い時刻 (0.6-1 fm/c) で熱化が起こり、流体力学的時間発展が進む。**
    → なぜ早い？
  - **高エネルギー重イオン衝突の初期条件**
    = グラズマ (古典ヤンミルズ場が主要)



Hydrodynamics vs Transport

- $\sqrt{s_{NN}}$ < 20 GeV → Transport model calculation seems to explain v2 data.
- RHIC (& LHC) → Hydrodynamics is successful.





M. Isse, AO, N. Otuka, P.K. Sahu, Y. Nara, PRC72 ('05) 064908 [nucl-th/0502058]

U. W. Heinz, AIP Conf.Proc. 739 ('05) 163 [nucl-th/0407067]

HIC Transport Models: Five Major Origins

- *Nuclear Mean Field Dynamics*
  - **Basic Element of Low Energy Nuclear Physics**
  - **TDHF → Vlasov → BUU**
- *NN two-body (residual) interaction*
  - **Main Source of Particle Production**
  - **Boltzmann equation → Intranuclear Cascade Models**
- *Partonic Interaction and String Decay*
  - **Main Source of high pT Particles at Collider Energies**
  - **JETSET + (previous) PYTHIA (Lund model) → (new) PYTHIA**
- *Relativistic Hydrodynamics*
  - **Most Successful Picture at RHIC**
- *Classical Field Dynamics*
  - **Classical Gluon Field = Initial condition of Hydro. at Collider Energies**

Nuclear Mean Field Dynamics

TDHF and Vlasov Equation

- **Time-Dependent Mean Field Theory (e.g., TDHF)** $i\hbar\frac{\partial\phi_i}{\partial t} = h\phi_i$
- **Density Matrix**

$$\rho(\boldsymbol{r},\boldsymbol{r}') = \sum_i^{\mathrm{Occ}} \phi_i(\boldsymbol{r})\phi_i^*(\boldsymbol{r}') \quad \to \rho_W = f \text{ (phase space dist.)}$$

- **TDHF for Density Matrix**

$$i\hbar\frac{\partial\rho}{\partial t} = [h,\rho] \quad \to \frac{\partial f}{\partial t} = \{h_W, f\} + \mathcal{O}(\hbar^2)$$

$$(AB)_W = A_W \exp(i\hbar\Lambda) B_W \ , \quad \Lambda = \boldsymbol{\nabla}'_r\cdot\boldsymbol{\nabla}_p - \boldsymbol{\nabla}'_p\cdot\boldsymbol{\nabla}_r \ (\boldsymbol{\nabla}' \text{ acts on the lef}$$

- **Wigner Transformation and Wigner-Kirkwood Expansion**

(Ref.: Ring-Schuck)

$$A_W(\boldsymbol{r},\boldsymbol{p}) = \int d^3s \exp(-i\boldsymbol{p}\cdot\boldsymbol{s}/\hbar)\,\langle \boldsymbol{r}+\boldsymbol{s}/2 \mid A \mid \boldsymbol{r}-\boldsymbol{s}/2\rangle$$

$$(AB)_W = A_W \exp(i\hbar\Lambda) B_W \quad \Lambda \equiv \nabla'_r\cdot\nabla_p - \nabla'_p\cdot\nabla_r \quad (\nabla' \text{acts on the left})$$

$$[A,B]_W = 2iA_W \sin(\hbar\Lambda/2) B_W = i\hbar\{A_W, B_W\}_{P.B.} + O(\hbar^3)$$

Test Particle Method

- **Vlasov Equation**

$$\frac{\partial f}{\partial t} - \{h_W, f\}_{P.B.} = \frac{\partial f}{\partial t} + v \cdot \nabla_r f - \nabla U \cdot \nabla_p f = 0$$

- **Classical Hamiltonian**

$$h_W(r,p) = \frac{p^2}{2m} + U(r,p)$$

- **Test Particle Method (C. Y. Wong, 1982)**

$$f(r,p) = \frac{1}{N_0} \sum_i^{AN_0} \delta(r - r_i)\,\delta(p - p_i) \quad \rightarrow \quad \frac{dr_i}{dt} = \nabla_p h_W, \quad \frac{dp_i}{dt} = -\nabla_r h_W,$$

Mean Field Evolution can be simulated by Classical Test Particles
→ Opened a possibility to Simulate High Energy HIC including Two-Body Collisions in Cascade

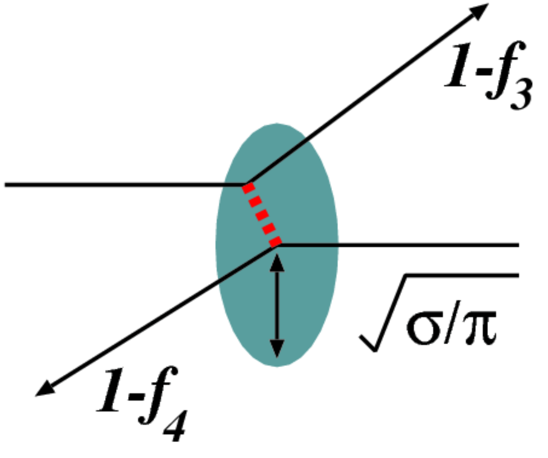
BUU (Boltzmann-Uehling-Uhlenbeck) Equation

- **BUU Equation** **(Bertsch and Das Gupta, Phys. Rept. 160(88), 190)**

$$\frac{\partial f}{\partial t} + v \cdot \nabla_r f - \nabla U \cdot \nabla_p f = I_{coll}[f]$$

$$I_{coll}[f] = -\frac{1}{2} \int \frac{d^3 p_2 \, d\Omega}{(2\pi\hbar)^3} v_{12} \frac{d\sigma}{d\Omega} \times [f f_2 (1-f_3)(1-f_4) - f_3 f_4 (1-f)(1-f_2)]$$

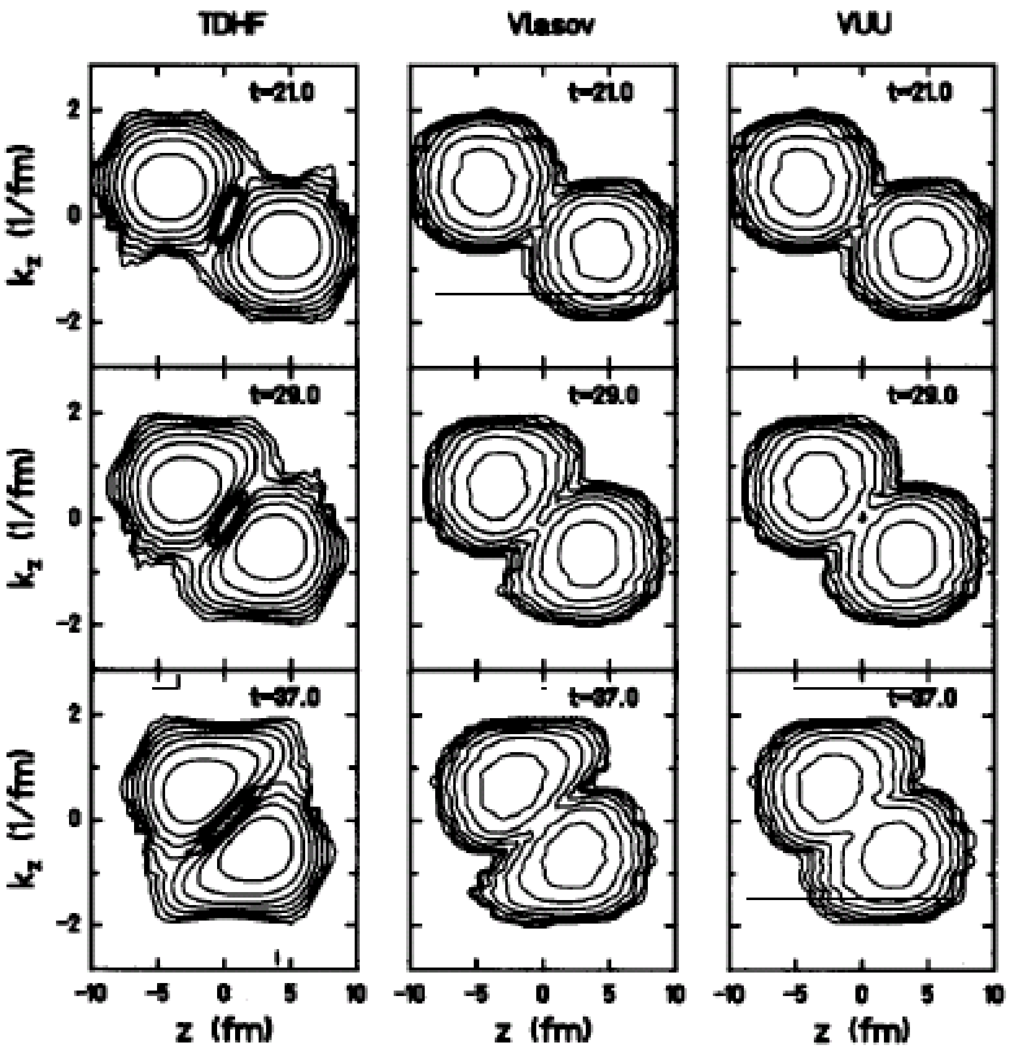
- **Incorporated Physics in BUU**
  - **Mean Field Evolution**
  - **(Incoherent) Two-Body Collisions**
  - **Pauli Blocking in Two-Body Collisions**



○ ***One-Body Observables (Particle Spectra, Collective Flow, ..)***
X ***Event-by-Event Fluctuation (Fragment, Intermittency, ...)***

Comarison of TDHF, Vlasov and BUU(VUU)

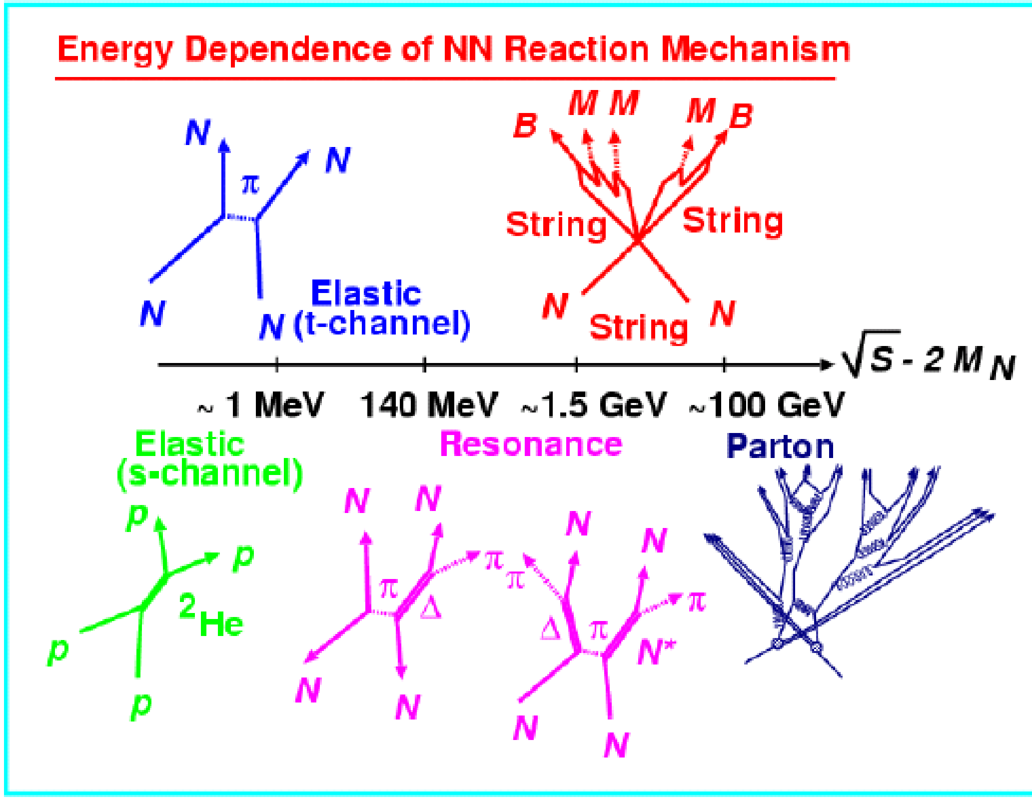
- **Ca+Ca, 40 A MeV (Cassing-Metag-Mosel-Niita, Phys. Rep. 188 (1990) 363).**



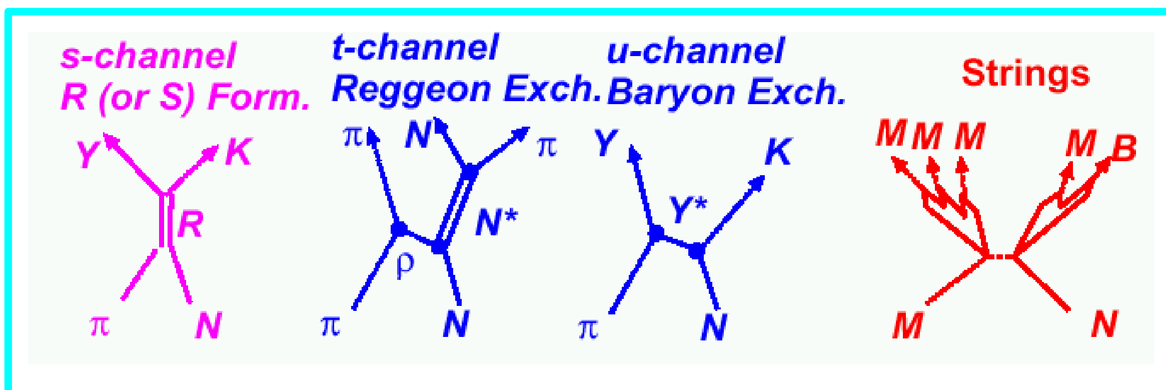
Collision Term and Particle Production

Baryon-Baryon and Meson-Baryon Collisions

- **NN collision mechanism**
  Elastic
  → Resonance
  → String
  → Jet

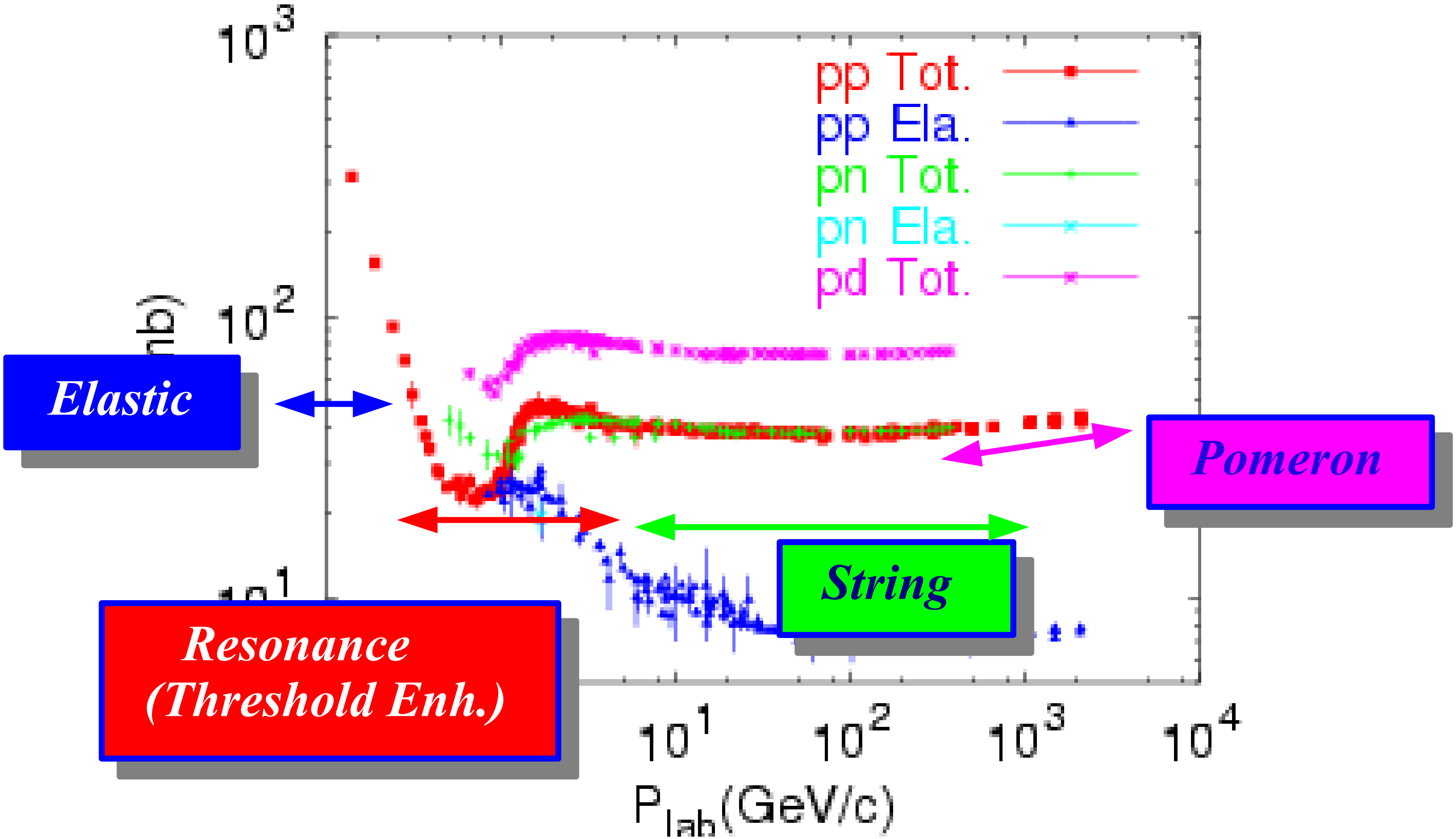


- **Meson-Nucleon Collision**
  → s-channel Resonance
  → t-(u-) channel Res.
  → String formation

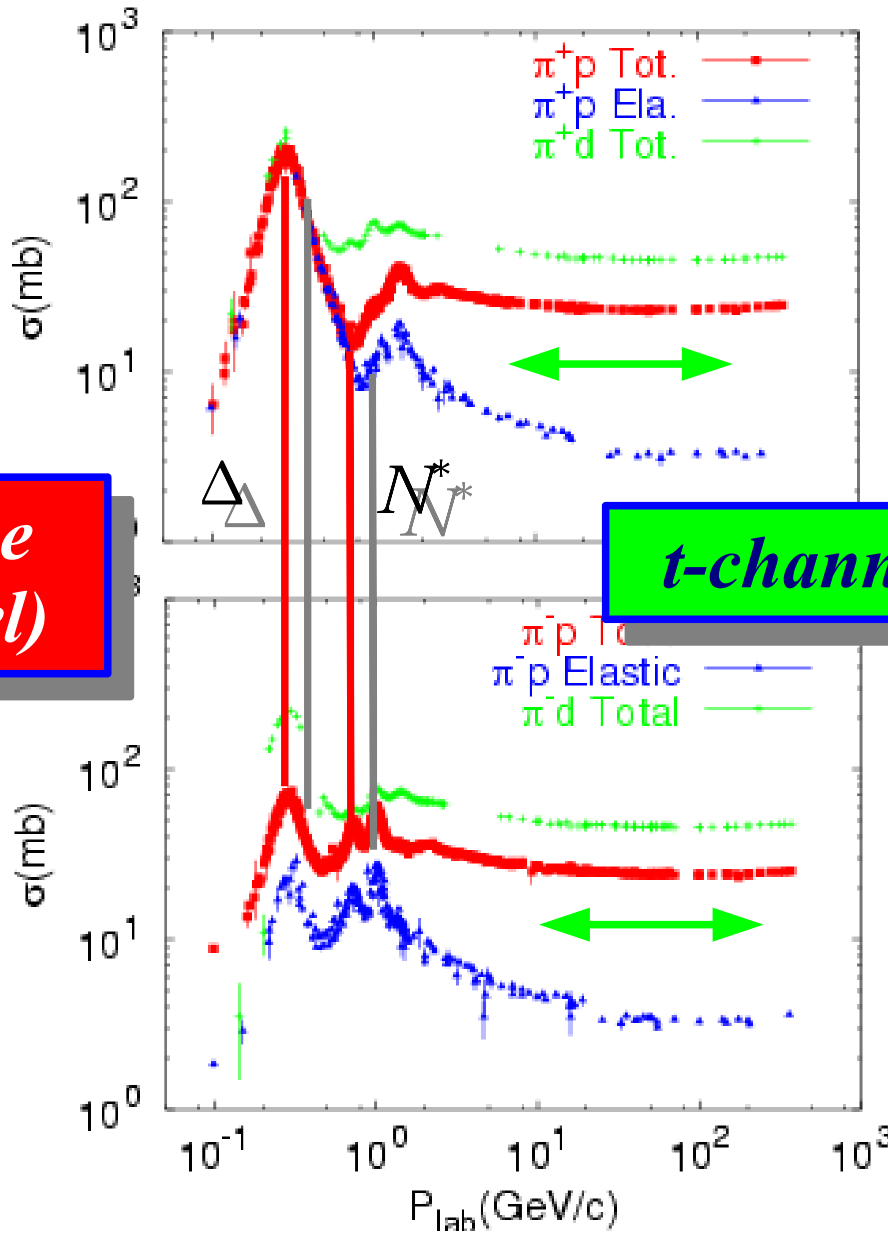


NN Cross Sections

From Particle Data Group



Meson-Baryon Cross Section



Resonance (s-channel)

t-channel and String

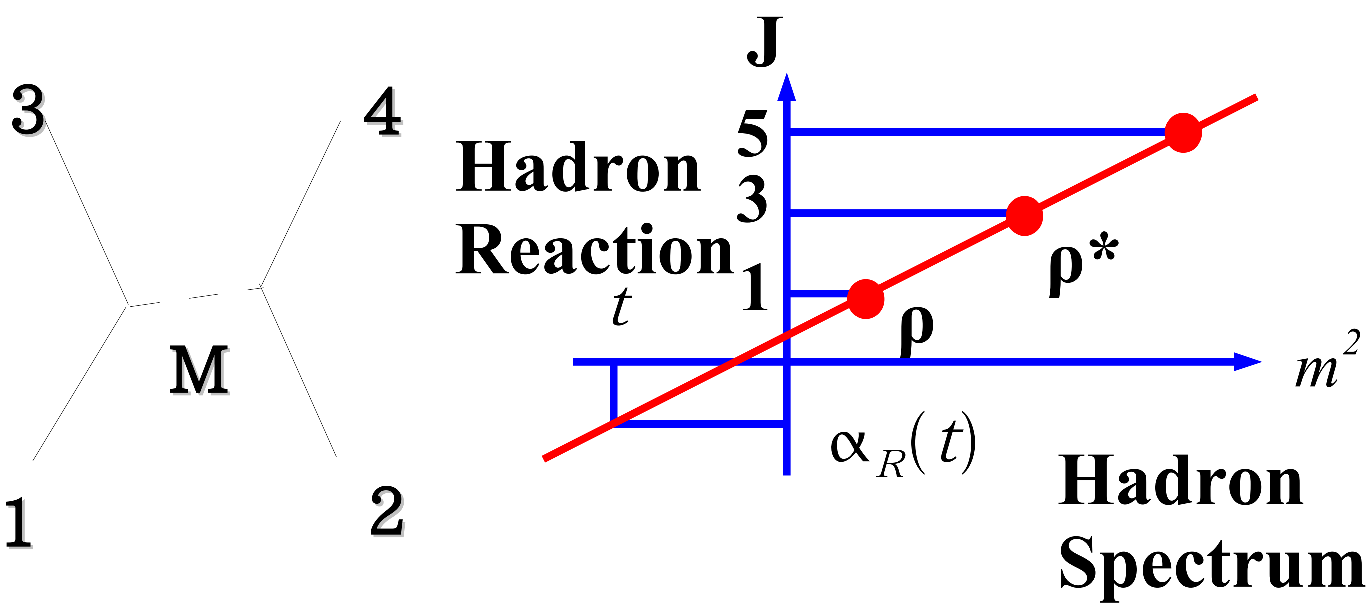
Reggeon Exchange

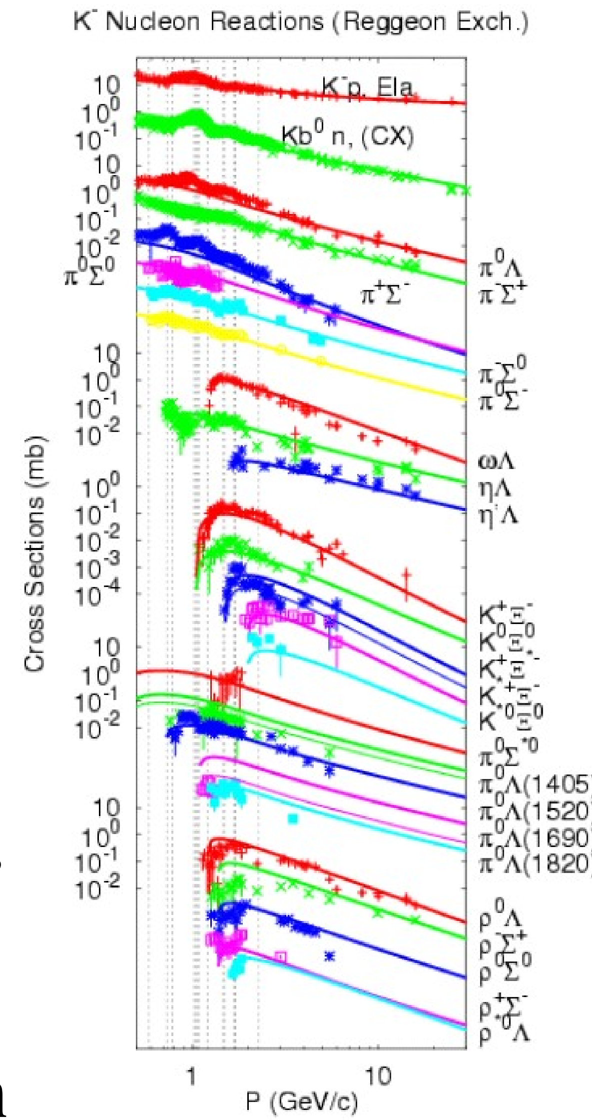
(Barger and Cline (Benjamin, 1969), H. Sorge, PRC (1995), RQMD2.1)

- **Regge Trajectory** $J = \alpha_R(t) \sim \alpha_R(0) + \alpha'_R(0)\, t$
- **2 to 2 Cross Section**

$$\frac{d\sigma}{d\Omega} = \frac{p_f}{64\pi\, s\, p_i} |M(s,t)|^2$$

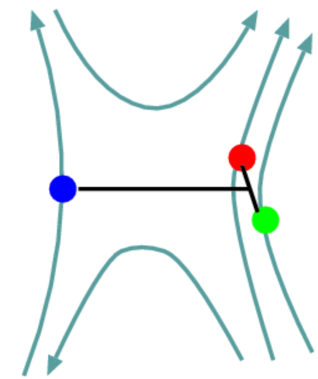
$$M(s,t) \sim \sum_R \frac{(p_i p_f)^J}{t - M_R} \sim F(t) \exp[\alpha_R(t) \log(s/s_0)]$$





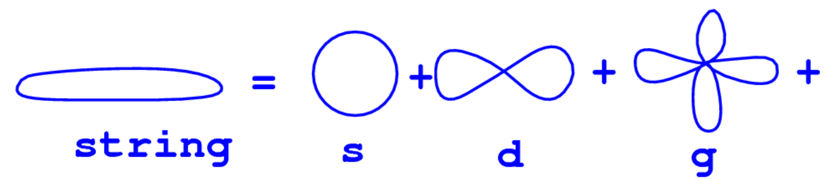
String formation and and decay

- What does the regge trajectory suggest ?
  → Existence of (color- or hadron-)String !



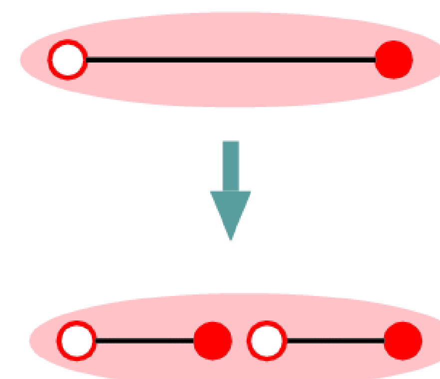
$$M=2\int_0^R \frac{\kappa\, dr}{\sqrt{1-(r/R)^2}}=\pi\kappa R \quad , \quad J=2\int_0^R r\times\frac{\kappa\, dr}{\sqrt{1-(r/R)^2}}\frac{r}{R}=\frac{\pi\kappa R^2}{2}\pi$$

$$\rightarrow \quad J=\frac{M^2}{2\pi\kappa}$$



- String Tension

$$\frac{1}{2\pi\kappa}=\alpha'_R(0)\approx 0.9\,\mathrm{GeV}^{-2} \quad \rightarrow \quad \kappa\approx 1\,\mathrm{GeV/fm}$$

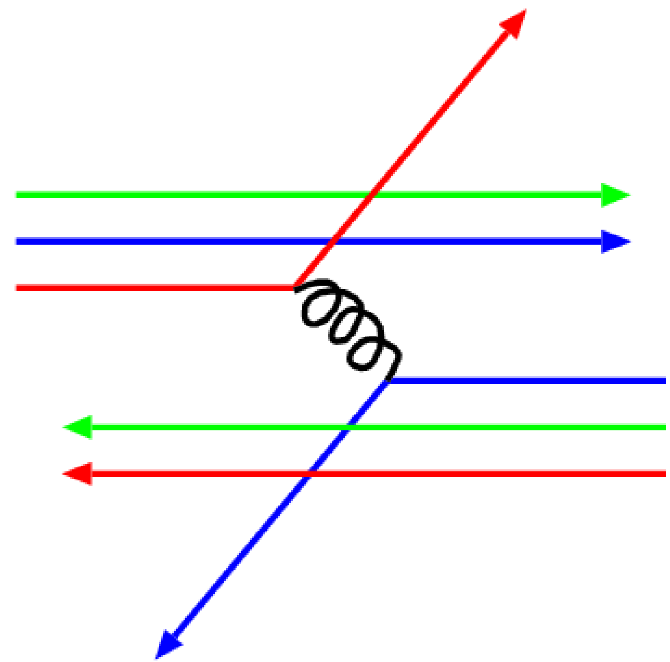


- String decay
  Extended String
  → Large E stored
  → q qbar pair creation (Schwinger mech.)

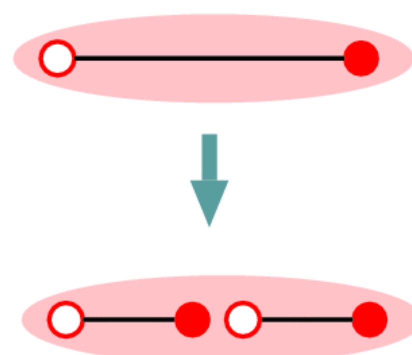
String = Coherent superposition of hadron resonances with various J

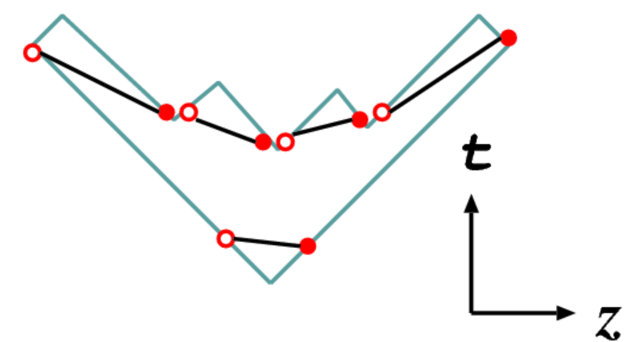
Jet Production

- **Elastic Scattering of Partons (mainly) with One Gluon Exch.**
- **Color Exch. between Hadrons**
  → Complex color flux starting from leading partons
  → many hadron production
  → Jet production



- ***PYTHIA***
  - **Event Generator of High Energy Reactions**
    → Jet production +String decay
    for QCD processes



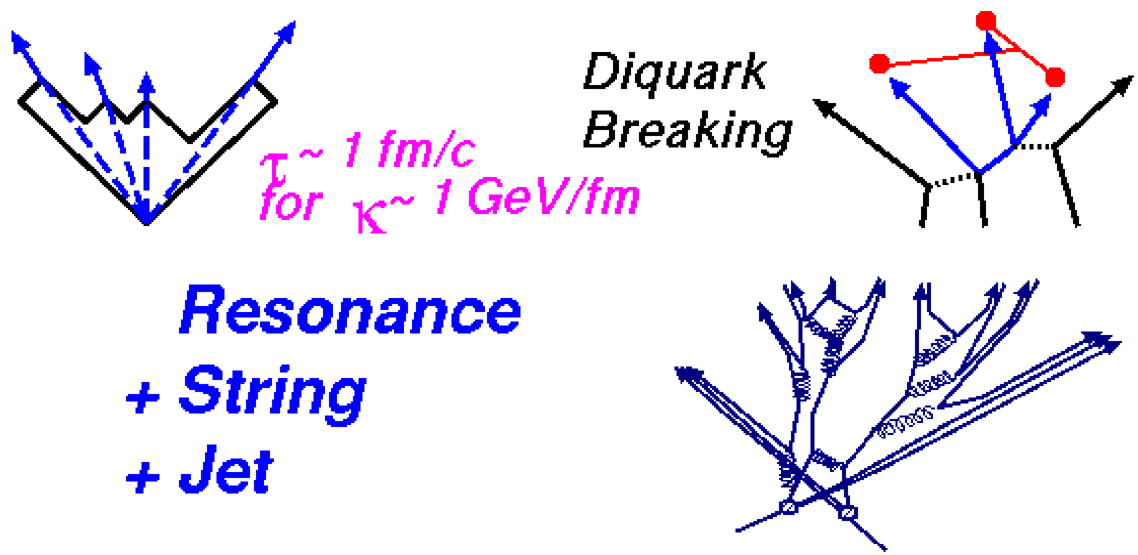


(T. Sjostrand et al., Comput. Phys. Commun. 135 (2001), 238.)

JAM (Jet AA Microscopic transport model)

Nara, Otuka, AO, Niita, Chiba, Phys. Rev. C61 (2000), 024901.

- **Hadron-String Cascade with Jet production**
  - **hh collision with Res. up to m < 2 GeV (3.5 GeV) for M (B)**
  - **String excitation and decay**
  - **String-Hadron collisions are simulated by hh collisions in the formation time.**
  - **jet production is incl. using PYTHIA**
  - **Secondary partonic int.:**
    NOT incl.
  - **Color transparency:**
    NOT taken care of



Relativistic Hydrodynamics

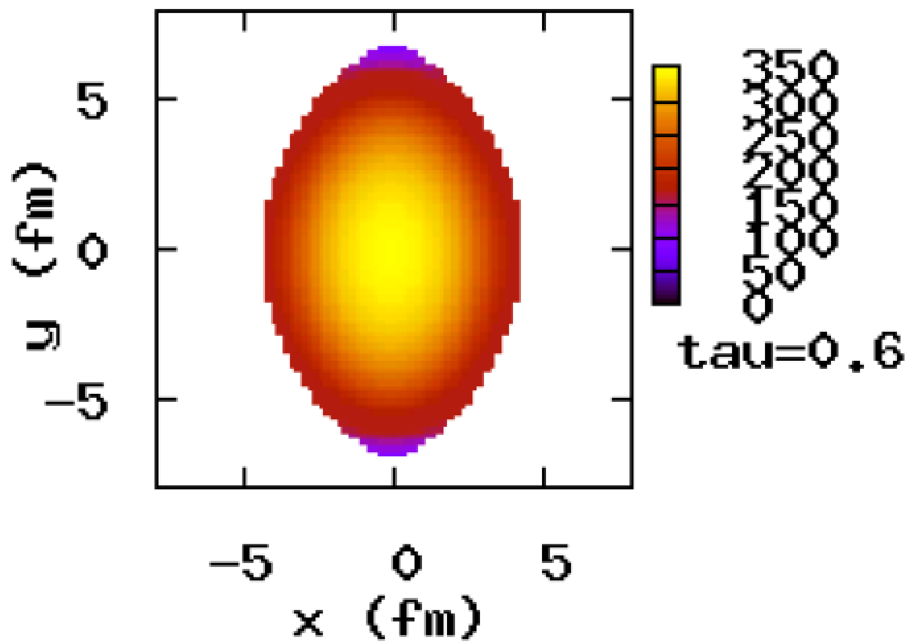
Relativistic Hydrodynamics

- **EOM: Conservation Laws**

$\partial_\mu T^{\mu\nu} = 0$ Energy Momentum Conservation
$\partial_\mu (n_i u^\mu) = 0$ Conservation of Charge (Baryon, Strangeness, ...)

$$T^{\mu\nu} = (e + P)u^\mu u^\nu - P g^{\mu\nu}$$

e : energy density, *P: pressure,*
u^μ: four velocity $\gamma(1,v)$,
n_i: number density

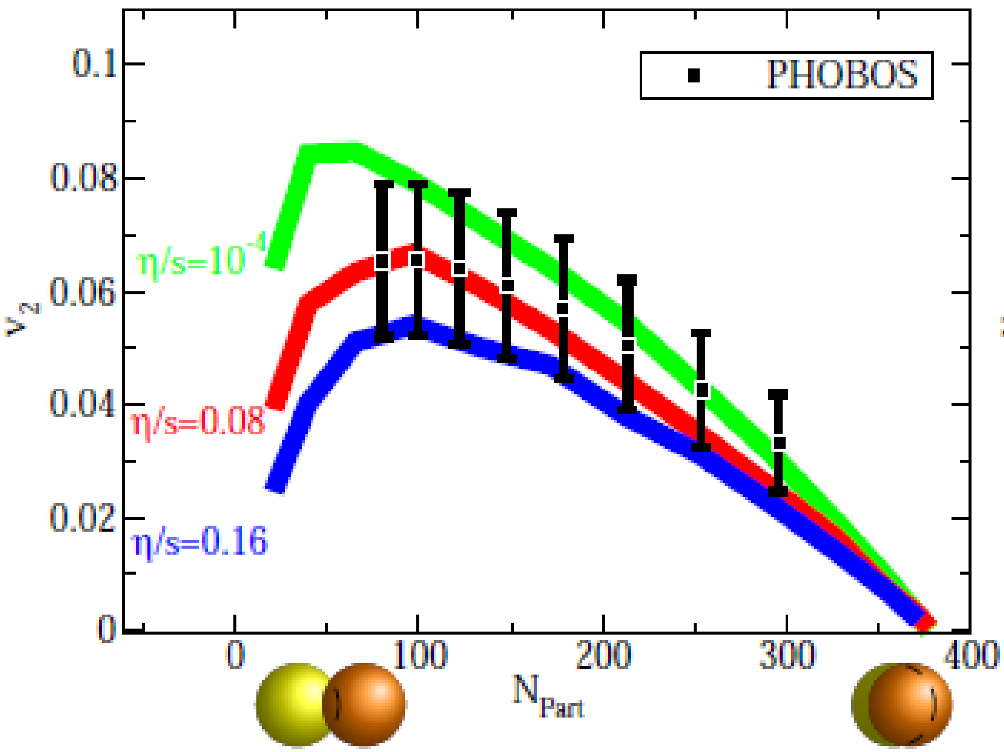


T. Hirano, Y. Nara, NPA743, 305 (2004)
T. Hirano, K. Tsuda, PRC 66, 054905(2002)

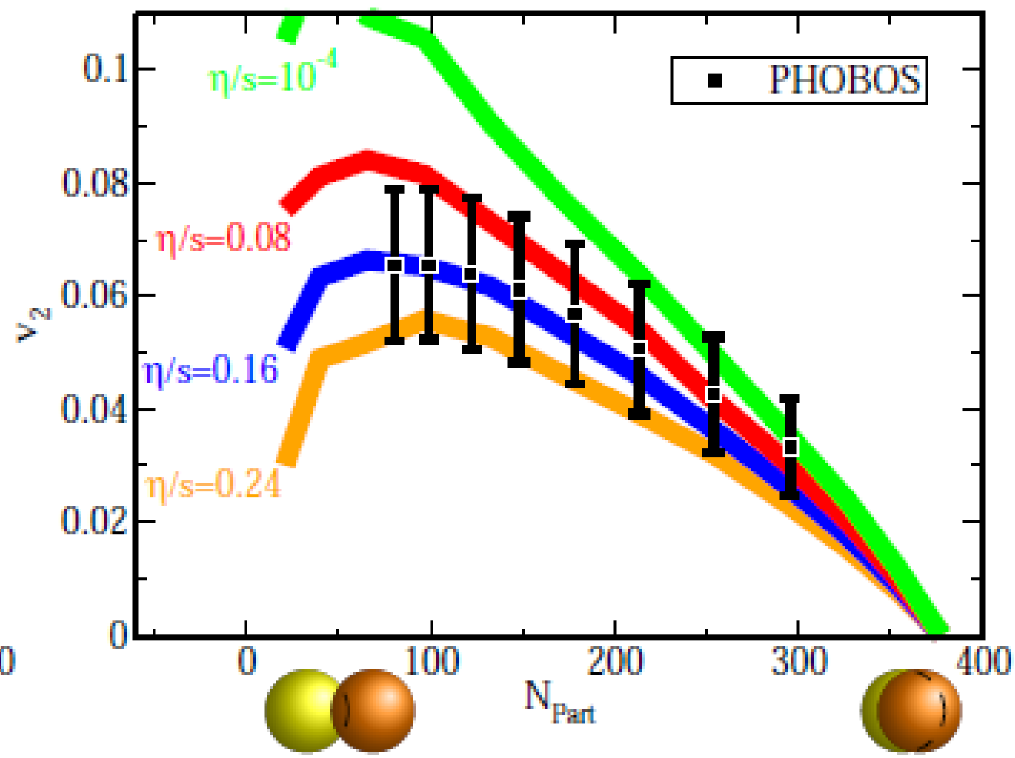
QM2012, Luzum

η/s from flow (historical)

“Glauber” initial conditions



“CGC” initial conditions



(ML & Romatschke, Phys.Rev. C78 (2008) 034915)

- Best extraction of η/s by comparing viscous hydro to flow data
- Largest uncertainty from unknown initial condition

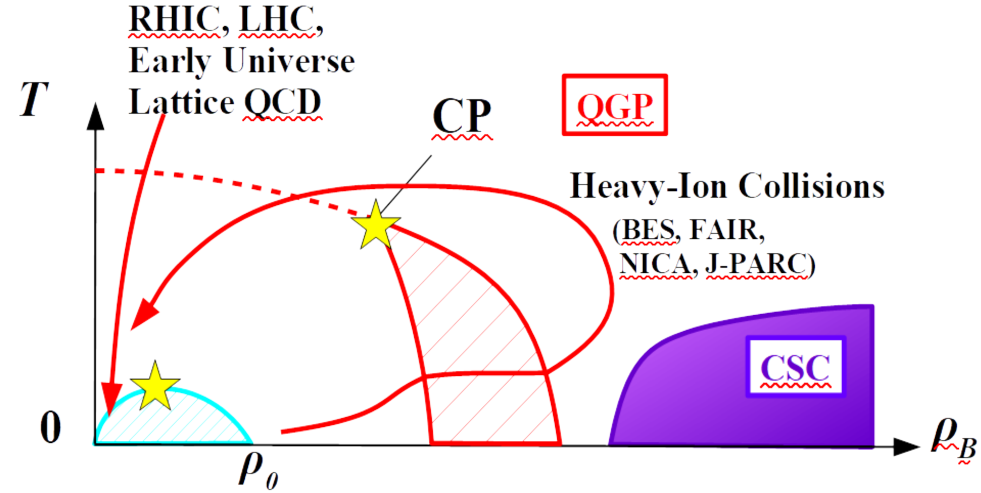
低エネルギーでのハドロン輸送模型の成功
＋高エネルギーでの流体模型の成功
→ QGP 生成の始まりは $\sqrt{s_{NN}}$=40 GeV 程度か？

$\sqrt{s_{NN}}$=(5-20)GeV で

QCD 相転移は見えるか？

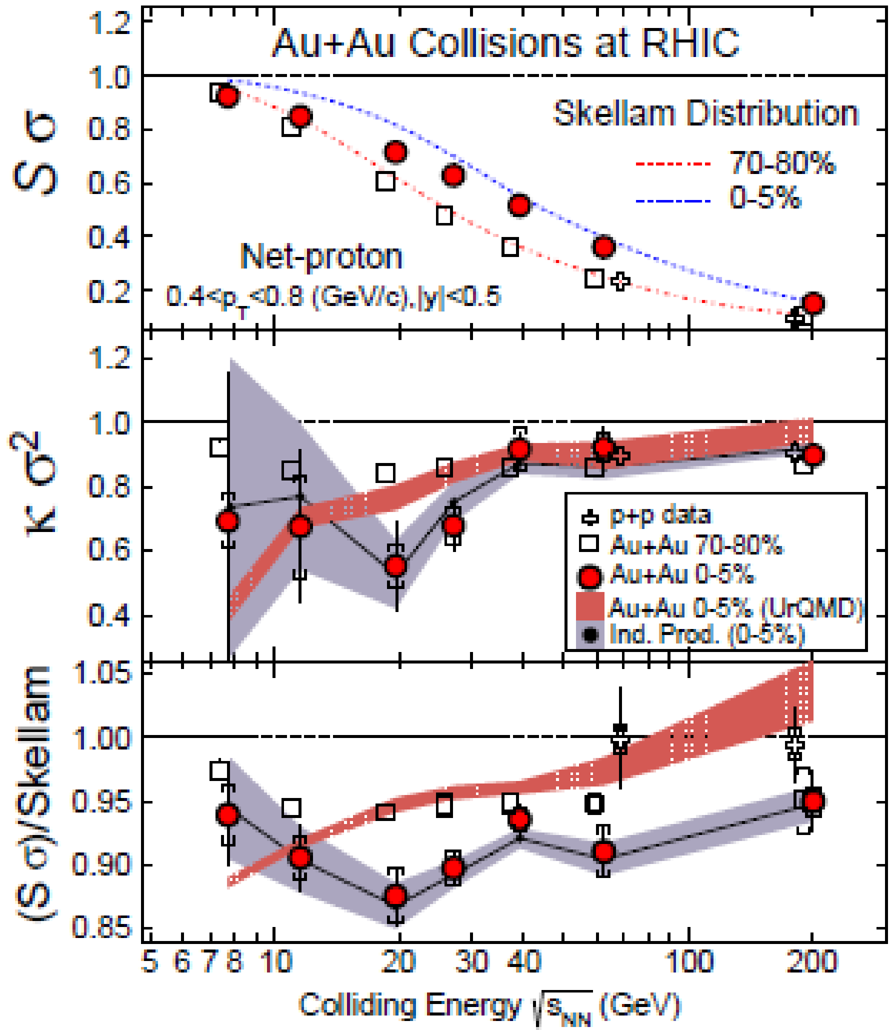
高エネルギー重イオン衝突の物理

- クォーク・グルーオン・プラズマ (QGP) の発見・物性
  - 流体模型とジェットによる記述の成功 → 精緻化へ (RHIC の最高エネルギー、LHC)
- 高密度領域における状態方程式 (EOS)・QCD 相転移
  - $\sqrt{s_{NN}}$=(5-20) GeV の衝突エネルギー領域 → $\rho=(5\text{-}10)\rho_0$
  - 高密度での EOS は決められるか？
  - 高密度での相転移は 1 次か？クロスオーバーか？

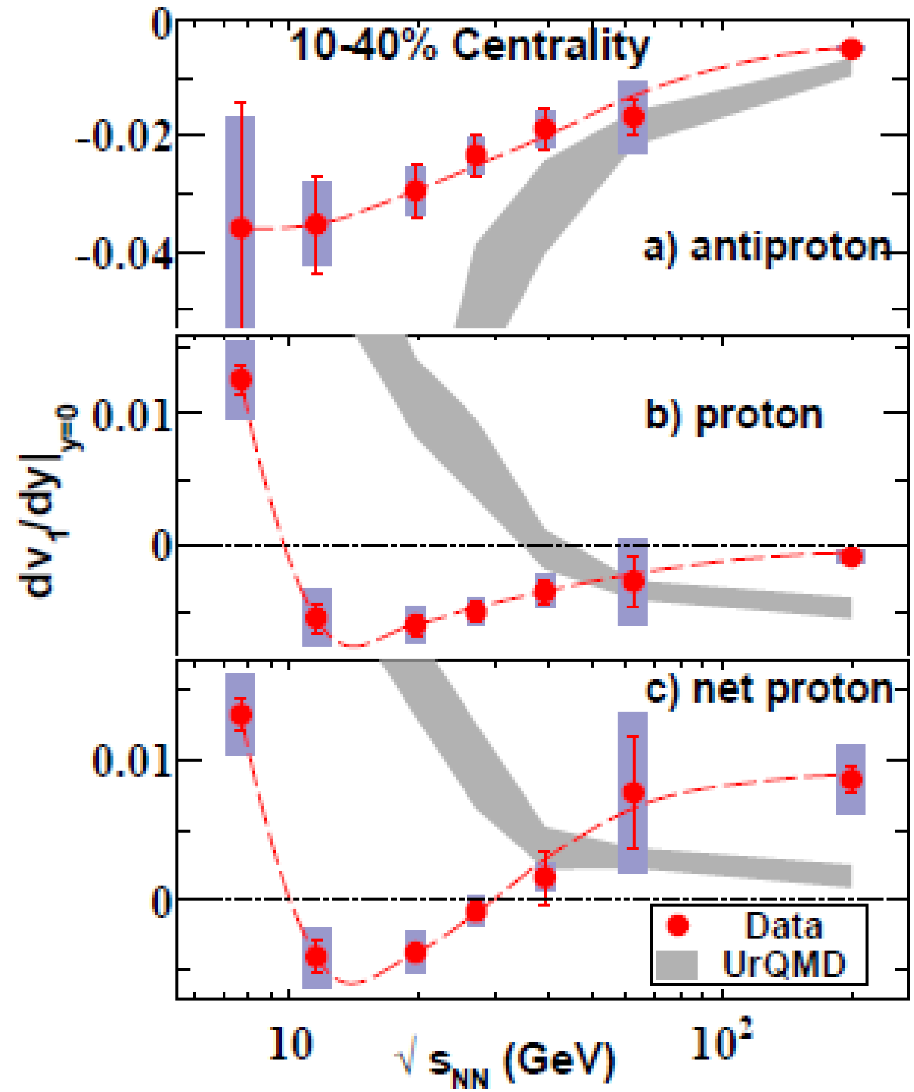


QCD 一次相転移は見えたか？

- **衝突エネルギー関数として非単調性が見えている ($\kappa\sigma^2$, dv_1/dy)**



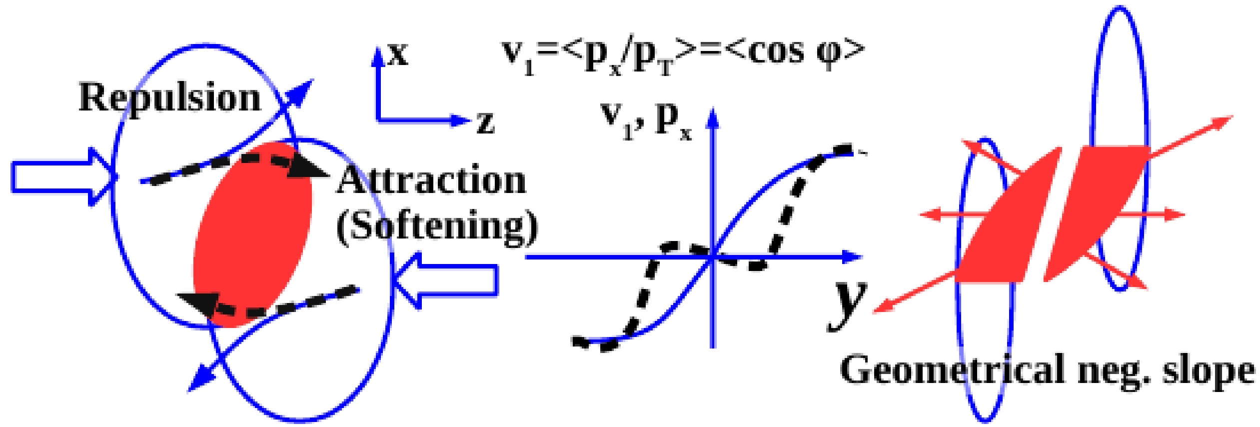
STAR Collab., PRL 112('14)032302.

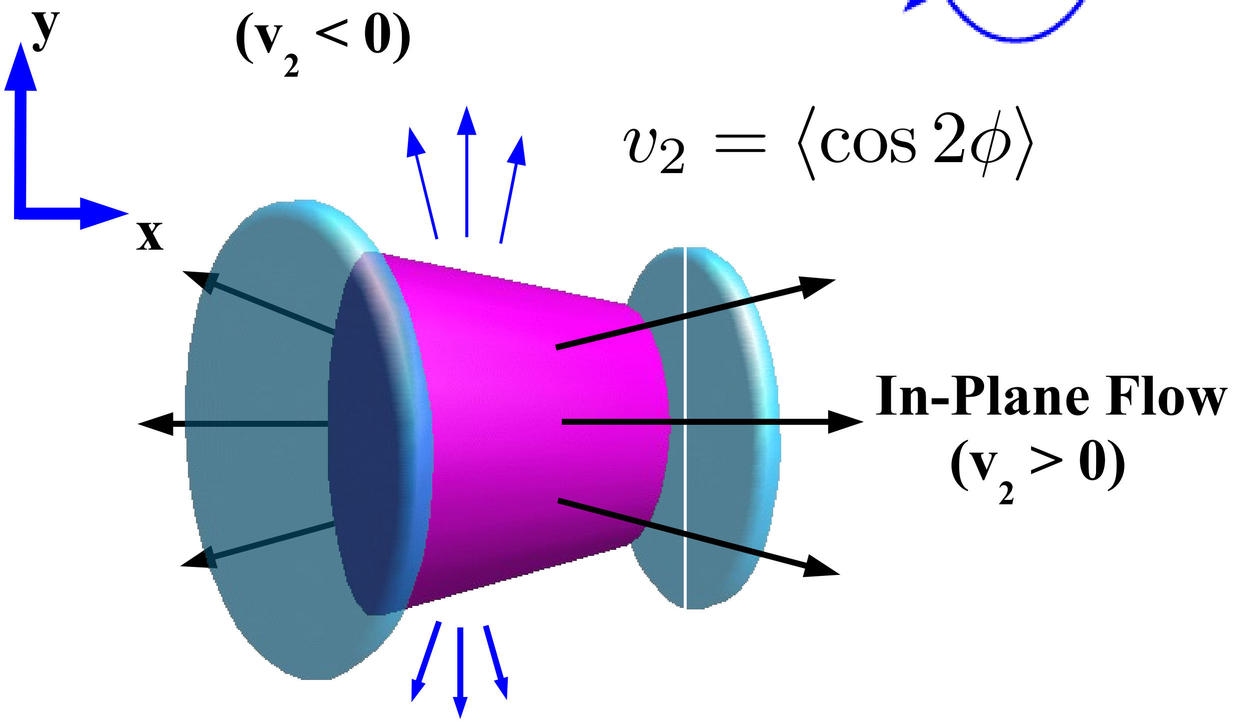


STAR Collab., PRL 112('14)162301.

集団フロー

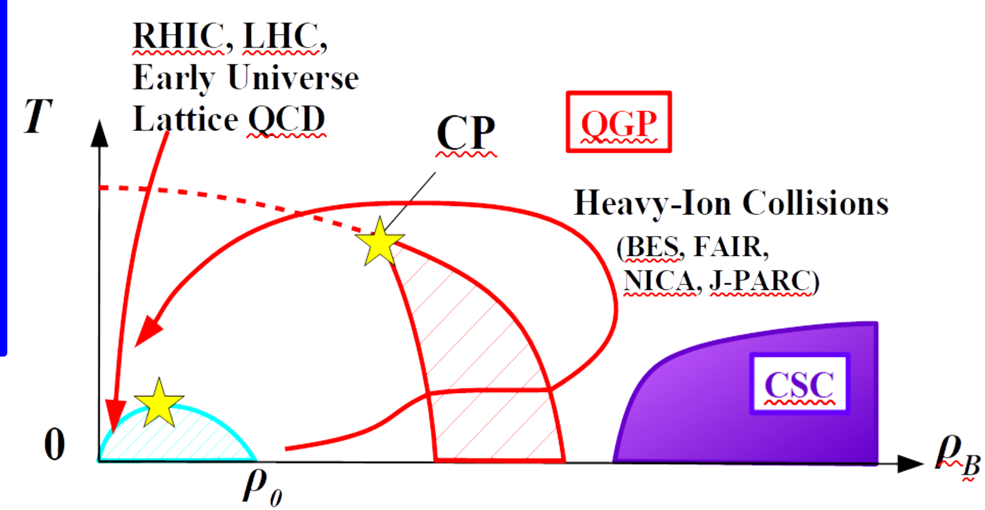
- Directed flow (v_1, <p_x>), Elliptic flow (v_2)
  → 衝突初期に作られ、高密度の状態方程式 (EOS) に敏感





高エネルギー重イオン衝突の物理

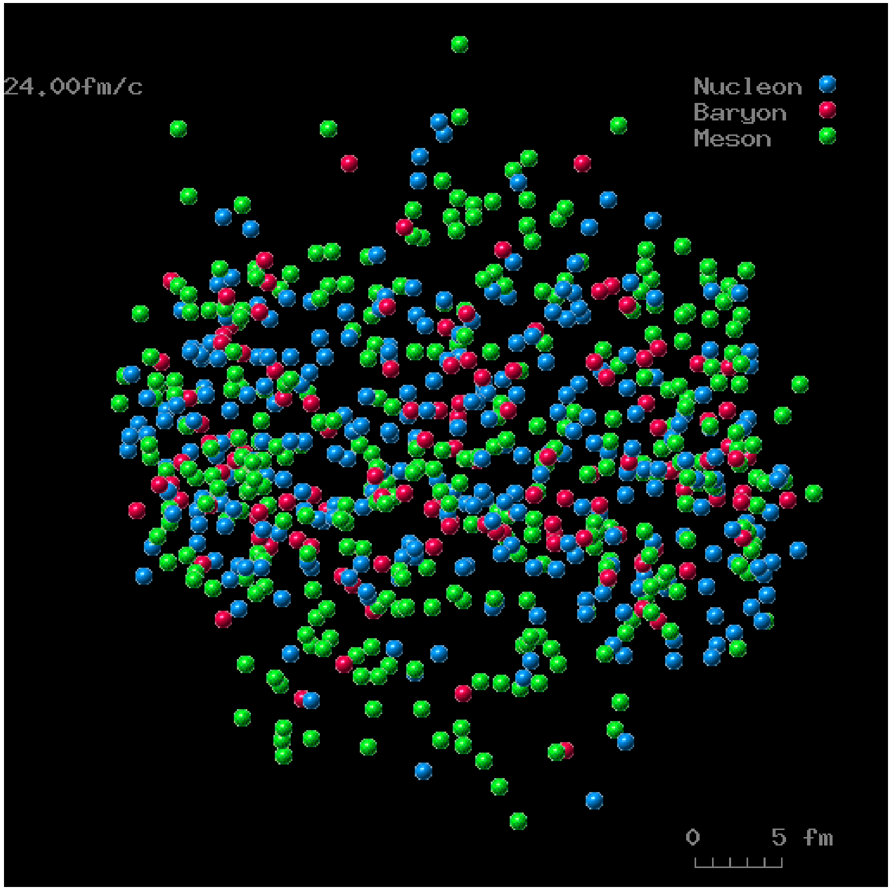
- クォーク･グルーオン･プラズマ (QGP) の発見･物性
  - 流体模型とジェットによる記述の成功 → 精緻化へ (RHIC の最高エネルギー、LHC)
- 高密度領域における状態方程式 (EOS)・QCD 相転移
  - $\sqrt{s_{NN}}$=(5-20) GeV の衝突エネルギー領域 → $\rho=(5\text{-}10)\rho_0$
  - 高密度での EOS は決められるか？
  - 高密度での相転移は 1 次か？クロスオーバーか？
- ハドロン輸送模型に基づく研究
  - ハドロン輸送模型 JAM
  - EOS の導入とフロー
  - 負の側方フローと EOS の軟化

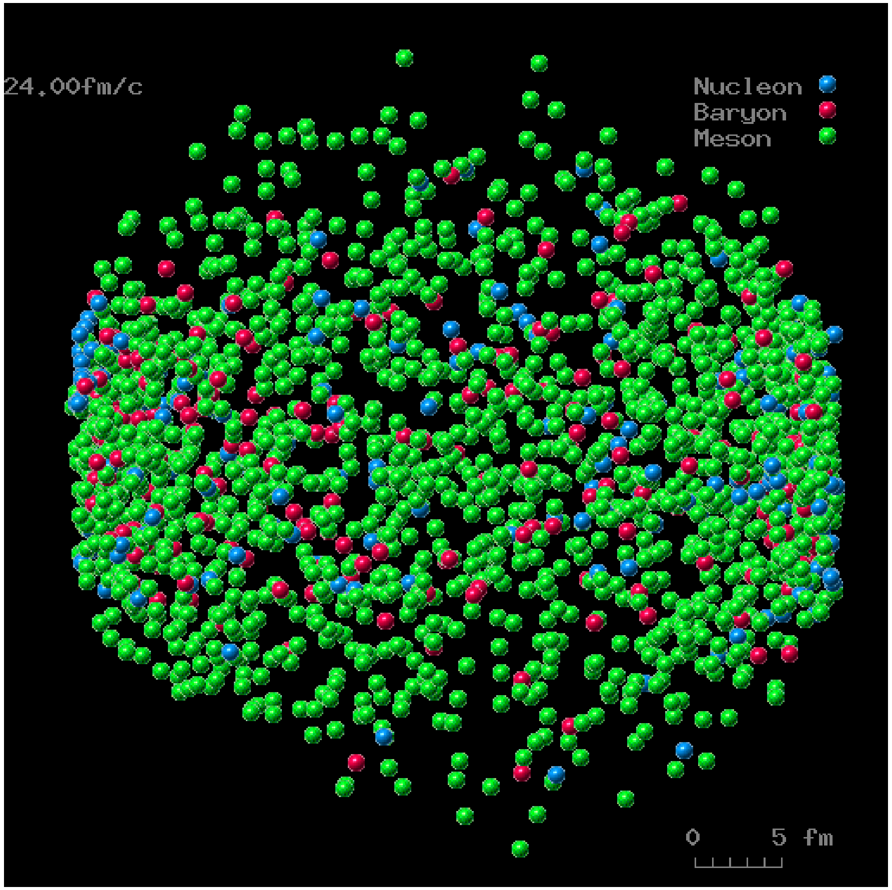


How do heavy-ion collisions look like ?

Au+Au, 10.6 A GeV

Pb+Pb, 158 A GeV





$\sqrt{s_{NN}}$ ~ 5 GeV

$\sqrt{s_{NN}}$ ~ 20 GeV

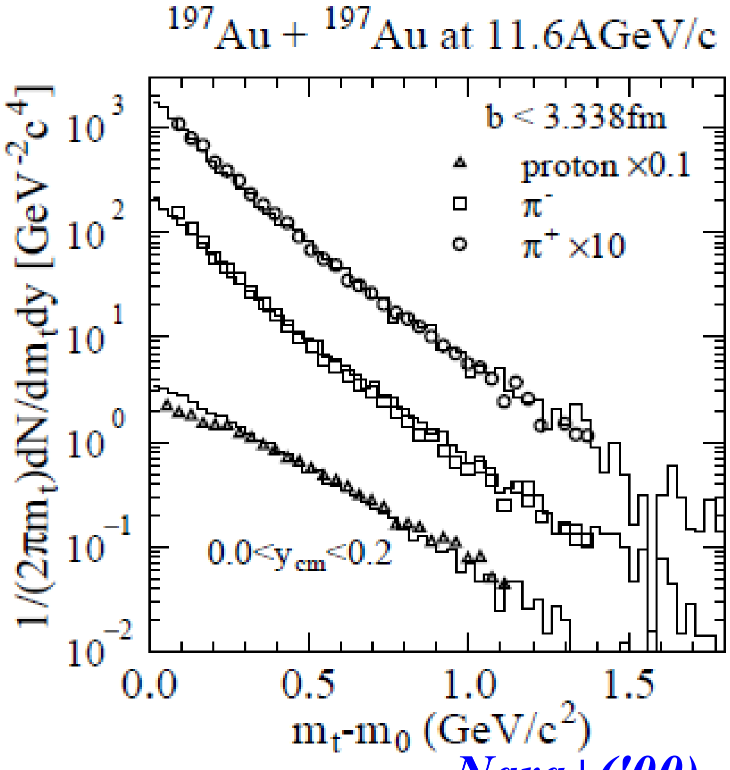
JAMming on the Web **http://www.jcprg.org/jow/**

YITP

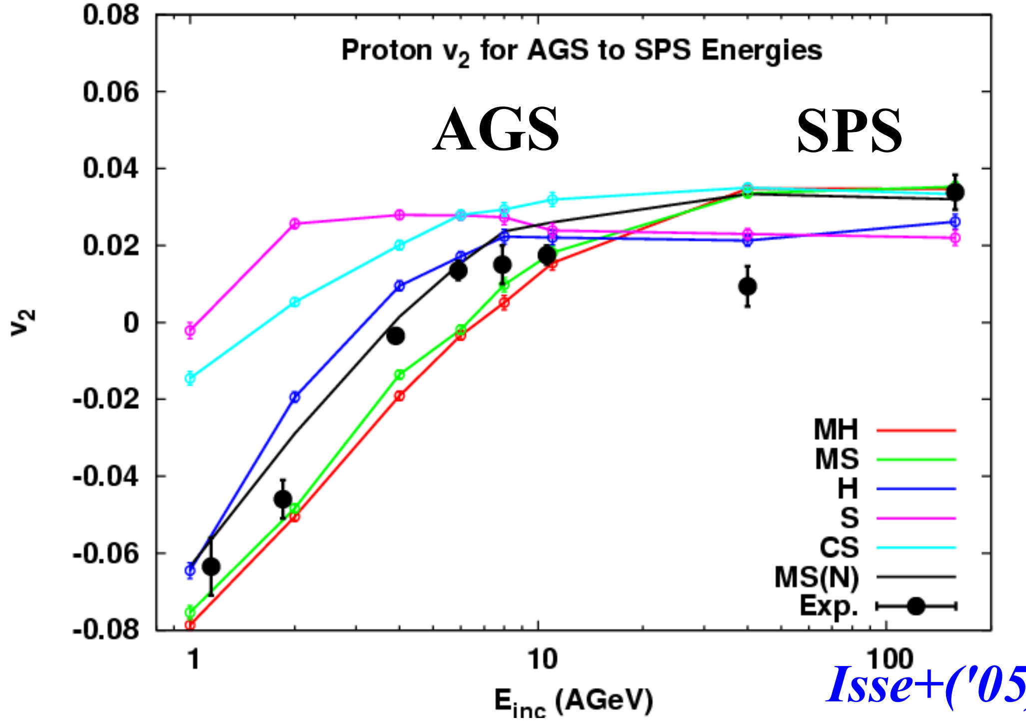
ハドロン輸送模型JAM

Y.Nara, N.Otuka, AO, K.Niita, S.Chiba, PRC61('00) , 024901
M.Isse, AO, N.Otuka, P.K.Sahu, Y.Nara, PRC72 ('05)064908

- **Event Generator: Jet AA Microscopic transport model (JAM)**
  - 多くの自由度・素過程を取り入れた輸送模型
  - 平均場を導入して E_{inc}=(1-158) AGeV でのフローをほぼ説明
  - しかし $\sqrt{s_{NN}}$=11.5 GeV での負のフローは説明できない



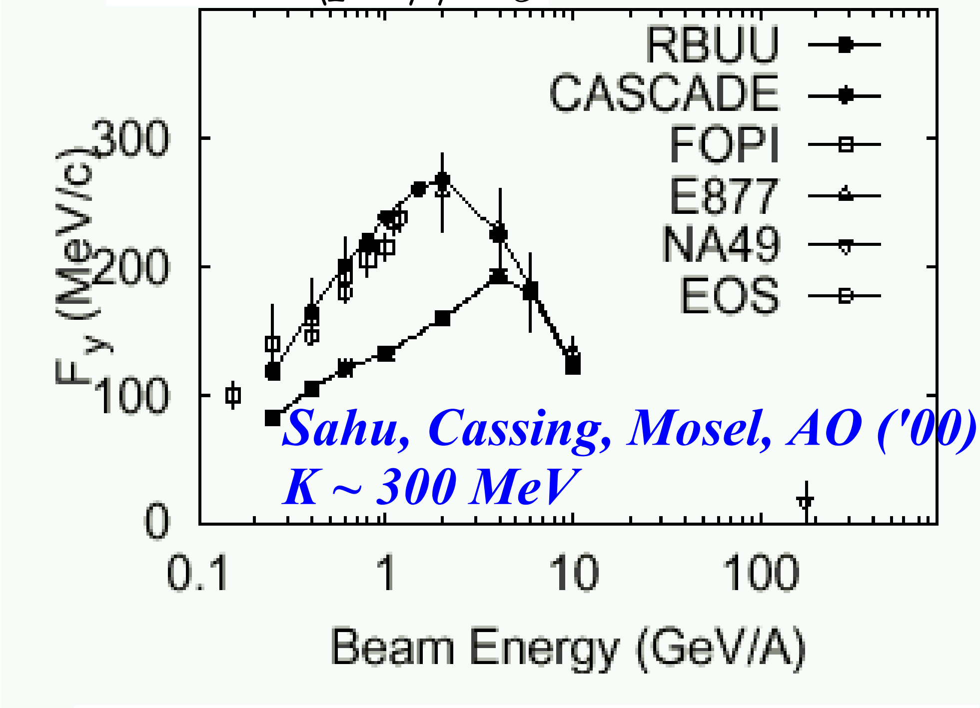
Nara+('00)

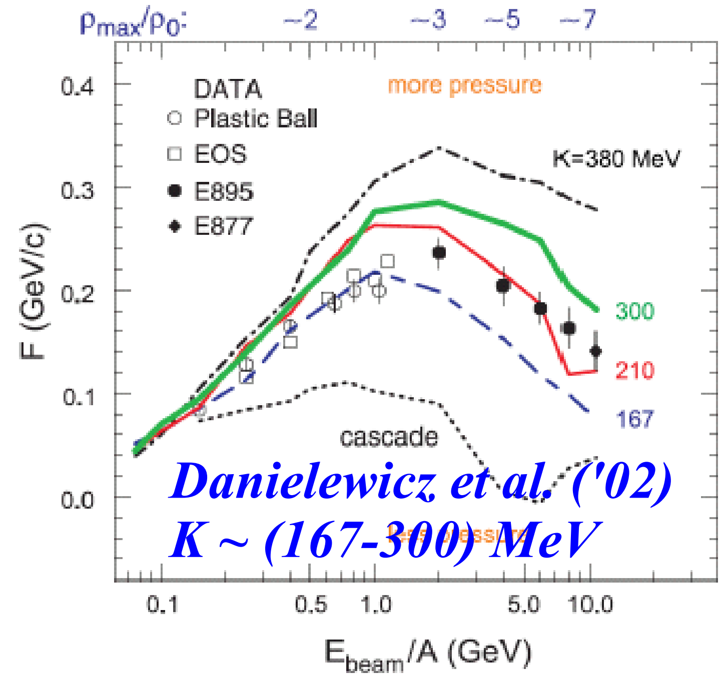


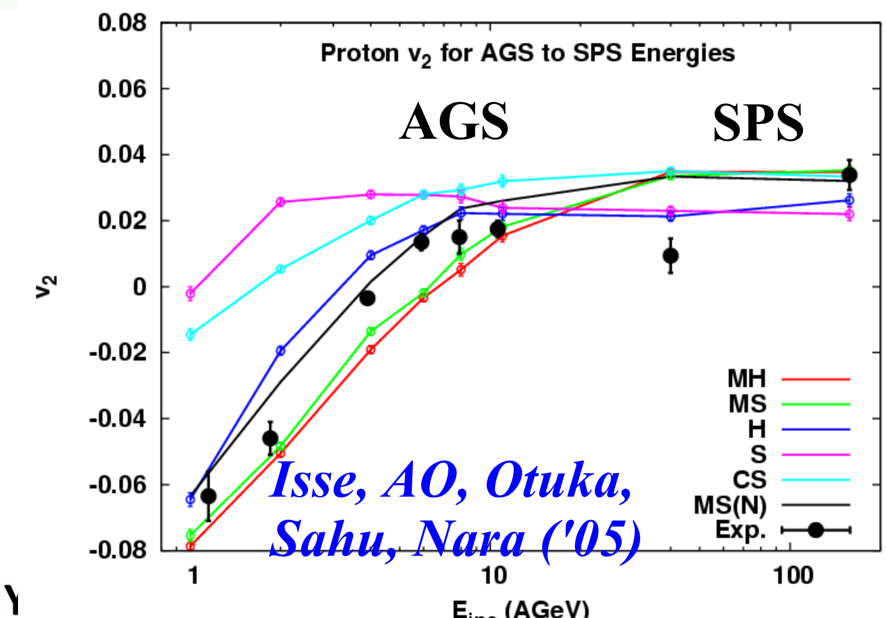
Isse+('05)

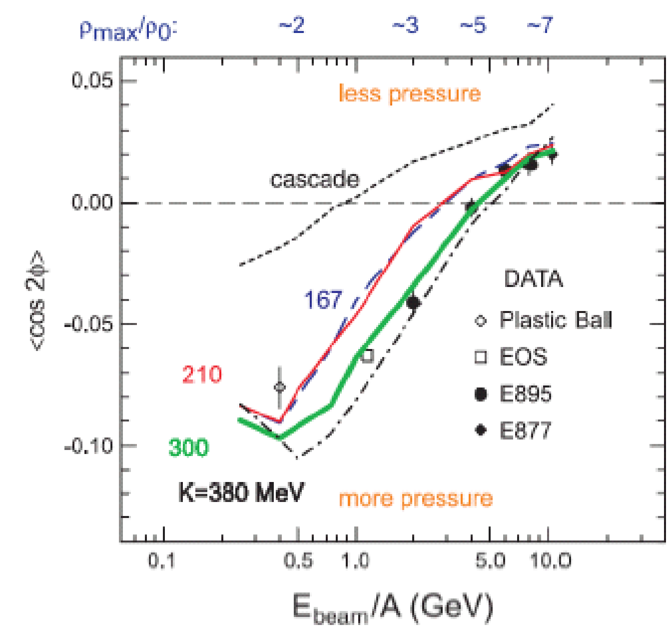
集団フローから状態方程式へ

$$F = d\langle p_x \rangle / dy$$





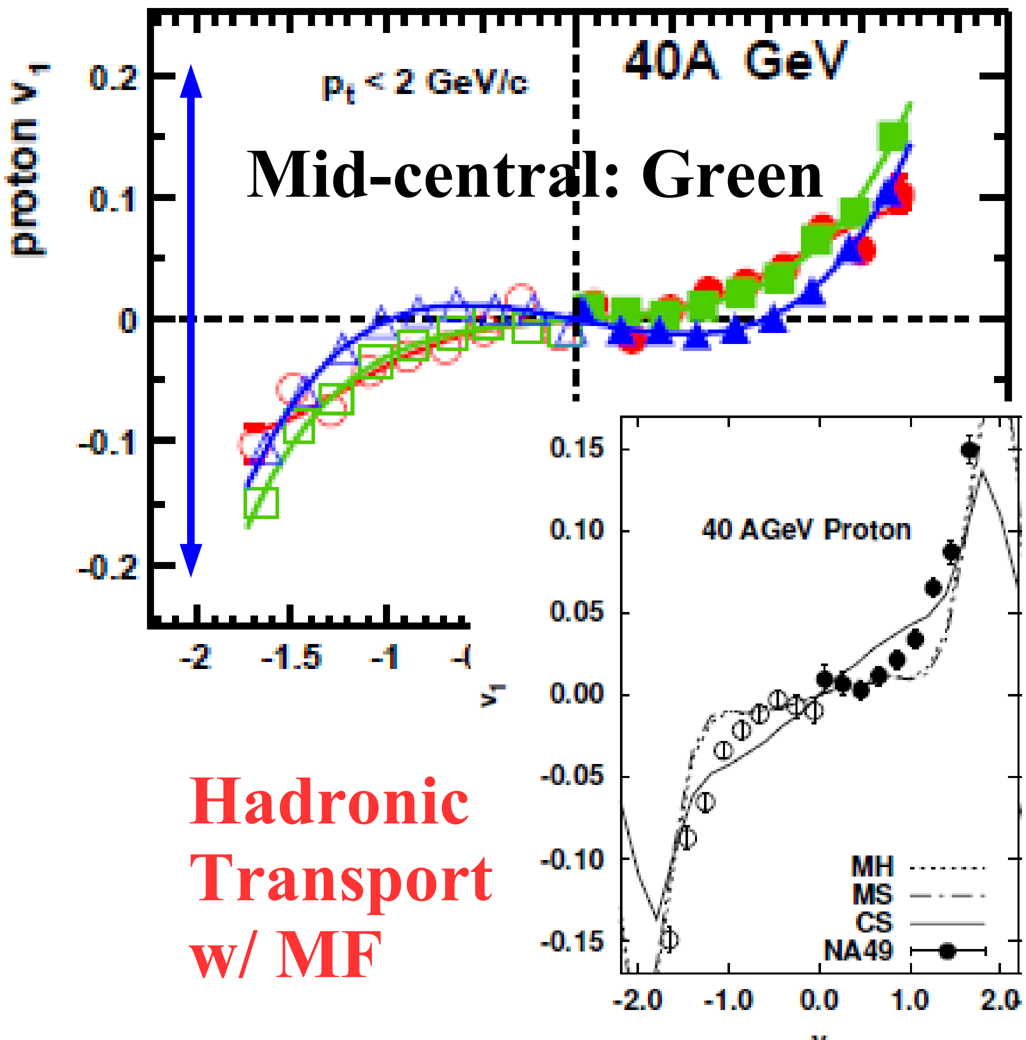




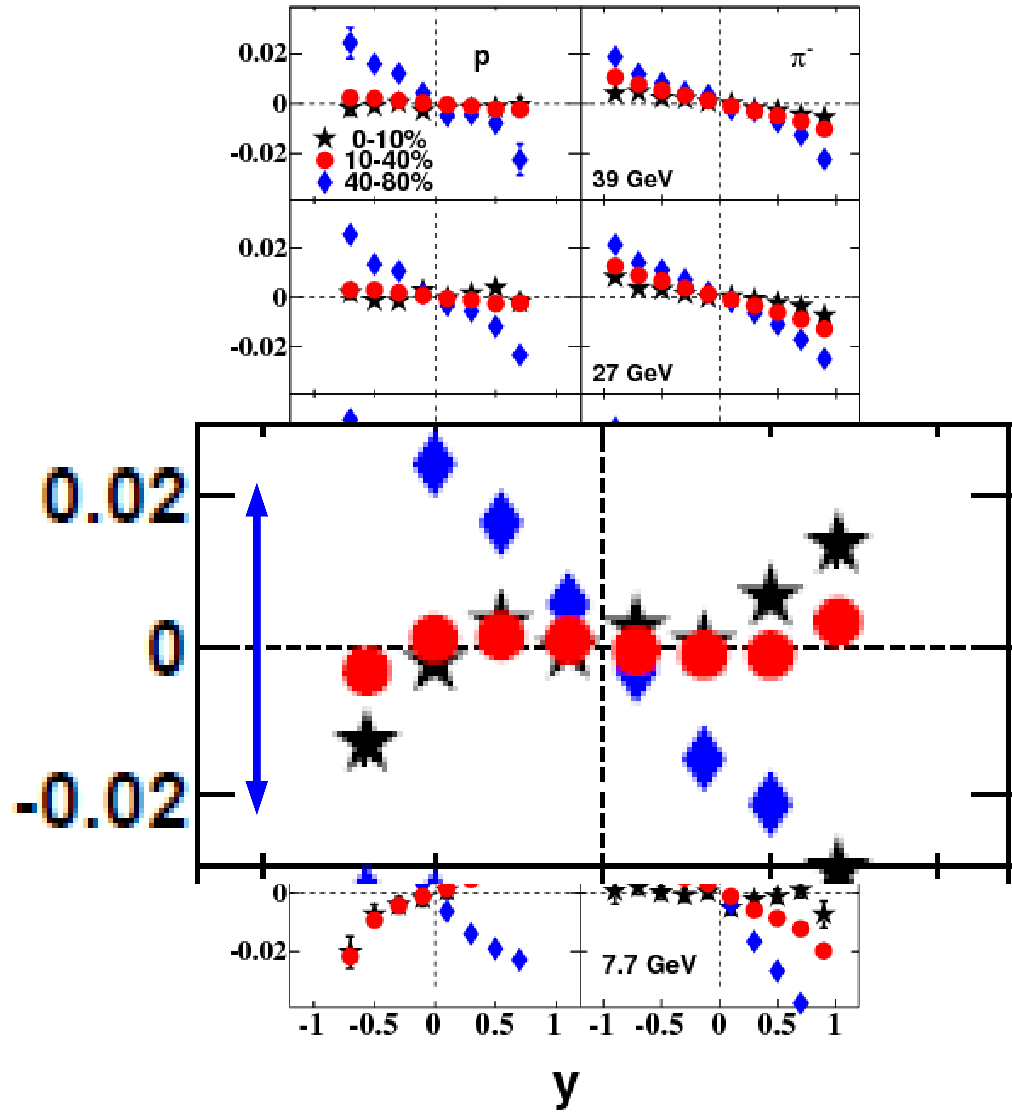
SPS(NA49) vs RHIC(STAR)

- SPS (NA49), $\sqrt{s_{NN}}$ = 8.9 GeV
- RHIC(STAR), 7.7-39 GeV

C. Alt et al. (NA49), PRC68 ('03) 034903



Hadronic Transport w/ MF

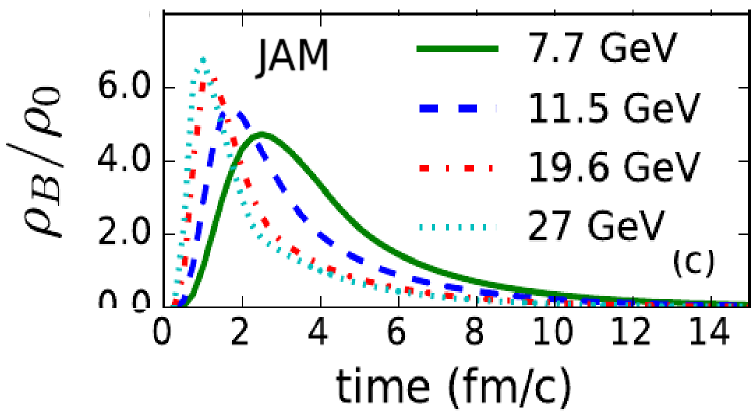


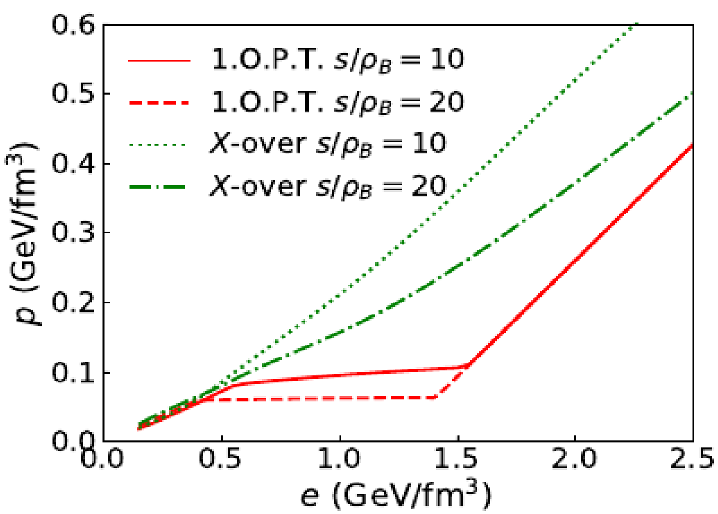
M.Isse,AO,N.Otuka,P.K.Sahu,Y.Nara, PRC72 ('05)064908

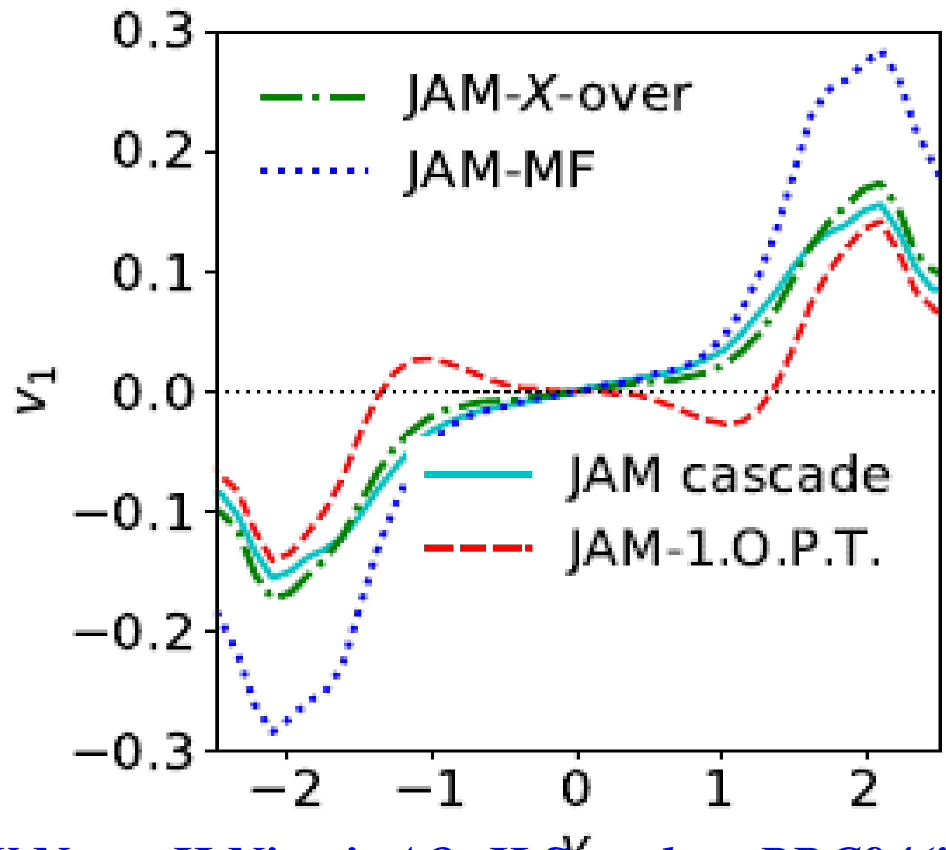
L. Adamczyk et al. (STAR), PRL 112(2014)162301

負のフローとEOS の軟化

- ビリアル定理を使って任意の EOS を取り込めるように理論を拡張
- $\sqrt{s_{NN}}$= 11.5 GeV で見られる負のフロー ($dv_1/dy<0$)
  → $(5\text{-}10)\rho_0$ において急激な EOS の軟化あれば説明可能







Y.Nara, H.Niemi, AO, H.Stoecker, PRC94('16)034906.
Y. Nara, H. Niemi, AO, J. Steinheimer, X.-F. Luo, H. Stoecker, EPJA 54 ('18)18

ビリアル定理

- **Virial**

$$G = \sum_i \mathbf{p}_i \cdot \mathbf{r}_i$$

$$\to \frac{dG}{dt} = \sum_i \mathbf{p}_i \cdot \mathbf{v}_i - \sum_i \nabla_i U \cdot \mathbf{r}_i + \boxed{\frac{1}{\Delta t} \sum_{\text{collision}} \mathbf{q}_i \cdot (\mathbf{r}_i - \mathbf{r}_j)} = 3VP$$

Kinetic **Potential** **Pressure from Collisions**

- **Attractive / Repulsive Orbit Scatterings**
  - **通常は散乱角はランダム → 衝突項の圧力への影響はゼロ**
  - **Attractive orbits → ΔP < 0 (softening)**
  - **Repulsive orbits → ΔP > 0 (hardening)**
- **Boltzmann Eq. simulating a given EOS**

  P > P(ε) → Attractive orbit, P > P(ε) → Repulsive orbit

  衝突が十分に頻繁であれば、ボルツマン方程式だけでポテンシャル効果をシミュレートできる！

YITP YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

$(5\text{-}10)\rho_0$ でQCD 相転移がありそう。
軟化が必要なことから(対称核物質では)
1 次相転移が想定される！

→ 輸送模型と流体力学を組み合わせた
理論開発が求められる。

レポート(5)

- 全部で 5-7 問程度出します。3 問程度以上レポートを出してください。レポート (5) の〆切は 1 月末

- (Report 6) Wigner 変換の性質

$$([A,B])_W = i\hbar\{A_W, B_W\}_{\rm PB} + \mathcal{O}(\hbar^3)$$

を示せ。
(Ring-Schuck text の Appendix 参照。面倒です。)

- (Report 7) 分布関数 f を用いてフェルミオン系のエントロピーを定義し、フェルミ統計を含む衝突項による時間発展において、エントロピーが増加することを示せ。

Backups

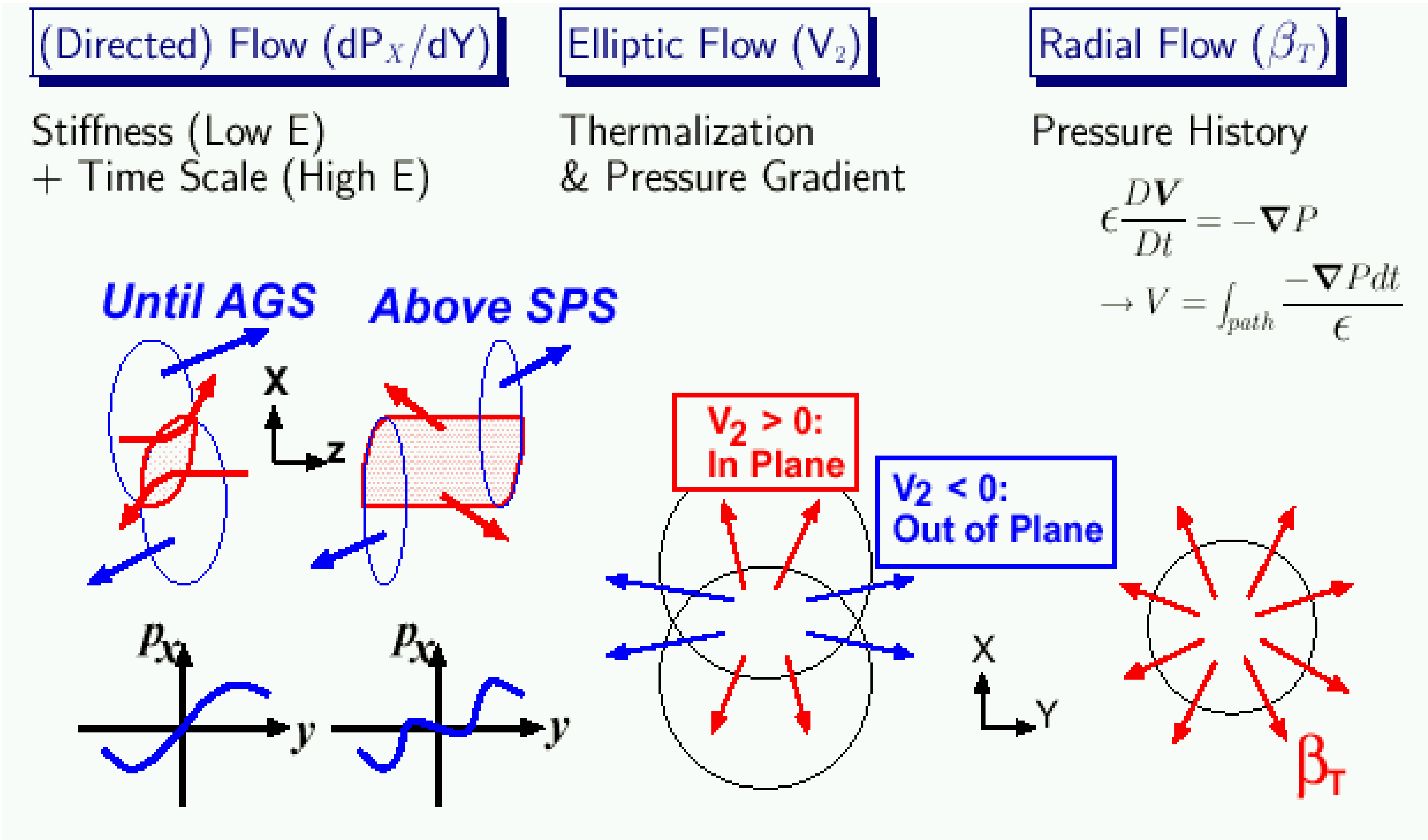
Collective Flows at AGS and SPS Energies

Collective Flow and EOS: Old Problem ?

- **1970's-1980's: First Suggestions and Measurement**
  - **Hydrodynamics suggested the Exsitence of Flow.**
  - **Strong Collective Flow suggests Hard EOS**
- **1980's-1990's: Deeper Discussions in Wider E_{inc} Range**
  - **Momentum Dep. Pot. can generate Strong Flows.**
  - **Einc deps. implies the importance of Momentum Deps.**
  - **Flow Measurement up to AGS Energies.**
- **2000's: Extention to SPS and RHIC Energies**
  - **EOS is determined with Mom. AND Density Dep. Pot. ?**

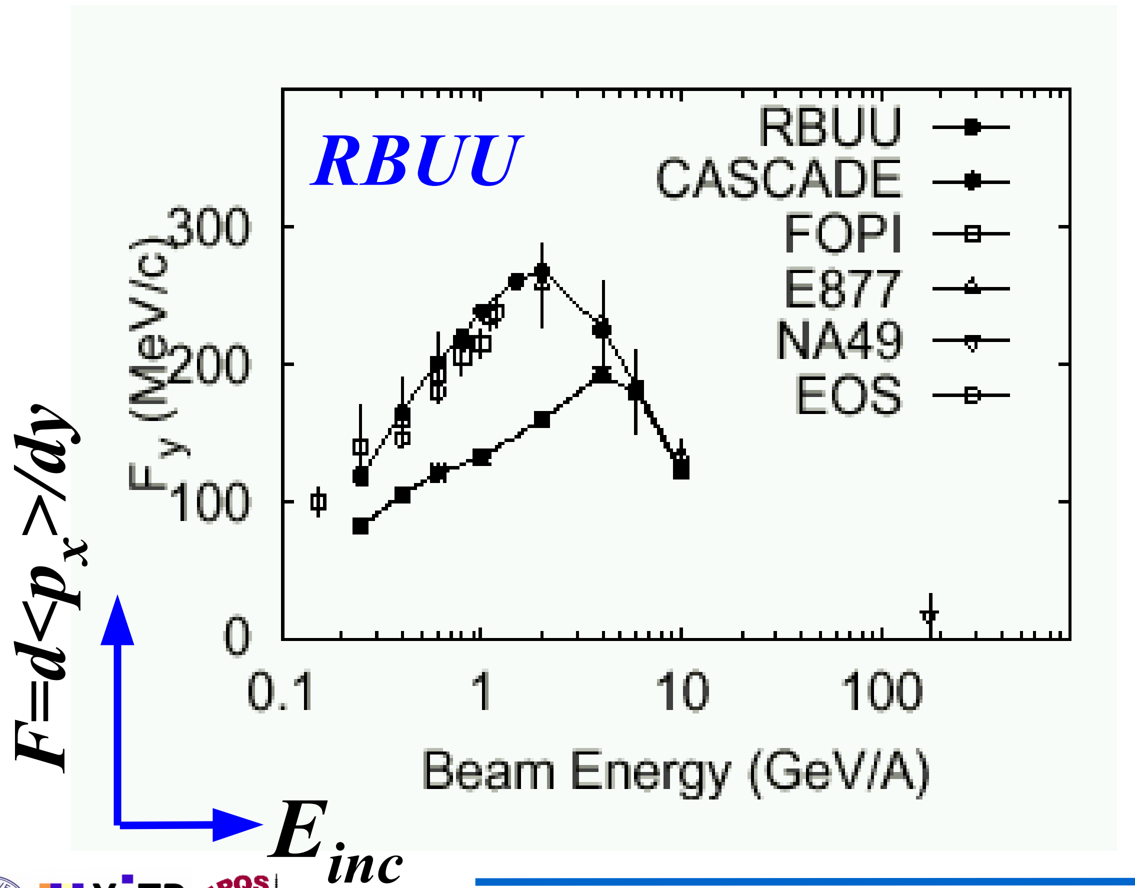
Old but New (Continuing) Problem !

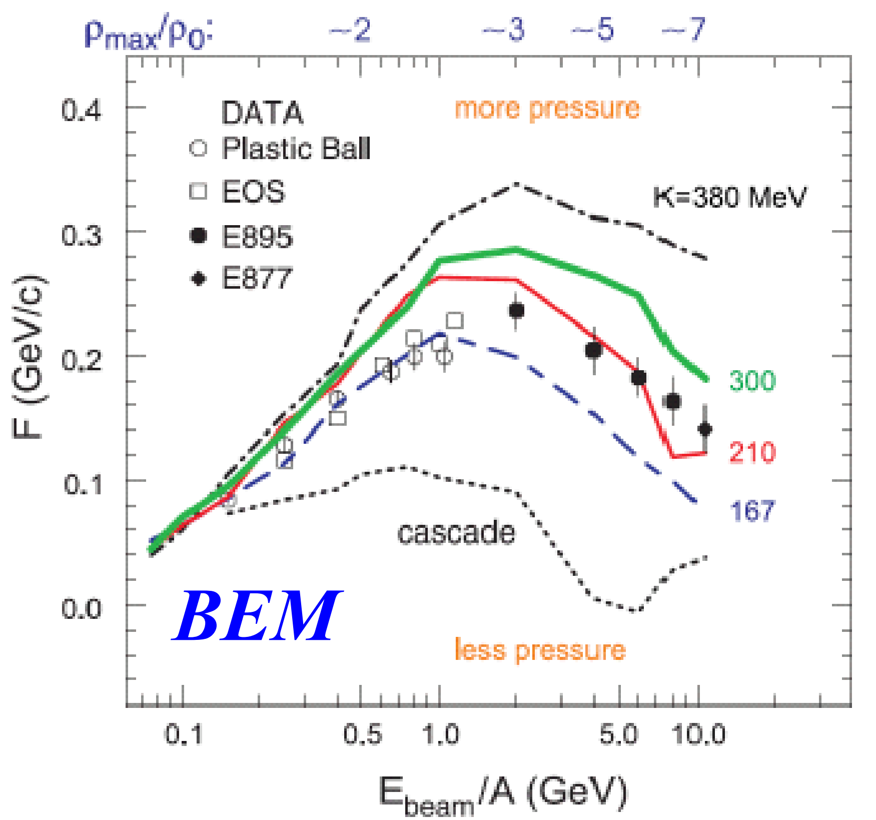
What is Collective Flow ?



Side Flow at AGS Energies

- **Relativistic BUU (RBUU) model:** ***K ~ 300 MeV***
  (Sahu, Cassing, Mosel, AO, Nucl. Phys. A672 (2000), 376.)
- **Boltzmann Equation Model (BEM):** ***K=167~210 MeV***
  (P. Danielewicz, R. Lacey, W.G. Lynch, Science 298(2002), 1592.)

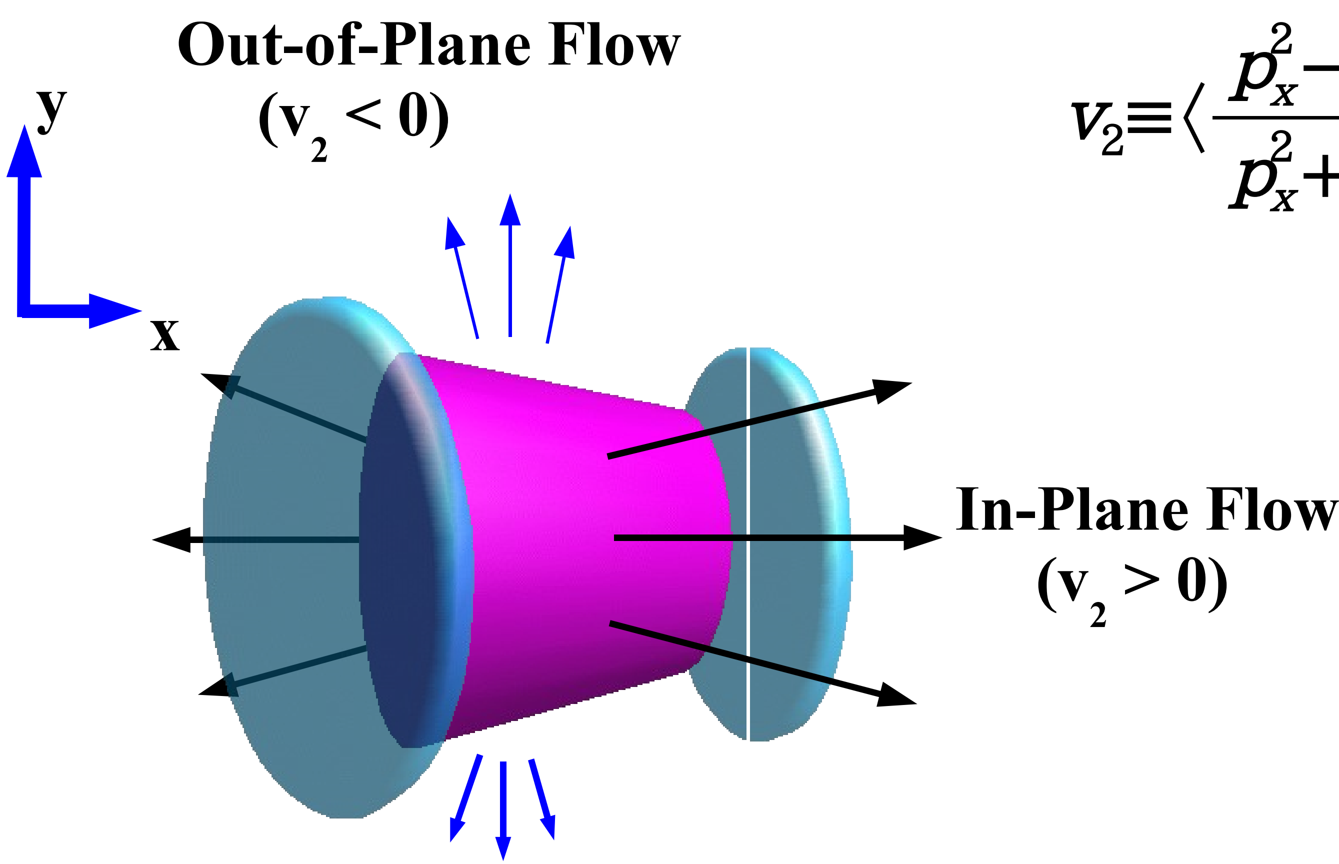




Elliptic Flow

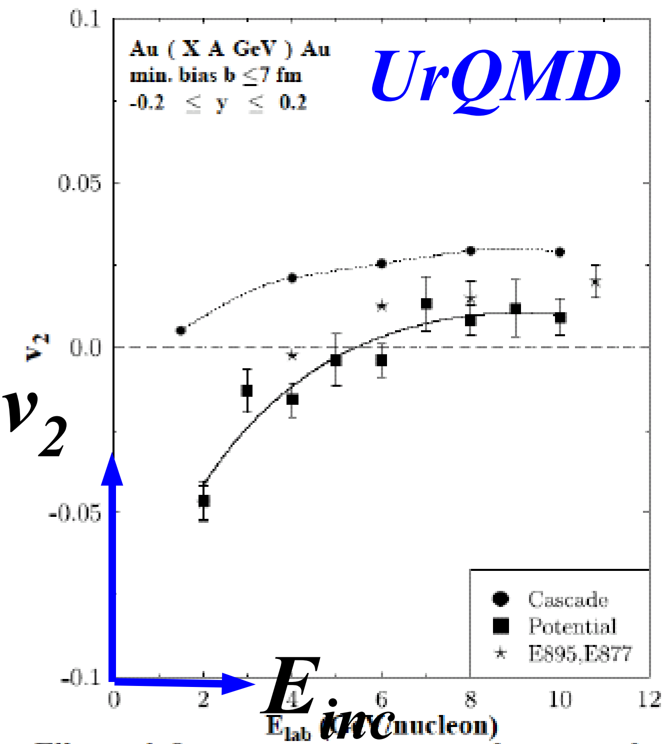
- **What is Elliptic Flow ? → Anisotropy in P space**
- **Hydrodynamical Picture**
  - **Sensitive to the Pressure Anisotropy in the Early Stage**
  - **Early Thermalization is Required for Large V2**

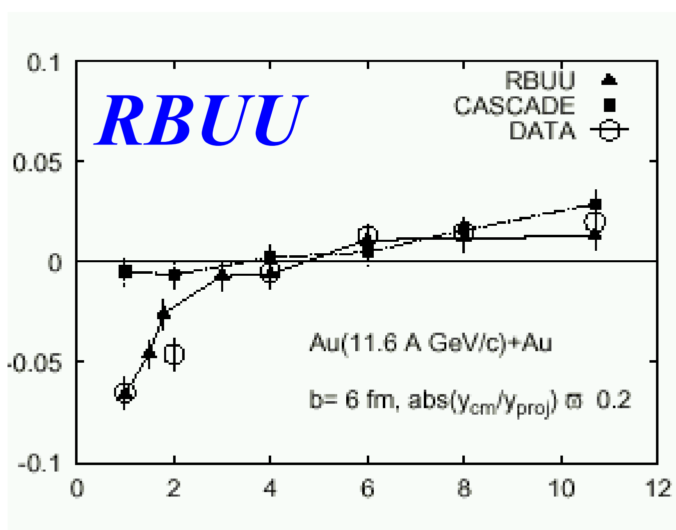
$$v_2 \equiv \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle = \langle \cos 2\phi \rangle$$

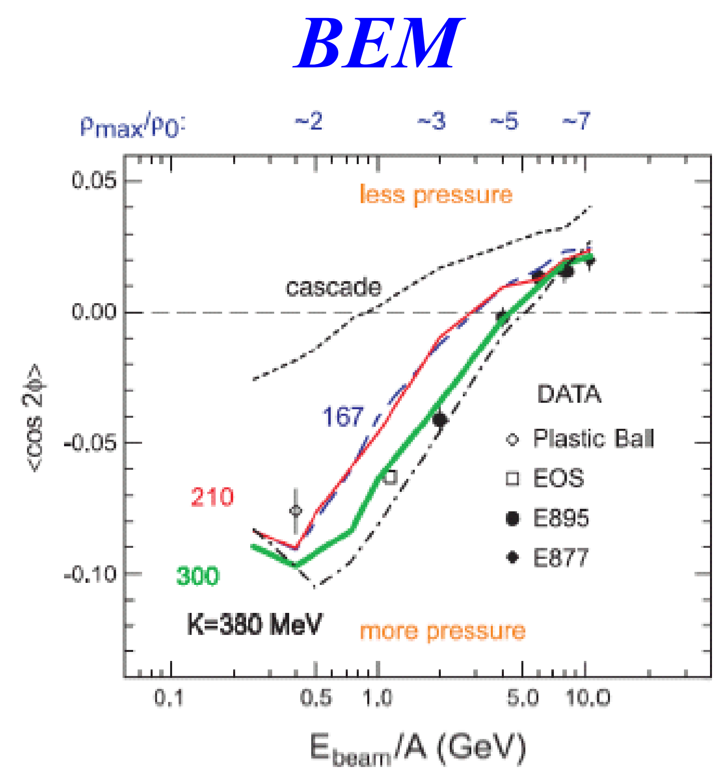


Elliptic Flow at AGS

- **Strong Squeezing Effects at low E (2-4 A GeV)**
  - **UrQMD: Hard EOS (S.Soff et al., nucl-th/9903061)**
  - **RBUU (Sahu-Cassing-Mosel-AO, 2000): K ~ 300 MeV**
  - **BEM(Danielewicz2002): K = 167 → 300 MeV**

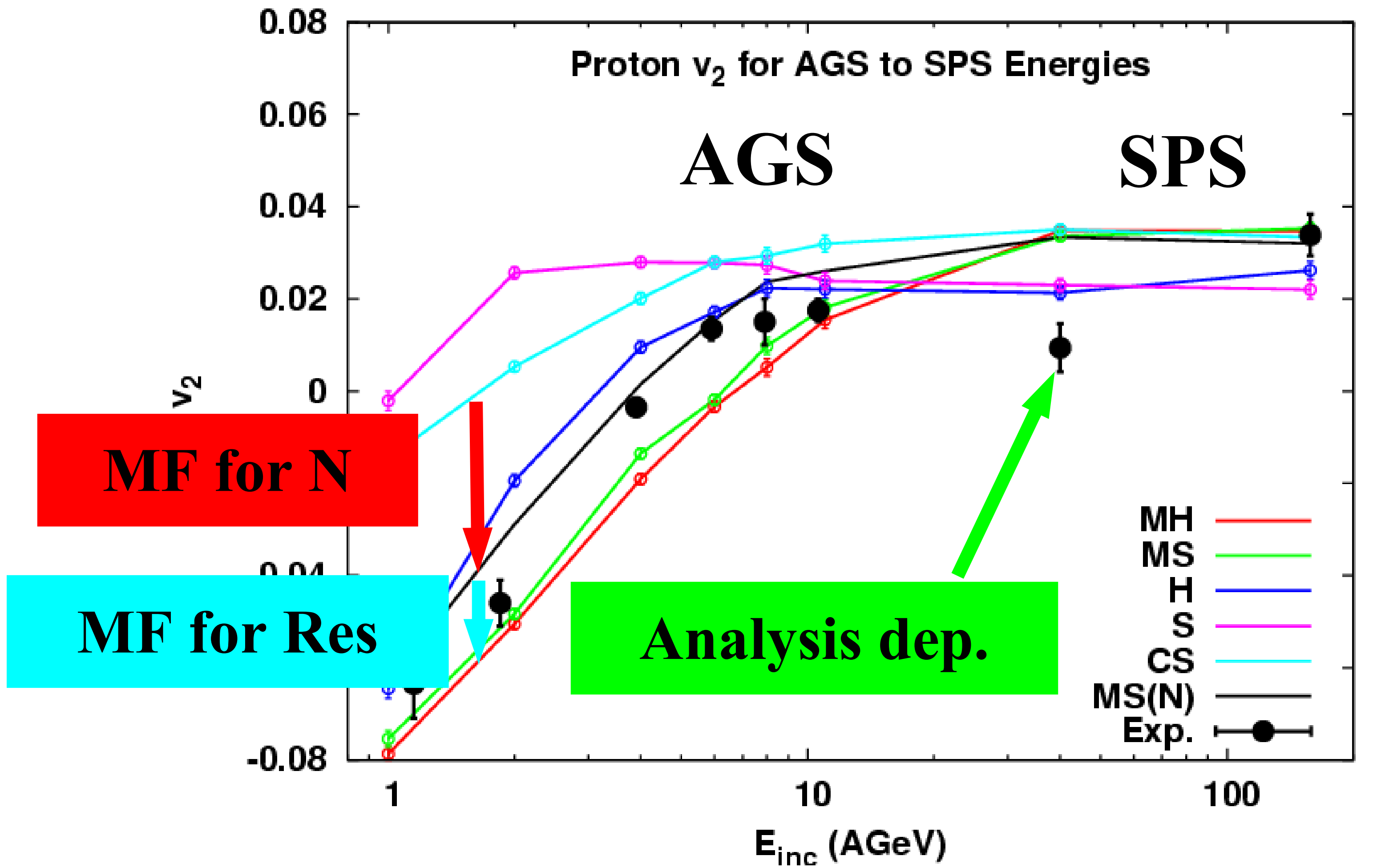






Elliptic Flow from AGS to SPS

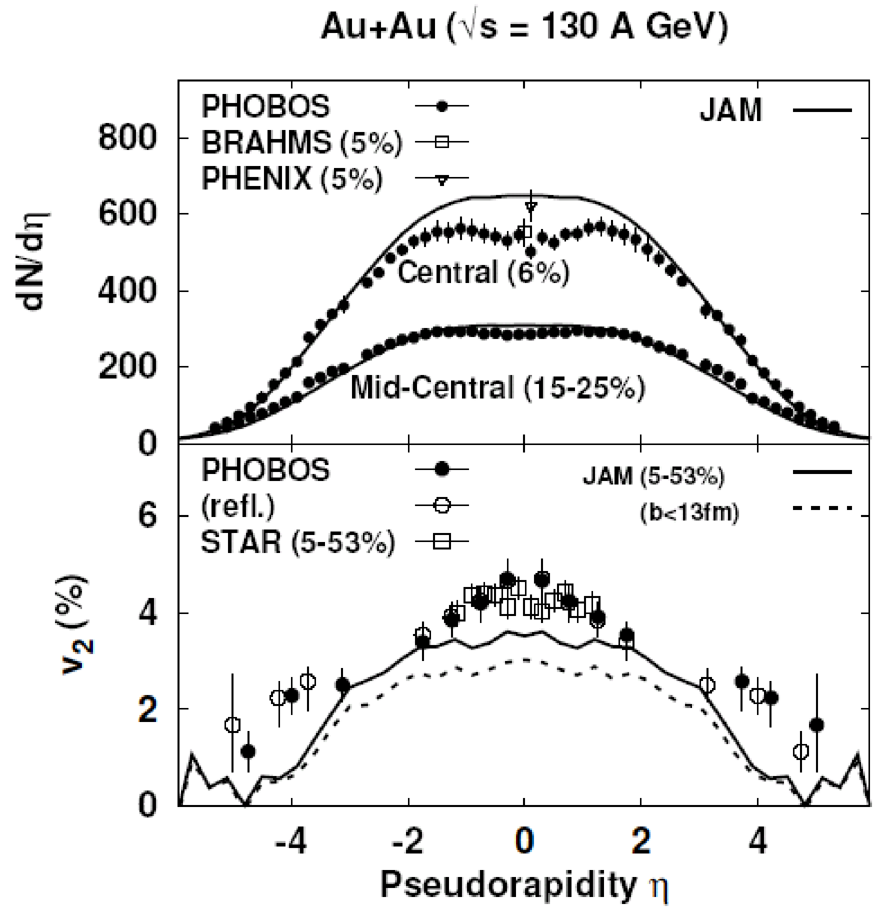
- **JAM-MF with p dep. MF explains proton v2 at 1-158 A GeV**
  - **v2 is not very sensitive to K (incompressibility)**
  - **Data lies between MS(B) and MS(N)**

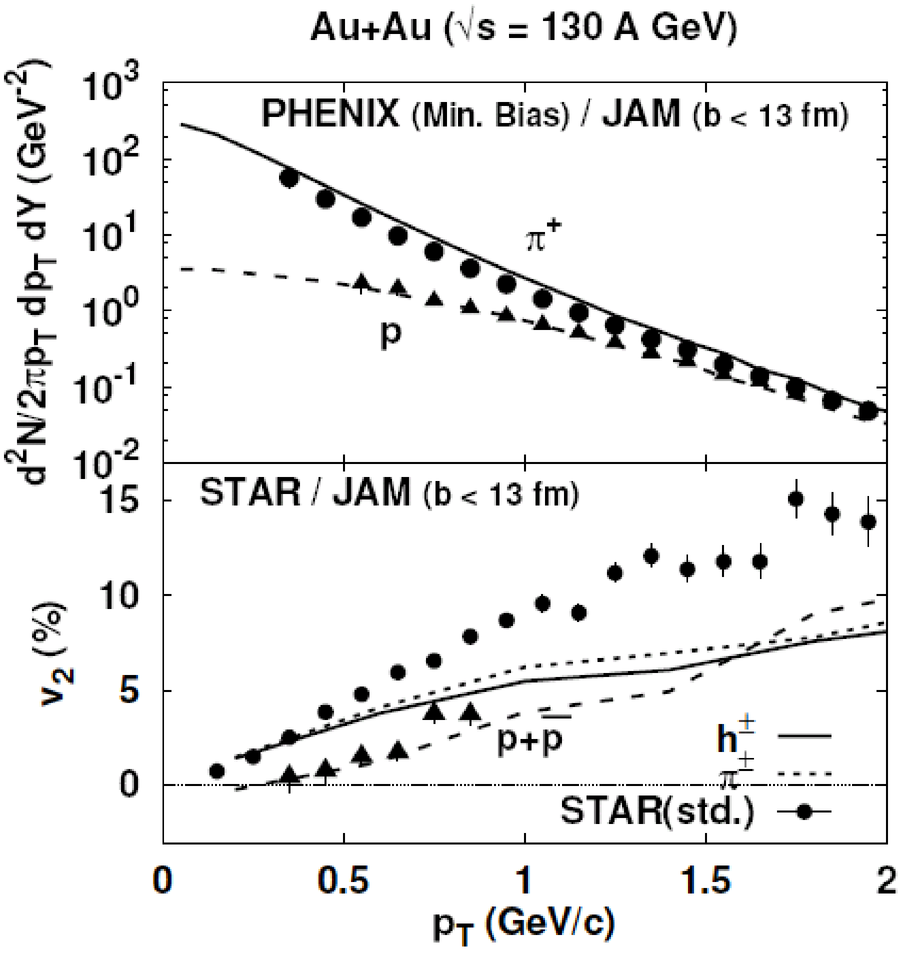


Elliptic Flow @ RHIC

Elliptic Flow in Hadron-String Cascade (I)

- **Hadron-String Cascade (JAM) @ RHIC**
  - **Hadron Yield is reasonably explained up to 2 GeV/c (10-20 % error)**
  - **v2 is underestimated (20-30 % (integrated), 50 % (p_T > 1 GeV)**

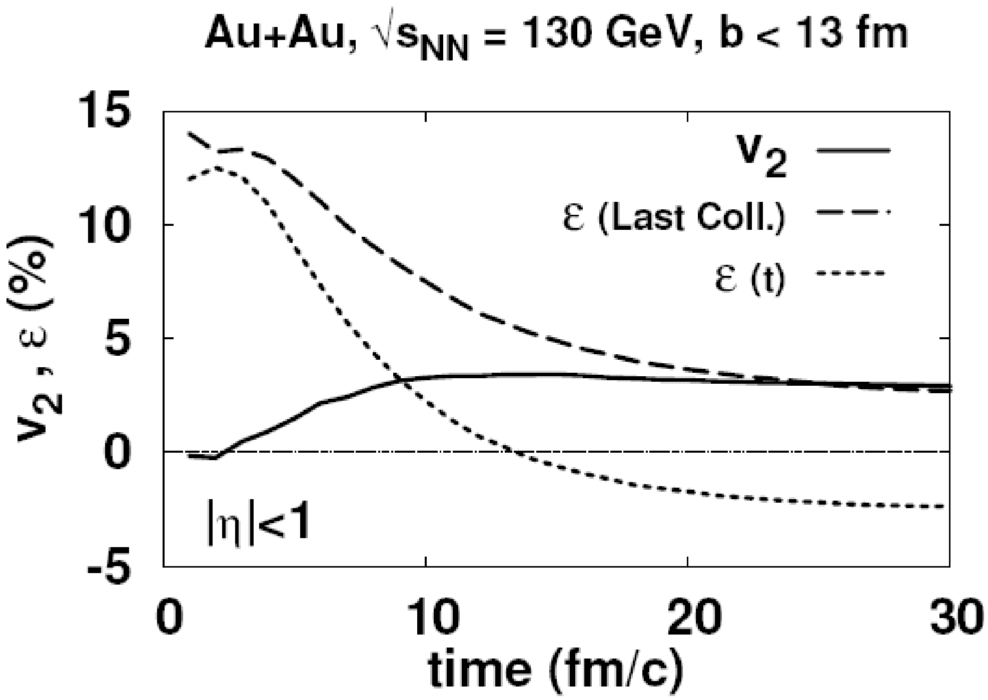


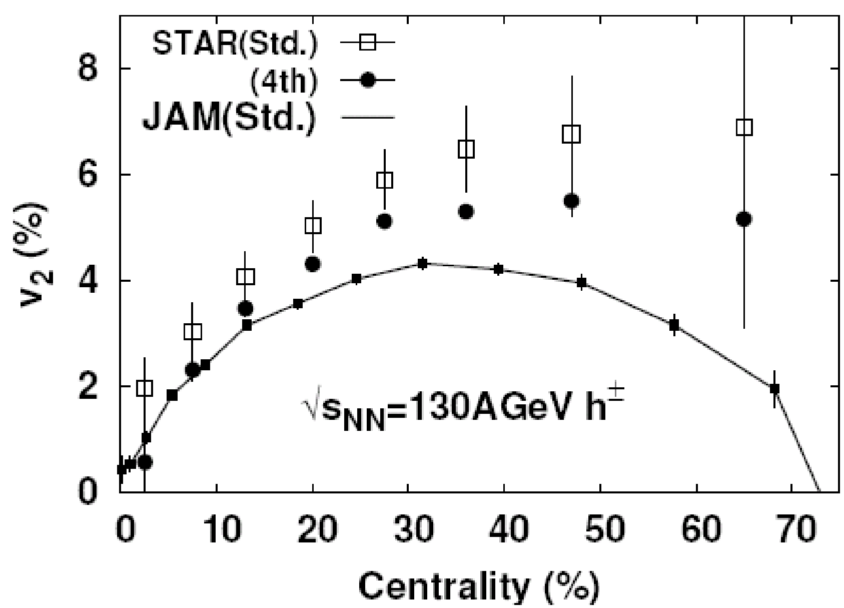


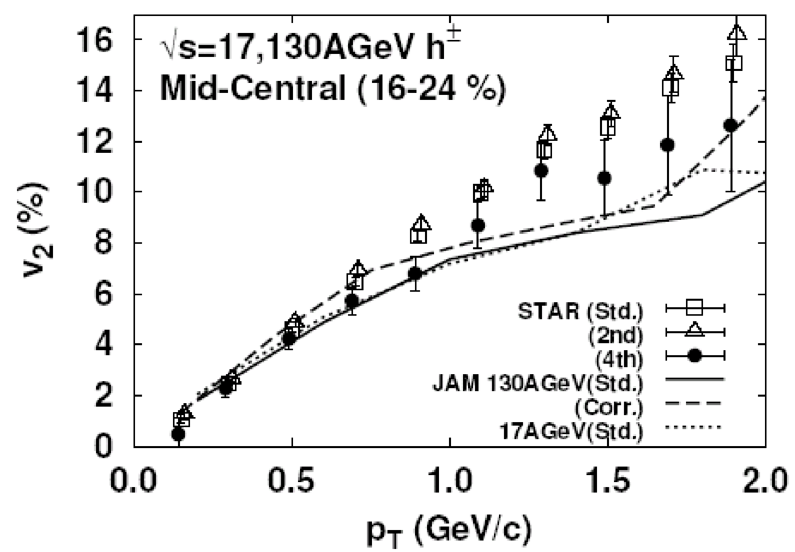
Elliptic Flow in Hadron-String Cascade (II)

- **Why do we underestimate v2 in Hadron-String Cascade ?**
  - **v2 growth time is long (~ 10 fm/c), due to hadron formation time (τ~ 1 fm/c). → much longer than hydro**

Sahu-Isse-AO-Otuka-Phatak 2006







Results of Parton Cascade

- **Unexpectedly high parton cross sections of σ =5-6 mb have to be assumed in parton cascades in order to reproduce the elliptic flow.**

ZI-WEI LIN AND C. M. KO

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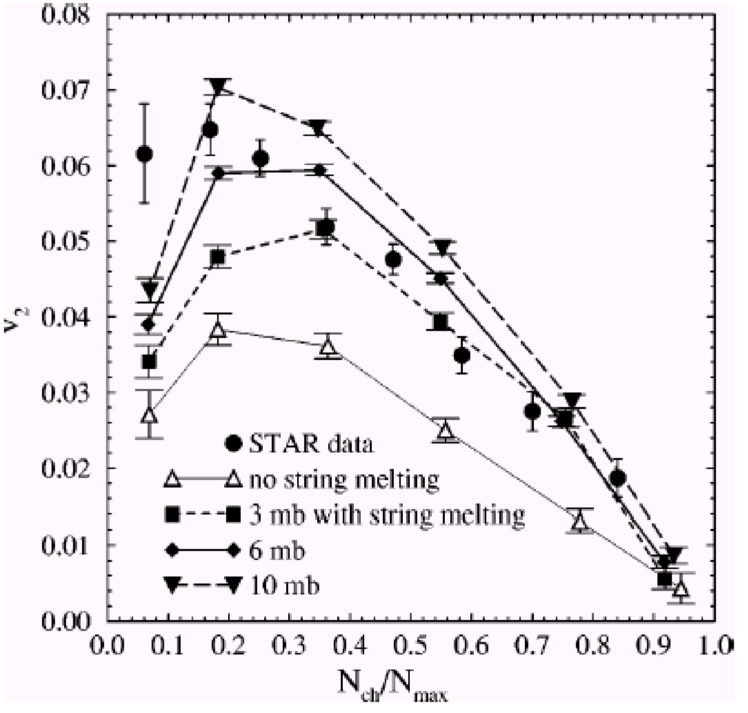


FIG. 3. Impact parameter dependence of elliptic flow at 130 *A* GeV. The data from the STAR collaboration [7] are shown by filled circles, while the theoretical results for different partonic dynamics are given by curves.

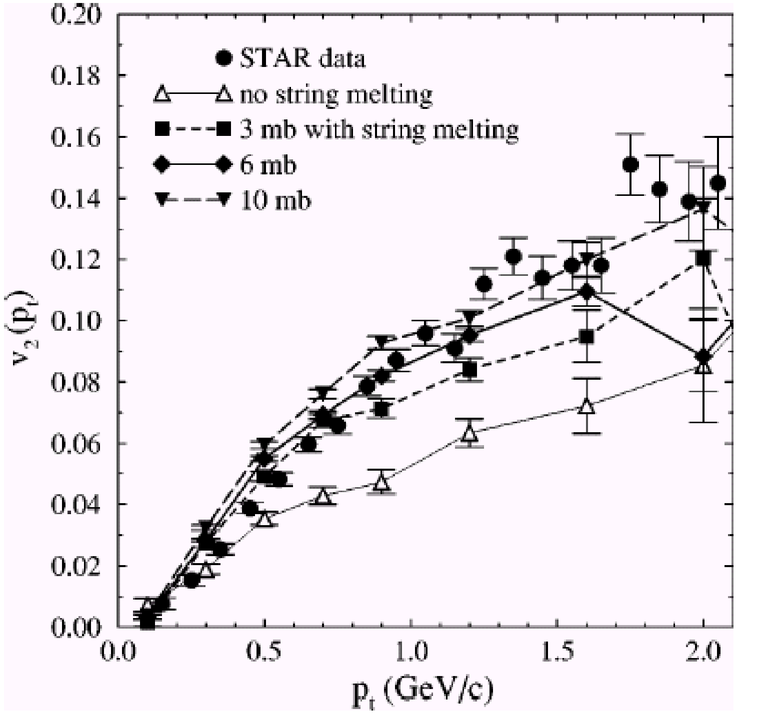
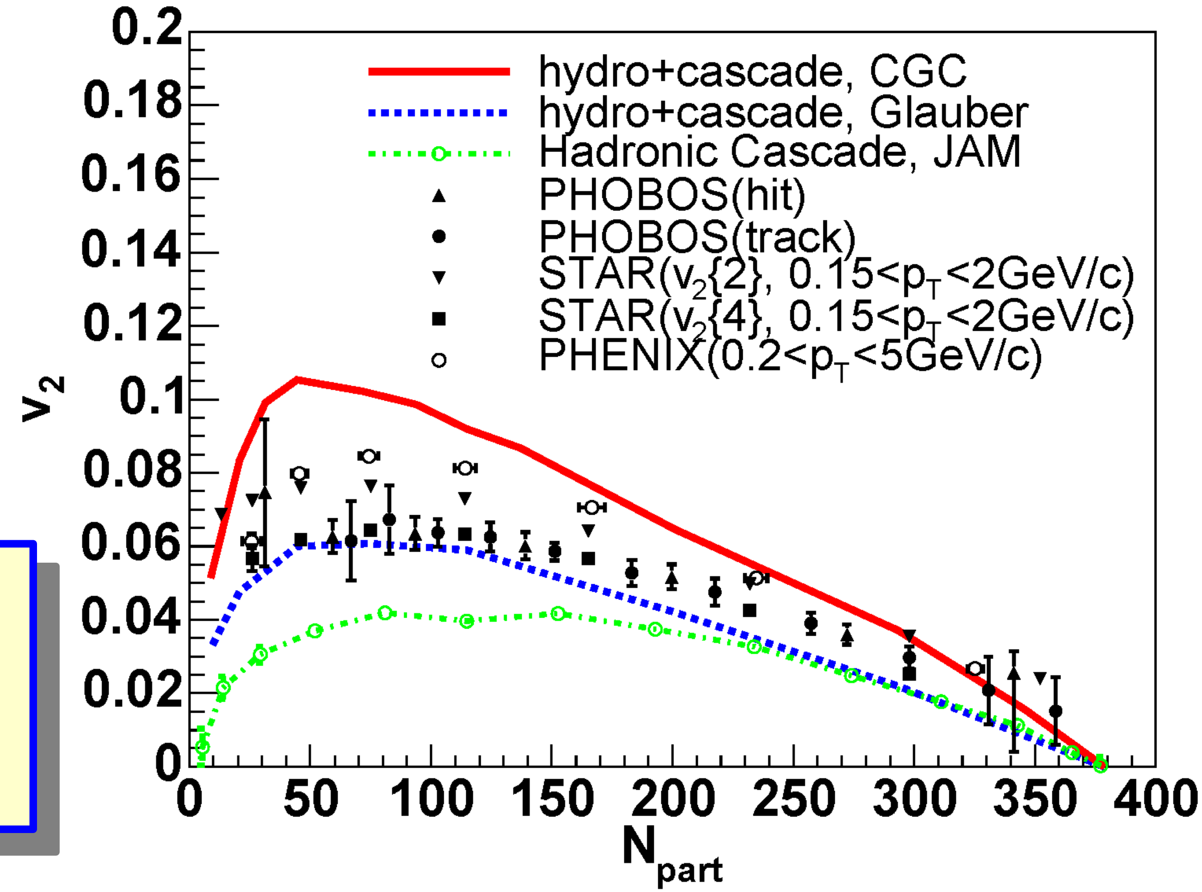


FIG. 4. Transverse momentum dependence of elliptic flow at 130 *A* GeV. Circles are the STAR data for minimum-bias Au+Au collisions [7], and curves represent the minimum-bias results for charged particles within $\eta \in (-1.3, 1.3)$ from the AMPT model.

Cascade vs Hydro @ RHIC: Au+Au

- **Comparison of v2 as a function of N_{part}**
  - **Cascade predict smaller v2 in peripheral collisions**
  - **Data lies between hydro results with two different initial condition CGC (Color Glass Condensate) and Glauber type initial condition.**



Hydro is better,
CGC may be realized
in central collisions.