高エネルギー重イオン反応で調べる
核物質の相図
--- 平均場、状態方程式、QGP ---

名古屋大学特別講義 ***(12/18-20)***

Akira Ohnishi
Department of Physics, Faculty of Science, Hokkaido University

based on the Collaboration work with
K. Yoshino, M. Isse (Osaka U.), N. Otuka (JAEA), Y. Nara (Frankfurt),
T. Hirano (U. Tokyo), P.K. Sahu (Bubhaneswar, India)

Contents

- Overview of QGP hunting at RHIC (12/18, AM)
- Basic ingredients of heavy-ion collision theory (12/18, PM & 12/19 AM)
  - Mean Field, Cascade, Jet and String, Hydrodynamics
- Phase diagram of quark and hadron matter (12/19, AM/PM)
  - Strong coupling limit of lattice QCD
  - Relativistic Mean Field model with chiral symmetry
- Collective flow in heavy-ion collisions (12/20, AM)
  - Mean field and DOF effects
  - Collective flows at AGS, SPS, and RHIC energies
- Unsolved Problems in RHIC physics (12/20, PM)
  - Hadronization, Thermalization, HBT puzzle, Mach Cone, ...

Part I:
Overview of QGP Hunting at RHIC

地上で作る小さなビッグバン
ー クォーク･グルーオン･プラズマの探索 ー

Abstract

この世界を構成している「最小」の粒子はクォークであり、クォークが３つ集まって陽子･中性子、そしてさらにこれらが集まって原子核を作っている。

これまではクォークは核子の中に閉じ込められ、単独でみることができなかったが、近年の実験において、クォークがばらばらになった状態、「クォーク･グルーオン･プラズマ(QGP)」が生成された。

初期宇宙では、このQGP状態を経て現在の宇宙の「真空」が作られており、人類は実験室で「小さなビッグバン」を作ったことになる。

本公演では、現在急速に実験研究が進行しているQGP生成研究と、そこで必要とされているシミュレーション計算の現状について紹介する。

(北海道大学･シミュレーションサロン、2006/06/23)

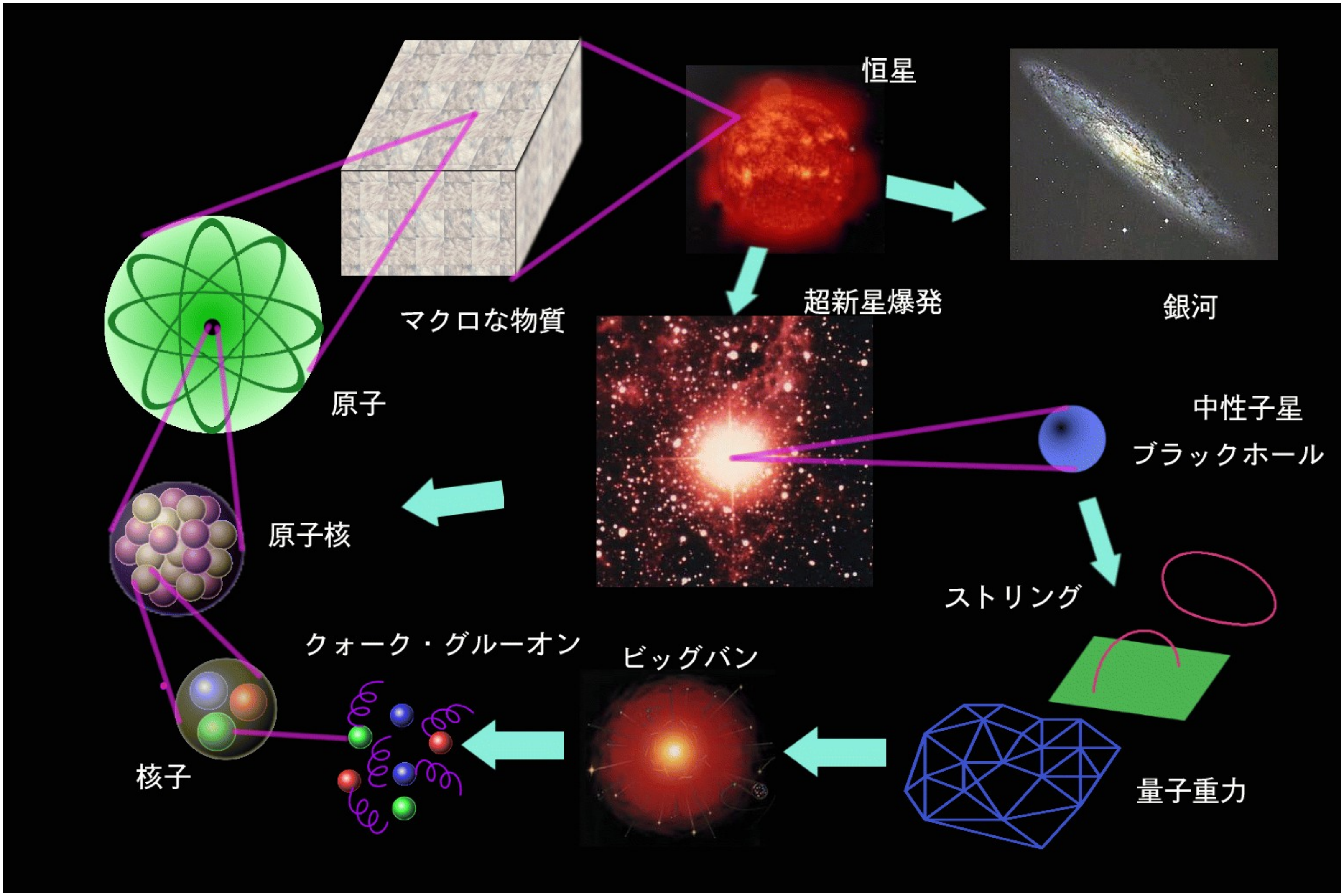
Contents

- Introduction
  - クォーク･グルーオン･プラズマ(QGP)とは何か？
- QGPは見つかったか？
  - ジェット抑制
  - 楕円型フロー
- QGP物性の探求へ向けて (詳細は Part V にて)
  - 格子QCD計算、流体力学計算、ジェット生成、流体と速いパートンの相互作用
- まとめ

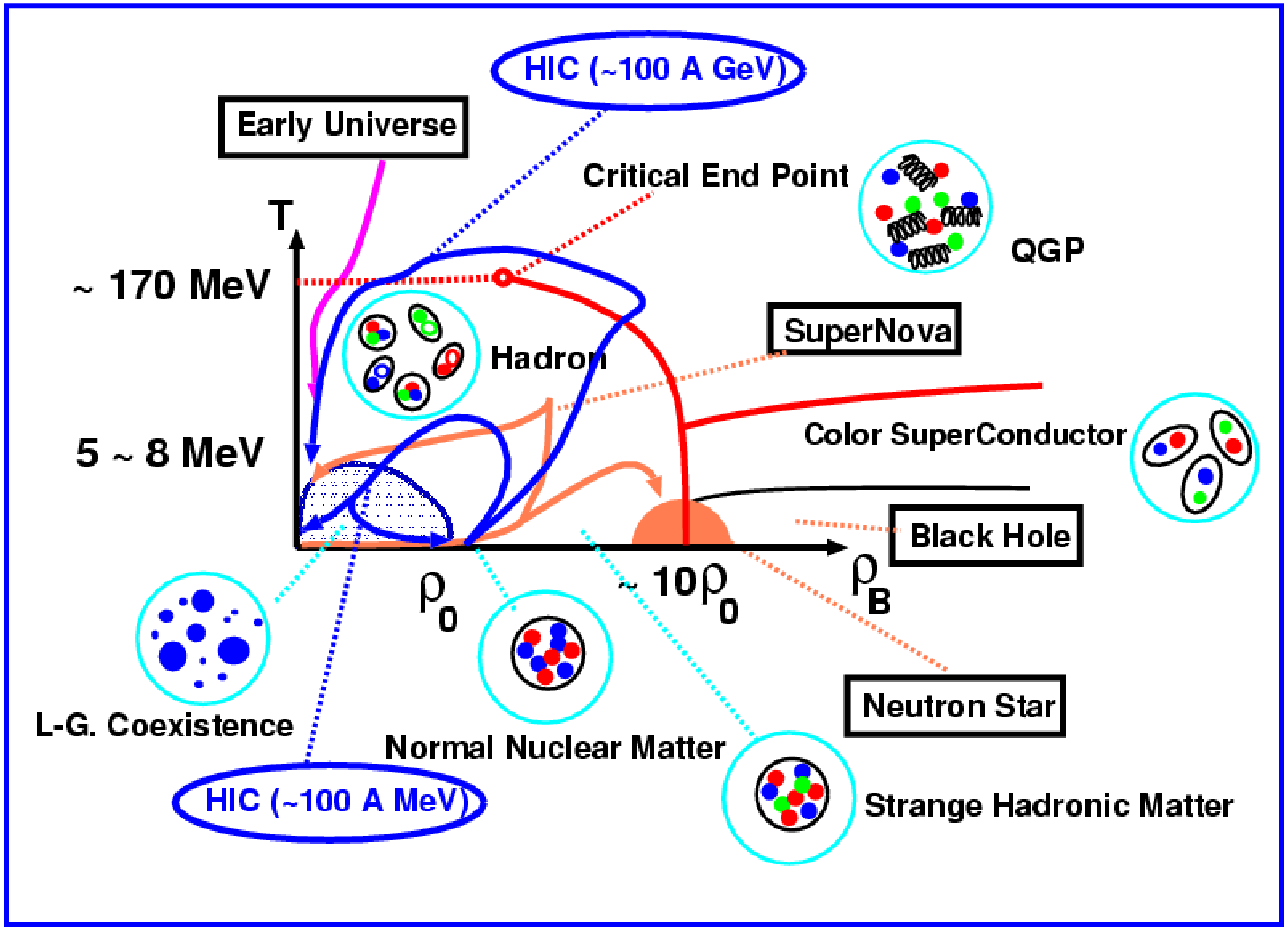
Introduction

物質は何からできているか？

- 原子 → 原子核 → 核子
→ クォーク(=現時点で「最小」と考えられている粒子)

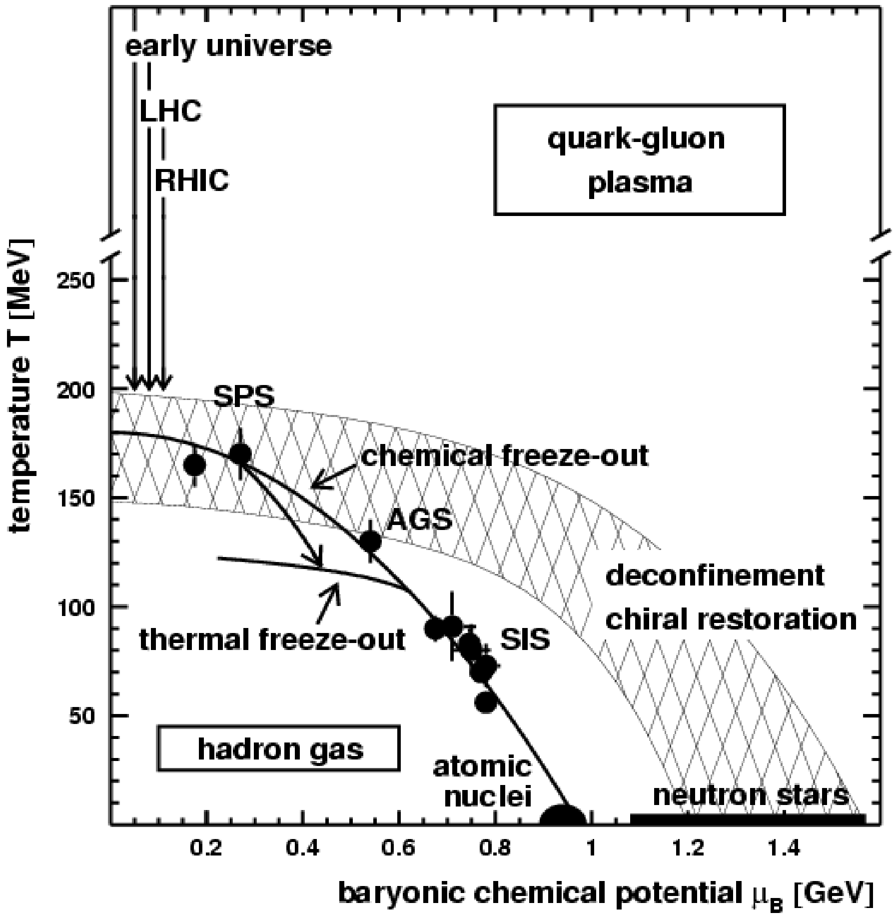


宇宙と地上でのクォーク物質相転移

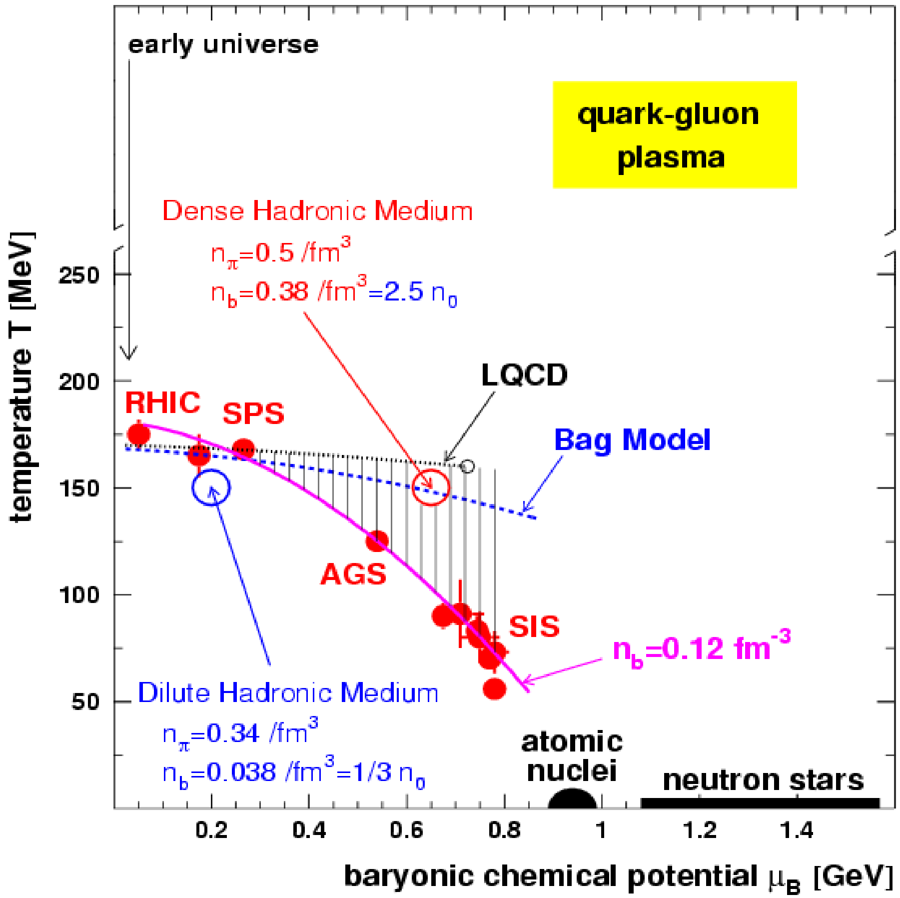


- **QGPからハドロン相への相転移(QCD 相転移)**
  ＝この宇宙最後の「真空相転移」である！

Experimentally Estimated Phase Diagram



J. Stachel et al., 1998



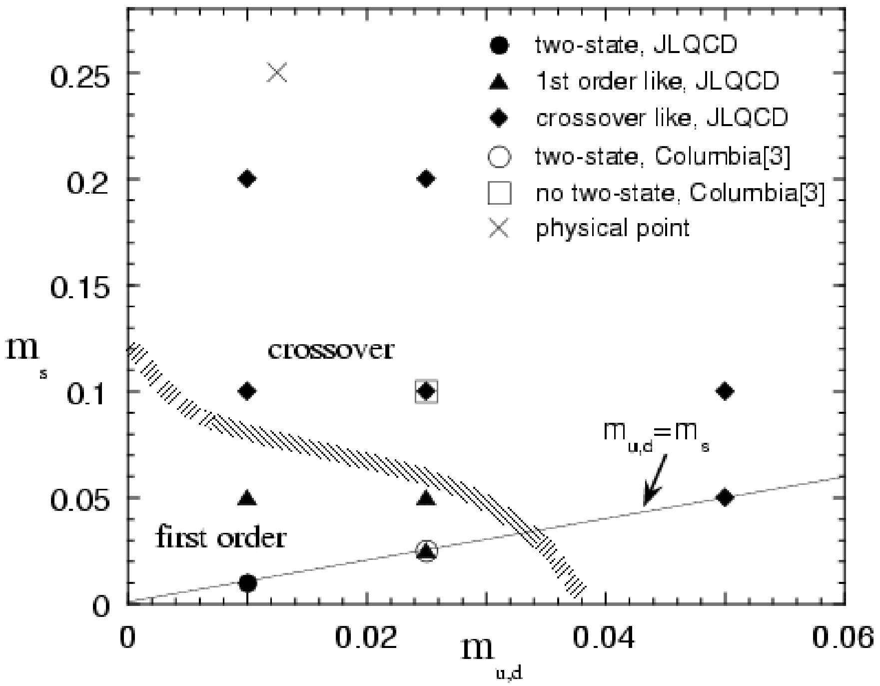
Braun-Munzinger et al., 2002

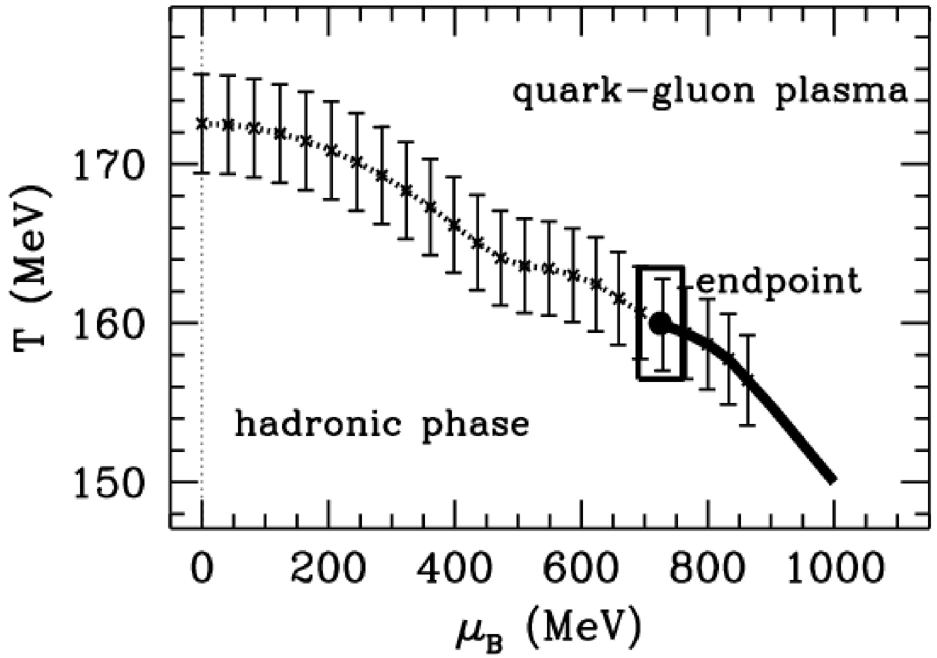
Chem. Freeze-Out Points are very Close to Expected QCD Phase Transition Boundary

Theoretically Expected QCD Phase Diagram

Zero Chem. Pot.

Finite Chem. Pot.





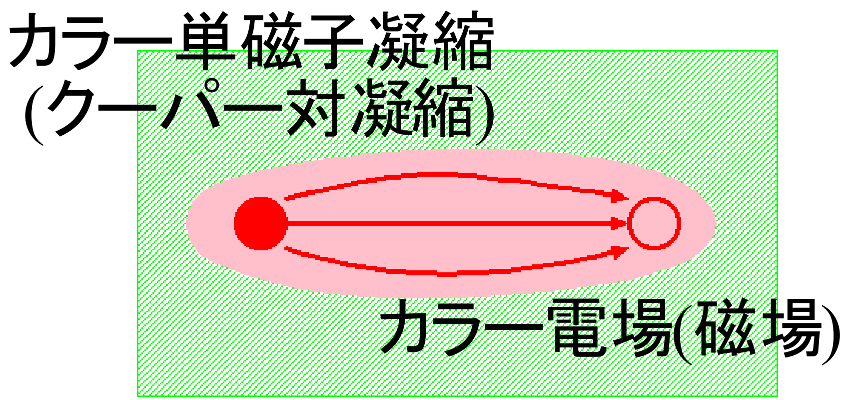
JLQCD Collab. (S. Aoki et al.), Nucl. Phys. Proc. Suppl. 73 (1999) 459.

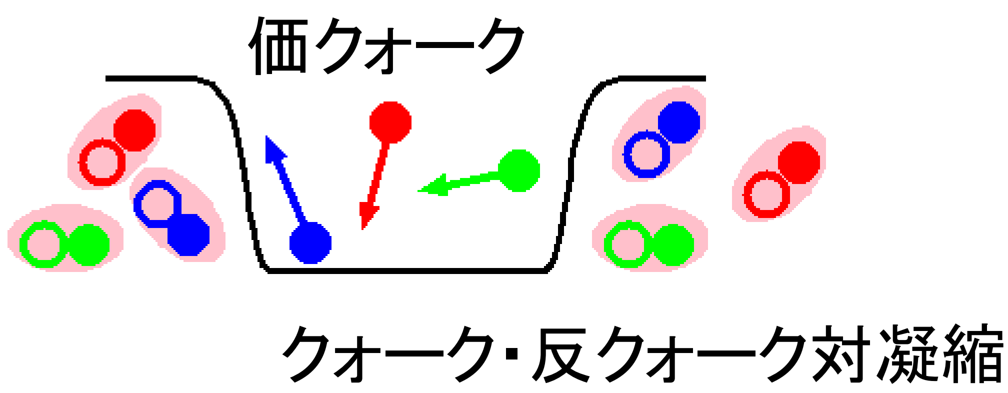
Finite μ: Fodor & Katz, JHEP 0203 (2002), 014.

Zero Chem. Pot. : Cross Over
Finite Chem. Pot.: Critical End Point

量子色力学 (QCD) の「真空」

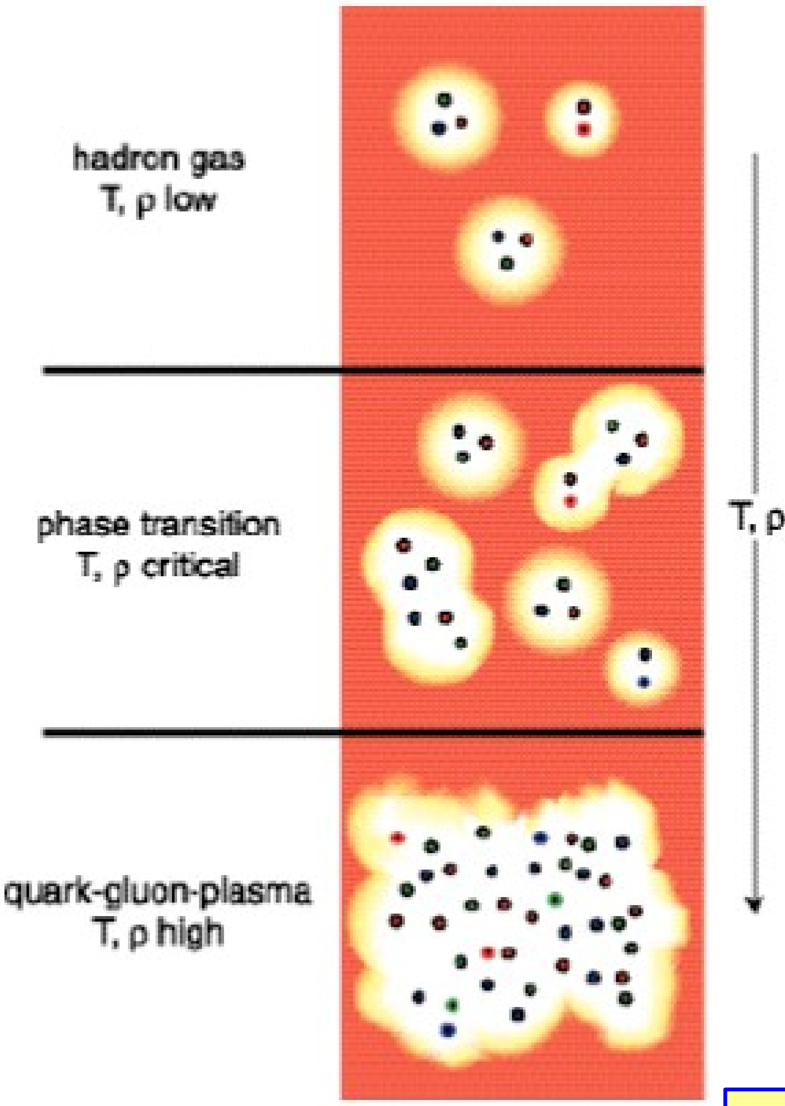
- 色の閉じ込め:クォーク間には「ひも」のような力が働く
  - クォーク間の電場はひも状に絞られている(⇔超伝導体での磁場)
  - 引き離そうとするとクォーク対が生成されて色は閉じ込められたまま。
- 質量の獲得: 核子は「モーゼの道」の中の3クォーク状態
  - QCD 真空ではクォーク・反クォーク対が凝縮
    → 凝縮体を「押しのける」のにエネルギーが必要
    → 5 MeVの質量のクォークが3つで1000 MeVの大きな質量





QCD真空には「カラー単磁子」と「クォーク・反クォーク対」の凝縮体

なぜ高温でQGPへの相転移がおこるか？(1)



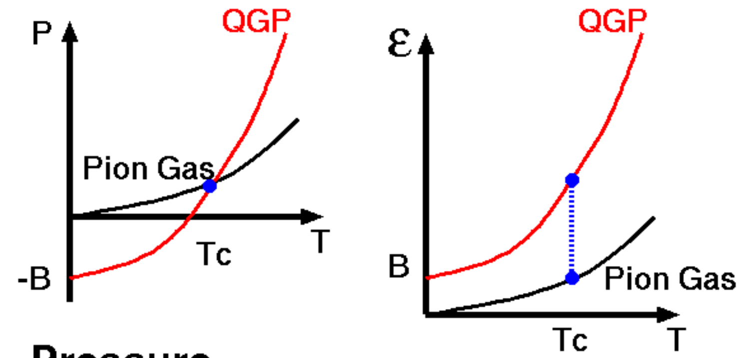
- ハドロン物質を熱する/圧縮するとどうなるか？
  - ハドロン(核子や中間子)は、1 fm 程度の大きさをもち、クォークと力を媒介するグルーオンからできている。(クォーク3つか、クォーク・反クォーク対)
  - 温度の増加により、多くの中間子が作られる
    → クォーク・反クォークの数が
    増えて、ハドロンが「重なる」
  - 核子内部の密度まで圧縮する
    → 核子同士が「重なる」

温度・密度を十分上げれば、
大きな体積でクォークが自由に動き回るはず

なぜ高温でQGPへの相転移がおこるか？ (2)

- 質量0の粒子の大自由エネルギー＝－(圧力)
  → ステファン・ボルツマン則 (T^4 に比例)

$$\Omega = -PV = -\frac{\pi^2 V}{90}\left(\sum_B g_B + \frac{7}{8}\sum_F g_F\right) T^4$$

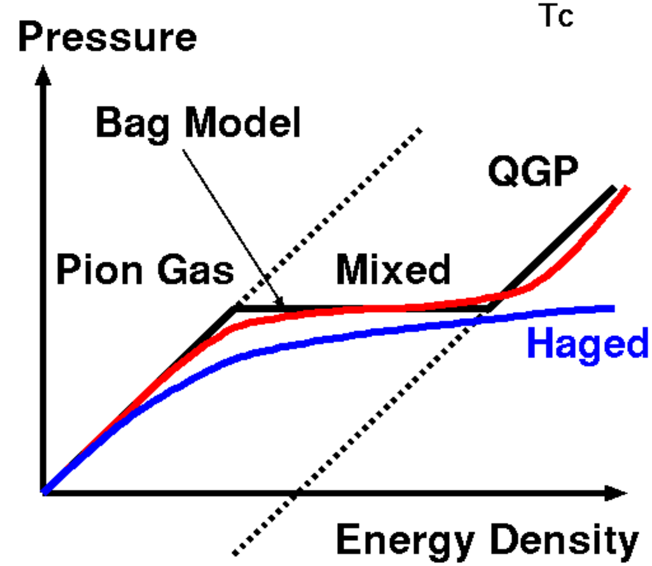


- ハドロン相 ~ 3種類の質量0のπ粒子

$$P_\pi = \frac{\pi^2}{30} T^4 \quad , \quad \epsilon_\pi = \frac{\pi^2}{10} T^4$$

- QGP ~ 質量0のクォーク･グルーオンと「真空」の負圧力

$$P_{QGP} = \frac{37\pi^2}{90} T^4 - B \qquad \epsilon_{QGP} = \frac{37\pi^2}{30} T^4 + B$$

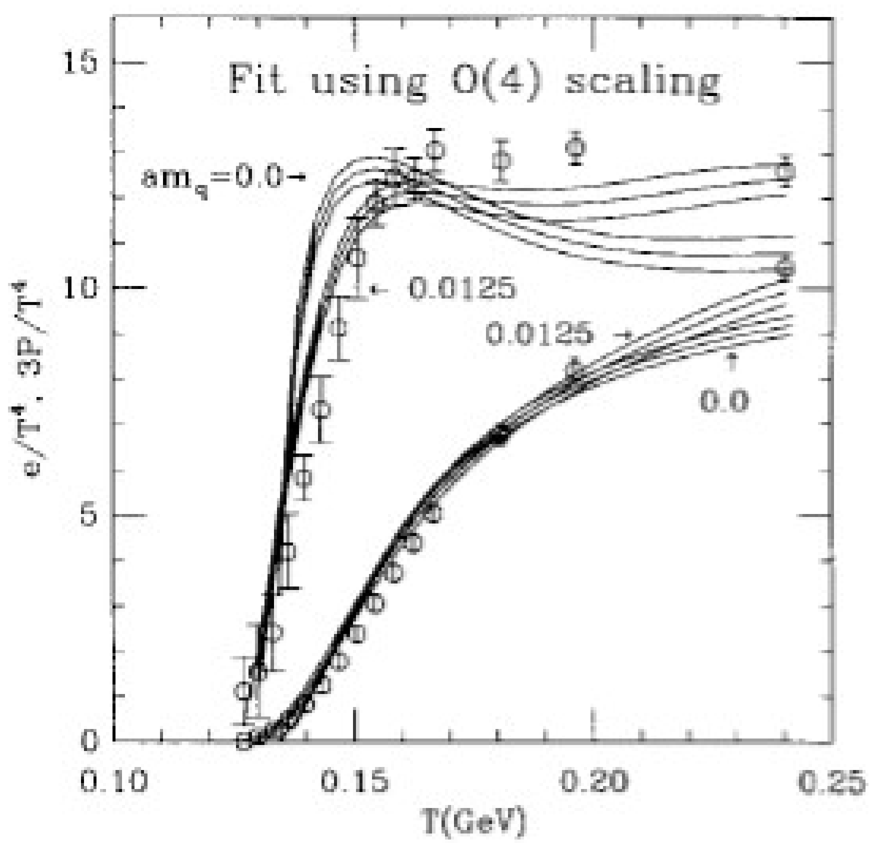


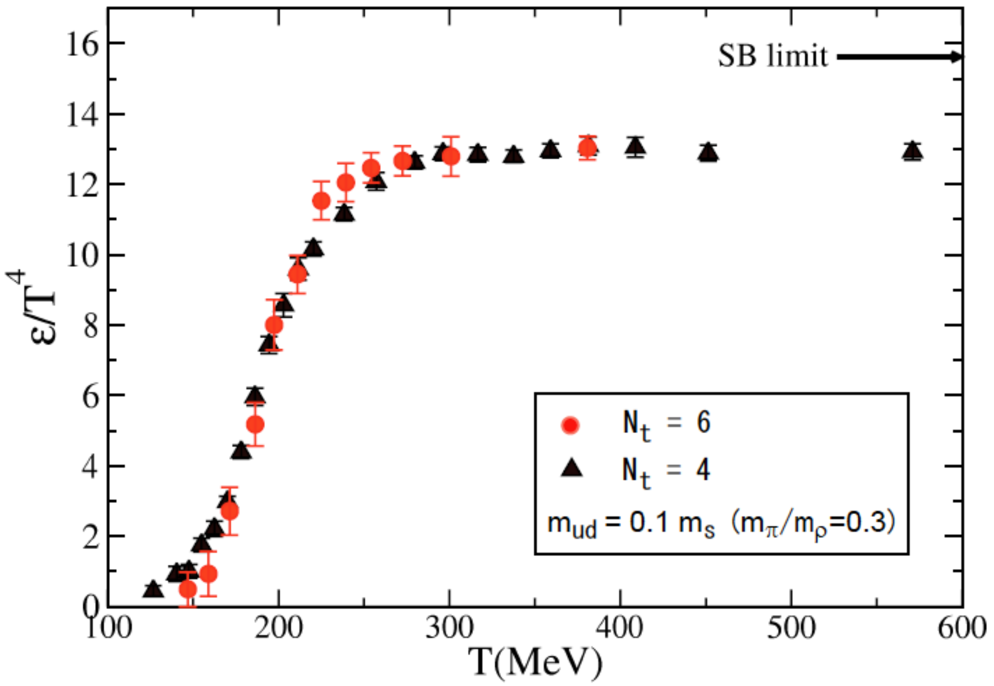
QGPへの相転移 = 自由度増加＋真空の変化

$$DOF = 2(\text{spin}) \times 2(q, \bar{q}) \times 3(\text{color}) \times 2(\text{flavor}) \times 7/8(\text{Fermion}) + 2(\text{spin}) \times 8(\text{color}) = 37$$

なぜ高温でQGPへの相転移がおこるか？ (3)

- **QCDに基づく第一原理計算＝格子QCD シミュレーション**
  - **T^4 で規格化したエネルギー密度と圧力 → T = 150-200 MeV で エネルギー密度は急激に変化、圧力はやや滑らかに増加 → QGP への相転移**

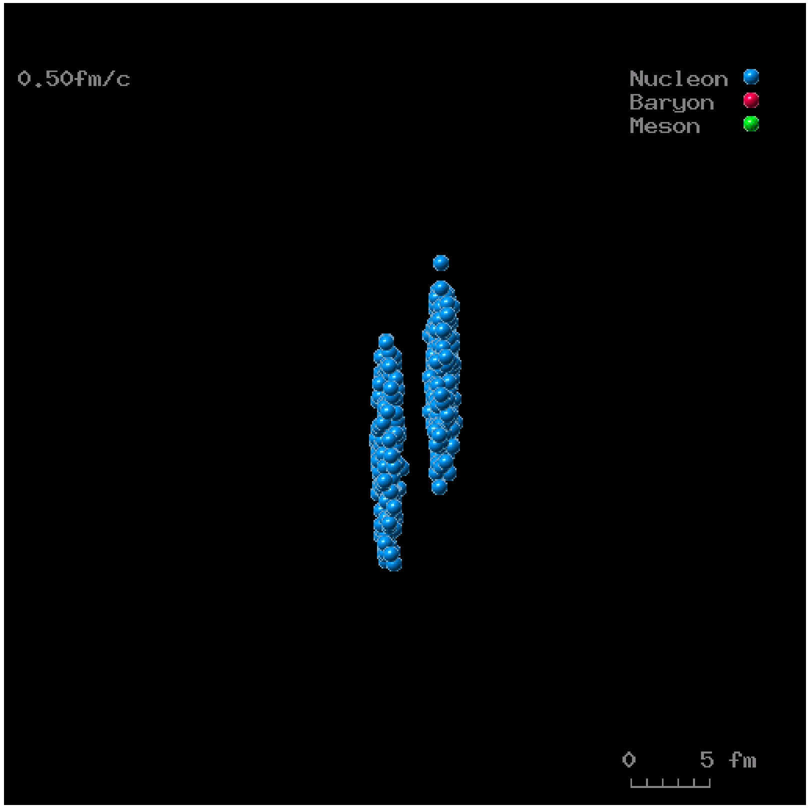




MILC Coll., hep-lat/061001
$O(a^2)$ improved action
N_s/N_t=2, inexact R-algorithm.

クォーク･グルーオン･プラズマを作るには？

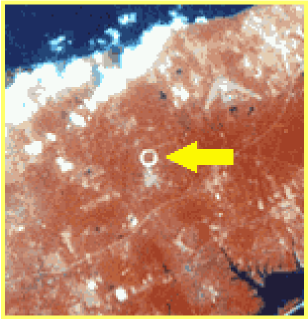
- クォーク･グルーオン･プラズマ (QGP)
  - 大きな体積中をクォークとグルーオンが閉じ込めから解放され、凝縮のない単純な真空を動き回っている状態
  - 初期宇宙等の「超高温状態」(~10^{12} K)や、中性子星中心部などの「超高密度状態」(~ 10^{15} g/cc)で実現
  - 実験室でのQGP生成
    → 高エネルギーの重イオン反応



高エネルギー原子核反応での QGP生成 =地上の "Big Bang" 再現実験

High Energy Heavy-Ion Collision Experiments

- ランダウの昔から核物理屋は重イオン反応でQGPを作りたかった！
  - LBL-Bevalac: 800 A MeV
  - GSI-SIS: 1-2 A GeV
  - BNL-AGS (1987-): 10 A GeV
  - CERN-SPS (1987-): 160 A GeV
  - BNL-RHIC (2000-): 100+100 A GeV
  - CERN-LHC (2008(?)-): 3 + 3 A TeV



QGPは見つかったか？
― ジェット抑制と楕円型フロー ―

QGP生成のシグナル

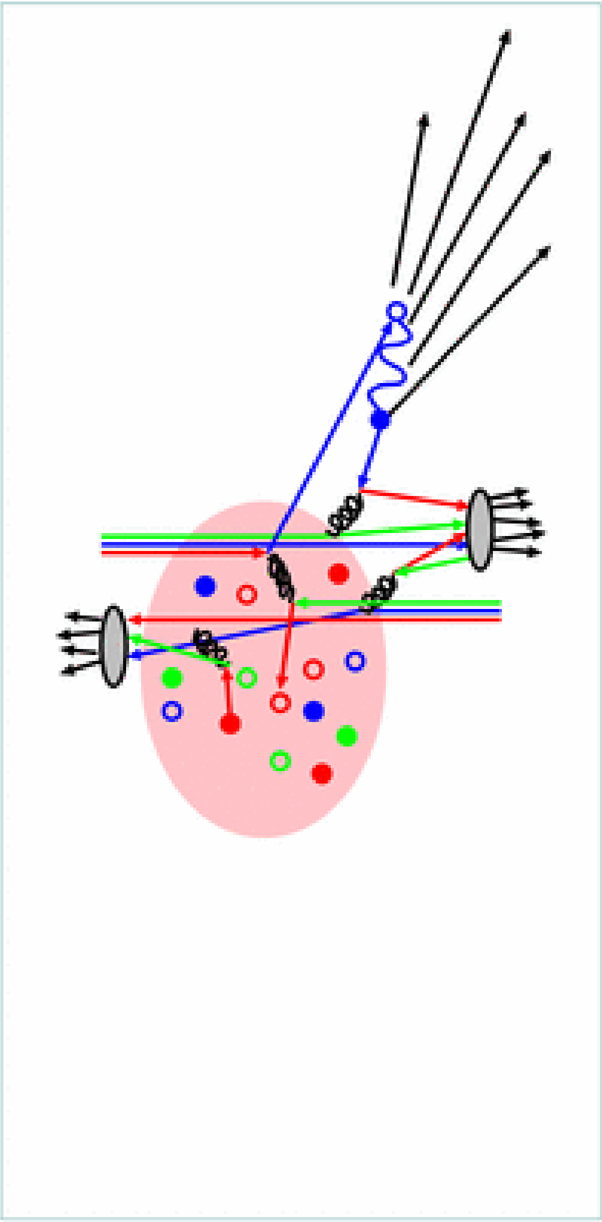
- QGP が作られると何が起こるか？
  - 初期の核子内のパートン
    (クォーク、グルーオン)の激しい散乱

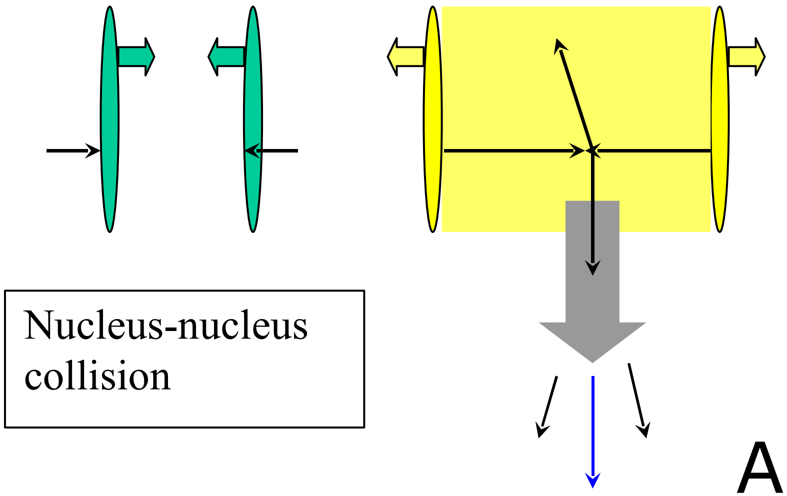
  → QGPが生成されると、カラー電荷
  （クォーク、グルーオン）が熱的に分布

  → *クォークやグルーオンが*
  エネルギーを損失
  （ジェット抑制、Jet Quenching)
  c.f.荷電粒子は電子と散乱して
  エネルギーを損失）

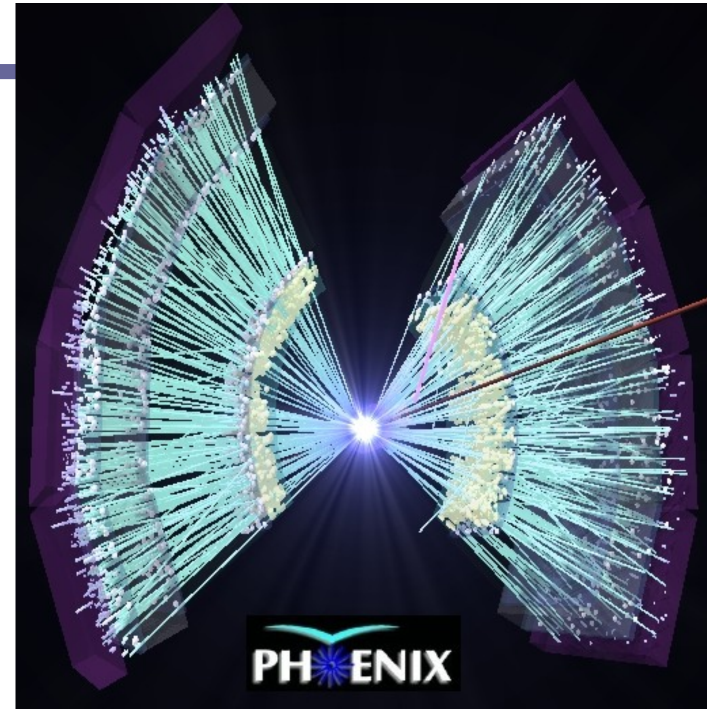
  - 早い段階で熱平衡化

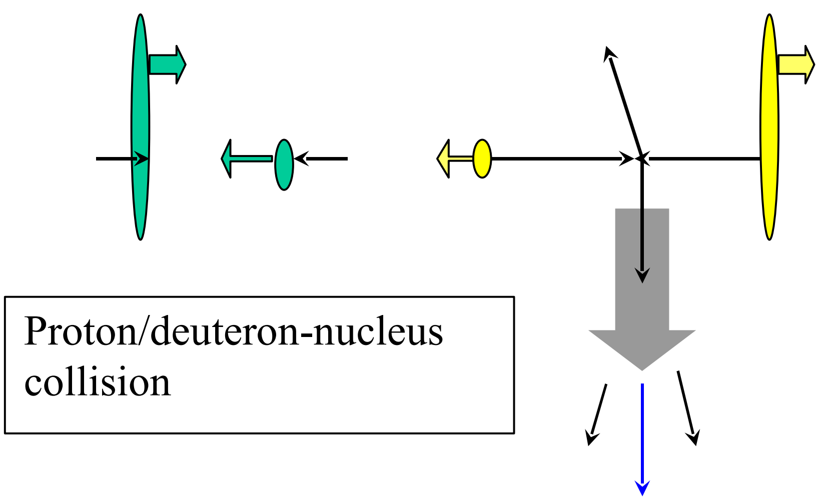
  → (熱平衡が仮定される）
  流体力学的振る舞い



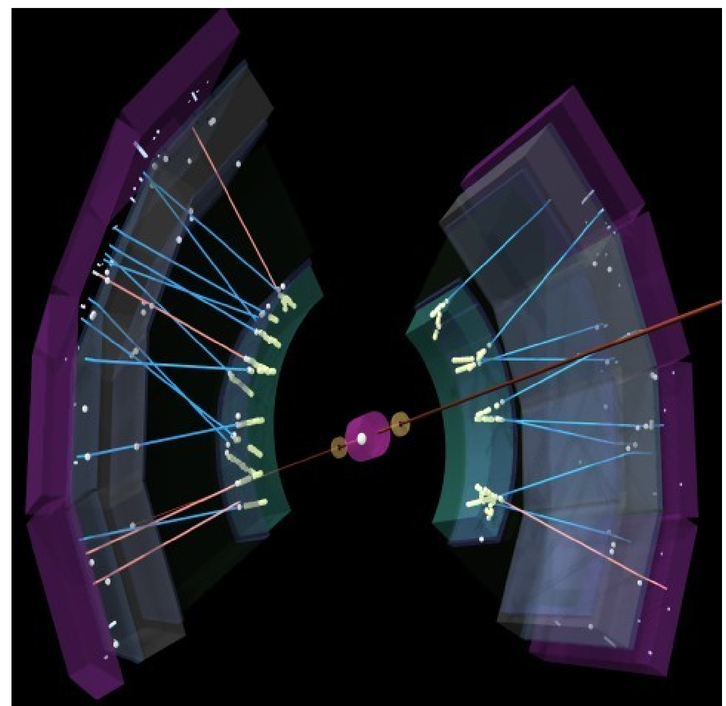

Nucleus-nucleus
collision

Au+Au


PHENIX


Proton/deuteron-nucleus
collision

d+Au

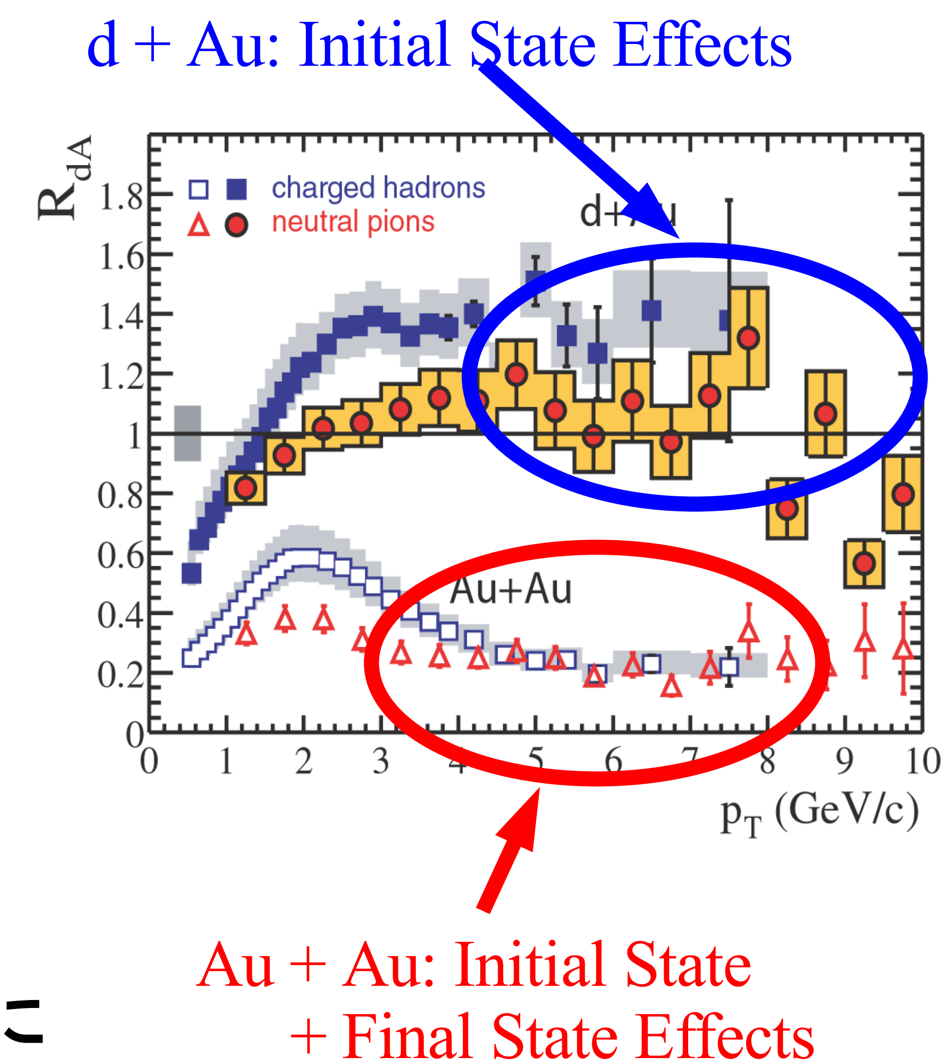


QGP生成の実験的証拠：ジェット抑制 (1)

- 原子核抑制因子 R_{AB}
  =核子衝突と比べた粒子生成率
  $R_{AB} \geqq 1$ (抑制なし)
  $R_{AB} < 1$ (抑制あり)

$$R_{AB}(p_T) = \frac{d^2N/dp_T d\eta}{T_{AB}\, d^2\sigma^{pp}/dp_T d\eta}$$

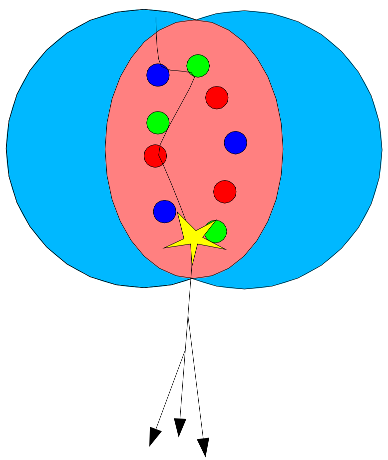
- 本当にジェット抑制は見えるか？
  - d+Au 衝突では No！
  - 大きな原子核衝突ではYes！
- エネルギー密度が大きくなったときにだけ、ジェット抑制が起こる
  → QGP の形成

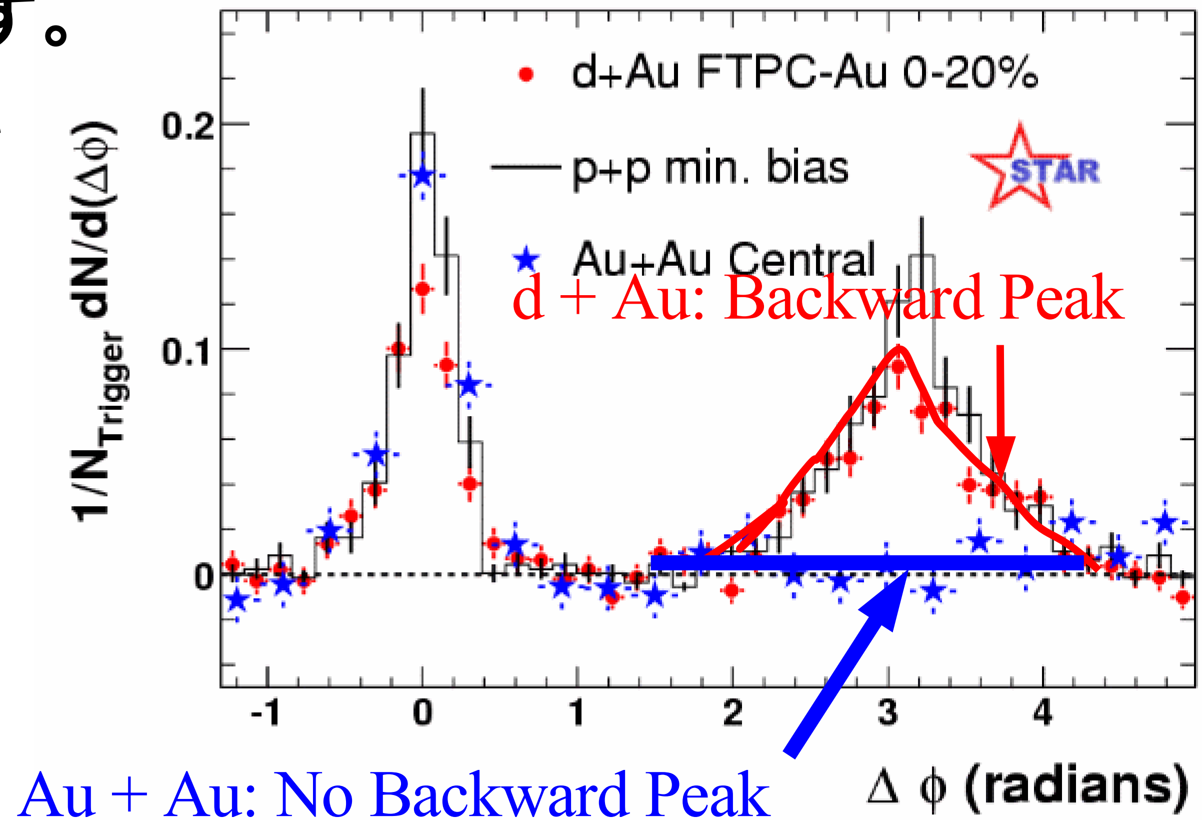


QGP生成の実験的証拠 :ジェット抑制 (2)

STAR

- ジェットが消えているなら、裏側の相関が消えるはず。
  - d+Au では消えていない
  - Au+Au では消える

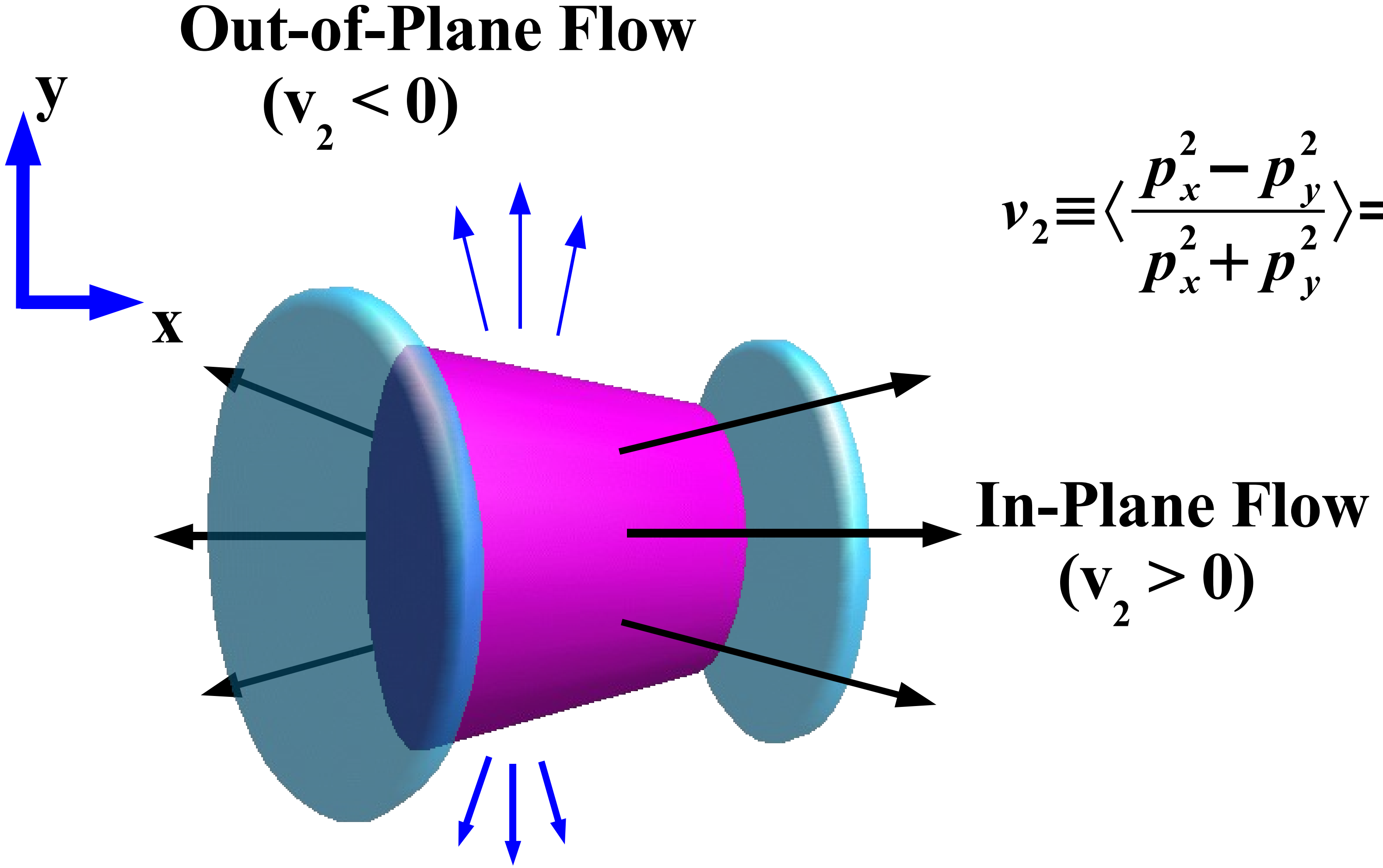




ジェットが抑制されると、裏側の相関が見えなくなる
→ ジェットが一本しか見えない
→ QGP 生成のシグナル

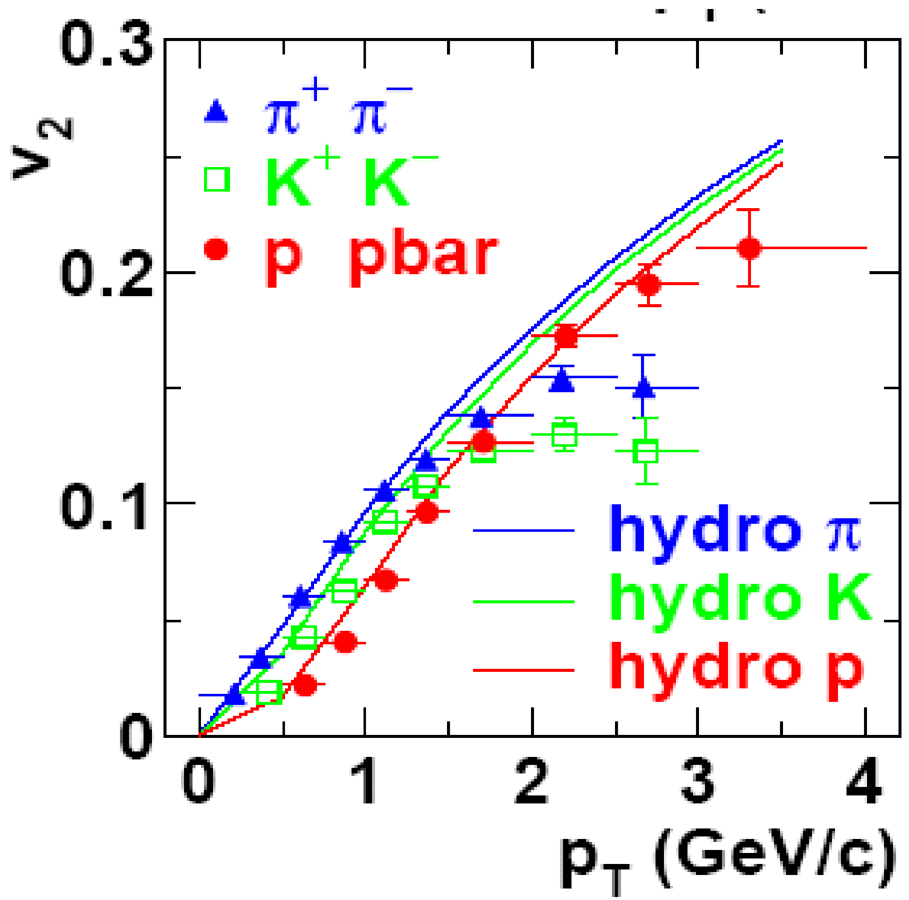
QGP生成の実験的証拠: 強い楕円型フロー(1)

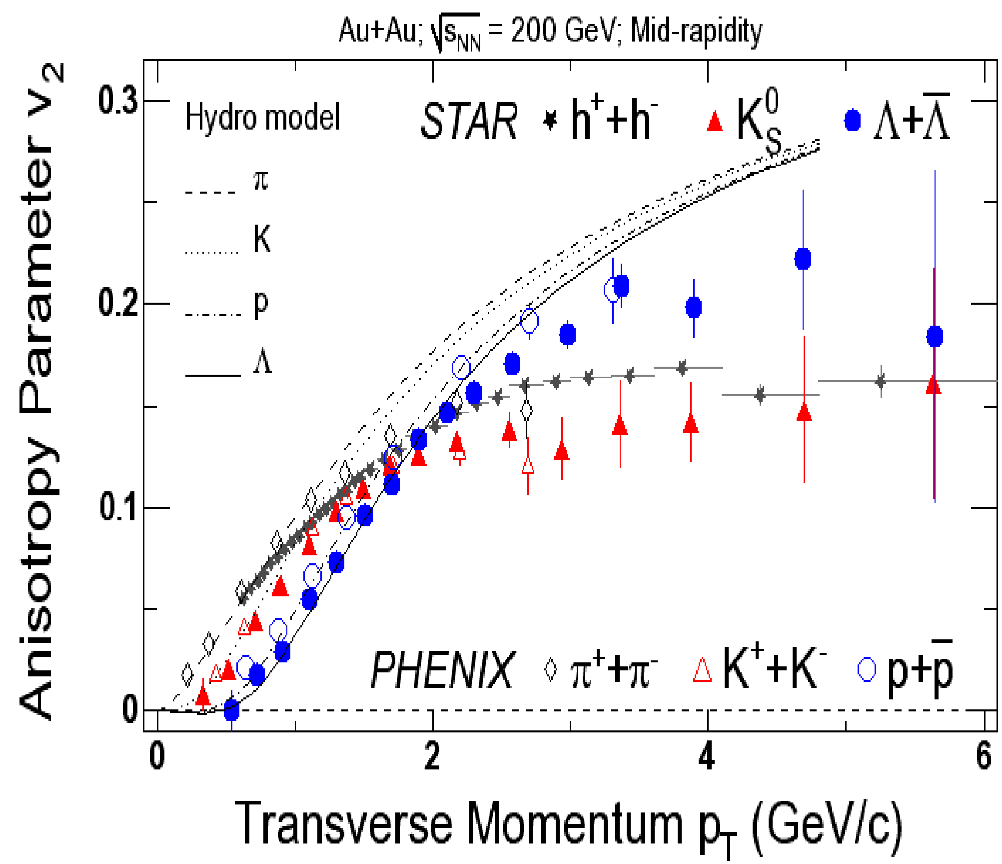
- 楕円型フロー=運動量の方位角異方性
  - 反応初期の「空間異方性」が起源
    →圧力勾配が作られる熱平衡化の速さに依存



$$v_2 \equiv \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle = \langle \cos 2\phi \rangle$$

QGP生成の実験的証拠: 強い楕円型フロー(2)

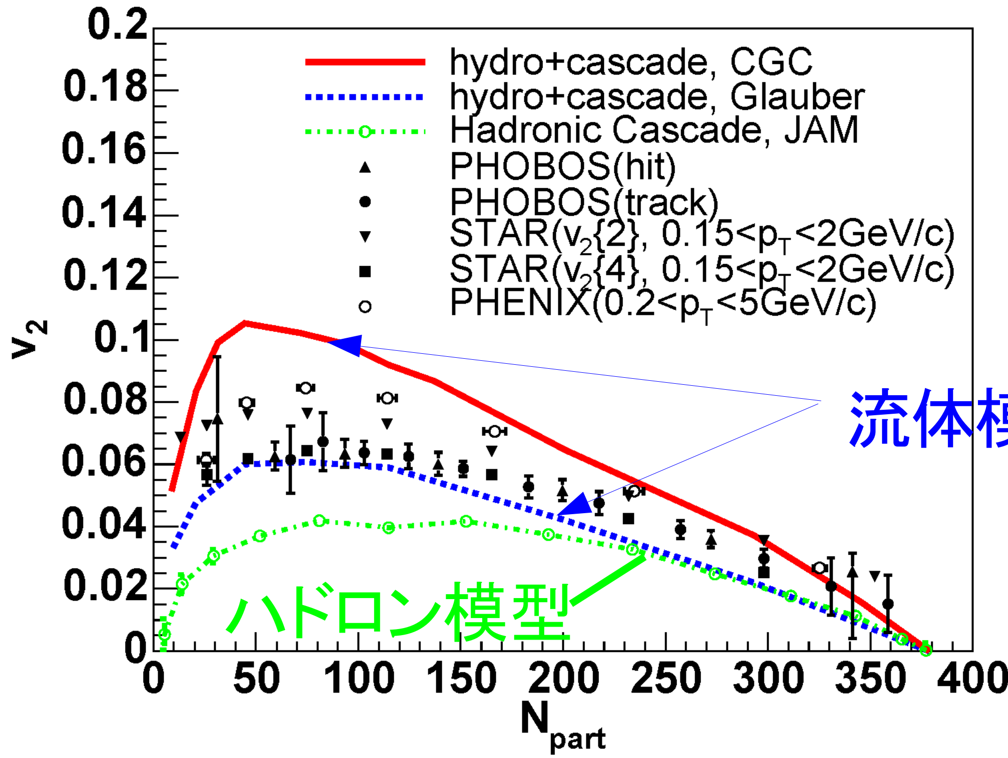


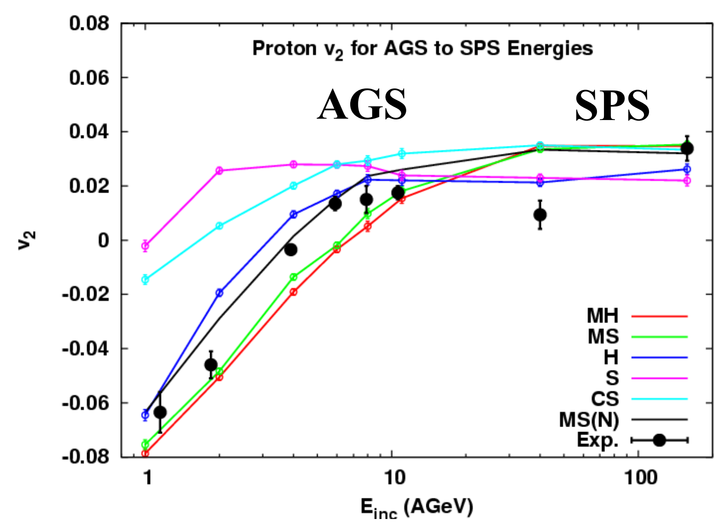


Low Momentum : Hydrodynamical calc. with Early Thermalization
High Momentum : Reduction from Hydro. calc.

QGP生成の実験的証拠:強い楕円型フロー(3)

- RHICエネルギーでは強い楕円型フローが見られる
  - Au+Au: v_2(Casc.) < v_2(hydro) ~ v_2(data)
    - QGP生成を仮定した流体力学模型と無矛盾
  - 低いエネルギーを説明するハドロン模型とは矛盾





低いエネルギーでの結果
→ハドロン模型の範囲内

Isse, AO, Otuka, Sahu, Nara, PRC 72 ('05), 064908

QGP物性の探求へ向けて

「QGP 生成の証拠」のまとめ

- *強いジェット抑制 → 色電荷を持つ粒子の分布*
  - Nucl. Mod. Factor (R_{AA})、ハドロン方位角相関でともに観測
  - pp 衝突、d+Au 衝突、SPS までの重イオン衝突で見られず、
    RHIC エネルギーの重イオン衝突でのみ観測
  - 中心衝突に近いほど強い抑制
- *大きな楕円フロー(v_2) → 早い熱平衡化*
  - 非常に早い(τ<1 fm/c) 段階での熱平衡化が必要
    (ハドロンの formation time ($\tau_f \sim 1$ fm/c) と同等の時間での平衡化)
  - SPS までの v_2 を説明するハドロン-ストリング輸送模型で足りず、
    早い熱平衡化 ($\tau\sim$ 0.6 fm/c) を仮定したQGP 流体模型で説明可
  - Constituent Quark Number Scaling ($v2^h(pT)= n\ v2^q(pT/n)$) が成立
    → クォーク段階でのフロー生成
- 化学凍結の温度・化学ポテンシャルが Lattice QCD の予言に近い

簡単な仮定のもとで "大雑把な性質" の解明はすすんだが...

多くのパズルが残されている...

- 例1: 相対論的な「粘性流体方程式」
  → 共変な定式化さえ、きちんとできていない
- 例2: 高い運動量領域での楕円型フローのデータ
  → 幾何学的な「極限」を越えている！
  - 「極限越え」の例: ジェットと流体パートンの融合によるストリング生成

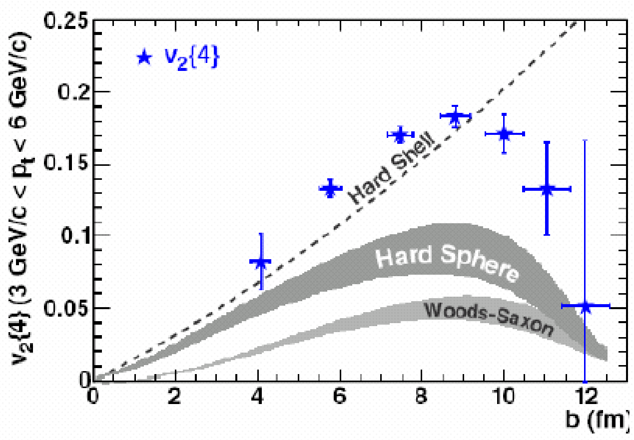
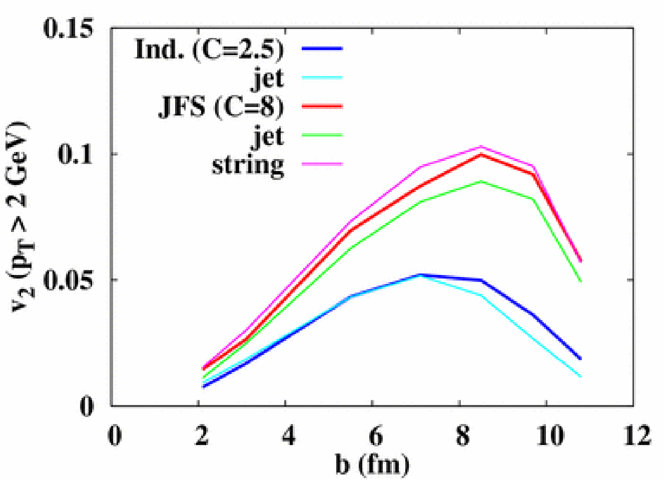
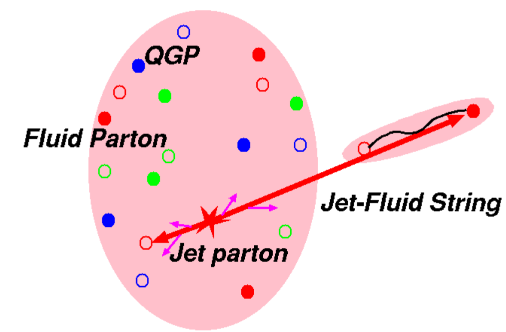


FIG. 4 (color online). v_2 at $3 \le p_t \le 6$ GeV/c versus impact parameter, b, compared to models of particle emission by a static source (see text).

STAR, PRL93, 252301('04)





Hirano,Isse,Nara,AO,Yoshino, in prep.

> ***粘性(非完全流体)、非平衡過程***
> ***→ より高いエネルギーの原子核衝突(LHC, 2008稼働)では重要***

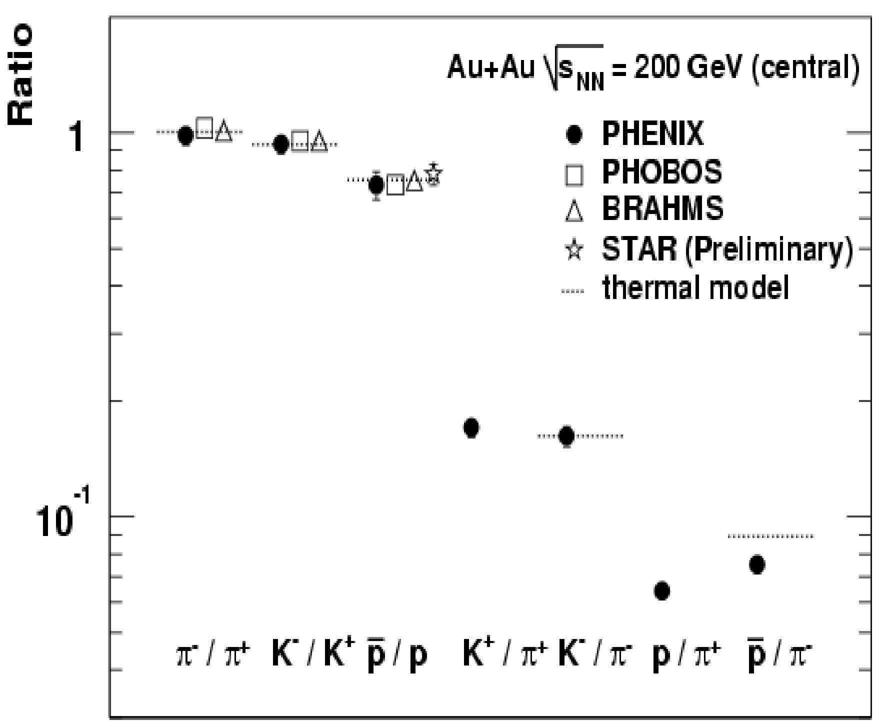
Summary

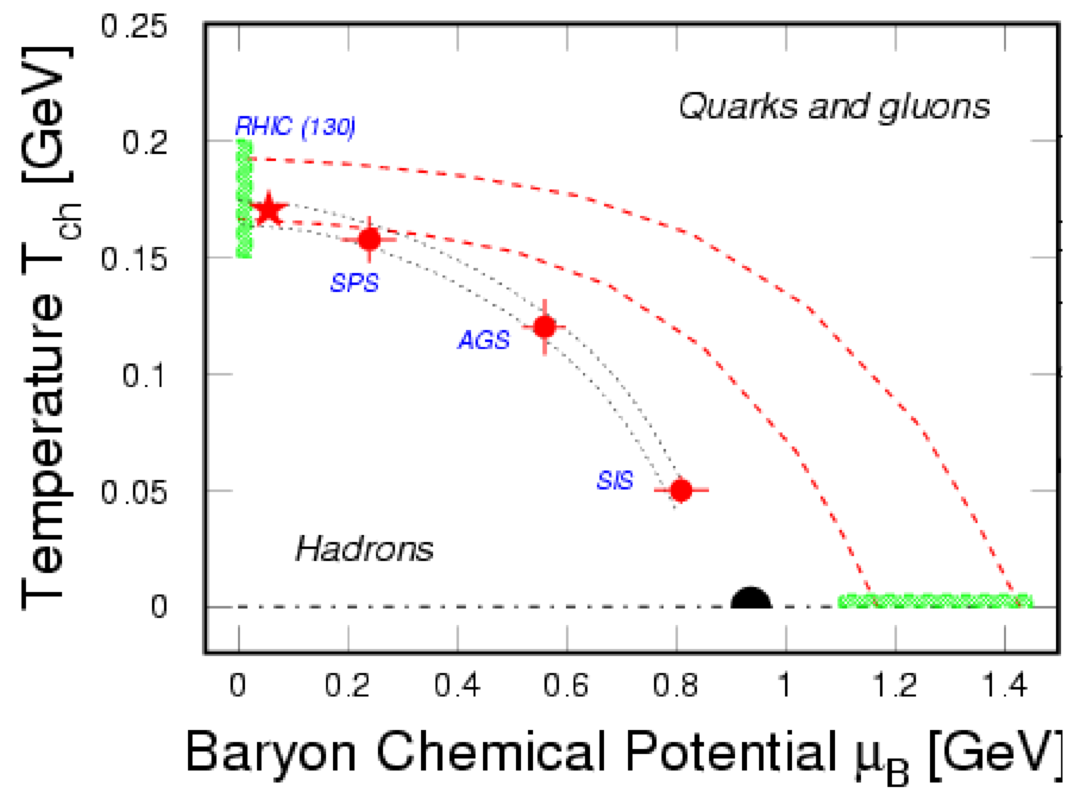
- *高エネルギー原子核衝突におけるクォーク・グルーオン・プラズマ (QGP)生成*
  ≈ 地上における小さなビッグバン(Little Bang)
  ≈ 宇宙最後の真空相転移のシミュレーション
  - 2000年6月稼動のRHICで人類は(おそらく)初めて生成 (21世紀に間に合いました！)
- QGP生成のシグナル
  - 「ジェット抑制」と「強い楕円型フロー」はハドロン模型で説明不可
  - 他にも楕円型フローのクォーク数スケーリング等のシグナルあり
- QGP物性の理解へ向けて
  - 「QGP 生成の証拠」を超える様々なデータが出てきている
    → high pT v2 問題、J/ψ 問題、Mach Cone、baryon 問題 (Part V へ)
  - 第一原理計算 (Lattice QCD, perturbative QCD)
    + 現象論 (Hydro, Jet, String, Color Glass Condensate, ...)
    の両面からの追求が今後も必要

Backups

Thermal Freeze-out Parameters from particle ratios

- **Almost complete reconstruction of particle ratios by the statistical thermal model.**
- **Thermal model prediction in AuAu 200 GeV central.**
  T_{ch} = 177 MeV, μ_B = 29 MeV



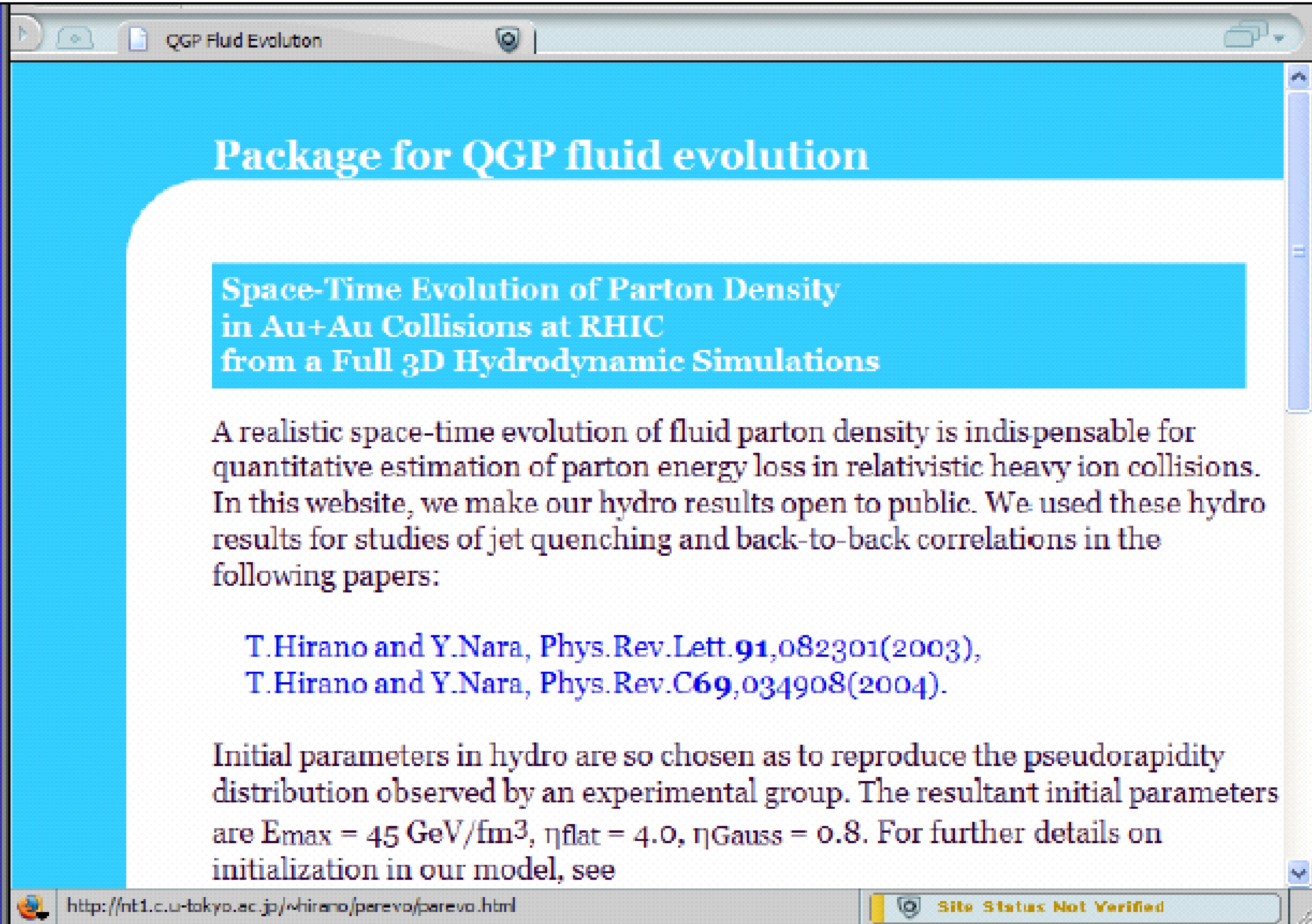


by Esumi, 2003

シミュレーション資産の共有

- ***格子QCD計算***
  - クォークの積分まで含めた計算
    → 行列のサイズが「格子点数x12」の行列式計算を、それぞれの
    　グルーオン配位に対して計算する必要性あり
    → 世界的なグルーオン配位の共有
- ***流体力学***
  - 2+1 (空間、時間)次元ではプログラムが公開
  - 3+1次元では「計算結果」を公開
- ***ジェット生成***
  - プログラムが公開され、メンテナンスされている

http://nt1.c.u-tokyo.ac.jp/~hirano/parevo/parevo.html



QGP Fluid Evolution

Package for QGP fluid evolution

Space-Time Evolution of Parton Density in Au+Au Collisions at RHIC from a Full 3D Hydrodynamic Simulations

A realistic space-time evolution of fluid parton density is indispensable for quantitative estimation of parton energy loss in relativistic heavy ion collisions. In this website, we make our hydro results open to public. We used these hydro results for studies of jet quenching and back-to-back correlations in the following papers:

T.Hirano and Y.Nara, Phys.Rev.Lett.**91**,082301(2003),
T.Hirano and Y.Nara, Phys.Rev.C**69**,034908(2004).

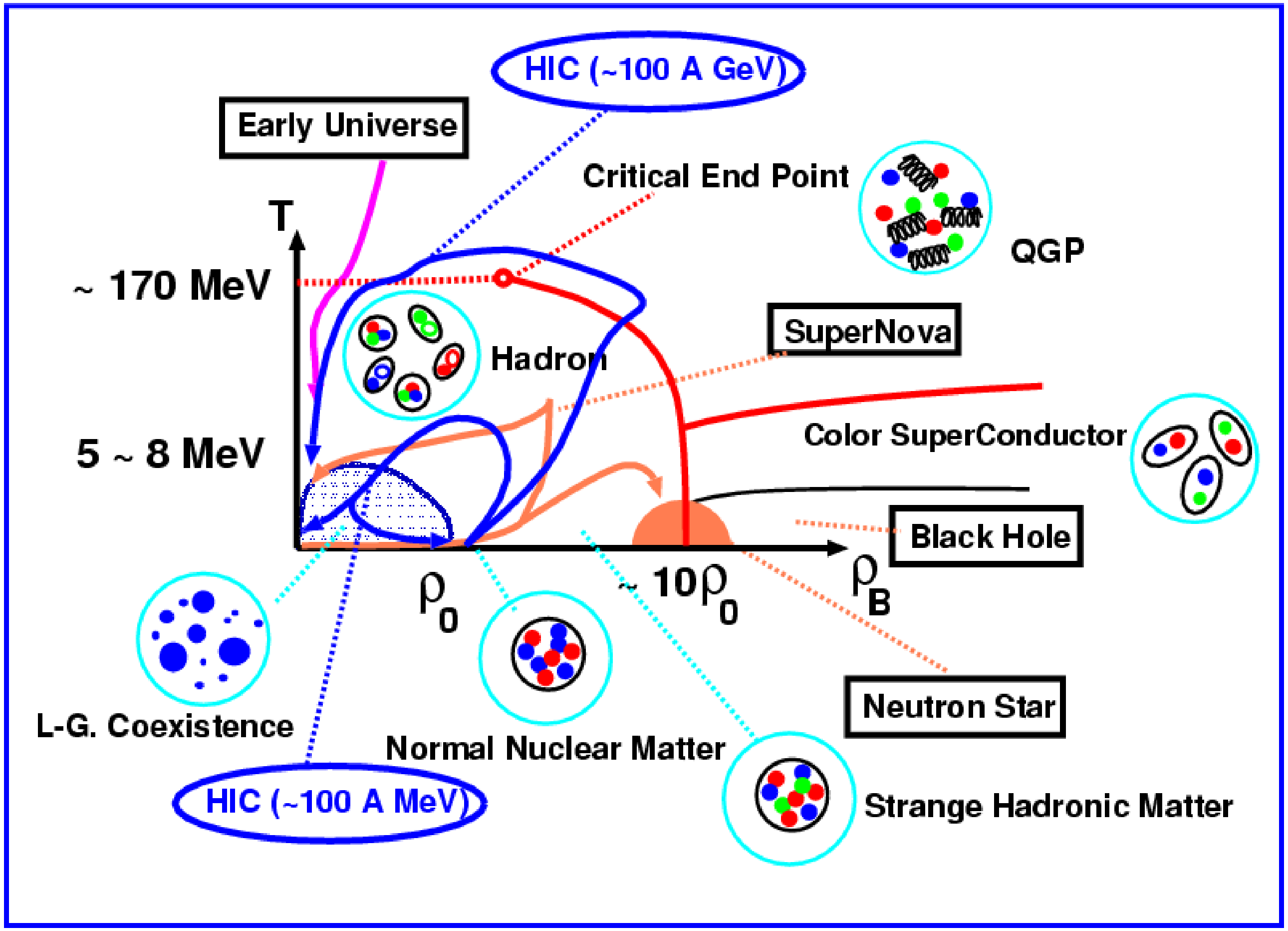
Initial parameters in hydro are so chosen as to reproduce the pseudorapidity distribution observed by an experimental group. The resultant initial parameters are E_{max} = 45 GeV/fm^3, η_{flat} = 4.0, η_{Gauss} = 0.8. For further details on initialization in our model, see

http://nt1.c.u-tokyo.ac.jp/~hirano/parevo/parevo.html

Site Status Not Verified

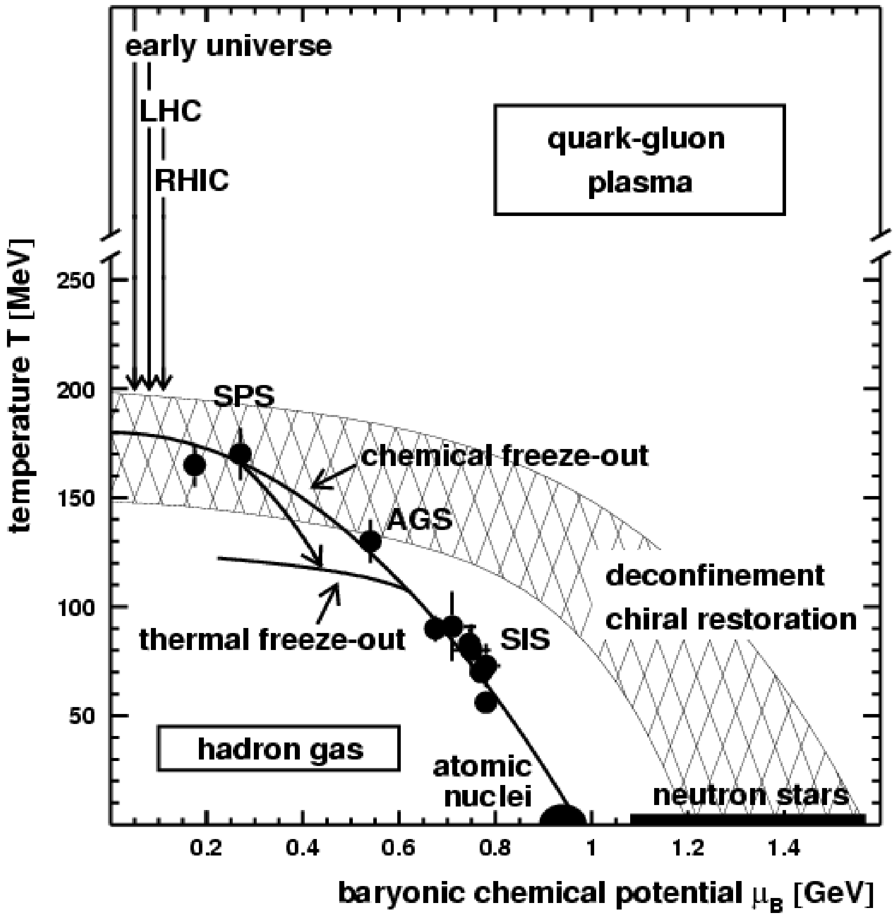
Part II: Basic Ingredients in Heavy-Ion Collision Theory

Hadronic Matter Phase Diagram

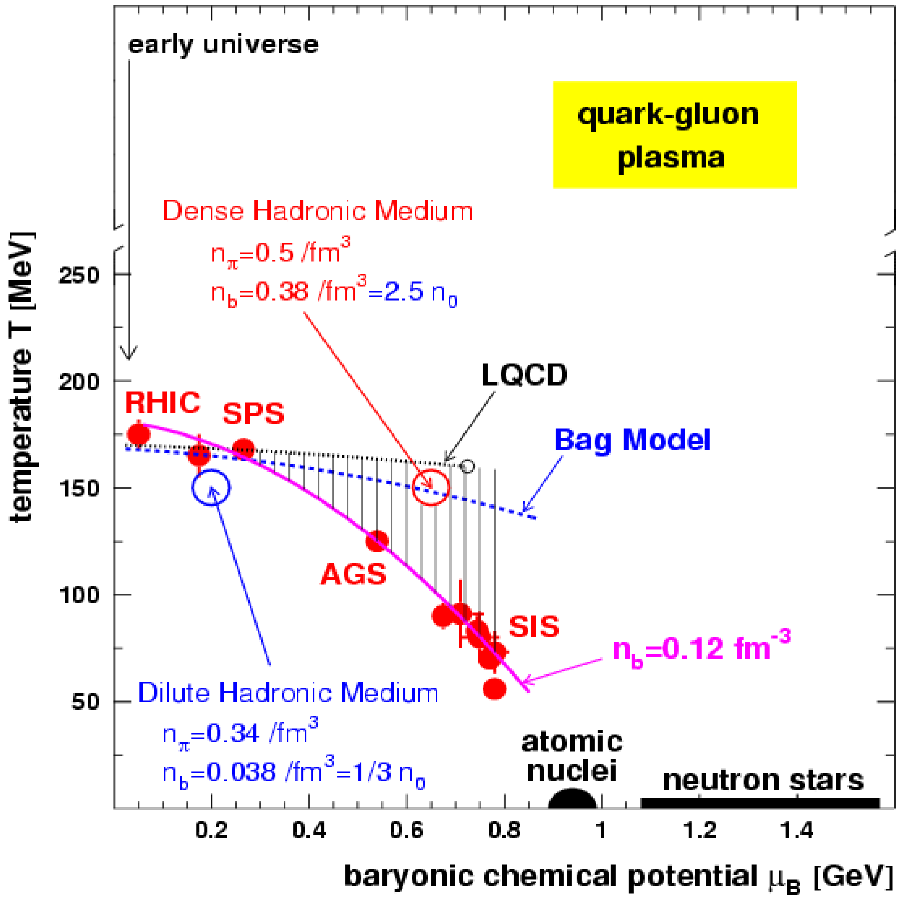


HIC (~ A few 100 A MeV) = Little Supernova
HIC (100+100 A GeV) = Little Big Bang

Experimentally Estimated Phase Diagram



J. Stachel et al., 1998

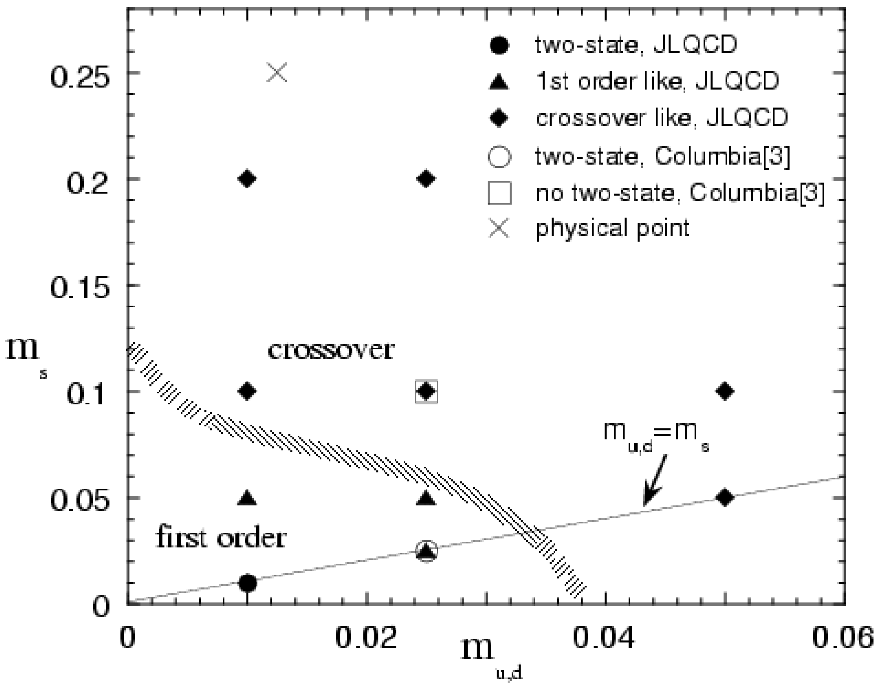


Braun-Munzinger et al., 2002

Chem. Freeze-Out Points are very Close to Expected QCD Phase Transition Boundary

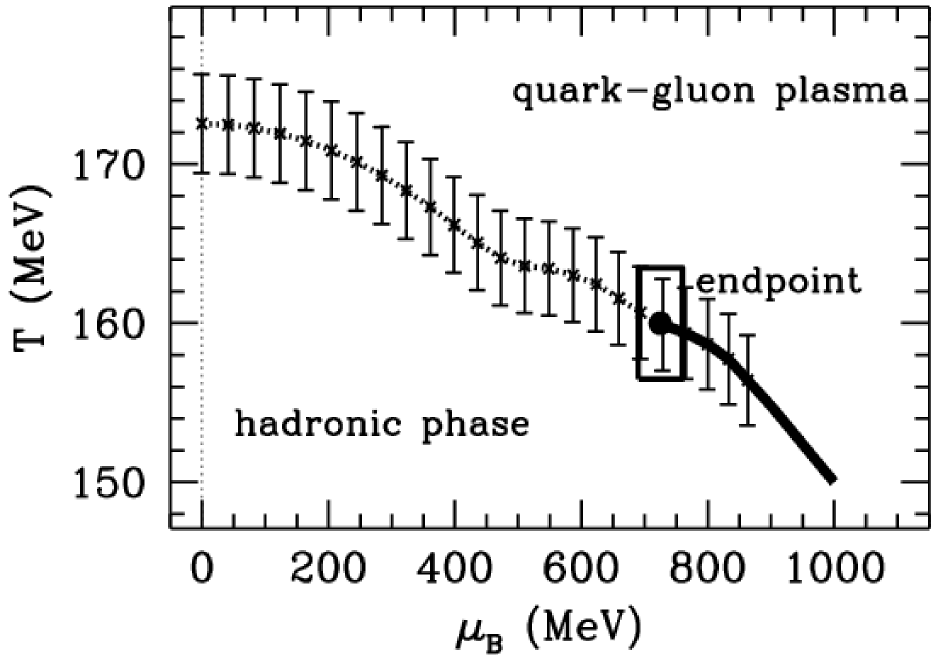
Theoretically Expected QCD Phase Diagram

Zero Chem. Pot.



JLQCD Collab. (S. Aoki et al.), Nucl. Phys. Proc. Suppl. 73 (1999) 459.

Finite Chem. Pot.



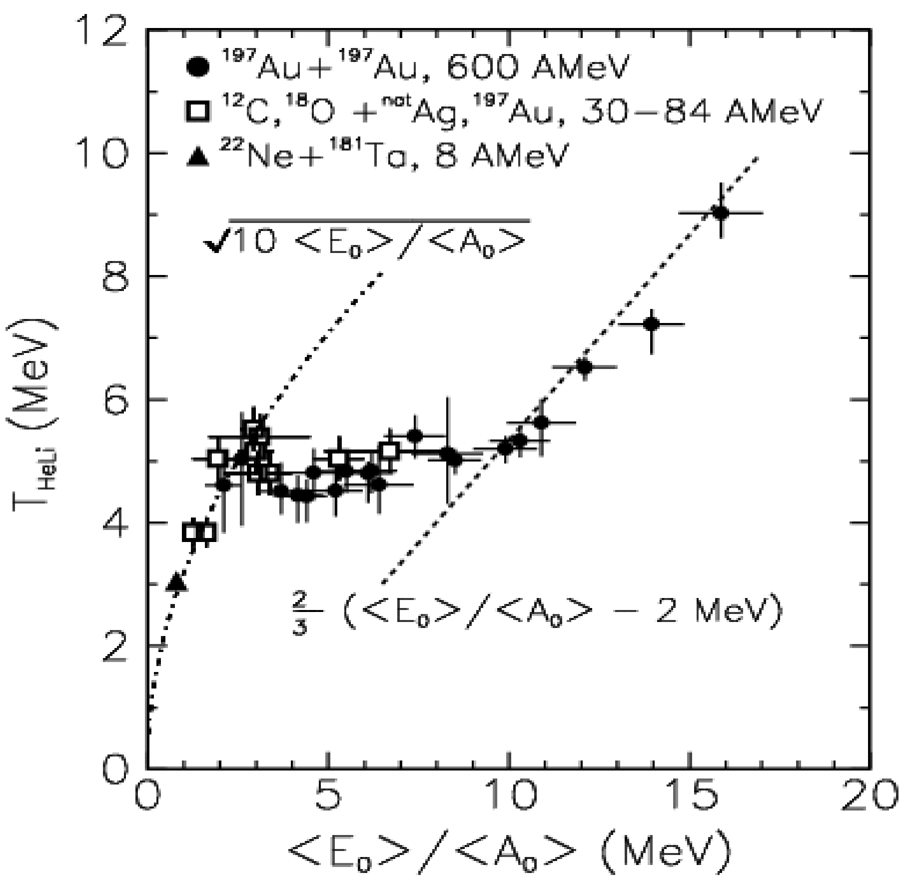
Finite μ: Fodor & Katz, JHEP 0203 (2002), 014.

Zero Chem. Pot. : Cross Over
Finite Chem. Pot.: Critical End Point

Nuclear Caloric Curve

J. Pochadzalla et al (GSI-ALLADIN collab.) ., PRL 75 (1995) 1040.

Boiling Temperature is Clearly Seen



Fragment Yields are assumed to follow Equilibrium Statistics

$$Y_f \propto g_f \exp((B_f + Z\mu_p + N\mu_n)/T)$$

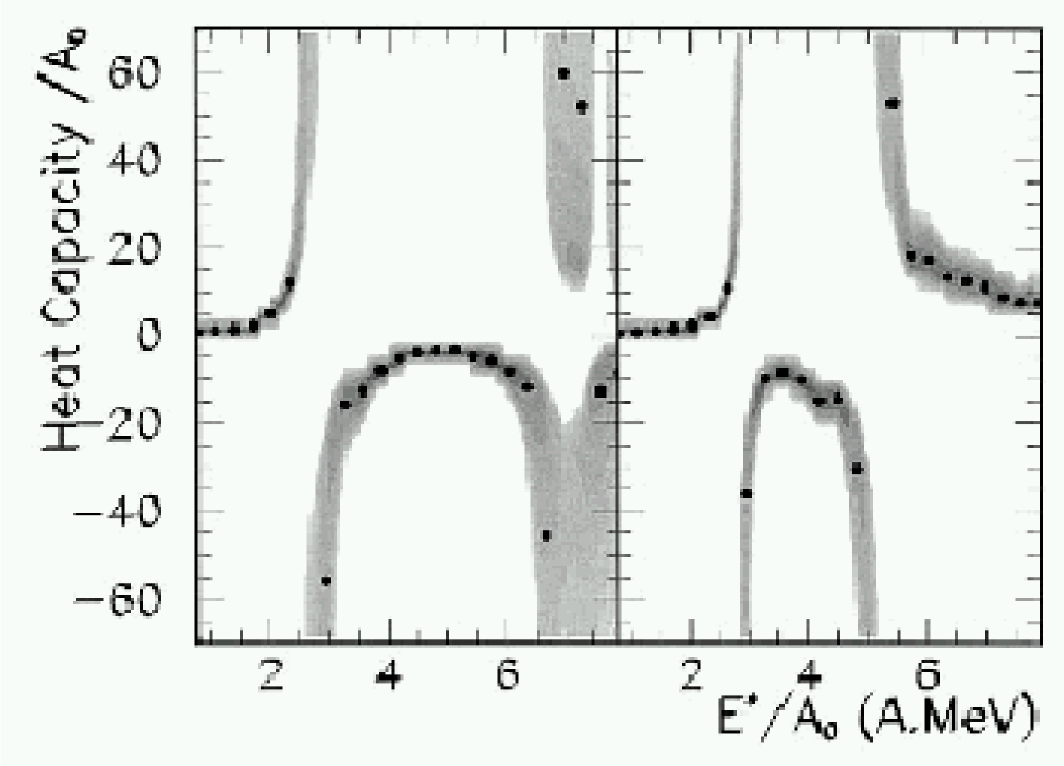
$$\to \frac{Y(^4He)/Y(^3He)}{Y(^7Li)/Y(^6Li)} \propto \exp(\Delta B/T)$$

Negative Heat Capacity

M. D Agostino et al.,(MSU Exp./INFN-IN2P3 Collab.) PLB 473 (2000) 219.

- **Negative Heat Capacity**
  → Evidence of the First Order Phase Transition

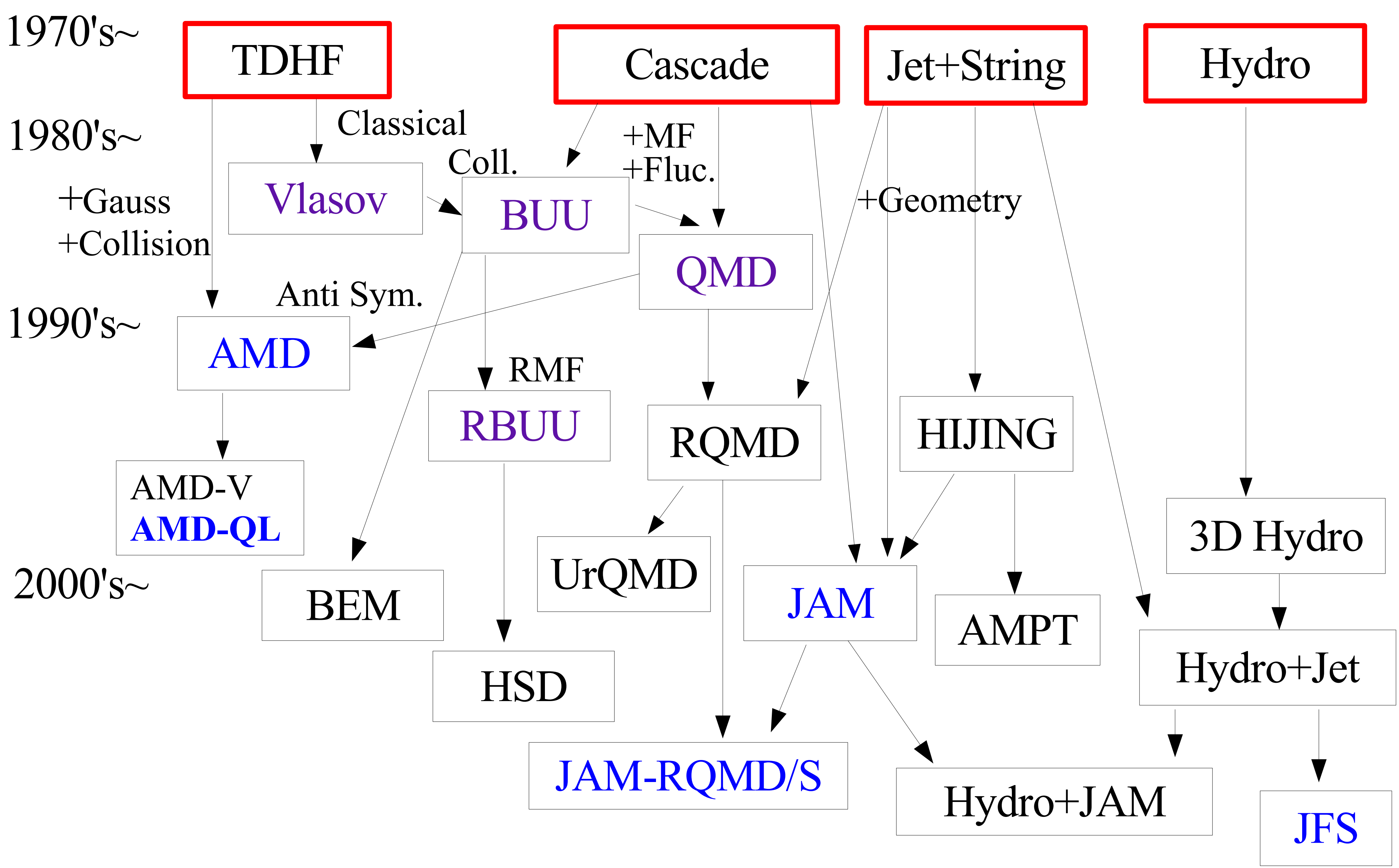
- ***T* and *E** are determined**
  from Fragment Multiplicity and Kinetic Energy
  based on Theoretical Model



HIC Models: Major Four Origins

- ***Nuclear Mean Field Dynamics***
  - **Basic Element of Low Energy Nuclear Physics, and Critically Determines High Density EOS / Collective Flows**
  - **TDHF → Vlasov → BUU**
- ***NN two-body (residual) interaction***
  - **Main Source of Particle Production**
  - **Intranuclear Cascade Models**
- ***Partonic Interaction and String Decay***
  - **Main Source of high pT Particles at Collider Energies**
  - **JETSET + (previous) PYTHIA (Lund model) → (new) PYTHIA**
- ***Relativistic Hydrodynamics***
  - **Most Successful Picture at RHIC**

HIC Models: History



Nuclear Mean Field Models for Heavy-Ion Collisions

TDHF and Vlasov Equation

- **Time-Dependent Mean Field Theory (e.g., TDHF)** $i\hbar \frac{\partial \phi_i}{\partial t} = h\phi_i$
- **Density Matrix**

$$\rho(r, r') = \sum_i^{Occ} \phi_i(r)\phi_i^*(r') \quad \rightarrow \quad \boldsymbol{\rho_W = f} \textbf{ (phase space density)}$$

- **TDHF for Density Matrix**

$$i\hbar \frac{\partial \rho}{\partial t} = [h, \rho] \quad \rightarrow \quad \frac{\partial f}{\partial t} = \{h_W, f\}_{P.B.} + O(\hbar^2)$$

- **Wigner Transformation and Wigner-Kirkwood Expansion**
  (Ref.: Ring-Schuck)

$$O_W(r, p) \equiv \int d^3 s \exp(-i p \cdot s/\hbar) < r + s/2 | O | r - s/2 >$$

$$(AB)_W = A_W \exp(i\hbar\Lambda) B_W \quad \Lambda \equiv \nabla'_r \cdot \nabla_p - \nabla'_p \cdot \nabla_r \quad (\nabla' \text{acts on the left})$$

$$[A, B]_W = 2\mathrm{i}\, A_W \sin(\hbar\Lambda/2) B_W = i\hbar\{A_W, B_W\}_{P.B.} + O(\hbar^3)$$

Test Particle Method

- **Vlasov Equation**

$$\frac{\partial f}{\partial t} - \{h_W, f\}_{P.B.} = \frac{\partial f}{\partial t} + v \cdot \nabla_r f - \nabla U \cdot \nabla_p f = 0$$

- **Classical Hamiltonian**

$$h_W(r, p) = \frac{p^2}{2\mathrm{m}} + U(r, p)$$

- **Test Particle Method (C. Y. Wong, 1982)**

$$f(r, p) = \frac{1}{N_0} \sum_i^{AN_0} \delta(r - r_i)\, \delta(p - p_i) \quad \rightarrow \quad \frac{dr_i}{dt} = \nabla_p h_w, \quad \frac{dp_i}{dt} = -\nabla_r h_w,$$

Mean Field Evolution can be simulated
by Classical Test Particles
→ Opened a possibility to Simulate High Energy HIC
including Two-Body Collisions in Cascade

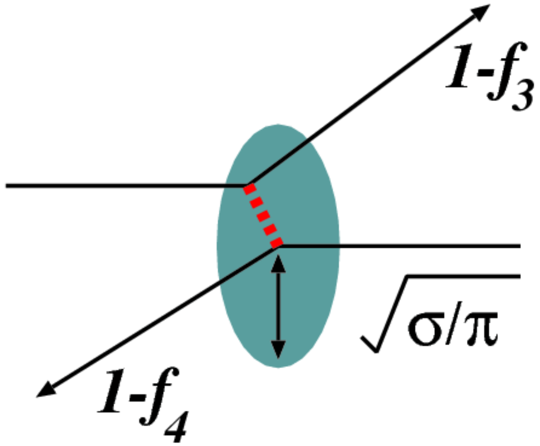
BUU (Boltzmann-Uehling-Uhlenbeck) Equation

- **BUU Equation (Bertsch and Das Gupta, Phys. Rept. 160(88), 190)**

$$\frac{\partial f}{\partial t} + v \cdot \nabla_r f - \nabla U \cdot \nabla_p f = I_{coll}[f]$$

$$I_{coll}[f] = -\frac{1}{2} \int \frac{d^3 p_2 d\Omega}{(2\pi\hbar)^3} v_{12} \frac{d\sigma}{d\Omega}$$
$$\times \quad [f f_2 (1-f_3)(1-f_4) - f_3 f_4 (1-f)(1-f_2)]$$

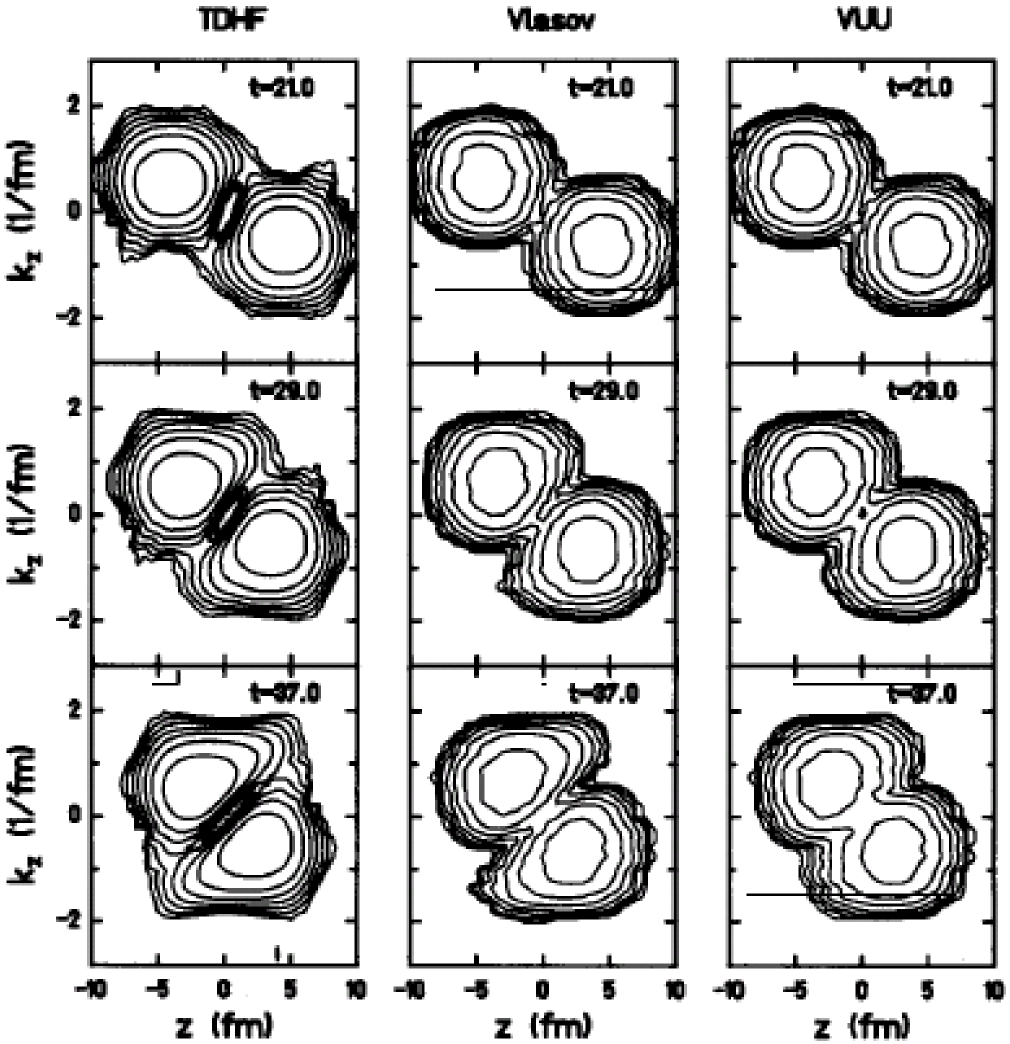
- **Incorporated Physics in BUU**
  - **Mean Field Evolution**
  - **(Incoherent) Two-Body Collisions**
  - **Pauli Blocking in Two-Body Collisions**



O ***One-Body Observables (Particle Spectra, Collective Flow, ..)***
X ***Event-by-Event Fluctuation (Fragment, Intermittency, ...)***

Comarison of TDHF, Vlasov and BUU(VUU)

- **Ca+Ca, 40 A MeV**
  (Cassing-Metag-Mosel-Niita, Phys. Rep. 188 (1990) 363).



Exercise (1)

- **Prove that the spatial integral of the Wigner function *f(x,p)* gives a momentum distribution of nucleons.**
- **Prove that the Wigner function with test particles satisfy the Vlasov equation when the test particle follows the classical EOM.**
- **Prove that the collision term becomes zero (i.e. gain and loss terms cancel) in equilibrium.**
- **Derive the collision term for bosons, which disappears in equilibrium.**
- ***(ADVANCED)* Prove the relation of the commutator and Poisson bracket. (It takes a long time ....)**
- ***(ADVANCED)* Prove that the Wigner function can be negative. (Therefore, the probability interpretation is not always possible.)**

Relativistic QMD/Simplified (RQMD/S)

- **RQMD = Constraint Hamiltonian Dynamics**
  (Sorge, Stocker, Greiner, Ann. of Phys. 192 (1989), 266.)
- **Constraints: φ ≈ 0 (Satisfied on the realized trajectory, by Dirac)**
  - **Variables in Covariant Dynamics = 8N phase space:** q_μ, p_μ
  - **Variables in EOM = 6N phase space**
    → We need 2N constraints to get EOM
- **On Mass-Shell Constraints**

$$H_i \equiv p_i^2 - m_i^2 - 2m_i V_i \approx 0$$

- **Time-Fixation in RQMD/S**

$$\chi_i \equiv \hat{a}\cdot(q_i - q_N) \approx 0 \, (i = 1, \sim N-1) \quad , \quad \chi_N \equiv \hat{a}\cdot q_N - \tau \approx 0$$

$$\hat{a} = \text{Time-like unit vector in the Calculation Frame}$$

(Tomoyuki Maruyama et al., Prog. Theor. Phys. 96(1996), 263.)

RQMD/S (cont.)

- **Hamiltonian is made of constraints**

$$H=\sum_i u_i\phi_i \quad (\phi_i=H_i(i=1\sim N), \chi_{i-N}(i=N+1\sim 2N))$$

- **Time Development** $\quad \dfrac{df}{d\tau}=\dfrac{\partial f}{\partial \tau}+\{f, H\} \quad , \quad \{q_\mu, p_\nu\}=g_{\mu\nu}$

- **Lagrange multipliers are determined to keep constraints**

  → ***We can solve obtain the multipliers analytically in RQMD/S***

$$\frac{d\phi_i}{d\tau}\approx 0 \;\rightarrow\; \delta_{i,2N}+\sum_j u_j\{\phi_i, \phi_j\}\approx 0$$

- **Equations of Motion**

$$H=\sum_i (p_i^2-m_i^2-2m_iV_i)/2p_i^0 \quad , \quad p_i^0=E_i=\sqrt{\vec{p}_i^2+m_i^2+2m_iV_i}$$

$$\frac{d\vec{r}_i}{d\tau}\approx-\frac{\partial H}{\partial \vec{p}_i}=\frac{\vec{p}}{p_i^0}+\sum_j \frac{m_j}{p_j^0}\frac{\partial V_j}{\partial \vec{p}_i} \quad , \quad \frac{d\vec{p}_i}{d\tau}\approx\frac{\partial H}{\partial \vec{r}_i}=-\sum_j \frac{m_j}{p_j^0}\frac{\partial V_j}{\partial \vec{r}_i}$$

We can include MF in an almost covariant way in molecular dynamics

Particle "DISTANCE"

$$r_{Tij}^{2} \equiv r_\mu r^\mu - \left(r_\mu P_{ij}^\mu\right)^2 / P_{ij}^2 = \vec{r}^{\,2} \qquad (in\ CM)$$

$$P_{ij} \equiv p_i + p_j \quad , \quad r \equiv r_i - r_j$$

Particle "Momentum Difference"

$$p_{Tij}^{2} \equiv p_\mu p^\mu - \left(p_\mu P_{ij}^\mu\right)^2 / P_{ij}^2 = \vec{p}^{\,2} \qquad (in\ CM)$$

$$p \equiv p_i - p_j$$

Lorentz Invariant, and Becomes Normal Distance in CM !

AMD (Antisymmetrized Molecular Dynamics)

Ono-Horiuchi-Maruyama-AO, 1992

- **Gaussian Approximation for single particle wave function**

$$|\Psi> = A\prod |\psi_i> \quad , \quad \psi_i = \phi(r;Z_i)\chi(\sigma,\tau) \quad , \qquad Z = \sqrt{\nu}\,D + \frac{i}{2\hbar\sqrt{\nu}}K$$

$$\phi(r;Z) = \left(\frac{2\nu}{\pi}\right)^{3/4} \exp\left(-\nu\left(r - Z/\sqrt{\nu}\right)^2 + Z^2/2\right) \;\propto\; \exp\left(-\nu(r-D)^2 + i\,K\cdot(r-D)/\hbar\right)$$

- **Time-dependent Variational Principle → Equations of Motion**

$$L = \frac{\langle\Psi|i\hbar\partial/\partial t - H|\Psi\rangle}{\langle\Psi|\Psi\rangle} \quad , \quad \frac{d}{dt}\frac{\partial L}{\partial(d\bar{Z}_i/dt)} - \frac{\partial L}{\partial \bar{Z}_i} = 0 \rightarrow \quad i\hbar C_{i\alpha,j\beta}\frac{dZ_i}{dt} = \frac{\partial H}{\partial \bar{Z}_i}$$

- **Ignoring Antisymmetrization**
  → Quantum Molecular Dynamics EOM (= Classical EOM)

$$C = \delta \;\rightarrow\; \frac{dD_i}{dt} = \frac{\partial H}{\partial K_i} \quad , \quad \frac{dK_i}{dt} = -\frac{\partial H}{\partial D_i}$$

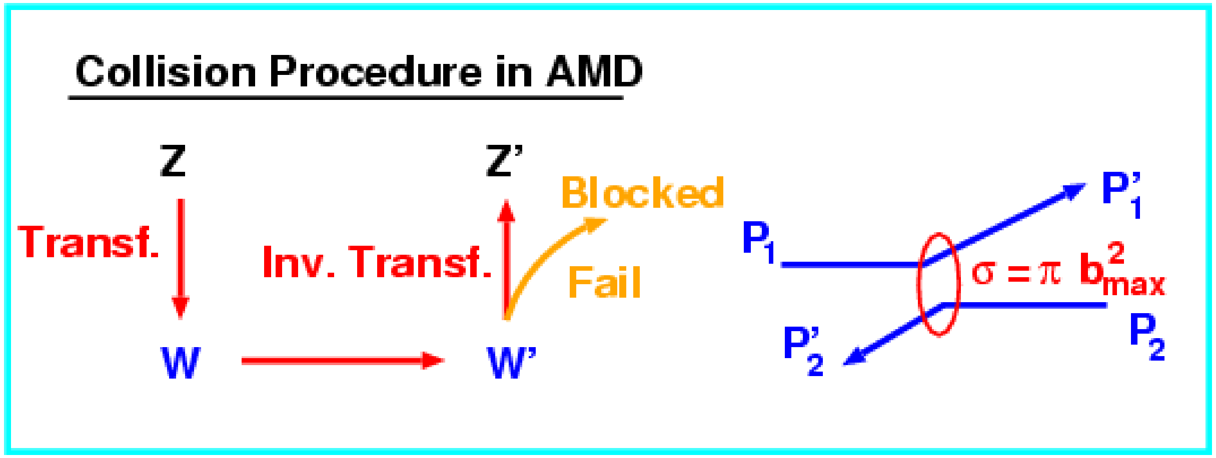
Classical-type EOM is obtained through Gaussian + TDVP

Collision Term in AMD

- **Approximate Canonical Variables**

$$W_i = \sqrt{Q}_{ij} Z_j = \sqrt{\nu} R_i + \frac{i}{\sqrt{\nu}\hbar} P_i \ , \quad Q_{ij} \equiv B_{ij} \ \ B_{ij}^{-1} \ , \quad B_{ij} = \langle \psi_i | \psi_j \rangle$$

$$Example \ \ \langle \mathbf{L} \rangle = \sum_{ij} B_{ji}^{-1} B_{ij} \frac{1}{i} \overline{Z}_i \times Z_j = \sum_i \overline{W}_i \times W_i$$



Physics included in AMD

Time Evolution of Anti-Symmetrized Wave Function

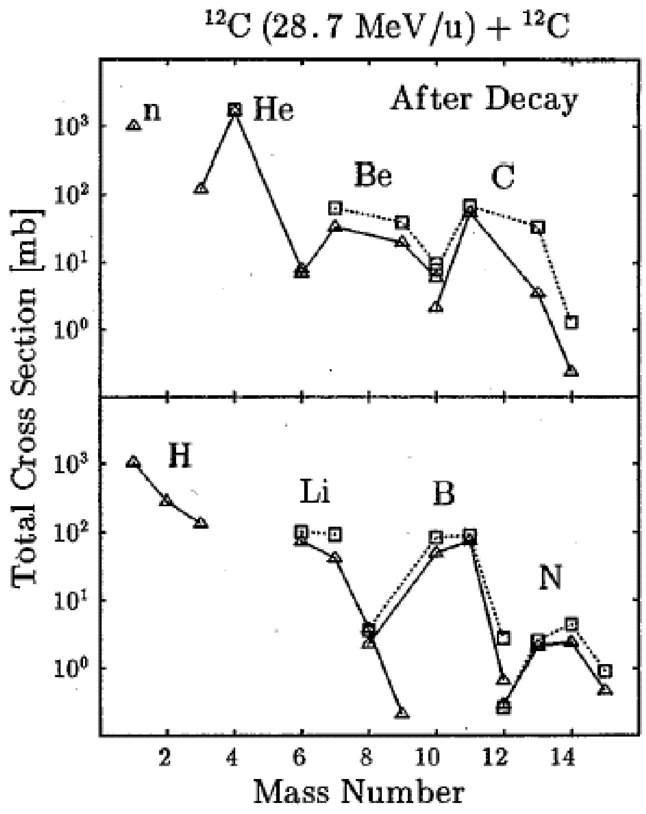
Collision Term = "Canonical" Variable + Classical Analogy

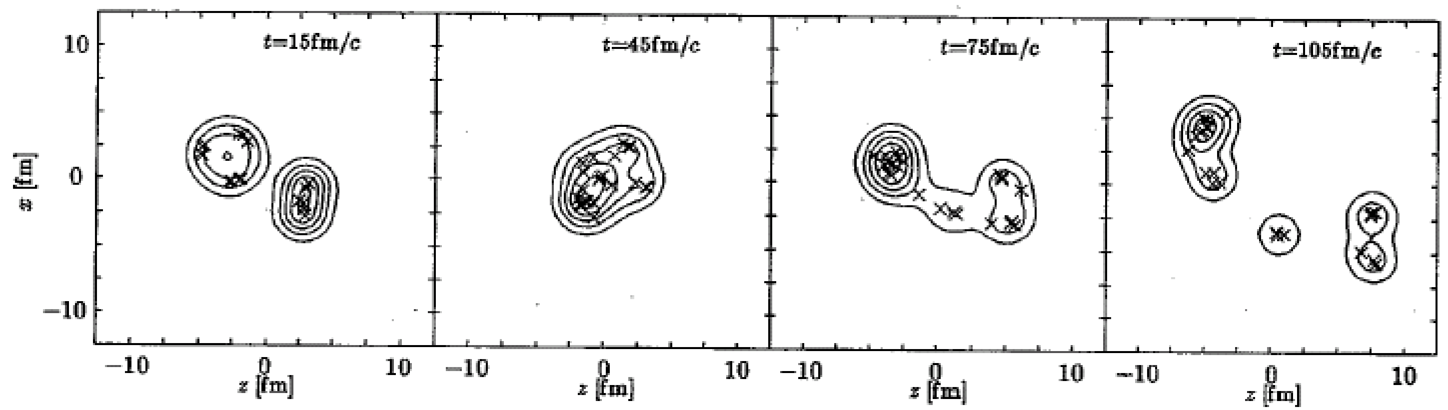
Event-by-Event Fluctuation

Problems: Non-Rela., Classical Analogy of Collision term,CPU cost

Fragment Formation in AMD

- ***Fluctuation is ESSENTIAL*** **to Fragment Formation !**
  - **Initial Orientation of Deformed Nuclei**
  - **Stochastic Two-Body Collision Term**
- **Fragment Formation is well described in *AMD + Statistical Decay***
  - **Exception: ^{13}C (Compared to Mirror ^{13}N, σ(^{13}C) is around 10 times larger in Data)**





Fragmentation: Low T and Low Density Matter

- **Experimental Evidence on the First Order LG Phase Transition**
  - **Two indep. exp. on two indep. Observables show**
  - **We need models, which describes both of**
    Nuclear Reactions and Statistics
    having Quantum Statistical nature $E \propto T^2$ in nuclei at low T
- **Molecular Dynamics with Quantum Fluctuation**
  - **AO & Randrup: Quantum Langevin Model**
    NPA565(1993), 474; PRL 75(1995), 596; Ann.Phys.253 (1997), 279; PLB394(1997), 260; PRA55(1997), 3315R
  - **Hirata, Nara, AO, Harada, Randrup; AMD-QL (PTP 102 (1999), 89)**
  - **Ono-Horiuchi: AMD-V (E.g., PRC53 (1996), 2958)**
  - **Sugawa-Horiuchi-Ono: AMD-MF (PRC60 (1999) 064607)**

Quantum Langevin Model

Wave Packet Statistics

- **Partition Function**

$$\mathcal{Z}_\beta \equiv \mathrm{Tr}\left(\exp(-\beta\hat{H})\right) = \int d\Gamma\ \mathcal{W}_\beta(\mathbf{Z})$$
$$\mathcal{W}_\beta(\mathbf{Z}) \equiv \langle\mathbf{Z}|\exp(-\beta\hat{H})|\mathbf{Z}\rangle \neq \exp(-\beta\langle\hat{H}\rangle)$$

- **Thermal Average**

$$\prec\hat{O}\succ_\beta \equiv \frac{1}{\mathcal{Z}_\beta}\mathrm{Tr}\left(\hat{O}\exp(-\beta\hat{H})\right) = \frac{1}{\mathcal{Z}_\beta}\int d\Gamma\ \mathcal{W}_\beta(\mathbf{Z})\ \mathcal{O}_\beta(\mathbf{Z})$$
$$\mathcal{O}_\beta(\mathbf{Z}) \equiv \frac{\langle\mathbf{Z}_{\beta/2}|\hat{O}|\mathbf{Z}_{\beta/2}\rangle}{\langle\mathbf{Z}_{\beta/2}|\mathbf{Z}_{\beta/2}\rangle} \neq \langle\hat{O}\rangle$$
$$|\mathbf{Z}_{\beta/2}\rangle \equiv \exp(-\beta\hat{H}/2)|\mathbf{Z}\rangle \neq |\mathbf{Z}\rangle$$

- **Harmonic Approximation**

$$\mathcal{W}_\beta(\mathbf{Z}) \approx \exp\left[-\frac{\mathcal{H}}{D}\left(1-e^{-\beta D}\right)\right] = \exp(-\beta\mathcal{H} + \beta^2\sigma_E^2/2 + \cdots)$$
$$D(\mathbf{Z}) \equiv \sigma_E^2/\mathcal{H}$$
$$\mathcal{H}_\beta(\mathbf{Z}) \equiv -\frac{\partial\log\mathcal{W}_\beta(\mathbf{Z})}{\partial\beta} \approx \mathcal{H}(\mathbf{Z})\ e^{-\beta D}$$

→ Improved β Expansion

Wave Packet Dynamics

Fokker-Planck Eq.

$$\frac{D\phi}{Dt} = \left[-\sum_i\frac{\partial}{\partial q_i}V_i + \sum_{ij}\frac{\partial}{\partial q_i}M_{ij}\frac{\partial}{\partial q_j}\right]\phi\ ,$$
$$V_i = -\sum_j M_{ij}\frac{\partial\mathcal{F}_\beta}{\partial q_j}\ ,$$

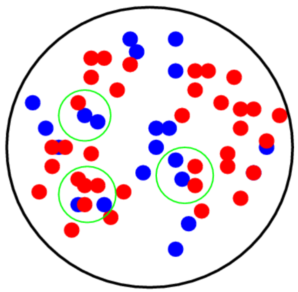
$$V_i = -\alpha\beta\sum_j M_{ij}\frac{\partial\mathcal{H}}{\partial q_j}\ ,\quad \alpha = \frac{1-\exp(-\beta D)}{\beta D}$$

Langevin Equation

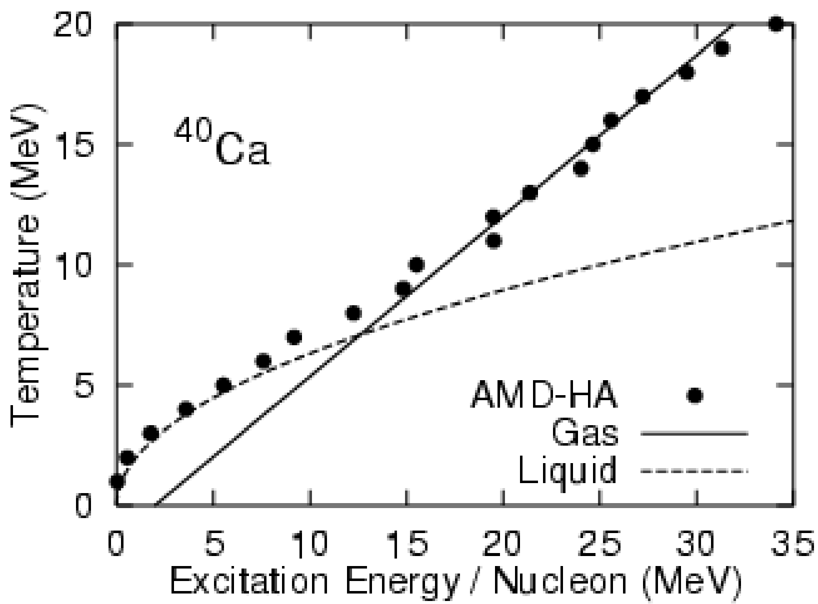
$$\dot{\boldsymbol{p}} = \boldsymbol{f} - \alpha\beta\boldsymbol{M}^p\cdot\boldsymbol{v} + \boldsymbol{g}^p\cdot\boldsymbol{\xi}^p\ ,$$
$$\dot{\boldsymbol{r}} = \boldsymbol{v} + \alpha\beta\boldsymbol{M}^r\cdot\boldsymbol{f} + \boldsymbol{g}^r\cdot\boldsymbol{\xi}^r\ ,$$
$$\boldsymbol{v} = \frac{\partial\mathcal{H}}{\partial\boldsymbol{p}}\ ,\quad \boldsymbol{f} = -\frac{\partial\mathcal{H}}{\partial\boldsymbol{r}}\ ,$$
$$\boldsymbol{M}^p = \boldsymbol{g}^p\cdot\boldsymbol{g}^p\ ,\quad \boldsymbol{M}^r = \boldsymbol{g}^r\cdot\boldsymbol{g}^r\ .$$

Sampled State ≠ Observing State
Dual State Structure make it possible
to Simulate Quantum Statistics in Molecular Dynamics

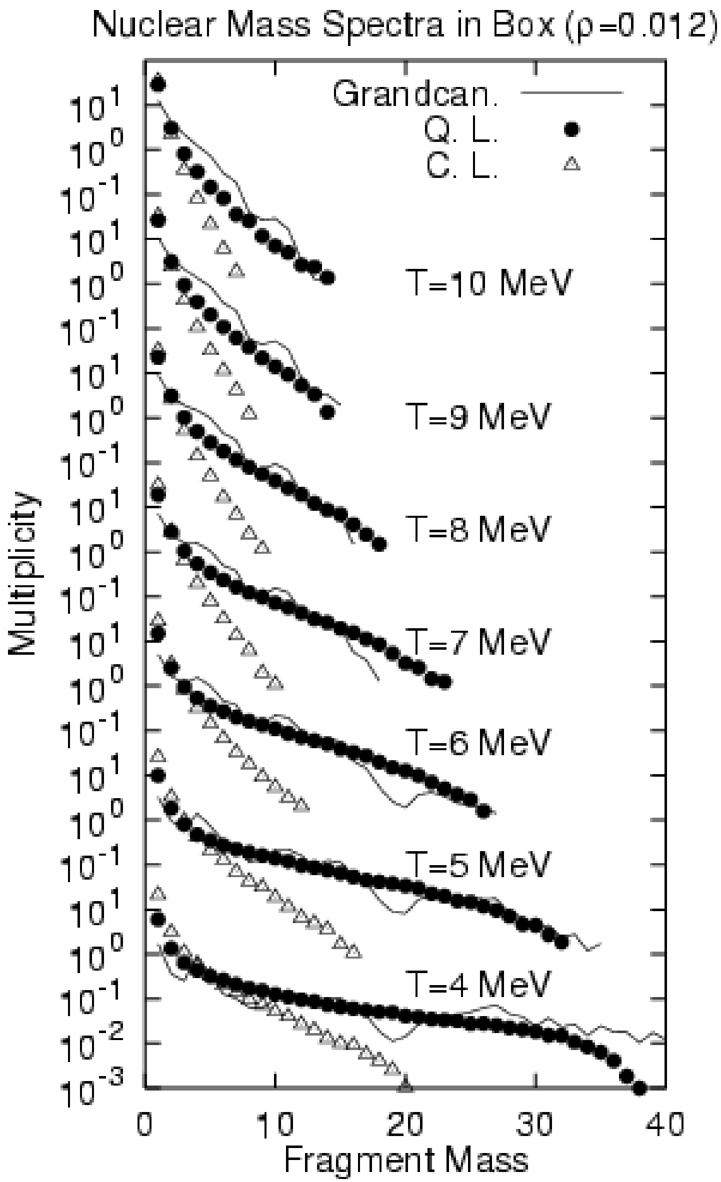
Wave Packet Statistics



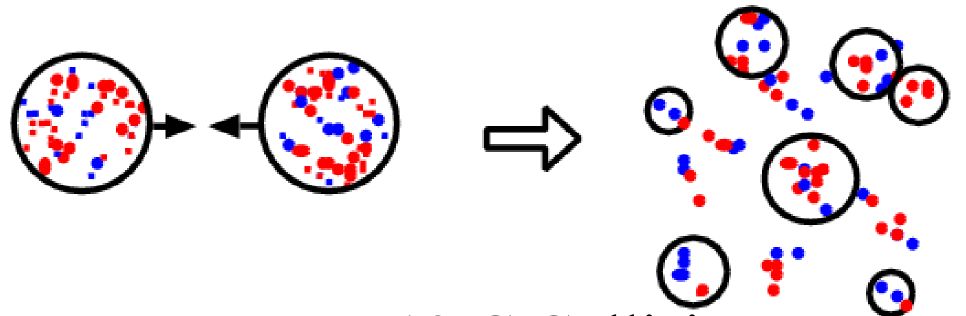
Caloric Curve



Mass Distribution

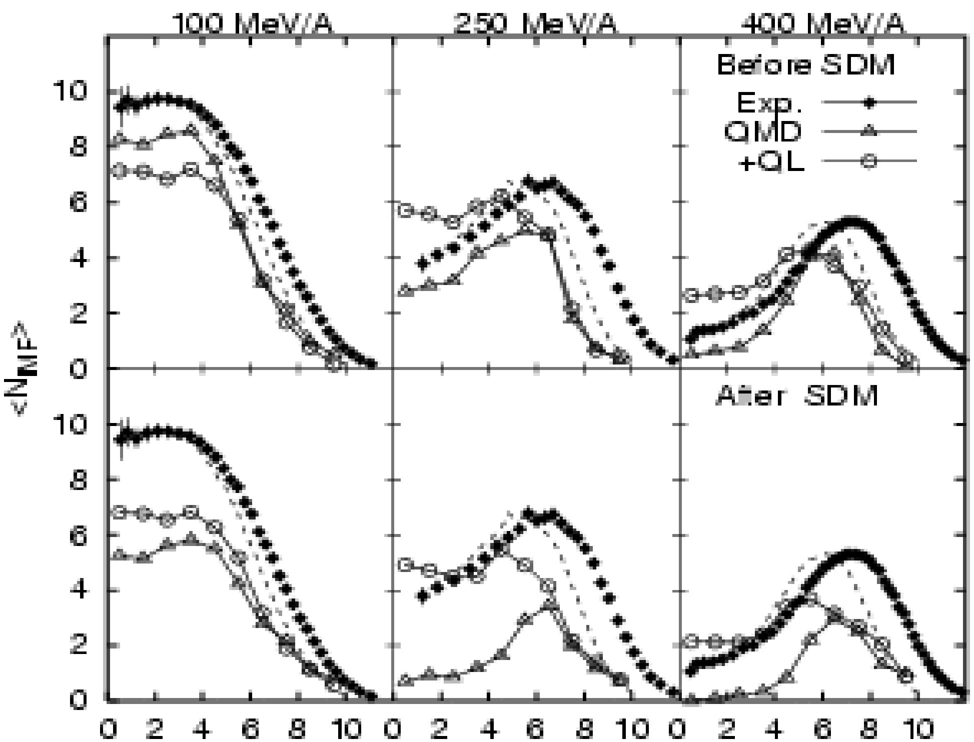


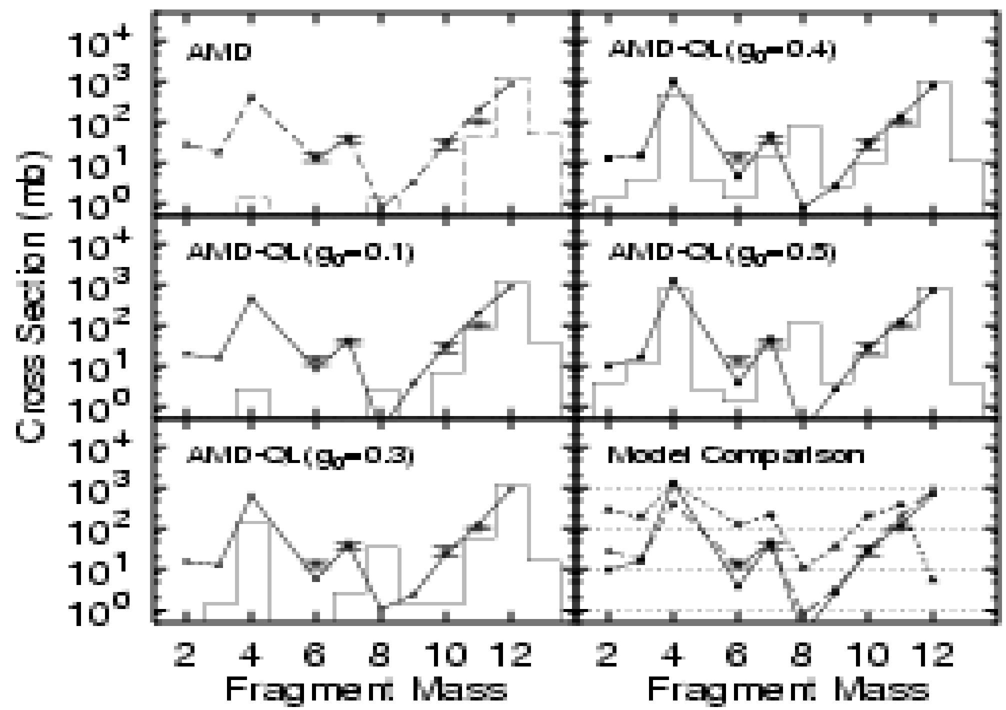
Wave Packet Dynamics



Au + Au Collisions

p + 12 C Collisions





AO-Randrup, 1997

Hirata et al. PTP102(99),89

What is Understood ?

- **What is understood ?**
  - **LG Phase Transition is of First Order (Exp.).**
  - **It can be understood in Microscopic MD qualitatively, e.g. Fragment Yield.**
- **What is NOT Understood ?**
  - **Direct Relation between Fragment Formation and the Properties of Nuclear Matter**
  - **Are Fragments Produced through LG Phase Transition ?**
  - **"Initial" Condition of Fragmenation At Which T and ρ Fragments are Formed ?**
  - **Is Equilibrium Reached in Heavy-Ion Collisions ?**

Simpler Cases: pA Reaction & Supernova Explosion !

Exercise (2)

- **Prove that the TDVP (time-dependent variational principle) gives the Schrodinger equation when the wave function is not restricted, for example to a Slater determinant.**
- ***(ADVANCED)*** **Prove that the AMD wave function is equivalent to harmonic oscillator shell model wave function when all Z's goes to zero. (This tells you why the Slater determinant of (s-wave) Gaussians can describe nuclei above s-shell.)**
- ***(ADVANCED)*** **Obtain the Lagrange multiplier in RQMD/S.**

Cascade Model
Hadron-Hadron Collisions

AA collisions at High E.

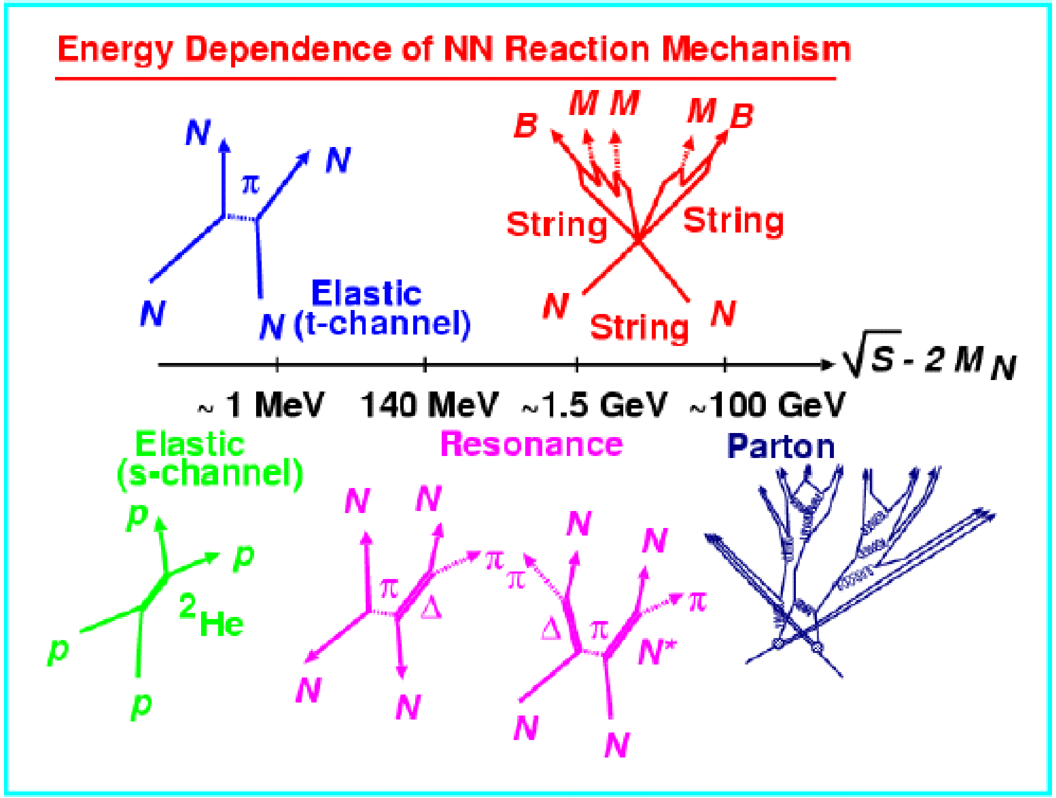
~ Sum of (Multistep) NN collisions (Cascade)

+ *Interesting Physics*

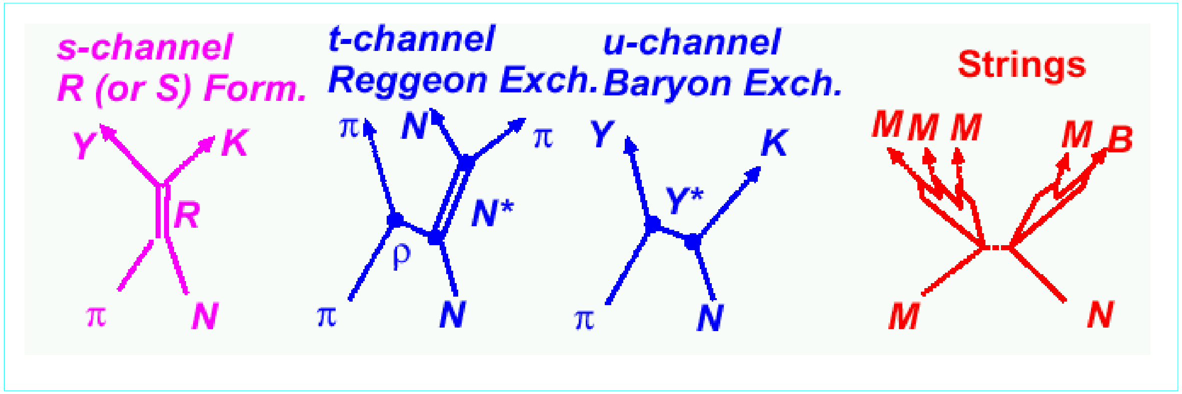
→ Cascade gives the "baseline" of evaluation !

Baryon-Baryon and Meson-Baryon Collisions

- **NN collision mechanism**
  Elastic
  → Resonance
  → String
  → Jet

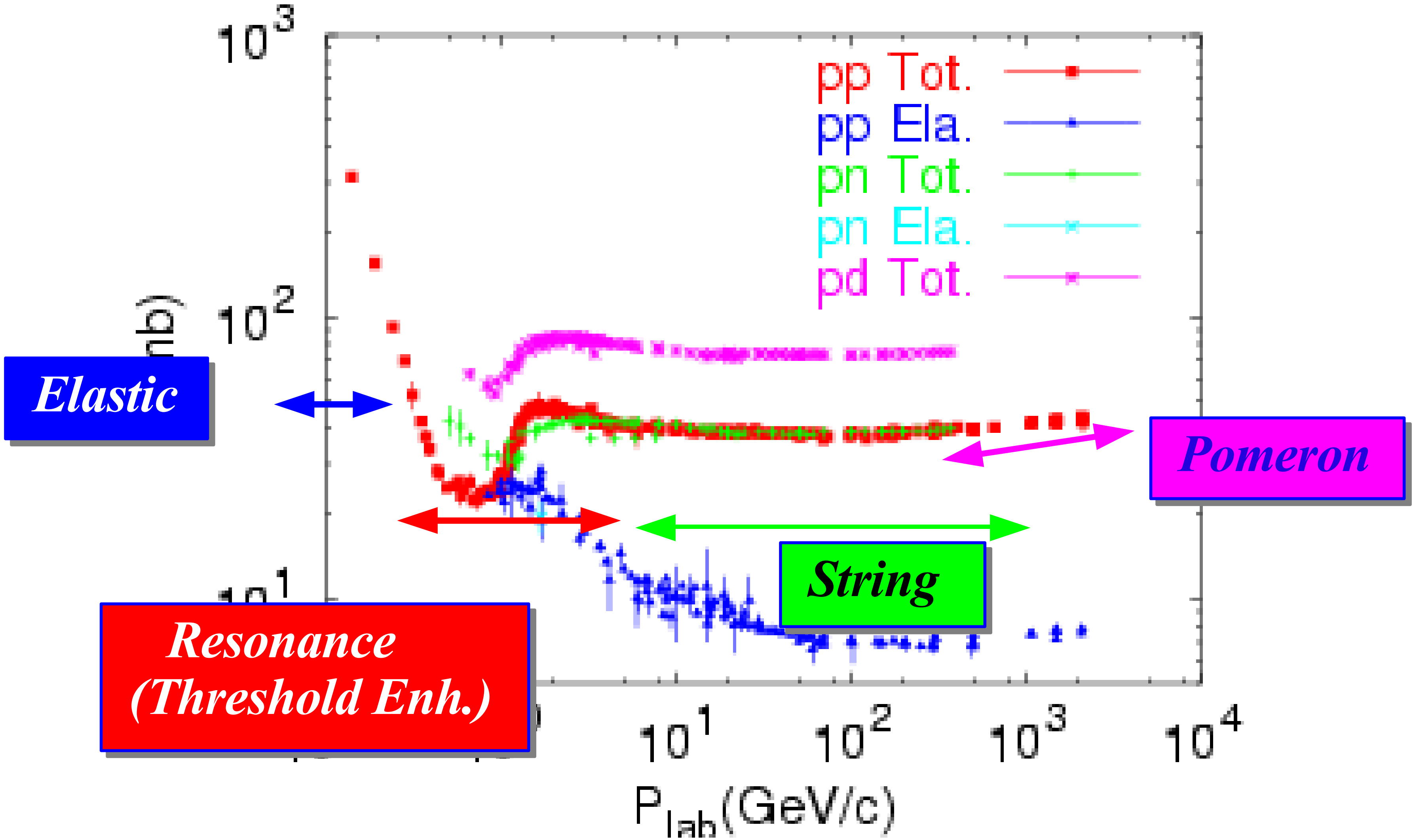


- **Meson-Nucleon Collision**
  → s-channel Resonance
  → t-(u-) channel Res.
  → String formation

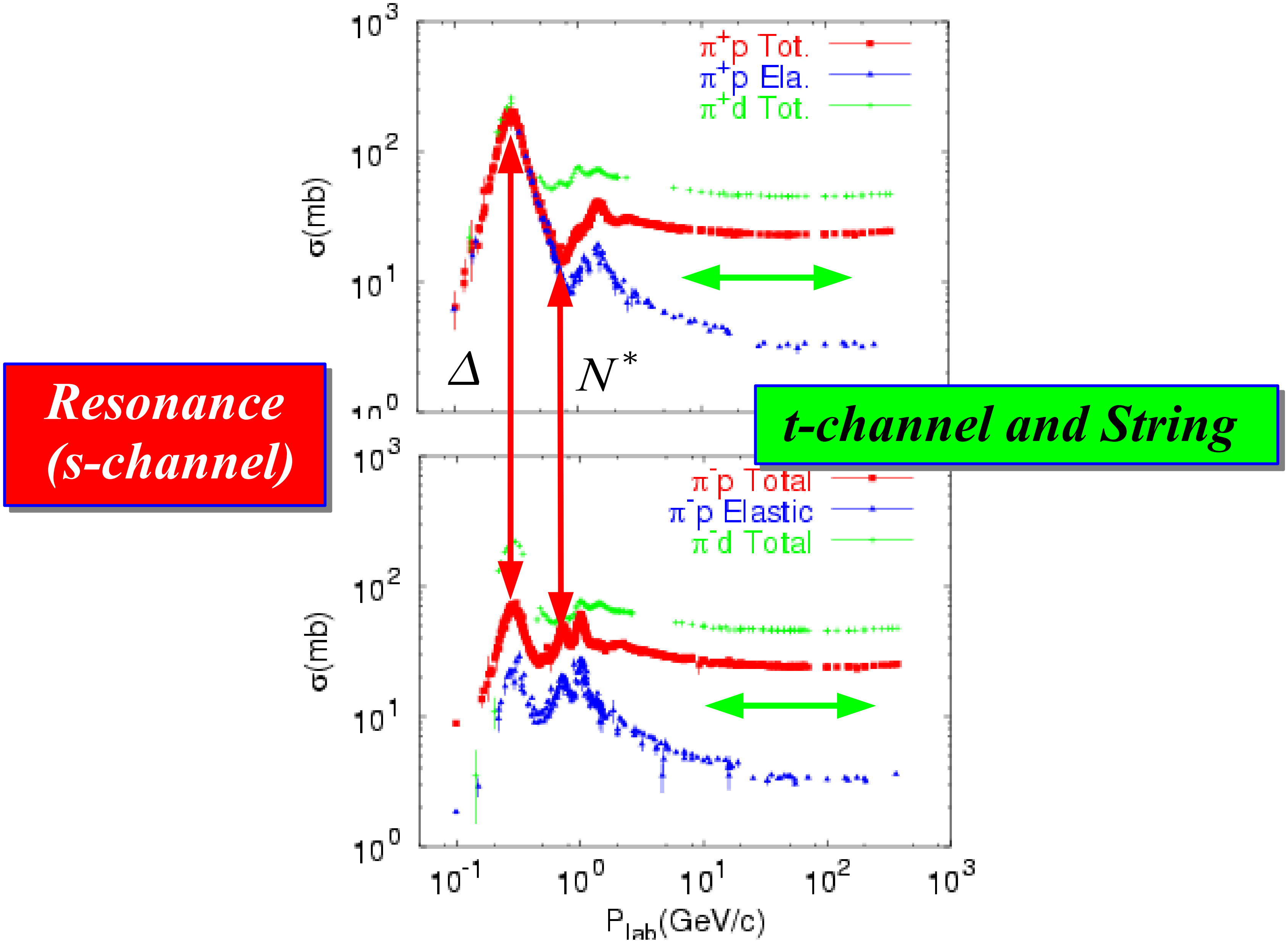


NN Cross Sections

From Particle Data Group



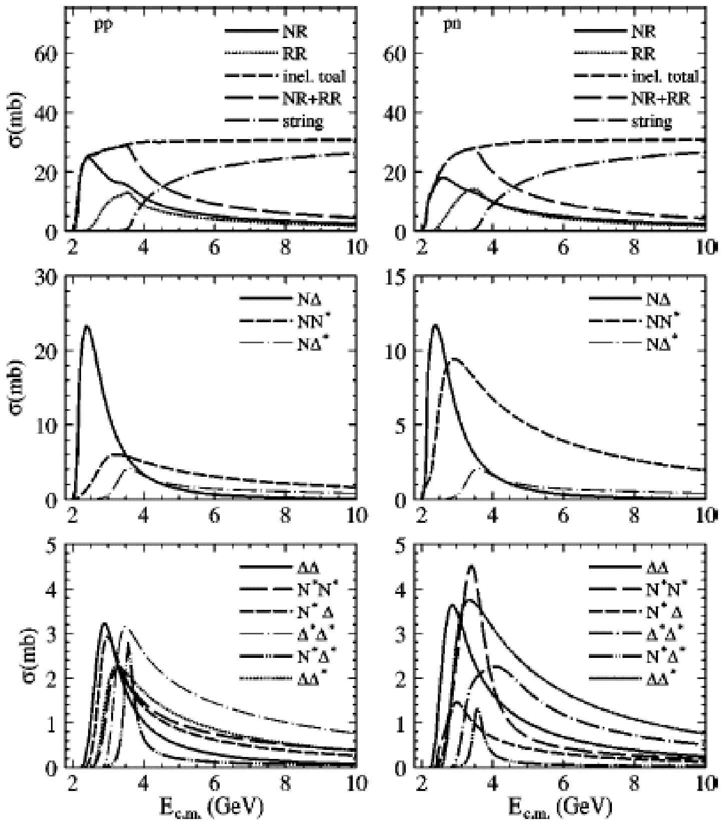
Meson-Baryon Cross Section

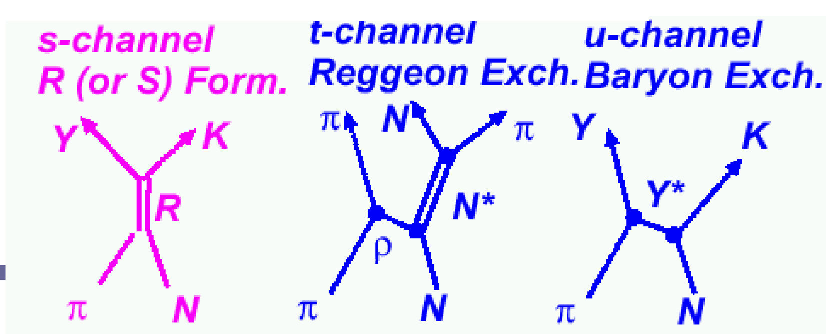


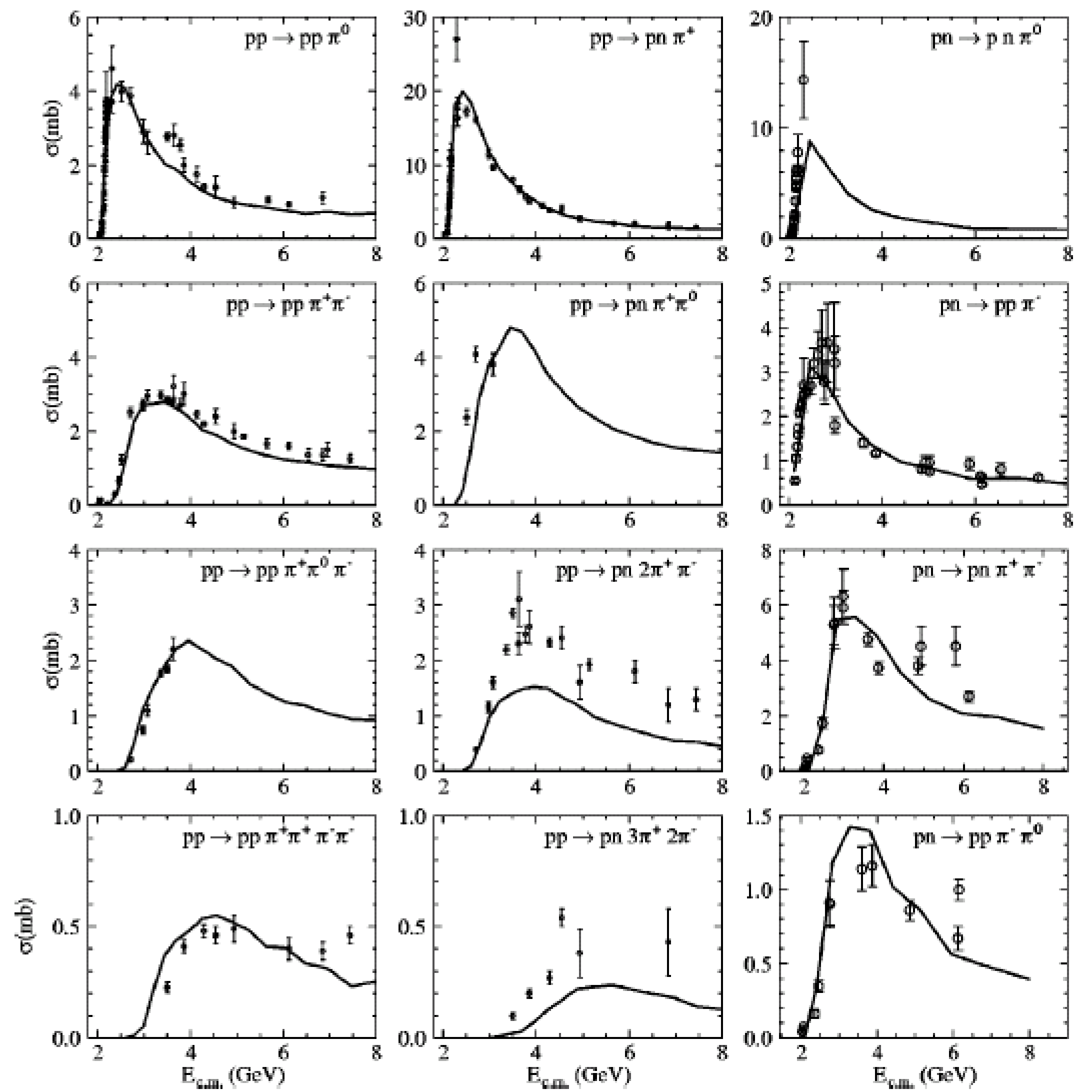
Exclusive Cross Sections

Nara, Otuka, AO, Niita, Chiba (JAM), PRC 61 (2000), 024901.

- **We need not only Total and Elastic Cross Sections, but also Cross Sections for *Each Channel* !**







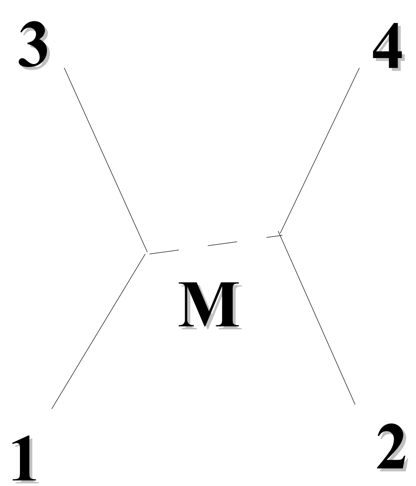
Regge, String, and Jet
--- High Energy hh Collisions ---

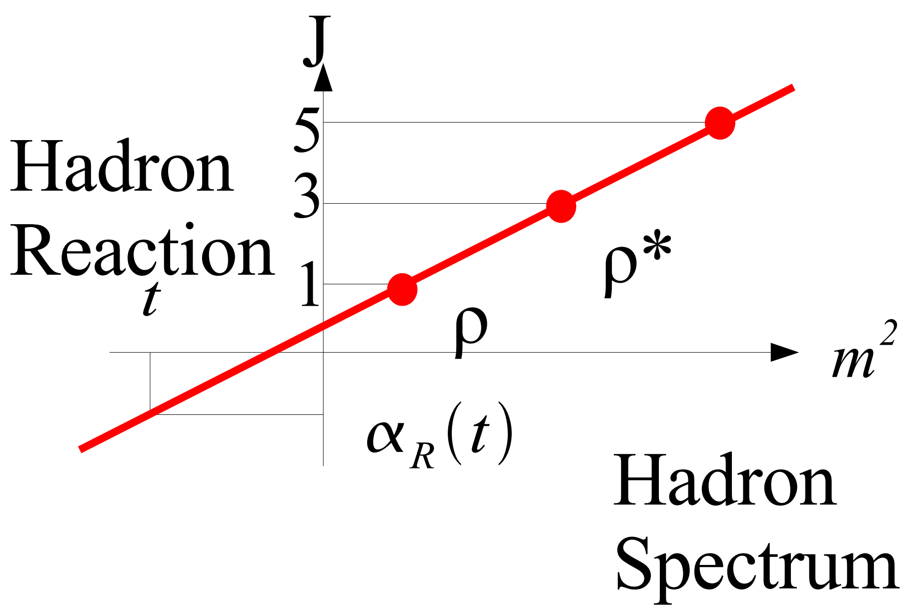
Barger and Cline (Benjamin, 1969), H. Sorge (RQMD), PRC (1995)

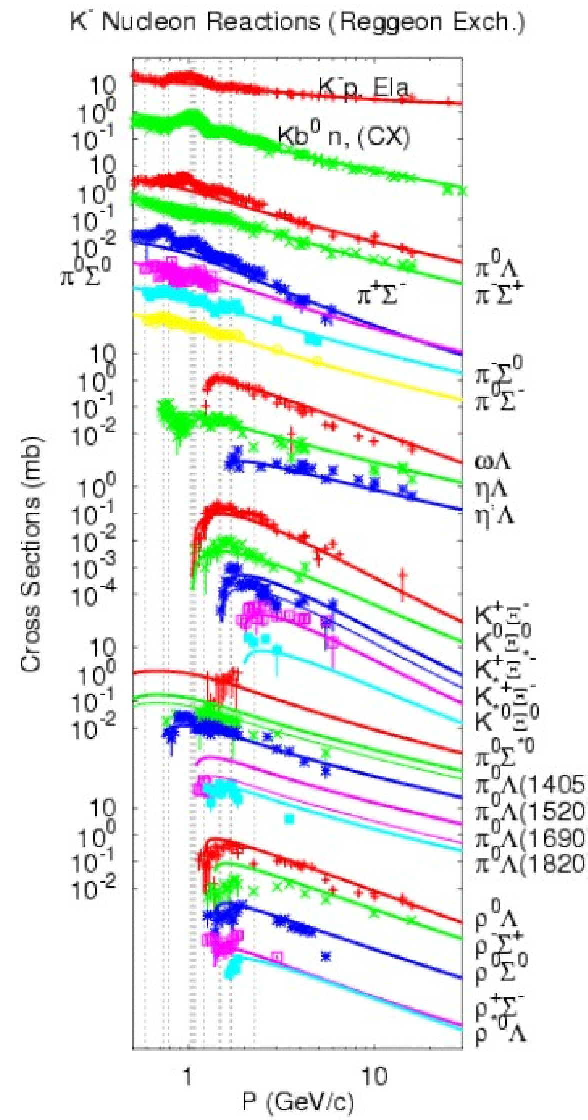
- **Regge Trajectory** $J=\alpha_R(t)\sim\alpha_R(0)+\alpha'_R(0)\,t$
- **2 to 2 Cross Section**

$$\frac{d\sigma}{d\Omega}=\frac{p_f}{64\pi s\, p_i}|M(s,t)|^2$$

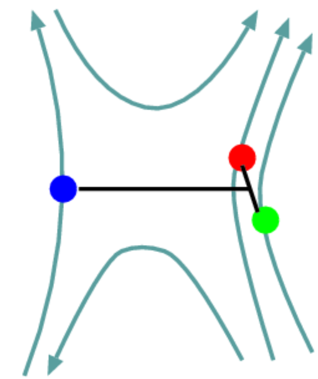
$$M(s,t)\sim\sum_R\frac{(p_i p_f)^J}{t-M_R}\sim F(t)\exp[\alpha_R(t)\log(s/s_0)]$$







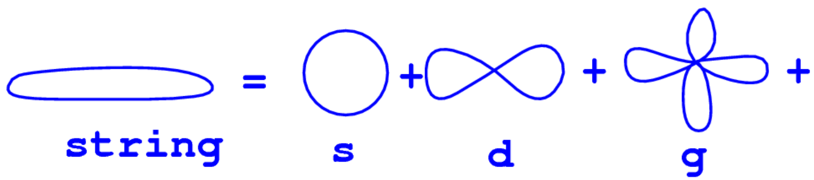
String formation and decay



- **What does the regge trajectory suggest ?**
  → Existence of (color- or hadron-)String !

$$M=2\int_0^R \frac{\kappa\, dr}{\sqrt{1-(r/R)^2}}=\pi\kappa R\ ,\quad J=2\int_0^R r\times\frac{\kappa\, dr}{\sqrt{1-(r/R)^2}}\frac{r}{R}=\frac{\pi\kappa R^2}{2}\pi$$

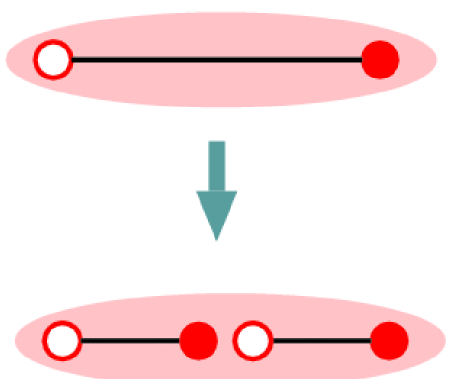
$$\rightarrow\ J=\frac{M^2}{2\pi\kappa}$$



- **String Tension**

$$\frac{1}{2\pi\kappa}=\alpha'_R(0)\approx 0.9\,GeV\ \rightarrow\ \kappa\approx 1\,GeV/fm$$

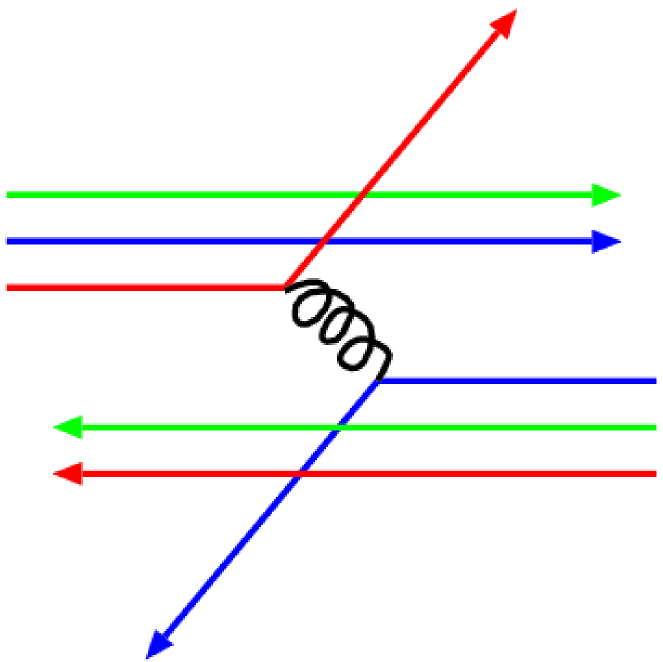
- **String decay**
  Extended String
  → Large E stored
  → q qbar pair creation (Schwinger mech.)



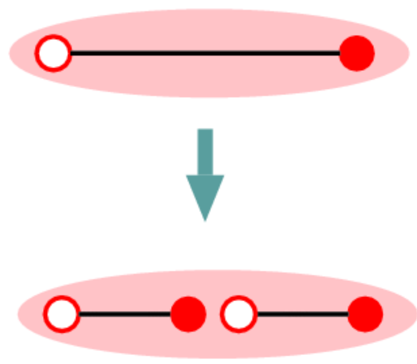
String = Coherent superposition of hadron resonances with various J

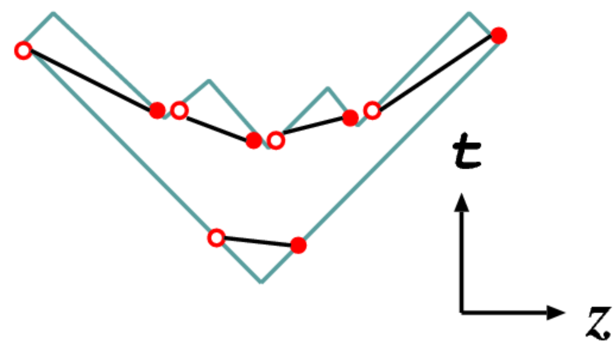
Jet Production

- Elastic Scattering of Partons (mainly) with One Gluon Exch.
- Color Exch. between Hadrons
  → Complex color flux starting
     from leading partons
  → many hadron production
  → Jet production



- ***PYTHIA***
  - Event Generator
    of High Energy Reactions
    → Jet production
    +String decay
    for QCD processes

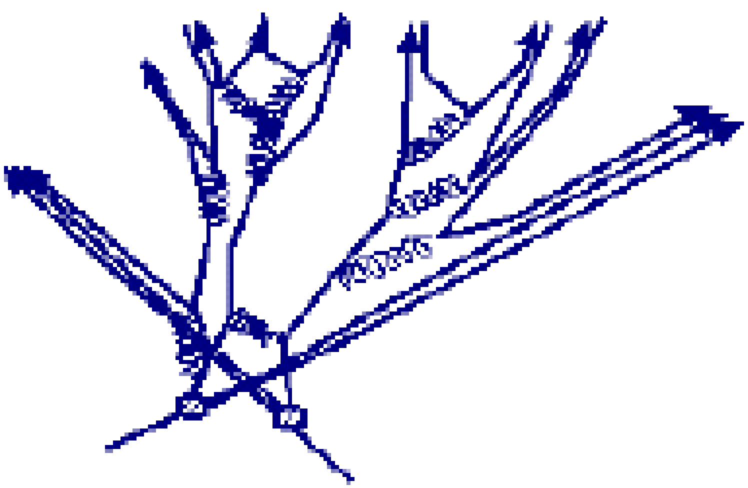


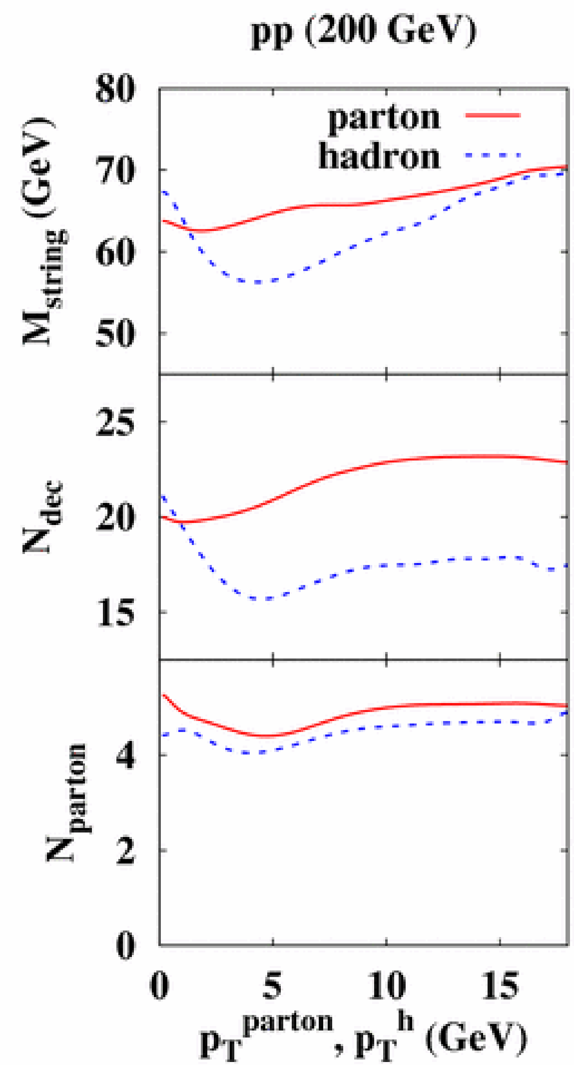


(T. Sjostrand et al., Comput. Phys. Commun. 135 (2001), 238.)

String Mass and p_T in Jet

- **In average, Jet Strings have 60-70 GeV masses, contain around 4-5 partons (q-g-g-g-qbar, ...). and decay into 20-25 hadrons.**
  - **Complex color flux starting from leading partons make strings heavy !**





JAM (Jet AA Microscopic transport model)

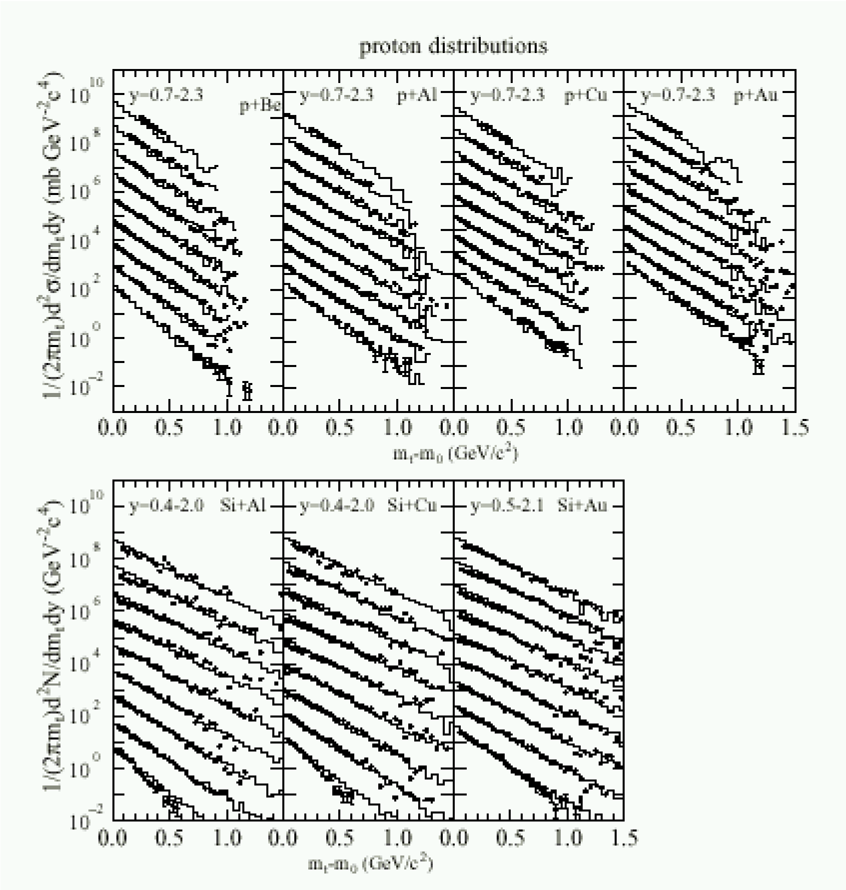
Nara, Otuka, AO, Niita, Chiba, PRC 61 (2000), 024901.

- **Hadron-String Cascade with Jet production**
  - **hh collision with Res. up to m < 2 GeV (3.5 GeV) for M (B)**
  - **String excitation and decay**
  - **String-Hadron collisions are simulated by hh collisions in the formation time.**
  - **jet production is incl. using PYTHIA**
  - **Secondary partonic int.: NOT incl.**
  - **Color transparency: NOT taken care of**

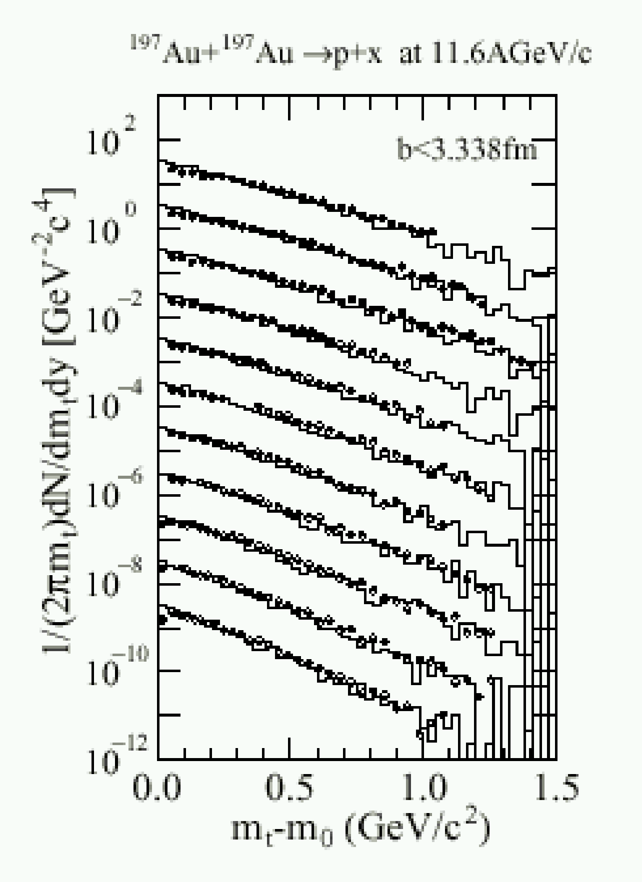
JAM Results @ AGS Energy

Nara, Otuka, AO, Niita, Chiba, PRC 61 (2000), 024901.

p-A Collision



Au+Au Collision



JAM explains AA collisions as well as pA collisions:
→ Good Elementary Cross Sections for MM, MN and NN

Exercise (3)

- **Prove that the sum of Mandelstam variables becomes a constant.**

  $$s = (p_1+p_2)^2,\quad t = (p_1-p_3)^2,\quad u = (p_1-p_4)^2,$$

  in 1+2 → 3+4 reaction.

- **Draw the Feynman diagram of $K^- + p \to \pi^+ + \Sigma^-$. You will be able to guess that the angular distribution becomes backward peaked due to the u-channel dominance.**

- **Explain why we have peak structures in MB collisions and we do not see peaks in BB collisions.**

- ***(If you already learned QCD,)*** **Obtain the squared Feynman amplitude of qq → qq in the tree level averaged over the color and spin. (You can ignore quark mass.) You will see the cross section is divergent at forward angle. Explain why we do not see this divergent behavior in NN collisions.**

Relativistic Hydrodynamics

Relativistic Hydrodynamics

- **EOM: Conservation Laws**

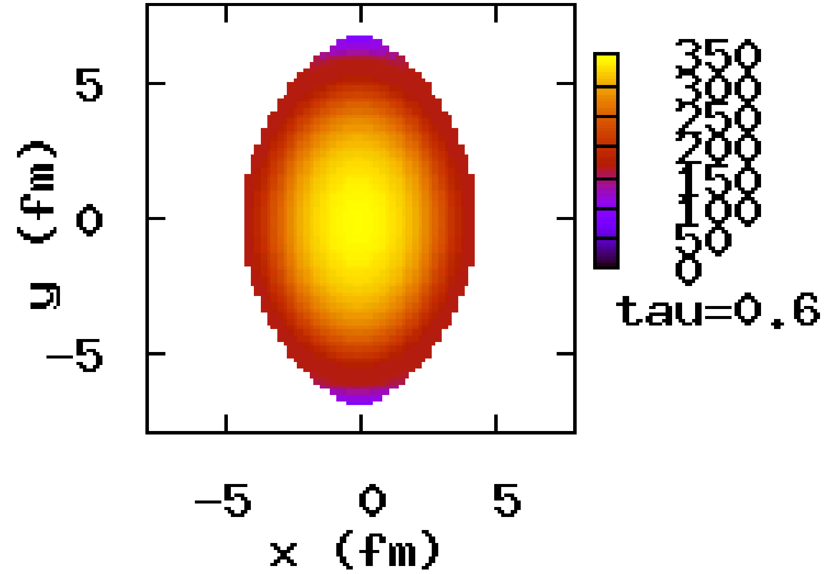
$\partial_\mu T^{\mu\nu} = 0$ Energy Momentum Conservation

$\partial_\mu (n_i u^\mu) = 0$ Conservation of Charge (Baryon, Strangeness, ...)

$$T^{\mu\nu} = (e + P) u^\mu u^\nu - P g^{\mu\nu}$$

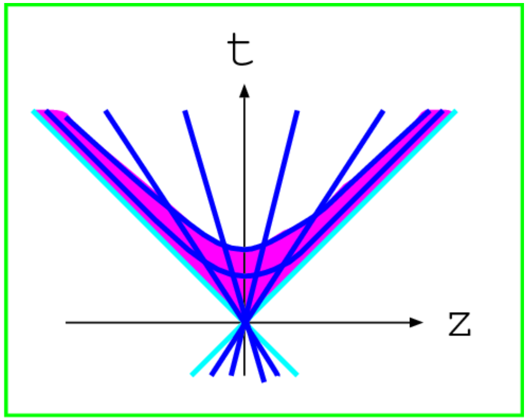
e **: energy density, *P: pressure,***

u^μ**: :four velocity $\gamma(1,v)$,** n_i**: :number density**



T. Hirano, Y. Nara, NPA743, 305 (2004)
T. Hirano, K. Tsuda, PRC 66, 054905(2002)

Relativistic Hydrodynamics (II)



- **One more condition is necessary**
  → *Equation of State $P = P(e, n_i)$ is needed*
  - **Independent Variables: e, P, v, n_i → 6**
  - **Independent Equations: 4+1 =5**
- **Solve Hydro. in Bjorken Variables (τ, η_s, x, y) → Save CPU a lot !**
  - **Most of the Dynamics is govered by τ during τ < 10 fm/c**
  - **η_s approximately corresponds to η, and fixed by inc. E.**
- **Parameters**
  - **τ_0 (thermalization time), T^{ch} (chem. F.O.) → Au+Au *dN/dη* fit**
  - **T^{th}: Free Parameter**
- **Initial Condition: Glauber type / Color Glass Condensate**

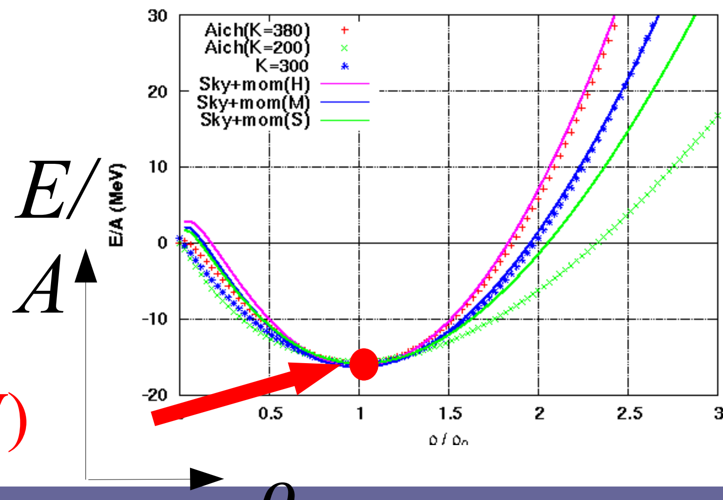
Nuclear Mean Field for HIC
--- Density and Momentum Deps. ---

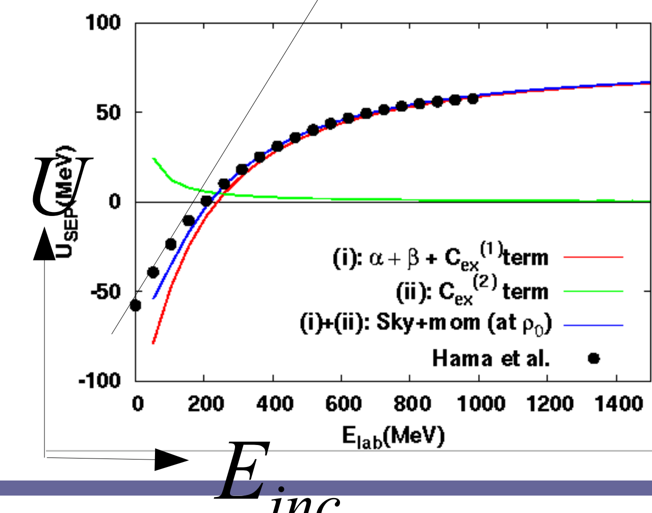
Nuclear Mean Field

- **MF has on both of ρ and p-deps.**
  - **ρ dep.: (ρ_0, E/A) = (0.15 fm^{-3}, -16.3 MeV) is known**
    Stiffness is not known well
  - **p dep.: Global potential up to E=1 GeV is known from pA scattering**
    $U(\rho_0, E) = U(\rho_0, E{=}0) + 0.3\,E$
- **Ab initio Approach; LQCD, GFMC, DBHF, G-matrix, ...**
  → Not easy to handle, Not satisfactory for phen. purposes
- **Effective Interactions (or Energy Functionals):**
  Skyrme HF, RMF, ...

$U(E)=U(0)+0.3E$

$(\rho_0, E/A)$
$=(0.15\ \mathrm{fm}^{-3}, -16.3\ \mathrm{MeV})$





Skyrme Hartree-Fock

See Ring-Schuck for details

- **Zero-Range Two- and Three-Body Interaction**

$$v_{ij} = t_0 \delta(r_i - r_j) + \frac{1}{2}\left[\delta(r_i - r_j) k^2 + k^2 \delta(r_i - r_j)\right]$$

$$+ t_2 k \delta(r_i - r_j) k + i W_0 \left[\sigma_i + \sigma_j\right] \times \delta(r_i - r_j) k$$

$$k = \frac{1}{2i}\left(\nabla_i - \nabla_j\right)$$

$$v_{ijk} = t_3 \delta(r_i - r_j) \delta(r_j - r_k)$$

- **Energy Density (Even-Even, N=Z)**

$$H(r) = \frac{\hbar^2}{2m^*(\rho)}\tau + \frac{3}{8} t_0 \rho^2 + \frac{1}{16} t_3 \rho^3 + Deriv.\ \ terms \to \rho\left[\frac{3}{5}\ \frac{\hbar^2 k_F^2}{2m^*(\rho)} + \frac{3}{8} t_0 \rho + \frac{1}{16} t_3 \rho^2\right]$$

$$\tau = \sum_i |\nabla \phi_i|^2 \ , \quad \frac{\hbar^2}{2m^*(\rho)} = \frac{\hbar^2}{2m} + \frac{1}{16}(3t_1 + 5t_2)\rho$$

Problems in Skyrme HF (in Dense Nuclear Matter/High Energy)
Repulsive Zero-Range 3-body Int.: → Ferromagnetism
Energy Dep. = Linear (m term) → Too Repulsive at High E*

Relativistic Mean Field (I)

Serot-Walecka, Walecka text book.

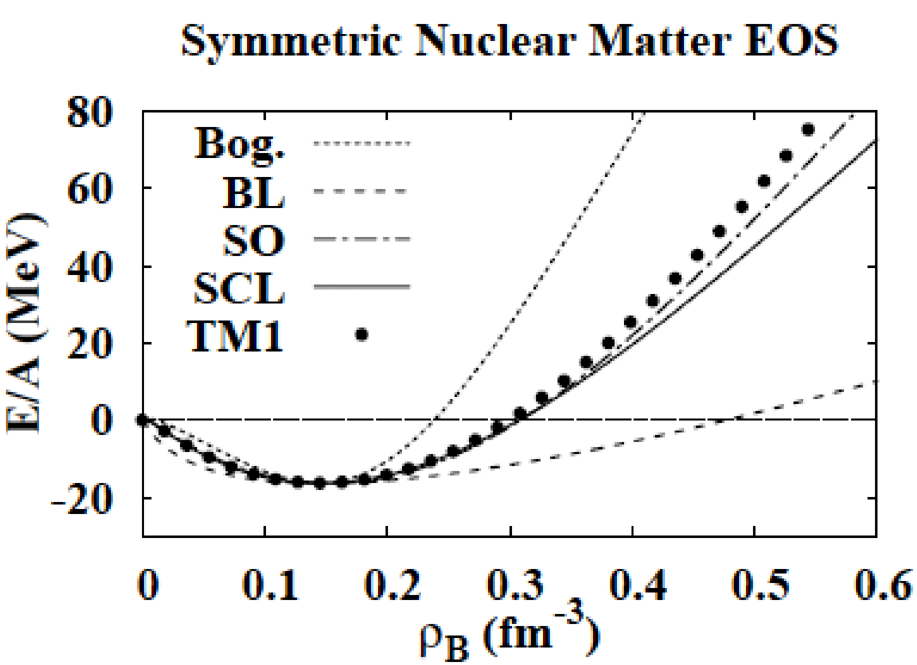
- **Describe nuclear energy functional in meson and baryon fields**
  - **Fit B.E. of Stable as well as Unstable (n-rich) Nuclei**
  - **Has been successfully applied to Supernova Explosion**
  - **Three Mesons (σ,ω,ρ) are included**
  - **Meson Self-Energy Term (σ,ω)**

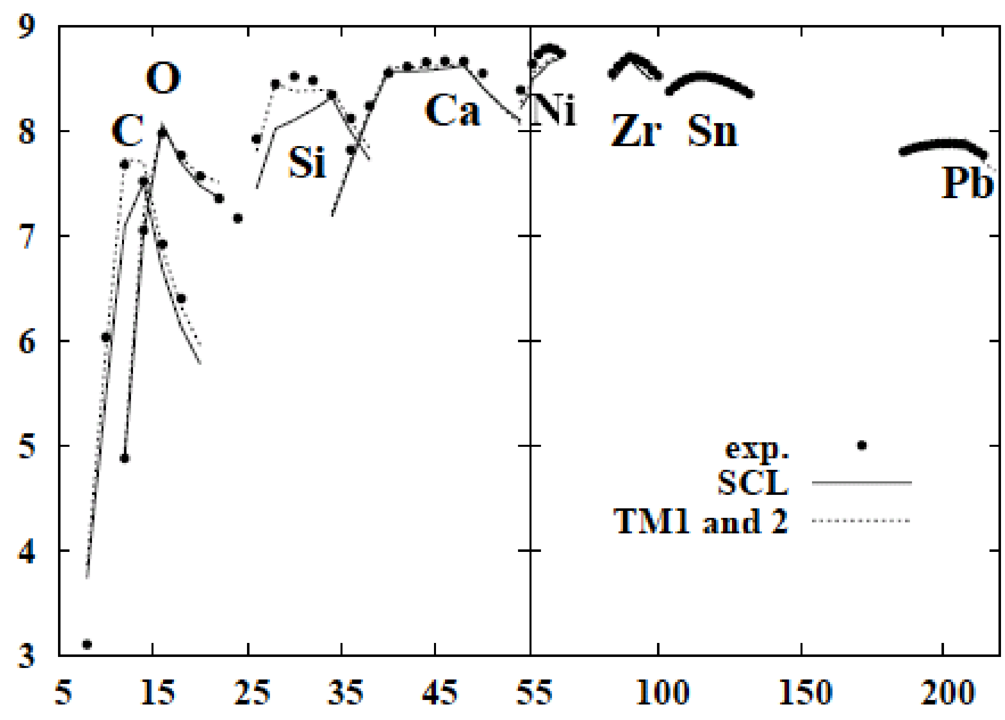
$$
\begin{aligned}
\mathcal{L} =& \bar{\psi}_N \left(i\partial\!\!\!/ - M - g_\sigma \sigma - g_\omega \omega\!\!\!/ - g_\rho \tau^a \rho\!\!\!/^a \right) \psi_N \\
&+ \frac{1}{2}\partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 \\
&- \frac{1}{4} W^{\mu\nu} W_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4} R^{a\mu\nu} R^a_{\mu\nu} + \frac{1}{2} m_\rho^2 \rho^{a\mu} \rho^a_\mu + \frac{1}{4} c_3 \left(\omega_\mu \omega^\mu \right)^2 \\
&+ \bar{\psi}_e \left(i\partial\!\!\!/ - m_e \right) \psi_e + \bar{\psi}_\nu i\partial\!\!\!/ \psi_\nu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \ , \\
W_{\mu\nu} =& \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \ , \\
R^a_{\mu\nu} =& \partial_\mu \rho^a_\nu - \partial_\nu \rho^a_\mu + g_\rho \epsilon^{abc} \rho^{b\mu} \rho^{c\nu} \ , \\
F_{\mu\nu} =& \partial_\mu A_\nu - \partial_\nu A_\mu \ . \qquad (2)
\end{aligned}
$$

Nuclear Matter EOS and Nuclear Binding E in TM

Sugahara-Toki, NPA579 (1994), 557.

- **Example: TM1 parameter set**
  - **Nuclear Matter: σ4 and ω4 terms soften EOS (K ~ 280 MeV)**
  - **Finite nuclei: Explains B.E. from C to Pb isotopes**





c.f. SCL=Chiral RMF with log σ term.

(K. Tsubakihara and AO, in preparation)

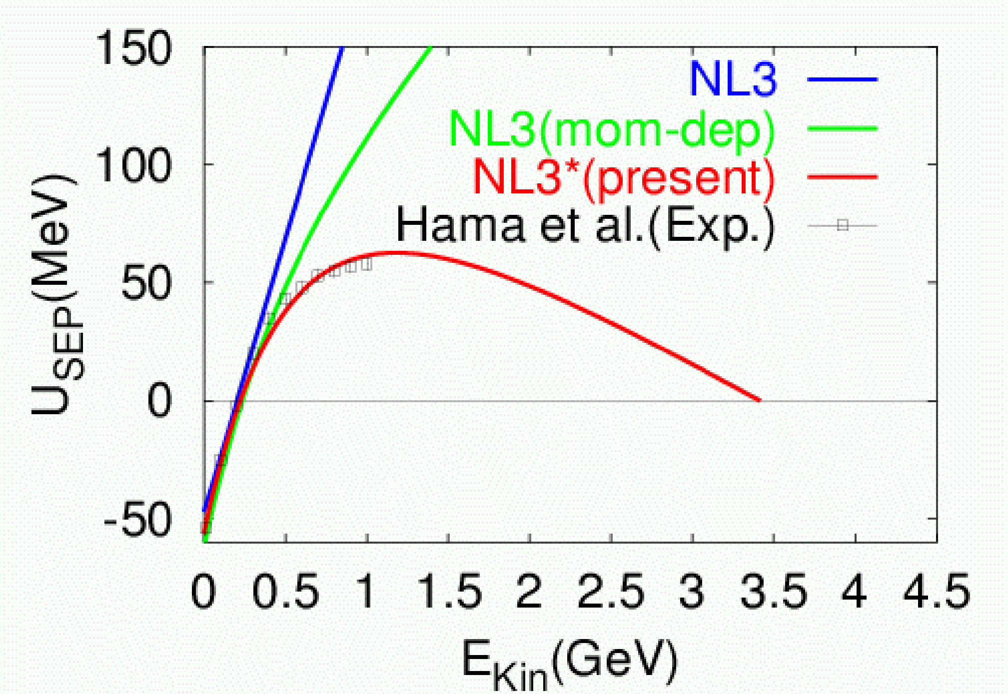
Relativistic Mean Field (II)

- **Dirac Equation** $\left(i\gamma\partial-\gamma^0 U_v - M - U_s\right)\psi=0\ ,\quad U_v=g_\omega\omega\ ,\quad U_s=-g_\sigma\sigma$
- **Schroedinger Equivalent Potential**

$$\begin{pmatrix} E-U_v-M-U_s & -i\sigma\cdot\nabla \\ i\sigma\cdot\nabla & -E+U_v-M-U_s \end{pmatrix}\begin{pmatrix} f \\ g \end{pmatrix}=0$$

$$U_{sep}\sim U_s+\frac{E}{m}U_V=-g_\sigma\sigma+\frac{E}{m}g_\omega\omega$$

$$=-\frac{g_\sigma^2}{m_\sigma^2}\rho_s+\frac{E}{m}\frac{g_\omega^2}{m_\omega^2}\rho_B$$



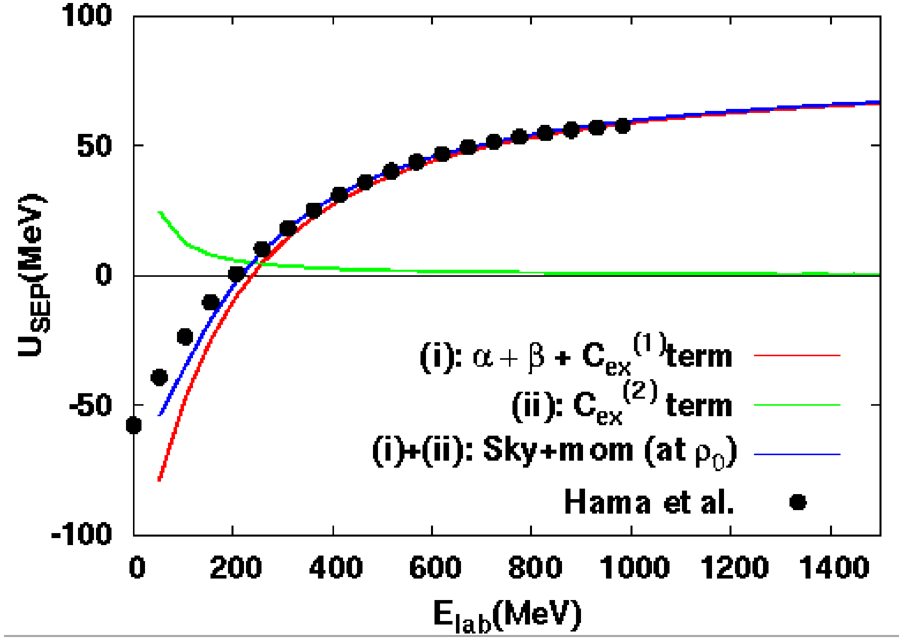
Saturation: -Scalar+Baryon Density
Linear Energy Dependence: Good at Low Energies,
Bad at High Energies (We need cut off !)

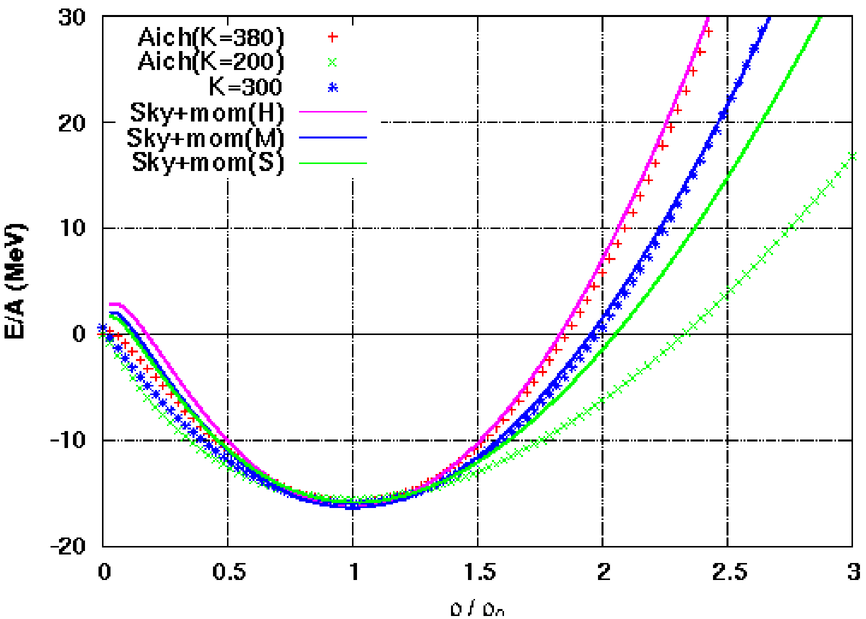
(Sahu, Cassing, Mosel, AO, Nucl. Phys. A672 (2000), 376.)

- **Skyrme type ρ-Dep. + Lorentzian p-Dep. Potential**

$$V=\sum_i V_i=\int d^3r\left[\frac{\alpha}{2}\left(\frac{\rho}{\rho_0}\right)^2+\frac{\beta}{\gamma+1}\left(\frac{\rho}{\rho_0}\right)^{\gamma+1}\right]$$

$$+\sum_k\int d^3r\,d^3p\,d^3p'\,\frac{C_{ex}^{(k)}}{2\rho_0}\frac{f(r,p)f(r,p')}{1+(p-p')^2/\mu_k^2}$$





Isse, AO, Otuka, Sahu, Nara, Phys.Rev. C 72 (2005), 064908

Exercise (4)

- **Prove that the single particle potential with Skyrme interaction has a linear dependence on energy. From NA elastic scattering, the energy dependence is found to be**

  $$U(\rho_0, E) \sim U(\rho_0, E=0) + 0.3\, E$$

  at low energies. Obtain the value of m*/m which explains the above energy dependence.

- **Obtain the form of the Schrodinger equivalent potential in RMF. You will find that the spin-orbit potential appears as a sum of scalar and vector potential.**

Summary

- **Basic ingredients in HIC models are explained.**
  - **Mean field dynamics**
  - **Two-body hadron-hadron collisions**
  - **String formation and Jet production**
  - **Hydrodynamics**
- **While nuclear MF at low energies are well investigated, it is not trivial how to apply these MFs to higher energy reactions.**
  At present, phenomenologically parametrized potentials are frequently used.
- **Students interested in HIC up to 1 A GeV should understand mean-field dynamics and NN cross sections (and π productions).**
  Students interested in RHIC physics should understand parton dynamics and strings, and hydrodynamics.

Part III:
Phase diagram of quark and hadron matter

強結合格子QCDでみた核物質と原子核

YKIS06 での Talk とほぼ同じです。

Strong Coupling QCD
→ Strong Coupling Limit/Region of Lattice QCD

Akira Ohnishi Hokkaido University, Sapporo, Japan

This talk is based on following Eprints

(1) Phase diagram at finite temperature and quark density in the strong coupling limit of lattice QCD for color SU(3)
N. Kawamoto, K. Miura, A. Ohnishi, T. Ohnuma,
Phys. Rev. D, in press; hep-lat/0512023

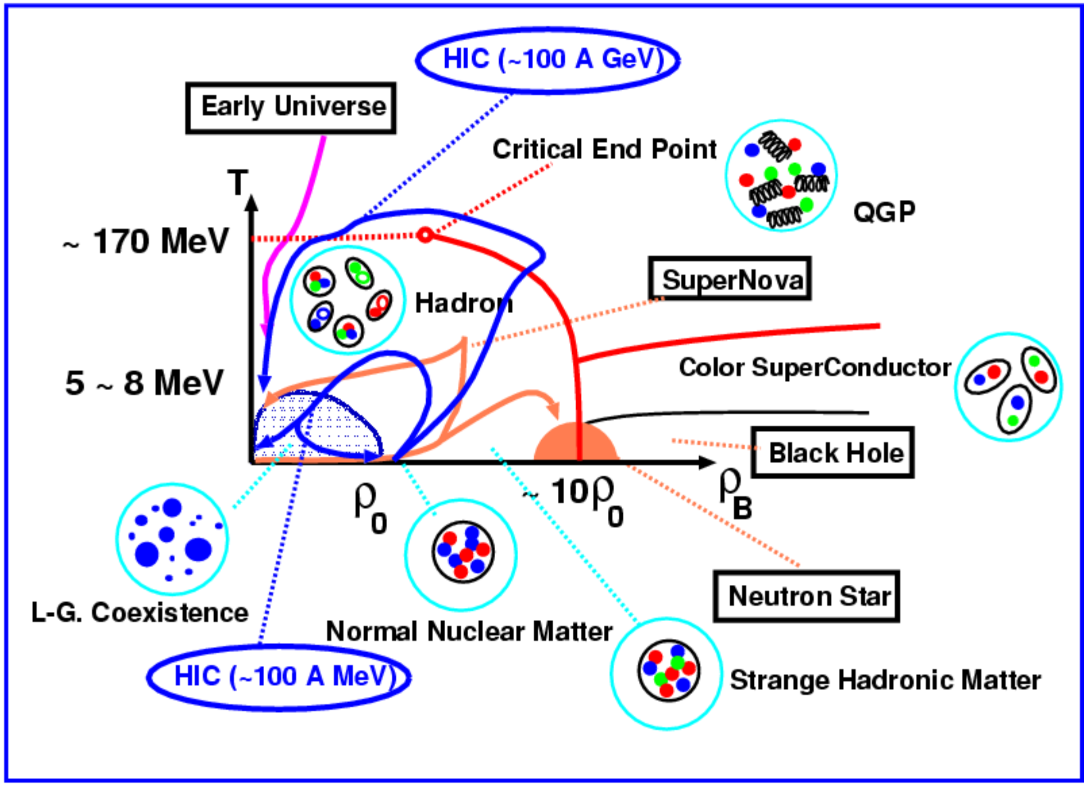
(2) A chiral symmetric relativistic mean field model with logarithmic sigma potential
K. Tsubakihara and A. Ohnishi, nucl-th/0607046

Outline

- **Introduction**
- **Strong coupling limit lattice QCD with baryon effects**
- **$1/g^2$ correction of Phase Diagram**
- **Chiral RMF with logarithmic σ potential**
- **Summary**

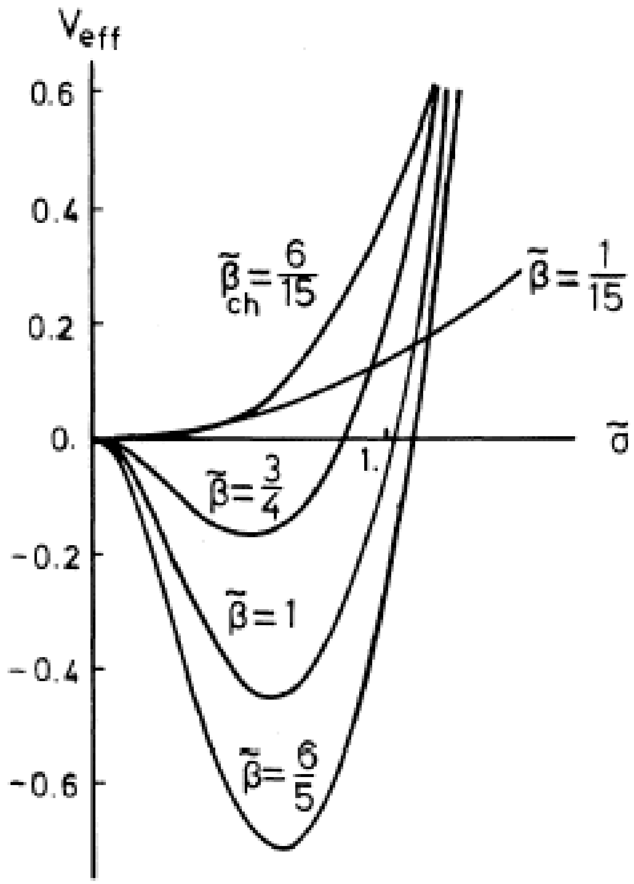
Quark and Hadronic Matter Phase Diagram

- **Dense quark & hadronic matter contains rich physics, but Lattice QCD simulation is not yet reliable.**
  → *Model/Approximate approaches are necessary !*
  - **Monte-Carlo calc. of Lattice QCD:**
    Improved ReWeighting Method (Fodor-Katz)
    Taylor Expansion in μ (Bielefeld-Swansea)
    Analytic Continuation (de Forcrand-Philipssen)
  - **Model / Phen. Approaches: (P)NJL, QMC, RMF, ...**
  - ***Strong Coupling Limit of Lattice QCD***

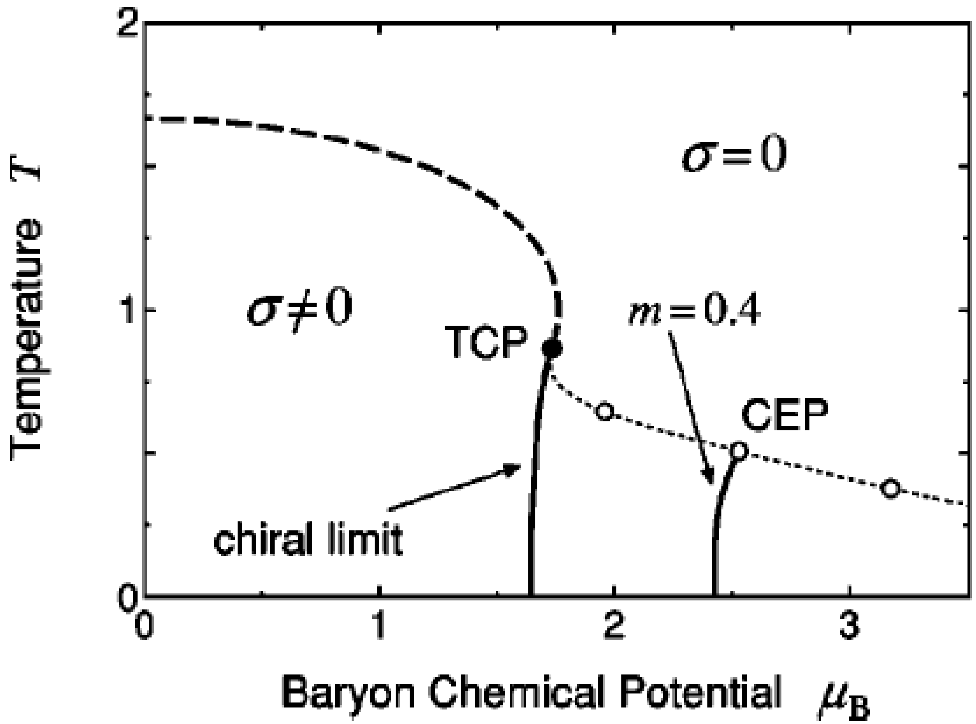


Strong Coupling Limit of Lattice QCD

- **Chiral Restoration at μ=0.**
  - **Damgaard, Kawamoto, Shigemoto, PRL53(1984),2211**



- **Phase Diagram with Nc=3**
  - **Nishida, PRD69, 094501 (2004)**



Previous Works in Strong Coupling Limit LQCD

- **Strong Coupling Limit Lattice QCD re-attracts interests**
  c.f. Nakamura @ JHF Symp. for high density matter (2001)

Ref	T	μ	Nc	Baryon	CSC	Nf
Damgaard-Kawamoto-Shigemoto('84)	*Finite*	***0***	*U(Nc)*	***X***	*X*	*1*
Damgaard-Hochberg-Kawamoto('85)	***0***	*Finite*	*3*	*Yes*	*X*	*1*
Bilic-Karsch-Redlich('92)	*Finite*	*Finite*	*3*	***X***	*X*	*1 ~ 3*
Azcoiti-Di Carlo-Galante-Laliena('03)	***0***	*Finite*	*3*	*Yes*	*Yes*	*1*
Nishida-Fukushima-Hatsuda('04)	*Finite*	*Finite*	***2***	*Yes (*)*	*Yes (*)*	*1*
Nishida('04)	*Finite*	*Finite*	*3*	***X***	*X*	*1~2*
Kawamoto-Miura-AO-Ohnuma('05)	***Finite***	***Finite***	***3***	***Yes***	***Yes (+)***	***1***

*: bosonic baryon=diquark in SU(2)
+: analytically included, but ignored in numerical calc.

- ***Baryon effects have been ignored in finite T treatments !***
  → This work: Baryonic effects at Finite T (and μ) for $SU_c(3)$

Strong Coupling Limit Lattice QCD

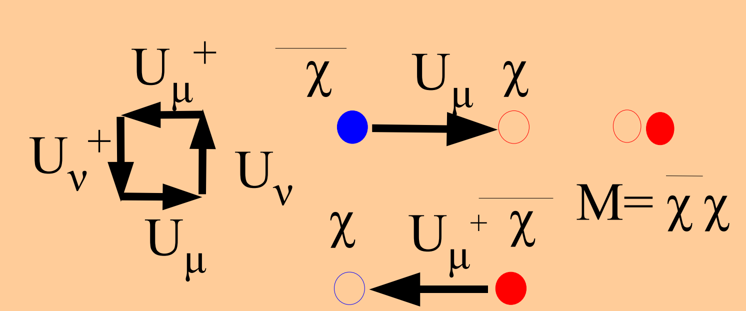
- **QCD Lattice Action**

$$Z \simeq \int D[\chi,\bar{\chi},U] \exp\left[-\left(S_G + S_F^{(s)} + S_F^{(t)} + m_0 M\right)\right]$$

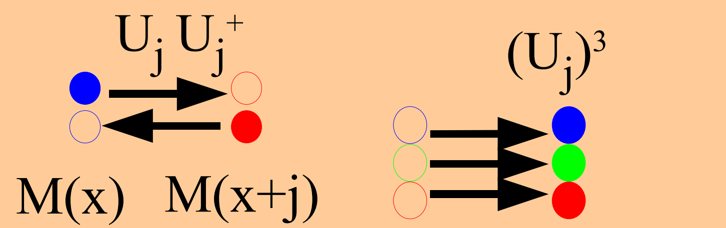
$$S_G = \frac{1}{g^2} \sum_{x\mu\nu} \left[\mathrm{Tr}\, U_{\mu\nu} + \mathrm{Tr}\, U^+_{\mu\nu}\right]$$

$$S_F^{(s)} = \frac{1}{2} \sum_{x,j} \eta_j(x) \left(\bar{\chi}_x U_j(x) \chi_{x+\hat{j}} - \bar{\chi}_{x+\hat{j}} U_j^+(x) \chi_x\right)$$

$$S_F^{(t)} = \frac{1}{2} \sum_{x} \left(e^{\mu} \bar{\chi}_x U_0(x) \chi_{x+\hat{0}} - e^{-\mu} \bar{\chi}_{x+\hat{0}} U_0^+(x) \chi_x\right)$$



- **Strong Coupling Limit: g→∞**
  - **We can ignore S_G and perform one-link integral after 1/d expansion.**



$\bar{B} = \epsilon \bar{\chi}\bar{\chi}\bar{\chi}/6 \quad B = \epsilon \chi\chi\chi/6$

$$\int dU\, U_{ab} U^+_{cd} = \frac{1}{N_c} \delta_{ad} \delta_{bc}$$

$$\int dU\, U_{ab} U_{cd} U_{ef} = \frac{1}{6} \epsilon_{ace} \epsilon_{bdf}$$

$$S_F^{(s)} \to -\frac{1}{2} (M V_M M) - (\bar{B} V_B B)$$

$$= -\frac{1}{4 N_c} \sum_{x,j>0} M_x M_{x+\hat{j}} + \sum_{x,j>0} \frac{\eta_j}{8} \left[\bar{B}_x B_{x+\hat{j}} - \bar{B}_{x+\hat{j}} B_x\right]$$

SCL-LQCD w/o Baryons

Damgaad-Kawamoto-Shigemoto 1984, Faldt-Petersson 1986, Bilic-Karsch-Redlich 1992, Nishida 2004, ...

- **Lattice Action (staggered fermion) in SCL**

$$Z \simeq \int D[\chi,\bar{\chi},U] \exp\left[-S_F^{(s)} - S_F^{(t)} - m_0 \bar{\chi}\chi - S_G\right]$$

- **Spatial Link Integral**

Strong Coupling

$$\simeq \int D[\chi,\bar{\chi},U_0] \exp\left[\frac{1}{2}(M, V_M M) + (\bar{B}, V_B B) - (\bar{\chi} G_0 \chi)\right]$$

1/d Expansion (1/√d)

- **Bosonization (Hubburd-Stratonovich transformation)**

$$\simeq \int D[\chi,\bar{\chi},U_{0,}\sigma] \exp\left[-\frac{1}{2}(\sigma, V_M \sigma) - (\sigma, V_M M) - (\bar{\chi} G_0 \chi)\right]$$

- **Quark and U_0 Integral**

$$(\bar{\chi} G(\sigma) \chi)$$

$$\simeq \exp\left(-N_S^3 N_\tau \left[\frac{1}{2} a_\sigma \sigma^2 - T \log G_U(\sigma)\right]\right) = \exp(-N_S^3 F_{\text{eff}}/T)$$

Local Bi-linear action in quarks → Effective Free Energy

SCL-LQCD with Baryons

$$M = \bar{\chi}_a \chi^a$$
$$B = \epsilon_{abc} \chi^a \chi^b \chi^c / 6$$

- **Effective Action up to O(1/√d)**

$$Z \simeq \int D[\chi, \bar{\chi}, U_0] \exp\left[\frac{1}{2}(M, V_M M) + (\bar{B}, V_B B) - (\bar{\chi} G_0 \chi)\right]$$
$$= \int D[\chi, \bar{\chi}, U_0, b, \bar{b}] \exp\left[\frac{1}{2}(M V_M M) - (\bar{b} V_B^{-1} b) + (\bar{b}, B) + (\bar{B}, b) - (\bar{\chi} G_0 \chi)\right]$$

- *Decomposition of bB by using diquark condensate (Azcoiti et al., 2004)*

$$\exp[(\bar{b}, B) + (\bar{B}, b)] = \exp\left[\frac{1}{6}(\bar{b}, \epsilon \chi \chi \chi) + \frac{1}{6}(\epsilon \bar{\chi} \bar{\chi} \bar{\chi}, b)\right]$$
$$= \int D[\phi_a, \phi_a^*] \exp\left[-\phi^* \phi + \phi^* \left(\frac{\gamma}{2} \epsilon \chi \chi + \frac{\bar{\chi} b}{3\gamma}\right) + \phi \left(\frac{\gamma}{2} \epsilon \bar{\chi} \bar{\chi} + \frac{\bar{b} \chi}{3\gamma}\right)\right]$$
$$\times \exp(-\gamma M^2/2 + M \bar{b} b / 9\gamma^2)$$

- *Decomposition of Mbb using baryon potential field ω*

$$\exp(M \bar{b} b / 9\gamma^2) = \int D[\omega] \exp\left[\frac{1}{2}\omega^2 - \omega\left(\alpha M + \frac{\bar{b} b}{9\alpha\gamma^2}\right) - \frac{\alpha^2}{2} M^2\right]$$

  - *note:* $(\bar{b} b)^2 = 0$ *with one species of staggered fermion !*

Effective Free Energy with Baryon Effects

- **Effective Action in local bilinear form of quarks**

$$S_F = -\frac{1}{2}(M\tilde{V}_M M) + \frac{1}{2}(\omega,\omega) + (\bar{b},\tilde{V}_B^{-1}(g_\omega \omega) b) + \alpha(\omega, M) + (\bar{\chi} G_0 \chi)$$

~~$+(\phi^* \phi) + (\phi^* D) + (D^+ \phi)$~~

Bosonization + MFA
+No diquark cond.

$$= \frac{N_s^3 N_\tau}{2}\left(a_\sigma \sigma^2 + \omega^2\right) + (a_\sigma \sigma + \alpha\omega, M) + (\bar{\chi} G_0 \chi) + (\bar{b}, \tilde{V}_B^{-1}(g_\omega \omega) b)$$

quark & gluon int. *b int.*

$$F_{\mathrm{eff}}(\sigma,\omega) = \frac{1}{2} a_\sigma \sigma^2 + \frac{1}{2}\omega^2 + F_{\mathrm{eff}}^{(q)}(a_\sigma \sigma + \alpha\omega) + F_{\mathrm{eff}}^{(b)}(g_\omega \omega)$$

$O(\omega^2)$ *$O(\omega^4)$*

$$= \frac{1}{2} a_\sigma \sigma^2 + \frac{1}{2} a_\omega \omega^2 + F_{\mathrm{eff}}^{(q)}(a_\sigma \sigma + \alpha\omega) + \Delta F_{\mathrm{eff}}^{(b)}(g_\omega \omega)$$

Linear Approx. *$(\omega \sim \alpha\sigma/a_\omega)$*

$$F_{\mathrm{eff}}(\sigma) = \frac{1}{2} b_\sigma \sigma^2 + F_{\mathrm{eff}}^{(q)}(b_\sigma \sigma) + \Delta F_{\mathrm{eff}}^{(b)}(g_\sigma \sigma)$$

Color Angle Average

- **Problem: Diquark Condensates induce quark-baryon coupling, and Baryon integral becomes difficult.**
  → Solution: *Color Angle Average*

$$D = \frac{\gamma}{2}\epsilon\chi\chi + \frac{\bar{\chi} b}{3\gamma}$$

  - **Integral of "Color Angle Variables"**

$$\int \mathcal{D}[\phi_a, \phi_a^\dagger] \exp\left\{\phi_a^\dagger D_a + D_a^\dagger \phi_a\right\} = \int \mathcal{D}[v] \exp\left\{\frac{v^2}{3} D_a^\dagger D_a + \frac{v^4}{162} M^3 \bar{b} b\right\}$$

  - **Three-Quark and Baryon Coupling is ReBorn !**

$$D_a^\dagger D_a = Y + \bar{b}B + \bar{B}b\ , \quad Y = \frac{\gamma^2}{2} M^2 - \frac{1}{9\gamma^2} M \bar{b} b$$

  - **Solve "Self-Consistent" Equaton**

$$\exp(\bar{b}B + \bar{B}b) \simeq \exp\left[-v^2 - Y + \frac{v^2}{3}(\bar{b}B + \bar{B}b + Y) + \frac{v^4}{162} M^3 \bar{b} b\right]$$

$$\simeq \exp\left[-\frac{v^2}{R_v} + \frac{v^4 M^3 \bar{b} b}{162 R_v} - Y\right] \quad (R_v = 1 - v^2/3)$$

Effective Free Energy with Diquark Condensate

- **Bosonization of** $M^k \bar{b} b$ → **Introduce k bosons**

$$\exp M^k \bar{b} b = \int d\omega_k \exp[-\frac{1}{2}(\omega_k + \alpha_k M + 1/\alpha_k M^{k-1} \bar{b} b)^2 + M^k \bar{b} b]$$

$$= \int d\omega_k \exp[-\omega_k^2/2 - \omega(\alpha_k M + 1/\alpha_k M^{k-1} \bar{b} b) - \alpha_k^2 M^2/2]$$

- **Effective Free Energy**

$$\mathcal{F}_{\mathrm{eff}}^{(Tbv)} = F_X(\sigma, v, \omega_i) + F_{\mathrm{eff}}^{(b)}(g_\omega \omega) + F_{\mathrm{eff}}^{(q)}(m_q)$$

$$F_X = \frac{1}{2}(a_\sigma \sigma^2 + \omega^2 + \omega_1^2 + \omega_2^2) + \frac{v^2}{R_v} \qquad m_q = a_\sigma \sigma + \alpha\omega + \alpha_1 \omega_1 + \alpha_2 \omega_2 + m_0$$

$$a_\sigma = \frac{1}{2} - \gamma^2 - \alpha^2 - \alpha_1^2 - \alpha_2^2 \qquad g_\omega = \frac{1}{9\alpha\gamma^2}\left[1 + \frac{\gamma^2 v^4 \omega_1 \omega_2}{18\alpha_1 \alpha_2 R_v}\right]$$

The same F_{eff} is obtained at v=0.
Diquark Effects in interaction start from v^4.
(No Stable CSC phase appears at g= ∞)

c.f. Ipp, Yamamoto

Effective Free Energy with Baryon Effects

(Kawamoto-Miura-AO-Ohnuma, hep-lat/0512023)

$$F_{\text{eff}}(\sigma)=\frac{1}{2}b_\sigma\sigma^2+F_{\text{eff}}^{(q)}(b_\sigma\sigma;T,\mu)+\Delta F_{\text{eff}}^{(b)}(g_\sigma\sigma)$$

is analytically derived based on many previous works, including

- **Strong Coupling Limit** *(Kawamoto-Smit, 1981)*
- **1/d expansion** *(Kluberg-Stern-Morel-Petersson, 1983)*
- **Lattice chemical potential** *(Hasenfratz-Karsch, 1983)*
- **Quark and time-like gluon analytic integral**
  (Damgaad-Kawamoto-Shigemoto, 1984, Faldt-Petersson, 1986)

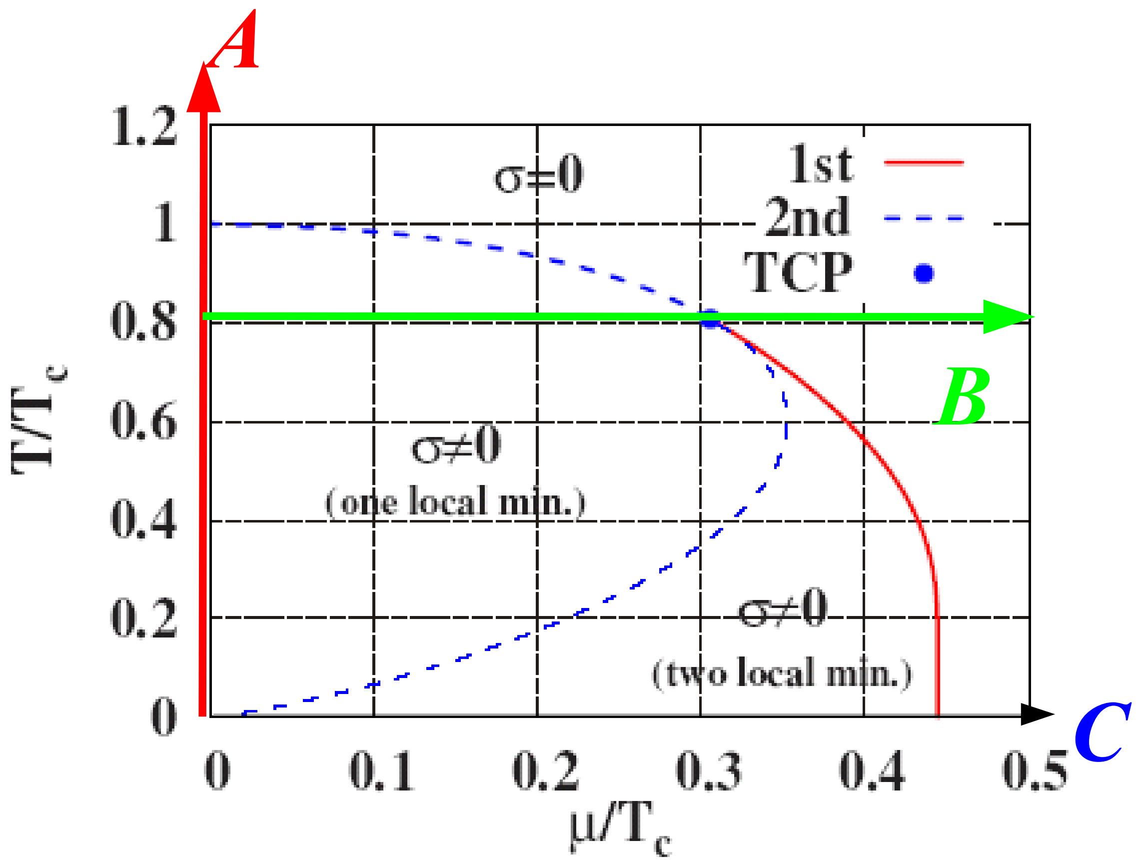
$$F_{\text{eff}}^{(q)}(\sigma;T,\mu)=-T\log\left(C_\sigma^3-\frac{1}{2}C_\sigma+\frac{1}{4}C_{3\mu}\right)\quad C_\sigma=\cosh(\sinh^{-1}\sigma/T)\quad C_{3\mu}=\cosh(3\mu/T)$$

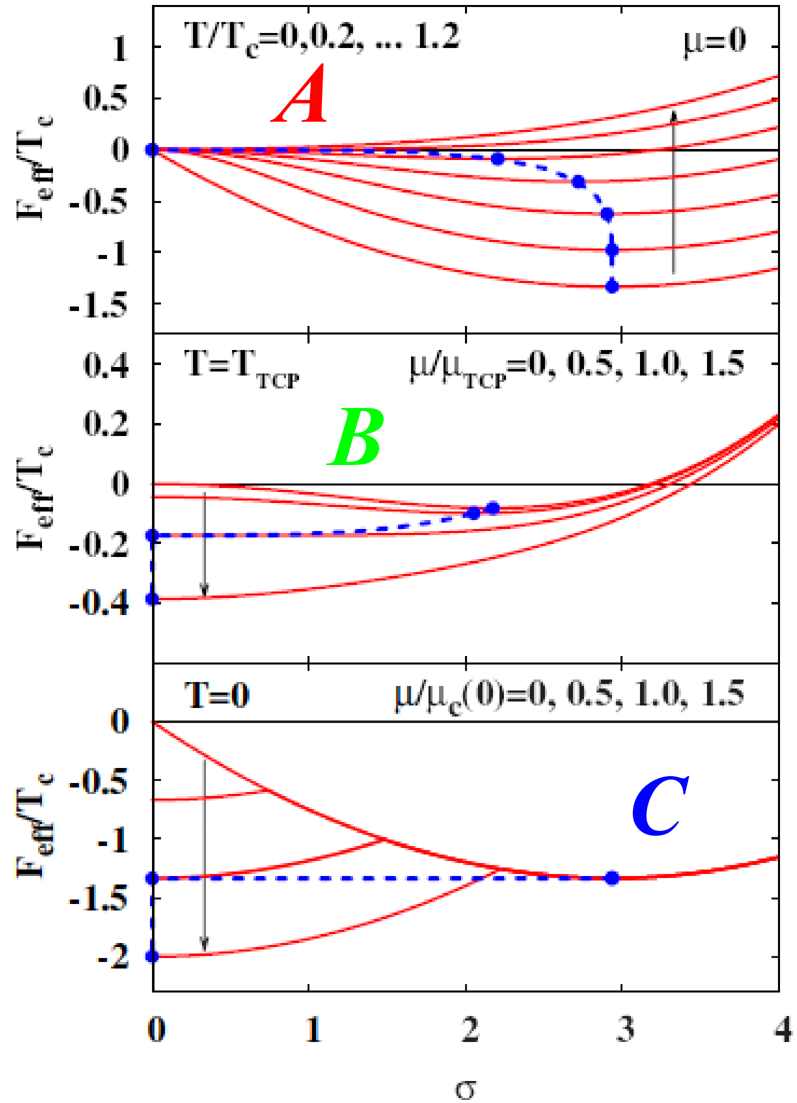
- **Decomposition of baryon-3 quark coupling**
  (Azcoiti-Di Carlo-Galante-Laliena, 2003)

and auxiliary baryon potential and baryon integral

Free Energy Surface and Phase Diagram

- **At μ ≠ 0, quark can gain Free Energy even at σ =0**
  → Two Min. Structure
  → First Order



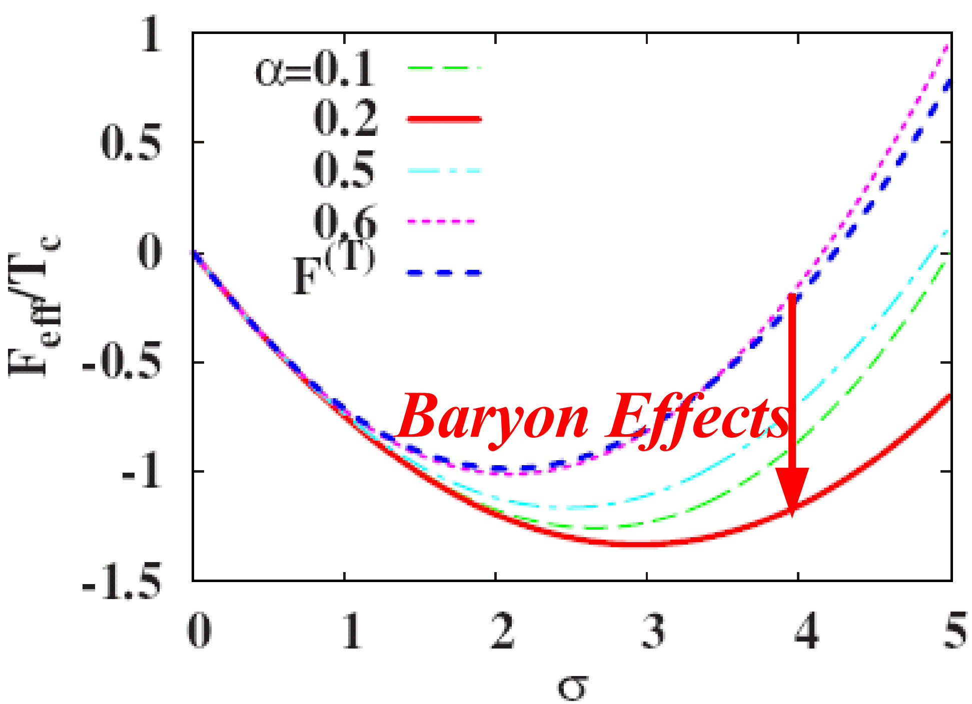


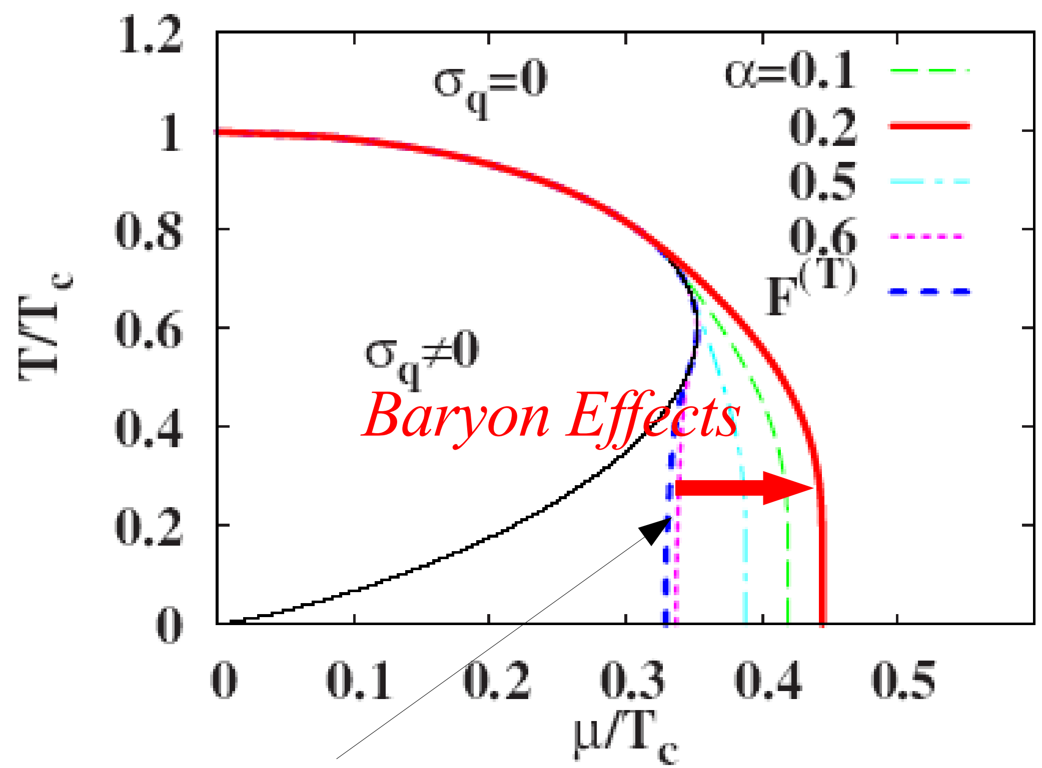
α =0.2

Phase diagram in SCL-LQCD with Baryons

(Kawamoto-Miura-AO-Ohnuma, hep-lat/0512023)

- **Baryon effects on phase diagram**
  - **Energy gain in larger condensates**
    → Extension of hadron phase to larger μ by around 30 %.





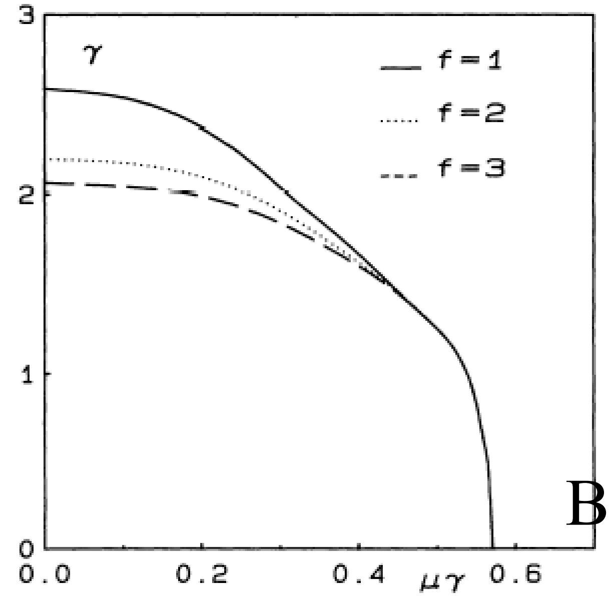
Nishida 2004(No B)

Discussions

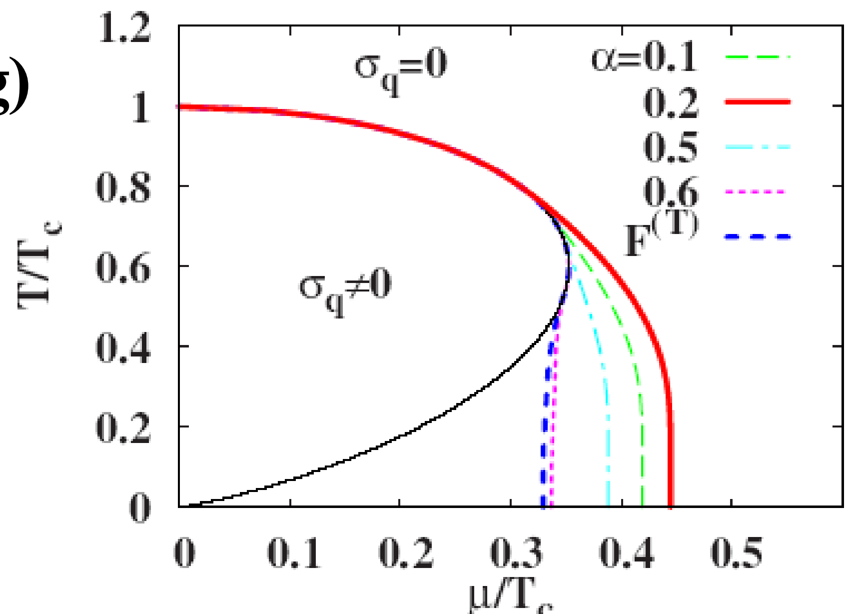
- **Present phase diagram ↔ real phase diagram**
  - **One species of staggered fermion ~ N_f=4. Should be 1st order !**
  - **Tc seems to be too high. μ_c/T_c(present) ~ 0.45 ↔ μ_c/T_c(real)~(2-3)**
  - **No stable CSC phase** *(Azcoiti et al., 2003)*
    ↔ Stable CSC phase at large μ *(Alford, Hands, Stephanov)*
- **Two parameters are introduced through identities (HS transf.)**
  - **The results should be independent from parameter choice !**
    → MFA may break the identity...
  - **How should we fix these parameters ?**
- **Is SCL-LQCD useful ? → We would like to answer "Yes" !**
  - **Chiral RMF derived in SCL-LQCD works well in Nuclear Physics (Tsubakihara, AO, nucl-th/0607046**
    Tsubakihara,Maekawa,AO, Proc. of HYP06, to appear)
  - **$1/g^2$ expansion may connect SCL-LQCD and real world.**

Small Critical μ : Common in SCL-LQCD ?

- **Finite *T* SCL-LQCD**
  - **No B:** $\mu_c(0)/T_c(0) \sim (0.2\text{-}0.35)$ **(Nishida2004, Bilic-Karsch-Redlich 1992, ...)**
  - **Present:** $\mu_c(0)/T_c(0) < 0.44$ **(Parameter dep.)**
- **Monte-Carlo:** $\mu_c(0)/T_c(0) > 1$
  - **Fodor-Katz (Improved Reweighting) Bielefeld (Taylor expansion), de Forcrand-Philipsen (AC), ...**
- **Real World:** $\mu_c(0)/T_c(0) > 2$
  - $T_c(0) \sim 170$ *MeV*, $\mu_c(0) > 330$ *MeV*



Bilic et al.

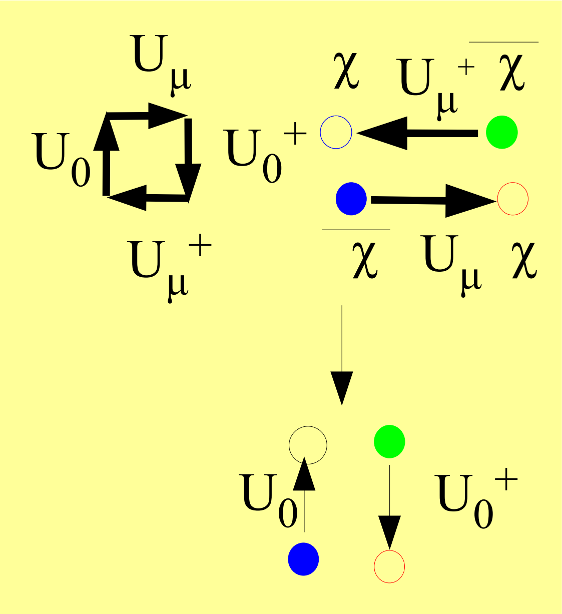


Present

$1/g^2$ expansion (w/o Baryon Effects)

- **T_c (μ=0) and μ_c (T=0): Which is worse ?**
  - **$1/g^2$ correction reduces T_c.** *(Bilic-Cleymans 1995)*
  - **Hadron masses are well explained in SCL.** *(Kawamoto-Smit 1981, Kawamoto-Shigemoto 1982)*

  → We expect Tc reduction with $1/g^2$ correction !



- **1/d expansion of plaquetts** *(Faldt-Petersson 1986)*
  - **Space-like plaquett**

$$\exp\left[\frac{1}{g^2}\sum_{x,i>j>0}\mathrm{Tr}\,U_{ij}(x)\right]\to\exp\left[\quad\frac{1}{8N_c^4 g^2}\sum_{x,k>j>0} M_x M_{x+\hat{j}} M_{x+\hat{k}} M_{x+\hat{k}+\hat{j}}\right]$$

  - **Time-like plaquett**

$$\exp\left[\frac{1}{g^2}\sum_{x,j>0}\mathrm{Tr}\,U_{0j}(x)\right]\to\exp\left[-\frac{1}{4N_c^2 g^2}\sum_{x,j>0}\left(V_x V_{x+\hat{j}}^+ + V_x^+ V_{x+\hat{j}}\right)\right]$$

$$\left(V_x=\bar{\chi}_x U_0(x)\chi_{x+\hat{0}}\right)$$

Plaquett Bosonization

- **Bosonization of Plaquetts** *(O(1/d, 1/g⁴) and Im(V) are ignored)* **+ MFA**

$$\exp\left(-S_F - S_g\right) \to \exp\left[-\frac{1}{2}\sum_x \left(e^{\mu} V_x - e^{-\mu} V_x^{+}\right) + \frac{1}{4N_c}\sum_{x,j>0} M_x M_{x+\hat{j}} - m_0 \sum_x M_x\right]$$

$$\times \exp\left[-\frac{\beta_t}{2}\varphi_t \sum_x \left(V_x - V_x^{+}\right) + \beta_s \varphi_s \sum_{x,j>0} M_x M_{x+\hat{j}}\right]$$

$$\times \exp\left[-L^3 N_\tau \left(\frac{\beta_t}{4}\varphi_t^2 + \frac{\beta_s d}{4}\varphi_s^2\right)\right] \quad \left(\beta_t = \frac{d}{2N_c^2 g^2}, \quad \beta_s = \frac{d-1}{8N_c^4 g^2}\right)$$

$$= \exp\left[-\frac{L^3}{T} F_\varphi - \frac{\alpha}{2}\sum_x \left(e^{\tilde{\mu}} V_x - e^{-\tilde{\mu}} V_x^{+}\right) + \frac{1}{2}\sum_{x,y} M_x \tilde{V}_M(x,y) M_y\right]$$

$$\alpha = 1 + \beta_t \varphi_t \cosh\mu \quad , \quad \tilde{\mu} = \mu - \beta_t \varphi_t \sinh\mu$$

$$\langle \varphi_t \rangle = \langle V^{+} - V \rangle \quad , \quad \langle \varphi_s \rangle = 2 \langle M_x M_{x+\hat{j}} \rangle$$

Time-like plaquetts modifies effective chemical potential

Effective Free Energy with $1/g^2$ Correction (w/o B)

- **After Quark and Time-like Link integral, we get *F* as**

$$F=\frac{d}{4N_c}\sigma^2(1+4N_c\beta_s\varphi_s)+\frac{\beta_t}{4}\varphi_t^2+\frac{\beta_s d}{4}\varphi_s^2-N_c\beta_t\varphi_t\cosh\mu+F_q(m_q;\tilde{\mu})$$

$$=\frac{d}{4N_c}\sigma^2+3\mathrm{d}\,\beta_s\sigma^4+\frac{\beta_t}{4}\tilde{\varphi}_t^2+F_q(m_q;\tilde{\mu})$$

$$\varphi_s=2\sigma^2\quad,\quad\varphi_t=\tilde{\varphi}_t+2N_c\cosh\mu$$

Time-like plaquetts remains finite at large μ (c.f., S. Hands' talk)

$$m_q=\frac{d}{2N_c}\sigma(1+4N_c\beta_s\varphi_s-\beta_t\varphi_t\cosh\mu)$$

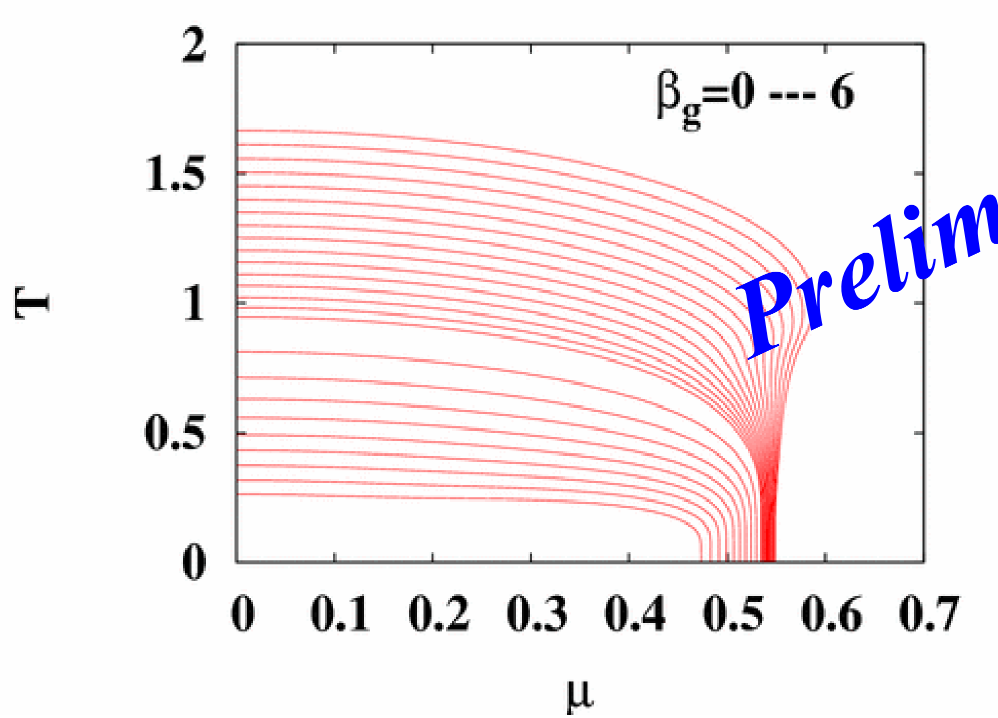
$$=\frac{d}{2N_c}\sigma(1-2N_c\beta_t\cosh^2\mu+8N_c\beta_s\sigma^4-\beta_t\tilde{\varphi}_t\cosh\mu)$$

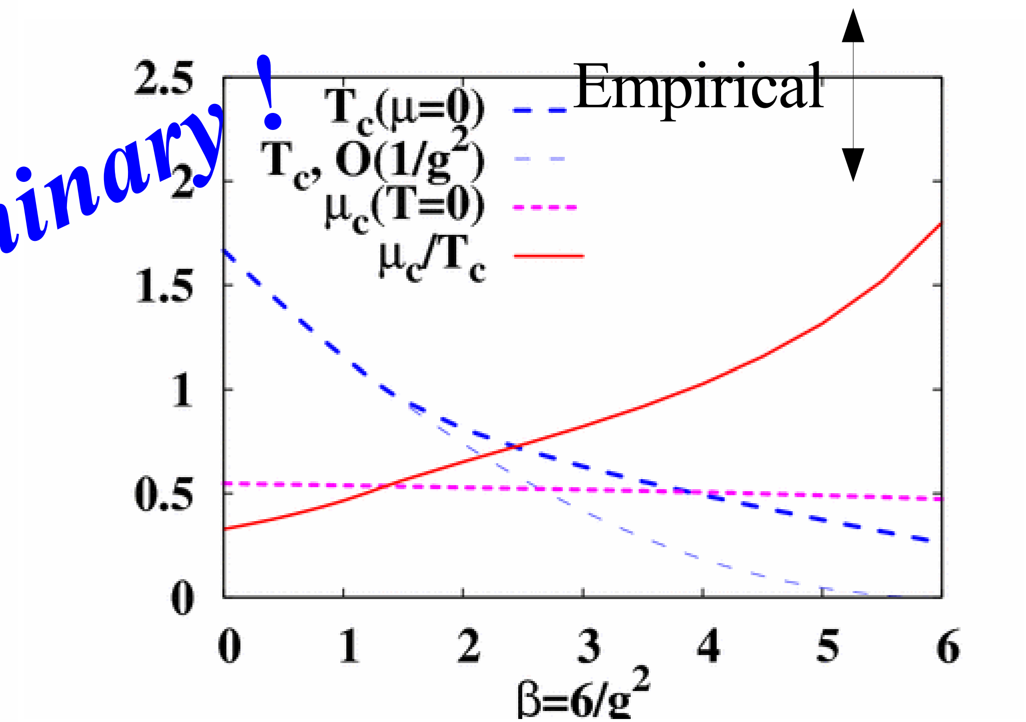
$$\tilde{\mu}=\mu-\beta_t\varphi_t\sinh\mu=\mu-2N_c\beta_t\cosh\mu\sinh\mu-\beta_t\tilde{\varphi}_t\sinh\mu$$

- **Space-like plaquett → Repulsive pot. ∝ σ^4, Enh. σ-quark couling**
- **Time-like plaquett → Reduces μ and σ-quark coupling (${\varphi}_t$ has to be determined to minimize F_{eff})**

Phase Boundary with $1/g^2$ correction

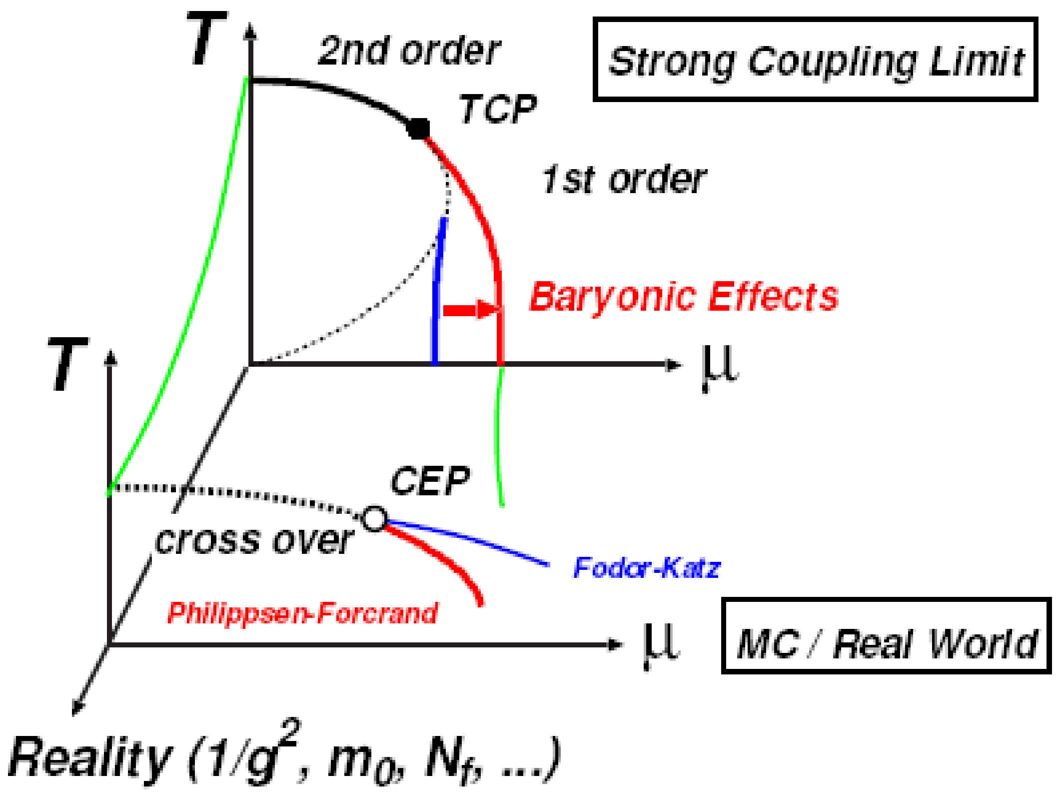
- **Rapid decrease of $T_c(\mu=0)$, and slow decrease of $\mu_c(T=0)$.**
  - **Similar reduction of σ-quark coupling and effective μ at small condensate → can be mimicked by the scaling of T**
    (c.f. Bilic-Claymans 1995 (T_c goes down), Arai-Yoshinaga (Poster, goes up).
- **Ratio $\mu_c/T_c \sim 1.8$ @ g=1.**
  - **with baryonic effects (~ 30 %), it may reach empirical value.**

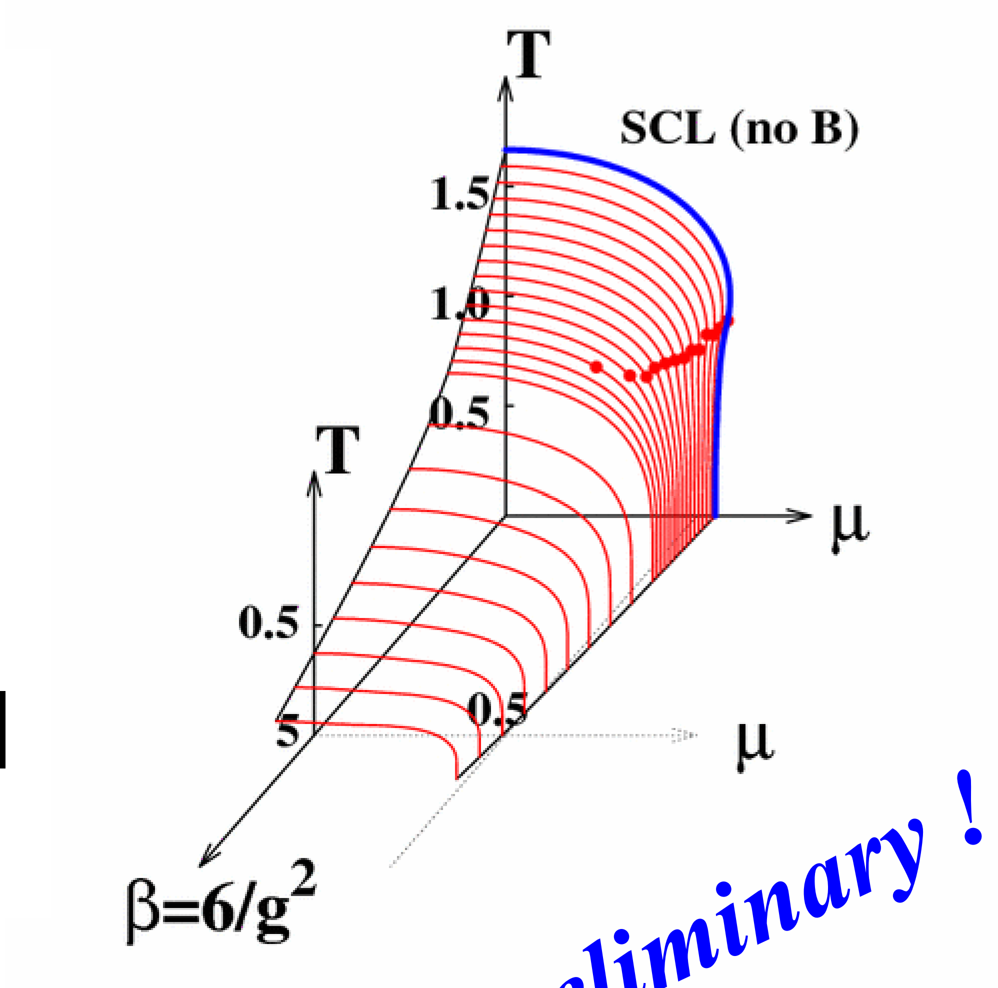




Evolution of Phase Diagram

- "Reality" Axis: $1/g^2$, n_f, m_0, ... would enhance μ_c/T_c ratio
- Example: $1/g^2$ correction enhances μ_c/T_c by a factor ~(2-3).





Preliminary !

Chiral symmetric RMF with logrithmic σ potential

K. Tsubakihara, AO, nucl-th/0607046
K. Tsubakihara, H. Maekawa, AO, Proc. of HYP06, to appear.

T.Tsubakihara shows a poster in the 3rd week.

RMF with Chiral Symmetry: Chiral Collapse

- **Naïve Chiral RMF models → Chiral collapse at low ρ** *(Lee-Wick 1974)*

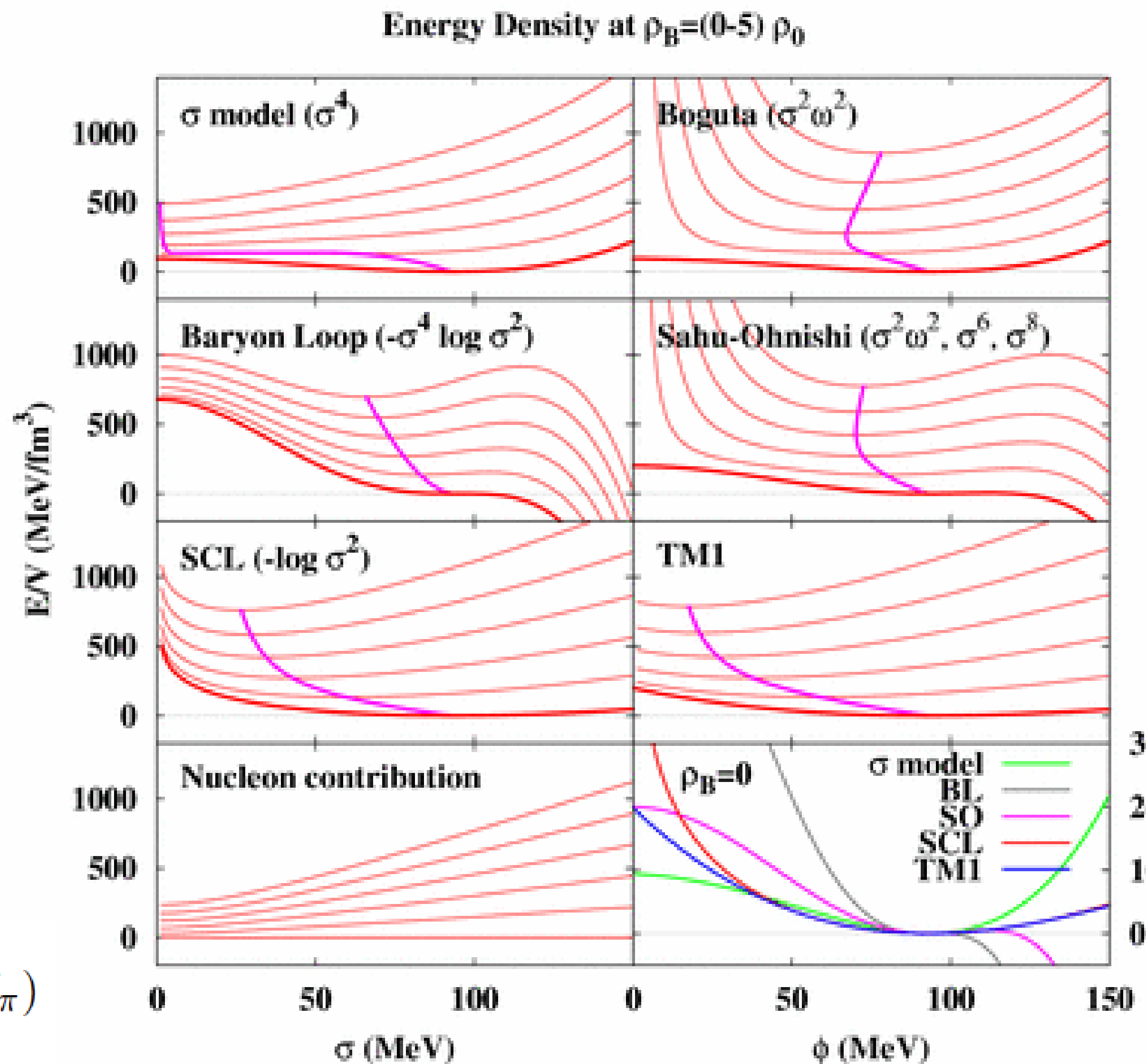
$$L = \frac{1}{2}\left(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \pi \partial^\mu \pi\right) - \frac{\lambda}{4}\left(\sigma^2 + \pi^2\right)^2 + \frac{\mu^2}{2}\left(\sigma^2 + \pi^2\right) + c\sigma$$

$$+ \overline{N} i \partial_\mu \gamma^\mu N - g_\sigma \overline{N}\left(\sigma + i\pi\tau\gamma_5\right) N$$

- **Prescriptions**
  - **σω coupling (too stiff EOS)** *(Boguta 1983, Ogawa et al. 2004)*
  - **Loop effects (unstable at large σ)** *(Matsui-Serot, 1982, Glendenning 1988, Prakash-Ainsworth 1987, Tamenaga et al. 2006)*
  - **Higher order terms (unstable at large σ)** *(Hatsuda-Prakash 1989, Sahu-Oh nishi 2000)*
  - ***Dielectric (Glueball) Field representing scale anomaly*** *(Furnstahl-Serot 199 3, Heide-Rudaz-Ellis 1994, Papazoglou et al.(SU(3)) 1998)*
  - **Different Chiral partner assignment** *(DeTar-Kunihiro 1989, Hatsuda-Prak ash 1989, Harada-Yamawaki 2001, Zschiesche-Tolos-Schaffner-Bielich-Pisa rski, nucl-th/0608044) → $SU_f(3)$ extention ?*
  - ***Nucleon Structure*** *(Saito-Thomas 1994, Bentz-Thomas 2001)*

Instability in Chiral Models

- **Linear σ Model**
  → Chiral restor.
  Below ρ_0.
- **Baryon Loop**
  & Sahu-Ohnishig
  models
  → Unstable
  at large σ
- **Boguta model**
  → Too Stiff EOS



$$V_\sigma^{\rm BL} = \frac{m_\sigma^2}{2f_\pi^2}(\phi^2 - f_\pi^2)^2 - M_N^4 f_{\rm BL}(\phi/f_\pi)$$

$$f_{\rm BL} = -\frac{1}{4\pi^2}\left[\frac{x^4}{2}\log x^2 - \frac{1}{4} + x^2 - \frac{3}{4}x^4\right]$$

RMF with σ Self Energy from SCL-LQCD

- σ Self Energy from simple Strong Coupling Limit LQCD

$$
\begin{aligned}
S &\to -\tfrac{1}{2}(M, V_M M) && (1/d \text{ expansion}) \\
&\to b\sigma^2 + (\bar{\chi}\ \sigma\chi) && (\text{auxiliary field}) \\
&\to b\sigma^2 \boxed{- a\log\sigma^2} && (\text{Fermion Integral})
\end{aligned}
$$

- RMF Lagrangian

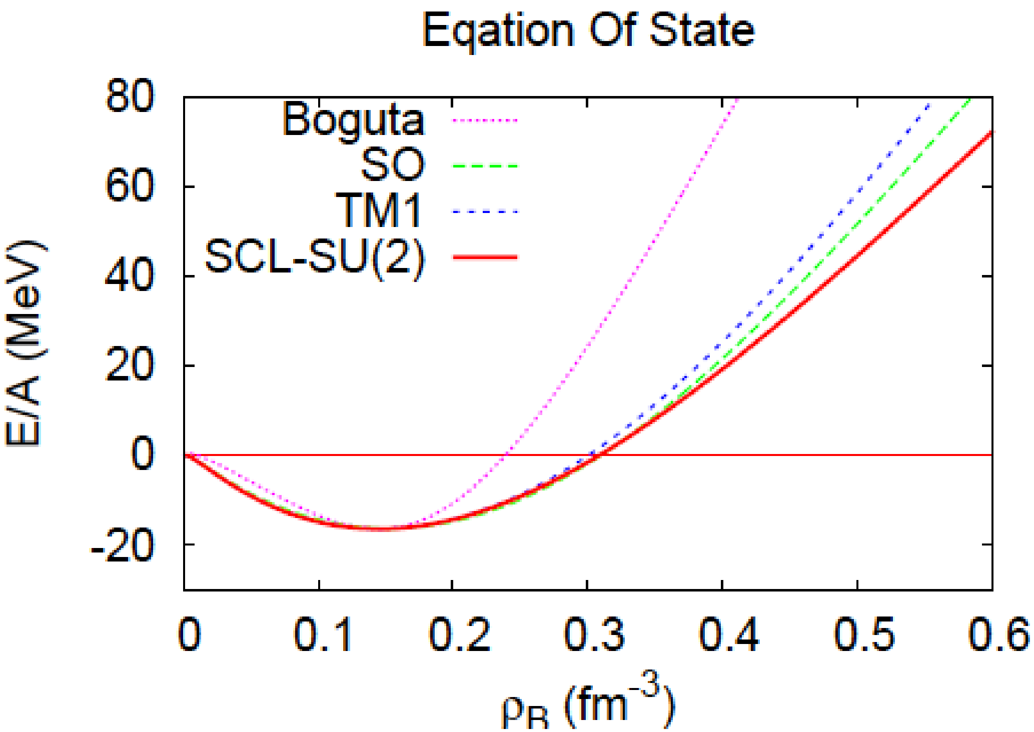
Non-Analytic Type σ Self Energy

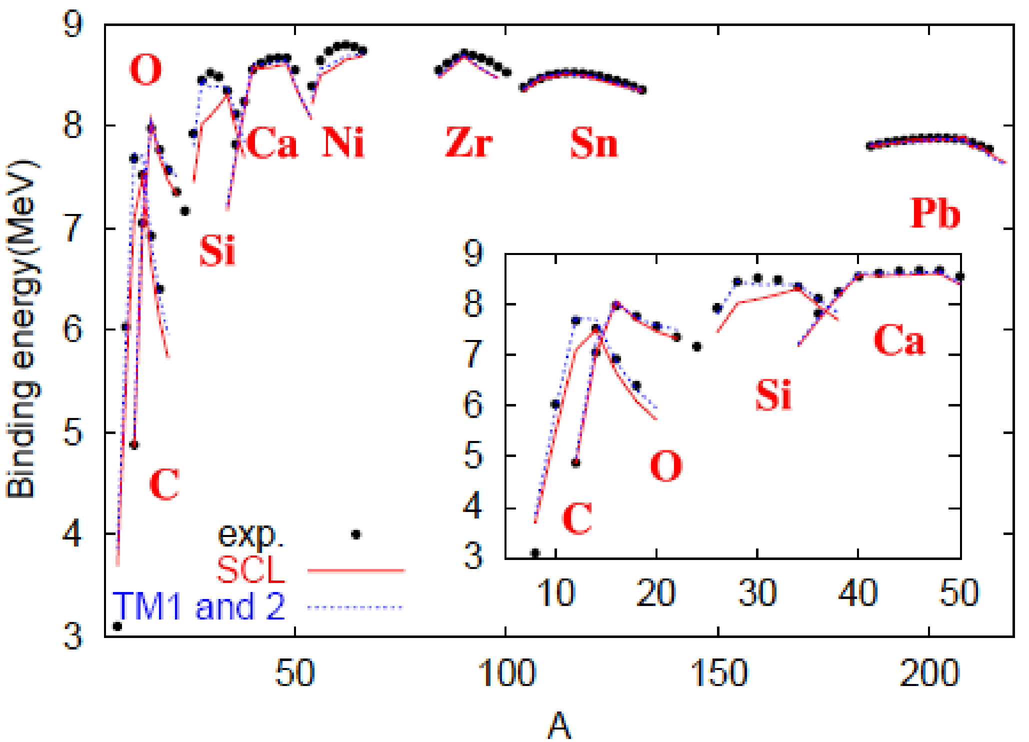
  - σ is shifted by f_π , and small explicit χ breaking term is added.

$$
\begin{aligned}
\mathcal{L} = \bar{\psi}\left(i\gamma^\mu\partial_\mu - \gamma^\mu V_\mu - M + g_\sigma\sigma\right)\psi + \mathcal{L}_\sigma^{(0)} + \mathcal{L}_\omega^{(0)} + \mathcal{L}_\rho^{(0)} \\
-U_\sigma + \frac{\lambda}{4}(\omega_\mu\omega^\mu)^2
\end{aligned}
$$

$$
U_\sigma(\sigma) = 2a\, f(\sigma/f_\pi), \quad f(x) = \frac{1}{2}\left[-\log(1+x) + x - \frac{x^2}{2}\right], \quad a = \frac{f_\pi^2}{2}\left(m_\sigma^2 - m_\pi^2\right)
$$

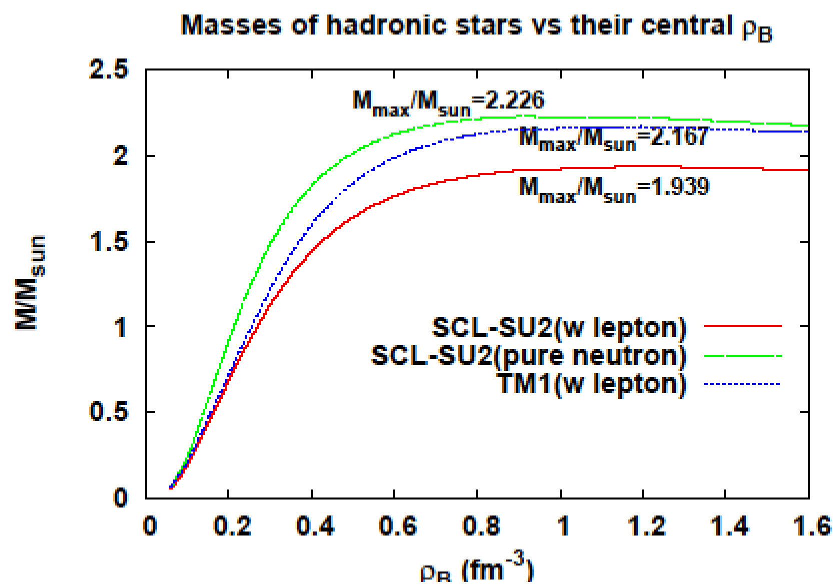
- **Nuclear Matter: By tuning λ, $g_{\omega N}$, m_σ, *EOS can be Soft !***
- **Finite Nuclei: By tuning $g_{\rho N}$, Global behavior of B.E. is reproduced, except for j-j closed nuclei (C, Si, Ni).**

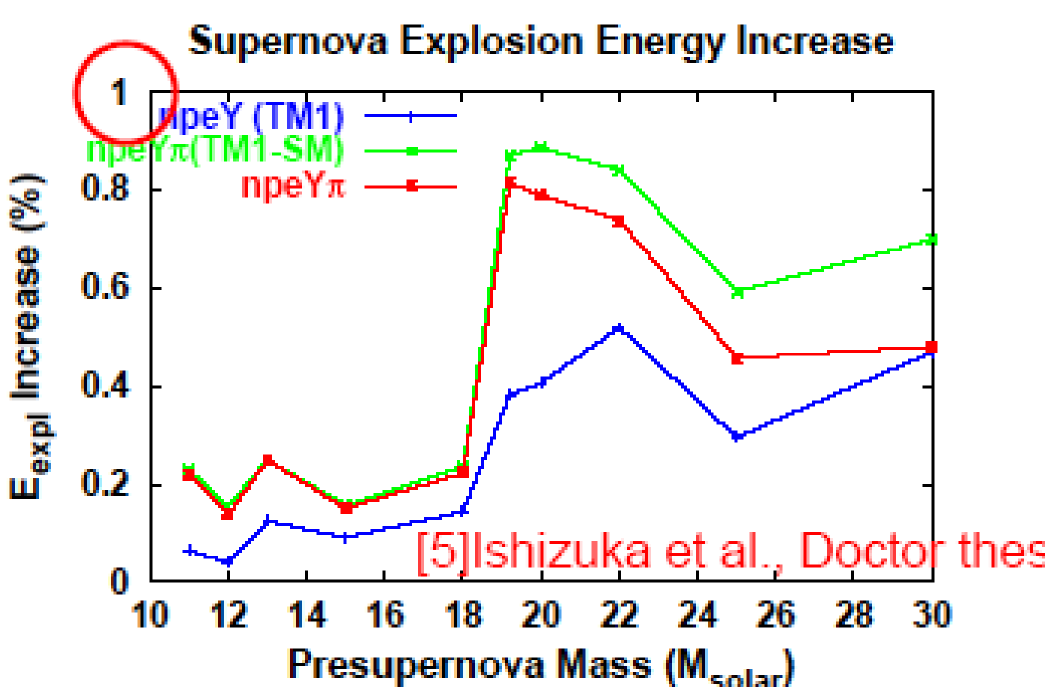


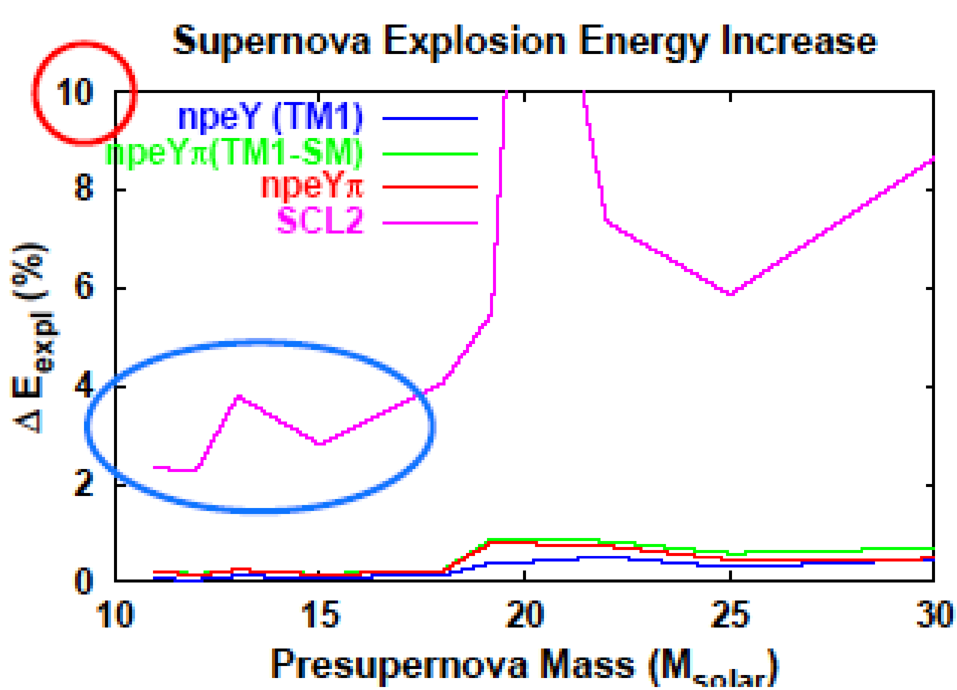


Astrophysical Applications

- **Neutron Stars**
  → **Supported up to 1.9 Msolar**
- **Supernova**
  → **Explision E. Enhancement of around 2-4 % compared to TM1**







Extention to Chiral SU(3)

- **Strong Coupling Limit LQCD guess**

- $$F_{eff} = b\, Tr(M^+ M) - a \log \det(M^+ M) - c_\sigma \sigma - c_\zeta \zeta + d(\det M^+ + \det M)$$

  Bosonization + Quark integral + Explicit breaking + $U_A(1)$ anomaly

$$M = \Sigma + i\Pi = diag(\sigma/\sqrt{2}, \sigma/\sqrt{2}, \zeta)\ (in\ MFA)$$

$$= a\left[2 f(\sigma/f_\pi) + \frac{1}{2} f(\zeta/f'_\zeta)\right] + \frac{m_\sigma^2}{2}\sigma^2 + \frac{m_\zeta^2}{2}\zeta^2 + \xi\sigma\zeta + const.$$

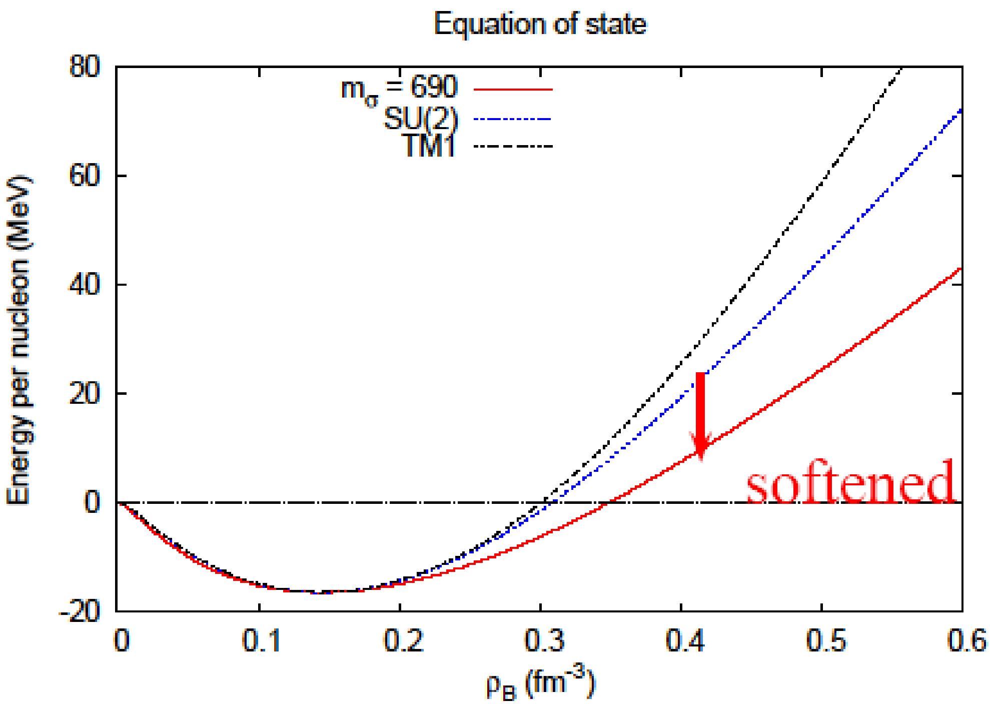
(after shifting $\sigma \to f_\pi + \sigma$, $\zeta \to f_\zeta + \zeta$)

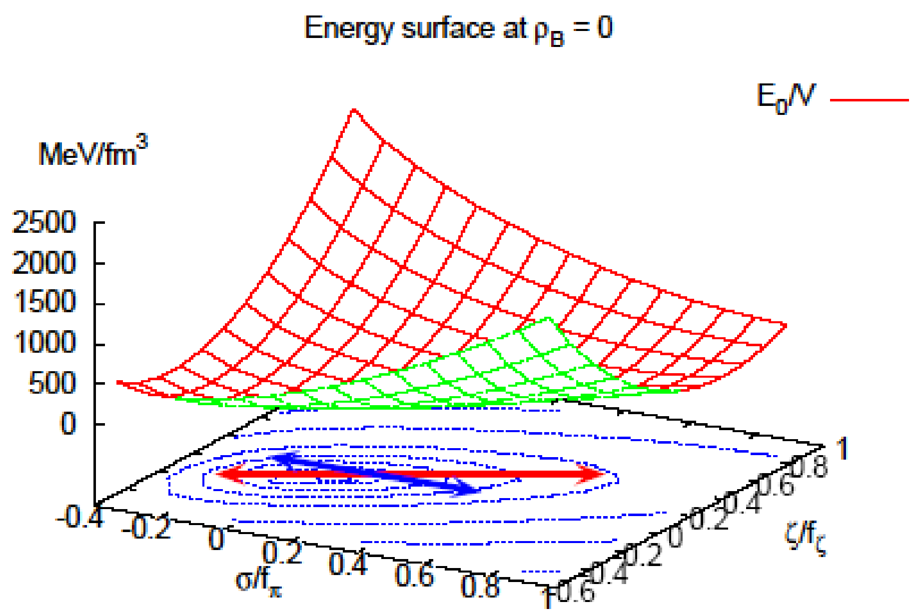
$$f(x) = \frac{1}{2}\left[-\log(1+x) + x + \frac{x^2}{2}\right], \quad a = \frac{f_\pi^2}{2}\left(m_\sigma^2 - m_\pi^2\right)$$

most of the parameters are determined to fit meson masses !
→ One parameter m_σ

Is it consistent with Nuclear Matter and Finite Nuclei ?

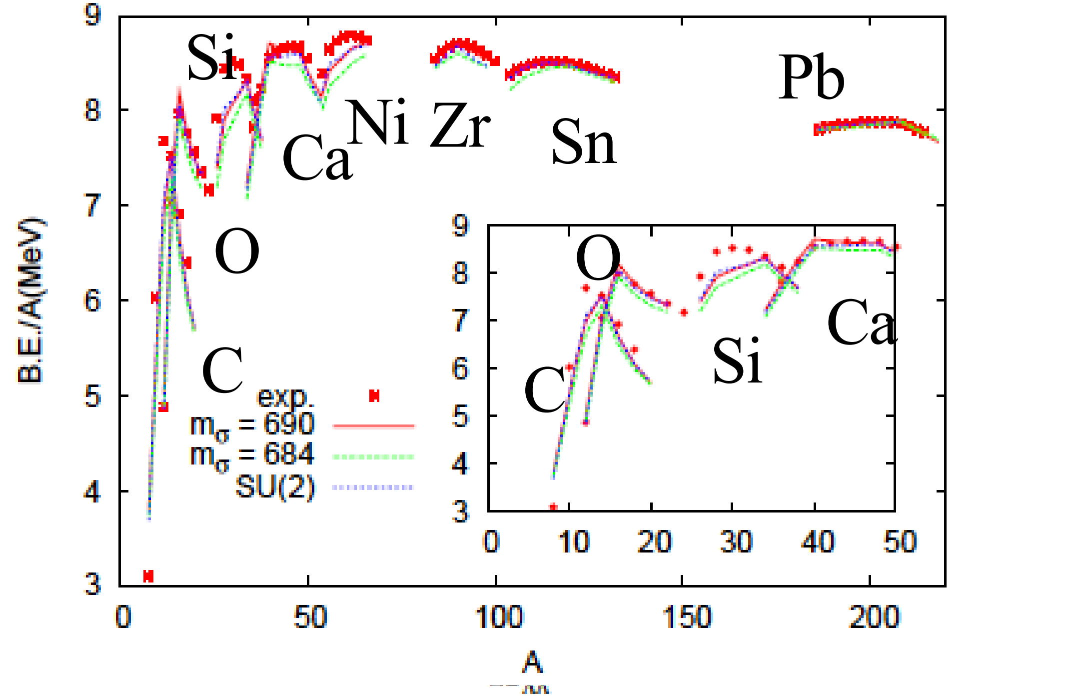
- **Soft EOS in Chiral SU(3) RMF**
  - **σ-ζ mixing → Evolution along σ-ζ valley**
  - **K= 216 MeV @ m_σ = 690 MeV**
    → Consistent with K=210 ± 30 MeV

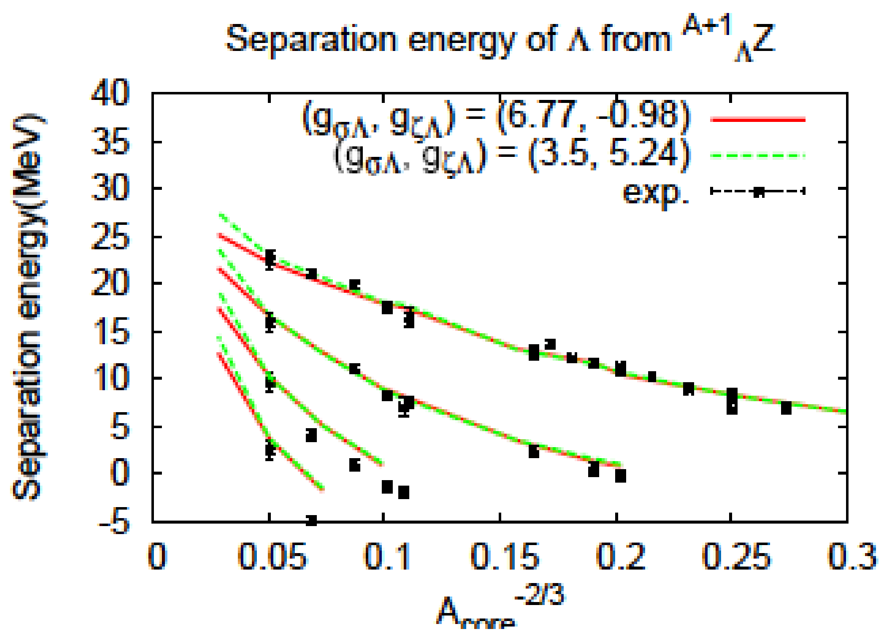


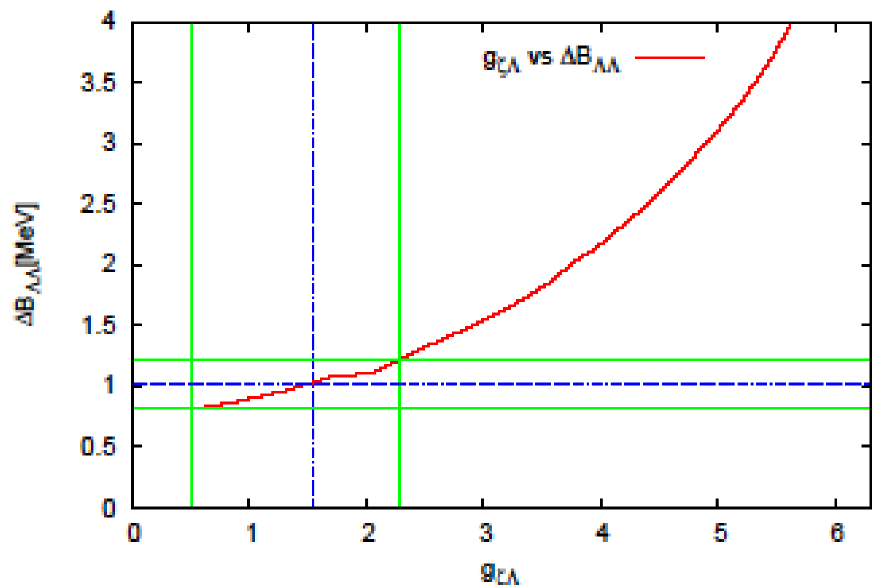


Finite Nuclei

- **Other Model Parameters**
  - $g_{\rho N}$ → Normal Nuclei
  - $(g_{\sigma\Lambda}, g_{\zeta\Lambda})$ → Single Λ Nuclei
  - $g_{\zeta\Lambda}$ → ${}^{6}_{\Lambda\Lambda}$He
    ($SU_V(3)$ is assumed for $g_{V\Lambda}$)







Summary

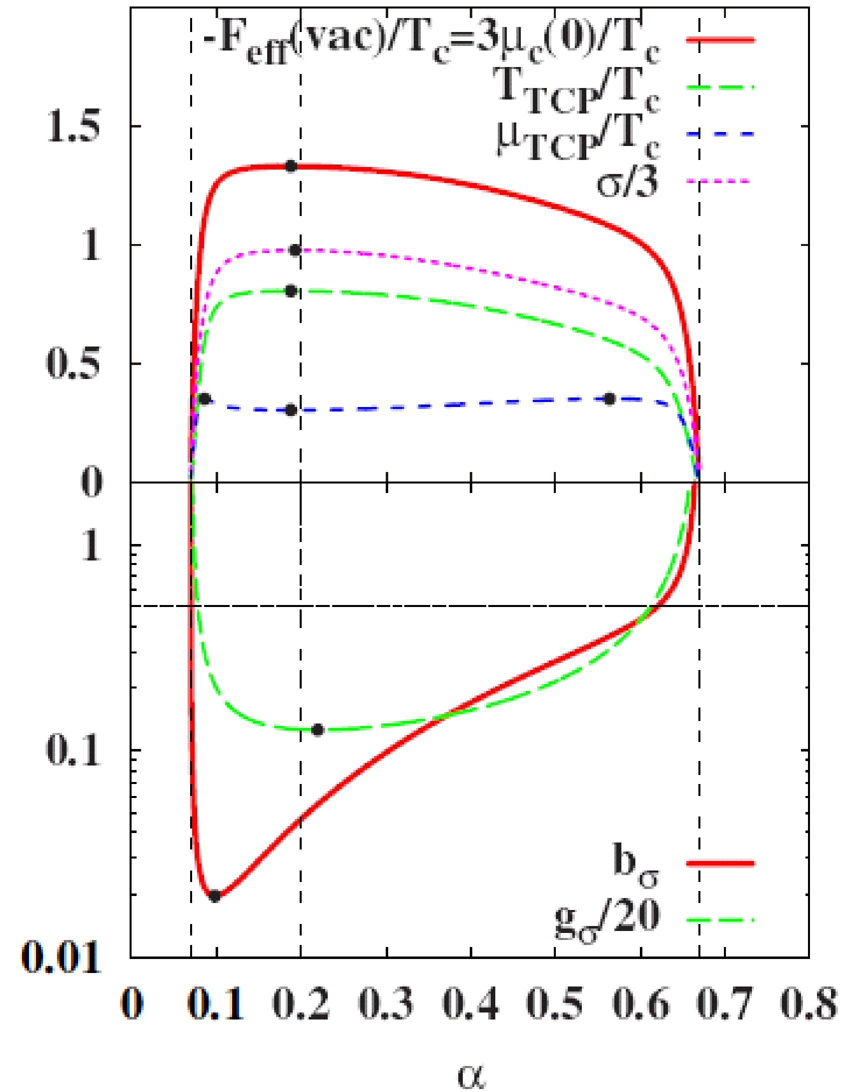
- **We obtain an analytical expression of effective free energy *at finite T and finite μ* with *baryonic composite* effects in the strong coupling limit of lattice QCD for color SU(3).**
  - *MFA, QG integral, 1/d expansion (NLO, O(1/√d)), bosonization with diquarks and baryon potential field using* $(\bar{b}b)^2=0$ *, Linear approx.,* ***zero diquark cond.(Color Angle Average), variational parameter choice***
- **Baryonic action is found to result in *Free Energy Gain* and *Extension of Hadron Phase to Larger μ* *by around 30 %.***
  - *Problem: Too small* μ_c/T_c *in the Strong Coupling Limit.*
- **Strong Coupling Limit is useful to understand Dense Matter**
  - *SCL gives a qualitative insight.*
  - *1/g² correction seems to work well (Do not believe us yet ...)*
  - *Application to chiral RMF (K. Tsubakihara, AO, nucl-th/0607046)*

Backups

Parameter Choice

- **In bosonization, two parameters (γ and α) are introduced through identities.**
  - **Major effects**
    → Modify the energy scale
  - **Minor effects**
    → Controls the higher order potential terms

→ *We have fixed them to minimize F_{eff}/T_c at vacuum*



Baryon Integral

- **Baryon integral can be evaluated in an almost analytic way !**

$$F_{\mathrm{eff}}^{(b)}(g_\omega \omega) = \frac{1}{\beta L^3} \log \mathrm{Det}\left[1 + g_\omega \omega V_B\right]$$

$$\simeq \frac{-a_0^{(b)}/2}{(4\pi\Lambda^3/3)} \int_0^\Lambda 4\pi k^2 dk \log\left[1 + \frac{g_\omega^2 \omega^2 k^2}{16}\right]$$

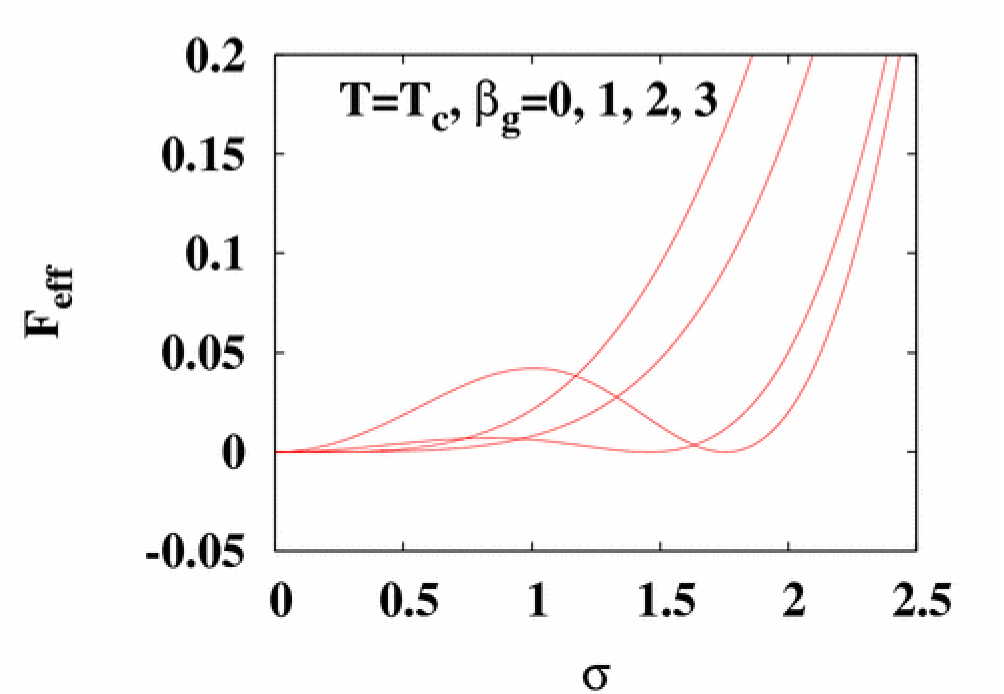
$$= -a_0^{(b)} f^{(b)}\left(\frac{g_\omega \omega \Lambda}{4}\right)$$

$$f^{(b)}(x) = \frac{1}{2}\log(1+x^2) - \frac{1}{x^3}\left[\arctan x - x + \frac{x^3}{3}\right]$$

$$a_0^{(b)} = 1.0055 \ , \ \ \Lambda = 1.01502 \times \pi/2$$

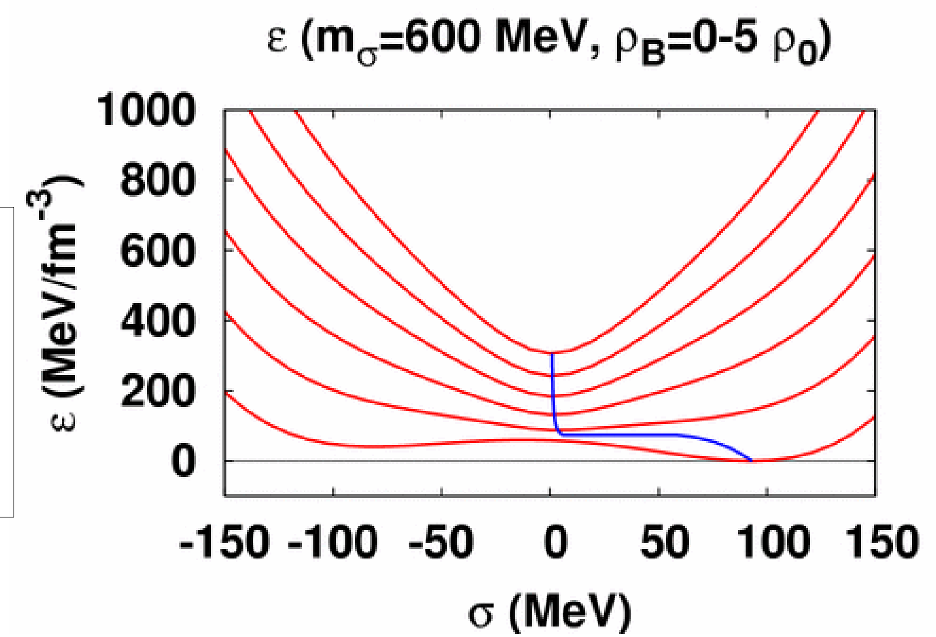
Disappearance of TCP

- Tri-Critical point disappears at around $\beta_g \sim 1.4$
  → 1st order phase transition even at μ=0.
  - One species of staggered fermion in the chiral limit ~ mass less quark flavor N_f=4
  - Need quarter-root treatment or Wilson fermion with finite s-quark mass
  - Reason: Space-like plaquett enhances σ-quark coupling at large condensate ???



Problems in RMF with Chiral Symmetry

- **Sudden Change of <σ>**

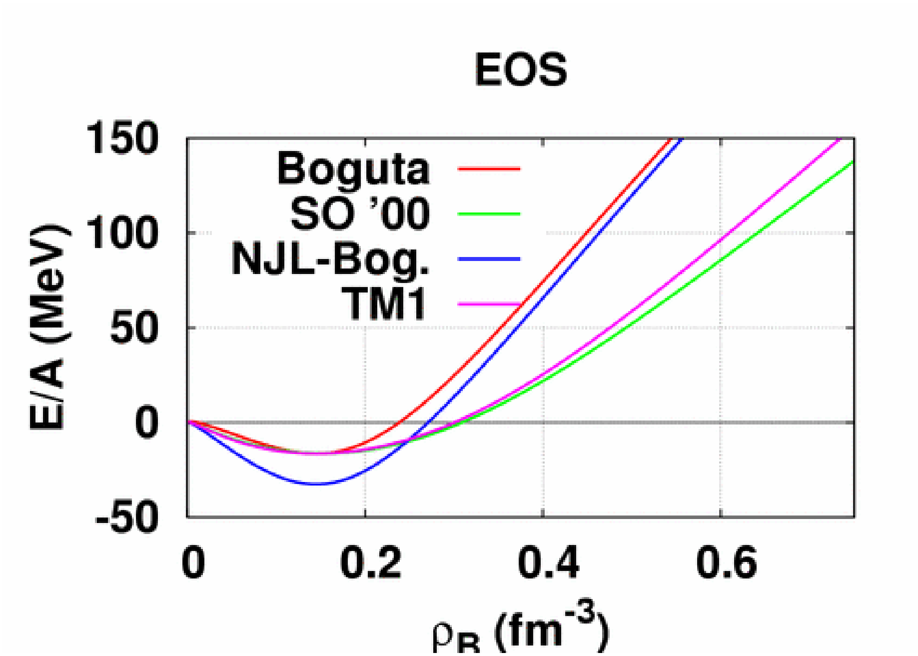


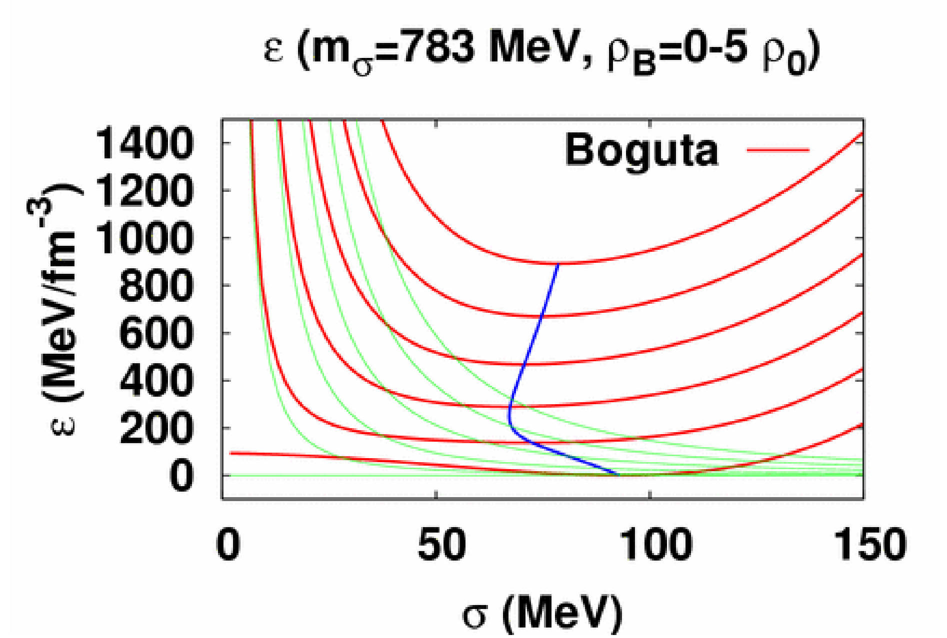
- **σ ω Coupling**

$$L_{\omega\sigma}=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}C_{\sigma\omega}\sigma^2\omega^2-g_\omega\overline{N}\gamma_\mu\omega^\mu N$$

$$\omega=g_\omega\rho_B/C_{\sigma\omega}\sigma^2 \quad \rightarrow \quad V_{\sigma\omega}=\frac{g_\omega^2\rho_B^2}{2C_{\sigma\omega}\sigma^2}$$

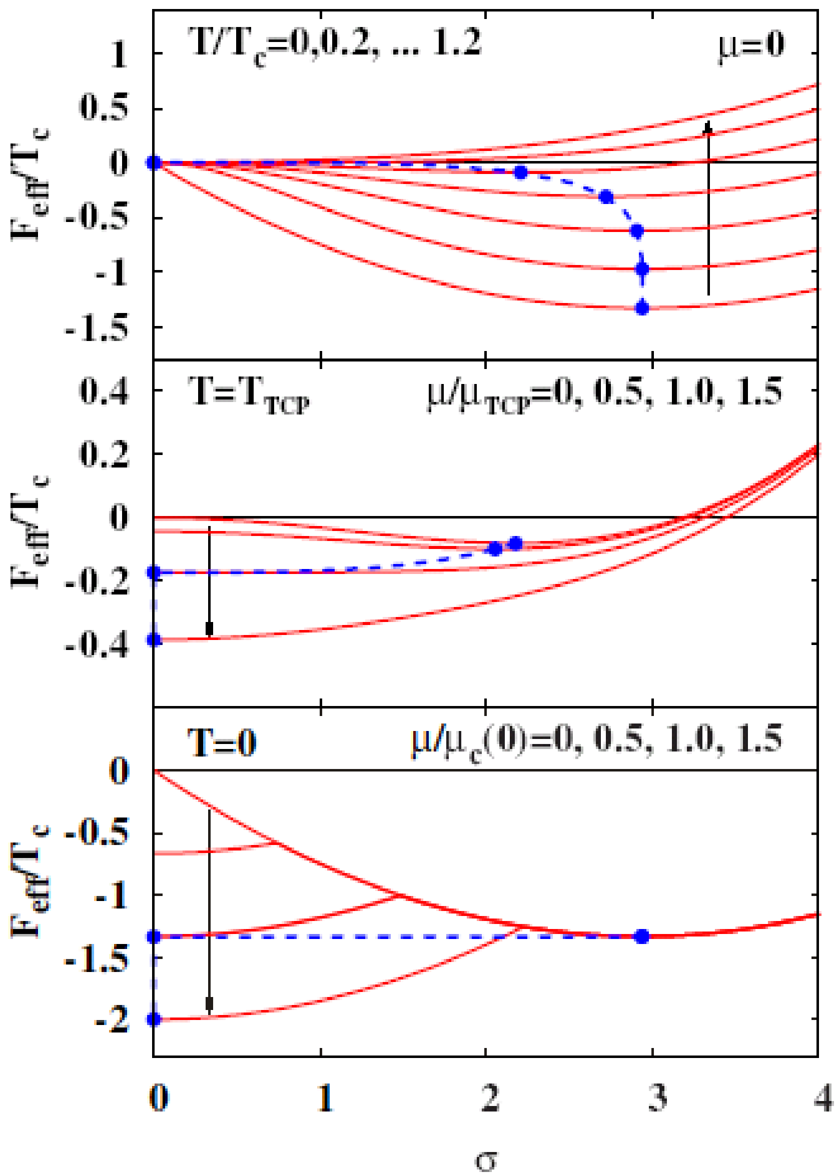
- **Stiff EOS**



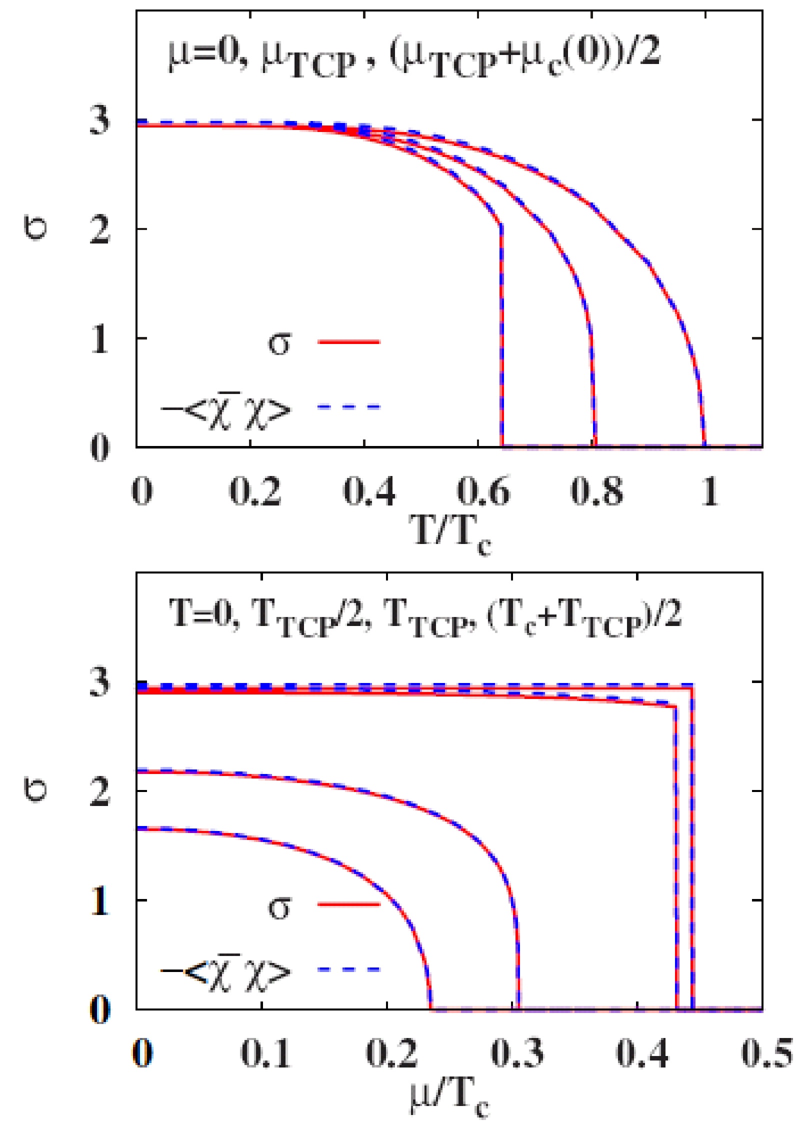


Figures

- **Energy surface**



- **Validity of "Linear" Approx.**



Part IV: Collective flows in heavy-ion collisions from AGS to RHIC energies

Akira Ohnishi @ Hokkaido Univ.
in Collaboration with
K. Yoshino (Hokkaido U.), M.Isse(Hokkaido U.→Osaka U.),
T.Hirano (U-Tokyo), Y.Nara (Frankfurt), P.K.Sahu (IOP, India)

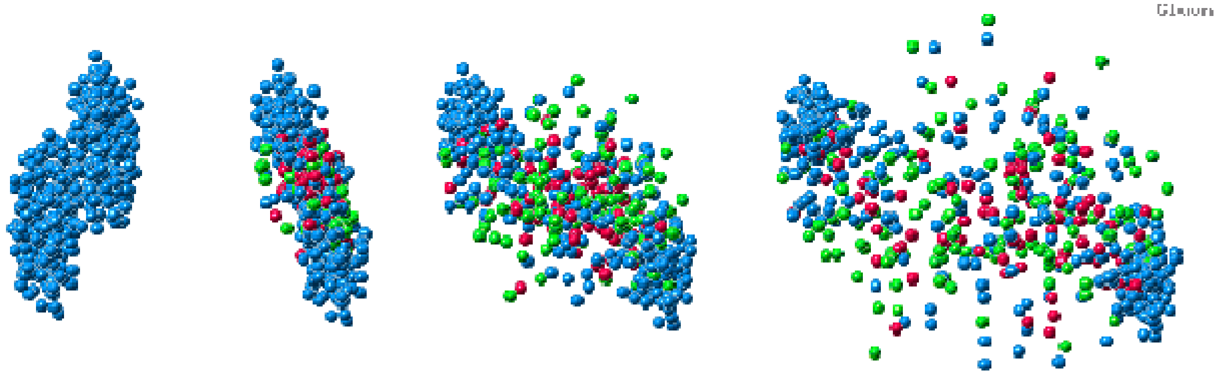
- **Collective Flows from AGS to SPS Energies**
  Isse, AO, Otuka, Sahu, Nara, Phys.Rev. C 72 (2005), 064908
- **Hydro. vs Cascade Comparison at RHIC**
  Hirano, Isse, Nara, AO, Yoshino, Phys. Rev. C 72(2005), 041901
  Sahu, Isse, Otuka, AO, Pramana, 2006, in press.
  Isse, Ph.D Thesis

Collective Flows at AGS and SPS Energies

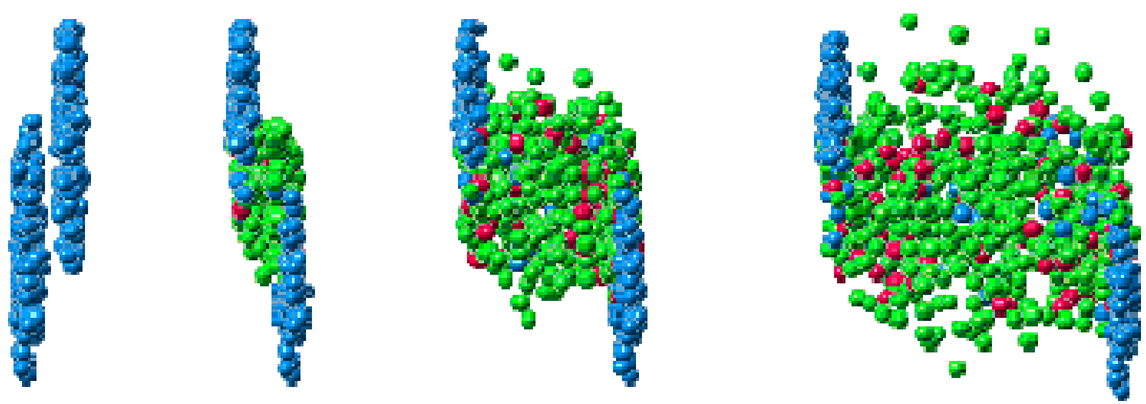
HIC at AGS and SPS Energies

JAMming on the Web, linked from http://www.jcprg.org/

AGS



SPS



What is Collective Flow ?

(Directed) Flow (dP_X/dY)

Stiffness (Low E)
+ Time Scale (High E)

Elliptic Flow (V_2)

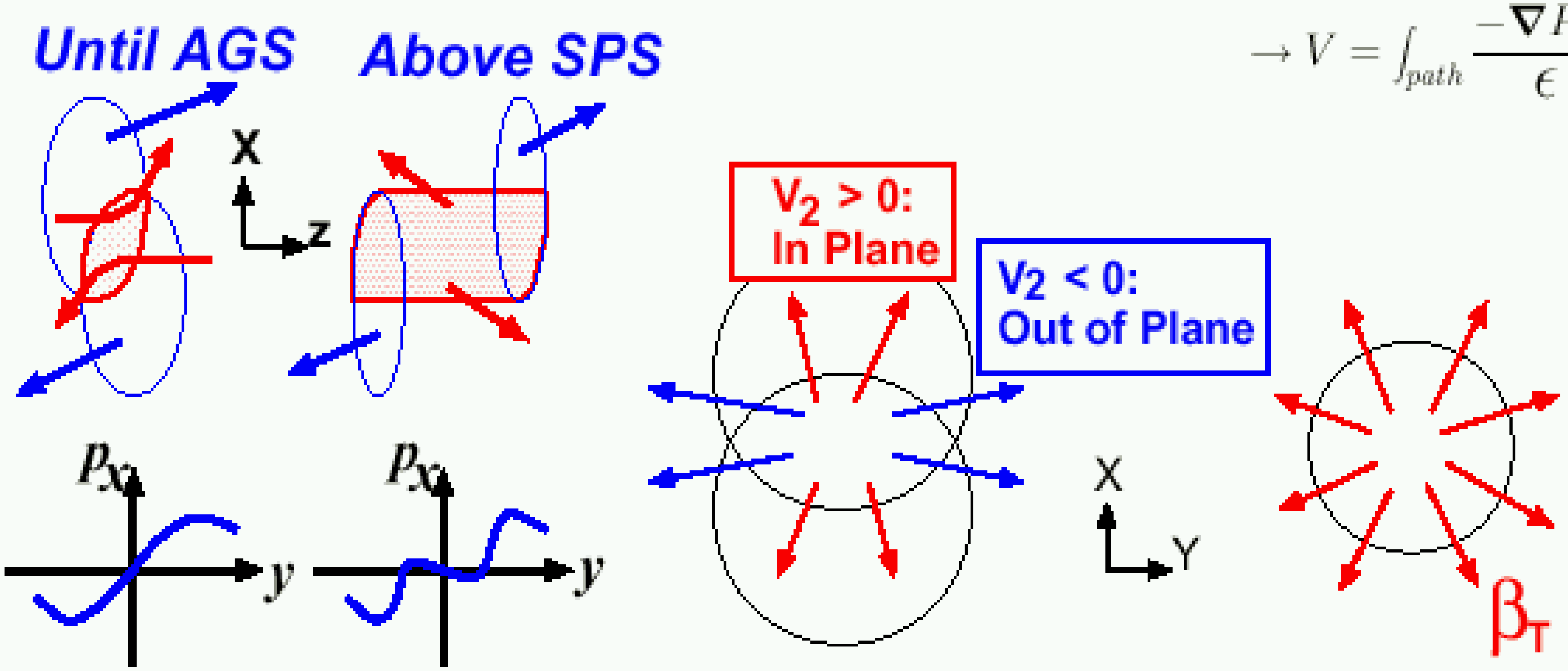
Thermalization
& Pressure Gradient

Radial Flow (β_T)

Pressure History

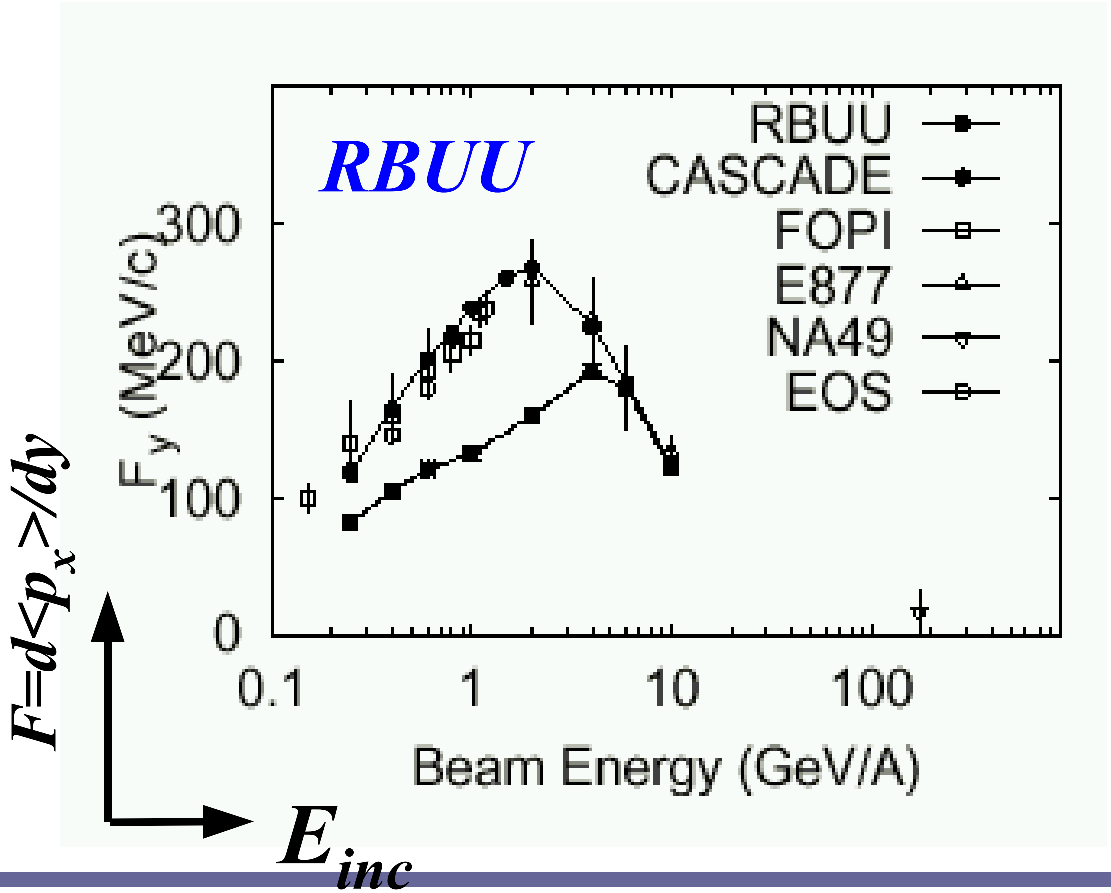
$$\epsilon \frac{D\mathbf{V}}{Dt} = -\nabla P$$

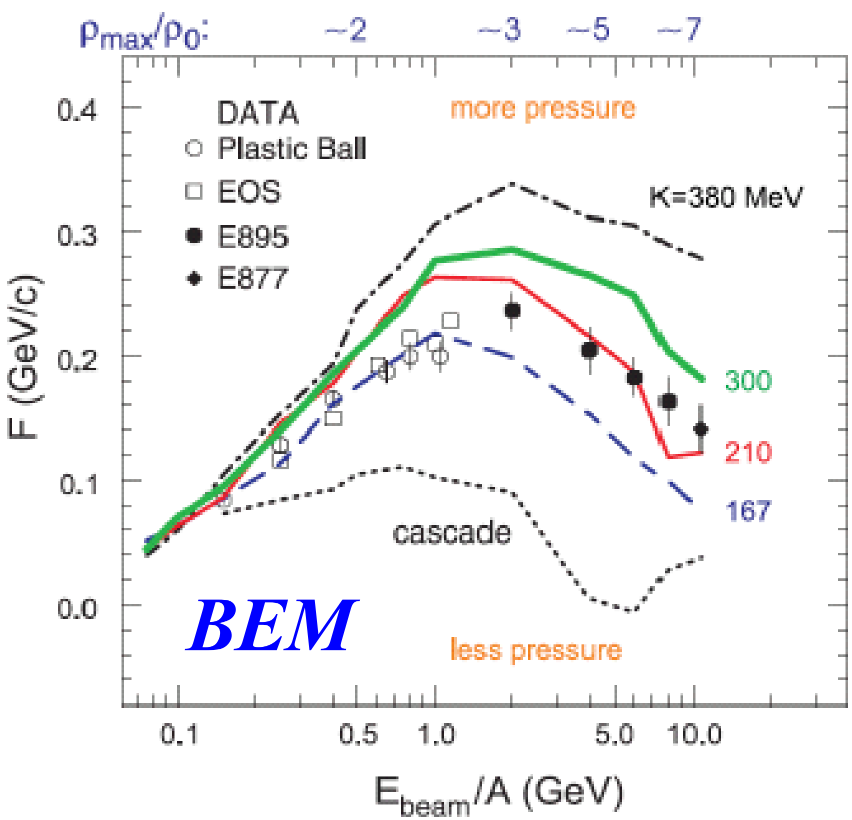
$$\to V = \int_{path} \frac{-\nabla P dt}{\epsilon}$$



Side Flow at AGS Energies

- **Relativistic BUU (RBUU) model:** ***K ~ 300 MeV***
  (Sahu, Cassing, Mosel, AO, Nucl. Phys. A672 (2000), 376.)
- **Boltzmann Equation Model (BEM):** ***K=167~210 MeV***
  (P. Danielewicz, R. Lacey, W.G. Lynch, Science 298(2002), 1592.)

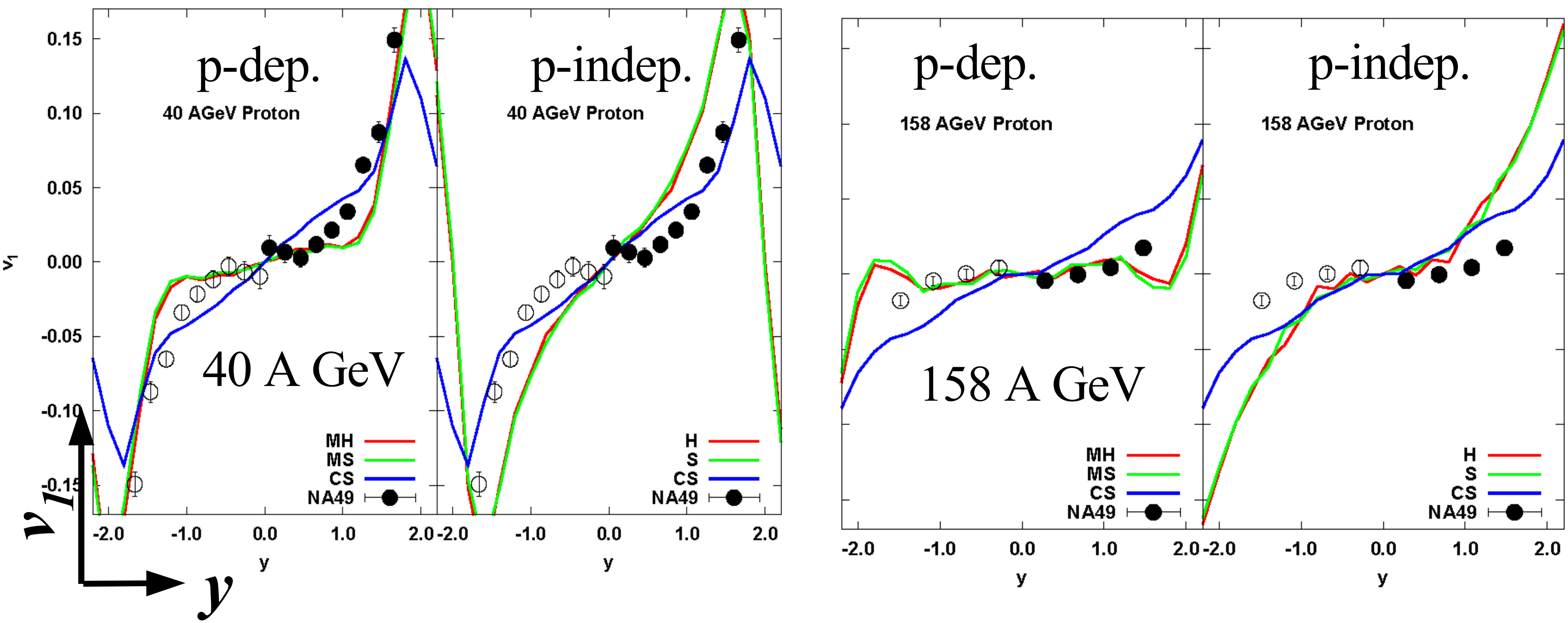




Directed flow v_1 at SPS

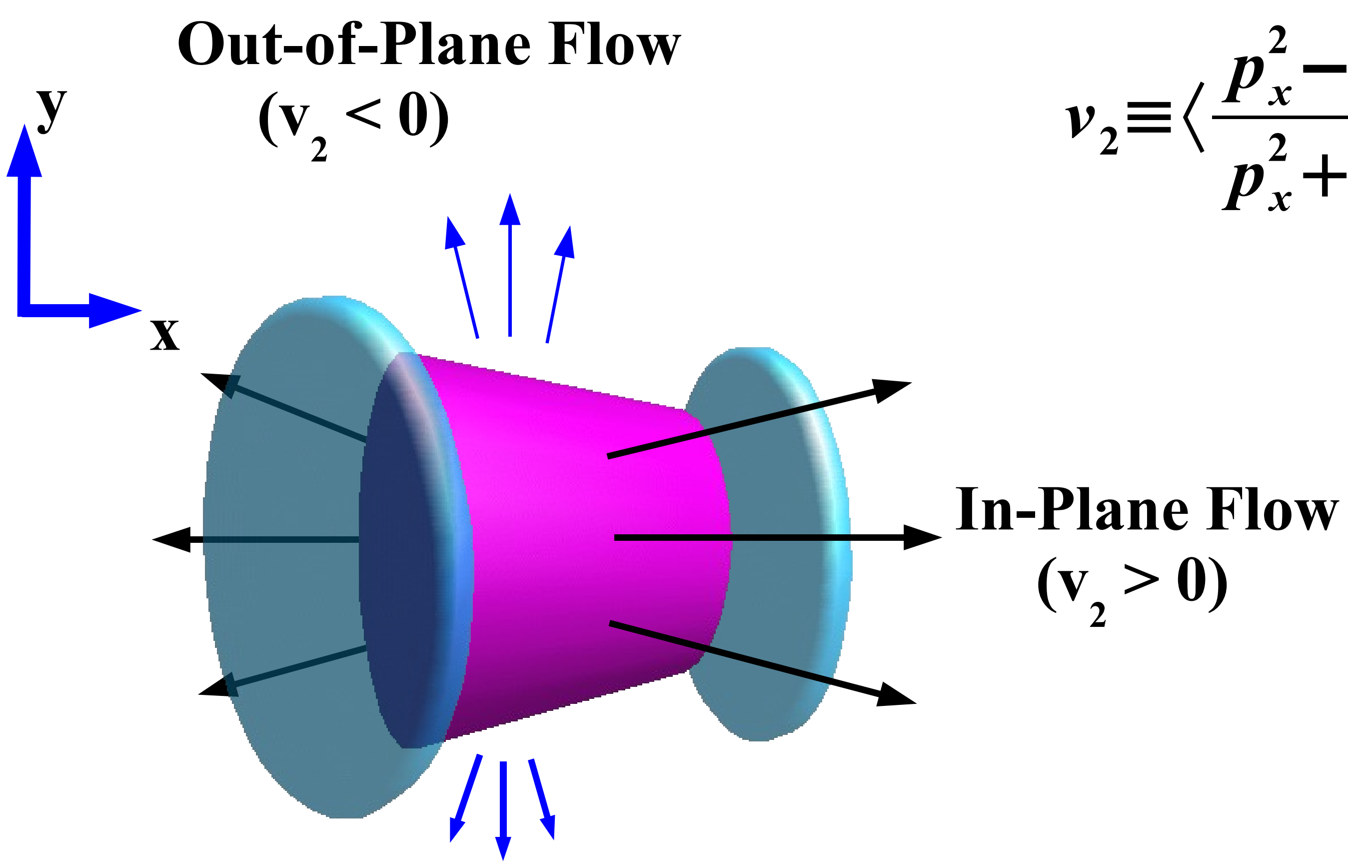
Isse, AO, Otuka, Sahu, Nara, PRC 72 (2005), 064908

- **JAM-RQMD/S**
  - **p-dep. (indep.) MF suppresses (enhances) v_1.** $v_1 = \langle \cos\phi \rangle = \langle p_x / p_T \rangle$
  - **"Wiggle" behavior appears with p-dep. MF at 158 A GeV.**



Elliptic Flow

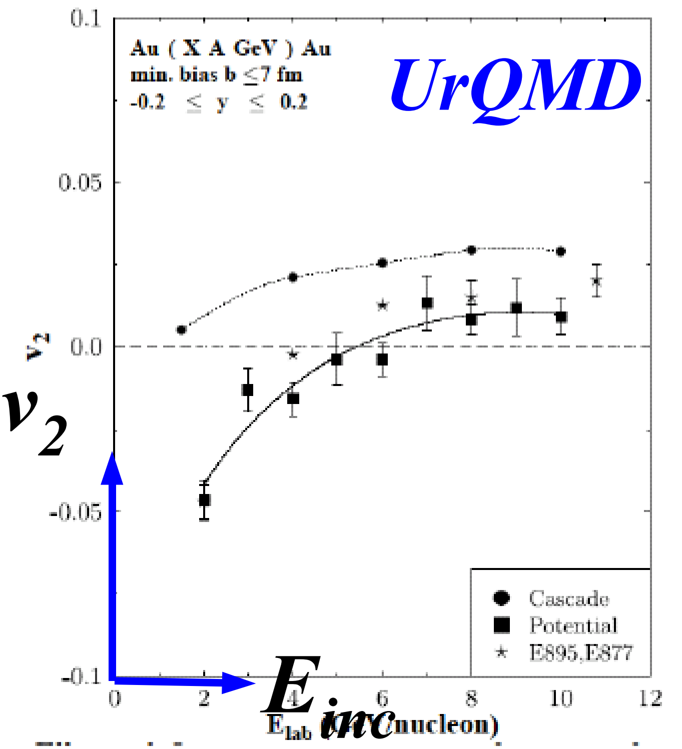
- What is Elliptic Flow ? → Anisotropy in P space
- Hydrodynamical Picture
  - Sensitive to the Pressure Anisotropy in the Early Stage
  - Early Thermalization is Required for Large V2

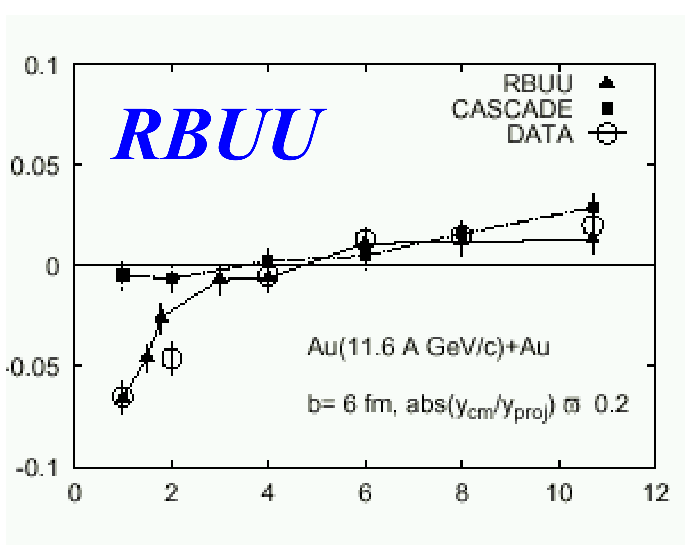


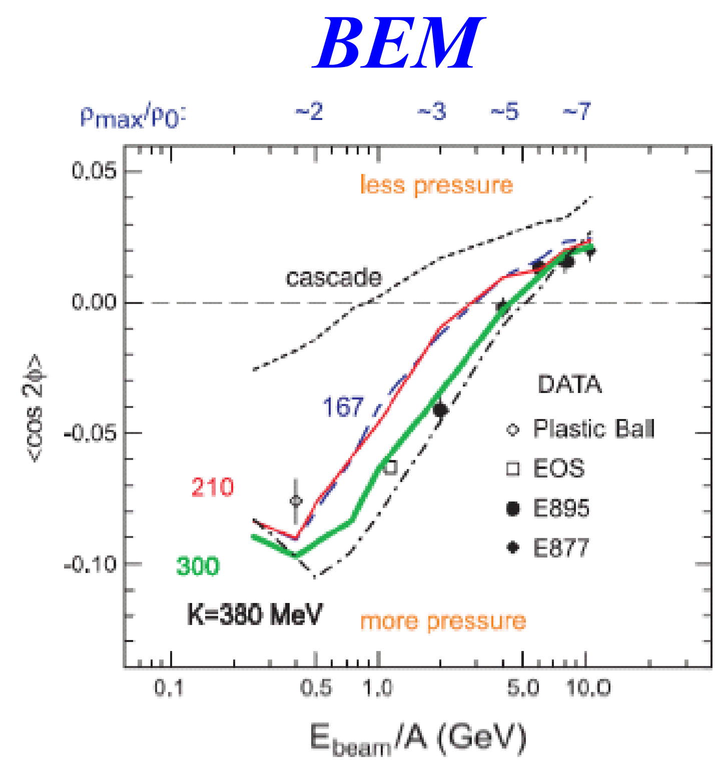
$$v_2 \equiv \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle = \langle \cos 2\phi \rangle$$

Elliptic Flow at AGS

- **Strong Squeezing Effects at low E (2-4 A GeV)**
  - **UrQMD: Hard EOS (S.Soff et al., nucl-th/9903061)**
  - **RBUU (Sahu-Cassing-Mosel-AO, 2000): K ~ 300 MeV**
  - **BEM(Danielewicz2002): K = 167 → 300 MeV**

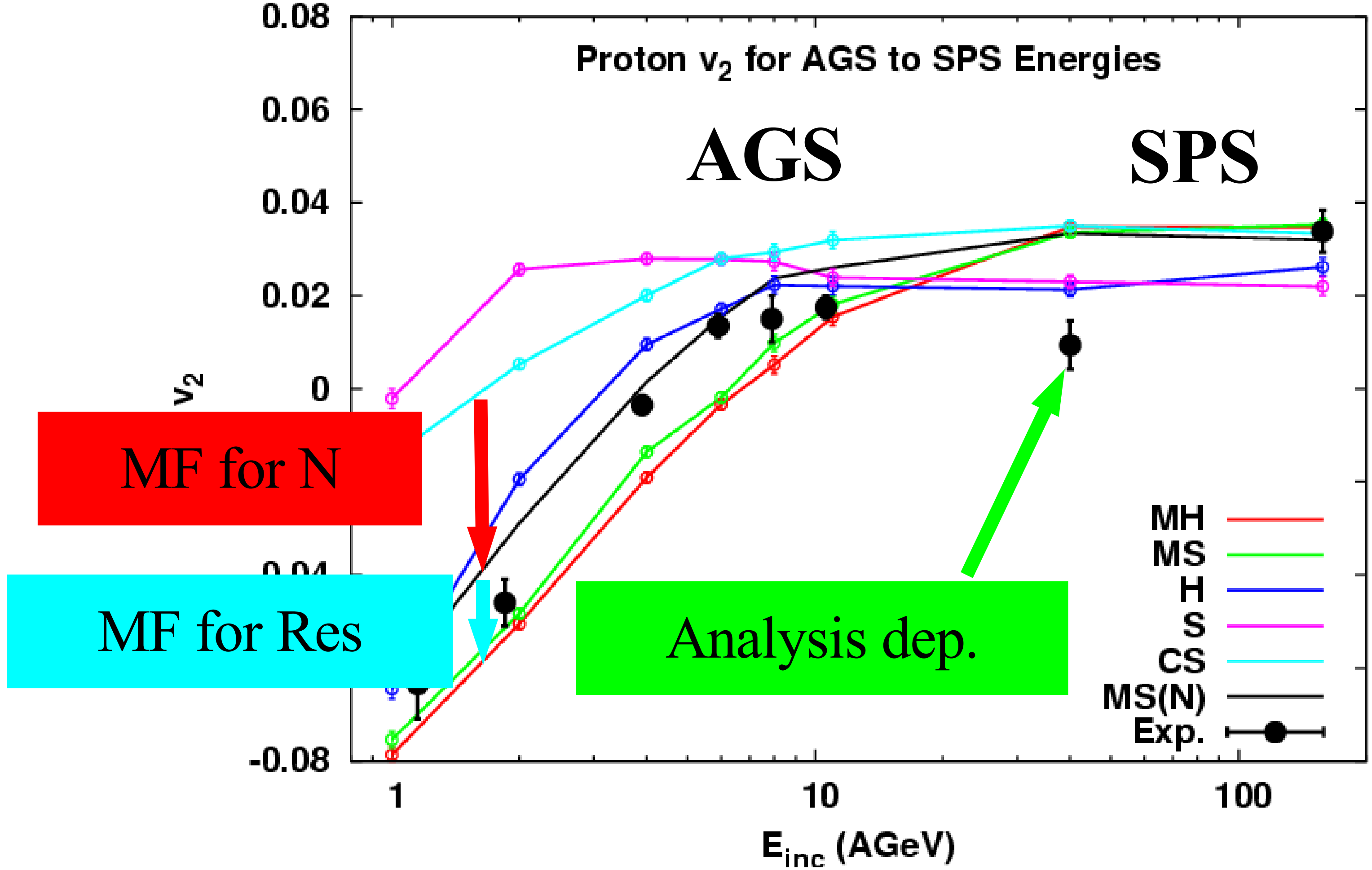






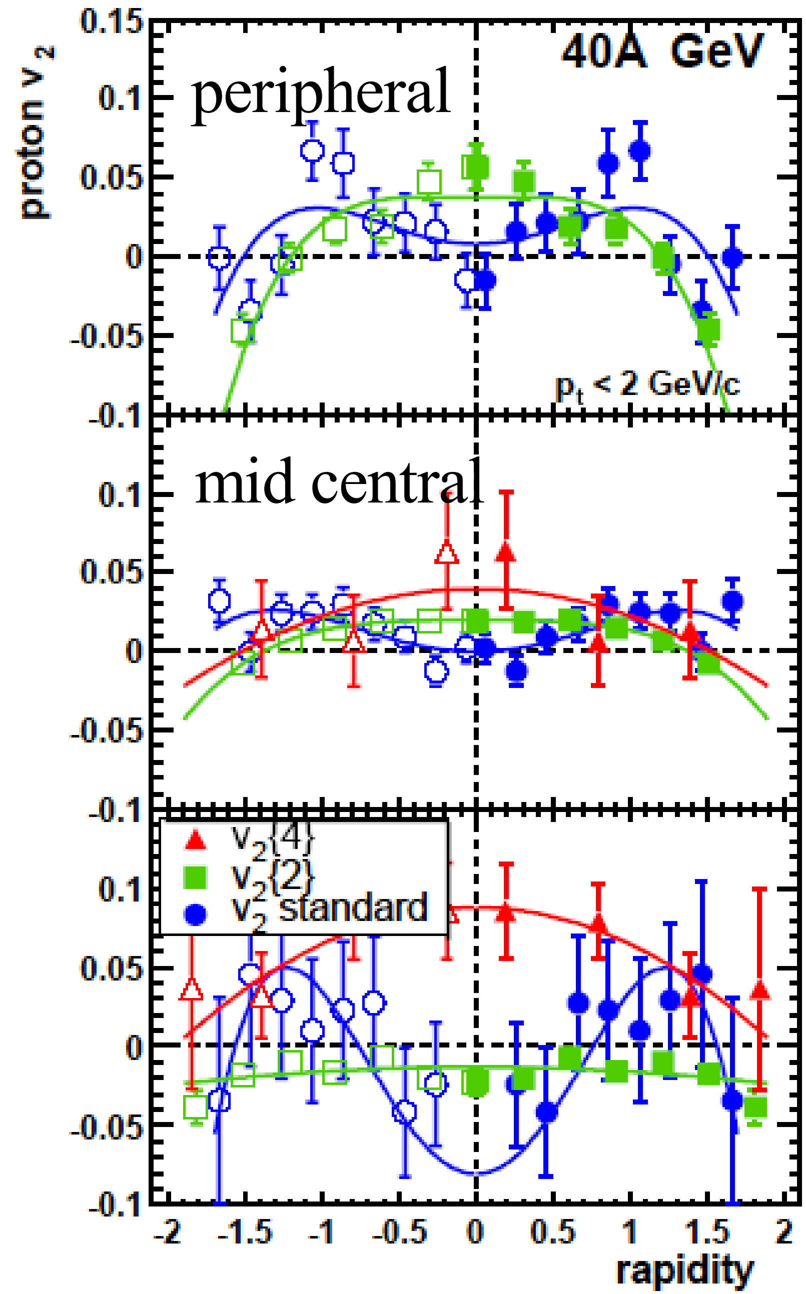
Elliptic Flow from AGS to SPS

- **JAM-MF with p dep. MF explains proton v2 at 1-158 A GeV**
  - **v2 is not very sensitive to K (incompressibility)**
  - **Data lies between MS(B) and MS(N)**



Dip of V_2 at 40 A GeV: Phase Transition ?

- Dip of V_2 at 40 A GeV may be a signal of QCD phase transition at high baryon density. (Cassing et al.)
- However, the data is too sensitive to the way of the analysis (reaction plane/two particle correlation).
  - We have to wait for better data.



Flow and EOS; to be continued

- **In addition to the ambiguities in in-medium cross sections, Res.-Res. cross sections, we have model dependence.**
  - **RBUU** *(e.g. Sahu, Cassing, Mosel, AO, Nucl. Phys. A672 (2000), 376.)*
    - **In RMF, Strong cut-off for meson-N coupling in RMF → Smaller EOS dep.**
  - **Scalar potential interpretation in BUU**
    Larionov,Cassing,Greiner,Mosel, PRC62,064611('00), Danielewicz, NPA673,375('00)

$$\varepsilon(\boldsymbol{p},\rho) = \sqrt{[m + U_s(\boldsymbol{p},\rho)]^2 + \boldsymbol{p}^2} \;\; = \sqrt{m^2 + \boldsymbol{p}^2} + U(\boldsymbol{p},\rho)$$

    - **Due to the Scalar potential nature, EOS dependence is smaller.**
  - **Scalar/Vector Combination** *Danielewicz, Lacey, Lynch, Science 298('02), 1592*

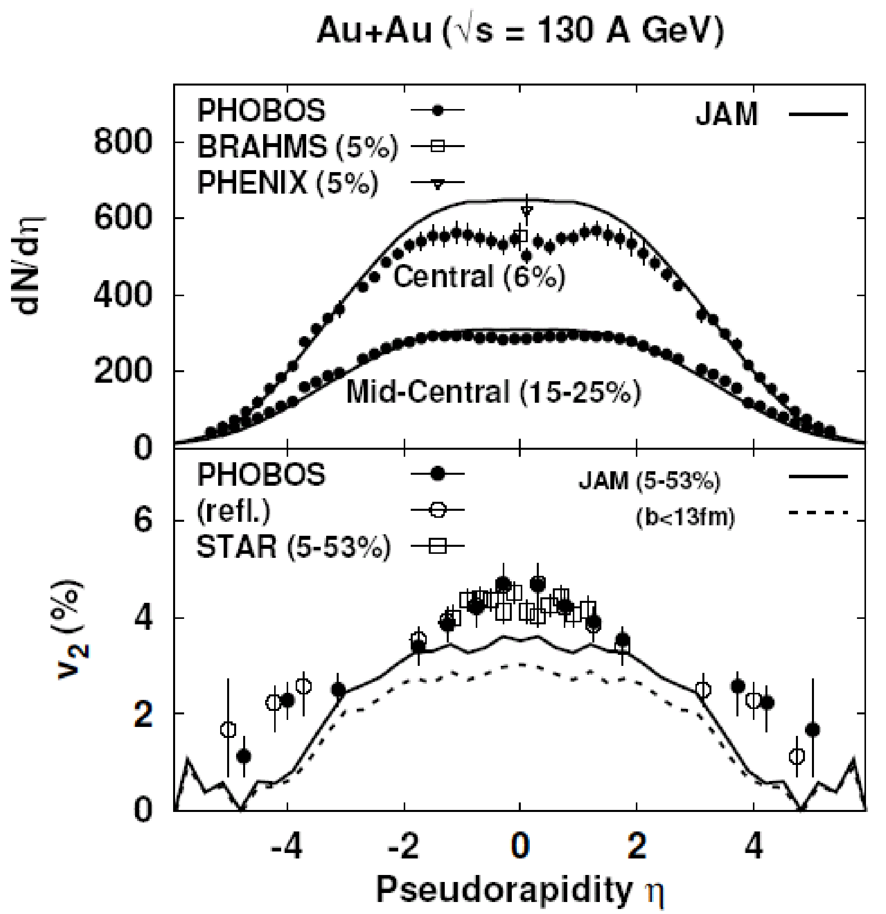
$$\varepsilon(p,\rho) = m + \int_0^p dp' v^*(p',\rho) + \widetilde{U}(\rho), \quad v^*(p,\rho) = \frac{p}{\sqrt{p^2 + [m^*(p,\rho)]^2}}.$$

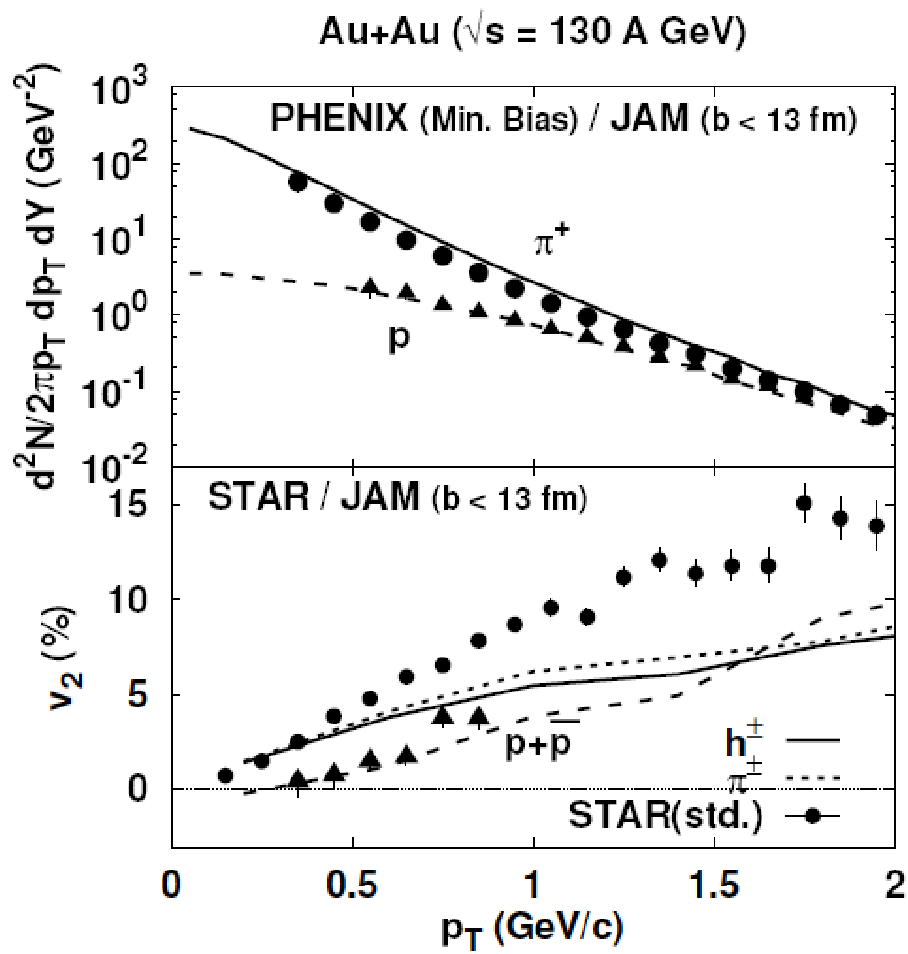
    - **Relatively Strong EOS dependence even at high energy**
  - **JAM-RQMD/S** *Isse, AO, Otuka, Sahu, Nara, PRC 72 (2005), 064908*
    - **Similar to the Scalar model BUU**

Elliptic Flow @ RHIC

Elliptic Flow in Hadron-String Cascade (I)

- **Hadron-String Cascade (JAM) @ RHIC**
  - **Hadron Yield is reasonably explained up to 2 GeV/c (10-20 % error)**
  - **v2 is underestimated (20-30 % (integrated), 50 % (p_T > 1 GeV)**

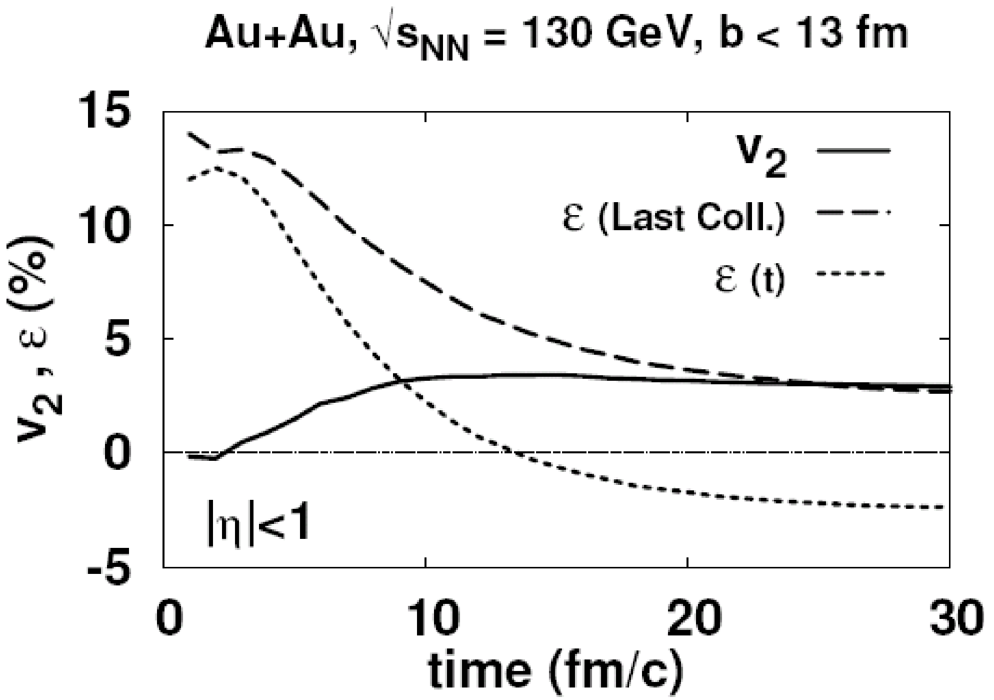


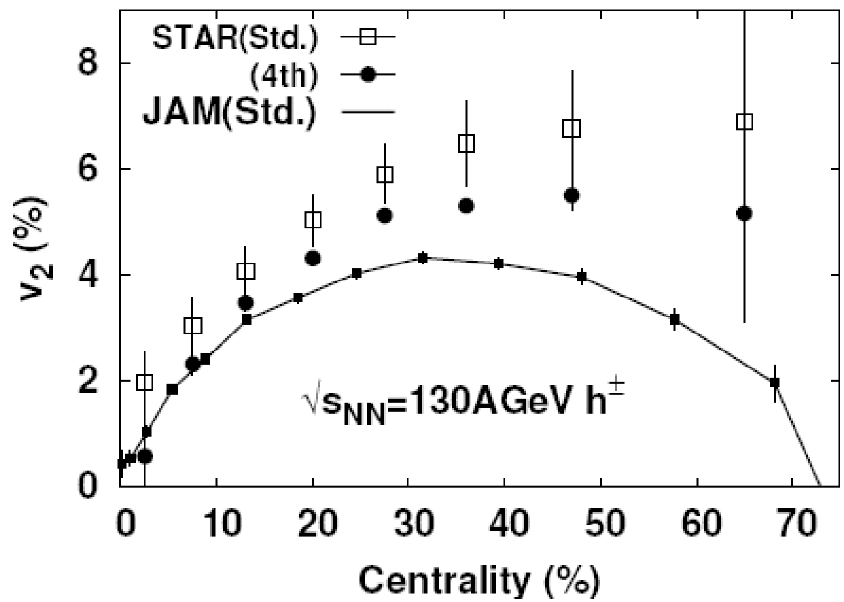


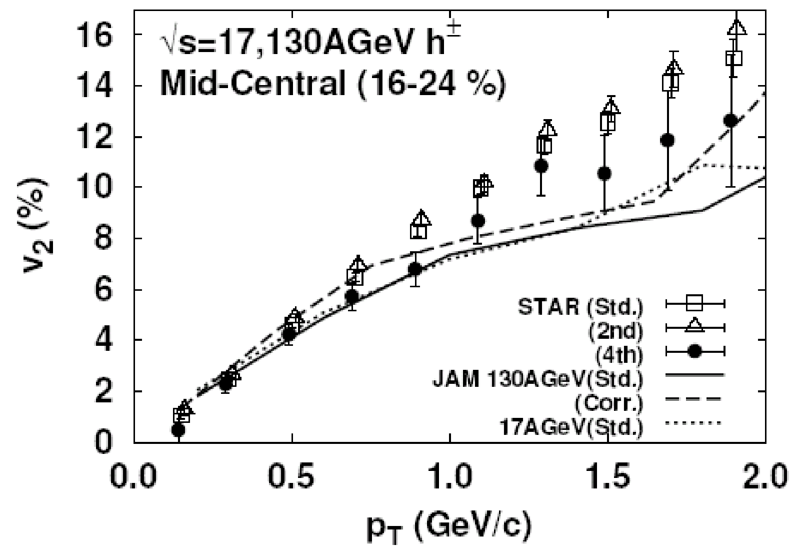
Elliptic Flow in Hadron-String Cascade (II)

- **Why do we underestimate v2 in Hadron-String Cascade ?**
  - **v2 growth time is long (~ 10 fm/c), due to hadron formation time (τ~ 1 fm/c).**
    → much longer than hydro

Sahu-Isse-AO-Otuka-Phatak 2006







Results of Parton Cascade

- **Unexpectedly high parton cross sections of σ =5-6 mb have to be assumed in parton cascades in order to reproduce the elliptic flow.**

ZI-WEI LIN AND C. M. KO PHYSICAL REVIEW C **65** 034904

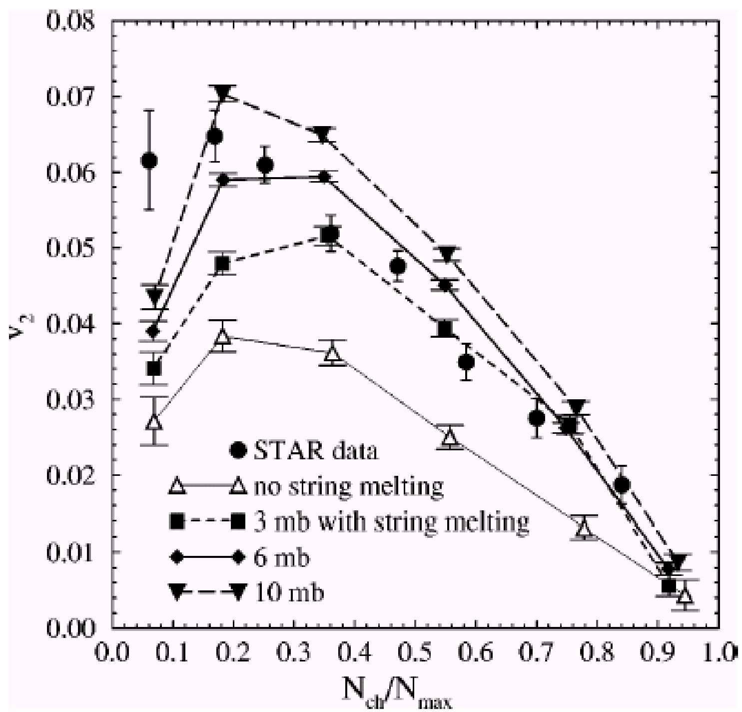


FIG. 3. Impact parameter dependence of elliptic flow at 130 *A* GeV. The data from the STAR collaboration [7] are shown by filled circles, while the theoretical results for different partonic dynamics are given by curves.

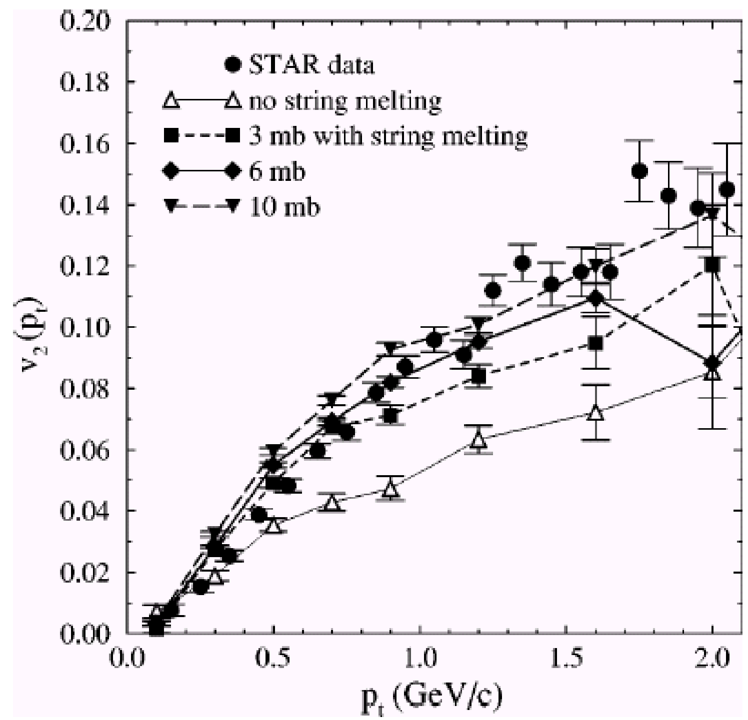


FIG. 4. Transverse momentum dependence of elliptic flow at 130 *A* GeV. Circles are the STAR data for minimum-bias Au+Au collisions [7], and curves represent the minimum-bias results for charged particles within $\eta \in (-1.3,1.3)$ from the AMPT model.

Hirano @ QM2006

Initial Conditions in Hydro

Glauber-BGK type

[Reference Initial Condition]

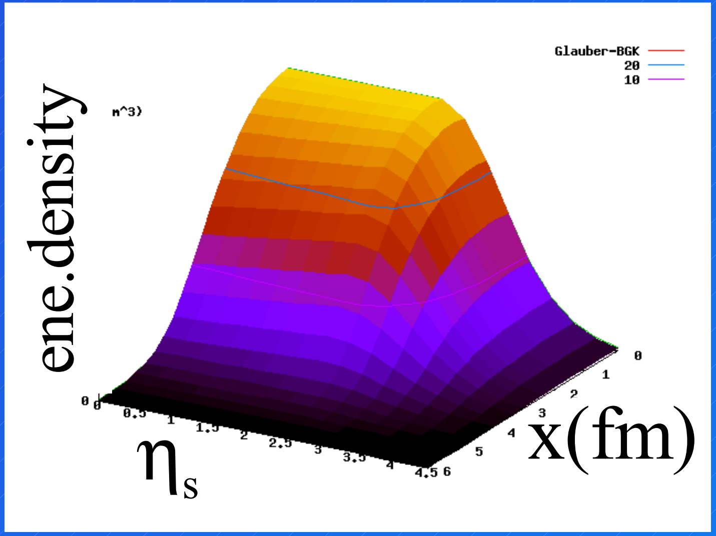
<u>Transverse profile</u>:

Entropy density

$$\propto a\rho_{\mathrm{part}} + b\rho_{\mathrm{coll}}$$

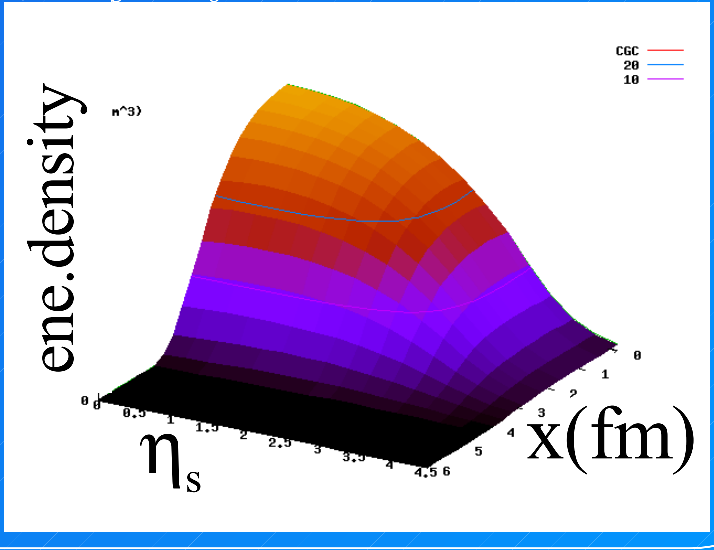
<u>Longitudinal Profile</u>:

Brodsky-Gunion-Kuhn triangle



Color Glass Condensate

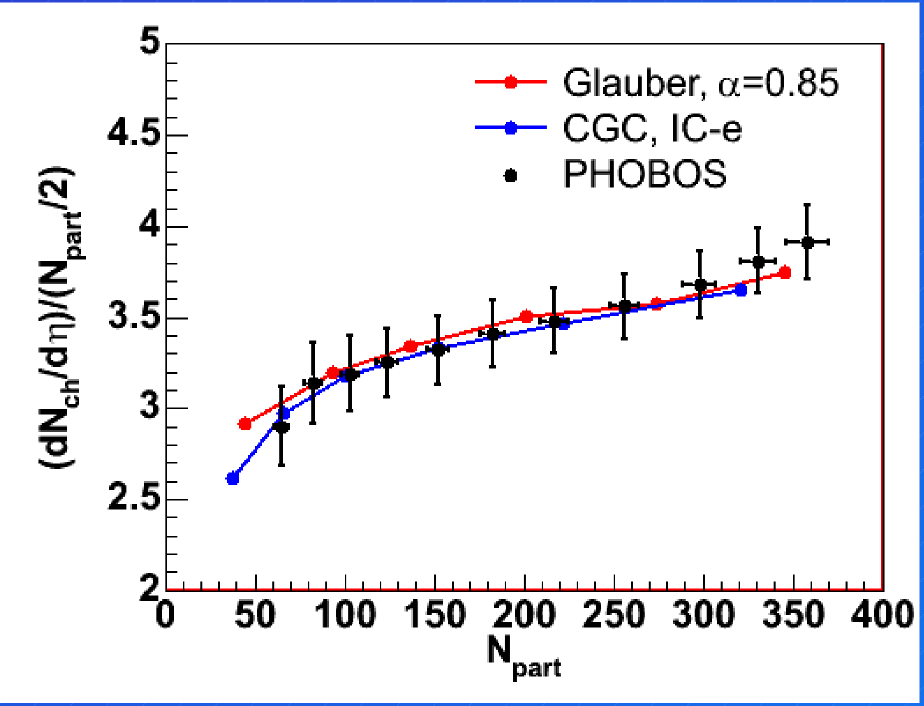
- Unintegrated gluon distribution a.la. Kharzeev, Levin, and Nardi
- Gluon production via k_T factorization formula
- Count deposited energy in dV at (τ_0, x, y, η_s), $\tau_0 = 0.6\mathrm{fm/c}$



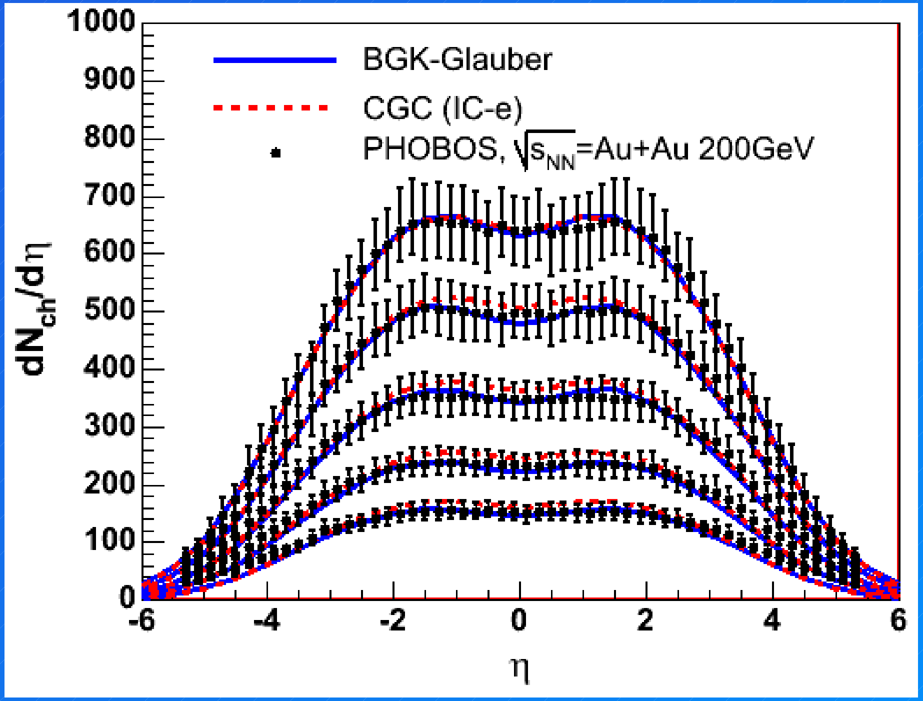
Hirano @ QM2006

Two Hydro Initial Conditions Which Clear the "First Hurdle"

Centrality dependence



Rapidity dependence



1. CGC model

Kharzeev,Levin, and Nardi

Implemented in hydro by TH and Nara

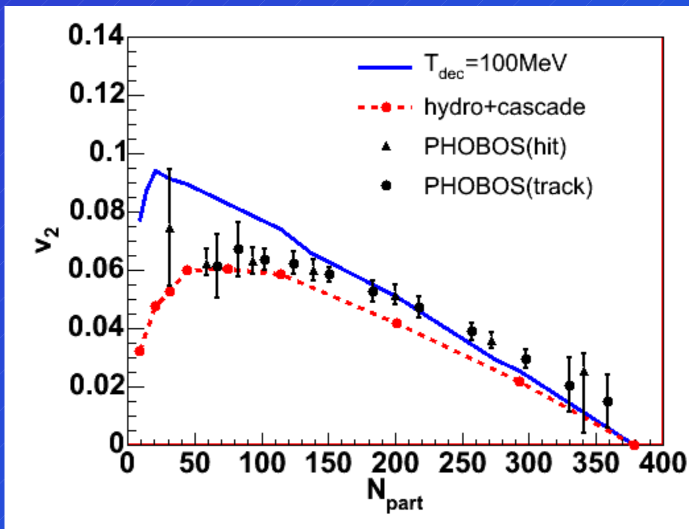
Matching I.C. via $e(x,y,\eta)$

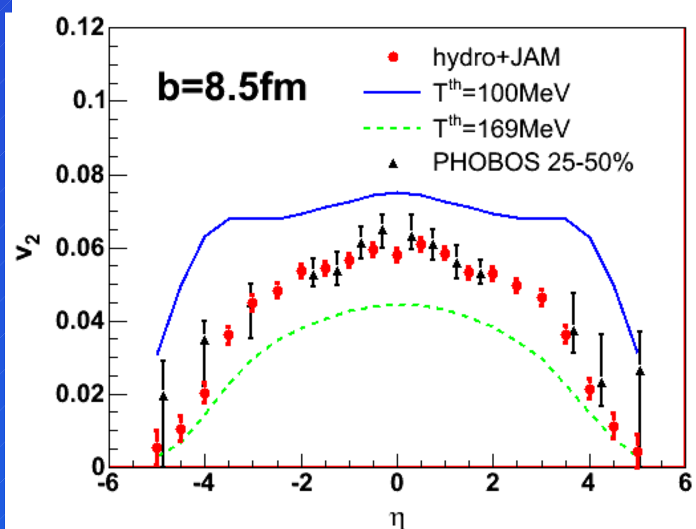
2. Glauber model (as a reference)

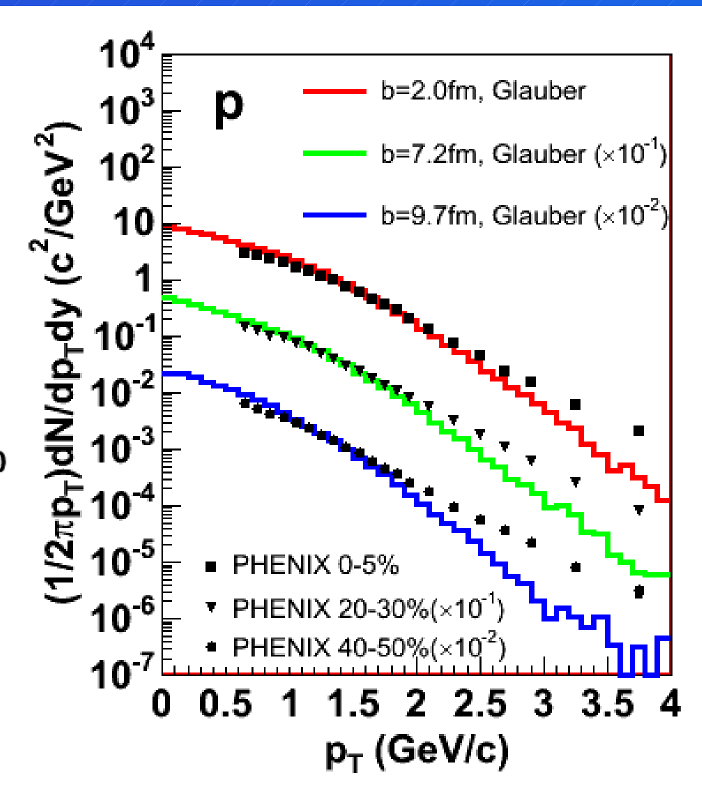
$N_{part}:N_{coll} = 85\%:15\%$

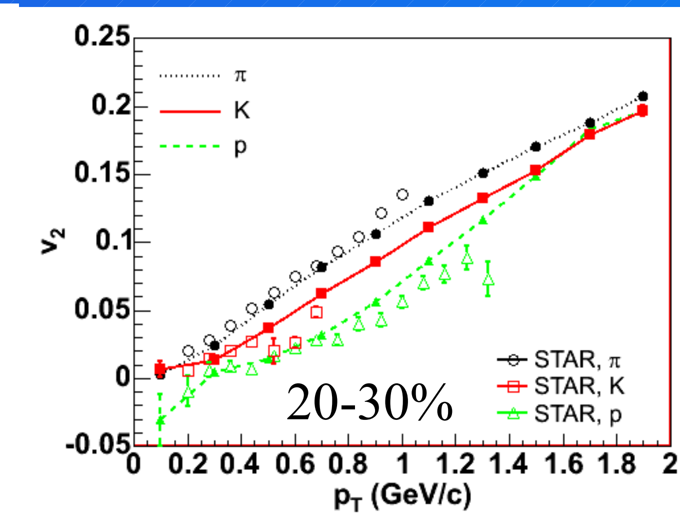
TH et al.('06); in preparation.

Highlights from Glauber + QGP Fluid + Hadron Gas Model









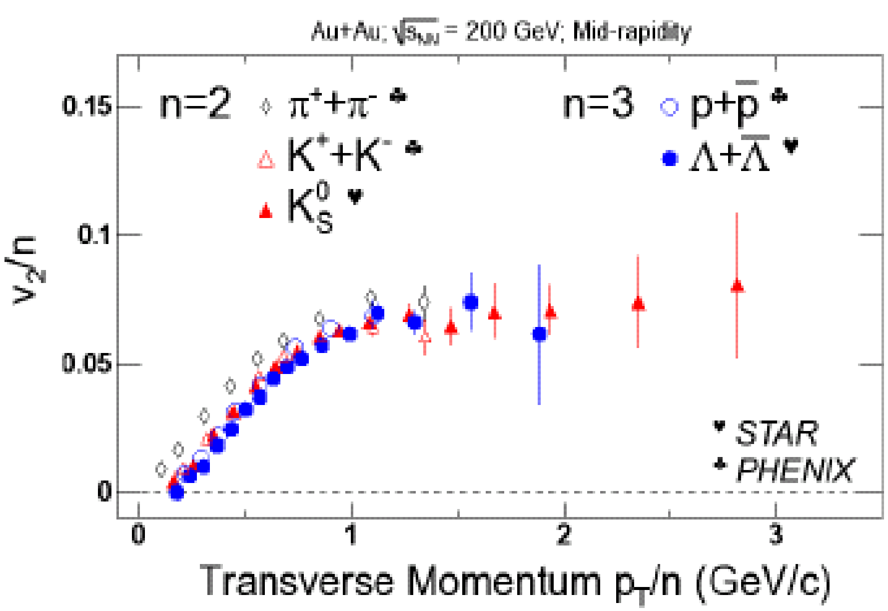
Good agreement for bulk (p_T<~1.5GeV/c)

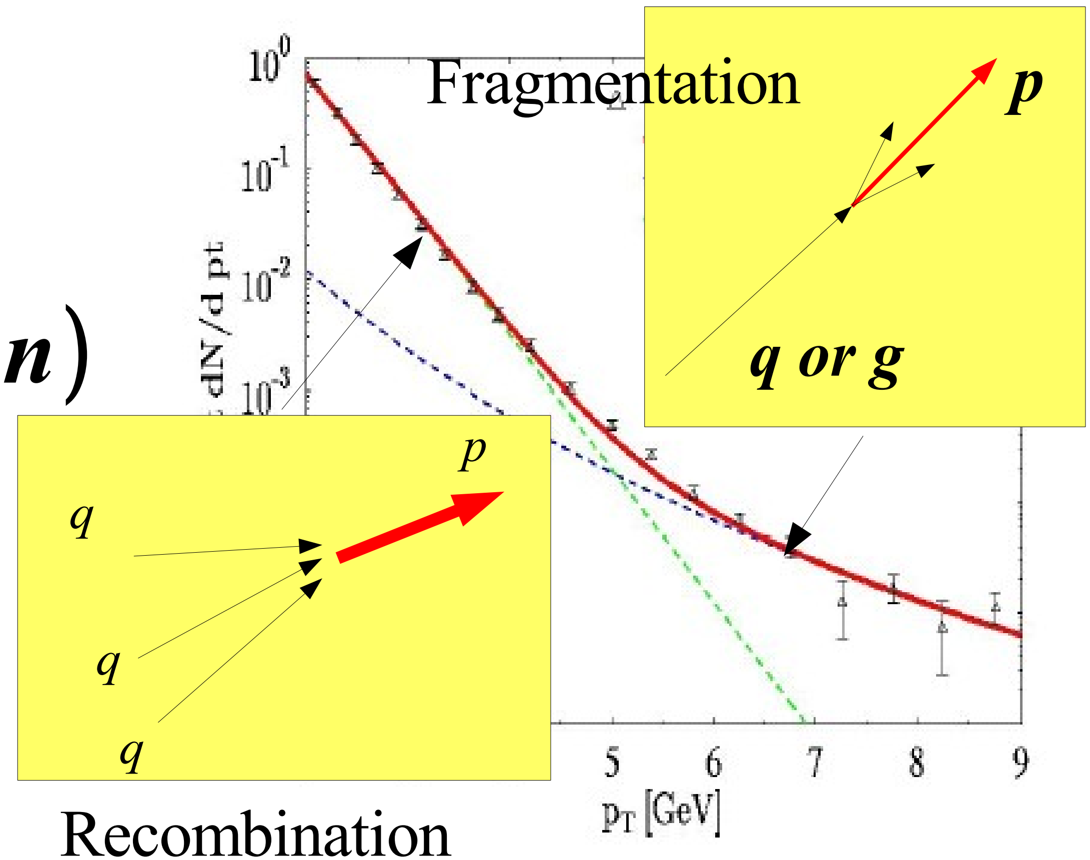
→ What happens to the CGC case?

QGP Signals: Quark Number Scaling

- **When *n* quarks recombines to a hadron, v2 is enhanced by *n* times.**

$$v_2^{Hadron}(P_T) = n\, v_2^{Parton}(P_T/n)$$





Fries et al. PRL 90 (2003), 202303
Nonaka et al., nucl-th/0308051

Recombination Picture seems to work well
... Parton Elliptic Flow

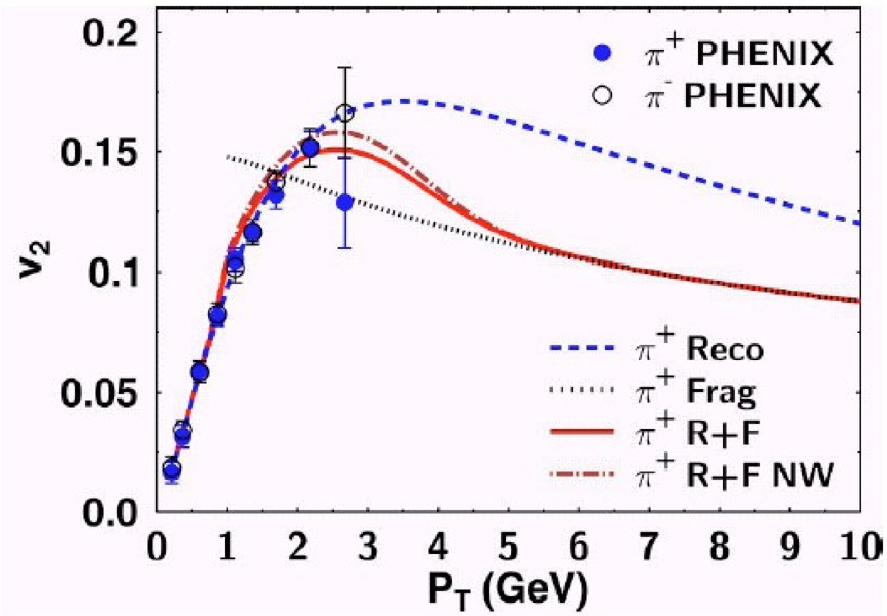
Recombination and Fragmentation

Fries,Muller,Nonaka,Bass, PRL90, 202303(2003); PRC68,044902 (2003)

- **Successes: quark number scaling, baryon/meson ratio**
  $\rightarrow v_2 \sim 0.10$ at high-pT .

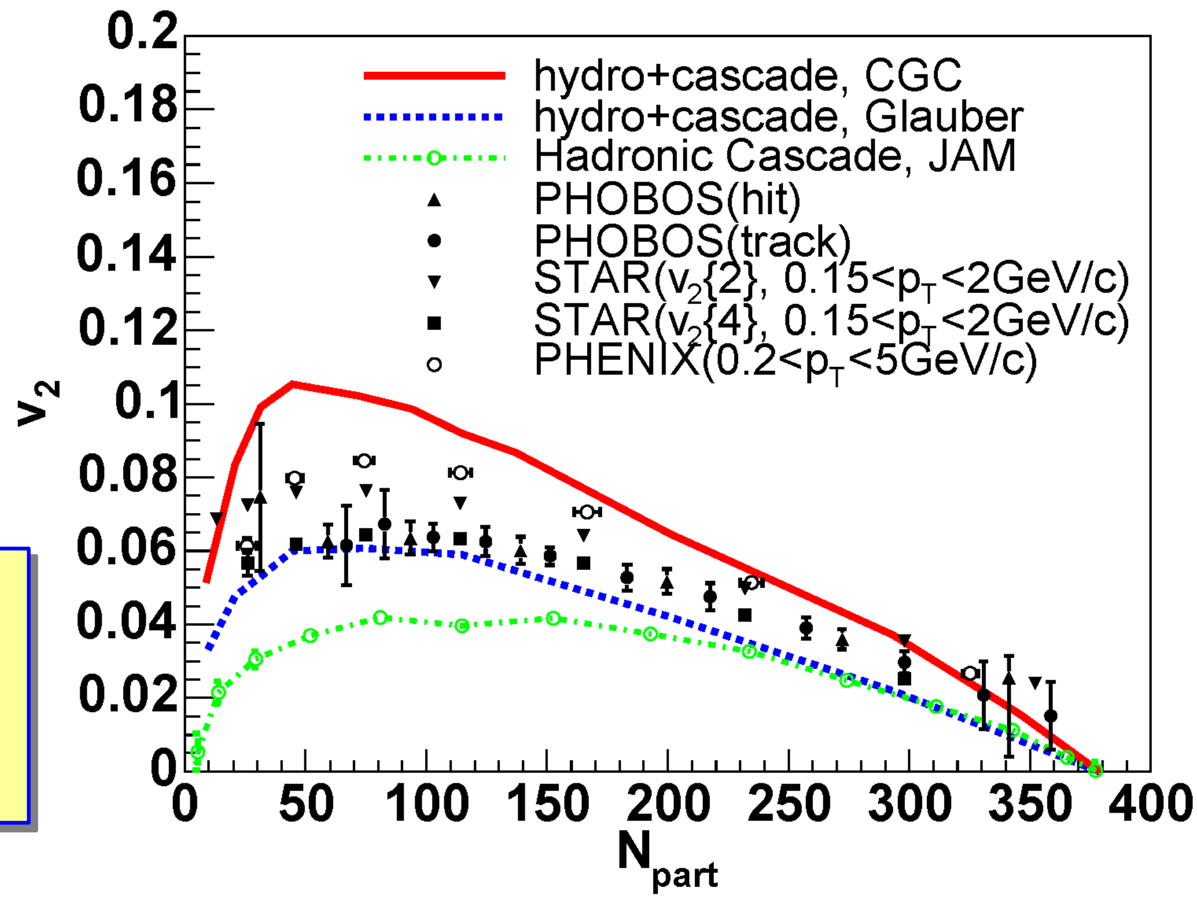
$$f(p, \varphi)=(1 + 2\, v_2(p/2) \cos \varphi)\ \mathrm{x}\ (1 + 2\, v_2(p/2) \cos \varphi)$$
$$\approx 1 + 2\ \mathrm{x}\ 2\, v_2(p/2)\ \cos \varphi$$

- **Problems: Sharply edged density dist. (Hard Sphere)**
  $\ell(b)=\sqrt{R_A^2 - (b/2)^2}$
  $\rightarrow$ E-loss $\propto \ell \rightarrow$ v2 ~ 0.10
  - **Woods-Saxon density distribution**
    $\rightarrow$ v2 ~ 0.05 : Half of H.S.



Cascade vs Hydro @ RHIC: Au+Au

- **Comparison of v2 as a function of N_{part}**
  - **Cascade predict smaller v2 in peripheral collisions**
  - **Data lies between hydro results with two different initial condition CGC (Color Glass Condensate) and Glauber type initial condition.**



Hydro is better,
CGC may be realized
in central collisions.

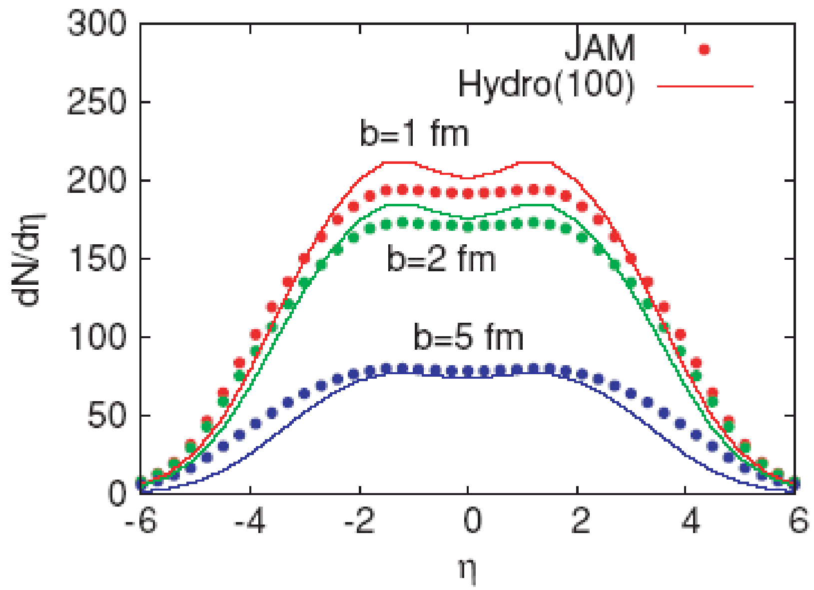
When and where is QGP formed ?

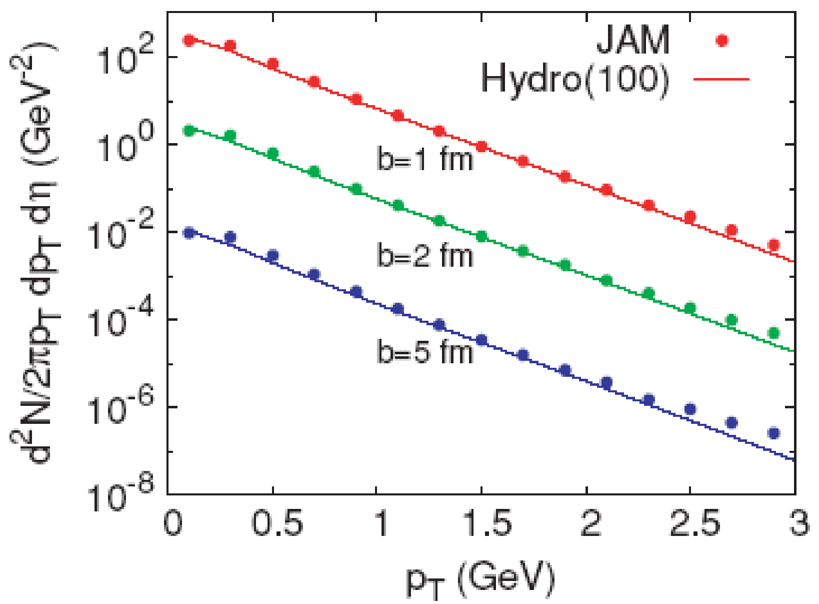
- **Incident Energy**
  - **AGS: Strangeness Enh. (High baryon ρ effect ?)**
  - **SPS:**
    J/ψ suppression (QGP?), Low mass dilepton enh. (chiral sym.)
    Hydro overestimate v_2 data
  - **RHIC:**
    Jet quenching, Strong v_2, Quark number scaling of v_2, ...
    Hadronic Cascade underestimate v_2 data

  → Bulk QGP formation seems to start between SPS and RHIC
- **Proj./Targ. Mass dependence**
  - **Au+Au: v_2(Casc.) < v_2(hydro) ~ v_2(data)**
  - **Cu+Cu: Recently Measured**

Predictions of Cu+Cu Collisions @ RHIC (I)

- **Single particle spectra**
  - **Cascade (JAM) and Hydro predict almost the same single particle spectra**
    $dN/d\eta$, $d^2N/p_T dp_T d\eta$
- **Surprising ?**
  - **Initial Cond. of Hydro is tuned to fit *$dN/d\eta$* (~ Energy per rapidity)**
  - **Cascade use fitted σ_{NN}**
  - **Themailzation is expected at Low p_T (long time before particle production)**
    → Coincidence may not be surprising

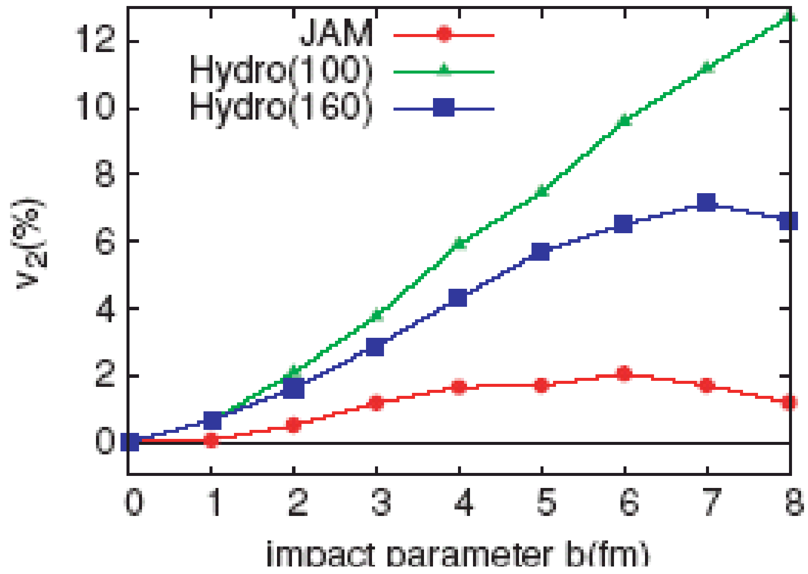


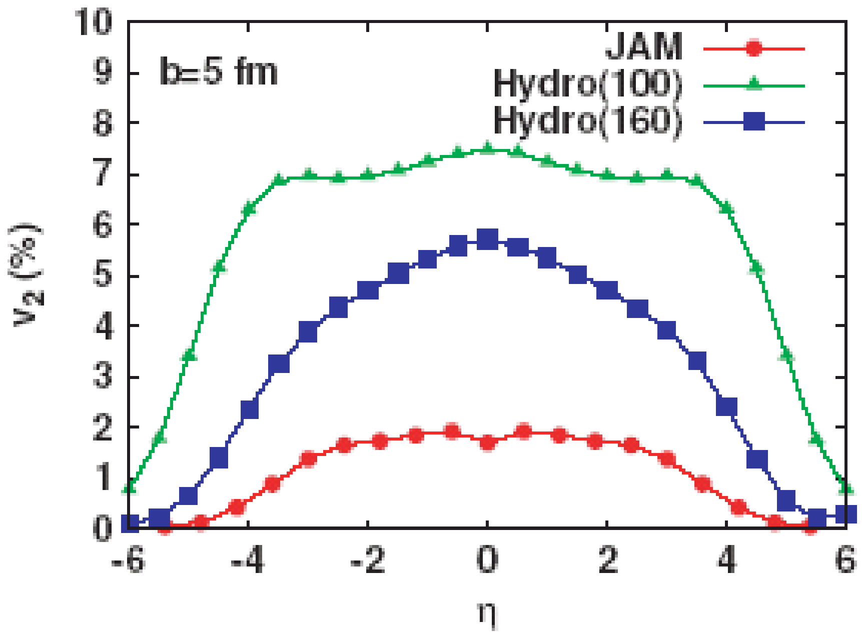


Hirano, Isse, Nara, AO, Yoshino, Phys. Rev. C 72(2005), 041901

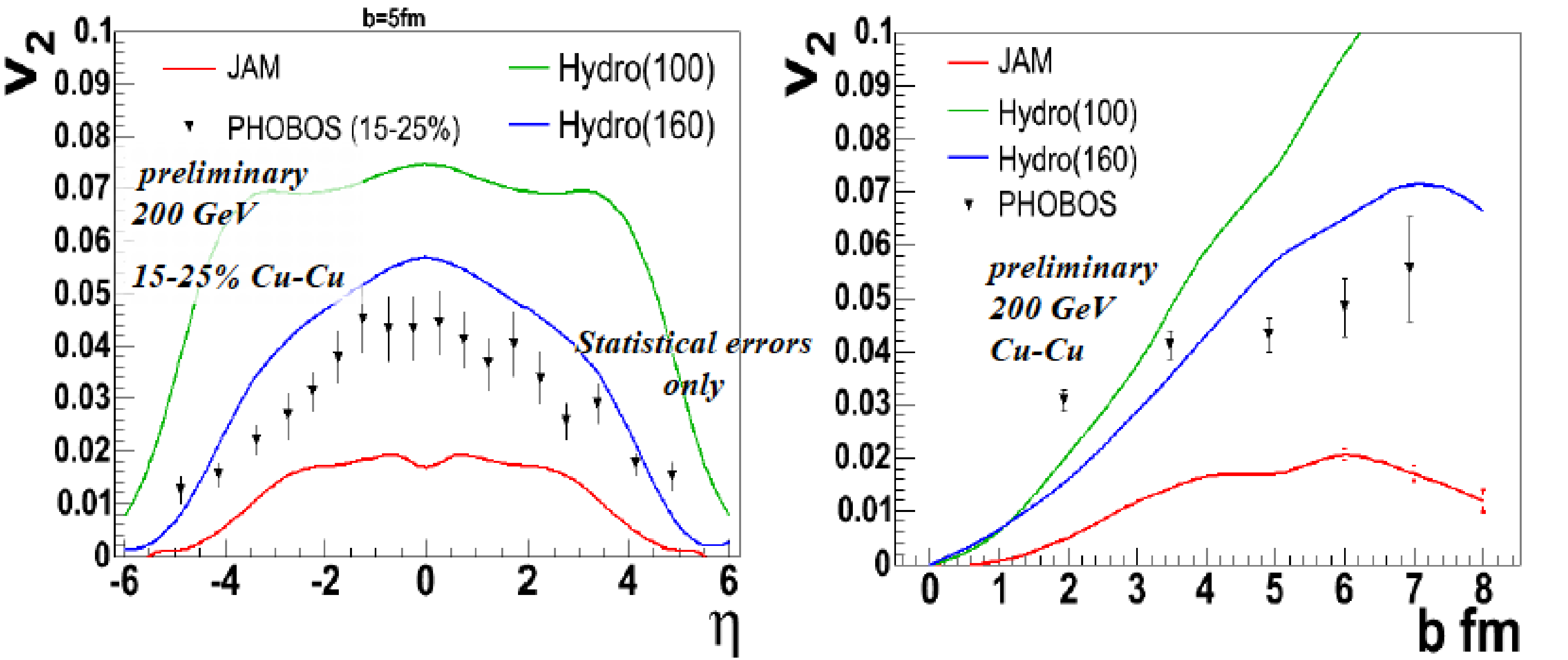
Predictions of Cu+Cu Collisions @ RHIC (II)

- **Calculations were done BEFORE the data are opened to public.**
- **Cascade and Hydro predict very different Elliptic Flow !**
  - **Cascade: small v2**
    → Small int. in the early stage
  - **Hydro: large v2**
    → Strong int. after $\tau=\tau_0 \sim 0.6$ fm/c
- **T^{th} dependence**
  - **T^{th} = 160 MeV ~ Tc = 170 MeV**
    → short time of expansion in the hadron phase
  - **T^{th} = 100 MeV < Tc = 170 MeV**
    → long time of expansion





Compared to JAM Model

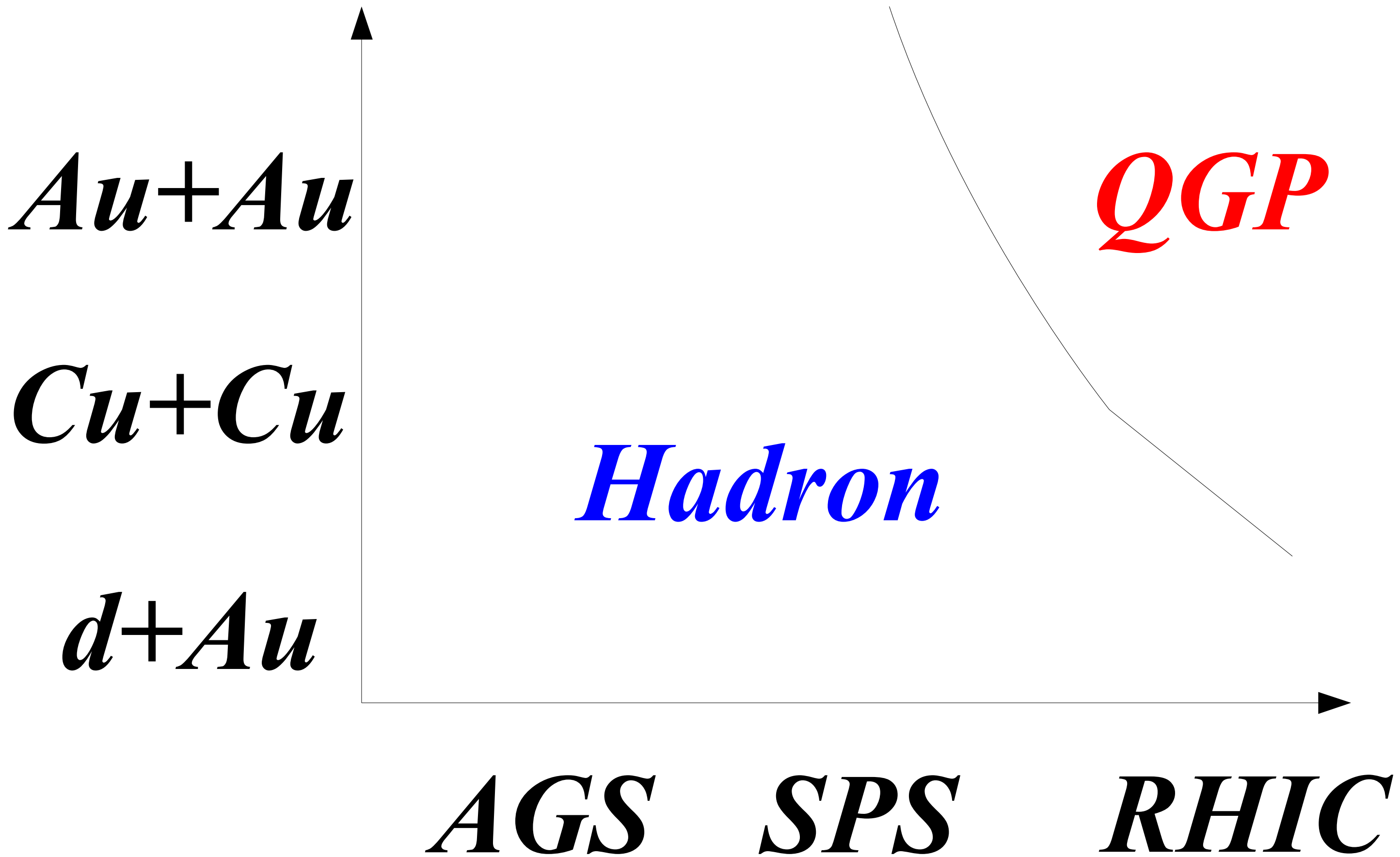


Cu-Cu more like Hydro than JAM hadron string cascade model

Here JAM uses a 1 fm/c formation time. Hydro (160) has kinetic freezeout temperature at 160 MeV

After Data are opened, ...

- Hydro wins Cascade at RHIC even for Cu+Cu collisions in the initial stage evolution.....
- "Reaction Phase Diagram" seems to be .....



Summary

- **Heavy-ion collisions up to SPS energies seems to be reasonably described by using hadron-string cascade such as JAM model, while HIC at RHIC requires earlier thermalization (larger anisotropic pressure) even in lighter nuclear collisions such as Cu+Cu collisions.**
- **There are many things to do in high-energy heavy-ion collision physics.**
  - **AGS-FAIR-SPS energies**
    Nuclear matter EOS, Baryon rich QGP, Strangeness enh., ...
  - **RHIC-LHC energies**
    Detailed studies of QGP properties have just started
    → Consistent understandings are not yet achieved,
    and we still have many puzzles

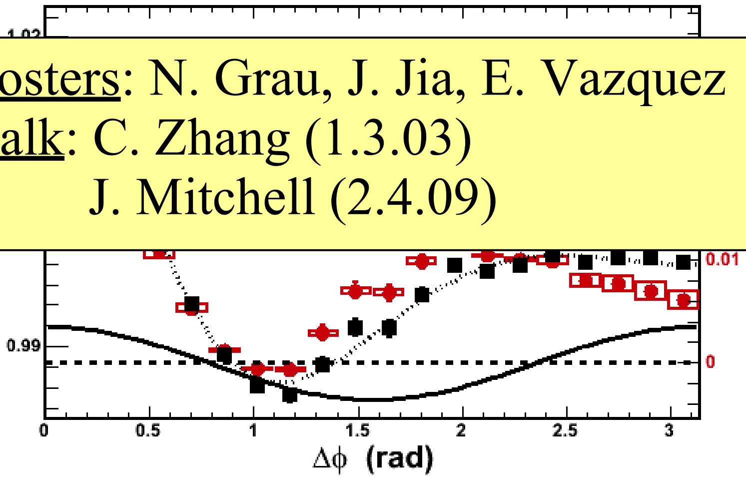
Part V:
Unsolved Problem in RHIC Physics

Unsolved (or NewlyFound) Problems at RHIC

- **Mach Cone / Color Cerenkov**
  - **Many low pT particles are observed along the Quenched Jet (Angle from Jet = 120 deg.)**
- **J/ ψ Production Mechanism**
  - **With the expected absorption ratio at SPS, J/ψ yield @ RHIC is underestimated.**
- **Baryon(Hyperon)-Hadron azimuthal angle correlation**
  - **Around the high pT baryon angle, many hadrons are observed as in the case of jet production → Baryons are also formed in jets.**
- **High p_T v_2 problem**
  - **With the energy loss explaining p_T spectrum, elliptic flow is calculated to be too small at high p_T.**
- **And Many....**

Jet Functions

nucl-ex/0611019
(submitted to Phys. Rev. Lett.)



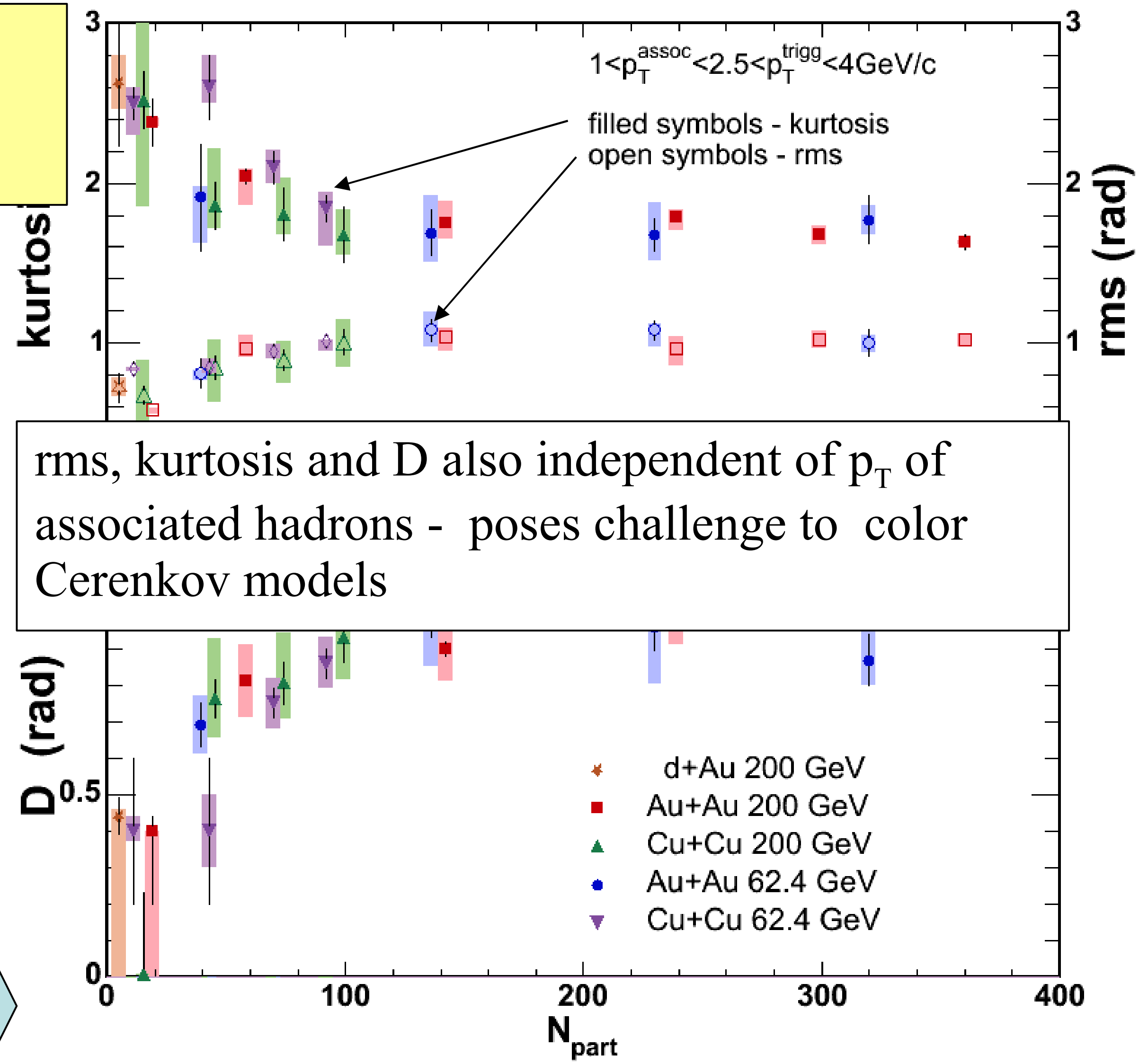
<u>Posters</u>: N. Grau, J. Jia, E. Vazquez
<u>Talk</u>: C. Zhang (1.3.03)
J. Mitchell (2.4.09)

$$\mu_n = \left\langle (\Delta\varphi - \pi)^n \right\rangle$$

$$rms = \sqrt{\mu_2}$$

$$kurtosis = \mu_4 / \mu_2^2$$

$$J(\Delta\varphi) = G(\Delta\varphi) + G(\Delta\varphi - \pi + D) + G(\Delta\varphi - \pi - D)$$

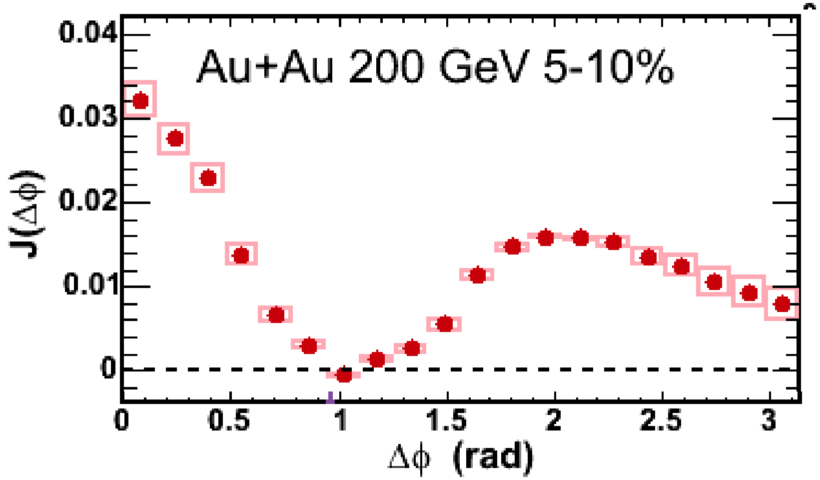


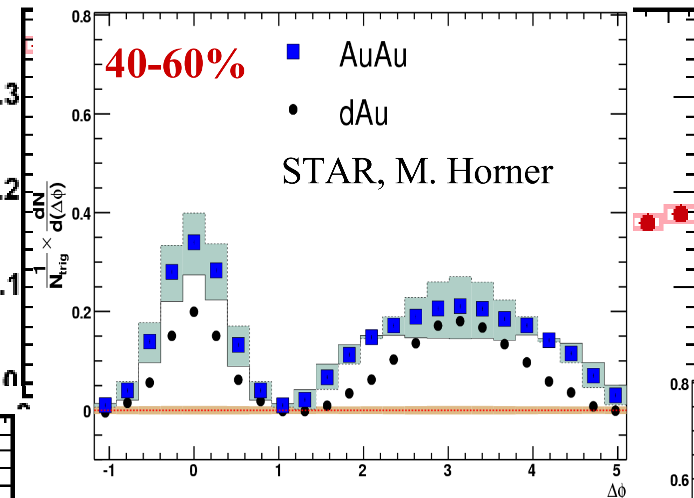
rms, kurtosis and D also independent of p_T of associated hadrons - poses challenge to color Cerenkov models

John Lajoie (PHENIX) @ QM2006

Di-hadrons: away-side shape

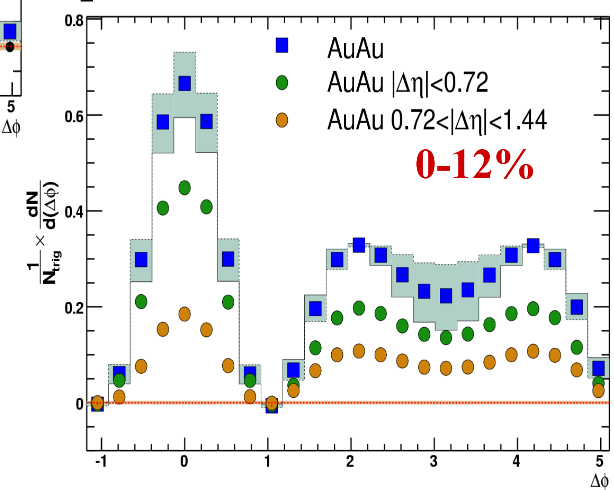
PHENIX: C. Zhang, N. Grau, J. Jia, E. Vazquez





$2.5 < p_{T,trig} < 4.0$ GeV/c
$1.0 < p_{T,assoc} < 2.5$ GeV/c

High statistics
Run IV data



Clear evolution peripheral → central:
Widening, flattening and ‘dip at π’

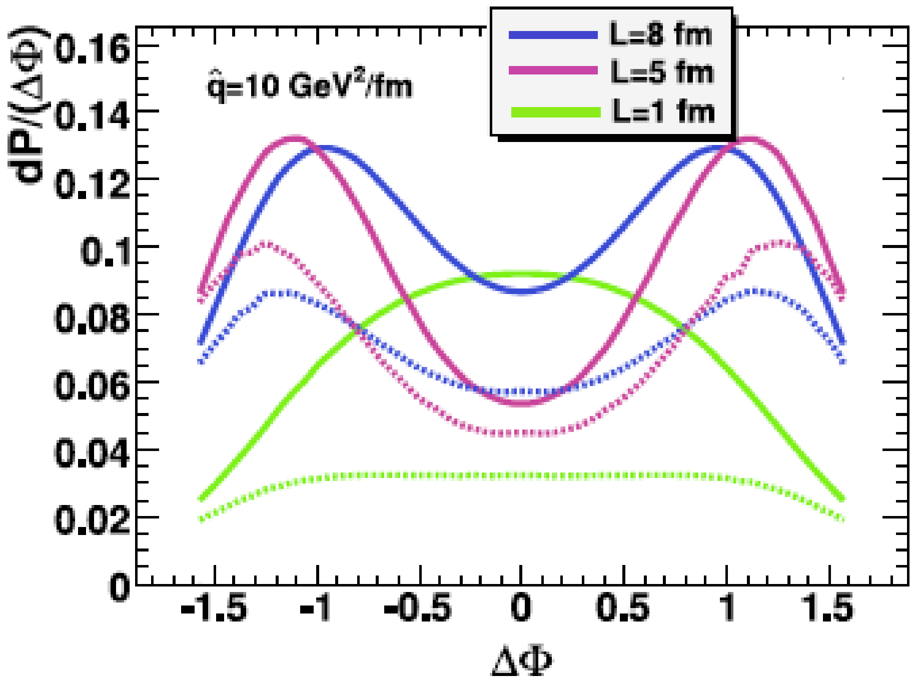
Jet Correlations: *Larry McLerran @ QM2006*

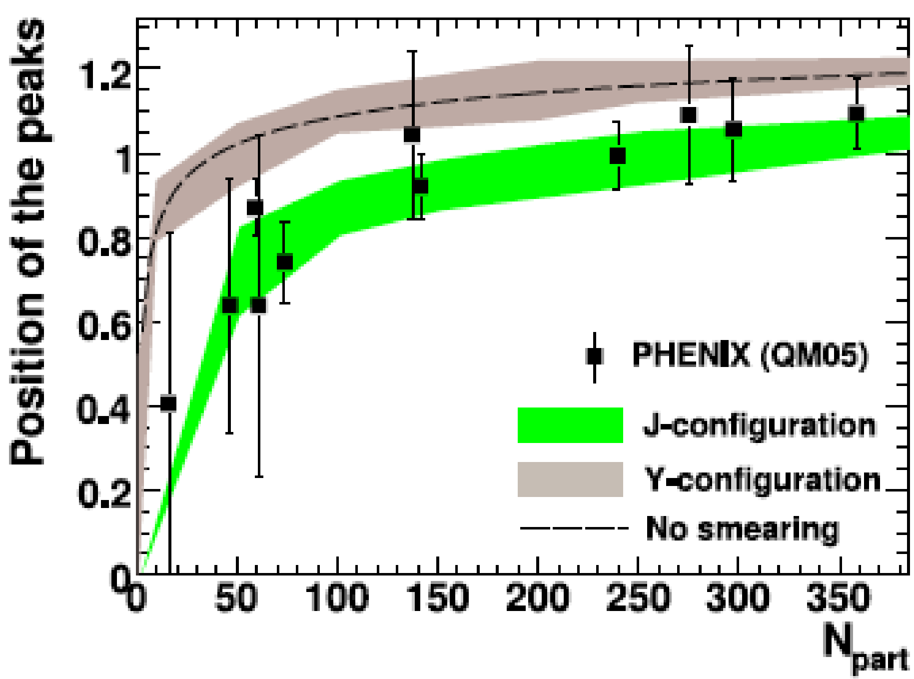
Mach cones one of earliest proposals for heavy ion collisions:
Greiner, Stocker and Frankfurt group

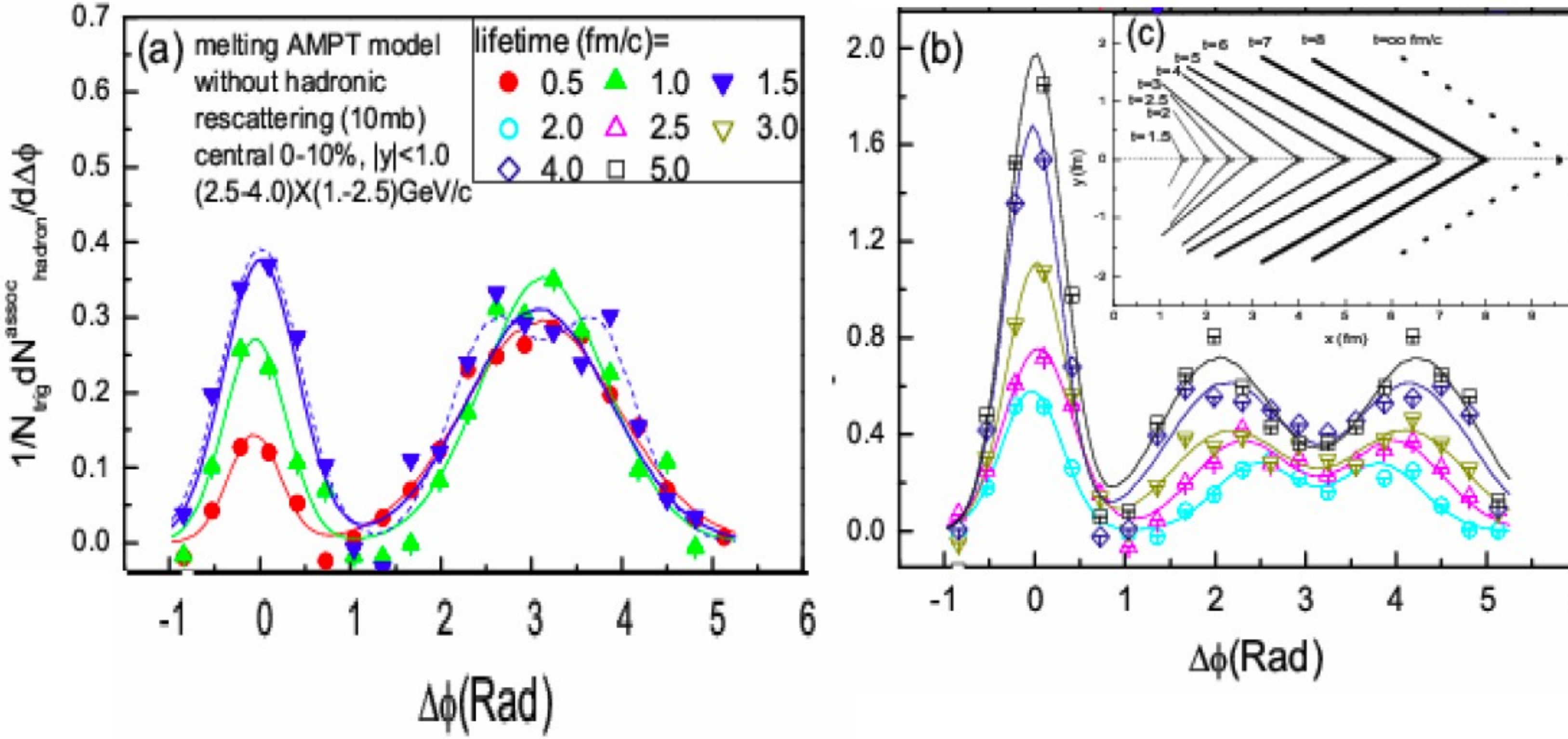
Cherenkov radiation and Mach cones possible,
but devil in the details

Possible explanation as Sudakov form factor for
jet emission by Salgado et. al?

Deflected jets al a Vitev?







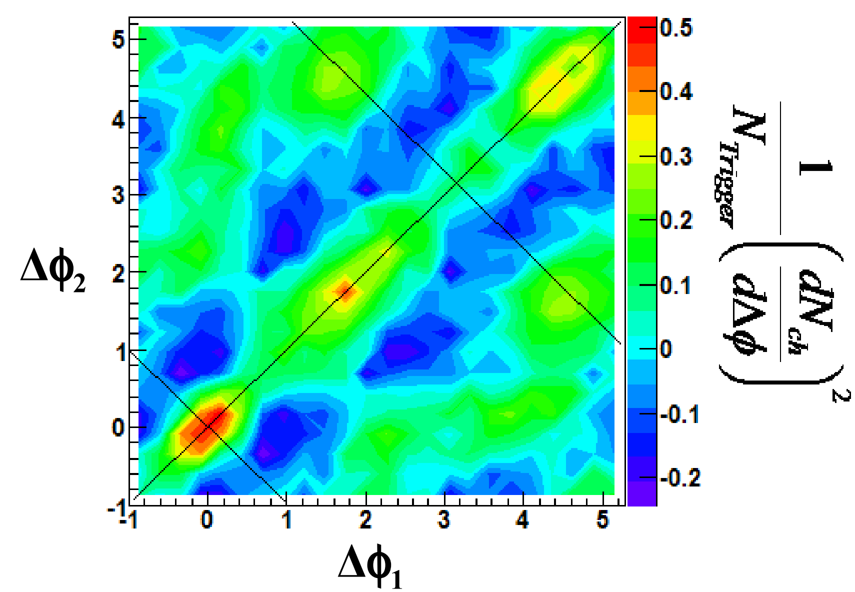
Mach Cone: $v_s^2 \sim 10^{-2}$

Radiation and scattering: No cone

Cerenkov: Wide angles

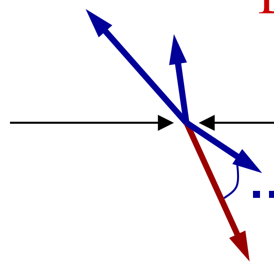
Larry McLerran @ QM2006

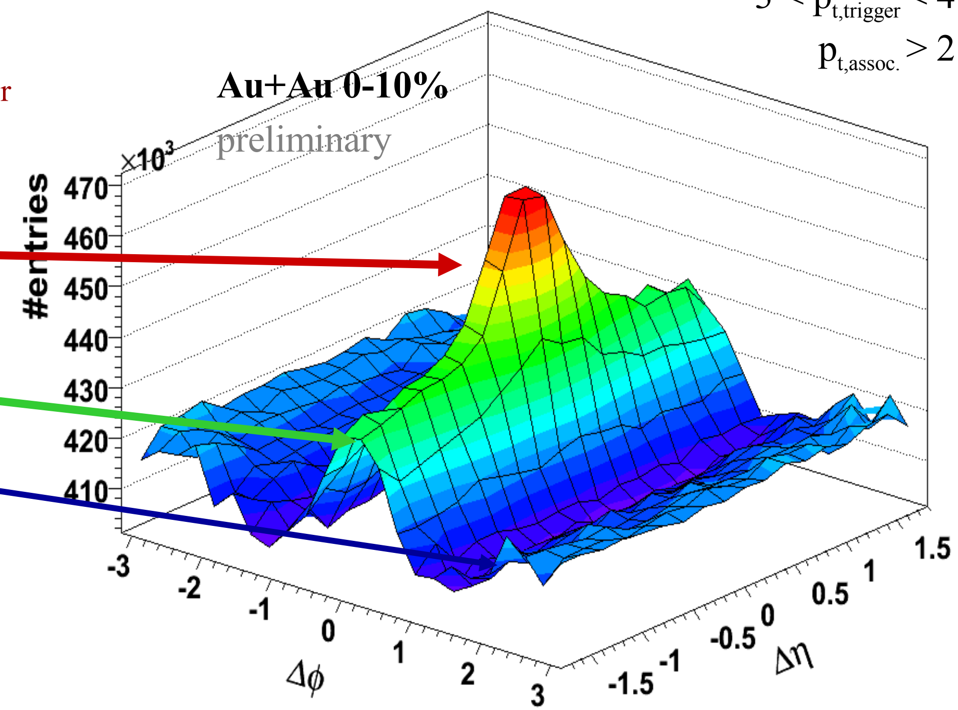
Au+Au central 0-12% ZDC



Fragmentation and energy loss I: near-side

Di-hadron correlations





$3 < p_{t,trigger} < 4$ GeV

$p_{t,assoc.} > 2$ GeV

M. Calderon, J. Putschke

Components

- **Near-side jet peak**
- **Near-side ridge**
- **Away-side (and v_2)**

Two distinct questions:

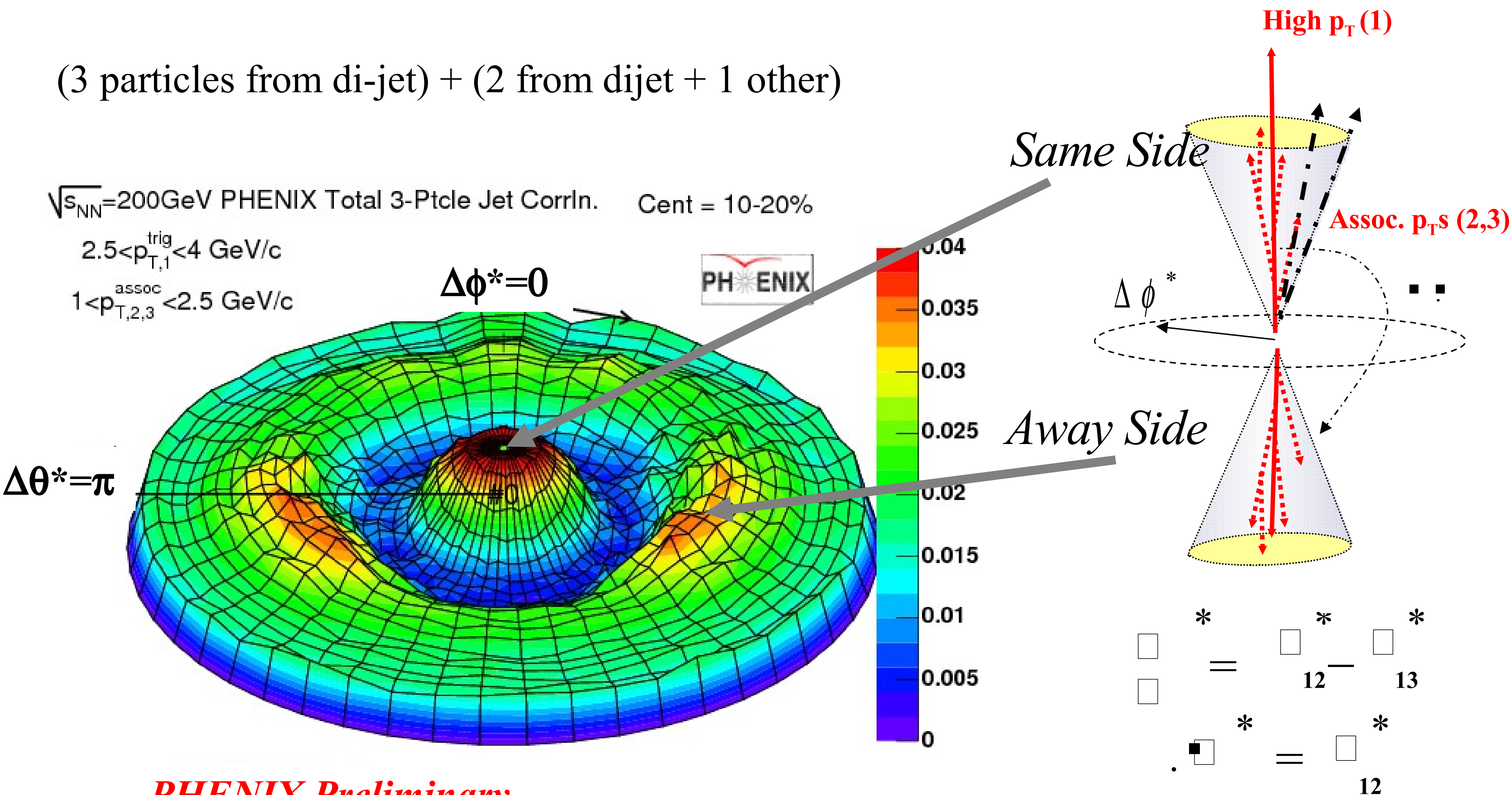
- What is it ?
  'something' coupling to long flow ?
  Can this quantify E-loss ?
- How to deal with it?
  Need to subtract for near-side studies?

Lesson: The near-side jet does interact with the medium

Marco van Leeuwen @ QM2006

3-Particle Correlations

(3 particles from di-jet) + (2 from dijet + 1 other)

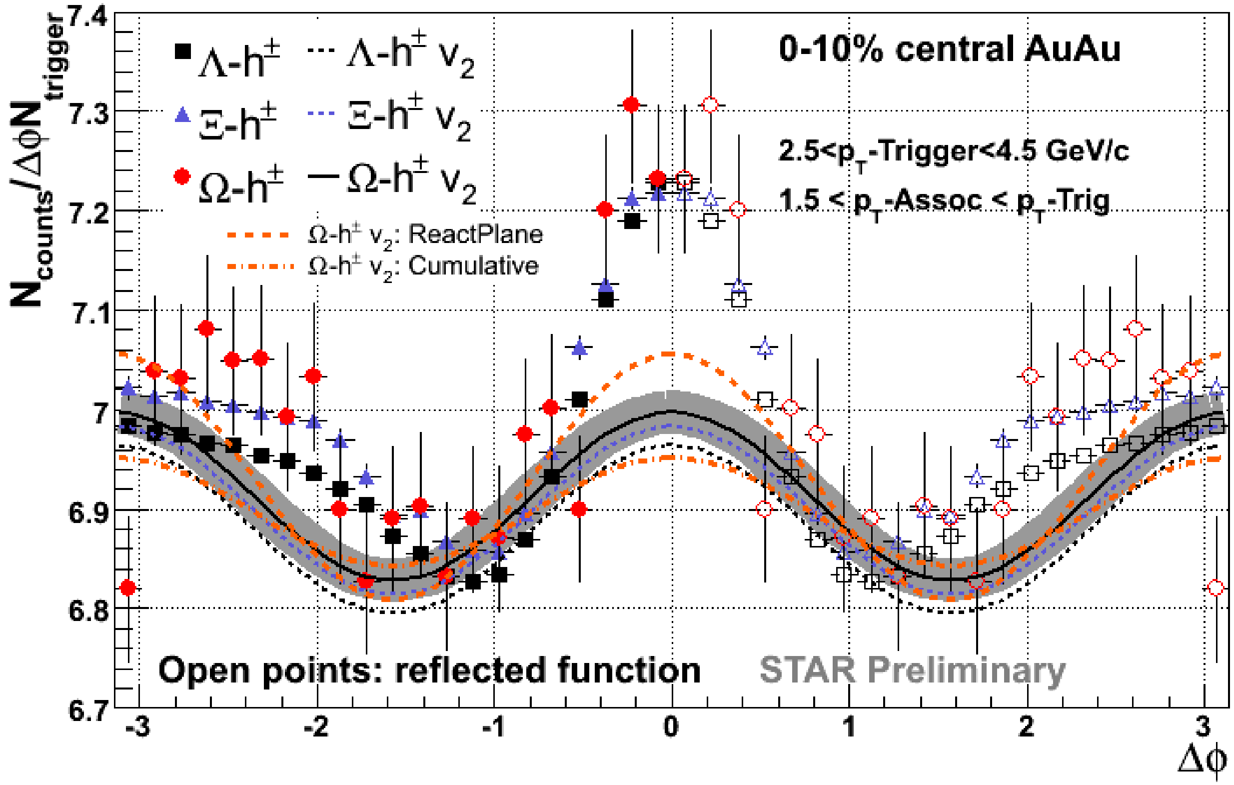


PHENIX Preliminary

John Lajoie (PHENIX) @ QM2006

Λ,Ξ,Ω-h correlation

J. Bielcikova



Near-side yield similar for Λ, Ξ, Ω triggered correlations

Initial expectation: Ω dominantly from TTT recombination, no associated yield

R. C. Hwa et al., nucl-th/0602024

Revisited (at QM06): possible large contribution from reheated medium

Experimental tests pending

Marco van Leeuwen @ QM2006

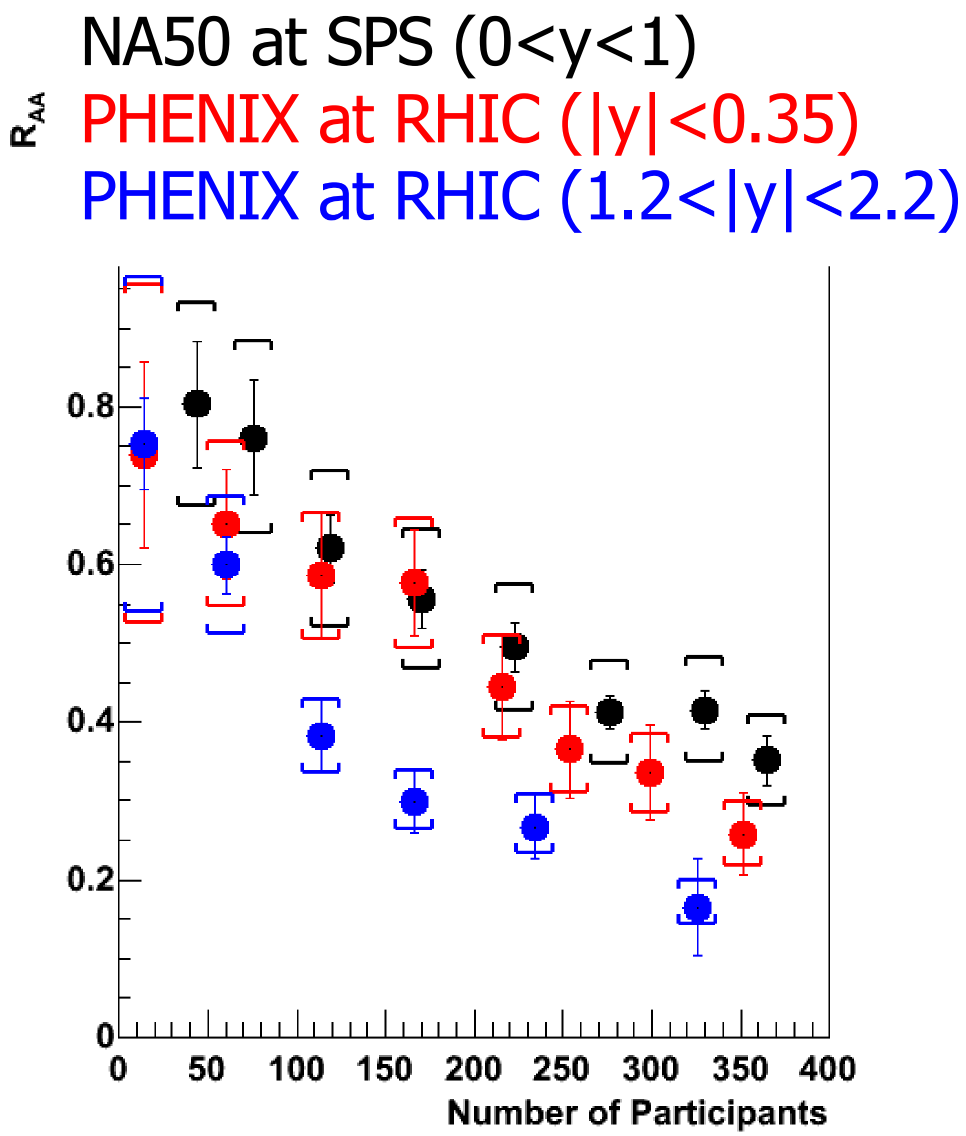
J/Ψ Suppression at SPS and RHIC

Suppression patterns are remarkably similar at SPS and RHIC!

Cold matter suppression larger at SPS, hot matter suppression larger at RHIC, balance?

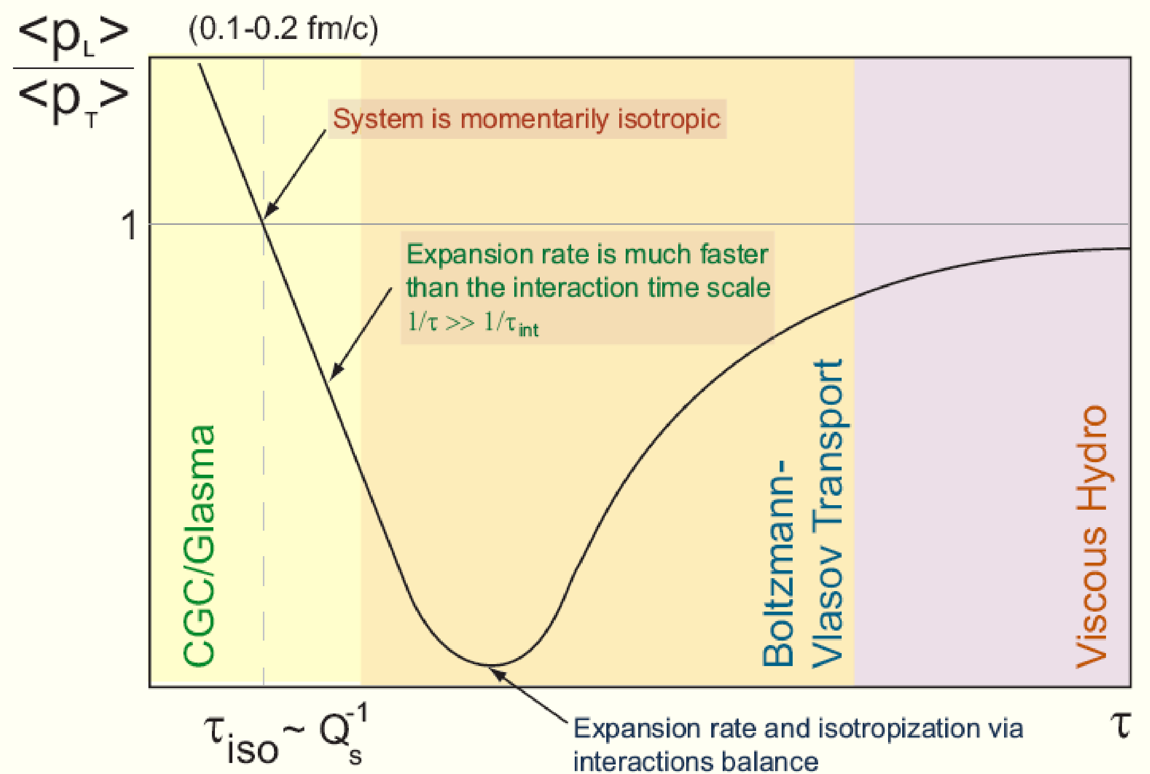
Recombination cancels additional suppression at RHIC?

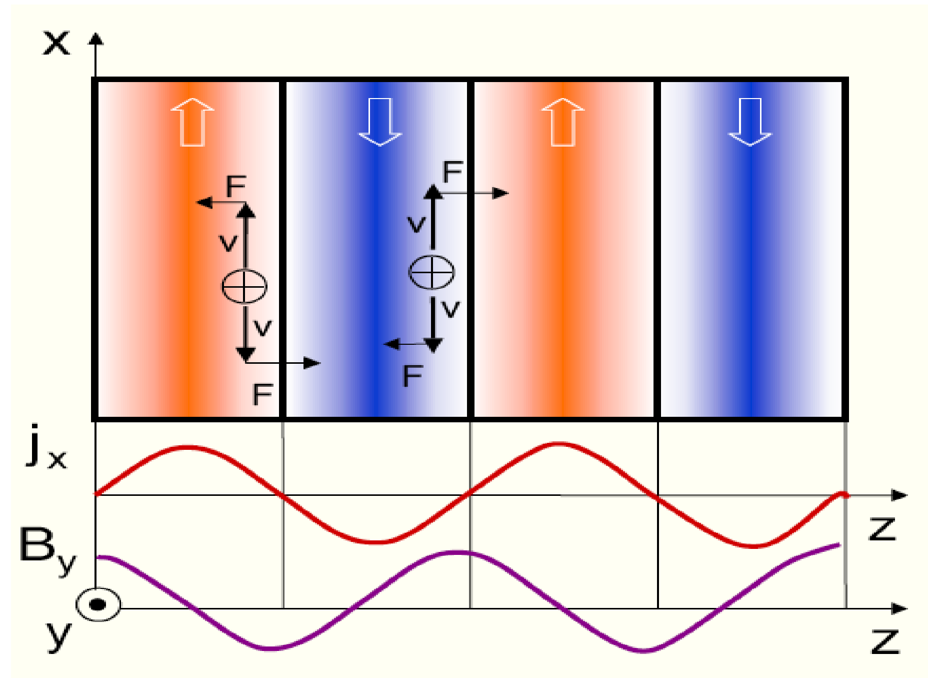
How did we get so "lucky"?



John Lajoie @ QM2006

Momentum Space Anisotropy Time Dependence

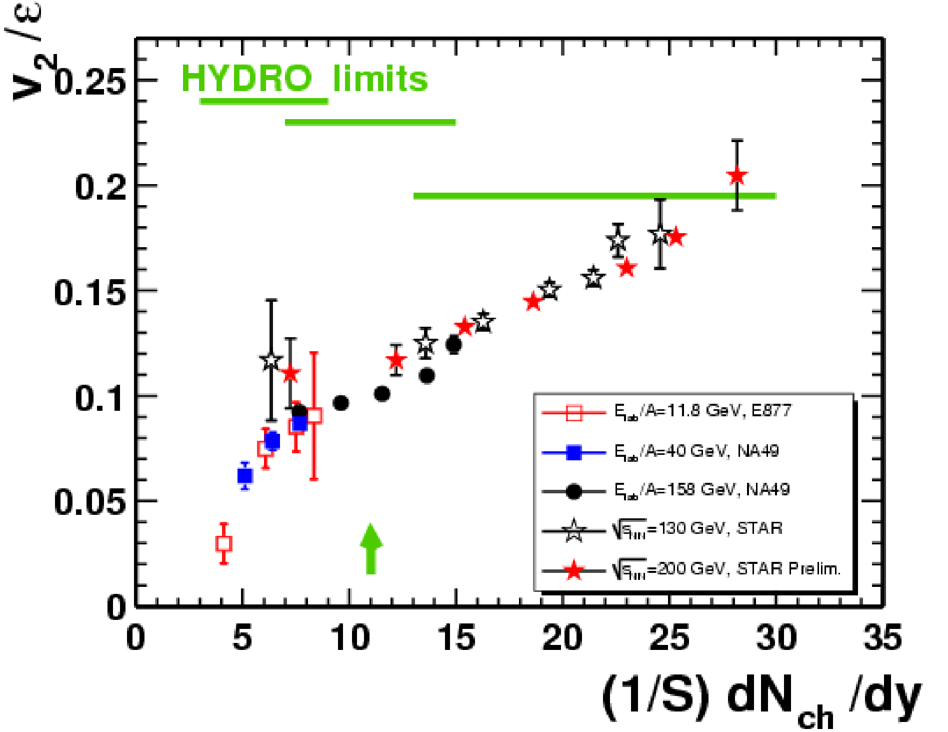




Instabilities driven by momentum anisotropy

Larry McLerran @ QM2006

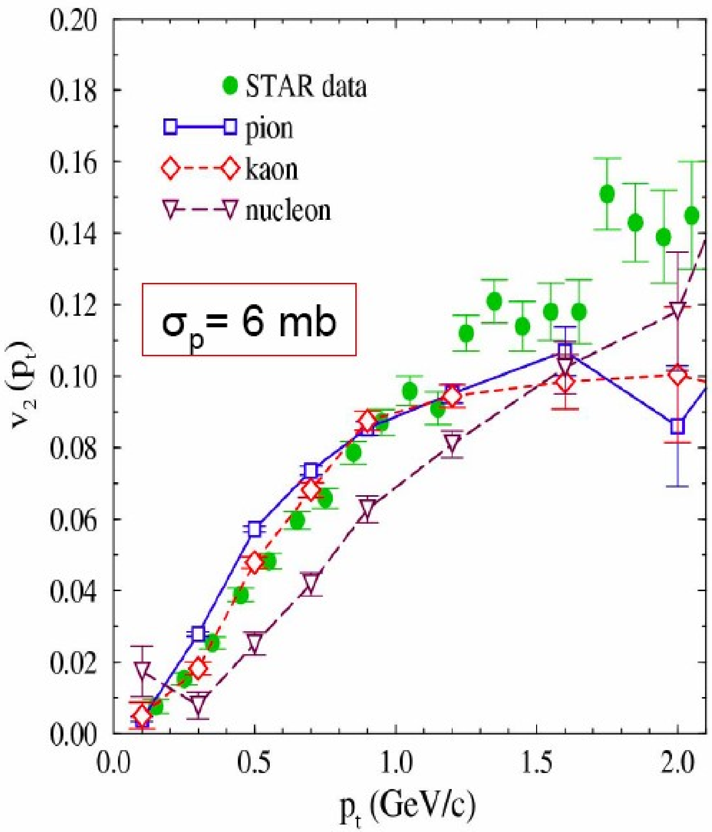
How Perfect is the sQGP?



CGC Initial Conditions allow for higher hydro limit.

LHC?

Larry McLerran @ QM2006



CGC Initial Conditions?

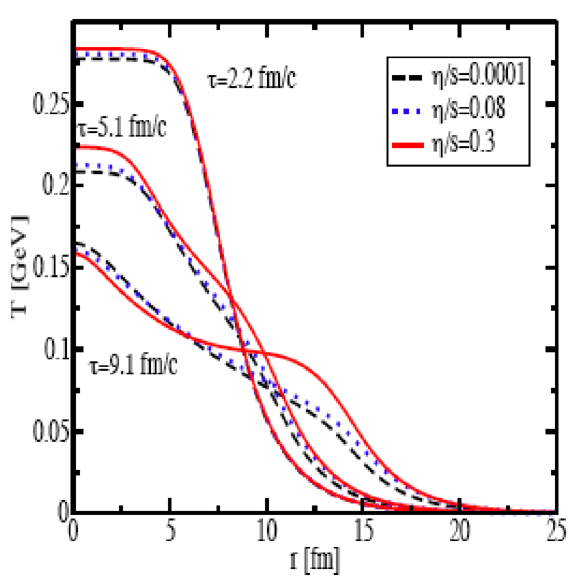
Large parton cross sections not required for flow.

Thermalization through mutligluon interactions?

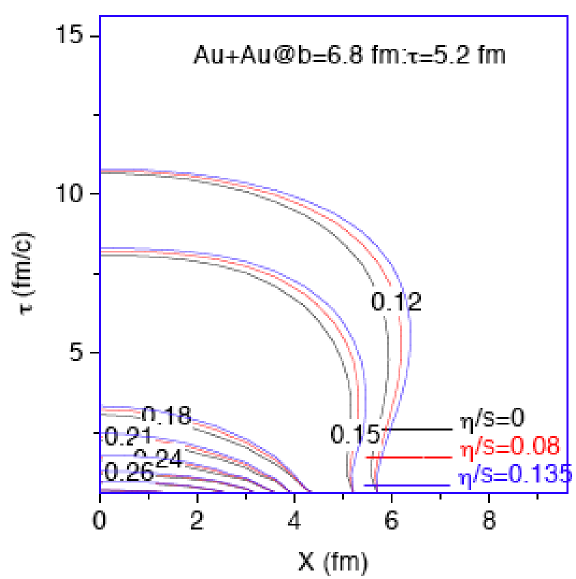
Plasma Instabilities?

Viscosity effects are unknown, computation is theoretical challenge.

- Temp. vs. Rad. for different τ



- Temp. contours in the τ, R plane



Viscous Hydrodynamics:

Becoming practical

Larry McLerran @ QM2006

Jet-Fluid String Formation and Decay at RHIC

Hirano, Isse, Mizukawa, Nara, AO, Yoshino, in preparation

Hadronization Mechanism at RHIC

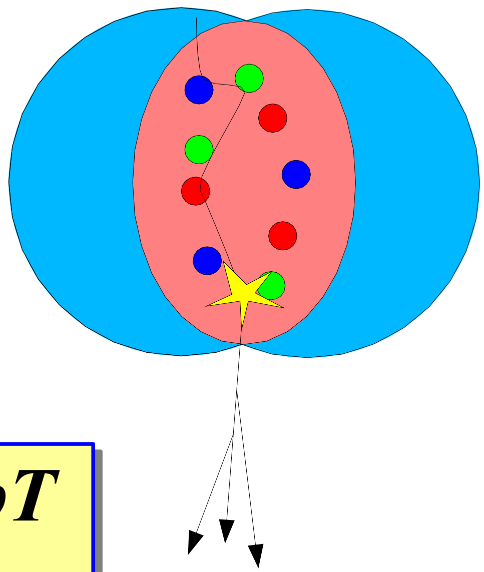
- *High p_T: Indep. Frag. of Jet Partons* *(E.g. Hirano-Nara)*
  - O Explains pT spectrum when E-loss is included.
  - X Elliptic Flow v_2 is small at high p_T ←*This Talk*
- *Medium p_T: Recombination* *(E.g. Duke-Osaka-Nagoya)*
  - O Explains Baryon Puzzle and Quark Number Scaling of v_2
  - X Hard sphere density profile is implicitly assumed
- *Low p_T: Equil. Fluid Hadronization* *(E.g. Hirano-Gyulassy)*
  - O Explains p_T spec. and v_2 at low p_T
  - X Results depends on the Freeze-Out Conditions

QGP Signals are understood separately,
and they are not necessarily consistent.
→ Further Ideas are required !

How can we get large v_2 at high p_T ?

- **Quark Recombination → Combined Objects have larger v2**

$$f(p, \varphi) = (1 + 2\, v_2(p/2) \cos\varphi) \times (1 + 2\, v_2(p/2) \cos\varphi)$$
$$\approx 1 + 2 \times 2\, v_2(p/2) \cos\varphi$$



- **Energy Loss in QGP generates v2**
  - **Large/Small suppression in y/x directions**

> ***Plausible Hadronization giving large v2 at high pT***
> - ***Combination of several partons***
> - ***Large Energy Loss***
>   ***→ Jet parton picks up Fluid parton***
>   ***and forms a string (Jet-Fluid String)***

Jet-Fluid String Formation and Decay

Jet production: pQCD(LO) x K-factor (PYTHIA6.3, K=1.8, *pp* fit)

$$\sigma_{jet} = K \sigma_{jet}^{pQCD(LO)}$$

Jet propagation in QGP

3D Hydro + Simplified GLV 1st order formula x *C*

(Hirano-Nara, NPA743('04)305, Hirano-Tsuda, PRC 66('02)054905. Web version!

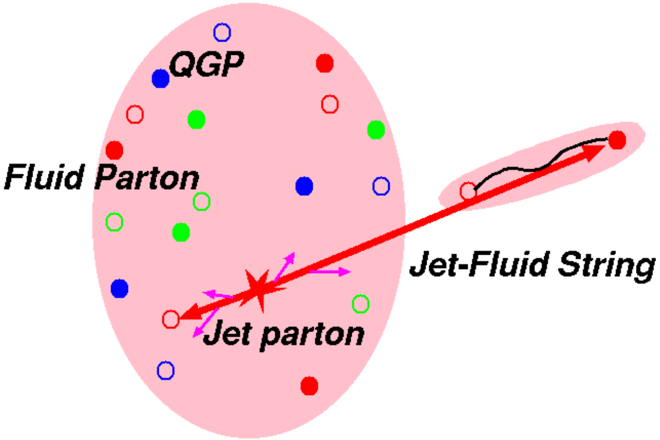
Gylassy-Levai-Vitev, PRL85('00)5535)

$$\Delta E = C \times 9\pi \frac{\alpha_s^3}{4} C_R \int d\tau (\tau - \tau_0) \rho_{\text{eff}} \log(\frac{2E_0}{\mu^2 L})$$

Jet-Fluid String formation

Fluid parton breaks color flux,
according to string spectral func.

$$P(\sqrt{s}) \propto \Theta(\sqrt{s} - \sqrt{s_0}) \quad (\sqrt{s_0} = 2\,\text{GeV})$$



Only g and light q (qbar) are considered.

http://nt1.c.u-tokyo.ac.jp/~hirano/parevo/parevo.html

QGP Fluid Evolution

Package for QGP fluid evolution

Space-Time Evolution of Parton Density in Au+Au Collisions at RHIC from a Full 3D Hydrodynamic Simulations

A realistic space-time evolution of fluid parton density is indispensable for quantitative estimation of parton energy loss in relativistic heavy ion collisions. In this website, we make our hydro results open to public. We used these hydro results for studies of jet quenching and back-to-back correlations in the following papers:

T.Hirano and Y.Nara, Phys.Rev.Lett.**91**,082301(2003),
T.Hirano and Y.Nara, Phys.Rev.C**69**,034908(2004).

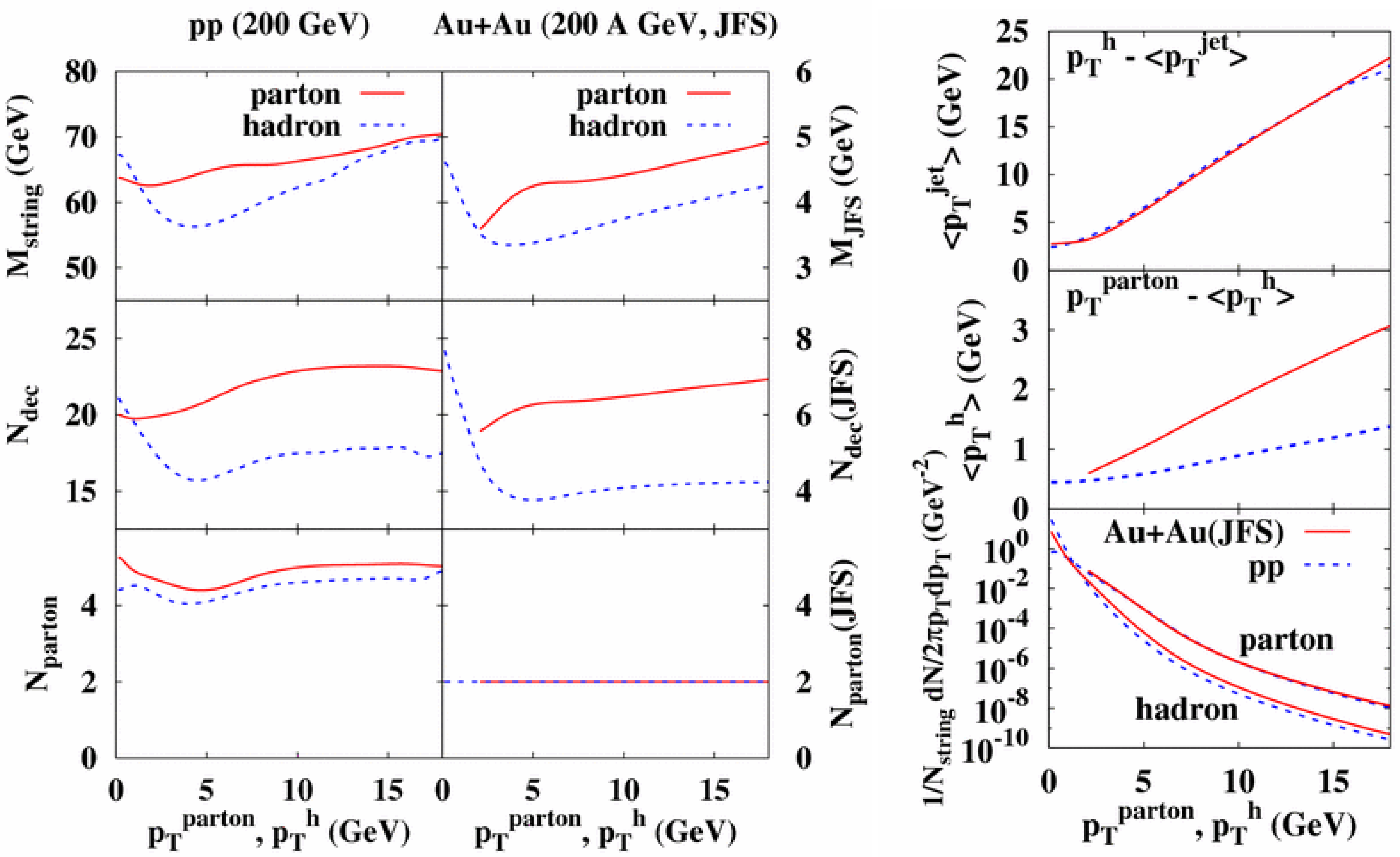
Initial parameters in hydro are so chosen as to reproduce the pseudorapidity distribution observed by an experimental group. The resultant initial parameters are Emax = 45 GeV/fm^3, ηflat = 4.0, ηGauss = 0.8. For further details on initialization in our model, see

http://nt1.c.u-tokyo.ac.jp/~hirano/parevo/parevo.html

Site Status Not Verified

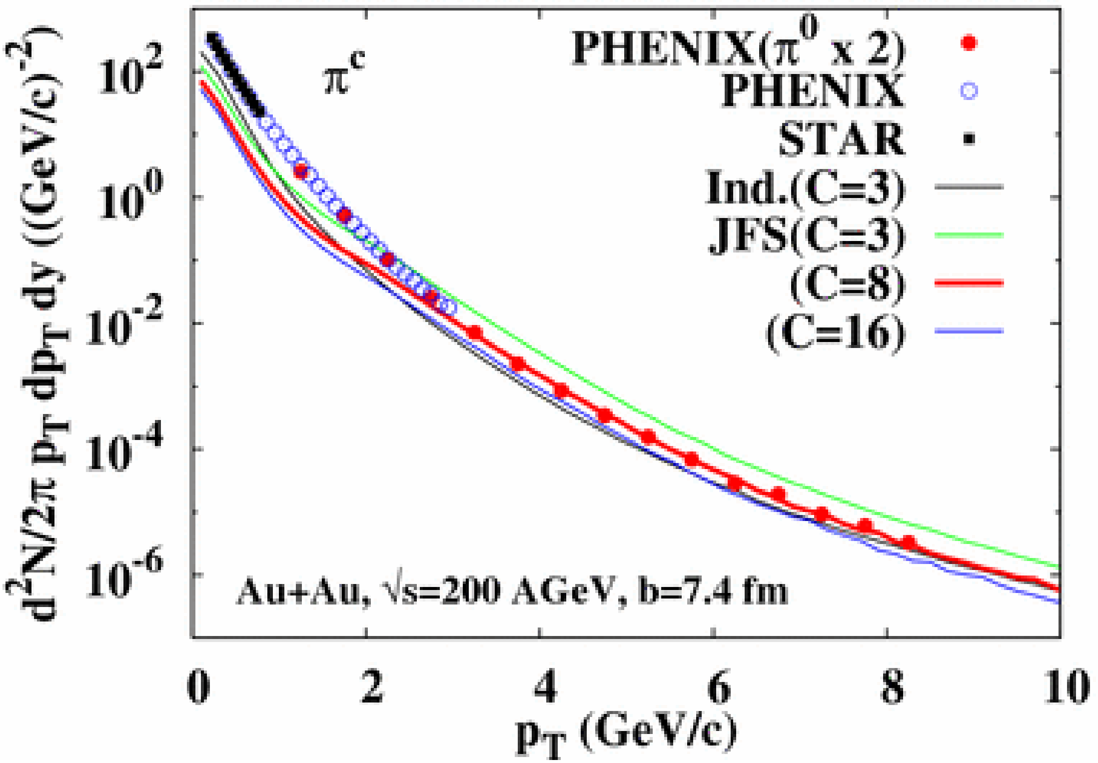
String Mass and p_T in JFS

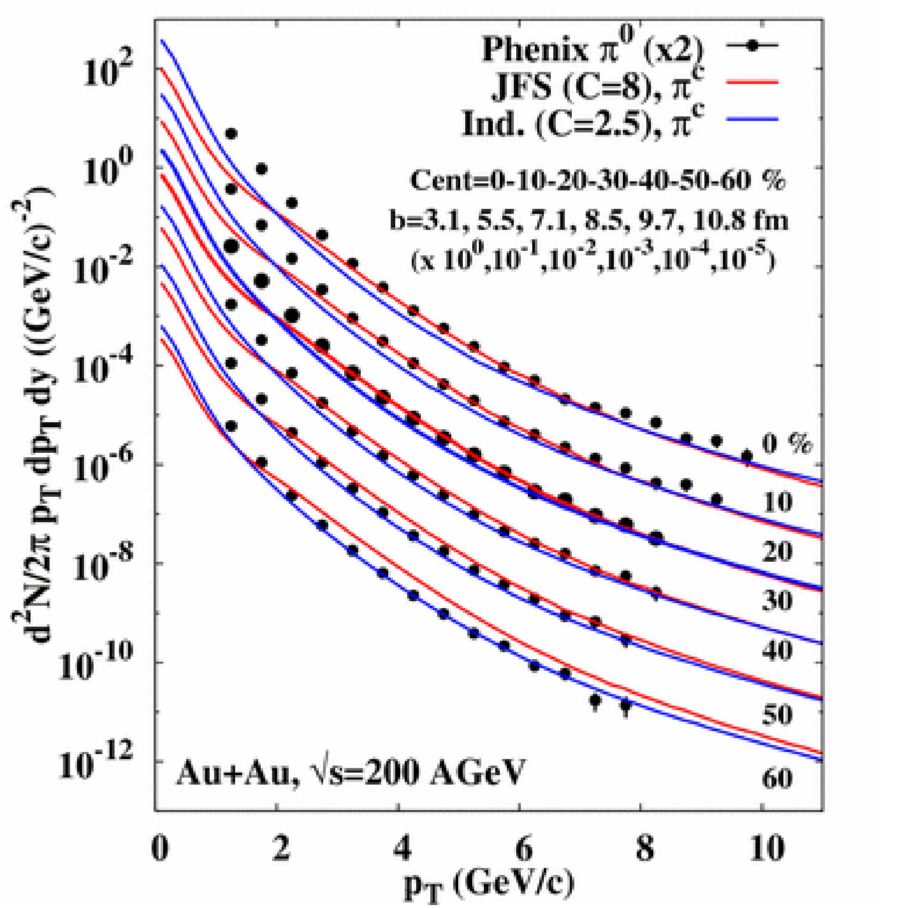
- Compared to pp collisions (and thus to Ind. Frag.), JFS has much smaller mass and decays into fewer hadrons.
  → high pT hadrons are enhanced → Larger E-loss is required



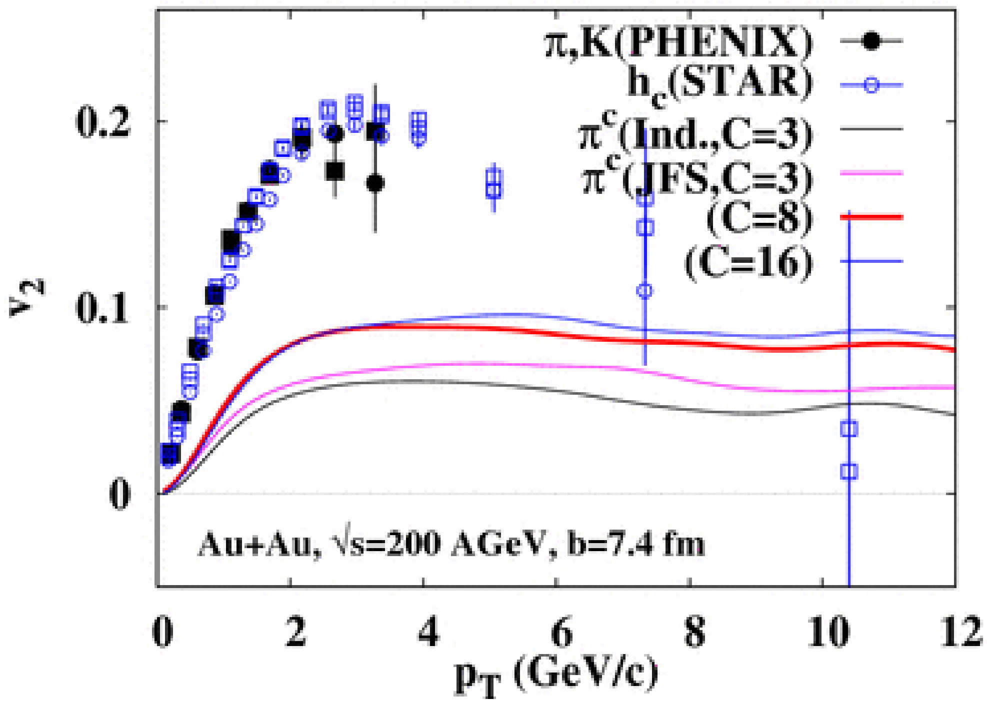
Energy Loss Factor C : p_T Spectrum Fit

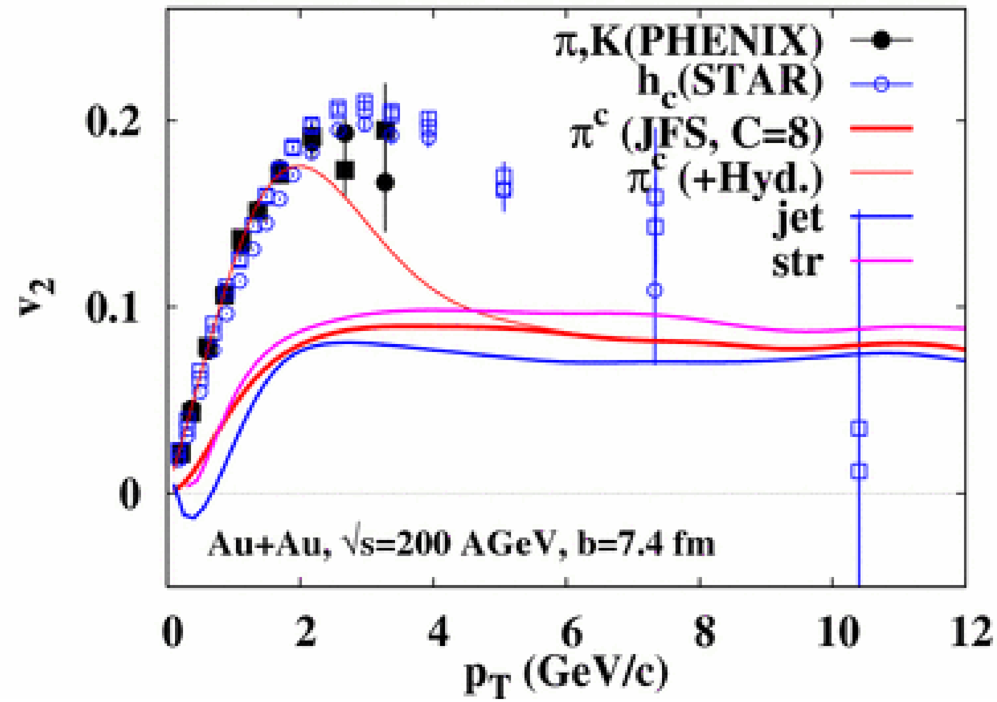
- **For the same C →** $dN_{JFS}(high\ p_T) > dN_{Ind}(high\ p_T)$
- p_T **spec. fit → Ind. Frag.:** *C ≈ (2.5-3)*, **JFS:** *C ≈ 8*
  → ***Large Energy Loss is necessary / allowed in JFS***





- **High pT v_2 : ~ 5 % in Ind. ($C = 3$) ↔ ~ 8 % in JFS ($C = 8$)**

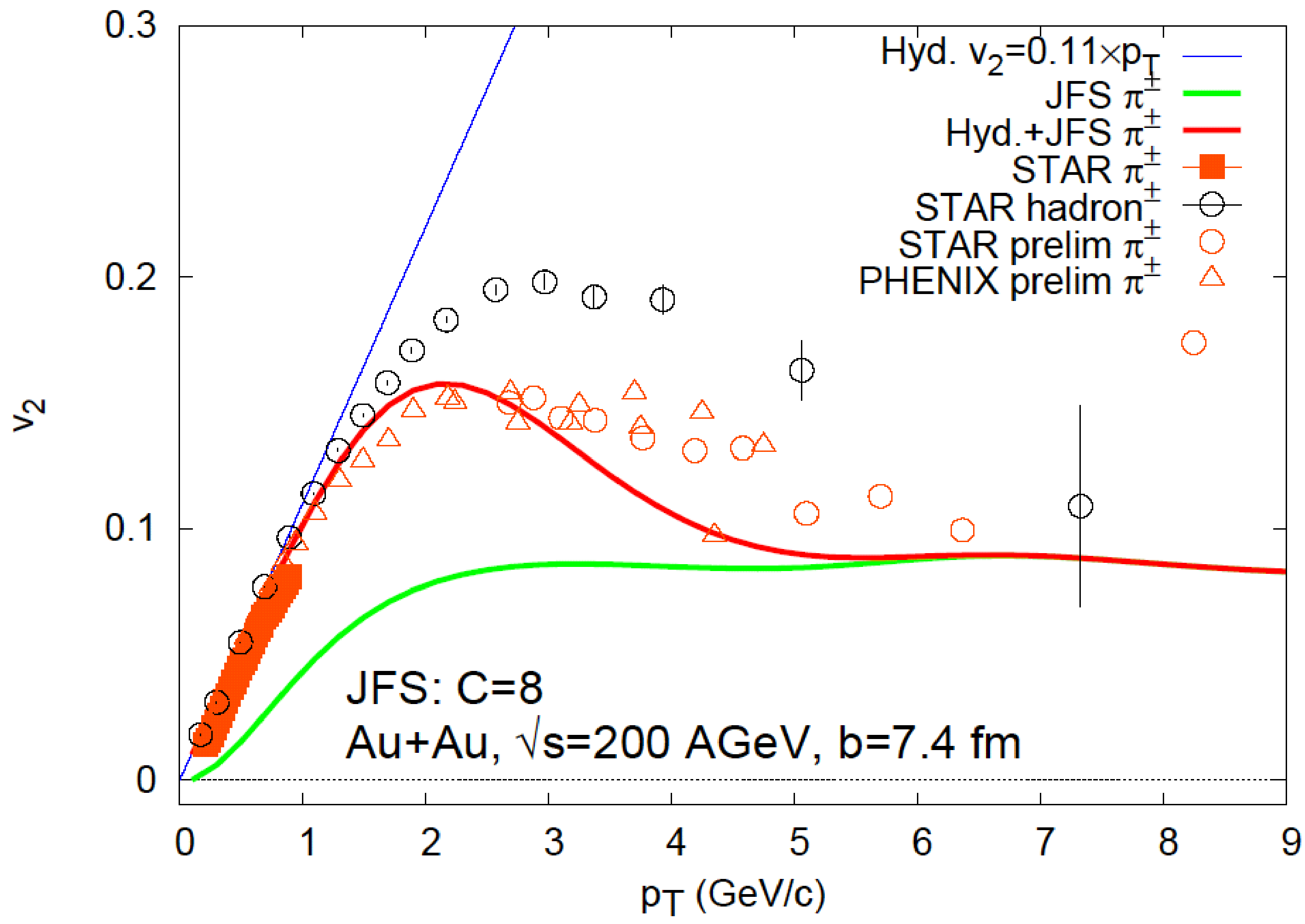




Origin of Large v_2 = Large E-loss factor C + Fluid parton v_2

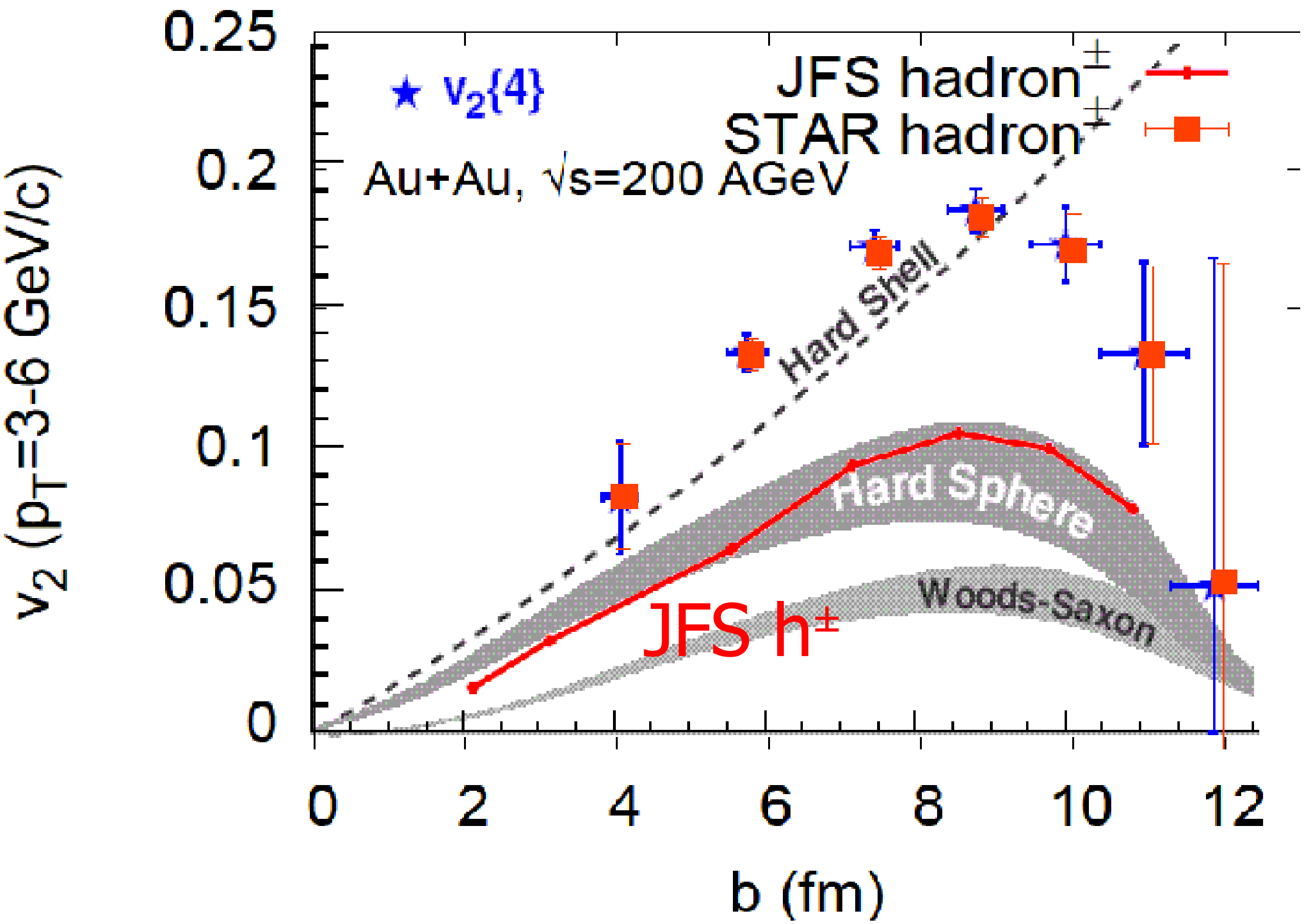
Elliptic Flow of pions

- **Observed pion v_2 at p_T > 5 GeV/c ~ 10 %**
  ↔ v2(JFS, p_T > 5 GeV/c) ~ 8 %



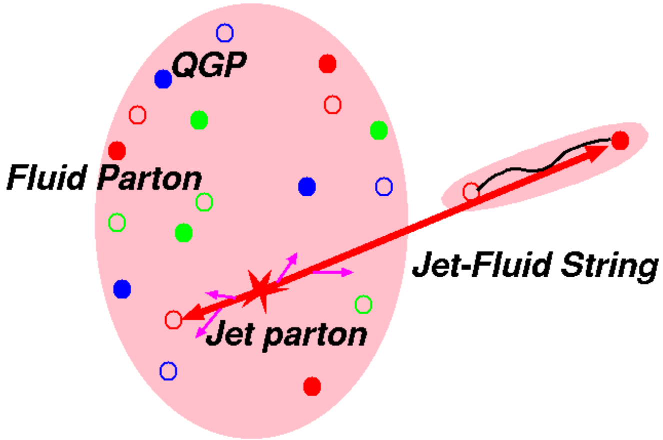
Impact Parameter Dependence

- **Mid-p_T v_2 ($3 < p_T < 6$ GeV/c) in JFS is larger than the "Strong E-loss Limit" with Woods-Saxon profile in Independent Fragmentation, but still smaller than Data.**



JFS Summary

- ***Jet-Fluid String (JFS) formation and decay*** **is proposed as a mechanism to produce high p_T hadrons.**
  - **Effecitve to produce high p_T hadrons**
  - **Event-by-Event Energy-Mom. conservation ↔ Ind. Frag.**
  - **Simple and small mass strings decaying into a few hadrons ↔ Ind. Frag.**
- **When we FIT p_T spectrum,** ***large v_2 emerges at high p_T***
  - **Large E-loss+fluid parton v_2**
- **Problems and Homeworks**
  - **Mechanism of large E-loss**
  - **d+Au fit → Cronin Effects**
  - **s-quarks, string spectral func.**



Backups

Comparison with Previous Works

- **J. Casalderrey-Solana, E.V. Shuryak, hep-ph/0305160**
  - **Quarks, diquarks and gluons in QGP cut color flux (~ JFS).**
  - **Large E-loss is generated by "phaleron"**
  - ***Large E-loss leads "surface emission" → large v_2***
- **Recombination (Duke-Osaka-(Minesota)-Nagoya)**
  - **Predicts large v_2 (~ 10 %) at high-pT**
    - **Sharply edged density dist. → E-loss ∝ L → $v_2 \approx 10$ %**
    - **Woods-Saxon density dist. → $v_2 \approx 5$ %**
  - **Entropy problem: S(QGP) ≈ S(H) requires Res. and Strings**
  - ***Spectral Func.: δ func. ↔ θ func. in JFS***

K-factor

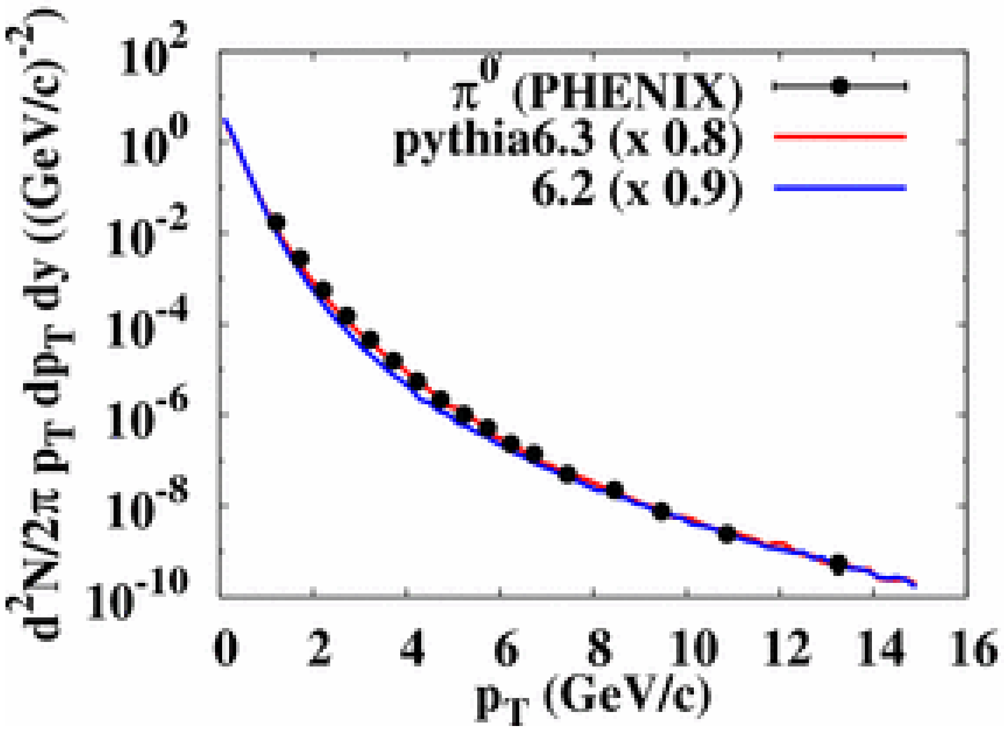
- K-factor → absolute value of σ_{jet}
- Experimental Data: pp → π^0 @ $\sqrt{s_{NN}}$ = 200 GeV (PHENIX)

$$\frac{1}{\sigma^{exp}} \frac{d^2\sigma^{exp}}{2\pi p_T d p_T dy} = K \frac{\sigma^{pQCD(1st)}}{\sigma^{exp}} \frac{d^2 N^{pQCD(1st)}}{2\pi p_T d p_T dy} \qquad A = K \frac{\sigma^{pQCD(1st)}}{\sigma^{exp}}$$

$\sigma^{Exp.}$ = 21.8 mb (trigger)

$\sigma^{pQCD(1st)}$ = 9.9 mb

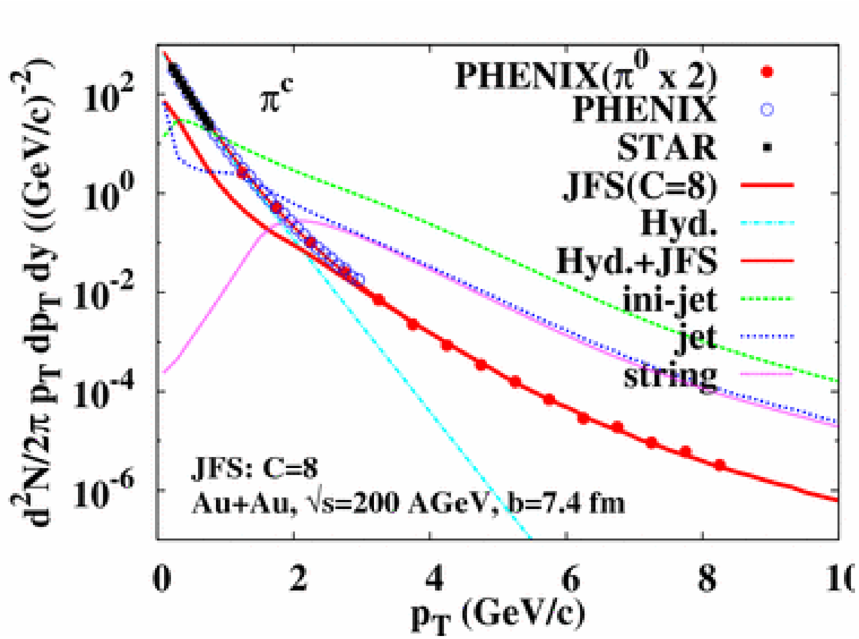
- pythia6.3 fit:
  A ≈ 0.8 → K = 1.8
  (σ_{jet} (p_T^{hard}>2Gev/c)≈ 17.5 mb)
- pythia6.2 fit:
  A ≈ 0.9 → K= 2.0
  (σ_{jet} ≈ 19.6 mb)

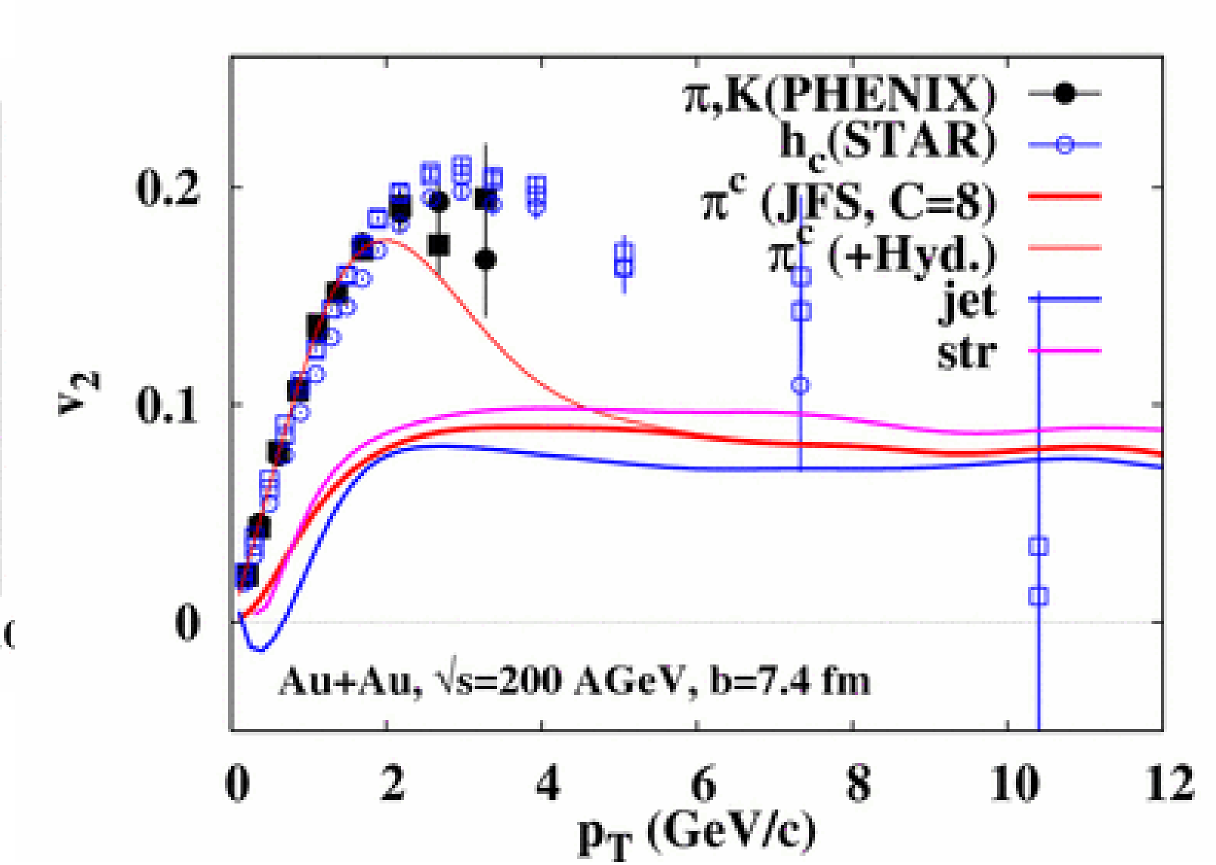


Combined with Low p_T spectrum

- Low pT spectrum is assumed and combined.

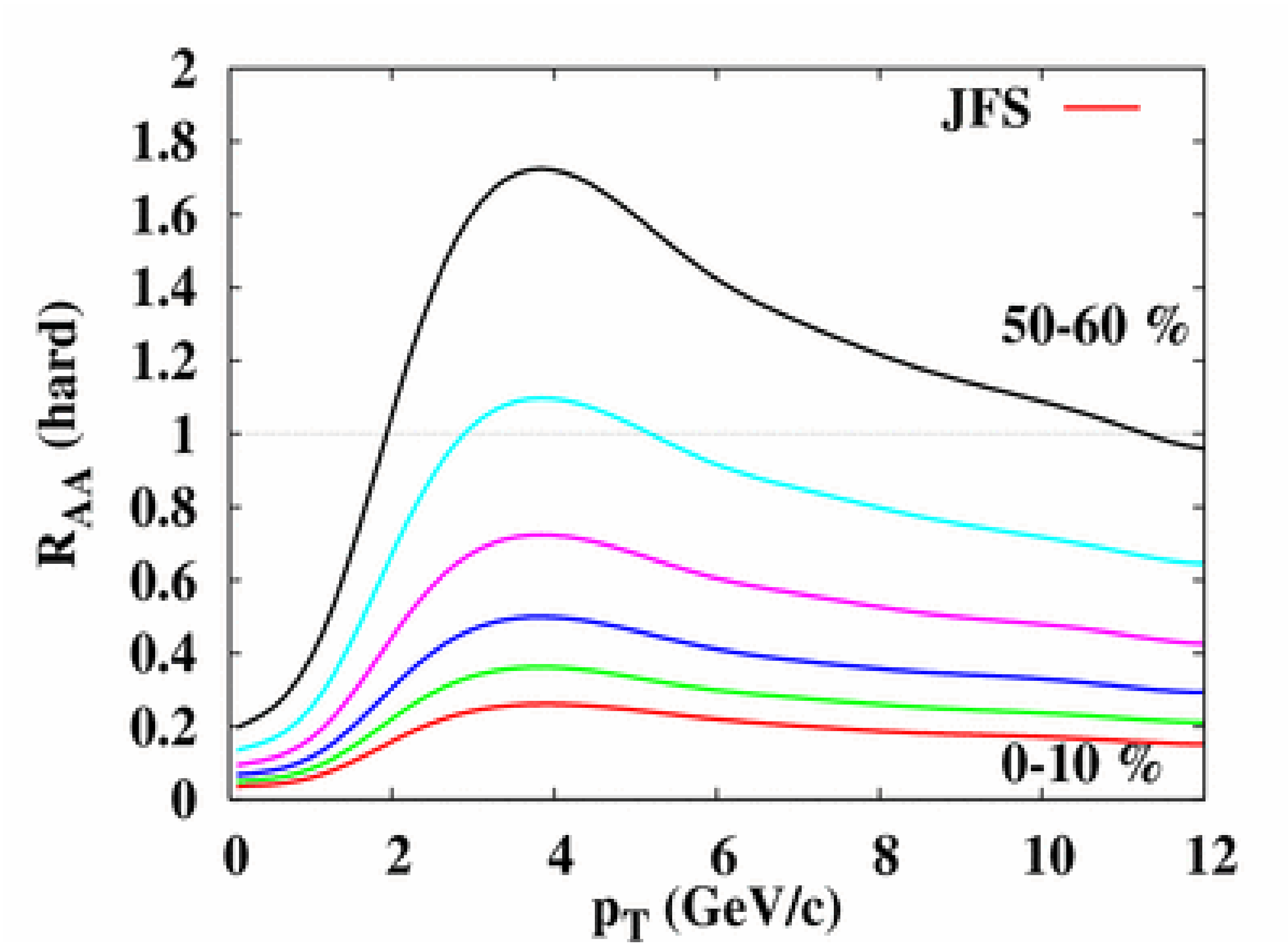
$$E\frac{d^3N_{Hyd}}{dp^3}(p_T)=A\exp(-p_T/T)(1+B/(1+(p_T/p_0)^8)) \qquad v_2^{Hyd}(p_T)=0.14\,p_T$$





Nuclear Modification Factor

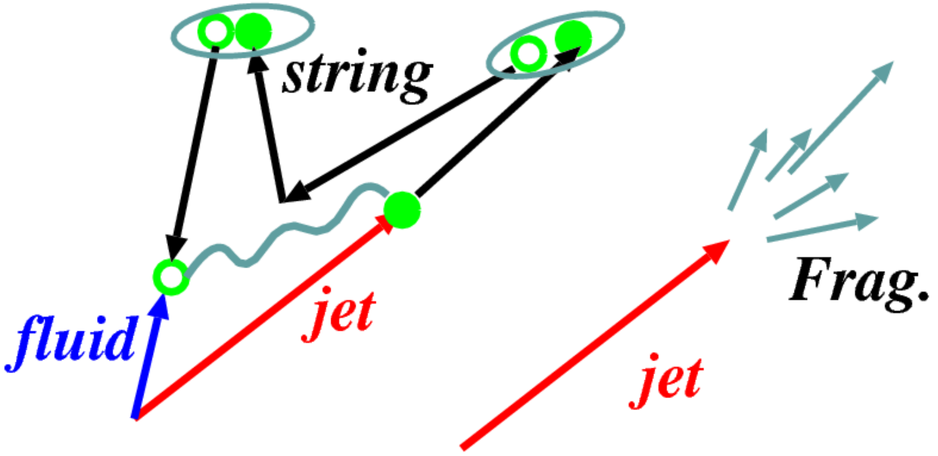
- p_T Deps.



Discussion

- Mechanism to produce high p_T hadrons in JFS
  - String Decay from Lorenz boosted fluid
  - Relative momentum is relatively small
    → Smaller number of hadrons with high p_T are formed

↔ Independent Frag. (Large no. of Low p_T hadrons)



Energy Loss Factor

- Additional Factor for Energy Loss → High p_T hadron yield
- Exp. Data: p_T spectra of π in Au+Au (PHENIX,STAR)

$$\frac{d^2 N^{Exp.}}{2\pi p_T d p_T dy} = N_{jet} \frac{1}{N_{jet}} \frac{d^2 N^{JFS}(C)}{2\pi p_T d p_T dy}$$

→ Determining N_{jet} is important !

Ncoll = 373 @ b=7.4 fm (PHENIX estimate)

σ^{NN}_{jet} = 17.5 mb (pp fit pythia 6.3), σ^{NN}_{tot} = 47. 4 mb (JAM)

$$N_{jet} = \sigma^{NN}_{jet} \int d^{2r}_T T_A(r_T + b/2) T_B(r_T - b/2) = \frac{\sigma^{NN}_{jet}}{\sigma^{NN}_{tot}} N_{coll}$$

$$T_A(r_T) = \int dz \rho(r_T, z)$$

Further Problems

- Very large energy loss is required to explain p_T spectrum.
  - C ≈ 8 in JFS ↔ C ≈ 2.7 in Hydro+Jet model (Hirano-Nara)

  Is it possible to justify this large energy loss ?
- Elliptic flow at medium pT is underestimated.
  → Fluid-Fluid String would be necessary to consider.
- Large baryon yield at medium pT may not be explained.
  → Three parton string ? (Jet-Fluid-Fluid, Fluid-Fluid-Fluid)
- String formation probability should be evaluated
  in pQCD matrix element + string level density.
- Strange hadrons