高密度物質と中性子星の物理：講義の内容

1. 中性子星の基本的性質
   - 質量･半径測定の概論など
2. 状態方程式を記述する理論模型
   - 平均場理論、第一原理計算手法、場の理論によるアプローチ
3. 対称エネルギーと非対称核物質の状態方程式
   - 対称エネルギーを決める実験手法、現在の制限
4. QCD 有効模型と高密度核物質の性質
   - 有限温度･密度の場の理論入門 (松原和･摂動論など)
   - NJL 模型による相転移と状態方程式の記述
5. ハイパー核物理と中性子星でのハイペロンパズル
   - ハイパー核実験の現状、ハイペロンパズルの解決に向けて

途中に入れます

- 談話会

  Symmetry Parameter Constraints
  from a Lower Bound on the Neutron-Matter Energy

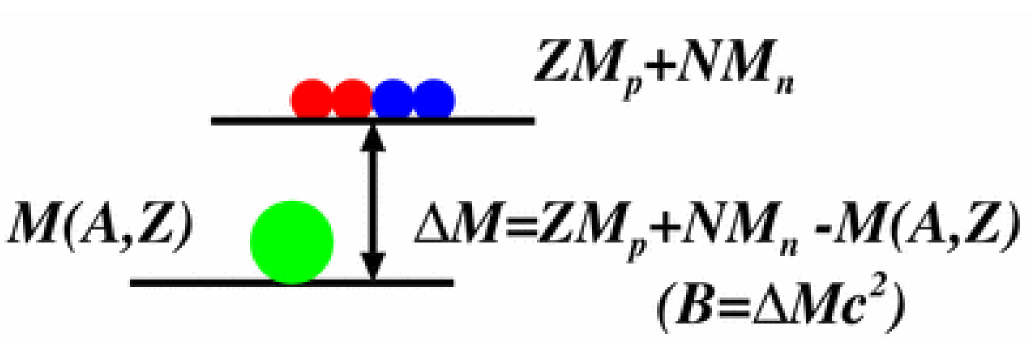
核物質の状態方程式

原子核の束縛エネルギー

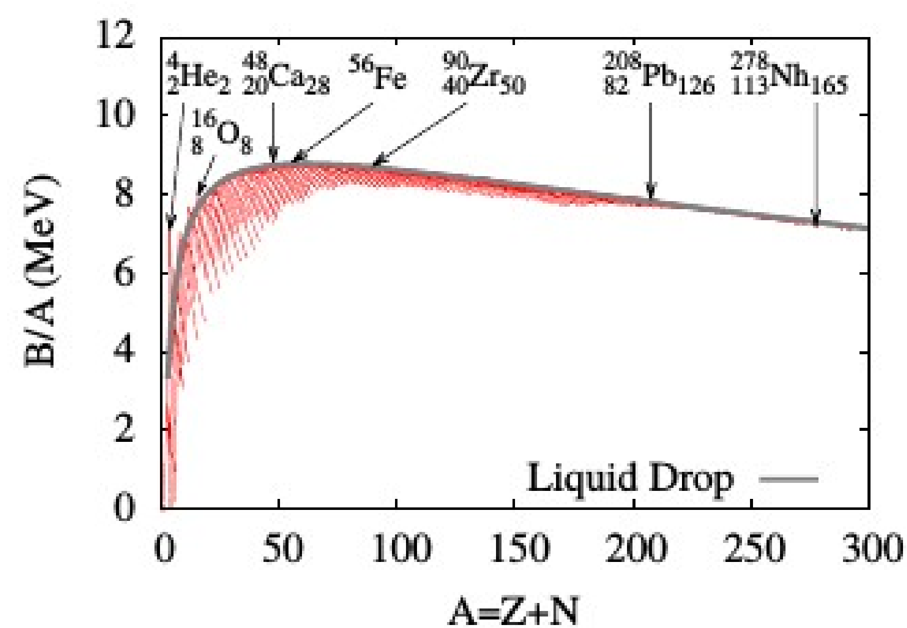
- 束縛エネルギー

$$B(A,Z) = ZM_p + NM_n - M(A,Z)$$

  - 陽子数 Z, 中性子数 N, 陽子質量 M_p, 中性子質量 M_n, 原子核質量 M(A,Z)
  - 原子核の質量は、核子の質量の和より小さい（質量欠損）
- 束縛エネルギーの観測値 : $16 \leq A \leq 240$ において、B/E ~ 8 MeV



質量欠損 = 核子質量の和 − 原子核の質量
束縛エネルギー = 質量欠損 x c^2



質量公式

- **Weizsäcker の半経験的質量公式**

$$B(A,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} + a_p \frac{\delta_p}{A^\gamma}$$

体積　表面　クーロン　対称エネルギー　対エネルギー

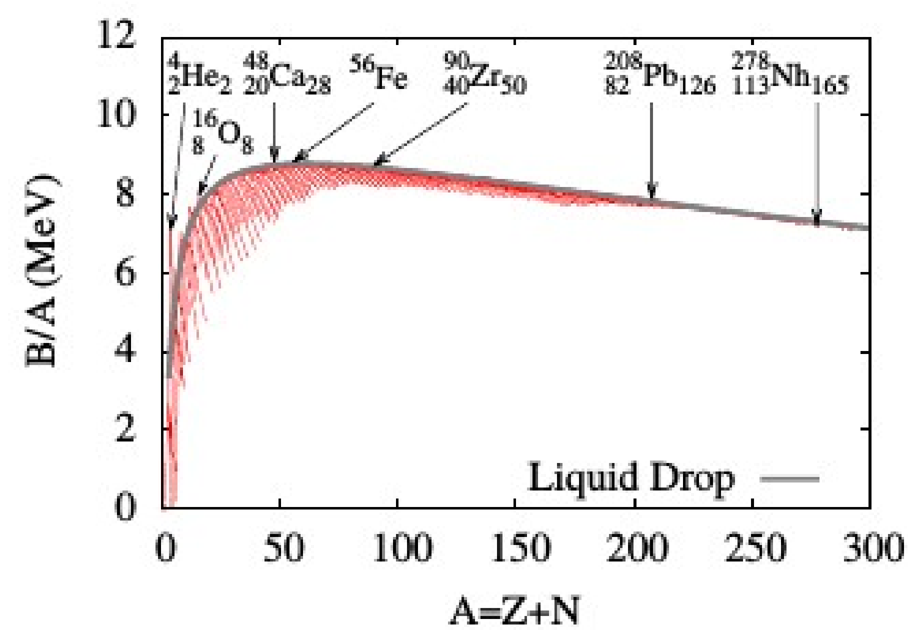
- **体積項、表面項 → 表面張力のある液滴**
- **一様帯電球 (半径 R = r_0 $A^{1/3}$, 電荷 Q=Ze) のクーロンエネルギー**

$$E_C = \frac{3}{5}\frac{\alpha\hbar c}{r_0}\frac{Z^2}{A^{1/3}}$$

- **対称エネルギー、対エネルギーは液滴描像からは出てこない。**

a_v	a_s	a_C	a_a	a_p
15.85	18.34	0.71	23.21	12.0

単位 MeV (γ=1/2 の場合)



質量公式と状態方程式

- **A → ∞ における核子あたりのエネルギー (クーロンエネルギーは無視)**

$$E = \lim_{A\to\infty} \frac{-B(A,Z)}{A} = \lim_{A\to\infty}\left[-a_v + a_s A^{-1/3} + a_a \frac{(N-Z)^2}{A^2} - a_p \frac{\delta_p}{A^{\gamma+1}}\right]$$

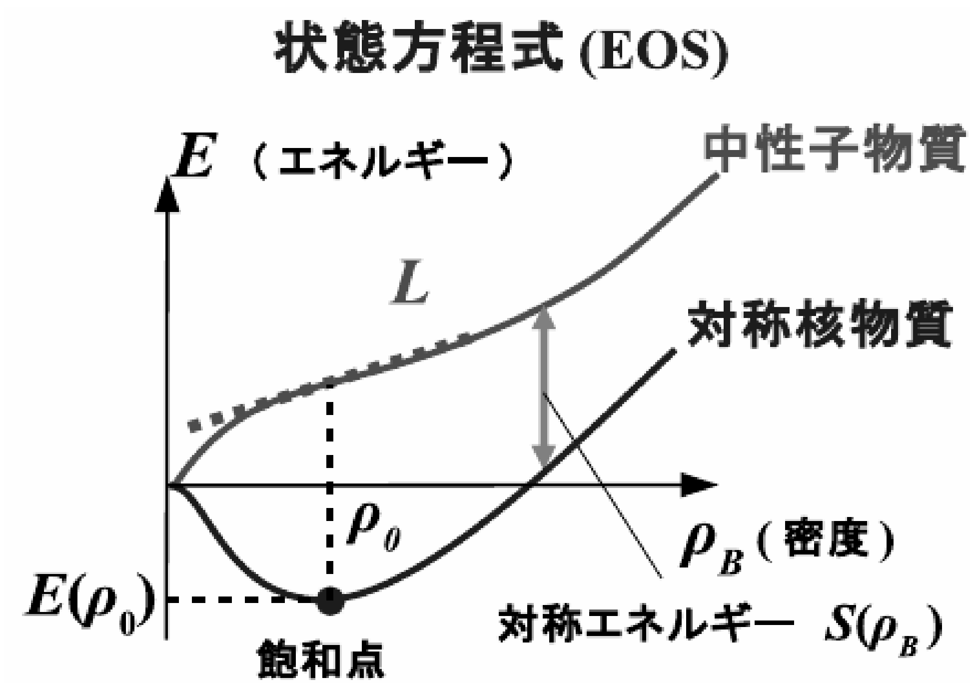
$$= -a_v + a_a\alpha^2 \quad (\alpha = (N-Z)/A)$$

- **非対称度が決まっているとき、基底状態では核子あたりのエネルギーが最小となる密度が実現する**

  → 核物質の飽和性

- **飽和点**

$$(\rho_0, E_0) \simeq (0.16\ \mathrm{fm}^{-3}, -16\ \mathrm{MeV})$$



対称エネルギー

- **非対称核物質 (N ≠ Z) のエネルギー**

$$E(\rho_\mathrm{B},\alpha) = E(\rho_\mathrm{B},\alpha=0) + S(\rho_\mathrm{B})\alpha^2$$

- **対称エネルギー**

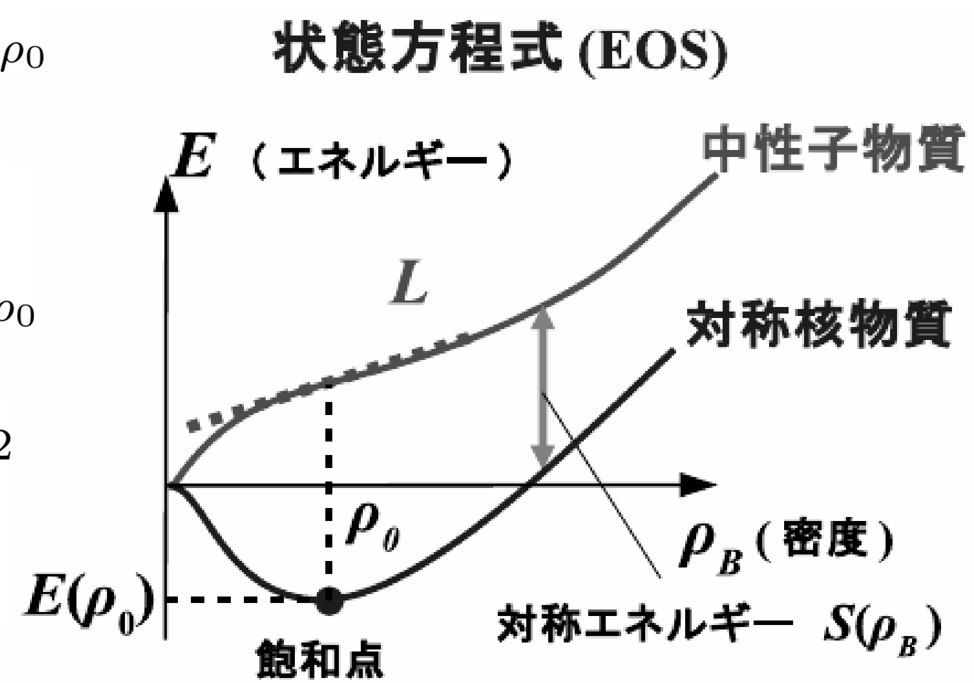
  $S(\rho_B)$ = E(中性子物質)- E(対称核物質)

- **飽和密度でのパラメータ**
  - **非圧縮率** $K \equiv 9\,\rho_0^2 \frac{\partial^2 E(\rho_\mathrm{B})}{\partial \rho_\mathrm{B}^2}\Big|_{\rho_\mathrm{B}=\rho_0}$
  - **対称エネルギーの値と微分**

$$S_0 \equiv S(\rho_0)\ , \quad L \equiv 3\rho_0 \frac{dS(\rho_\mathrm{B})}{d\rho_\mathrm{B}}\Big|_{\rho_\mathrm{B}=\rho_0}$$

$$E(\rho_\mathrm{B},\alpha) \simeq E_0 + S_0\,\alpha^2 + \frac{L}{3}\,x\,\alpha^2 + \frac{K}{18}\,x^2$$

$$(x = (\rho_\mathrm{B} - \rho_0)/\rho_0)$$

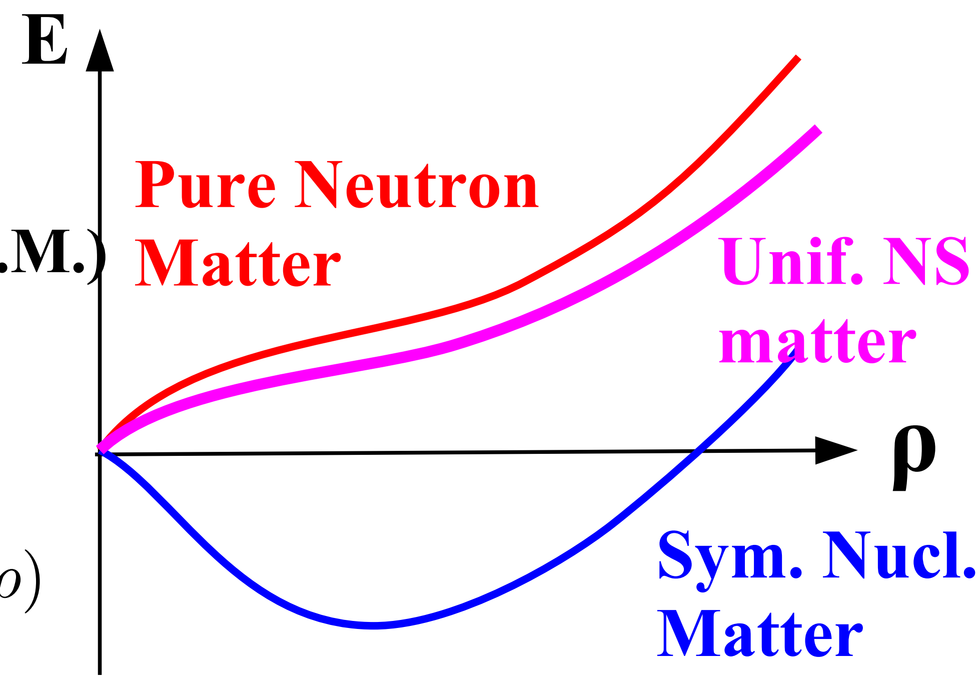


Neutron Star Matter EOS

- **What happens in low-density uniform neutron star matter ?**
  - **Constituents = proton, neutron and electron**
  - **Charge neutrality → # of electons= # of protons ($\rho_e=\rho_p=\rho(1-\alpha)/2$)**

$$E_{\rm NSM}(\rho) = E_{\rm NM}(\rho,\alpha) + E_e(\rho_e=\rho_p)$$
$$= E_{\rm SNM}(\rho) + \alpha^2 S(\rho) + \frac{\Delta M}{2}\alpha + \frac{3}{8}\hbar k_F (1-\alpha)^{4/3}$$

(electron mass neglected, neutron-proton mass diff. incl. k_F= Fermi wave num. in Sym. N.M.)



- **δ is optimized to minimize energy per nucleon**

$$E_{\rm NSM}(\rho) \leq E_{\rm NM}(\rho,\alpha=1) = E_{\rm PNM}(\rho)$$

中性子星物質 EOS

- **核物質 → 中性子星物質**
  (電子のエネルギーも考慮、電子質量・np の質量差無視)

$$E_{nsm}(u) = E_{snm}(u) + \alpha^2 S(u) + \frac{3}{8}\hbar(3\pi^2 n_0 u/2)^{1/3}(1-\alpha)^{4/3}$$

$$Y_p(u) = (1-\alpha(u))/2 = \left[(A(u)+1)^{1/3} - (A(u)-1)^{1/3}\right]^3 /4$$

$$A(u) = \sqrt{1 + \pi^2\hbar^3 n_0 u/288 S^3(u)}$$

対称核物質エネルギーと対称エネルギーが分かれば、
(電子質量とnp 質量差を無視すれば)
中性子星物質 EOS は3 行で書ける！

現象論的な核物質状態方程式

- **相互作用エネルギー**

$$V_{2B} = \frac{1}{2}\int d^3r\, d^3r' \rho_{\rm B}(\mathbf{r}) v(\mathbf{r}-\mathbf{r}') \rho_{\rm B}(\mathbf{r}') \to A \times \frac{\alpha}{2}\left(\frac{\rho_{\rm B}}{\rho_0}\right)$$

$$V_{3B} = \frac{1}{3}\int d^3r\, d^3r'\, d^3r''\, v(\mathbf{r},\mathbf{r}',\mathbf{r}'') \rho_{\rm B}(\mathbf{r}) \rho_{\rm B}(\mathbf{r}') \rho_{\rm B}(\mathbf{r}'') \to A \times \frac{\beta}{3}\left(\frac{\rho_{\rm B}}{\rho_0}\right)^2$$

（一様密度、ゼロレンジの2体力・3体力）

- **現象論的な状態方程式**
  - **対称核物質**

$$E(\rho_{\rm B}) = \frac{3}{5}E_F(\rho_{\rm B}) + \frac{\alpha}{2}\left(\frac{\rho_{\rm B}}{\rho_0}\right) + \frac{\beta}{2+\gamma}\left(\frac{\rho_{\rm B}}{\rho_0}\right)^{1+\gamma}$$

  - **対称エネルギー**

$$S(\rho_{\rm B}) = \frac{1}{3}E_F(\rho_{\rm B}) + \alpha_{\rm sym}\left(\frac{\rho_{\rm B}}{\rho_0}\right) + \beta_{\rm sym}\left(\frac{\rho_{\rm B}}{\rho_0}\right)^{\gamma_{\rm sym}}$$

Simple parametrized EOS

- **Skyrme int. motivated parameterization**

$$E_{\rm SNM} = \frac{3}{5}E_F(\rho) + \frac{\alpha}{2}\left(\frac{\rho}{\rho_0}\right) + \frac{\beta}{2+\gamma}\left(\frac{\rho}{\rho_0}\right)^{1+\gamma}$$

$$\alpha = \frac{2}{\gamma}\left(E_0(1+\gamma) - \frac{E_F(\rho_0)(1+3\gamma)}{5}\right) \ , \quad \beta = \frac{2+\gamma}{\gamma}\left[-E_0 + \frac{1}{5}E_F(\rho_0)\right] \ .$$

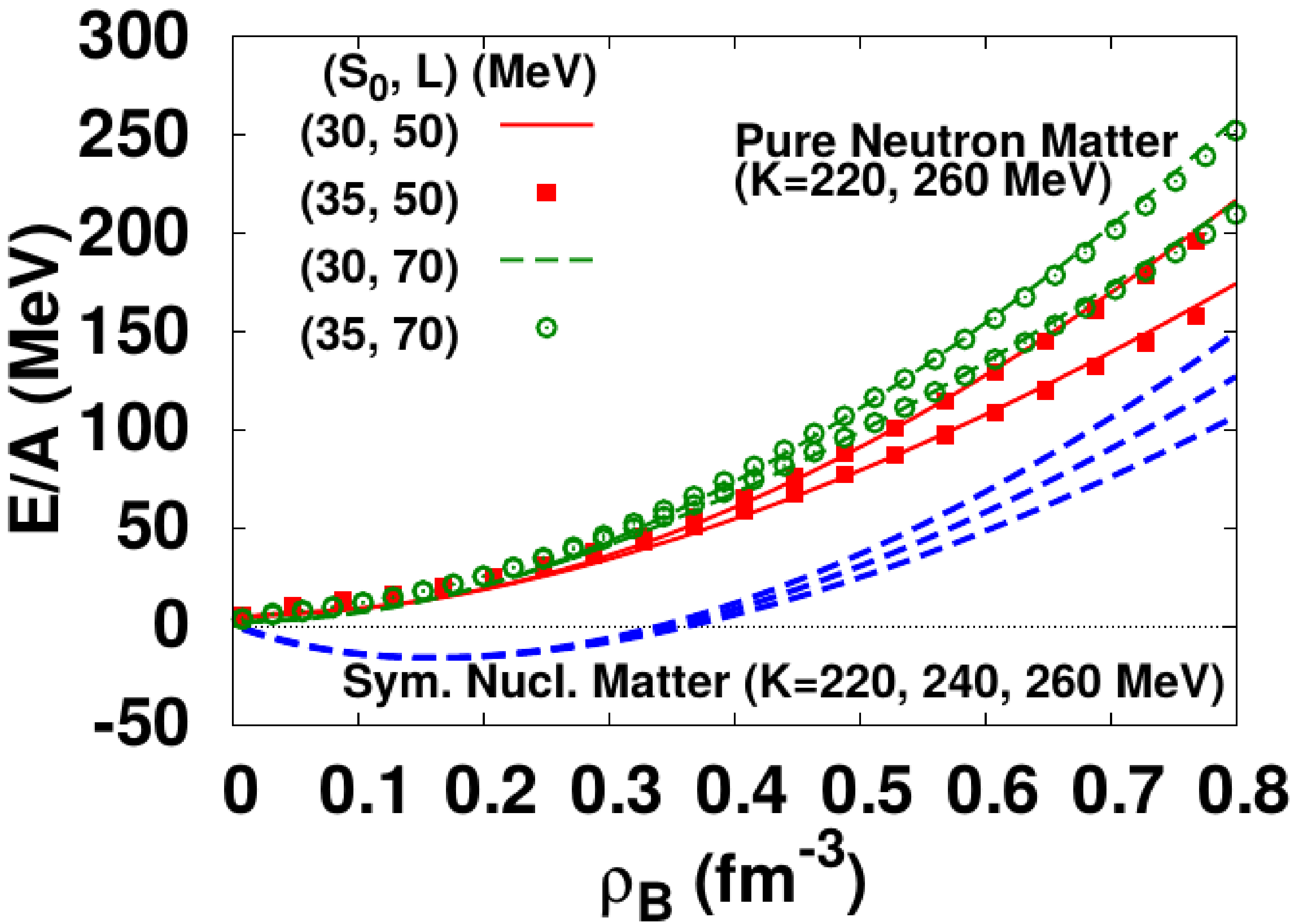
$$K = \frac{3(1+3\gamma)}{5}E_F(\rho_0) - 9E_0(1+\gamma) \ .$$

- **Symmetry energy parameterization**

$$S(\rho) = \frac{1}{3}E_F(\rho) + \left[S_0 - \frac{1}{3}E_F(\rho_0)\right]\left(\frac{\rho}{\rho_0}\right)^{\gamma_{\rm sym}}$$

$$\gamma_{\rm sym} = \frac{L - \frac{2}{3}E_F(\rho_0)}{3S_0 - E_F(\rho_0)}$$

現象論的な核物質状態方程式



飽和密度近辺での不定性は少ないが、
中性子物質・高密度では大きな不定性

さて、このように「現象論的」にEOSを
密度の関数として与えることに意味はあるのか？

→ 多分 Yes. その正当化の根拠は「密度汎関数理論」

状態方程式と原子核の構造

- 密度汎関数理論
  - 与えられた密度における多体系の基底状態エネルギーは密度の汎関数で与えられる。 *P. Hohenberg, W. Kohn ('64)*

$$E_{gs} = \min_{\Psi} \langle \Psi \mid \hat{H} \mid \Psi \rangle = \min_{\rho} \left[\min_{\Psi} \langle \Psi \mid \hat{H} \mid \Psi \rangle_{\rho} \right]$$

$$= \min_{\rho} \; F[\rho]$$

  - 相互作用する系の密度は、一体ポテンシャル中を運動する自由粒子系の密度として計算できる。 *W.Kohn, L.J.Sham ('65)*

$$-\frac{\hbar^2}{2m} \nabla^2 \varphi_i(\mathbf{r}) + U_{\rm eff}(\mathbf{r}) \varphi_i(\mathbf{r}) = \varepsilon_i \varphi_i(\mathbf{r})$$

$$F[\rho] = E_{\rm H}[\rho] + \underline{E_{\rm xc}[\rho]} \;, \quad U_{\rm eff}(\mathbf{r}) = U_{\rm H}(\mathbf{r}) + \frac{\delta E_{\rm xc}}{\delta \rho(\mathbf{r})}$$

Exchange-Correlation E.

  - 密度汎関数 → 状態方程式

$$E(\rho) = \min_{\rho'} F[\rho']|_{\rho'_{\rm ave} = \rho}$$

スキルム力による密度汎関数

- **Skyrme interaction (c.f. Ring-Schuck text)**
  - **ゼロレンジ相互作用 + 微分 (有限レンジ効果)+ 密度依存力**

$$
\begin{aligned}
v(\mathbf{r}_1,\mathbf{r}_2) =& t_0(1+x_0P_\sigma)\delta(\mathbf{r}_1-\mathbf{r}_2) \\
&+\frac{1}{2}t_1(\mathbf{k}^2\delta(\mathbf{r}_1-\mathbf{r}_2)+\delta(\mathbf{r}_1-\mathbf{r}_2)\mathbf{k}^2)+t_2\mathbf{k}\delta(\mathbf{r}_1-\mathbf{r}_2)\mathbf{k} \\
&+\frac{1}{6}t_3\delta(\mathbf{r}_1-\mathbf{r}_2)\rho^\alpha((\mathbf{r}_1+\mathbf{r}_2)/2) \\
&+iW_0(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\mathbf{k}\times\delta(\mathbf{r}_1-\mathbf{r}_2)\mathbf{k} \\
\mathbf{k} =& (\boldsymbol{\nabla}_1-\boldsymbol{\nabla}_2)/2i
\end{aligned}
$$

  - **エネルギー密度 (spin, isospin が飽和している場合)**

$$
\begin{aligned}
H(\mathbf{r}) =& \frac{\hbar^2}{2m}\tau(\mathbf{r})+\frac{3}{8}t_0\rho^2+\frac{1}{16}t_3\rho^{2+\alpha}+\frac{1}{16}(3t_1+5t_2)\rho\tau \\
&+\frac{1}{64}(9t_1-5t_2)(\boldsymbol{\nabla}\rho)^2-\frac{3}{4}W_0\rho\boldsymbol{\nabla}\cdot\mathbf{J}+\frac{1}{32}(t_1-t_2)\mathbf{J}^2
\end{aligned}
$$

$$
\tau=\sum|\boldsymbol{\nabla}\varphi_i|^2\ ,\quad \mathbf{J}=-i\sum\varphi_i^*(\mathbf{r},s,t)\boldsymbol{\nabla}\varphi_i(\mathbf{r},s',t)\times\boldsymbol{\sigma}_{ss'}
$$

広い質量領域の原子核の性質からパラメータを決定
→ 密度汎関数 F=∫ dr H(r)

メモ：スピン・アイソスピン・ファクター

- **3/8 等のファクターはどこから現れる？**
  → spin, isospin についての和

$$V_{\rm HF}(t_0{\rm term}) = \frac{1}{2} t_0 \sum_{i,j} \int d\mathbf{r}_1 d\mathbf{r}_2 \langle ij \mid (1 + x_0 P_\sigma)\delta(\mathbf{r}_1 - \mathbf{r}_2) \mid ij - ji \rangle$$

$$= \frac{1}{2} t_0 \sum_{n,m} \int d\mathbf{r} |\varphi_n(\mathbf{r})|^2 |\varphi_m(\mathbf{r})|^2 \times X = \frac{X}{32} t_0 \int d\mathbf{r} \rho^2$$

$$X = \sum_{s,t} \langle s_1 t_1 s_2 t_2 \mid (1 + x_0 P_\sigma) | s_1 t_1 s_2 t_2 - s_2 t_2 s_1 t_1 \rangle$$

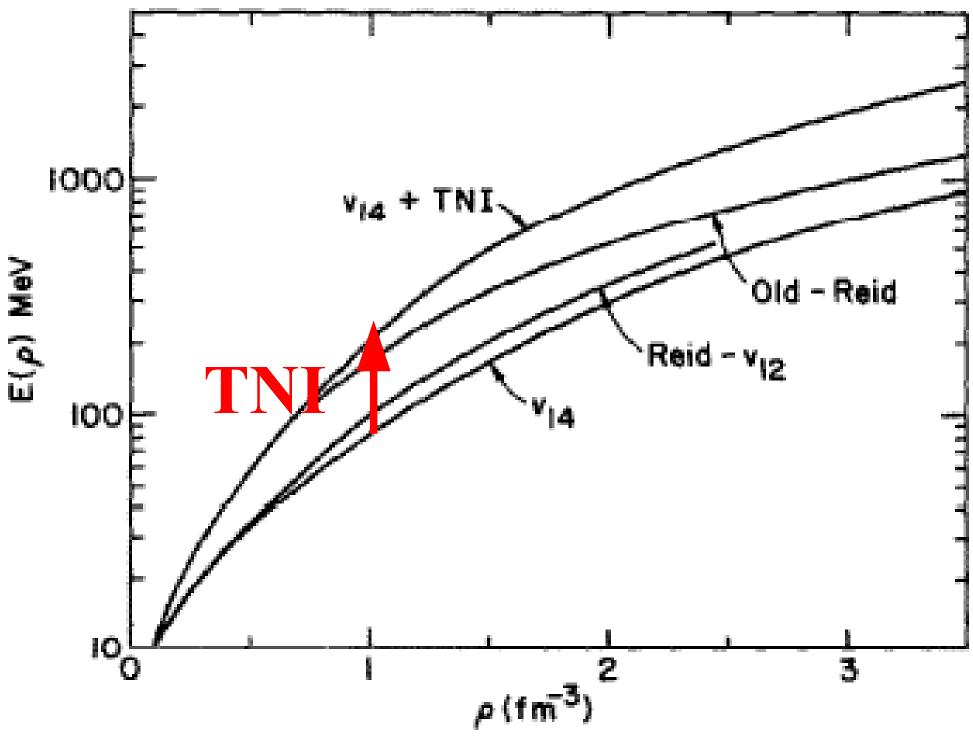
$$= 4 \times (4 + 2x_0 - 1 - 2x_0) = 12$$

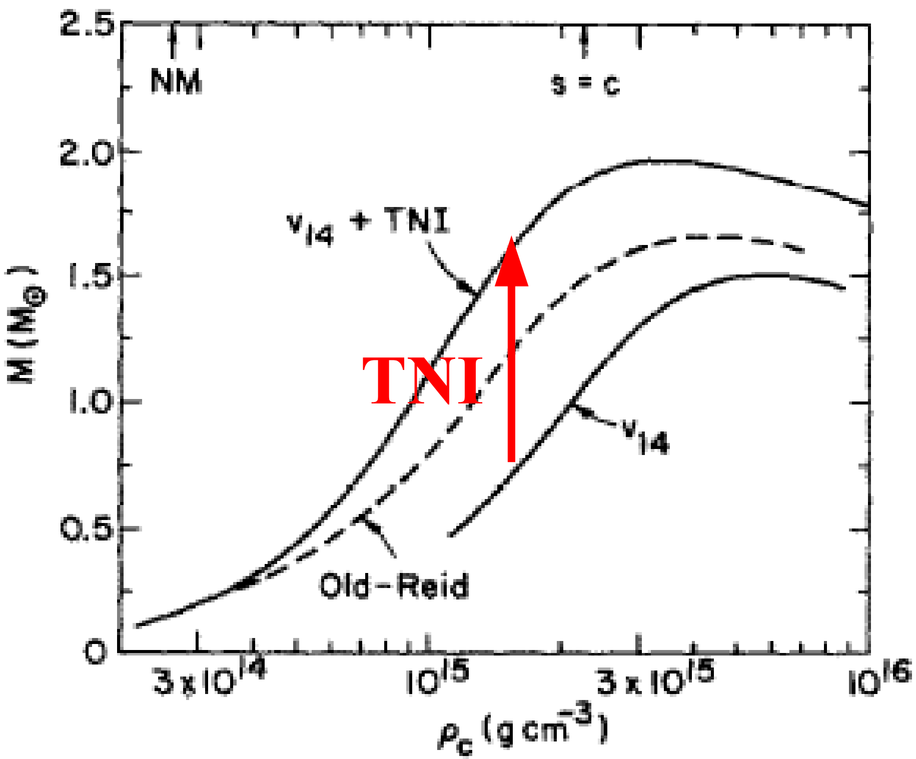
状態方程式を記述する理論の枠組み

Theories/Models for Nuclear Matter EOS

- **Ab initio Approaches: Start from QCD or bare NN force**
  - **Lattice QCD (sign problem),**
    Green's Function Monte-Carlo (GFMC), Variational methods,
    Bruckner Hartree-Fock (G-matrix), Dirac-Bruckner HF,
    Many-body perturbation with Chiral Effective Field Theory, ...
- **Mean Field from Effective Interactions ~ Nucl. Dens. Fuctionals**
  - **Skyrme Hartree-Fock(-Bogoliubov)**
    - **Non.-Rel.,Zero Range, Two-body + Three-body (or ρ-dep. two-body)**
    - **In HFB, Nuclear Mass is very well explained (Total B.E. ΔE ~ 0.6 MeV)**
    - **Causality is violated at very high densities.**
  - **Relativistic Mean Field**
    - **Relativistic, Meson-Baryon coupling, Meson self-energies**
    - **Successful in describing pA scatering (Dirac Phenomenology)**

- **Variational Calculation starting from bare nuclear force**
  B. Friedman, V.R. Pandharipande, NPA361('81)502
  - **Argonne v14 + TNI (TNR+TNA)**
    (TNI/TNR/TNA: three-nucleon int./repulsion/attraction)



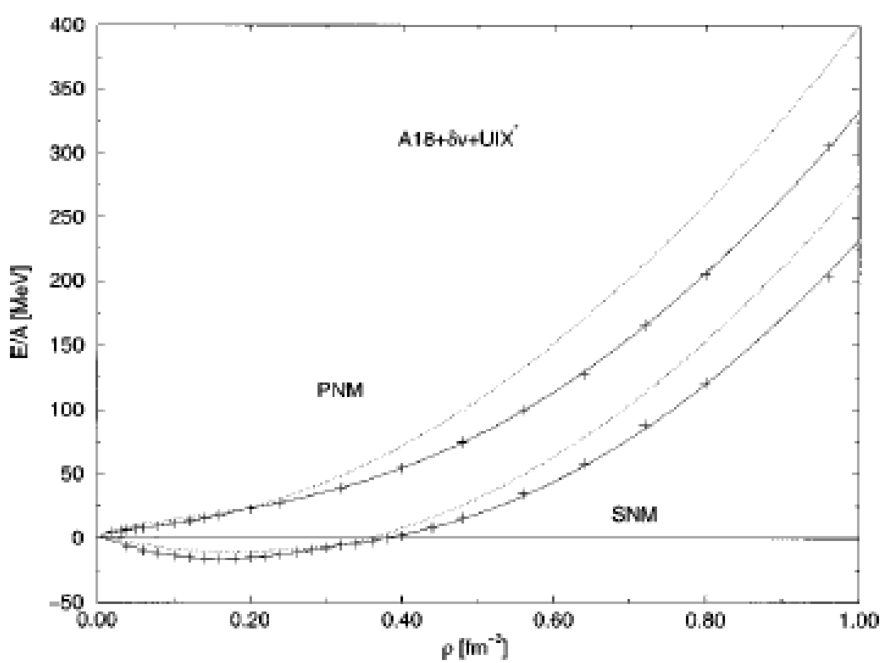


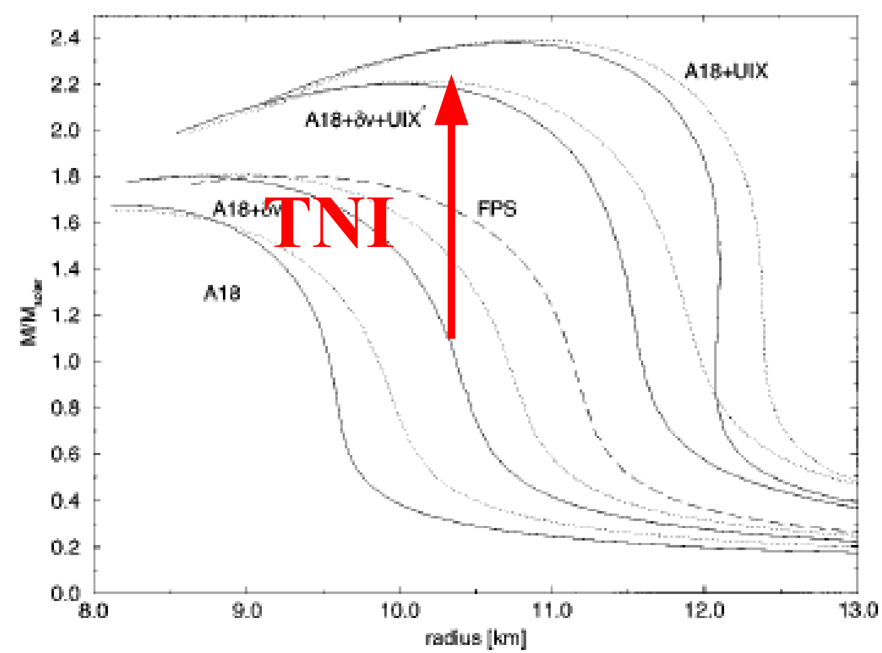
Variational Calculation (2)

- **Variational chain summation method**

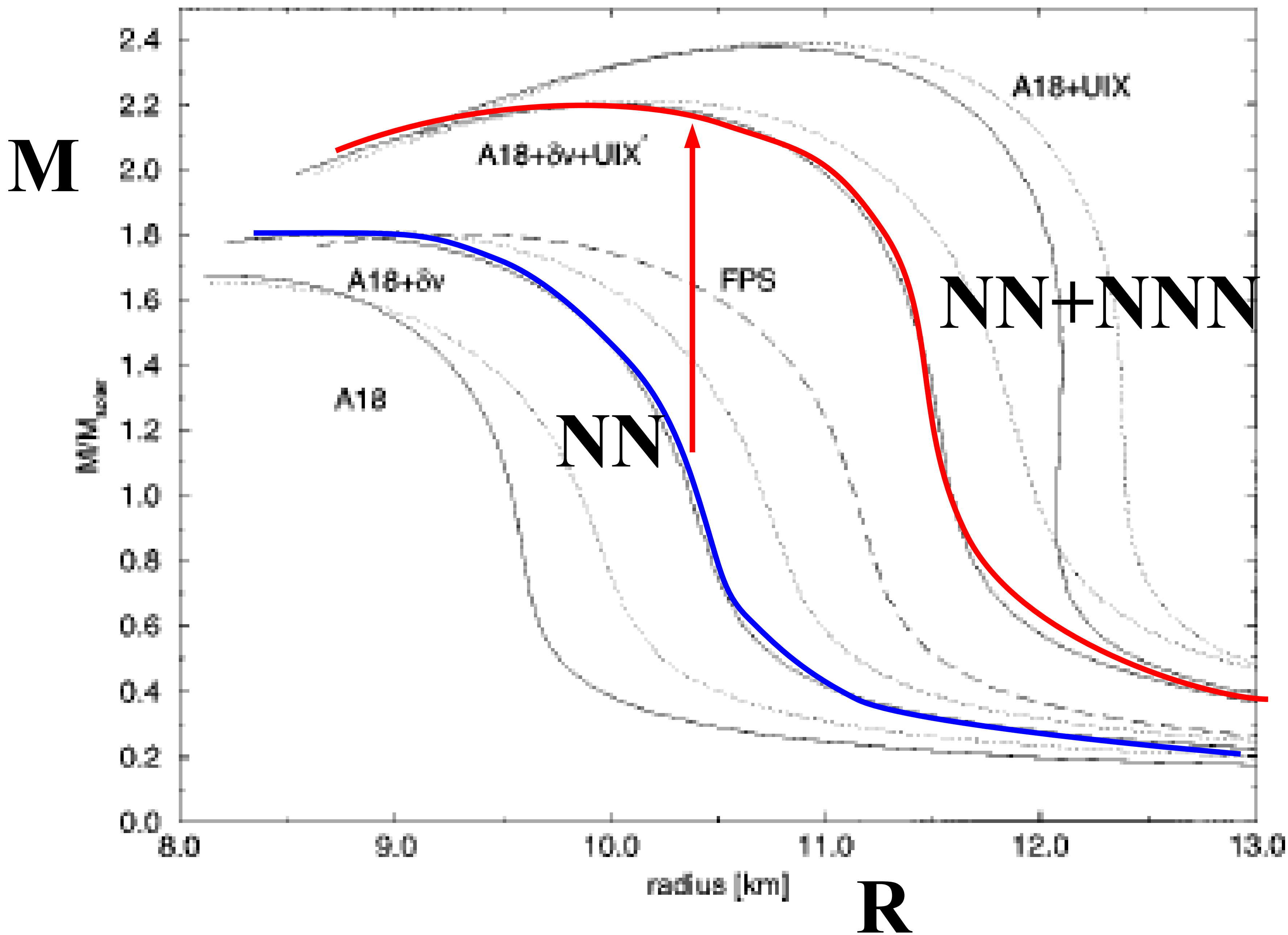
  A. Akmal, V.R.Pandharipande, D.G. Ravenhall, PRC58('98)1804

  - **v18, relativistic correction, TNI**
  - **Existence of neutral pion condensation at $\rho_B > 0.2\ \mathrm{fm}^{-3}$**



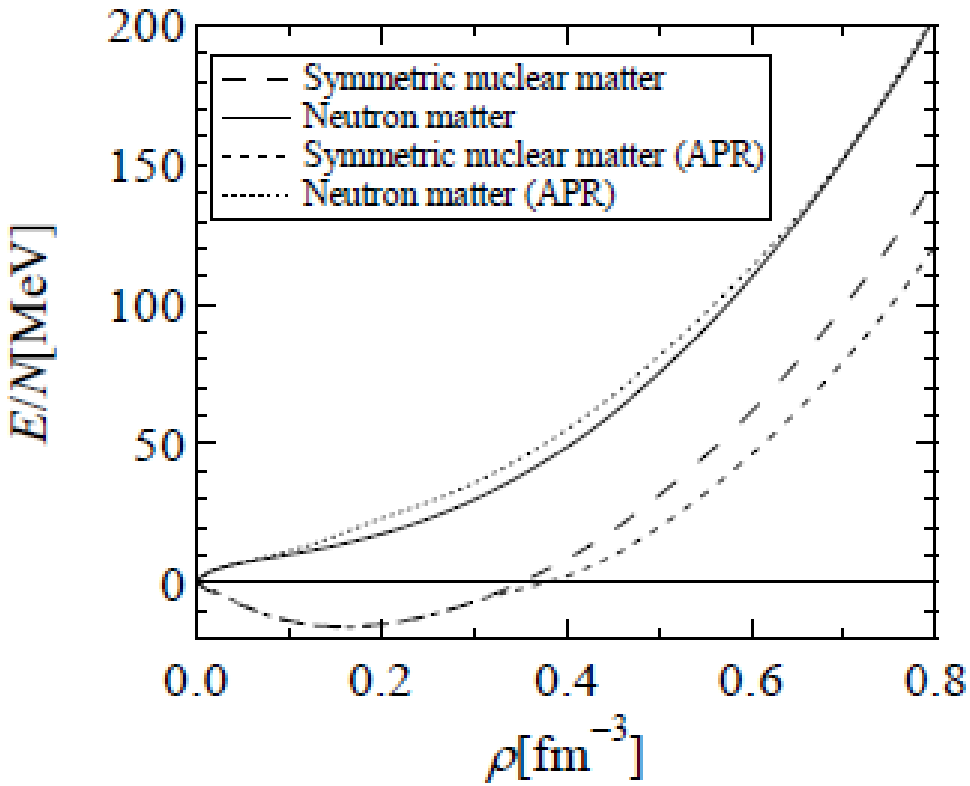


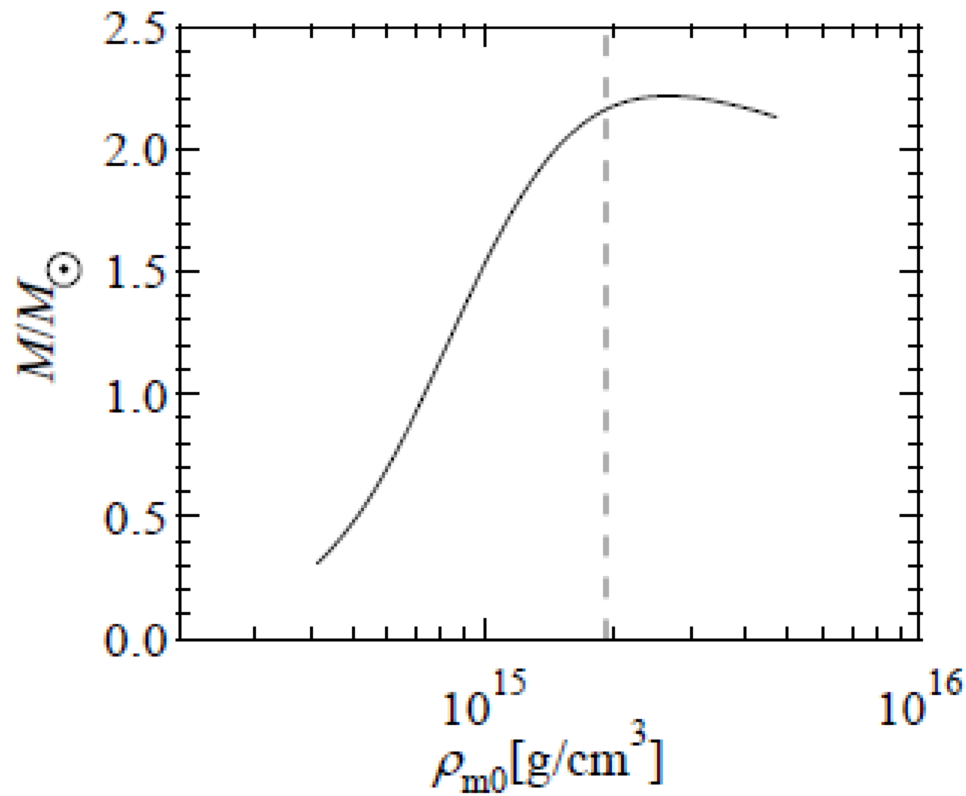
APR


M
R
NN
NN+NNN
A18+UIX
A18+δv+UIX*
A18+δv
FPS
A18
M/M_solar
radius [km]
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
8.0
9.0
10.0
11.0
12.0
13.0

Variational Calculation (3)

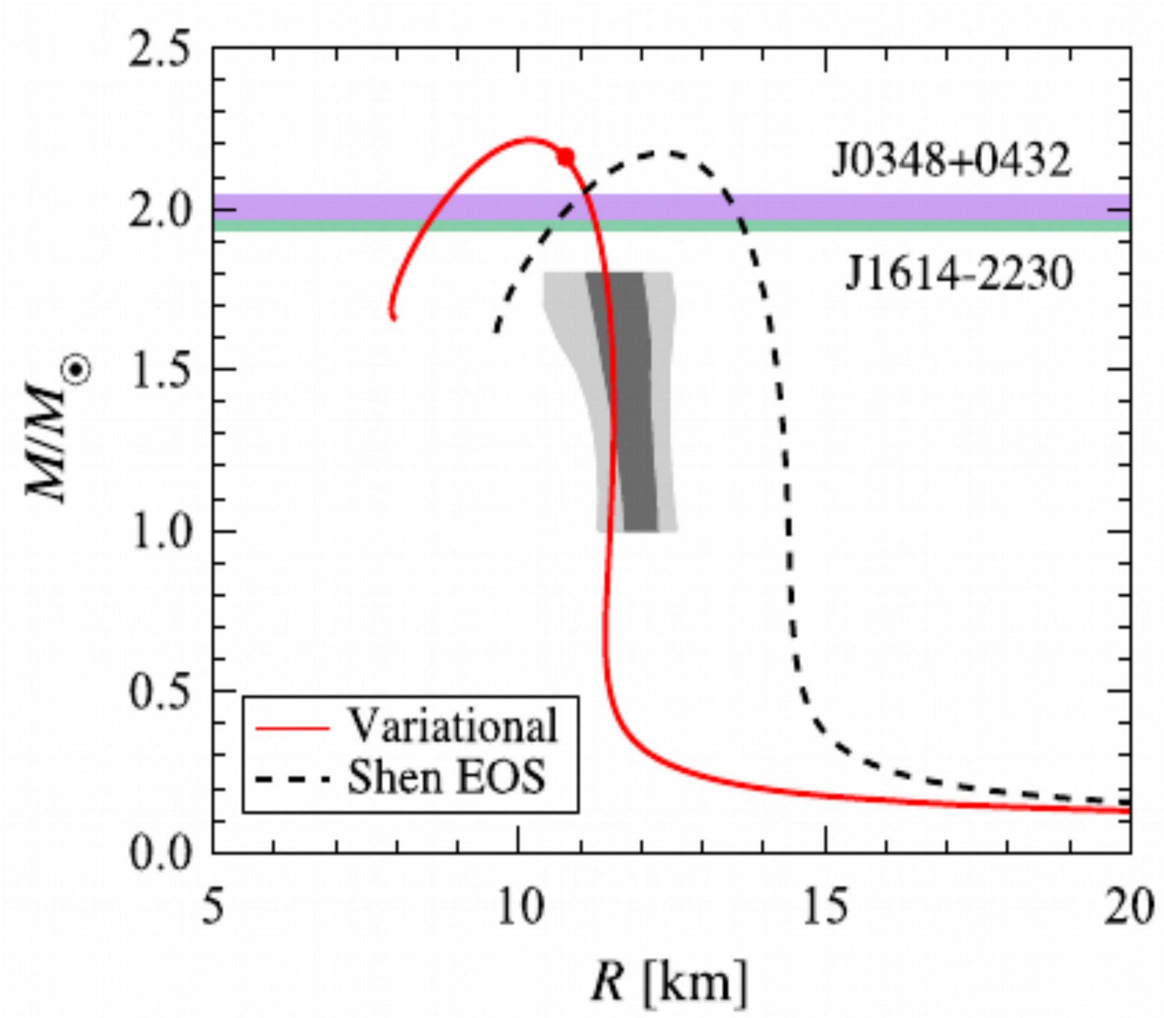
- **Variational Calculation using v18+UIX**
  H. Kanzawa, K. Oyamatsu, K. Sumiyoshi, M. Takano, NPA791 ('07) 232
  - **Similar to APR, but healing-distance condition is required.**
    → no π^0 condensation





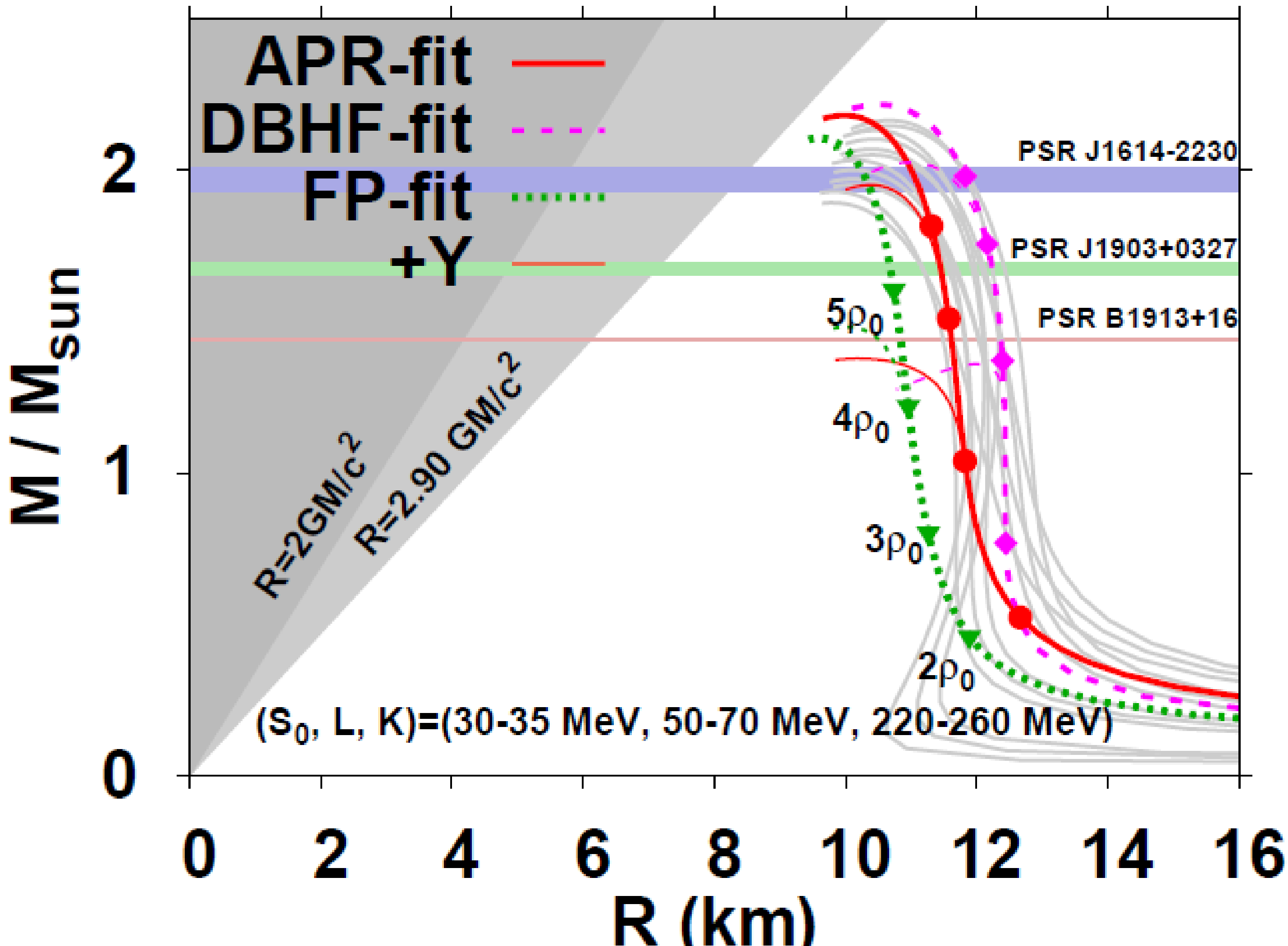
Variational Calculation (4)

- **Variational Calculation using v18+UIX (cont.)**
  H. Togashi, K. Nakazato, Y. Takehara, S. Yamamuro, H. Suzuki, M. Takano. NPA961 ('17)78
  - **NS crust EOS is included in the same way as Shen EOS**
  - **Extended to finite T**



Ab initio & 現象論的EOS でのMR 曲線

- **現象論的状態方程式から推測されるMR 曲線(灰色)**
  - **半径 R=(11-13) km (M= 1.4 $M_{\odot}$)、最大質量 M_{max} = (1.9-2.2) $M_{\odot}$ (核子のみの場合)**



Hartree-Fock Theory

- 平均場理論 = 多体問題の基本

$$\delta \frac{\langle \Phi | H | \Phi \rangle}{\langle \Phi | \Phi \rangle} = 0, \qquad |\Phi\rangle = \det\{\phi_1 \cdots \phi_N\}: \text{Slater} \quad \text{determinant}$$

- 電子系ではエネルギーをほぼ再現
- 原子核ではナイーブな HF は破綻
  - 短距離での斥力コア → エネルギー = ∞
  - 2 体相関が決定的

$$\rho_2(\mathbf{r}_1, \mathbf{r}_2) = 0, \quad \text{for} \ |\mathbf{r}_1 - \mathbf{r}_2| < c$$

→ Brueckner 理論 (G-matrix)

原子・分子など、電子系

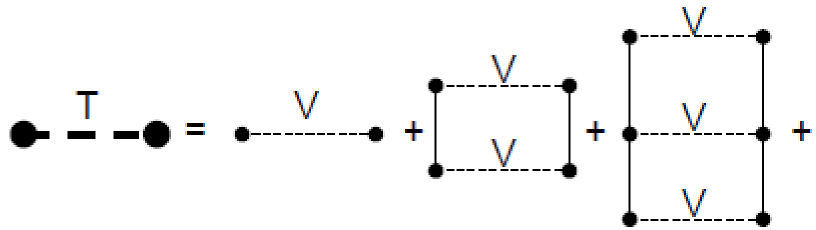
	HF	Exp
He	−2.86	−2.90
Li	−7.43	−7.48
Ne	−128.55	−128.94
Ar	−526.82	−527.60

原子単位 (27.2 eV)

Brueckner Theory

- **Lippmann-Schwinger Eq.**

$$T = V + VG_0 T$$



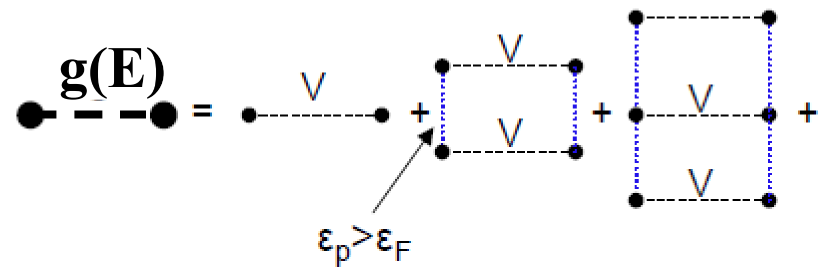
  - **V が singular でも T は有限**

$$V\left|\Psi_{\mathbf{k}}^{(+)}\right\rangle = T\left|\mathbf{k}\right\rangle$$

- **原子核中での 2 体散乱**
  → パウリ原理

$$g(E) = V + V\frac{Q}{E - H_0}g(E)$$

$$Q = 1 - \sum_{i,j<F}\left|ij\right\rangle\left\langle ij\right|$$



  - **原子核中では中間状態で**
    フェルミエネルギー以上の状態のみ
    伝播可能

$$V|\ \rangle = g(E)|\ \rangle$$

- **核内での散乱行列 =g-matrix**

2 体相関を含む複雑な状態

2 体相関の無い状態 (E.g. Slater det.)

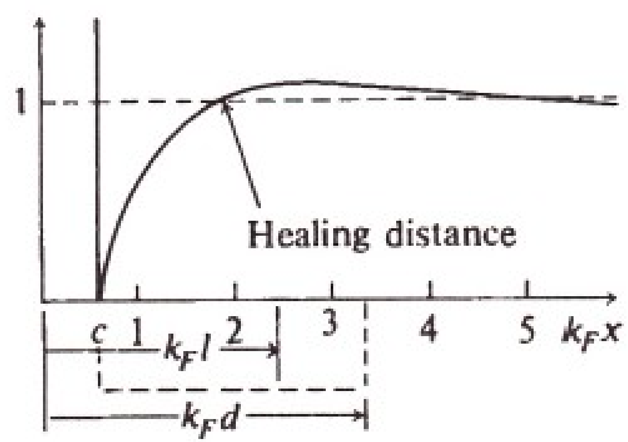
Healing distance

- (波動関数についての)
  Bethe-Goldstone 方程式

$$g_{12} = v_{12} + v_{12} \frac{Q_{12}}{E - (t_1 + t_2 + U_1 + U_2)} g_{12}$$

$$\to \left[E - (t_1 + t_2 + U_1 + U_2) \right] \Psi_{12} = Q_{12} v_{12} \Psi_{12}$$

- BG 方程式の解は、$k_F\, l \sim 1.9$ 程度の距離で通常の平面波にほぼ一致する。
  (Healing distance)
  → 独立粒子描像



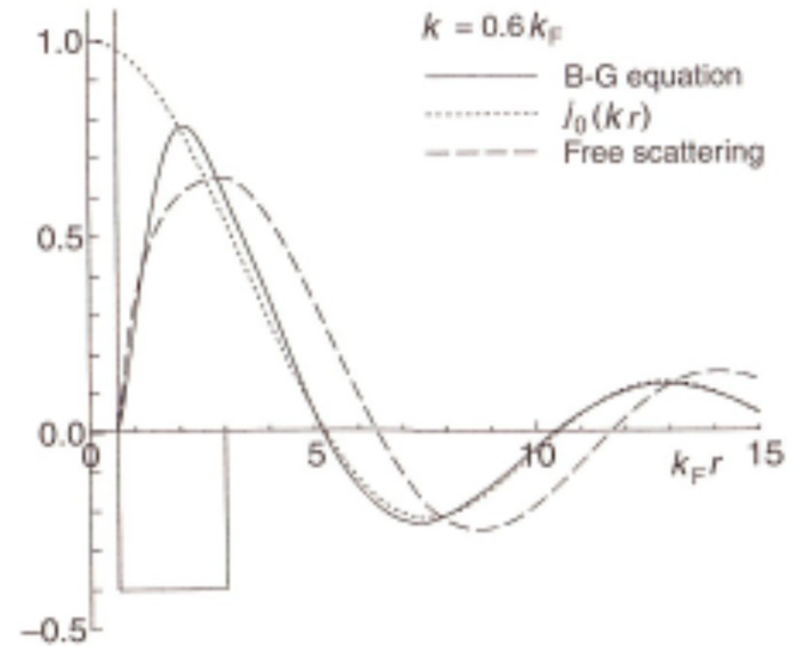
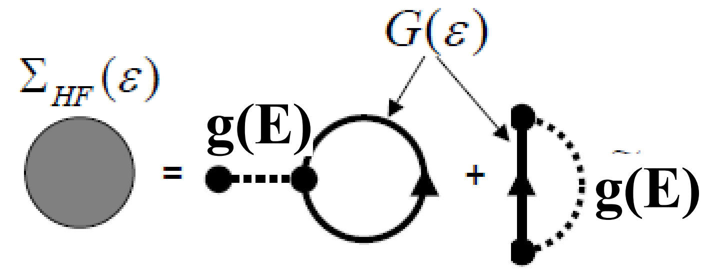


図 2.17 $k = 0.6 k_F$ の場合の Bethe-Goldstone 方程式の解 (実線) と，自由空間内の 2 粒子散乱 (破線) および自由粒子の相対波動関数 (点線) の比較

$k_F = 1.27\ \mathrm{fm}^{-1}$，芯半径は $k_F r_c = 0.62$，井戸型ポテンシャルの半径は $k_F r_a = 3.0$，有効質量は $M^*/M = 0.6$ ととられている．

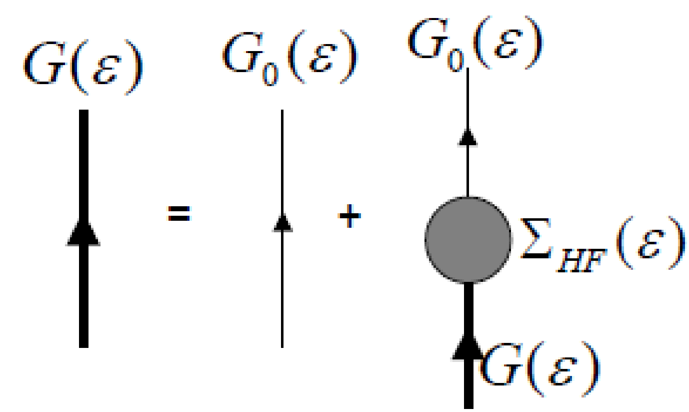
Brueckner-Hartree-Fock theory

- **g-matrix を 2 体相互作用とする HF**
  = Brueckner-Hartree-Fock



$$H = H_0 + V, \ V = \frac{1}{2}\sum_{i\neq j} V_{ij}$$

$$H_0 = \sum_i \left[-\frac{\hbar^2}{2m}\nabla_i^2 + U_i \right]$$



$$g(E) = V + V\frac{Q}{E - H_0}g(E)$$

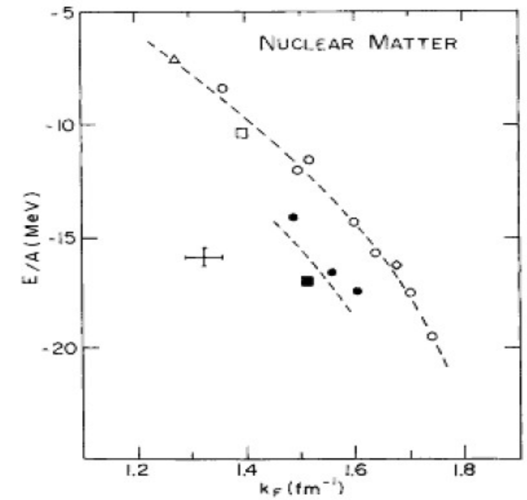
$$U_i(\varepsilon_i) = \sum_j \left[g_{ij,ij}(\varepsilon_i + \varepsilon_j) - g_{ij,ji}(\varepsilon_i + \varepsilon_j) \right]$$

$$E_{\mathrm{BHF}} = \sum_i^{\mathrm{occ}} \left\langle i \,\middle|\, -\frac{\hbar^2}{2m}\nabla^2 \,\middle|\, i \right\rangle + \frac{1}{2}\sum_{i\neq j}^{\mathrm{occ}} \langle ij \mid g(\varepsilon_i + \varepsilon_j) \mid ij - ji \rangle$$

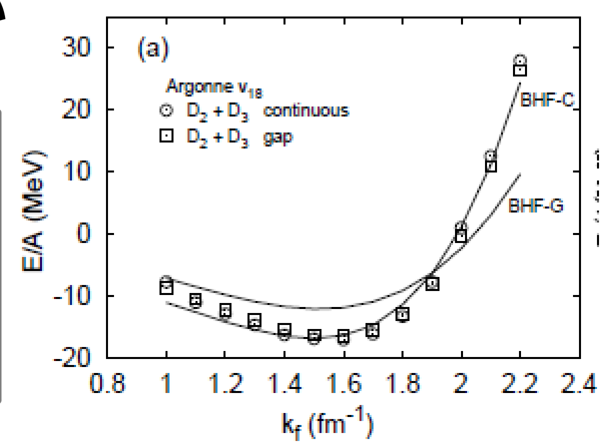
  - **Self-consistent treatment**
    U → g-matrix & φ (s.p.w.f) → U

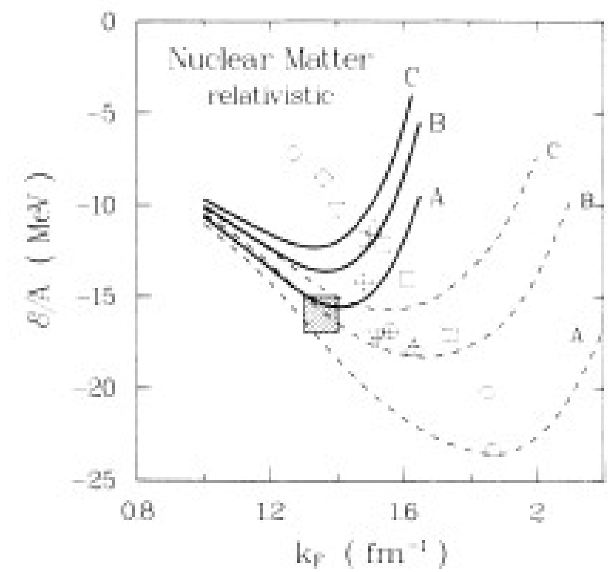
Brueckner-Hartree-Fock theory (cont.)

- **成功点**
  - **核物質の飽和性を定性的に説明**
  - **殻模型(独立粒子描像)の基礎を与える**
  - **有効核力の状態依存性を説明**
- **問題点**
  - **飽和点 (飽和密度、飽和エネルギー) の 定量的理解 (Coester line)→ Relativity or 3 体力**
  - **展開の高次項→ Continuum choice では 3 体クラスター効果は小さい**
  - **スピン軌道力が足りない**



問題は残っているが、 現実的核力から出発して 多体問題に適用する 有効な手法

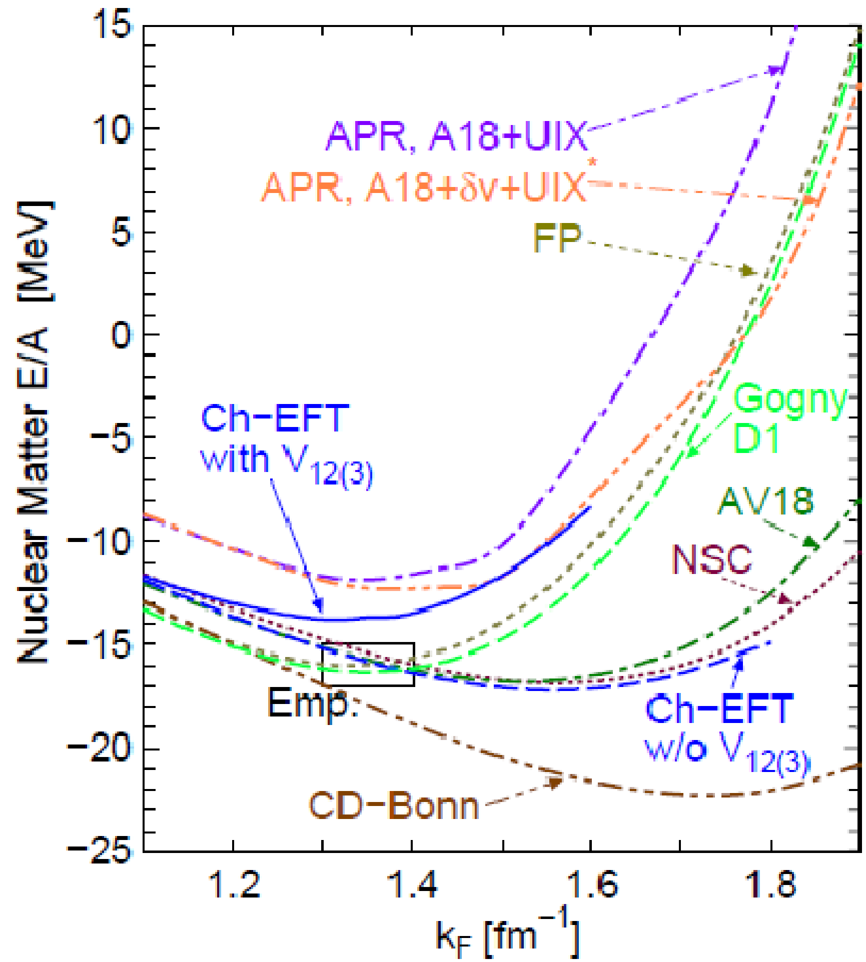




Ch-EFT EOS

- **Phen. models need inputs from**
  Experimental Data and/or Microscopic (Ab initio) Calc.
- **Recent Ch-EFT EOS is promising !**
  NN (N3LO)+3NF(N2LO)

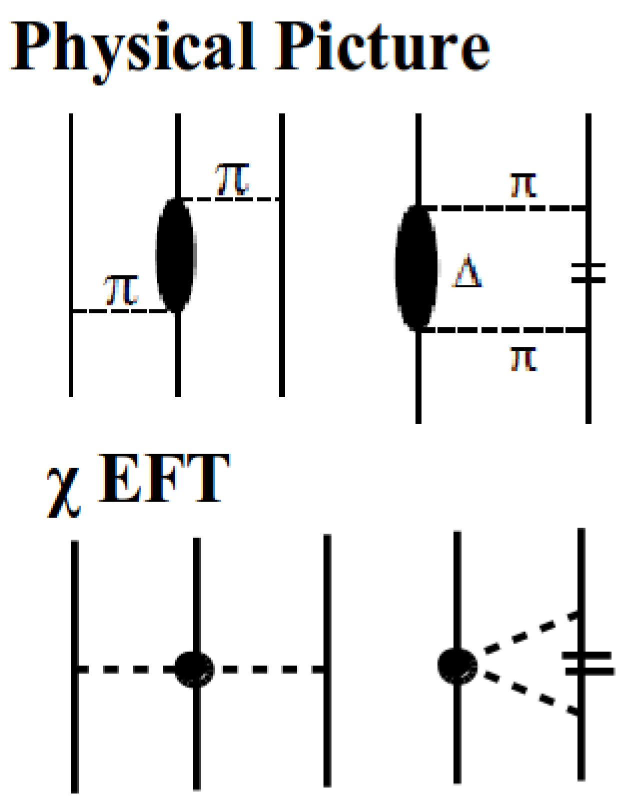
M.Kohno ('13)



M. Kohno, PRC 88 ('13) 064005

"Universal" mechanism of "Three-body" repulsion

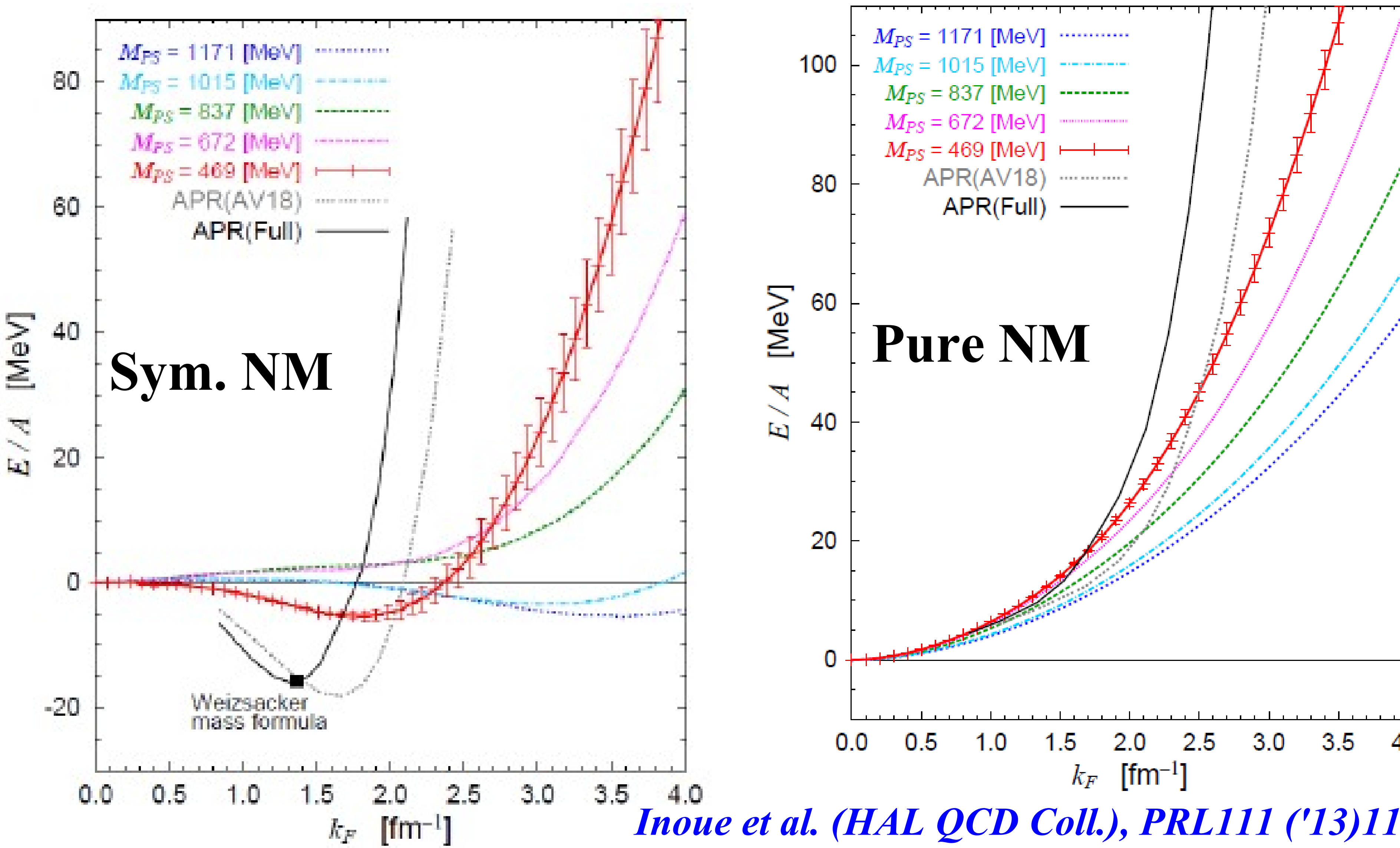
- **"Universal" 3-body repulsion is necessary to support NS. Nishizaki, Takatsuka, Yamamoto ('02)**
- **Mechanism of "Universal" Three-Baryon Repulsion.**
  - **"σ"-exchange ~ two pion exch. w/ res.**
  - **Large attraction from two pion exchange is suppressed by the Pauli blocking in the intermediate stage. Kohno ('13)**



> ***"Universal" TBR***
> - ***Coupling to Res. (hidden DOF)***
> - ***Reduced "σ" exch. pot. ?***

EOS from lattice NN force

- **格子 QCD 核力を用いた高密度状態方程式 (LQCD+BHF)**
  NN force: $^1S_0$, $^3S_1$, $^3D_1$ only



Inoue et al. (HAL QCD Coll.), PRL111 ('13)112503

Relativistic Mean Field

Relativistic Mean Field (1)

- **Effective Lagrangian of Baryons and Mesons + Mean Field App.**

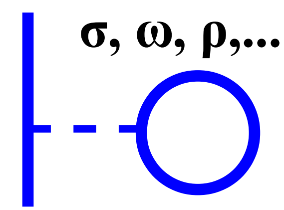
  B.D.Serot, J.D.Walecka, Adv.Nucl.Phys.16 ('86), 1

$$L = L_B^{\rm free} + L_M^{\rm free} + L_{BM} + L_M^{\rm Int}$$

$$L_M^{\rm Int} = -U_\sigma(\sigma) + \frac{1}{4} c_\omega (\omega_\mu \omega^\mu)^2 + \cdots$$

$$L_{BM} = -\sum_{B,S} g_{BS} \bar{\psi}_B \varphi_S \psi_B - \sum_{B,V} g_{BV} \bar{\psi}_B \gamma^\mu V_\mu \psi_B$$

$$L_B^{\rm free} = \bar{\psi}_B (i\gamma^\mu \partial_\mu - M_B) \psi_B \ , \quad L_M^{\rm free} = \sum_S [\frac{1}{2}\partial^\mu \varphi_S \partial_\mu \varphi_S - \frac{1}{2} m_S^2 \varphi_S^2] + \sum_V [-\frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \frac{1}{2} m_v^2 V_\mu V^\mu]$$



- **Baryons and Mesons: B=N, Λ, Σ, Ξ, ..., S= σ, ς, ..., V= ω, ρ, φ, …**
- **Based on Dirac phenomenology & Dirac Bruckner-Hatree-Fock**

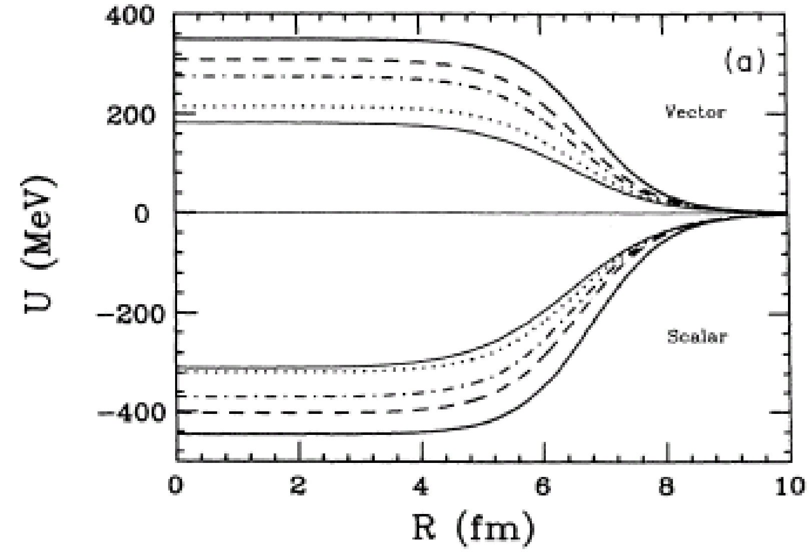
  E.D. Cooper, S. Hama, B.C. Clark, R.L. Mercer, PRC47('93),297
  R. Brockmann, R. Machleidt, PRC42('90),1965
- **Large scalar (att.) and vector (repl.) → Large spin-orbit pot.**
  Relativistic Kinematics → Effective 3-body repulsion
- **Non-linear terms of mesons → Bare 3-body and 4-body force**

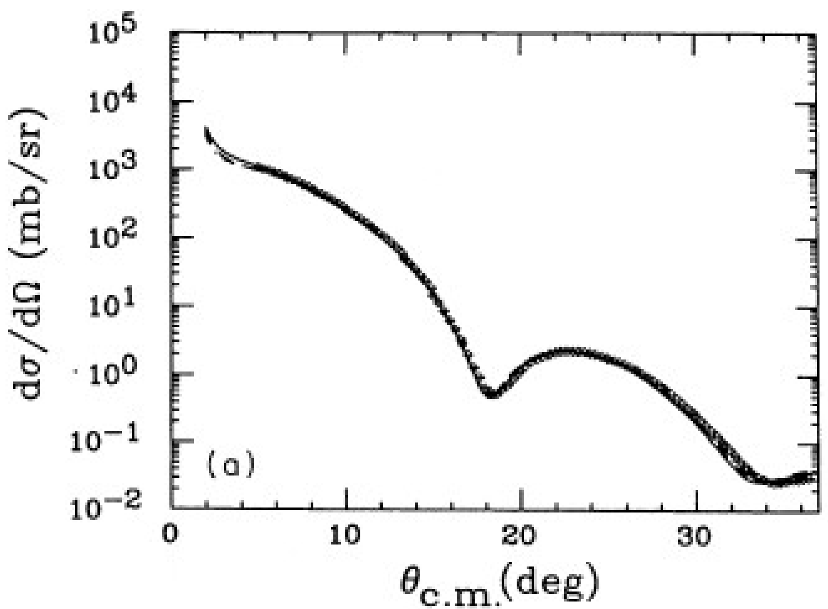
  Boguta, Bodmer ('77), NL1:Reinhardt, Rufa, Maruhn, Greiner, Friedrich ('86), NL3: Lalazissis, Konig, Ring ('97),TM1 and TM2: Sugahara, Toki ('94), Brockmann, Toki ('92)

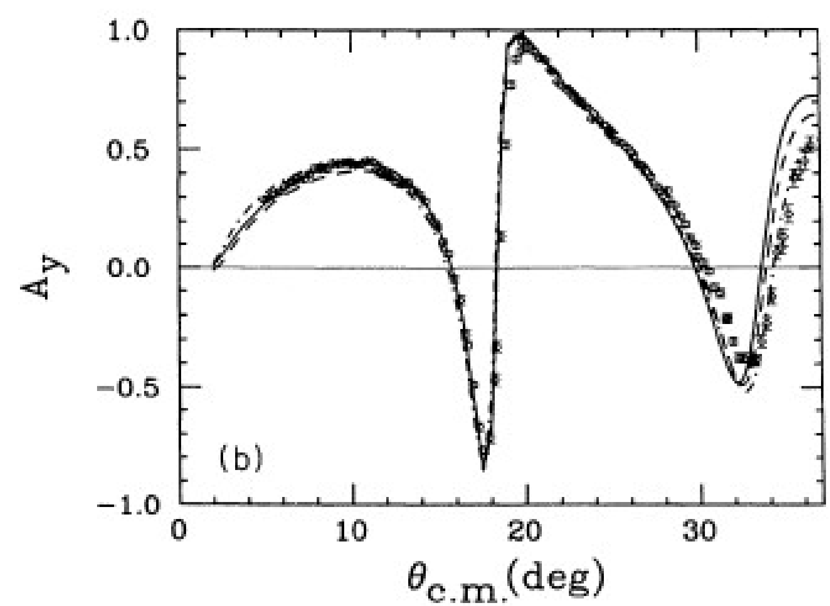
Dirac Phenomenology

E.D. Cooper, S. Hama, B.C. Clark, R.L. Mercer, PRC47('93),297

- **Dirac Eq. with**
  Scalar + Vector pA potential
  (-400 MeV + 350 MeV)
  → Cross Section, Spin Observables



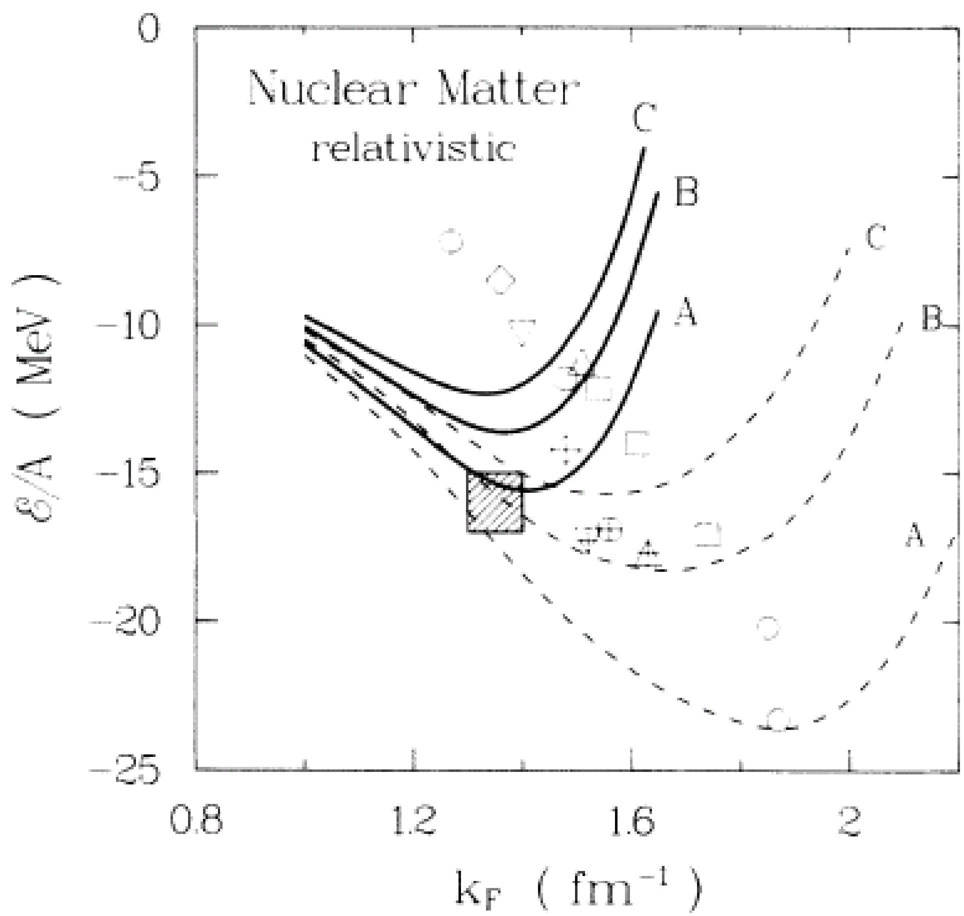




EOS in Dirac-Brueckner-Hartree-Fock

R. Brockmann, R. Machleidt, PRC42('90),1965

- **Non Relativistic Brueckner Calculation**
  → Nuclear Saturation Point cannot be reproduced (Coester Line)
- **Relativistic Approach (DBHF)**
  → Relativity gives additional repulsion, leading to successful description of the saturation point.



Relativistic Mean Field (2)

- **Mean Field treatment of meson field operator**
  = Meson ield operator is replaced with its expectation value
  $\varphi(r) \to \langle\varphi(r)\rangle$
  Ignoring fluctuations compared with the expectation value may be a good approximation at strong condensate.
- **Which Hadrons should be included in RMF ?**
  - **Baryons (1/2+) p, n, Λ, Σ, Ξ, Δ, ...**
  - **Scalar Mesons (0+) σ(600), f_0(980), a_0(980), ...**
  - **Vector Mesons (1-) ω(783), ρ(770), ϕ(1020), ...**
  - **Pseuso Scalar (0-) π , K, η, η', ...**
  - **Axial Vector (1+) a_1, ...**

  We require that the meson field can have uniform expectation values in nuclear matter.
  → Scalar and Time-Component of Vector Mesons (σ , ω, ρ, ...)

$\sigma\omega$ Model (1)

Serot, Walecka, Adv.Nucl.Phys.16 (1986),1

- **Consider only σ and ω mesons**
- **Lagrangian**

$$L = \bar{\psi}(i\gamma^\mu \partial_\mu - M + g_s \sigma - g_v \gamma^\mu \omega_\mu)\psi + \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_s^2 \sigma^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m_v^2 \omega_\mu \omega^\mu$$

$$(F_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu)$$

- **Equation of Motion**
  - **Euler-Lagrange Equation**

$$\frac{\partial}{\partial x^\mu}\left[\frac{\partial L}{\partial(\partial_\mu \phi_i)}\right] - \frac{\partial L}{\partial \phi_i} = 0$$

$$\sigma : \left[\partial_\mu \partial^\mu + m_s^2\right]\sigma = g_s \bar{\psi}\psi$$

$$\omega : \partial_\mu F^{\mu\nu} + m_v^2 \omega^\nu = g_v \bar{\psi}\gamma^\nu \psi \quad \to \quad \left[\partial_\mu \partial^\mu + m_v^2\right]\omega^\nu = g_v \bar{\psi}\gamma^\nu \psi$$

$$\psi : \left[\gamma^\mu \left(i\partial_\mu - g_v V_\mu\right) - (M - g_s \sigma)\right]\psi = 0$$

EOM of ω (for beginners)

- **Euler-Lagrange Eq.**

$$\partial_\mu F^{\mu\nu} + m_v^2 \omega^\nu = g_v \bar{\psi} \gamma^\nu \psi$$

- **Divergence of LHS and RHS**

$$\partial_\nu \partial_\mu F^{\mu\nu} + m_v^2 (\partial_\nu \omega^\nu) = m_v^2 (\partial_\nu \omega^\nu) = g_v (\partial_\nu \bar{\psi} \gamma^\nu \psi) = 0$$

LHS: derivatives are sym. and $F_{\mu\nu}$ is anti-sym.

RHS: Baryon Current = Conserved Current

- **Put it in the Euler-Lagrange Eq.**

$$\partial_\mu F^{\mu\nu} = \partial_\mu (\partial^\mu \omega^\nu - \partial^\nu \omega^\mu) = \partial_\mu \partial^\mu \omega^\nu - \partial^\nu (\partial_\mu \omega^\mu) = \partial_\mu \partial^\mu \omega^\nu$$

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Schroedinger Eq. for Upper Component (1)

- **Dirac Equation for Nucleons**

$$\left(i\gamma\partial-\gamma^0 U_v - M - U_s\right)\psi=0 \quad ,$$
$$U_v = g_\omega \omega \quad , \quad U_s = -g_\sigma \sigma$$

- **Decompose 4 spinor into Upper and Lower Components**

$$\begin{pmatrix} E-U_v-M-U_s & i\sigma\cdot\nabla \\ -i\sigma\cdot\nabla & -E+U_v-M-U_s \end{pmatrix}\begin{pmatrix} f \\ g \end{pmatrix}=0$$

$$g=\frac{-i}{E+M+U_s-U_V}(\sigma\cdot\nabla)f$$
$$(E-M-U_v-U_s)f=-i(\sigma\cdot\nabla)g$$

Schroedinger Eq. for Upper Component (2)

- **Erase Lower Component (assuming spherical sym.)**

$$-i(\sigma\cdot\nabla)g=-(\sigma\cdot\nabla)\frac{1}{X}(\sigma\cdot\nabla)f=-\frac{1}{X}\nabla^2 f-\frac{1}{r}\left[\frac{d}{dr}\frac{1}{X}\right](\sigma\cdot r)(\sigma\cdot\nabla)f$$

$$=-\nabla\frac{1}{X}\nabla f+\frac{1}{r}\left[\frac{d}{dr}\frac{1}{X}\right](\sigma\cdot l)f$$

$$(\sigma\cdot r)(\sigma\cdot\nabla)=(r\cdot\nabla)+i\sigma\cdot(r\times\nabla)=r\cdot\nabla-\sigma\cdot l$$

- **"Schroedinger-like" Eq. for Upper Component**

$$-\nabla\frac{1}{E+M+U_s-U_v}\nabla f+\left(U_s+U_v+U_{LS}(\sigma\cdot l)\right)f=(E-M)f$$

$$U_{LS}=\frac{1}{r}\left[\frac{d}{dr}\frac{1}{E+M+U_s-U_v}\right]<0 \quad \text{on surface}$$

$(U_s,U_v)\sim$ (-350 MeV, 280 MeV)

$\rightarrow$Small Central(U_s+U_v), Large LS (U_s-U_v)

Various Ways to Evaluate Non.-Rel. Potential

- **From Single Particle Energy**

$$\left(\gamma^0(E-U_v)+i\gamma\cdot\nabla-(M+U_s)\right)\psi=0 \;\rightarrow\; (E-U_v)^2=p^2+(M+U_s)^2$$

$$\rightarrow E=\sqrt{p^2+(M+U_s)^2}+U_v \approx E_p+\frac{M}{E_p}U_s+U_v+\frac{p^2}{2E_p^3}U_s^2$$

$$(E_p=\sqrt{p^2+M^2})$$

- **Schroedinger Equivalent Potential (Uniform matter)**

$$-\frac{\nabla^2}{2}M f+\left[U_s+\frac{E}{M}U_v+\frac{U_s^2-U_v^2}{2}M\right]f=\frac{E+M}{2}M(E-M)f$$

$$U_{\mathrm{SEP}}\approx U_s+\frac{E}{M}U_v$$

Anyway, slow baryons feel Non.-Rel. Potential,

$$U\approx U_s+U_v=-g_s\sigma+g_v\omega$$

Nuclear Matter in σω Model

Serot, Walecka, Adv.Nucl.Phys.16 (1986),1

- **Uniform Nuclear Matter**

$$E/V = \gamma_N \int^{P_F} \frac{d^3p}{(2\pi)^2} E^* + \frac{1}{2} m_s^2 \sigma^2 - \frac{1}{2} m_v^2 \omega^2 + g_v \rho_B \omega$$

$$\sigma = \frac{g_s}{m_s^2} \rho_s = \frac{g_s}{m_s^2} \int^{P_F} \frac{\gamma_N \, d^3p}{(2\pi)^2} \frac{M^*}{E^*}$$

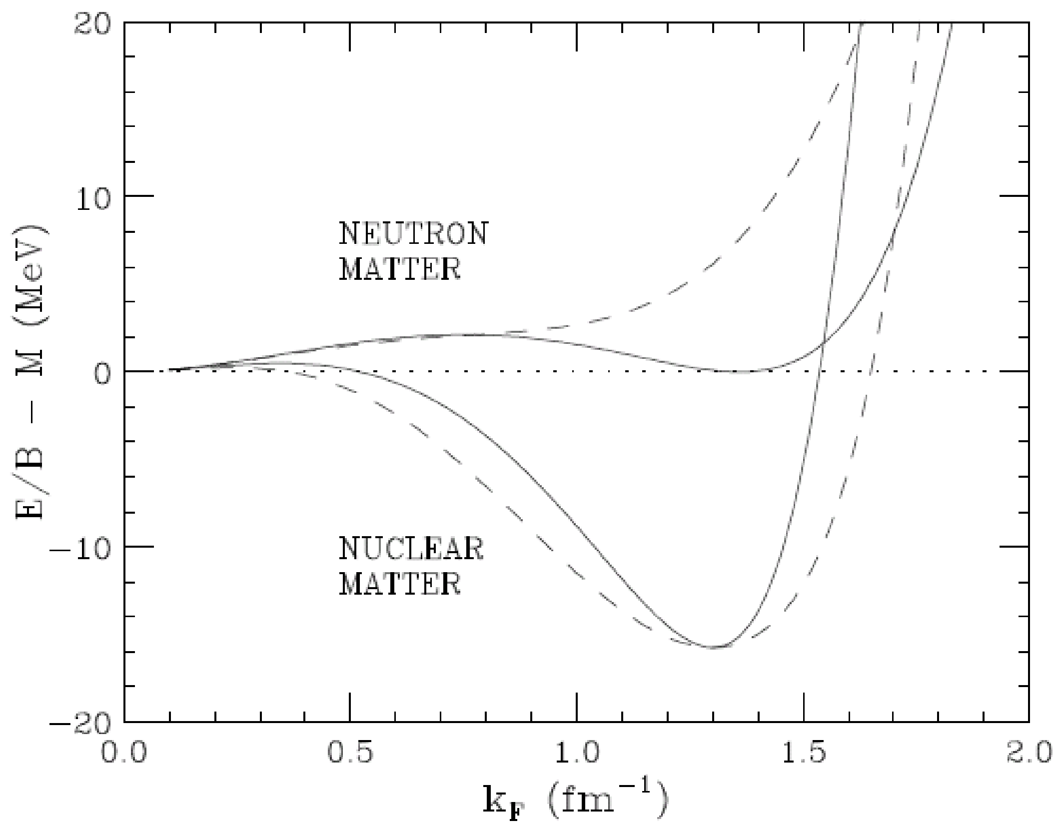
$$(M^* = M + U_s = M - g_s \sigma, \quad E^* = \sqrt{p^2 + M^{*2}})$$

$$\omega = \frac{g_v}{m_v^2} \rho_B = \gamma_N \frac{g_v}{m_v^2} \int^{P_F} \frac{d^3p}{(2\pi)^3}$$

γ_N = Nucleon degeneracy

(=4 in sym. nuclear matte

Problem: EOS is too stiff
K ~ (500-600) MeV !
→ How can we avoid it ?



RMF with Non-Linear Meson Int. Terms

Boguta, Bodmer ('77), NL1:Reinhardt, Rufa, Maruhn, Greiner, Friedrich ('86), NL3: Lalazissis, Konig, Ring ('97),TM1 and TM2: Sugahara, Toki ('94), Brockmann, Toki ('92)

- **Too stiff EOS in the simplest RMF (σω model) is improved by introducing non-linear terms (σ^4, ω^4)**
  - **Fit B.E. of Stable as well as Unstable (n-rich) Nuclei**
  - **Three Mesons (σ,ω,ρ) are included**
  - **Meson Self-Energy Term (σ,ω)**

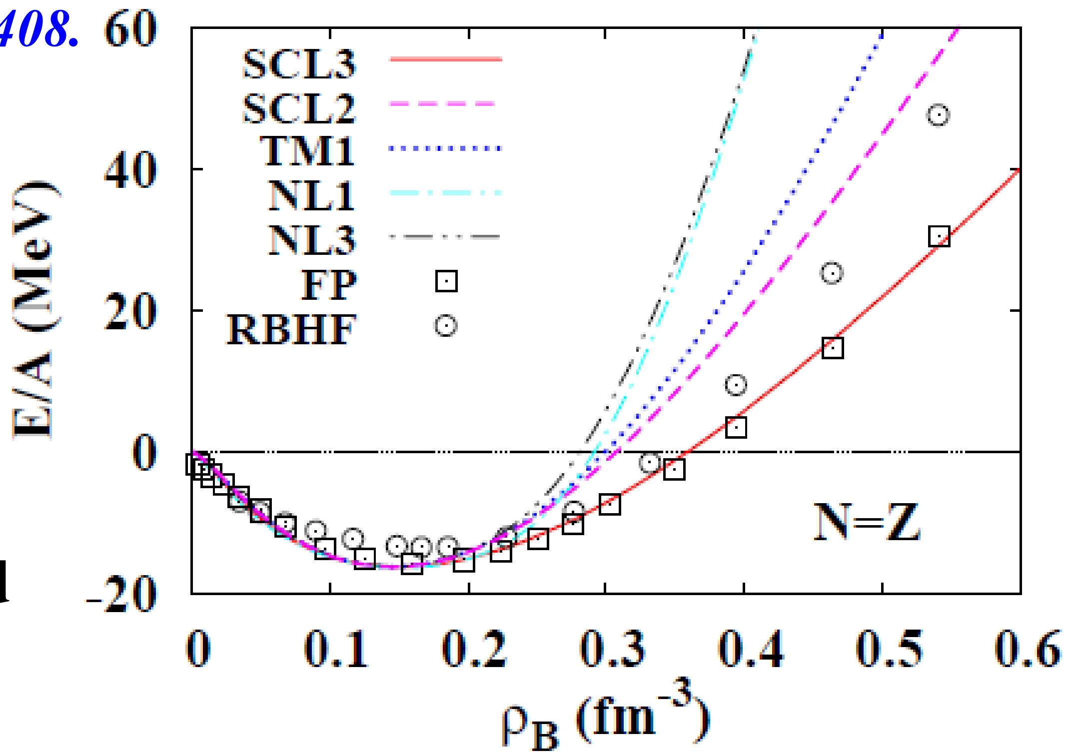
$$
\begin{aligned}
\mathcal{L} =& \bar{\psi}_N \left(i\partial\!\!\!/ - M - g_\sigma \sigma - g_\omega \omega\!\!\!/ - g_\rho \tau^a \rho\!\!\!/^a \right) \psi_N \\
&+ \frac{1}{2}\partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 \\
&- \frac{1}{4} W^{\mu\nu} W_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4} R^{a\mu\nu} R^a_{\mu\nu} + \frac{1}{2} m_\rho^2 \rho^{a\mu} \rho^a_\mu + \frac{1}{4} c_3 \left(\omega_\mu \omega^\mu \right)^2 \\
&+ \bar{\psi}_e \left(i\partial\!\!\!/ - m_e \right) \psi_e + \bar{\psi}_\nu i\partial\!\!\!/ \psi_\nu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} ,
\end{aligned}
$$

$$
\begin{aligned}
W_{\mu\nu} &= \partial_\mu \omega_\nu - \partial_\nu \omega_\mu , \\
R^a_{\mu\nu} &= \partial_\mu \rho^a_\nu - \partial_\nu \rho^a_\mu + g_\rho \epsilon^{abc} \rho^{b\mu} \rho^{c\nu} , \\
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu .
\end{aligned}
$$

RMF models with Non-Linear Meson Int. Terms

- Variety of the RMF models
  → MB couplings, meson masses, meson self-energies
  - σN, ωN, ρN couplings are well determined
    → almost no model deps. in Sym. N.M. at low ρ
  - ω^4 term is introduced to simulate DBHF results of vector pot.
    TM1&2: Y. Sugahara, H. Toki, NPA579('94)557;
    R. Brockmann, H. Toki, PRL68('92)3408.
  - σ^3 and σ^4 terms are introduced to soften EOS at ρ_0.
    J. Boguta, A.R.Bodmer NPA292('77)413,
    NL1:P.-G.Reinhardt, M.Rufa, J.Maruhn, W.Greiner, J.Friedrich, ZPA323('86)13.
    NL3: G.A.Lalazissis, J.Konig, P.Ring, PRC55('97)540.

    → Large differences are found at high ρ



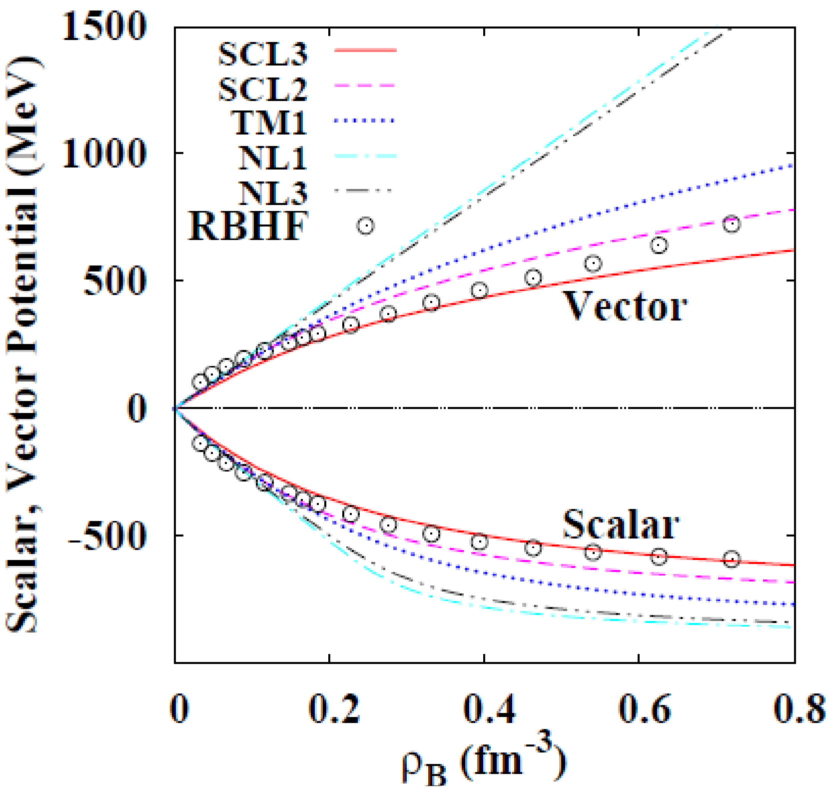
K. Tsubakihara, H. Maekawa, H. Matsumiya, AO,PRC81('10)065206.

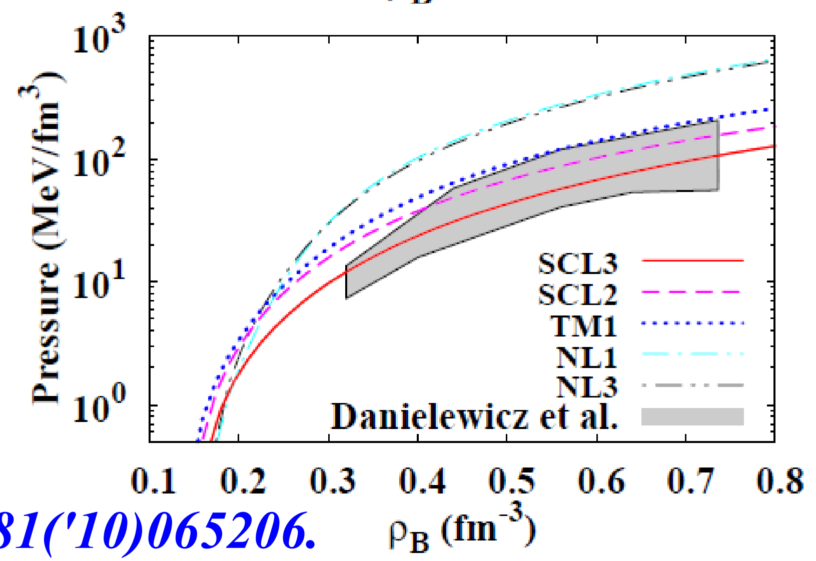
Vector potential in RMF

- **Vector potential from ω dominates at high density !**

$$U_v(\rho_B) = g_\omega \omega \sim \frac{g_\omega^2}{m_\omega^2} \rho_B$$

  - **Dirac-Bruckner-Hartree-Fock shows suppessed vector potential at high ρ_B.**
    R. Brockmann, R. Machleidt, PRC42('90)1965.
  - **Collective flow in heavy-ion collisions suggests pressure at high ρ_B.**
    P. Danielewicz, R. Lacey, W. G. Lynch, Science298('02)1592.
  - **Self-interaction of ω ~ $c_\omega(\omega_\mu \omega^\mu)^2$**
    → DBHF results & Heavy-ion data

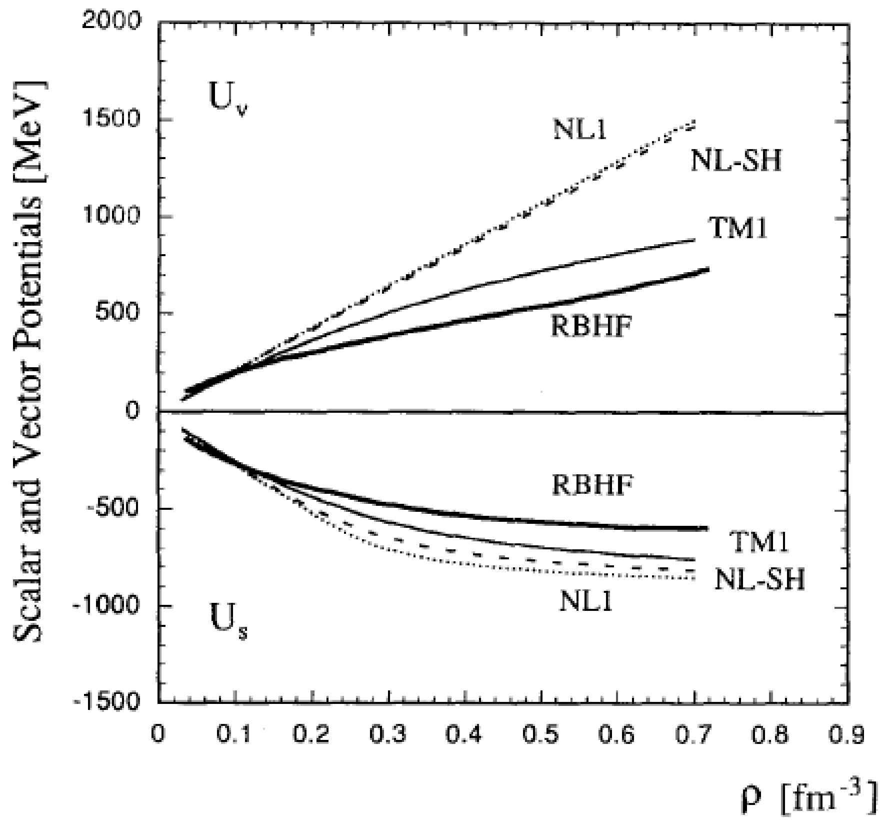


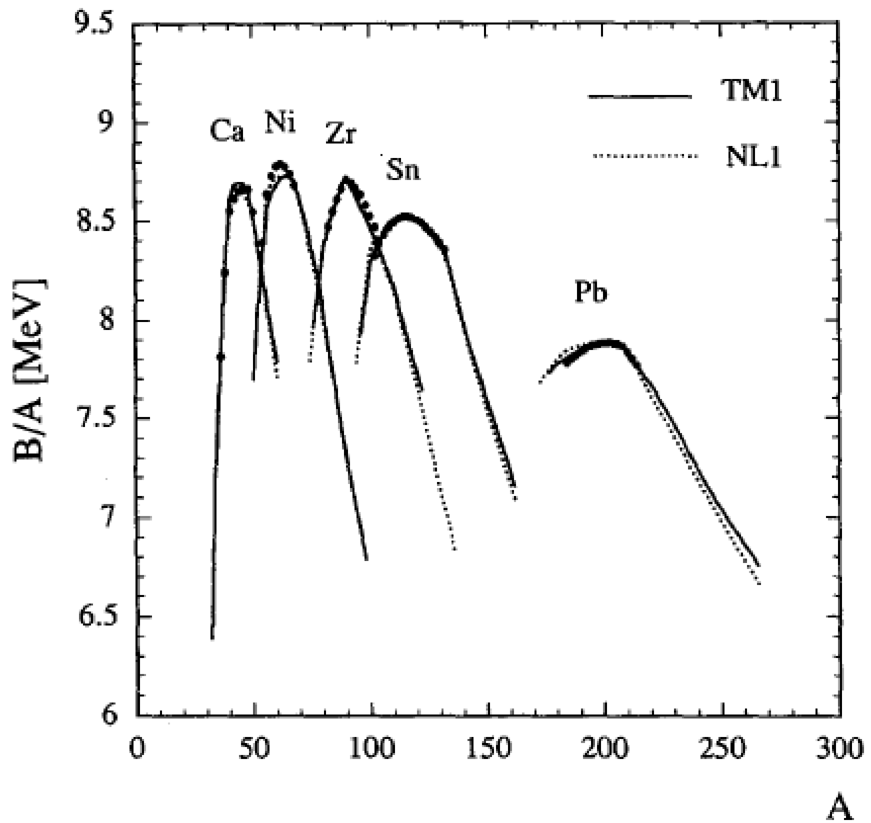


K. Tsubakihara, H. Maekawa, H. Matsumiya, AO,PRC81('10)065206.

TM1

- **TM1 Sugahara, Toki ('94)**
  - **Fit vector potential in RBHF by introducing ω^4 term.**
  - **Fit binding energies of neutron-rich nuclei**

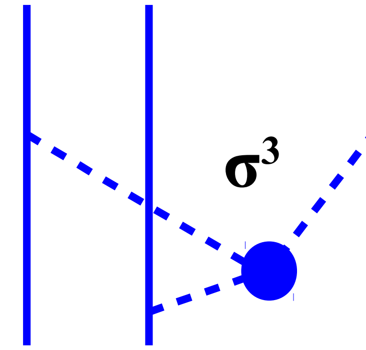


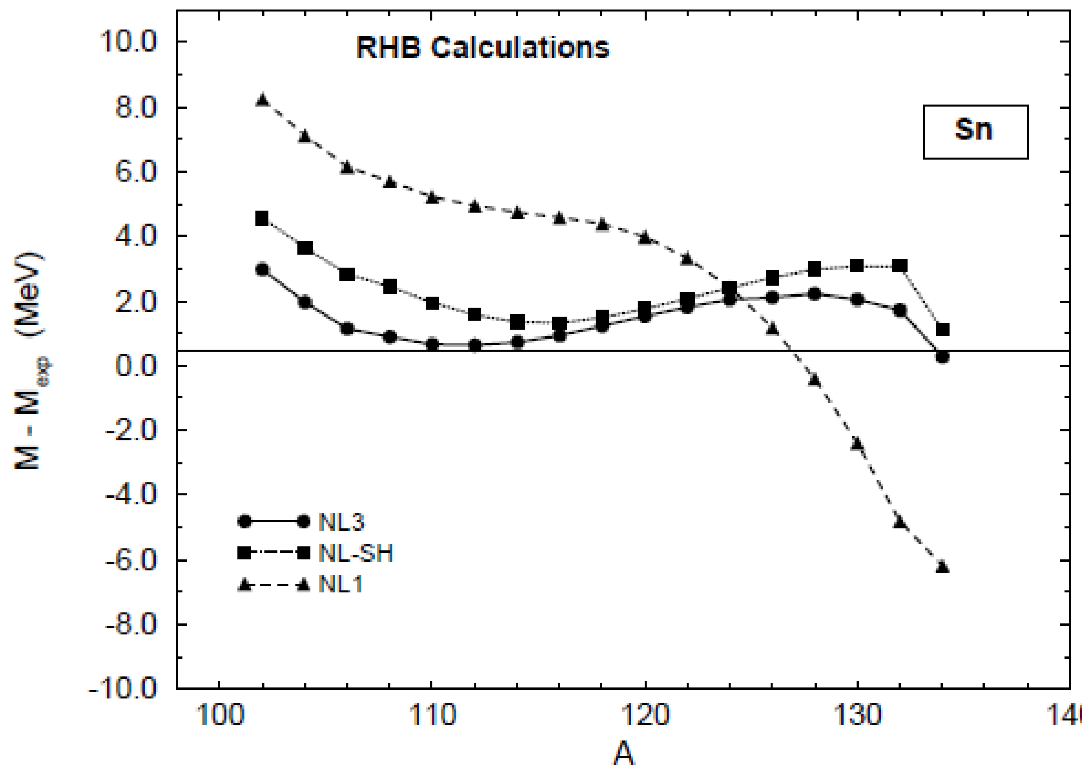


TM1: Sugahara, Toki ('94)

High Quality RMF models

- いくつかの RMF パラメータによる計算は、
  「質量公式」に迫る精度で原子核質量を記述！
  → High Quality RMF models.
  TM, NL1, NL3, ….
  - 全質量で 1-2 MeV の誤差 (NL3)
  - Linear coupling (σN, ωN, ρN), self-energy in σ, ω
  - 場合によっては結合定数の密度依存性を導入。





NL3: Lalazissis, Konig, Ring, PRC55 ('97)540

RMF with Non-Linear Meson Int. Terms

- **Are the Lagrangian parameters are well determined ?**

$$\mathcal{L} = \mathcal{L}_{\rm free}(\psi, \sigma, \omega, \rho, ...) + \bar{\psi}\left[g_\sigma \sigma - g_\omega \gamma^0 \omega - g_\rho \tau_z \gamma^0 \rho\right]\psi + c_\omega \omega^4/4 - V_\sigma(\sigma)\ , \quad (3)$$

$$V_\sigma = \begin{cases} \frac{1}{3} g_3 \sigma^3 + \frac{1}{4} g_4 \sigma^4 & ({\rm NL1,\ NL3,\ TM1}) \\ -a_\sigma f_{\rm SCL}(\sigma/f_\pi) & ({\rm SCL}) \end{cases}\ , \quad (4)$$

  - **Linear terms, Meson-Nucleon Coupling → Well determined**
  - **Negative Coef. of $\sigma^4 < 0$ in some of RMF models→ Vacuum is unstable**

TABLE II: RMF parameters

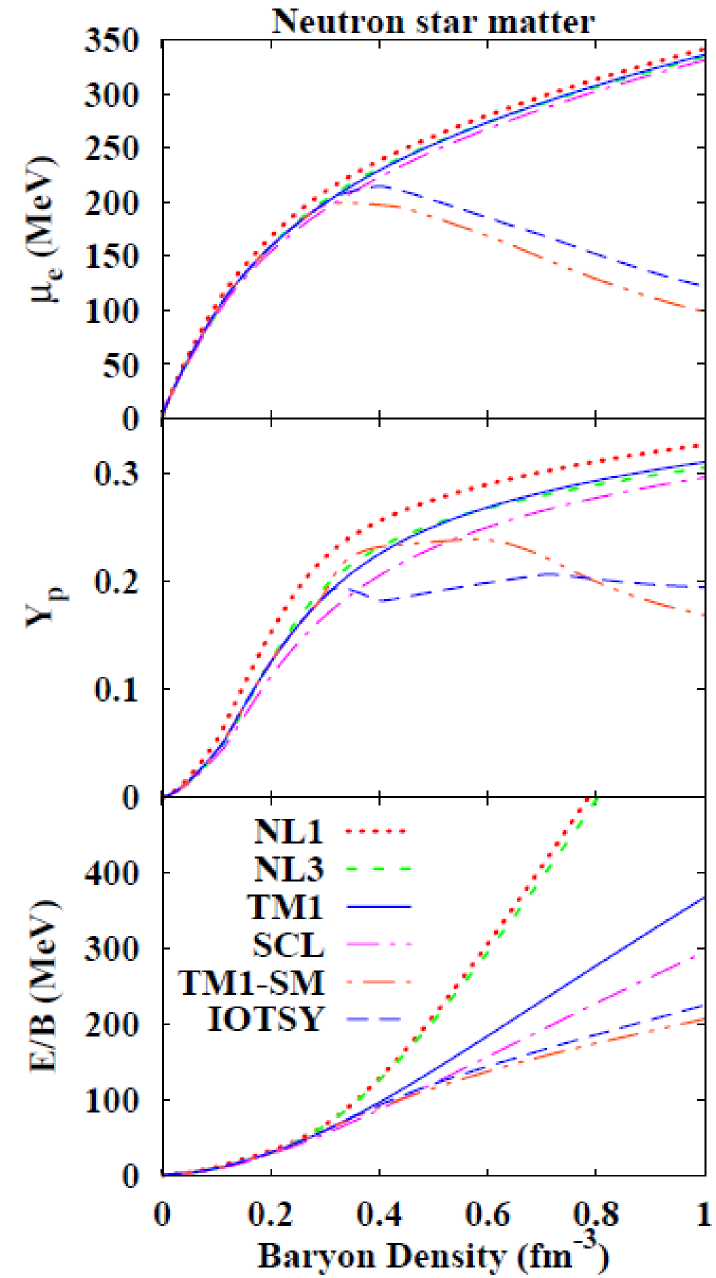
	$g_{\sigma N}$	$g_{\omega N}$	$g_{\rho N}$	g_3(MeV)	g_4	c_ω	m_σ(MeV)	m_ω(MeV)	m_ρ(MeV)
NL1[18]	10.138	13.285	4.976	2401.9	-36.265	0	492.25	795.359	763
NL3[19]	10.217	12.868	4.474	2058.35	-28.885	0	508.194	782.501	763
TM1[6]	10.0289	12.6139	4.6322	1426.466	0.6183	71.3075	511.198	783	770
SCL[20](*1)	10.08	13.02	4.40	1255.88	13.504	200	502.63	783	770

(*1): g_3 and g_4 are from the expansion of $f_{\rm SCL}$.

AO, Jido, Sekihara, Tsubakihara (2009)

Neutron Star Matter EOS

- **Difference in non-linear meson terms generate different predictions of EOS at high densities**



How can we fix non-linear terms ?

AO, Jido, Sekihara, Tsubakihara, Phys. Rev. C 80 (2009), 038202.

Lec. 2 のまとめ

- 状態方程式 (EOS) は原子核・核物質・中性子星物質を理解するための重要な概念
  - 状態方程式は質量公式の拡張！
  - 対称核物質と純中性子物質の EOS がわかれば、ある程度の近似 (電子質量・陽子中性子の質量差無視) すれば、中性子星物質の EOS が得られる。
- 状態方程式を記述する理論の枠組み
  - 第一原理計算、平均場理論 (非相対論・相対論) 等、様々な枠組
  - 「現象論的 EOS 」の基盤は密度汎関数
  - 「カイラル EFT+ 多体論」は期待できそう (promising)
- 相対論的平均場 (Relativistic Mean Field; RMF)
  - ハドロン物理からみて望ましい。ただし不定性は大。

Thank you !