高密度物質と中性子星の物理：講義の内容

1. 中性子星の基本的性質
   - 質量･半径測定の概論など
2. 状態方程式を記述する理論模型
   - 平均場理論、第一原理計算手法、場の理論によるアプローチ
3. 対称エネルギーと非対称核物質の状態方程式
   - 対称エネルギーを決める実験手法、現在の制限
4. QCD 有効模型と高密度核物質の性質
   - 有限温度･密度の場の理論入門 (松原和･摂動論など)
   - NJL 模型による相転移と状態方程式の記述
5. ハイパー核物理と中性子星でのハイペロンパズル
   - ハイパー核実験の現状、ハイペロンパズルの解決に向けて

- 談話会

  Symmetry Parameter Constraints
  from a Lower Bound on the Neutron-Matter Energy

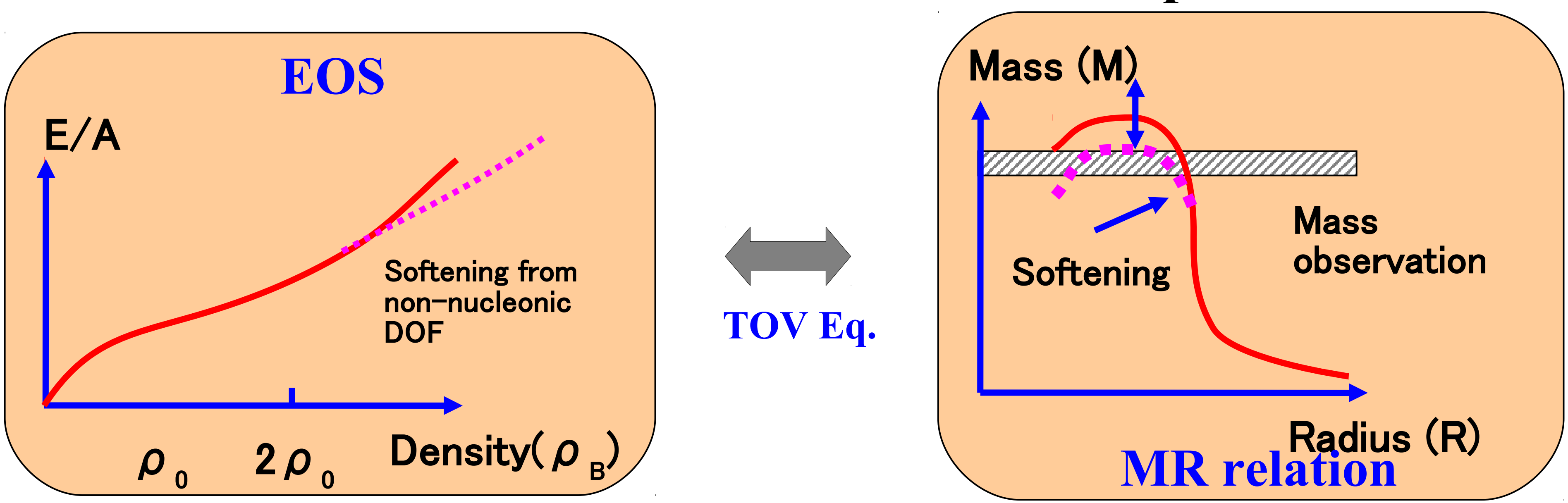
Nuclear Symmetry Energy – Overview –

M-R Relation and EOS

- **Solving TOV eq.**
  starting from the "initial" condition, $\varepsilon(r=0) = \varepsilon_c$ = given
  until the "boundary" condition P(r)=0 is satisfied.
  → M and R are the functions of $\varepsilon(r=0)$
  and functionals of EOS, $P=P(\varepsilon)$.

$$M = M(\varepsilon_c)[P(\varepsilon)],\ R = R(\varepsilon_c)[P(\varepsilon)]$$

→ M-R curve and NS matter EOS : 1 to 1 correspondence



Nuclear Mass

- **Bethe-Weizsacker mass formula**
  Nuclear binding energy is roughly given by Liquid drop.
  Nuclear size measurement → $R = r_0 A^{1/3}$

$$B(A,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} + a_p \frac{\delta_p}{A^\gamma}$$

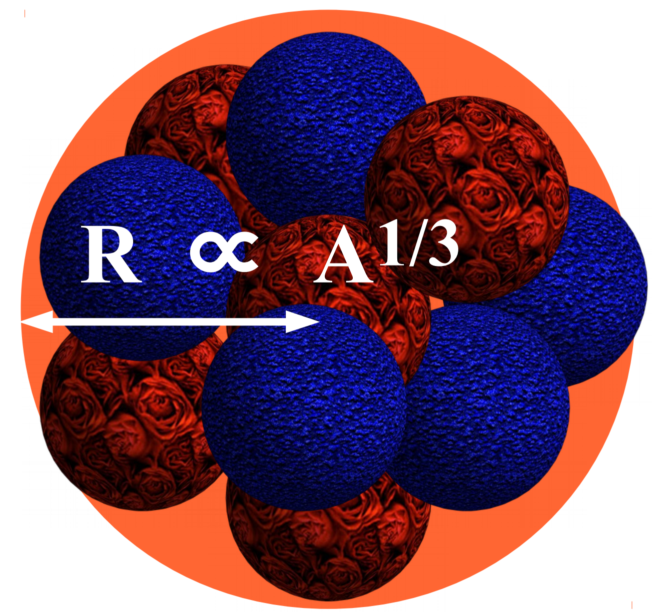
Volume **Surface** **Coulomb** **Symmetry** **Paring**

$A \propto \frac{4\pi}{3} R^3$ $A^{2/3} \propto 4\pi R^2$ $\propto \frac{Q^2}{R}$

- **Ignore Coulomb, consider A → ∞,**

$$B/A = a_v(\rho) - a_a(\rho)\alpha^2 \quad (\alpha = (N-Z)/A)$$

$$a_v \simeq 16 \text{ MeV}, a_a \simeq 30 \text{ MeV}$$



Coef. may depend on the number density ρ
→ Nuclear Matter EOS

Nuclear Matter EOS

- **Energy per nucleon in nuclear matter**

$$E_{\rm NM}(\rho, \alpha) = E_{\rm SNM}(\rho) + S(\rho)\alpha^2 \quad (\alpha = (N - Z)/A)$$

- **Saturation point (ρ_0, E_0)**

$$\rho_0 \simeq 0.16\ {\rm fm}^{-3}$$

$$E_0 = -a_v \simeq -16\ {\rm MeV}$$

(nuclear radius and mass)

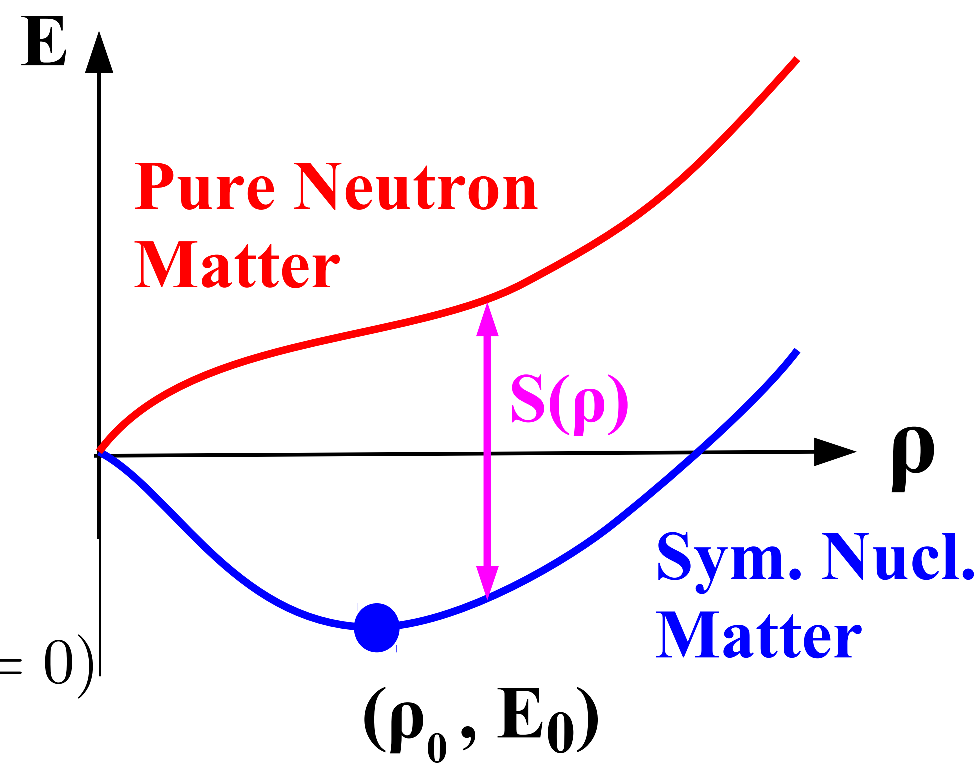
- **Symmetry energy**

$$S(\rho) = E_{\rm PNM}(\rho) - E_{\rm SNM}(\rho)$$

$$= E_{\rm NM}(\rho, \alpha = 1) - E_{\rm NM}(\rho, \alpha = 0)$$

$$S_0 = S(\rho_0) \simeq 30\ {\rm MeV}$$

(Mass formula, volume term)



Nuclear Matter EOS can be, in principle, determined by terrestrial (laboratory) nuclear physics experiments !

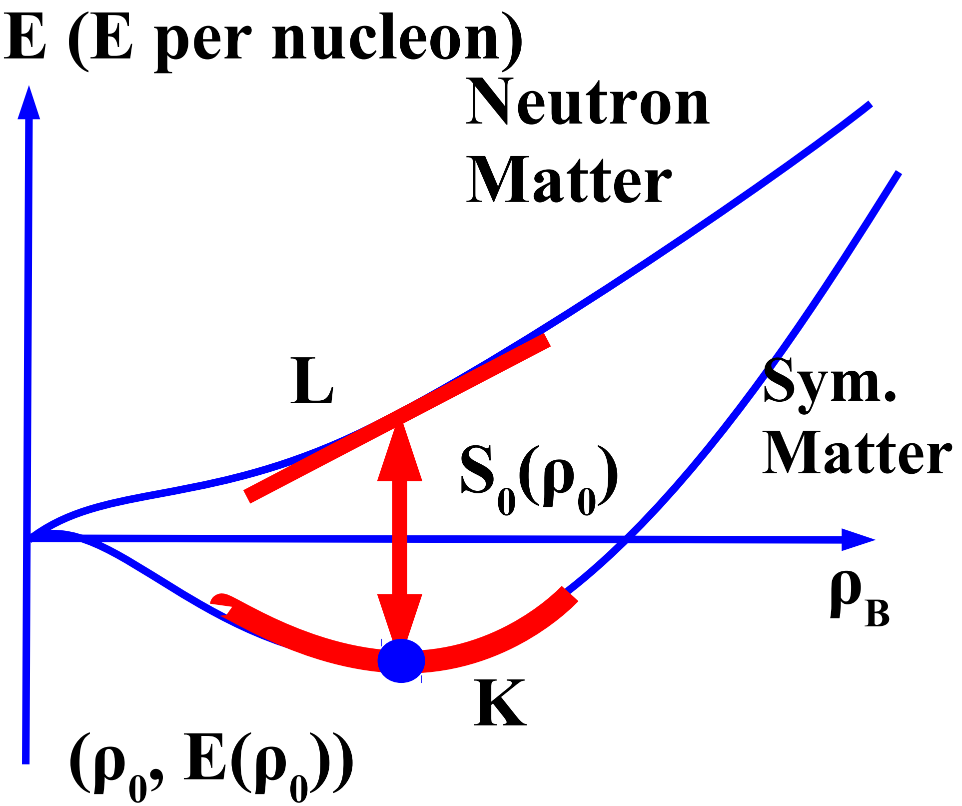
Nuclear Matter EOS

- **Additional two important parameters: K and L**
- **Pressure is given by the derivative of E via ρ**

$$P = \rho^2 (\partial E / \partial \rho)$$

At ρ_0, L determines P

$$P = \rho_0 L/3 \,(\text{at} \;\; \rho = \rho_0)$$



Neutron Star Matter EOS

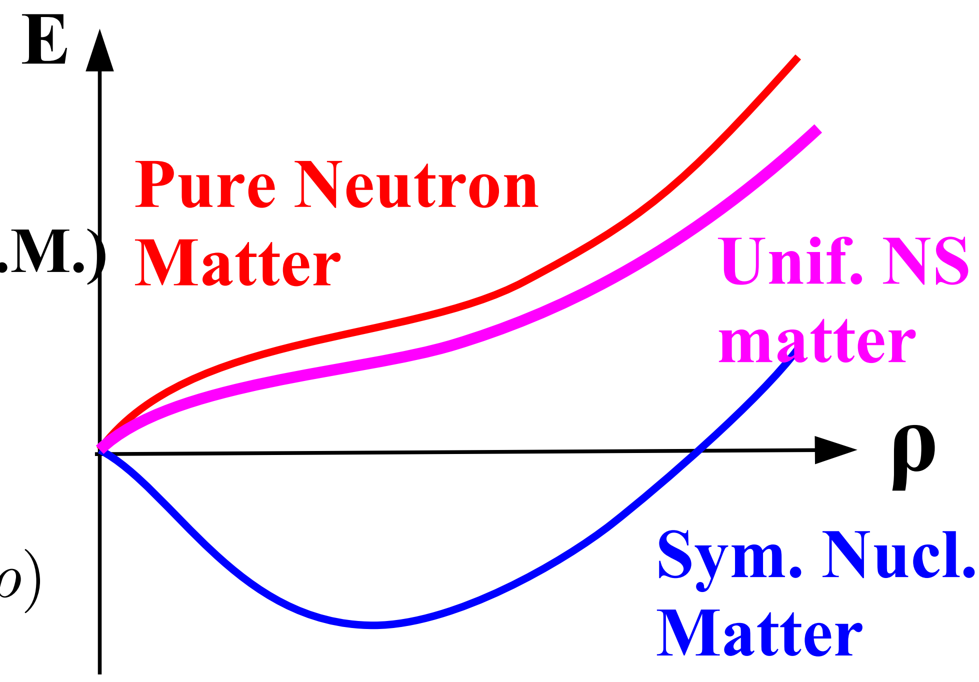
- **What happens in low-density uniform neutron star matter ?**
  - **Constituents = proton, neutron and electron**
  - **Charge neutrality → # of electons= # of protons ($\rho_e=\rho_p=\rho(1-\delta)/2$)**

$$E_{\mathrm{NSM}}(\rho) = E_{\mathrm{NM}}(\rho,\alpha) + E_e(\rho_e=\rho_p)$$
$$= E_{\mathrm{SNM}}(\rho) + \alpha^2 S(\rho) + \frac{\Delta M}{2}\alpha + \frac{3}{8}\hbar k_F (1-\alpha)^{4/3}$$

(electron mass neglected, neutron-proton mass diff. incl. k_F= Fermi wave num. in Sym. N.M.)

  - **δ is optimized to minimize energy per nucleon**

$$E_{\mathrm{NSM}}(\rho) \le E_{\mathrm{NM}}(\rho,\alpha=1) = E_{\mathrm{PNM}}(\rho)$$



対称エネルギーの起源

- Fermi Gas model での核子あたりの運動エネルギー

$$E_{\mathrm{sym},K} = \frac{Z}{A}\frac{3}{5}\frac{\hbar^2 k_{Fp}^2}{2m} + \frac{N}{A}\frac{3}{5}\frac{\hbar^2 k_{Fn}^2}{2m} = \frac{3}{5}E_F\frac{1}{2}\left[(1-\delta)^{5/3}+(1+\delta)^{5/3}\right]$$

$$\simeq \frac{3}{5}E_F + \frac{1}{3}E_F\delta^2 + \mathcal{O}(\delta^4)$$

a_{sym} (FG)=$E_F/3 \sim 11$ MeV となり、質量公式の $a_{\mathrm{sym}} \sim 23$ MeV (surface を考えると a_{sym} (vol) ~ 30 MeV) と比べて 1/3-1/2 程度。残りは相互作用。

- 残りの半分の対称エネルギーを RMF で評価してみましょう。

$$\Delta E_{\mathrm{sym},\rho} = \frac{1}{4}\frac{m_\rho^2 R^2}{\rho_B} = \frac{1}{4}\frac{g_\rho^2}{m_\rho^2}\rho_B\delta^2 = \Delta a_{\mathrm{sym}}\delta^2 \quad \left(R = \frac{g_\rho(\rho_n-\rho_p)}{m_\rho^2} = \frac{g_\rho\rho_B\delta}{m_\rho^2}\right)$$

$$g_\rho^2 = \frac{2m_\rho^2\Delta a_{\mathrm{sym}}}{\rho_B} \simeq (4.3)^2 \quad (a_{\mathrm{sym}} = 30\,\mathrm{MeV})$$

← RMF par. より少し小さめ

$$L \simeq E_F + 3\Delta a_{\mathrm{sym}} \simeq 90\,\mathrm{MeV}$$

← Optimal value より少し大きい

Simple parametrized EOS

- **Skyrme int. motivated parameterization**

$$E_{\mathrm{SNM}} = \frac{3}{5}E_F(\rho) + \frac{\alpha}{2}\left(\frac{\rho}{\rho_0}\right) + \frac{\beta}{2+\gamma}\left(\frac{\rho}{\rho_0}\right)^{1+\gamma}$$

$$\alpha = \frac{2}{\gamma}\left(E_0(1+\gamma) - \frac{E_F(\rho_0)(1+3\gamma)}{5}\right) \; , \quad \beta = \frac{2+\gamma}{\gamma}\left[-E_0 + \frac{1}{5}E_F(\rho_0)\right] \; .$$

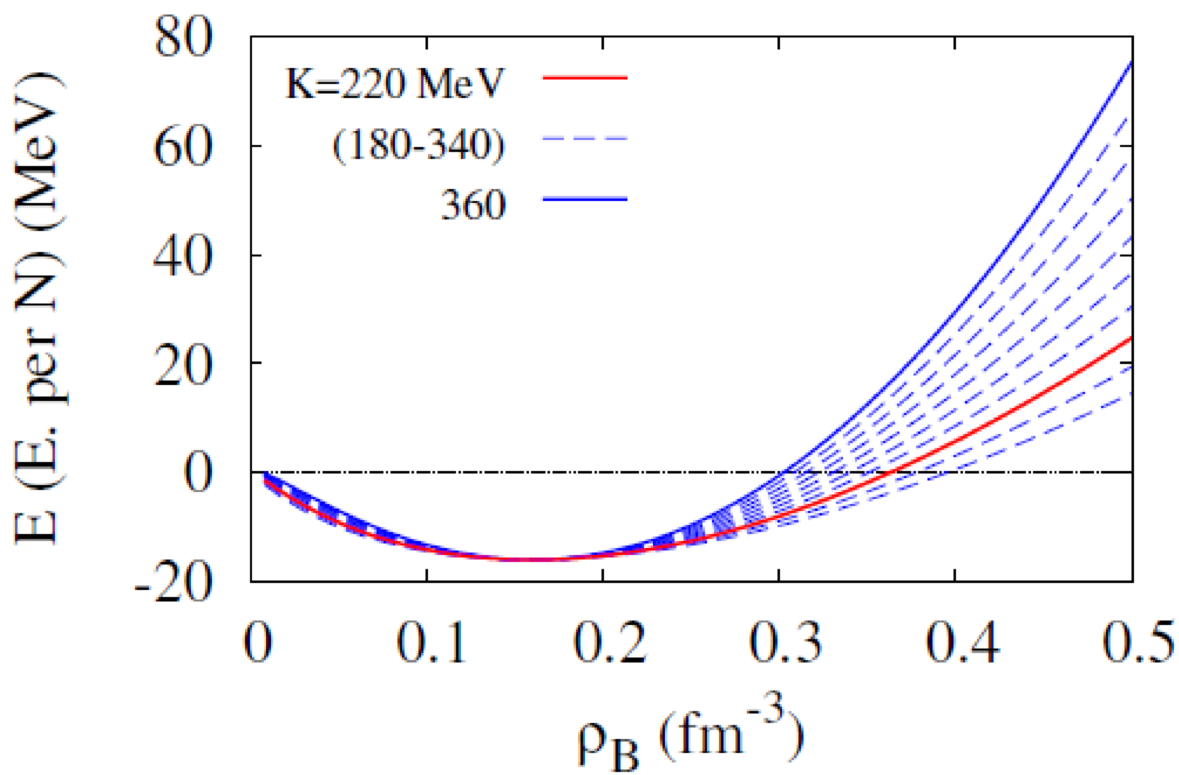
$$K = \frac{1+3\gamma}{5}E_F(\rho_0) - 3E_0(1+\gamma) \; .$$

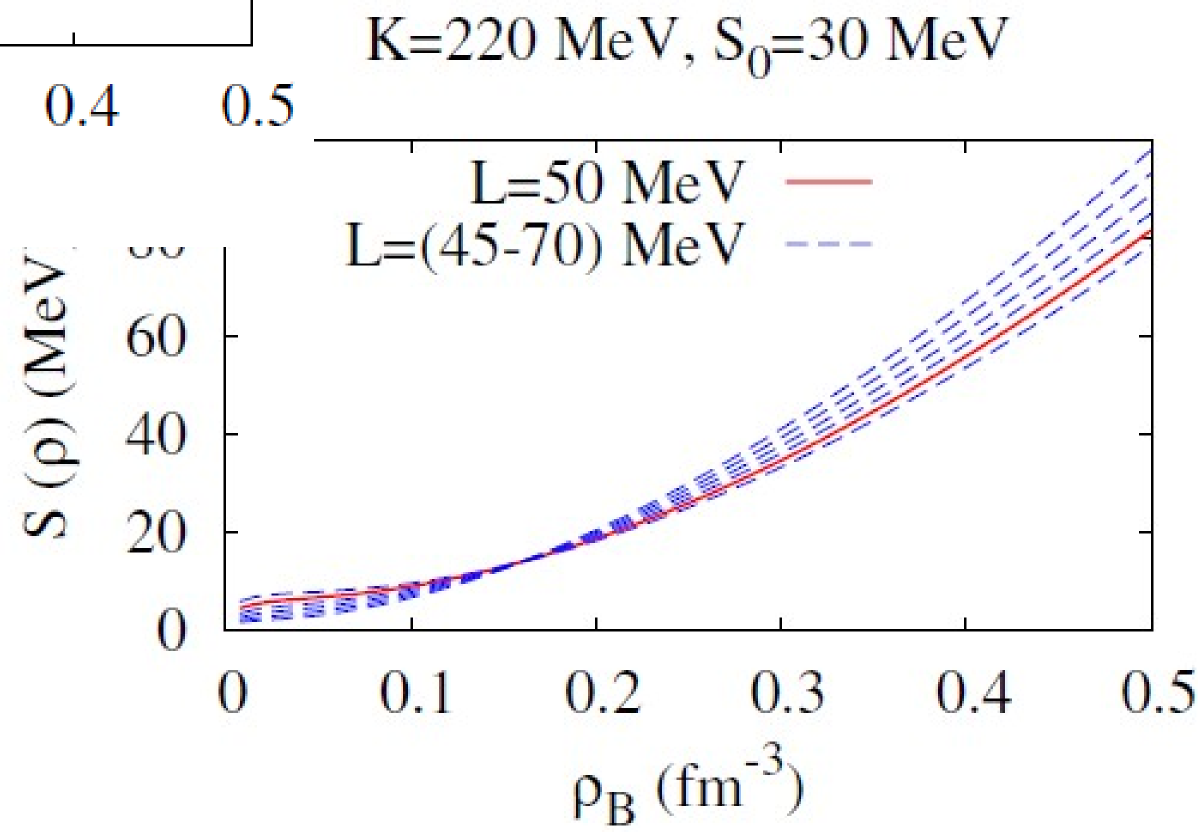
- **Symmetry energy parameterization**

$$S(\rho) = \frac{1}{3}E_F(\rho) + \left[S_0 - \frac{1}{3}E_F(\rho_0)\right]\left(\frac{\rho}{\rho_0}\right)^{\gamma_{\mathrm{sym}}}$$

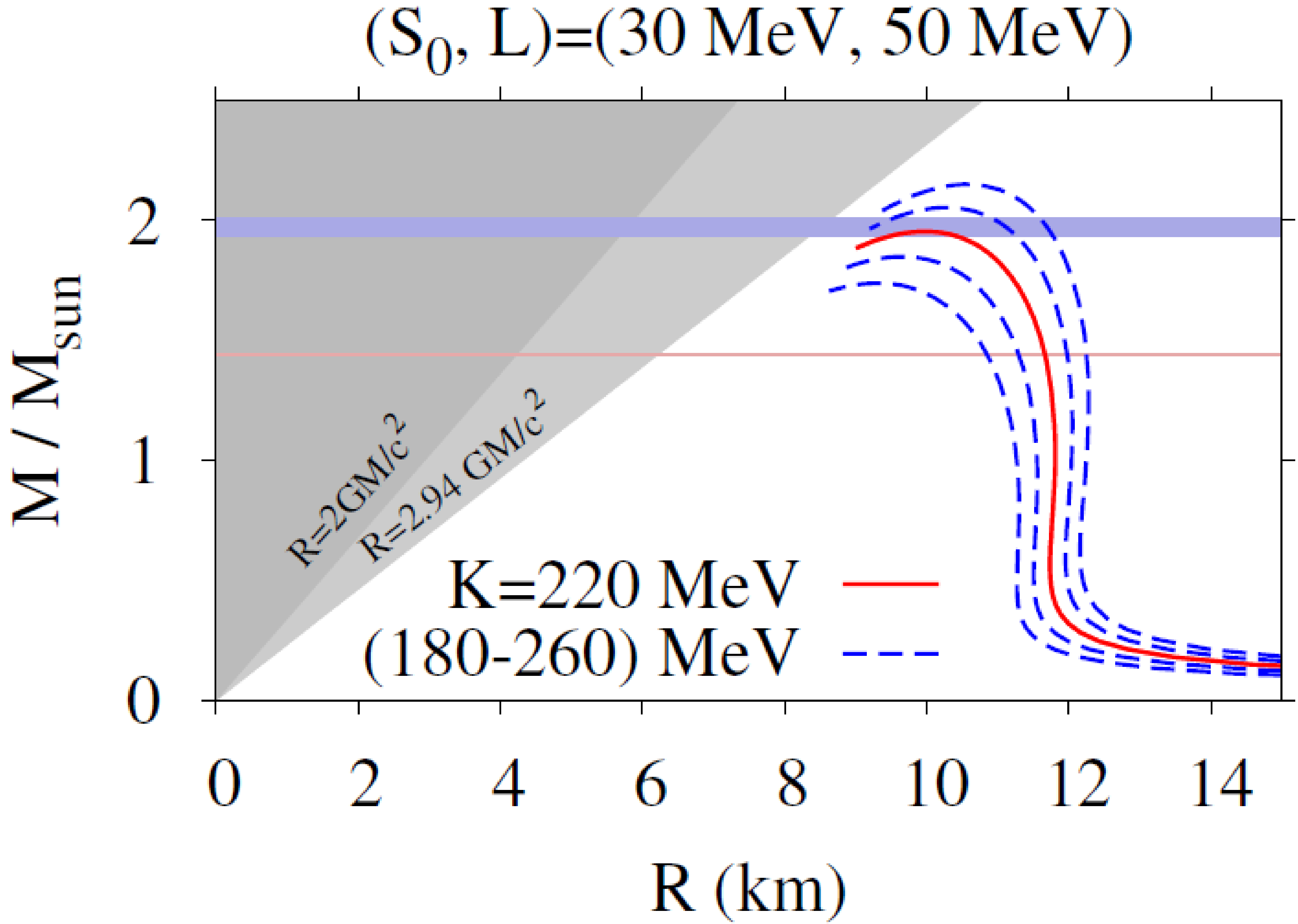
$$\gamma_{\mathrm{sym}} = \frac{L - \frac{2}{3}E_F(\rho_0)}{3S_0 - E_F(\rho_0)}$$

Simple parametrized EOS

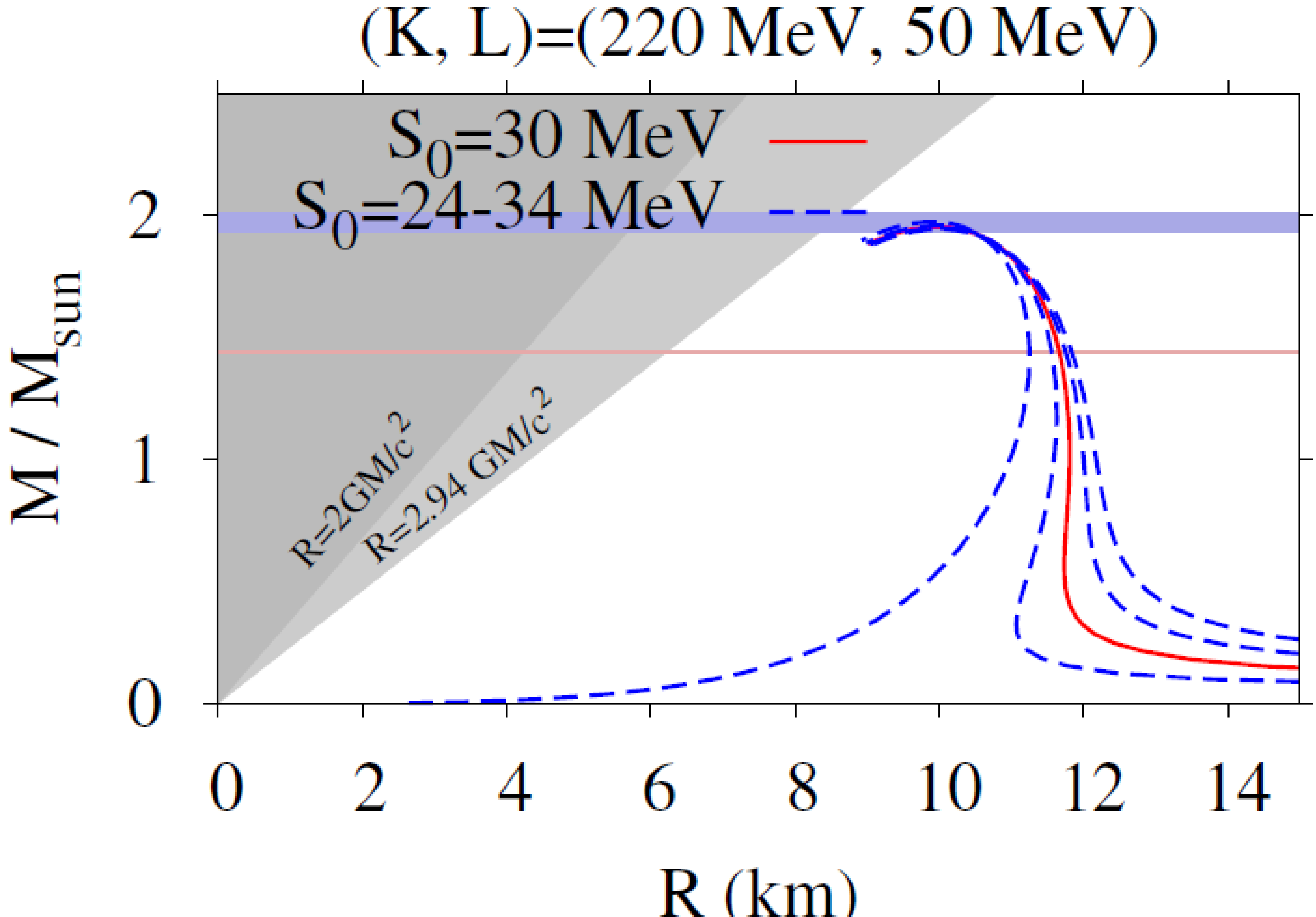




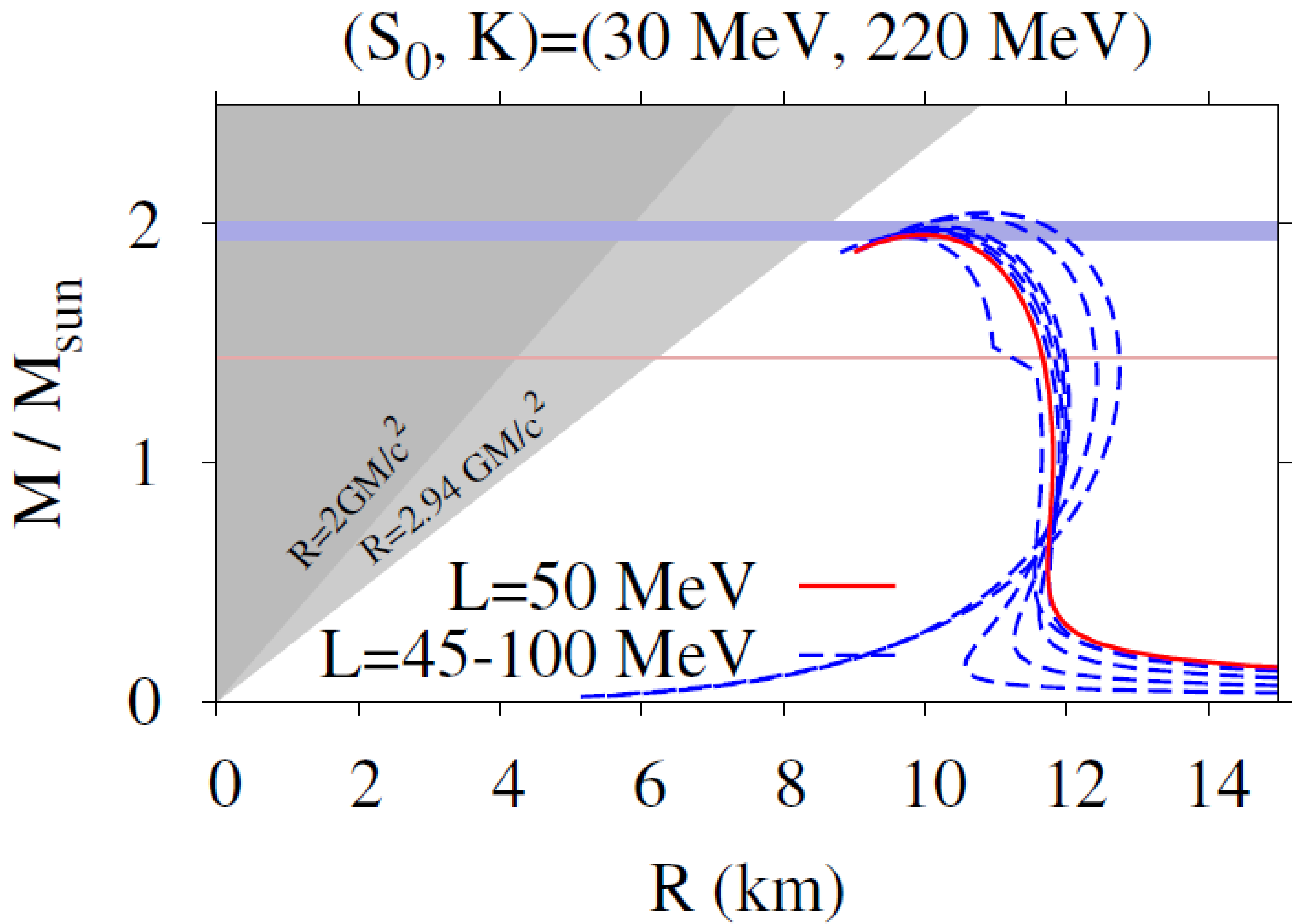
Simple parametrized EOS



Simple parametrized EOS

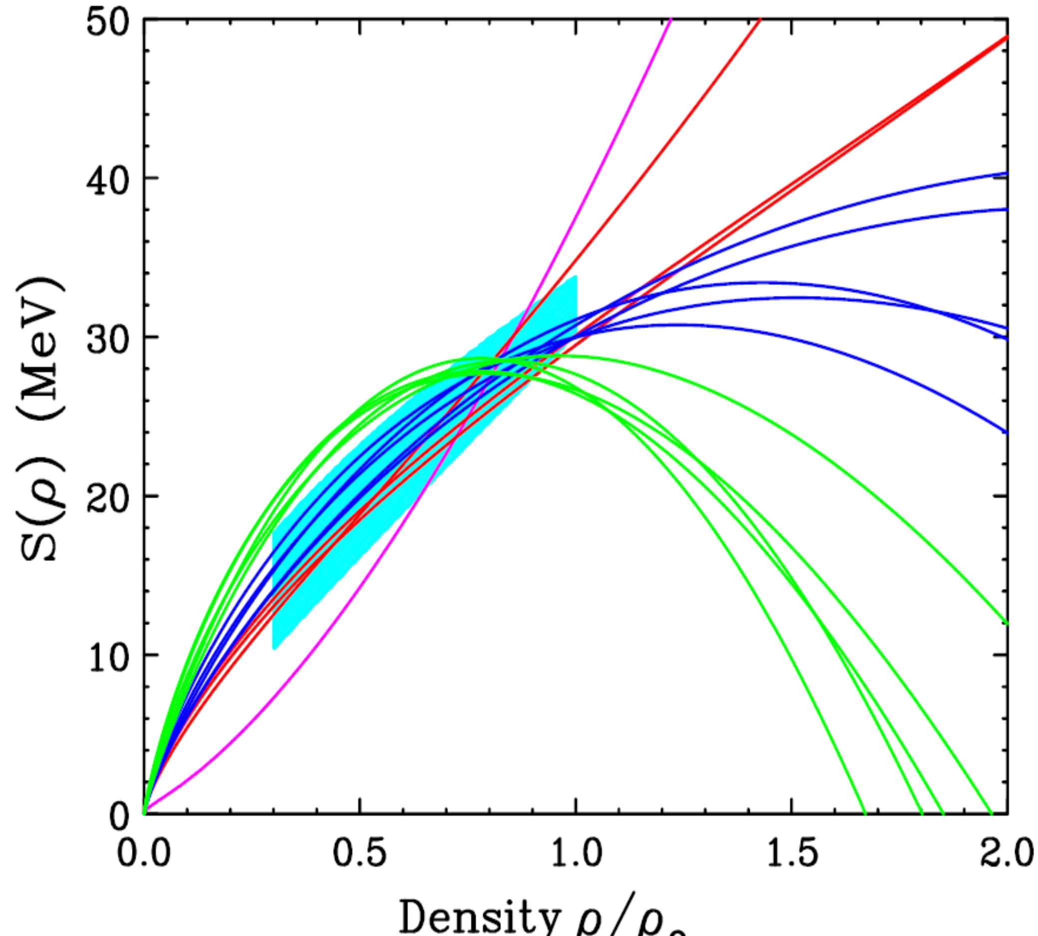


Simple parametrized EOS



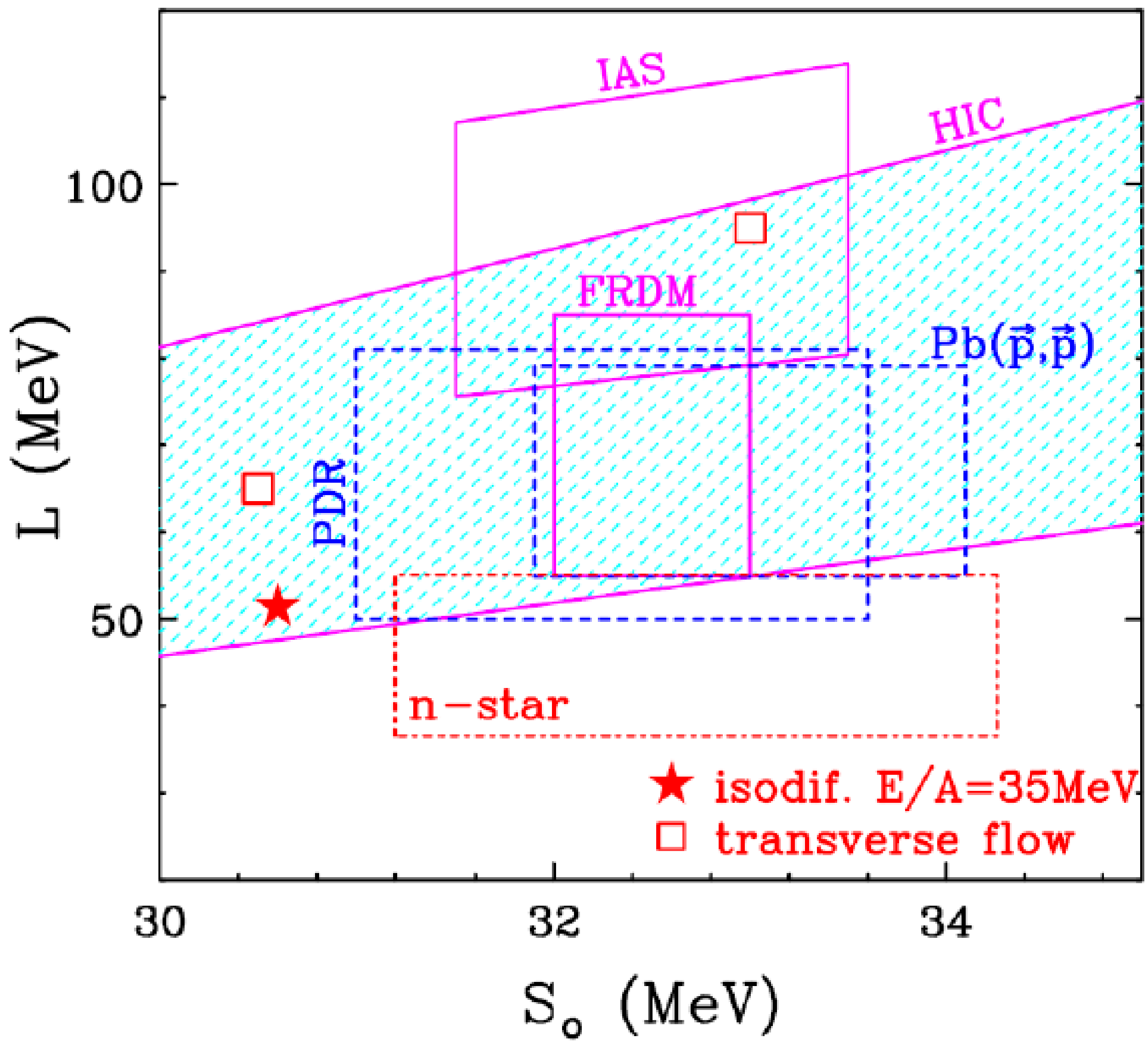
Symmetry Energy

- Summary of Nuclear Symmetry Energy workshop NuSym11 http://www.smith.edu/nusym11
  $E_{sym}(\rho_0)$ = 31-34 MeV, L = 50-110 MeV
  extracted from various observations.
  - Mass formula *Moller ('10)*
  - Isobaric Analog State *Danielewicz, Lee ('11)*
  - Pygmy Dipole Resonanc *Carbone+ ('10)*
  - Isospin Diffusion *Tsang et al. ('04)*
  - Neutron Skin thickness *J.Zenihiro+ ('10)*
- これらの多くは ρ_0 以下の密度での Esym に敏感。



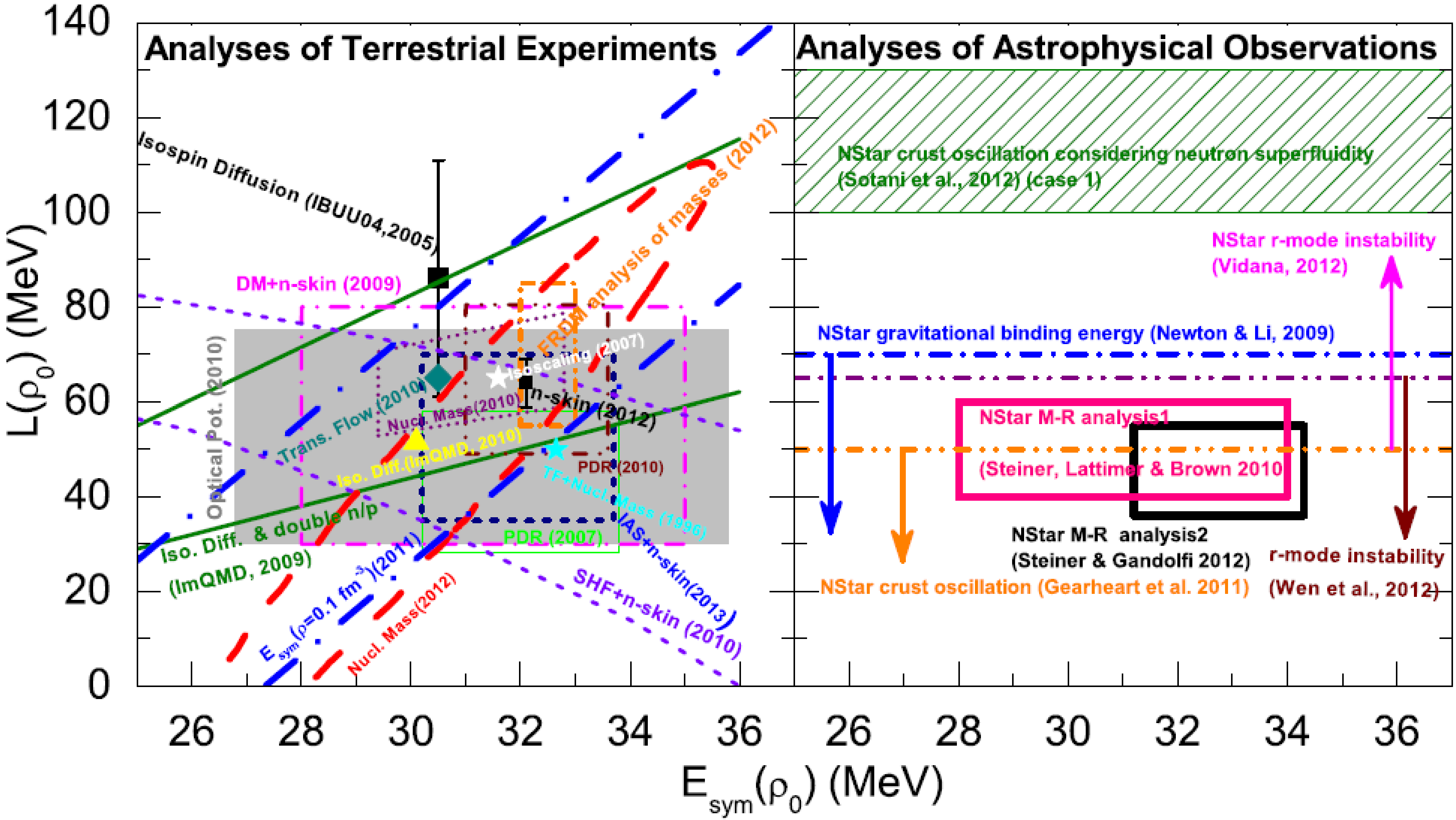
M. B. Tsang et al., Phys. Rev. C 86 (2012) 015803.

Nuclear Symmetry Energy (NuSYM 2011)



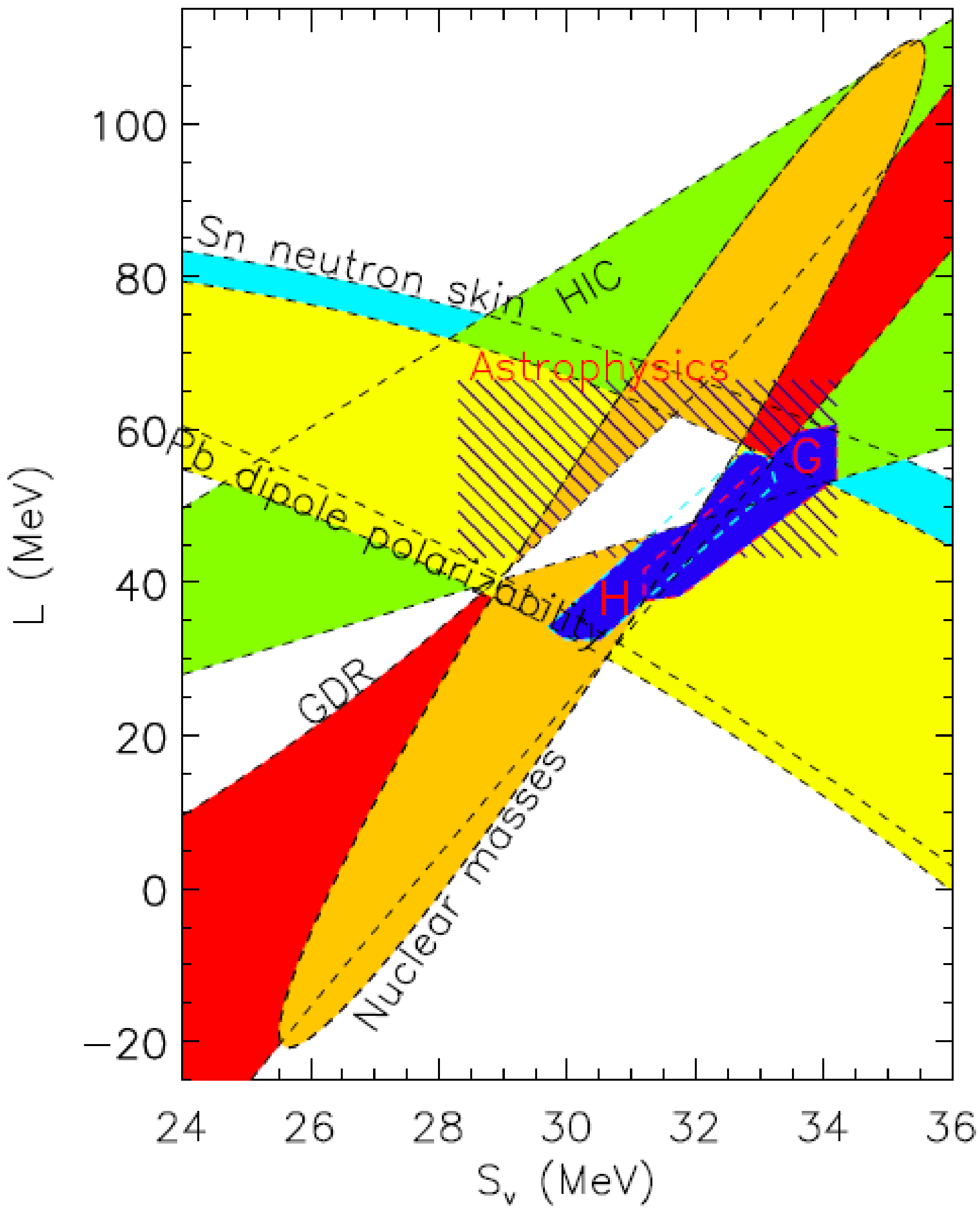
Tsang et al. ('12): NuSYM 2011

Nuclear Symmetry Energy (NuSYM 2013)



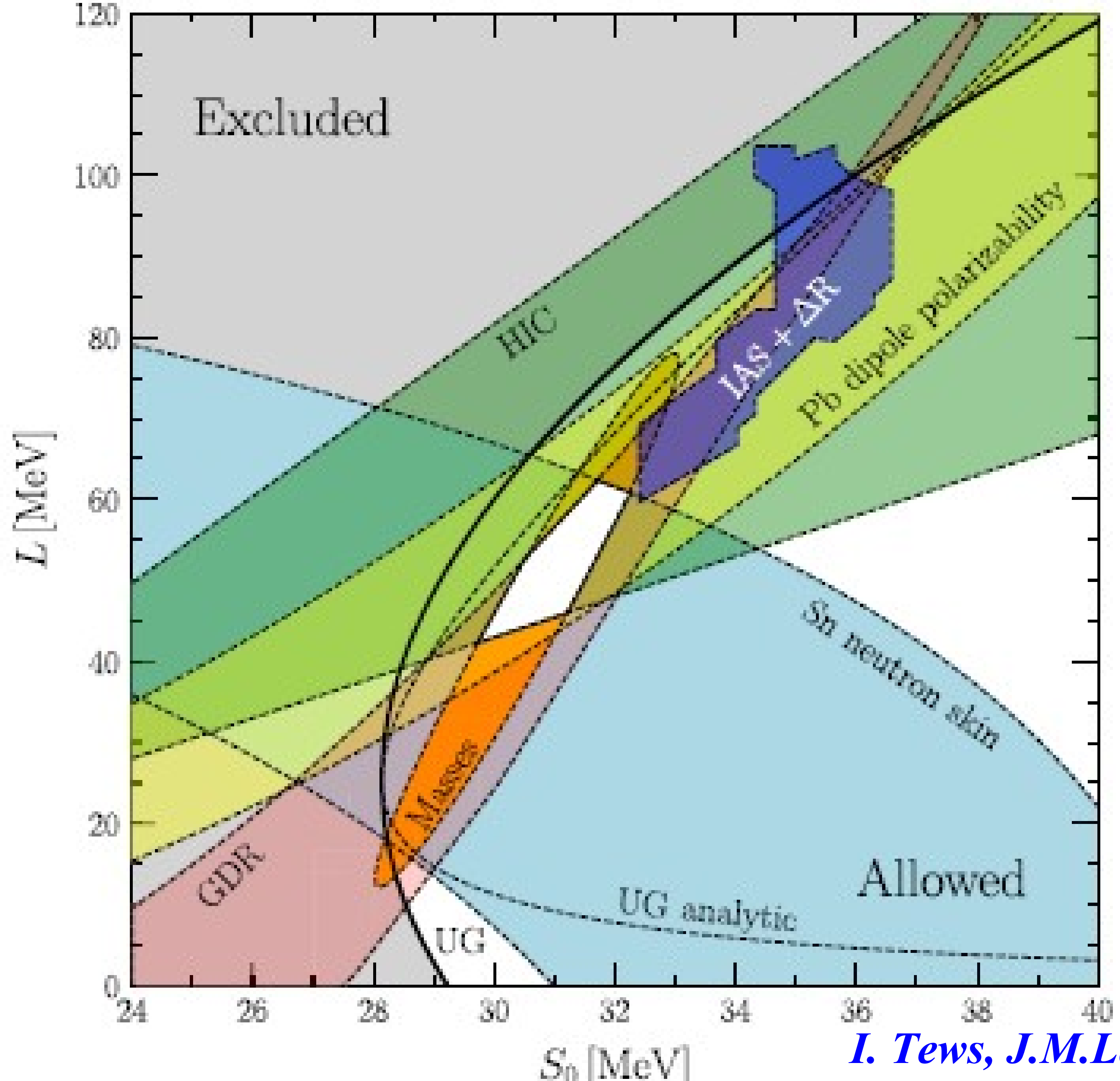
B. A. Li et al. ('13)

Nuclear Symmetry Energy (Lattmier-Lim, 2013)



Lattimer, Lim ('13)

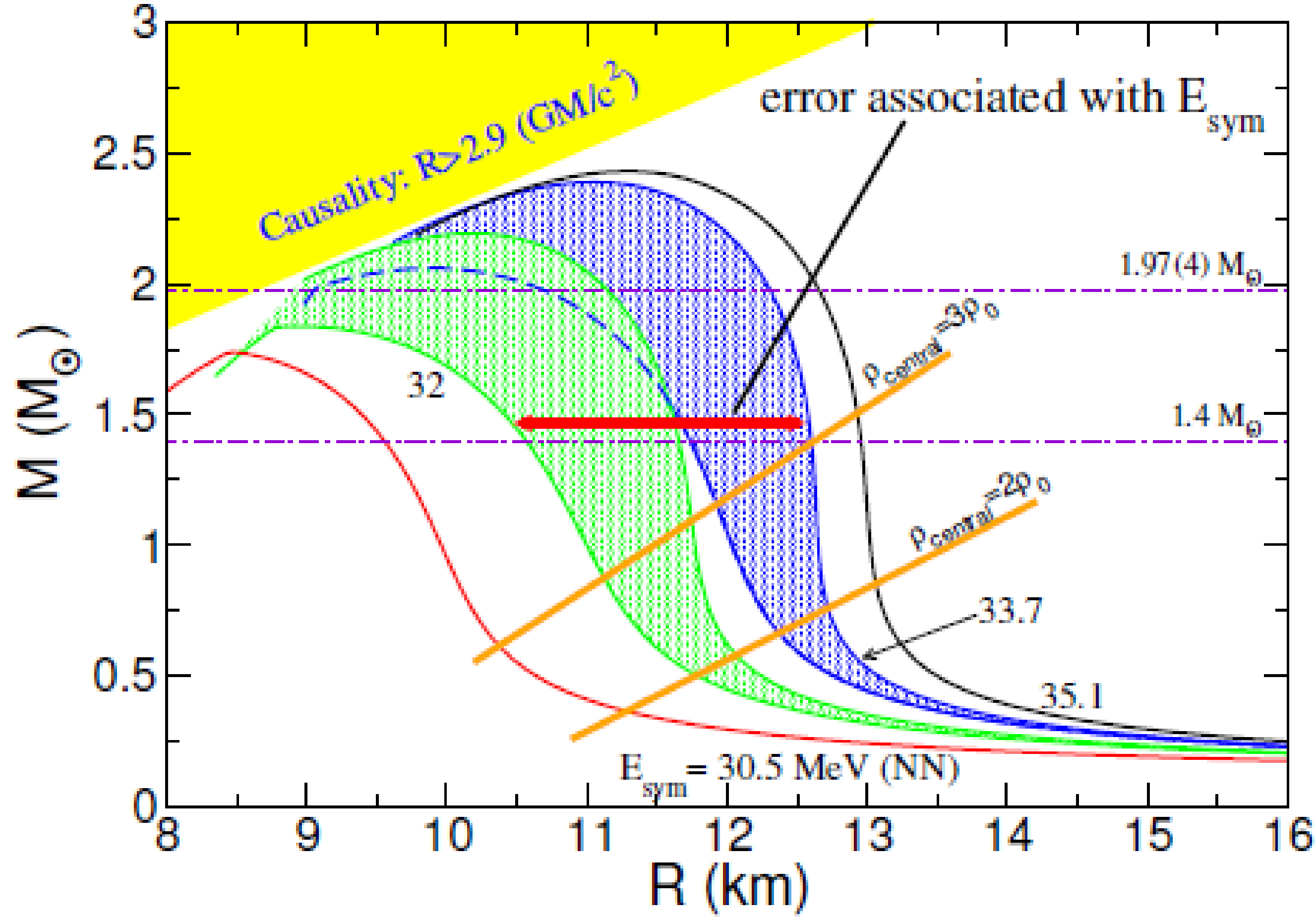
Nuclear Symmetry Energy (Tews+, 2017)



I. Tews, J.M.Lattimer, AO, E.E.Kolomeitsev (TLOK),ApJ 848 ('17)105

Symmetry Energy affects MR Relation of NS

- **Nuclear pressure at ρ_0 comes ONLY from Esym, then Esym dominates pressure around ρ_0 !**
- **5 MeV Difference in Esym results in (3-4) km difference in R_{NS} prediction.**



Gandolfi, Carlson, Reddy, PRC 032801, 85 (2012).

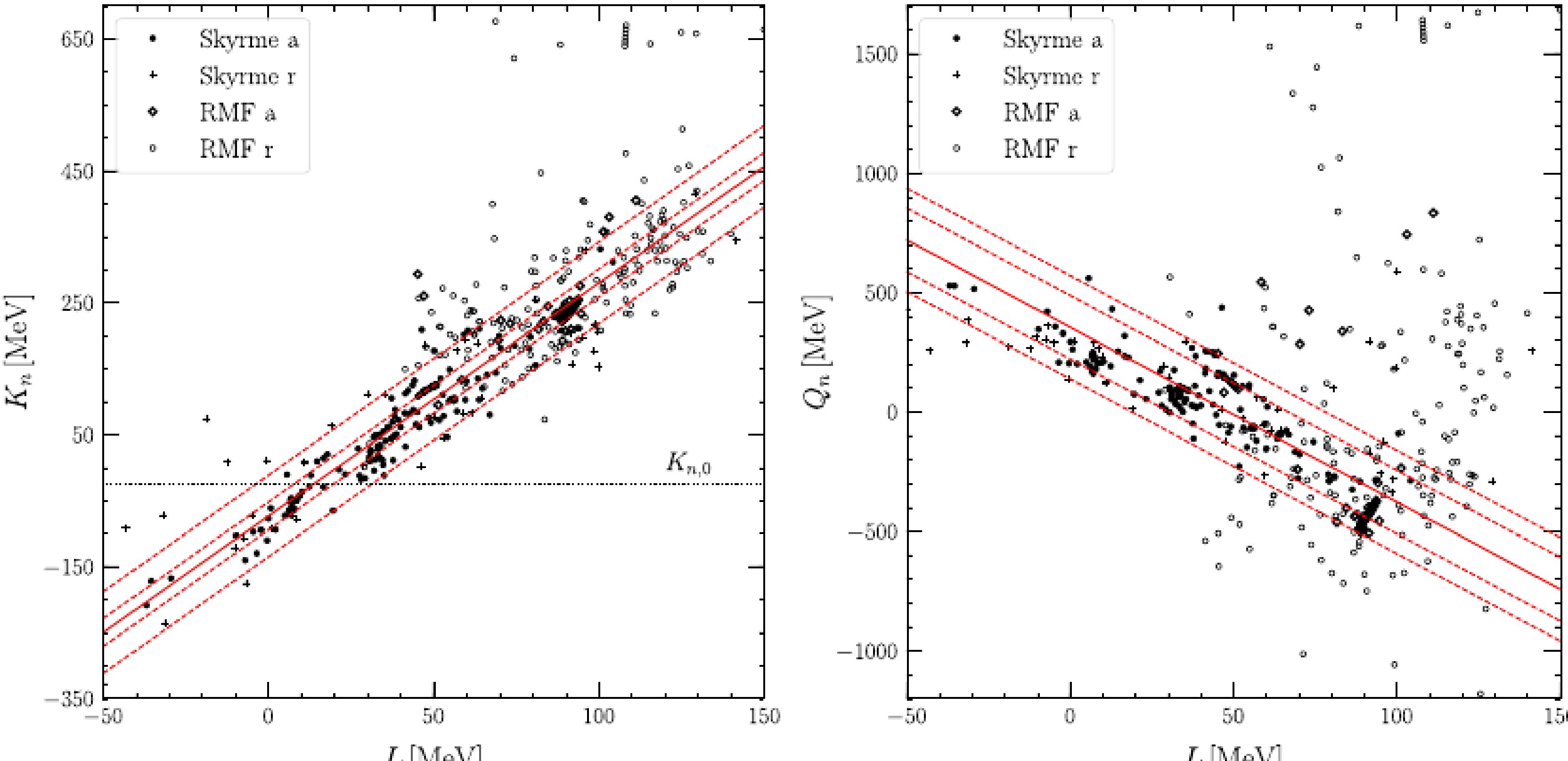
Higher-Order Symmetry Energy Parameter Effects on Nuclear Matter EOS, and Neutron Star Radius

Further Constraints on Higher-Order Sym. E. parameters

- **K_n and Q_n are correlated with L in "Good" theoretical models.**

$$K_n = 3.534L - (74.02 \pm 21.17)\mathrm{MeV}$$

$$Q_n = -7.313L + (354.03 \pm 133.16)\mathrm{MeV}$$



Regard theoretical models as data !

I. Tews, J.M.Lattimer, AO, E.E.Kolomeitsev (TLOK), ApJ 848 ('17)105

Fermi momentum (k_F) expansion

TLOK

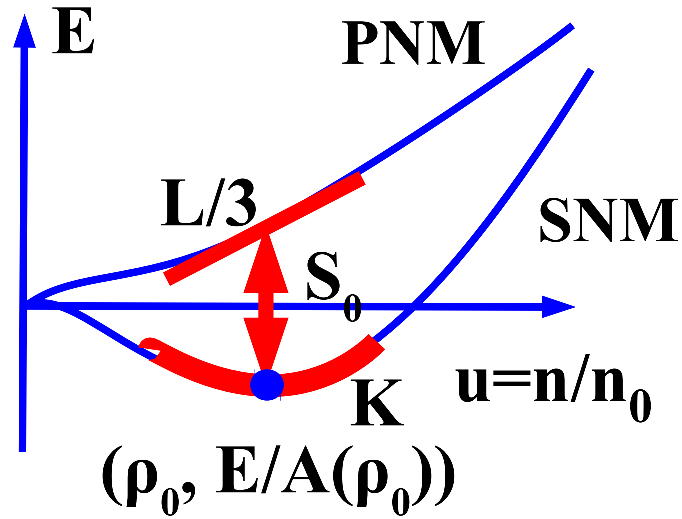
- **Saturation & Symmetry Energy Parameters**

$$E_{\rm NM}(u,\alpha) = E_{\rm SNM}(u) + \alpha^2 S(u)$$

$$E_{\rm SNM}(u) \simeq E_0 + \frac{K_0}{18}(u-1)^2 + \frac{Q_0}{162}(u-1)^3$$

$$S(u) \simeq S_0 + \frac{L}{3}(u-1) + \frac{K_s}{18}(u-1)^2 + \frac{Q_s}{162}(u-1)^3$$

$$(u = n/n_0, \alpha = (n_n - n_p)/n)$$



Energy does not approach zero at n → 0.

- **Fermi momentum expansion (~ Skyrme type EDF)**
  - **Generated many-body force is given by** $k_F \propto u^{1/3}$

$$E_{\rm SNM}(u) \simeq T_0\, u^{2/3} + a_0 u + b_0 u^{4/3} + c_0 u^{5/3} + d_0 u^2$$

$$S(u) \simeq T_s u^{2/3} + a_s u + b_s u^{4/3} + c_s u^{5/3} + d_s u^2$$

m* (c terms)

Kin. E. (T terms) **Two-body** (a terms) **Density-dep. pot.** (b, c, d terms)

Expansion Coefficients

- **Coefficients (a,b,c,d) are represented by Saturation and Symmetry Energy Parameters**

TLOK

$$
\begin{aligned}
a_0 &= -4T_0 && +20E_0 && +K_0 && -Q_0/6 \\
b_0 &= 6T_0 && -45E_0 && -5K_0/2 && +Q_0/2 \\
c_0 &= -4T_0 && +36E_0 && +2K_0 && -Q_0/2 \\
d_0 &= T_0 && -10E_0 && -K_0/2 && +Q_0/6
\end{aligned}
$$

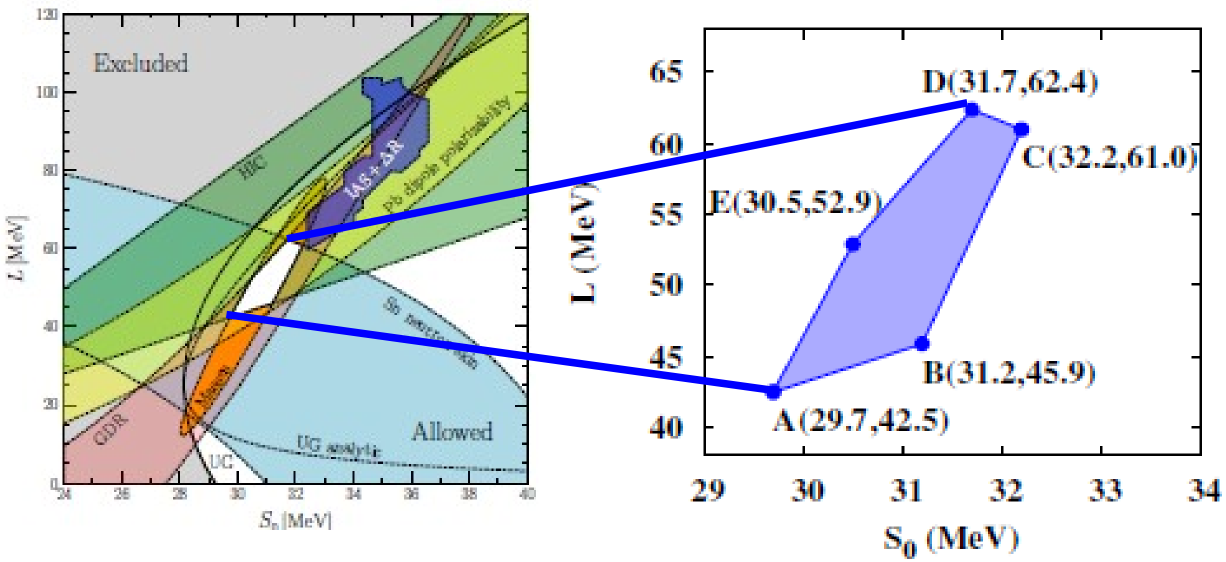
$$
\begin{aligned}
a_s &= -4T_s && +20S_0 - 19L/3 && +K_s && -Q_s/6 \\
b_s &= 6T_s && -45S_0 + 15L && -5K_s/2 && +Q_s/2 \\
c_s &= -4T_s && +36S_0 - 12L && +2K_s && -Q_s/2 \\
d_s &= T_s && -10S_0 + 10L/3 && -K_s/2 && +Q_s/6
\end{aligned}
$$

$$\left(T_0 = \frac{3}{5}\frac{\hbar^2 k_F(n_0)^2}{2m},\quad T_s = T_0(2^{1/3}-1)\right)$$

Tedious but straightforward calc.

TLOK+2M$_\odot$ constraints

- **TLOK constraints**
  - **(S_0, L) is in Pentagon.**
  - **(K_n, Q_n) are from TLOK constraint.**
  - **K_0=(190-270) MeV**
  - **(n_0,E_0) is fixed**
    n_0=0.164 fm-3, E_0=-15.9 MeV (small uncertainties)
  - **Q_0 is taken to kill d_0 parameter**
    (Coef. of u^2. Sym. N. M. is not very stiff at high-density)

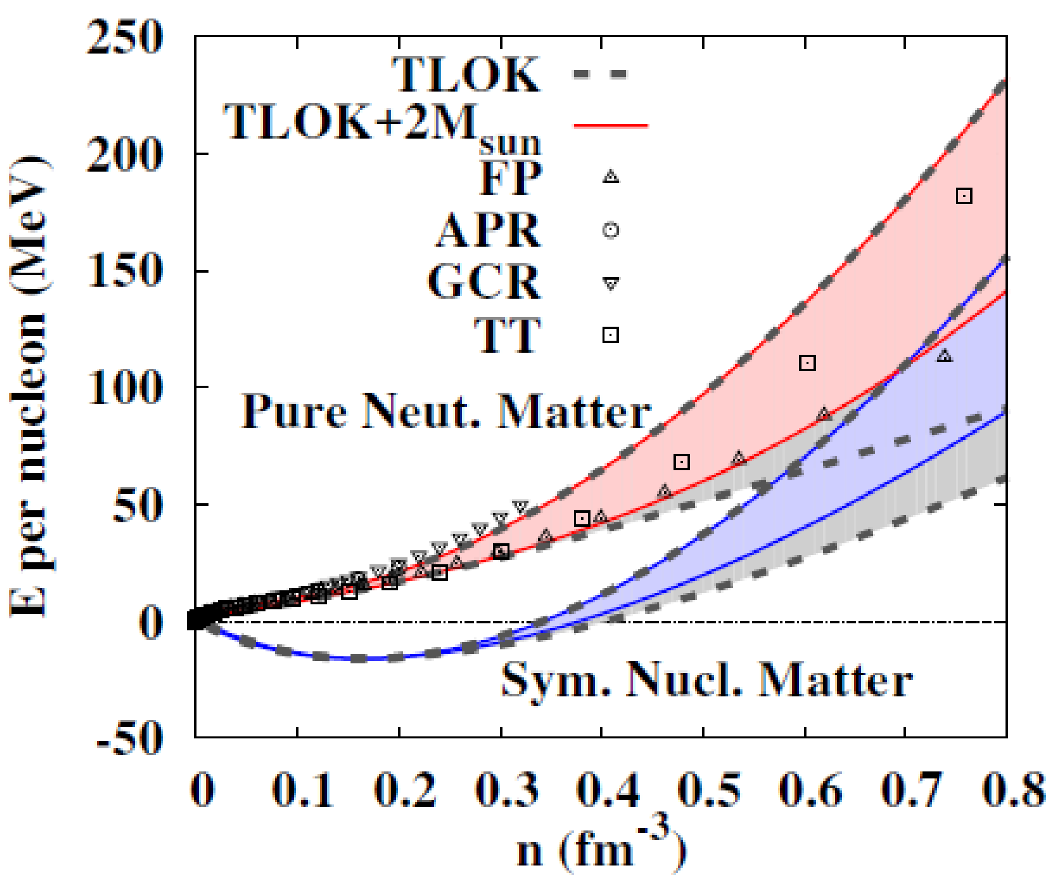


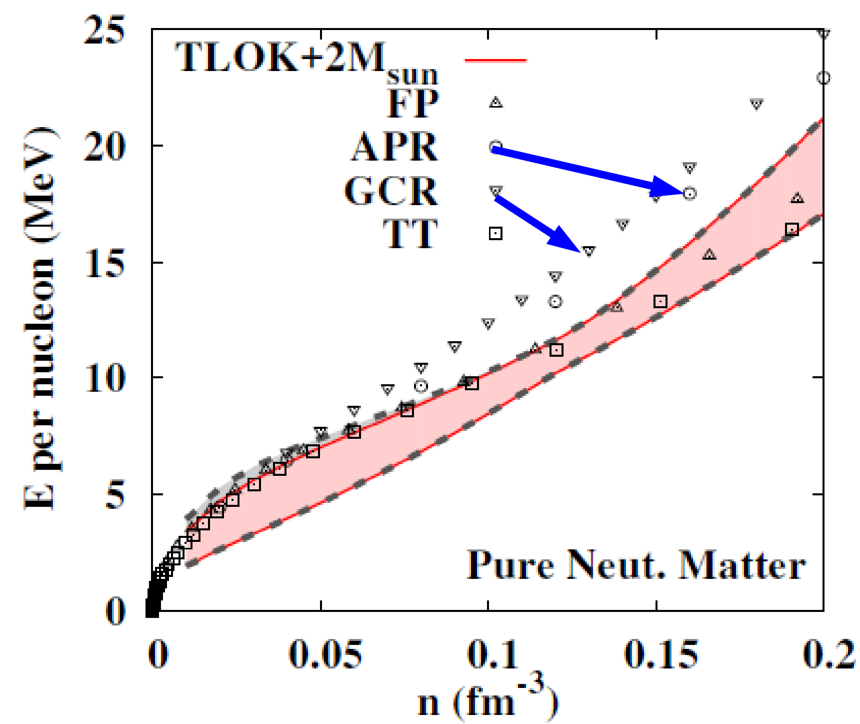
- **2 M$_\odot$ constraint**
  - **EOS should support 2 M$_\odot$ neutron stars.**

AO, Kolomeitsev, Lattimer, Tews, Wu (OKLTW), in prog.

TLOK+2M$_\odot$ constraints on EOS

- $2M_\odot$ constraint narrows the range of EOS.
- Consistent with FP and TT(Togashi-Takano) EOSs.
- APR and GCR(Gandolfi-Carlson-Reddy) EOSs seems to have larger S_0 values.

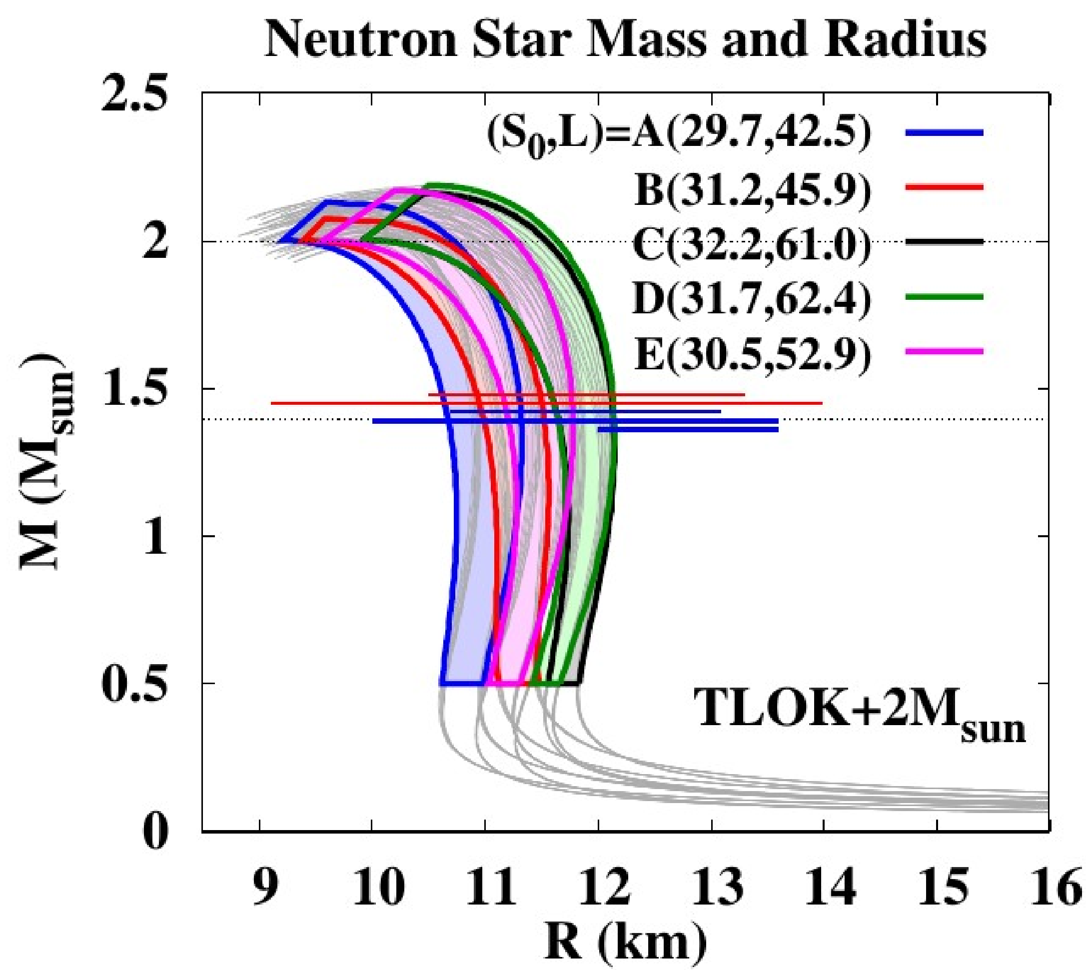




OKLTW, in prog.

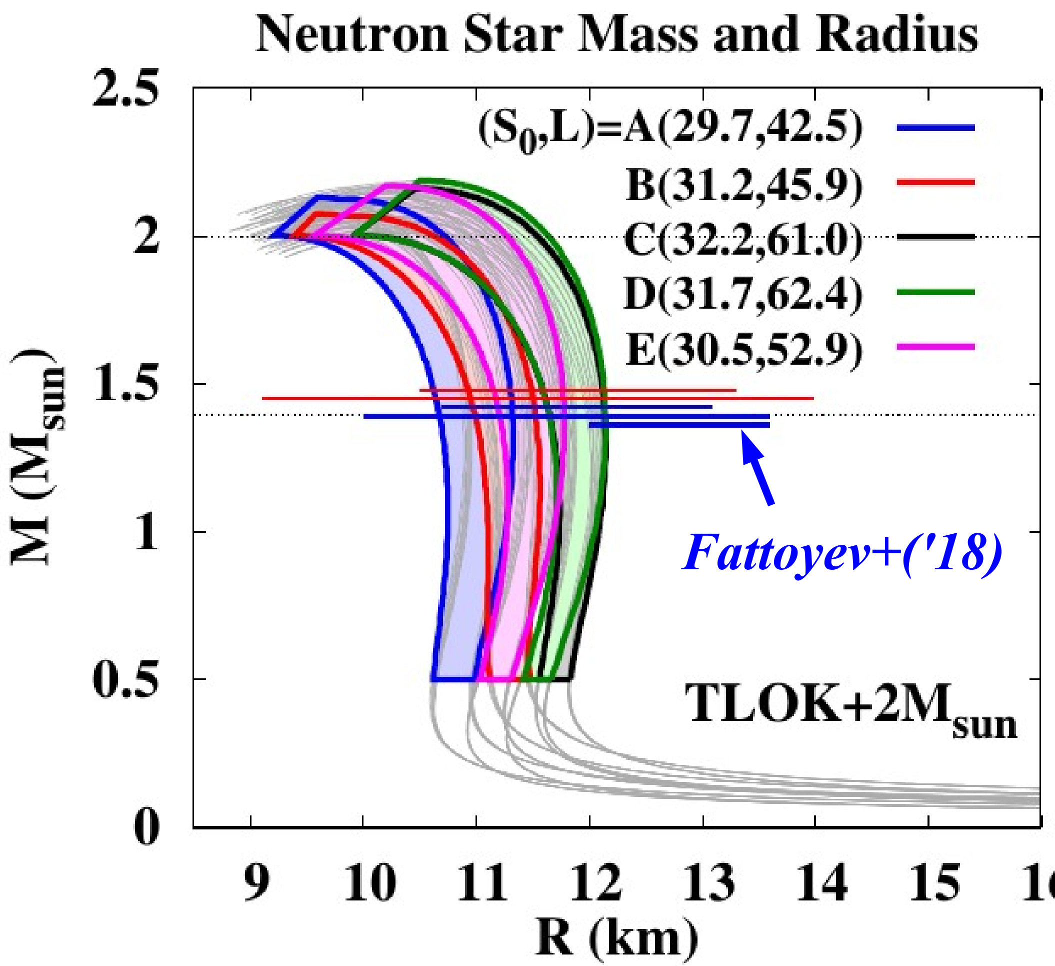
Neutron Star MR curve

- **TLOK + 2 $M_\odot$ constraints → $R_{1.4}$=(10.6-12.2) km** *OKLTW, in prog.*
  - **E and P are linear fn. of Sat. & Sym. E. parameters → Min./Max. appears at the corners of pentagon (ABCDE).**
  - **For a given (S_0, L), unc. of $R_{1.4}$ ~ 0.5 km = unc. from higher-order parameters**
  - **Unc. from (S_0, L) ~ 1.1 km → We still need to fix (S_0, L) more precisely.**

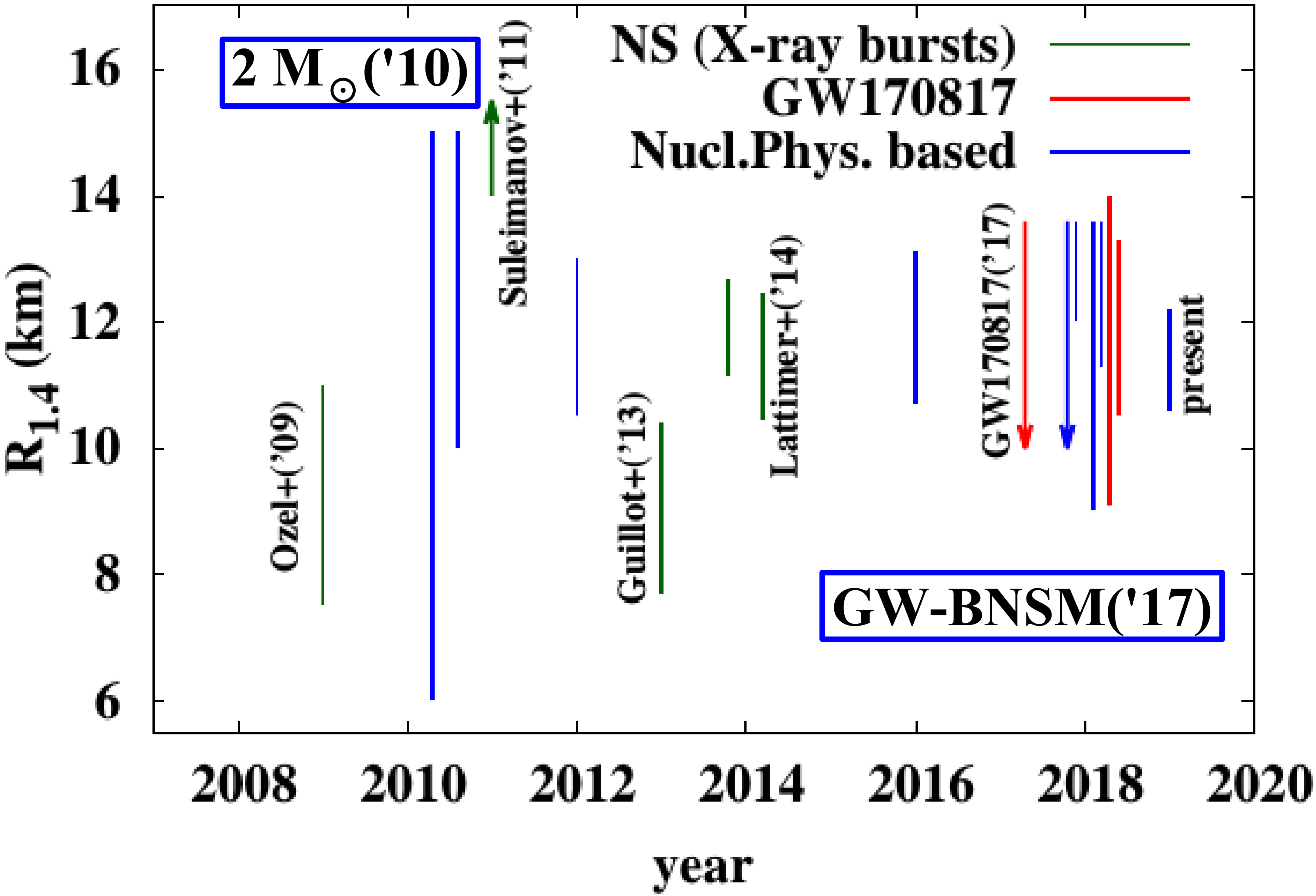


Neutron Star MR curve

- **Our constraint is consistent with many of previous ones.**
  - **$R_{1.4}$=(10.6-12.2) km** *Present work (TLOK + 2 $M_\odot$)* *OKLTW, in prog.*
  - **LIGO-Virgo (Tidal deformability Λ from BNSM)**
    (10.5-13.3) km *Abbott+('18b)*
    (9.1-14.0) km *De+('18) (Λ)*
  - **Theoretical Estimates**
    (10.7-13.1) km
    Lattimer+, PRep.621('16)127
    (10.0-13.6) km
    Annala+,PRL120('18)172703
    (9-13.6) km
    Tews+, PRC98 ('18)045804
    (12.0-13.6) km
    F.J.Fattoyev+(PREX), PRL120 ('18)172702



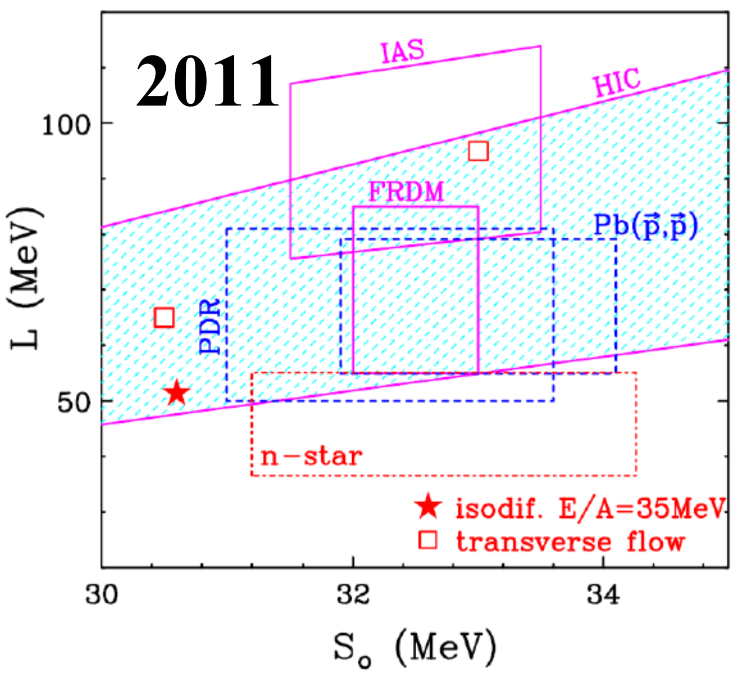
Time dependence of Neutron Star Radius ($R_{1.4}$)



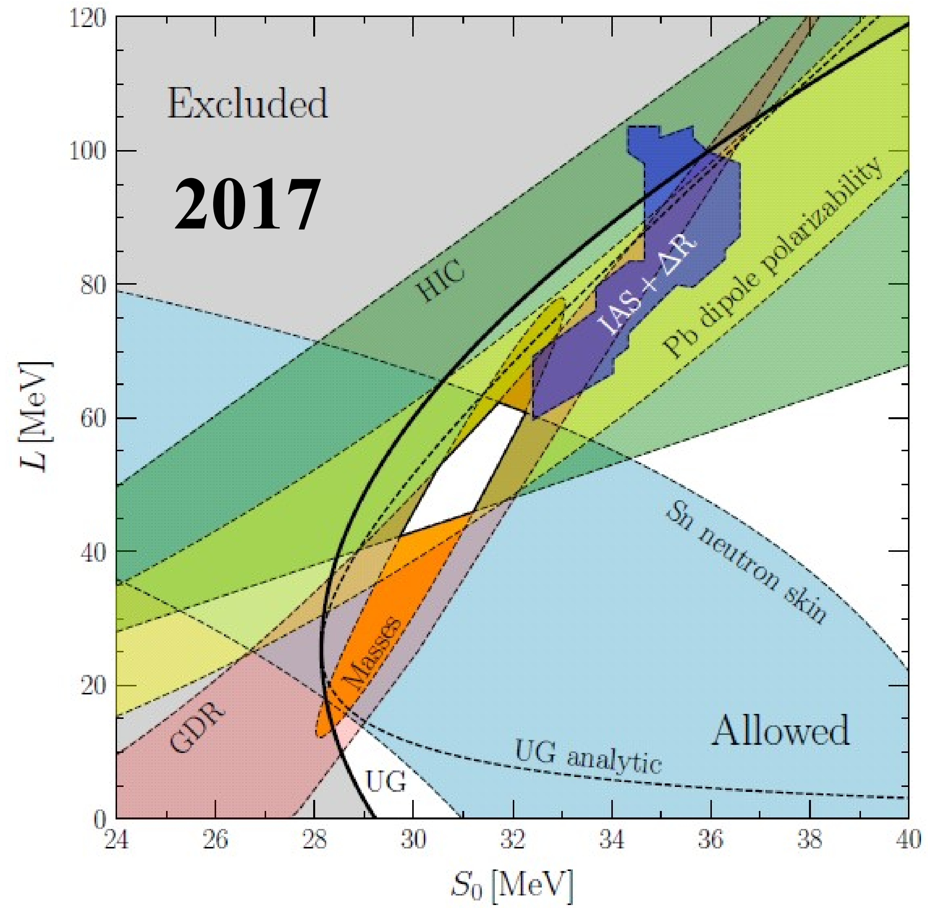
対称エネルギーパラメータ

- **Symmetry Energy Parameters**

$$S(u) = S_0 + \frac{L}{3}(u-1) + \frac{K_{\rm sym}}{18}(u-1)^2 + \mathcal{O}[(u-1)^3] \qquad (u=n/n_0)$$

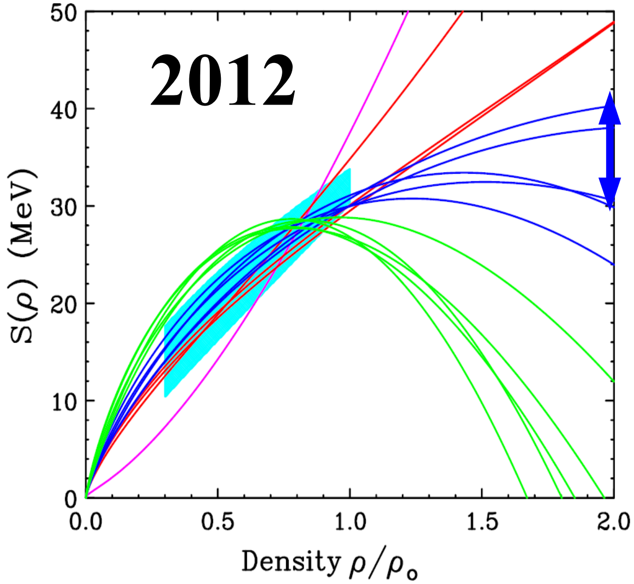


Tsang et al. ('12): NuSYM 2011

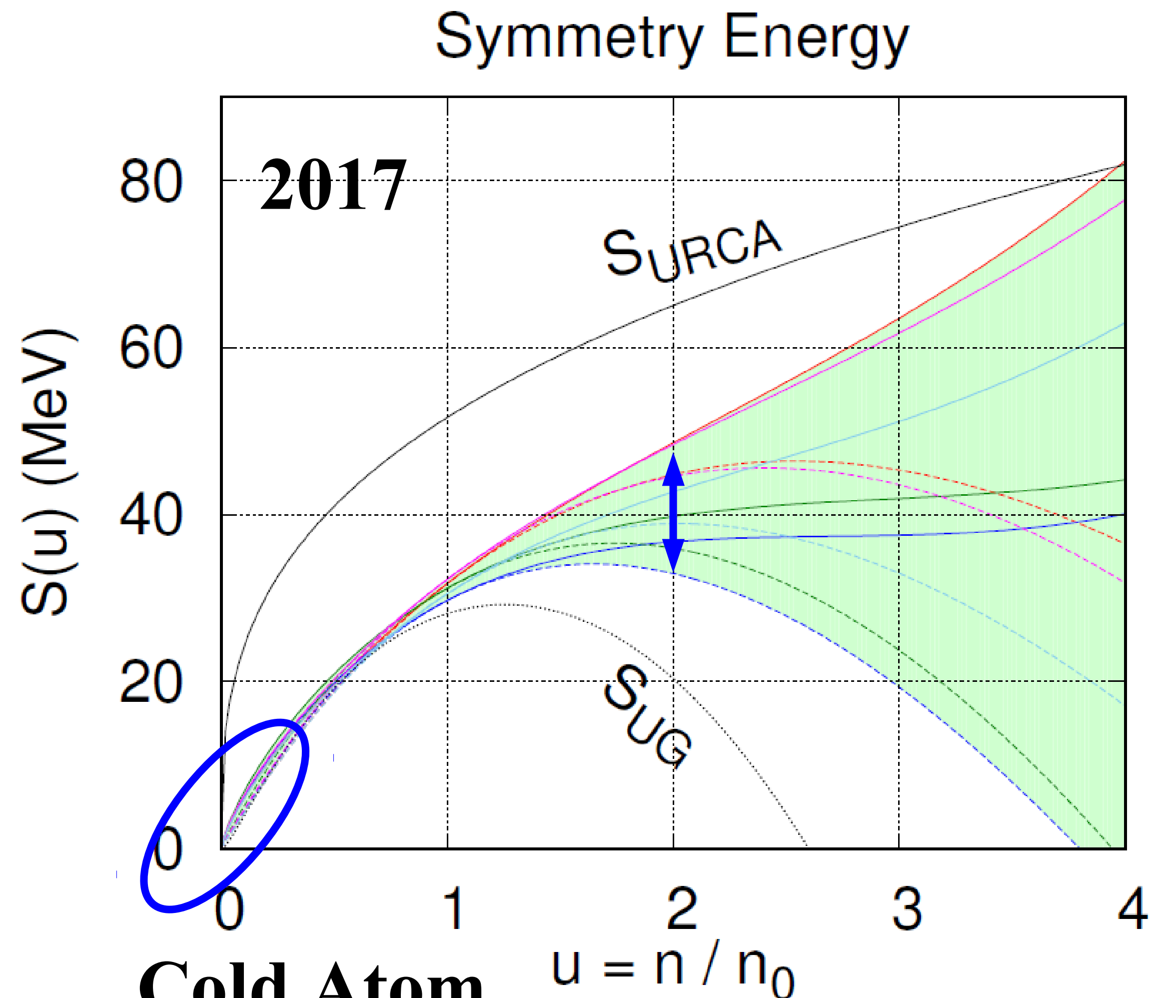


Lattimer, Lim ('13), Lattimer, Steiner ('14)
Tews, Lattimer, AO, Kolomeitsev ('16)

対称エネルギー



M. B. Tsang et al., PRC86 ('12) 015803.



Cold Atom

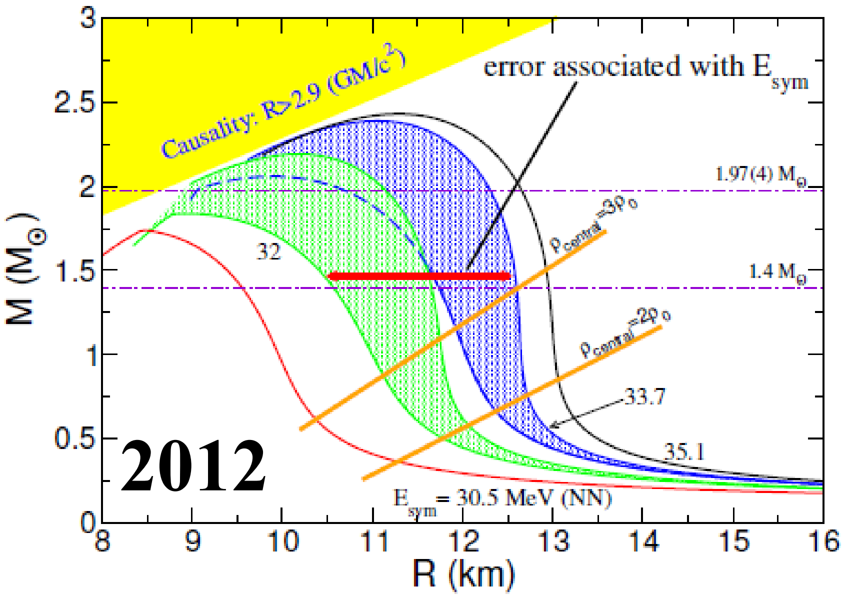
仮定

- (S_0, L) が 5 角形の中
- $k_F{}^n$ (n=2,3,4,5,6) で展開
- 2, 3 次の係数 (Ksym, Qsym) が L と相関 (模型からの推定)

Based on Tews et al. ('17)

OKLTW, in prog.

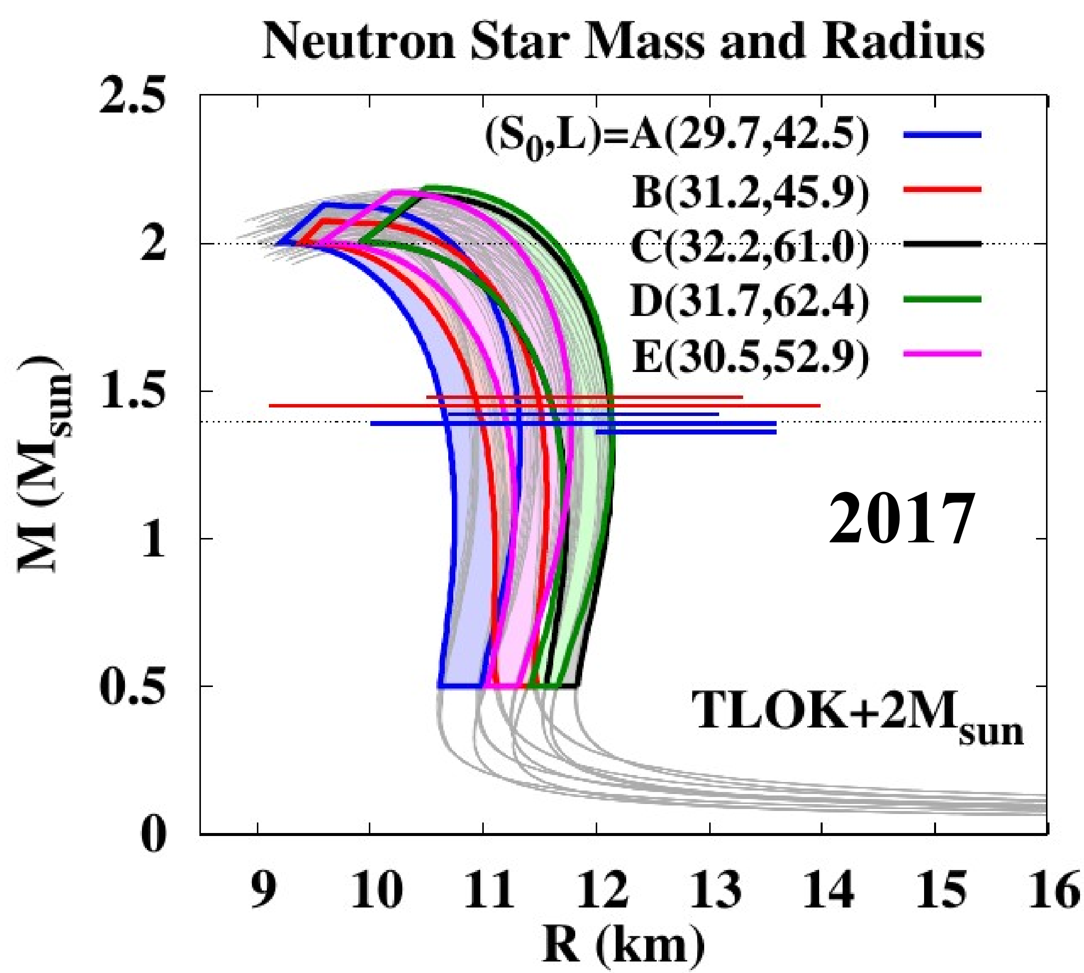
中性子星のMR曲線



Gandolfi, Carlson, Reddy, PRC85('12) 032801.

Based on Tews et al. ('17)
OKLTW, in prog.

R(1.4)=(10.6-12.2) km (unif. matter)



Experiments & Theories towards Nuclear Symmetry Energy

Nuclear Mass

- **Larger symmetry energy → B.E. of n-rich nuclei become smaller.**
  - **Volume and surface symmetry energy**

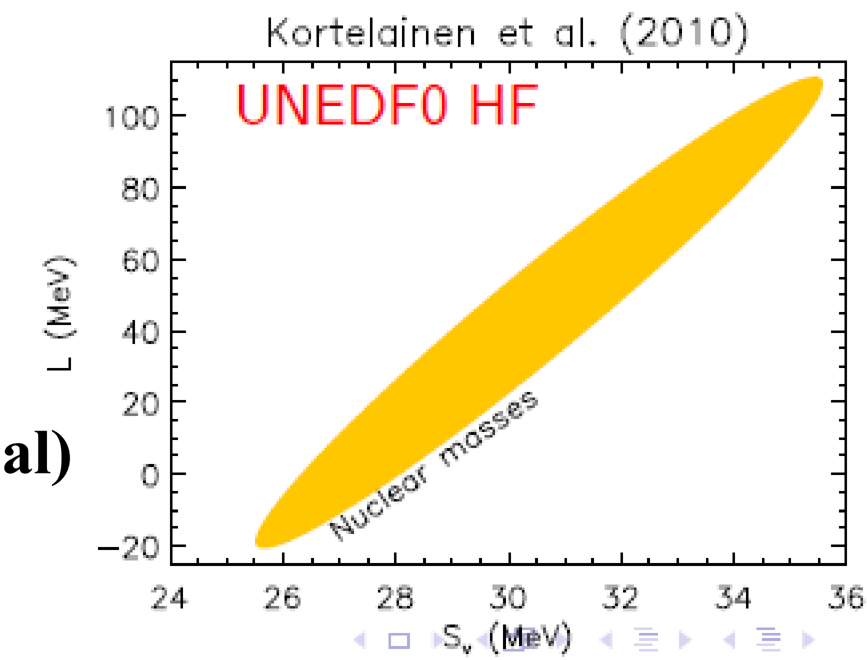
$$S(A) = a_a(A) = S_{\rm vol} - S_{\rm surf} A^{-1/3}$$

$$B(A,Z) = a_v A - a_s A^{2/3} - a_C \frac{Z^2}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} + a_p \frac{\delta_p}{A^\gamma}$$

  - **Finite Range Droplet Model**
    P.Moller, W.D.Myers, H.Sagawa, S.Yoshida, Phys. Rev. Lett. 108, 052501 (2012)

$$S_v = 32.5 \pm 0.5 \text{ MeV}, L = 70 \pm 15 \text{ MeV}$$

  - **Density Functional (Universal Nuclear Density Functional)**
    Kortelainen et al. (2010)



Lattimer, Lim ('13)

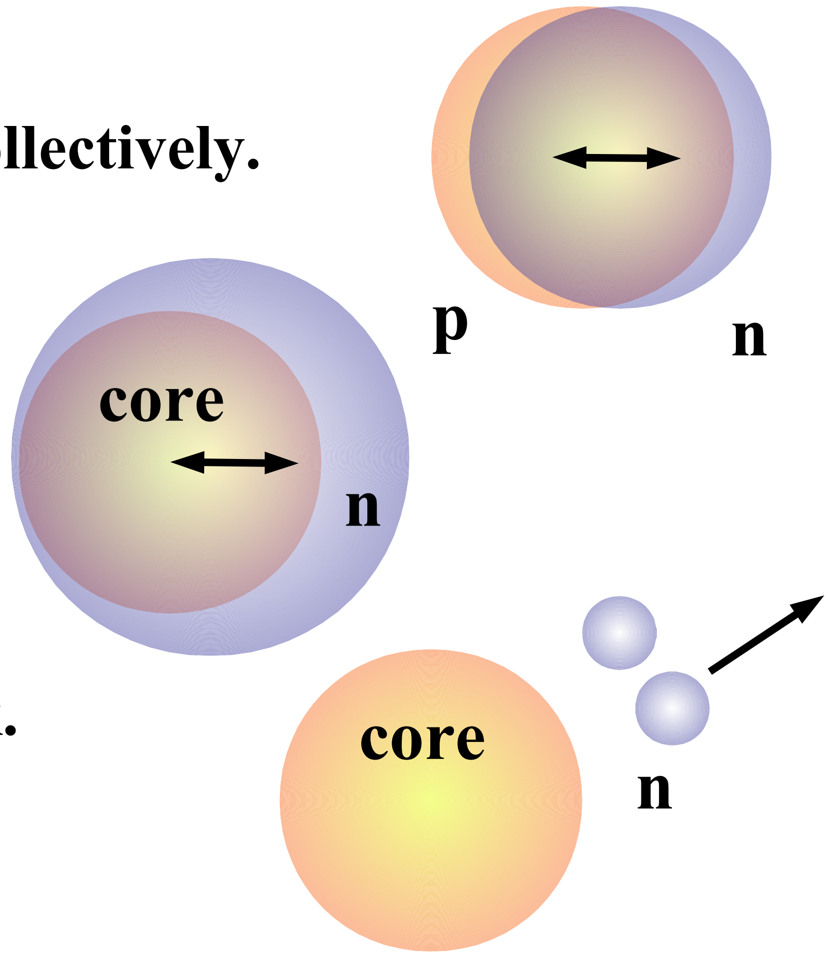
Well, relax (rough idea)

- **When L and Sv are linearly correlated as**
  $L = a S_v + b$,
  Symmetry energy is given as
  $E_{sym}(x)= Sv + L(x-1)/3=Sv (1+a(x-1)/3) + const.$ (indep. of Sv)
  ($x= \rho/ \rho_0$)
  → That observable determines symmetry energy
  most effectively at $x = 1 – 3/a$.

- **Nuclear mass: a ~ 14**
  原子核質量は $x \sim 1 – 3/14 = 0.78$ 近辺の対称エネルギーをよく決める

Pigmy Dipole Resonance

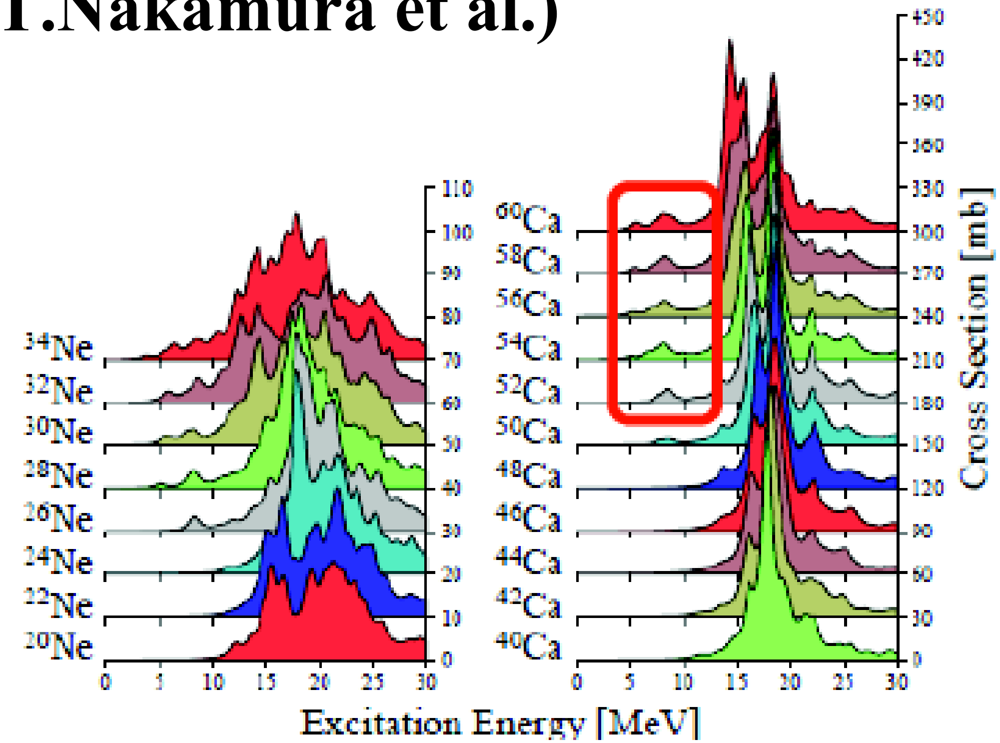
- **E1 response of nuclei**
  - **Giant Resonance: p and n oscillates collectively.**
    $E^* \sim 80\ A^{-1/3}$ MeV
  - **Pigmy Dipole Resonance:**
    Core oscillates in neutron skin/halo
    $E^* \sim (5\text{-}10)$ MeV
  - **Soft E1 excitation**
    When n wf is extended,
    direct dissociation σ also shows a peak.



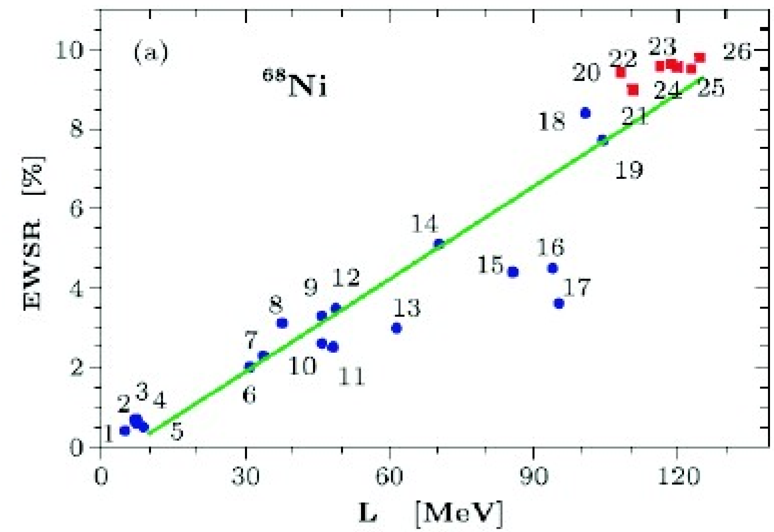
PDR would be sensitive to Esym

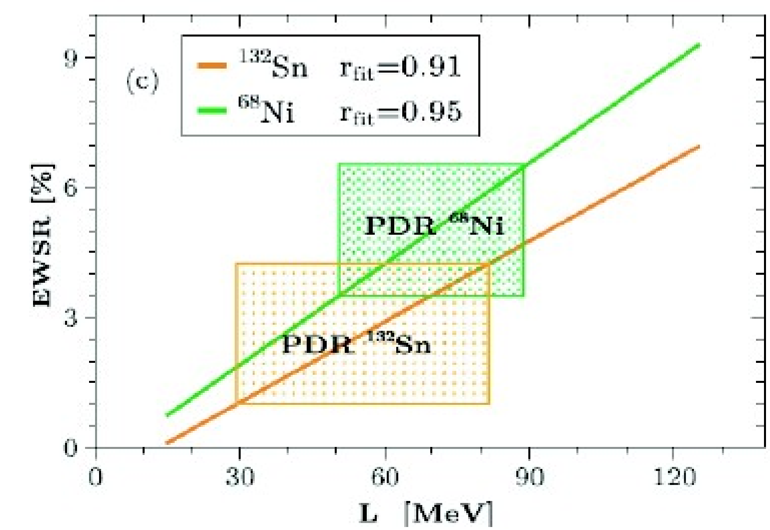
PDR of neutron skin nuclei

- Energy Weighted Sum Rule value of PDR would has linear dep. on L
- PDR of very neutron rich nuclei will be measured at RIBF-SAMURAI (T.Nakamura et al.)



T.Inakura, T.Nakatsukasa, K.Yabana Phys.Rev. C 84, 021302(R) (2011)





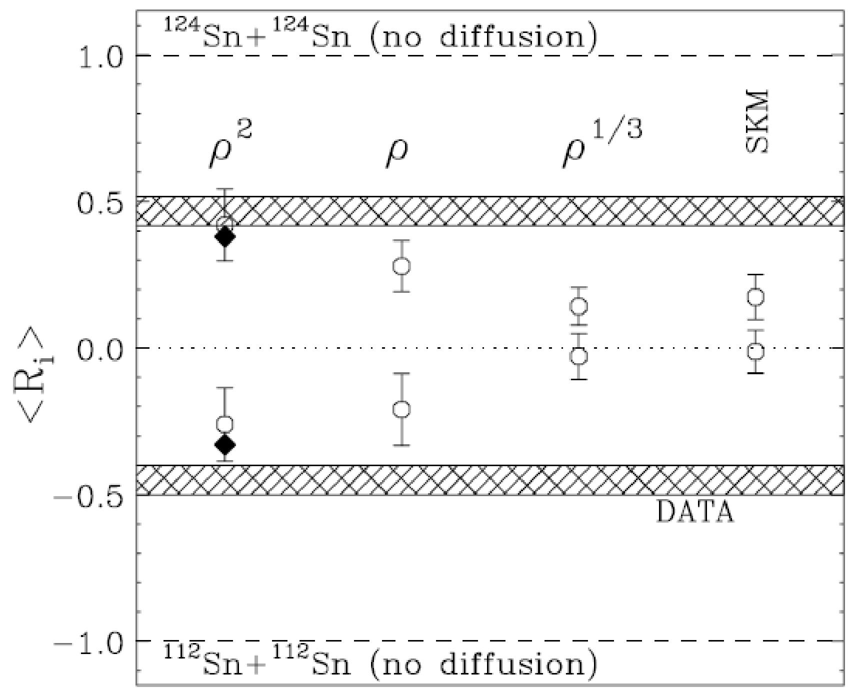
P. Adrich et al., PRL 95, 132501 (2005) .(GSI) 130,132Sn
O.Wieland et al., PRL 102, 092502 (2009) . (GSI) 68Ni

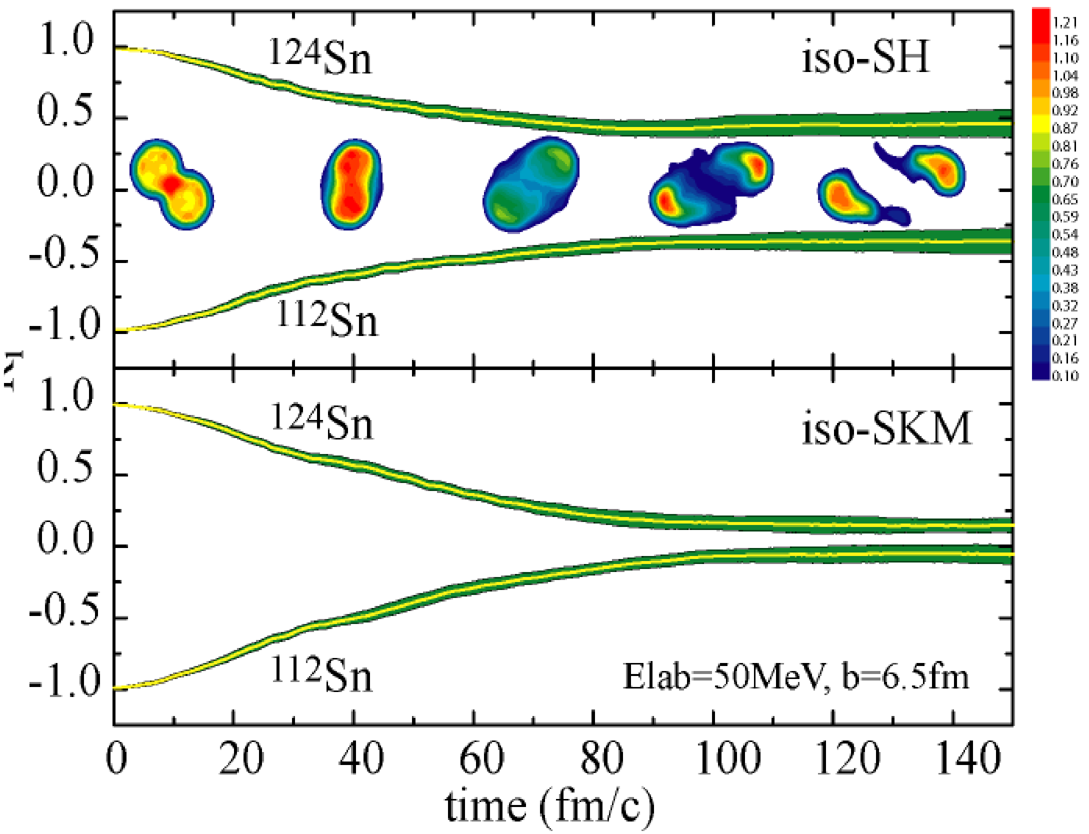
HIC (Isospin Diffusion)

- **Collision of nuclei having different n/p ratio**
  → fragments with medium n/p ratio will be formed (Isospin diffusion)

- **Driving force of isospin diffusion**
  = Symmetry energy

$$R_i = \frac{2x - x_{124+124} - x_{112+112}}{x_{124+124} - x_{112+112}}$$





Tsang et al. ('04)

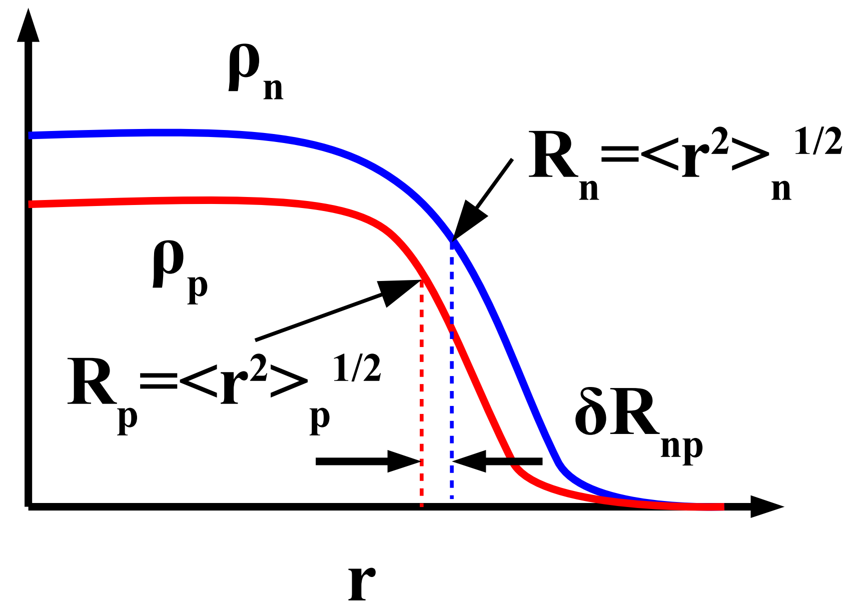
Skin Thickness & Dipole Polarizability

- **Skin Thickness δR_{np}**
  - **Larger L**

    → Small E_{sym} at low ρ

    Large E_{sym} at high ρ

    → Larger δR_{np}



- **Electric Dipole Polarizability α**

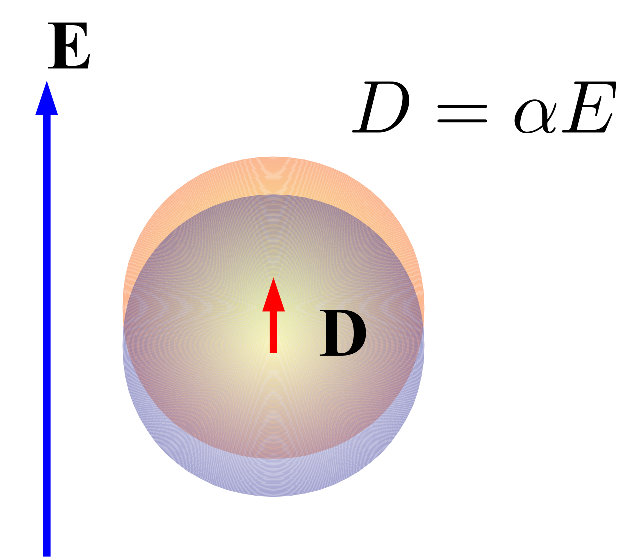
$$H = H_0 - eE \sum_{i \in p} x_i = H_0 - E\hat{D}$$

$$|\psi\rangle = |0\rangle - \sum_{n>0} \frac{|n\rangle\langle n \mid V \mid 0\rangle}{E_n - E_0} + O(E^2)$$

$$D = \langle\psi|\hat{D}|\psi\rangle = E \times \boxed{2\sum_{n>0} \frac{\langle 0|\hat{D}|n\rangle\langle n|\hat{D}|0\rangle}{E_n - E_0}}$$

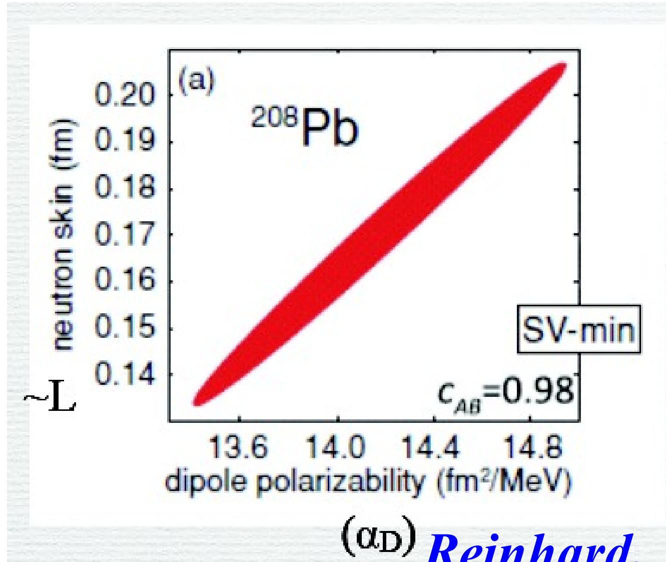
α

$$\alpha = \frac{8\pi}{9} \int \frac{dB(E1)}{\omega}$$

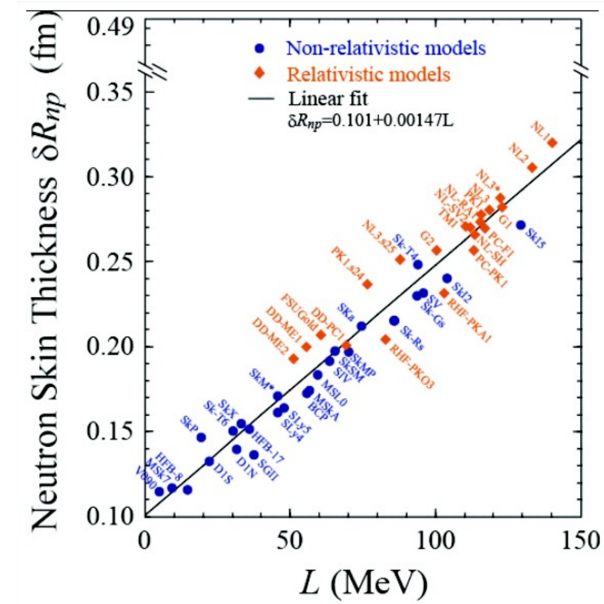


Skin Thickness & Dipole Polarizability

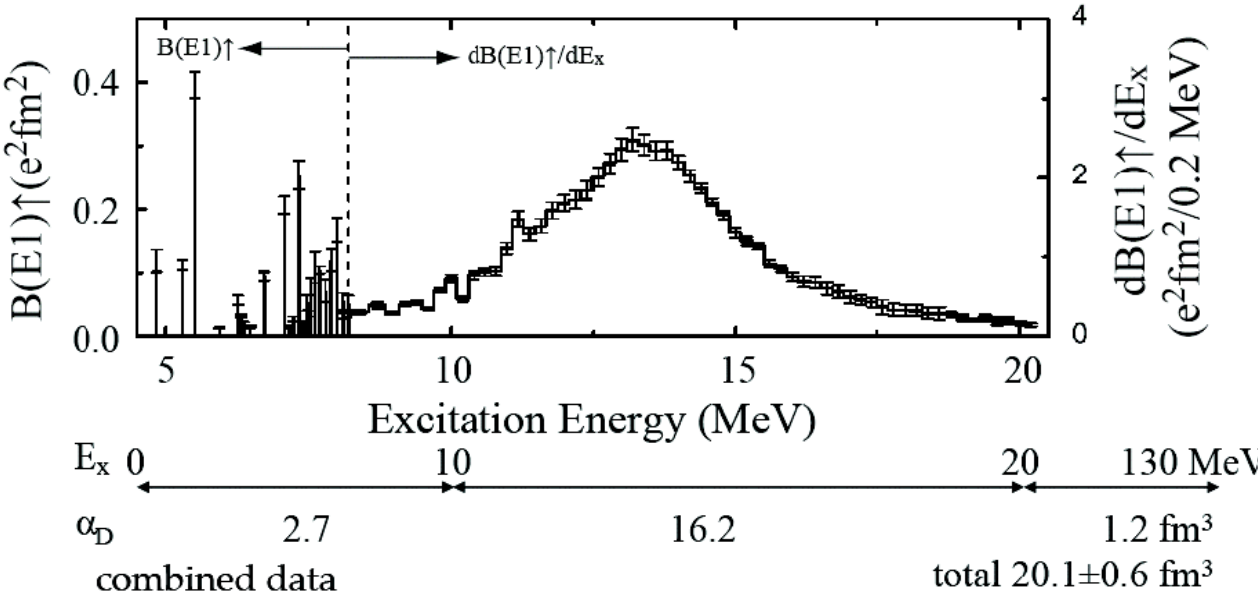
- **Strong corr. btw α and skin thickness (smaller restoring force → soft)**
- **Skin thickness is also correlated with L.**
- **Precise data from RCNP**

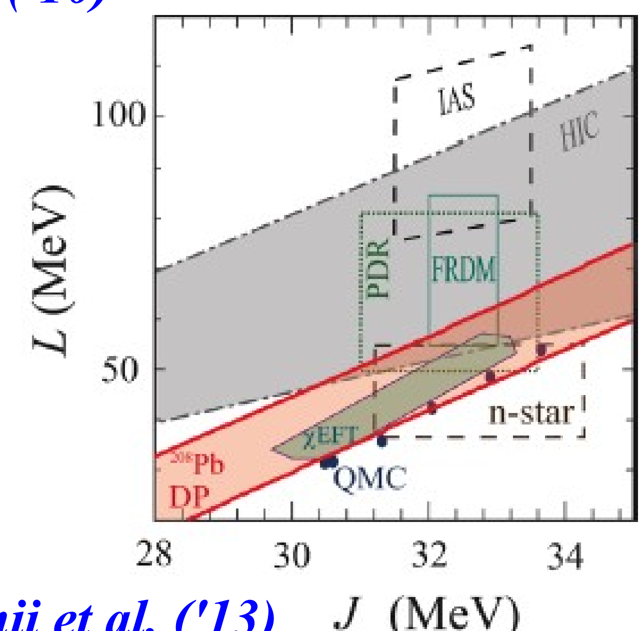


Reinhard, Nazarewicz ('10)



Roca-Maza et al. ('11)

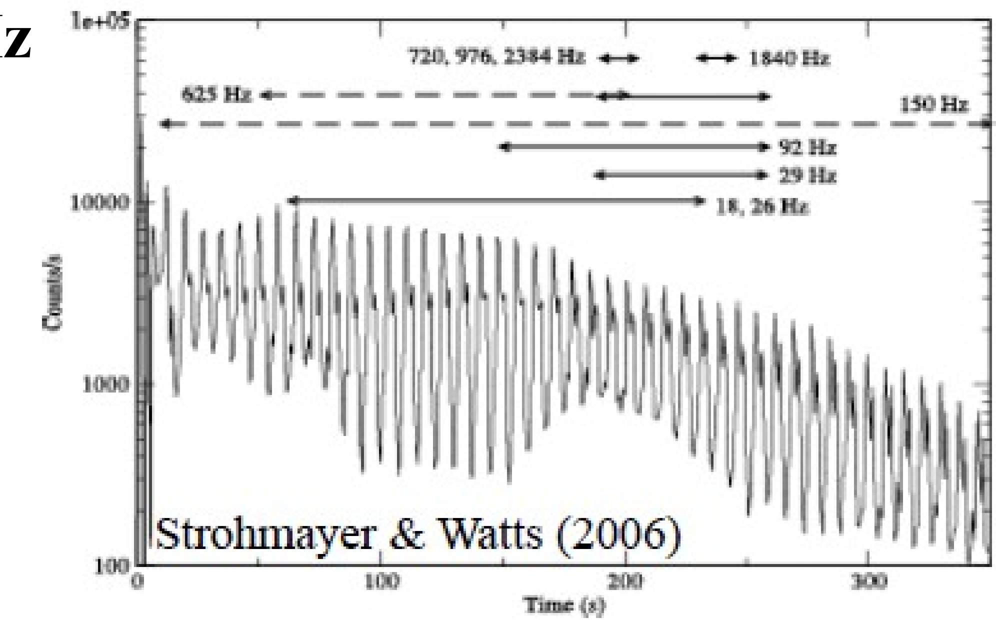




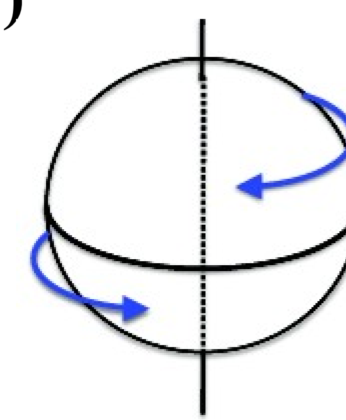
Tamii et al. ('13)

Quasi Periodic Oscillation of Neutron Stars

- **QPOs in afterglow of giant flares from soft-gamma repeaters (SGRs)** *(Barat+ 83, Israel+ 05, Strohmayer & Watts 05, Watts & Strohmayer 06)*
  - **SGR 0526-66 (5th/3/1979) : 43 Hz**
  - **SGR 1900+14 (27th/8/1998) :**
    28, 54, 84, 155 Hz
  - **SGR 1806-20 (27th/12/2004) :**
    18, 26, 30, 92.5, 150,
    626.5, 1837 Hz



- **Asteroseismology**
  - **From star quake to stellar properties (M, R, B, EOS …)**
  - **Low frequency (e.g. 28 Hz) requires long wave mode.**
    → Torsional oscillations of the crust



QPO and Symmetry Energy

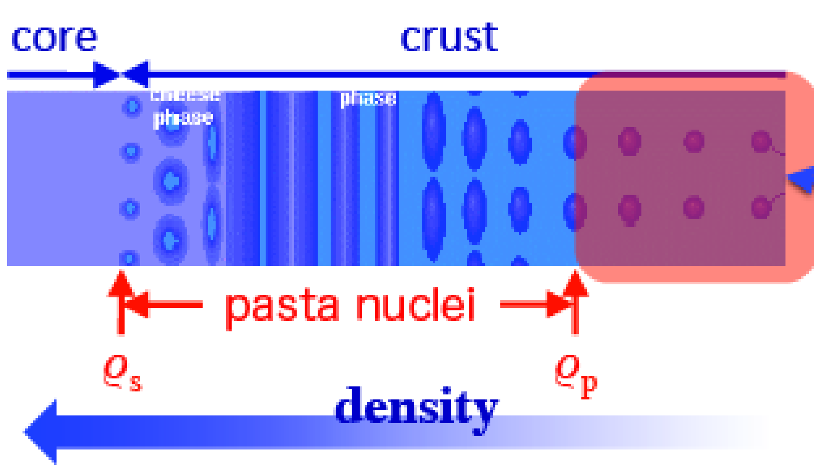
- **Torsional oscillations (ねじれ振動)**
  - **incompressible (no density perturbations)**
  - **Frequency**

$$ {}_l t_0 = \sqrt{l(l+1)}\,\frac{v_s}{2\pi R}, \quad v_s = \sqrt{\mu/\rho} $$

μ: shear modulus, v_s: shear velocity

(Hansen & Cioff 1980)

- **Shear modulus of bcc lattice depends on nuclear charge**
  → Dependence on the symmetry energy



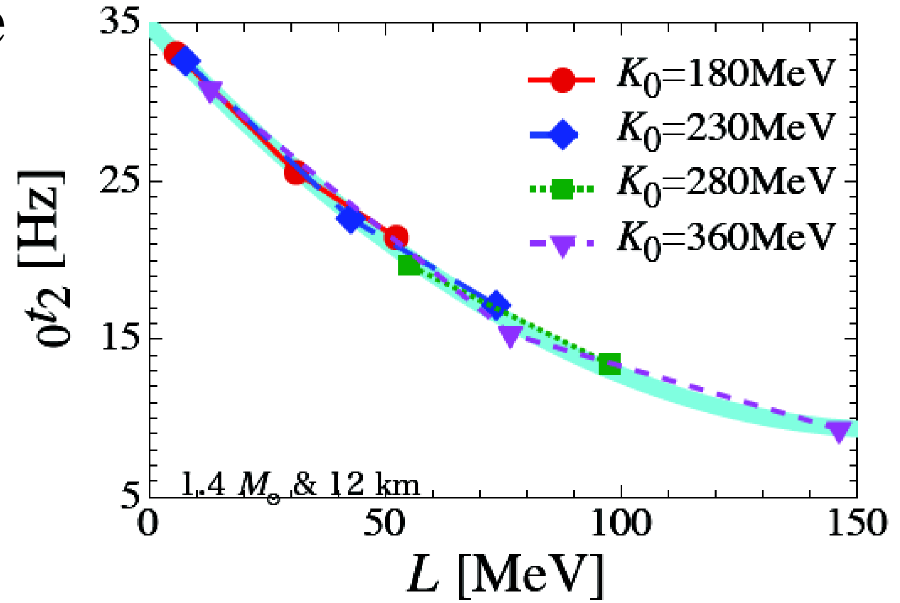
for bcc lattice (Strohmayer+ 1991)

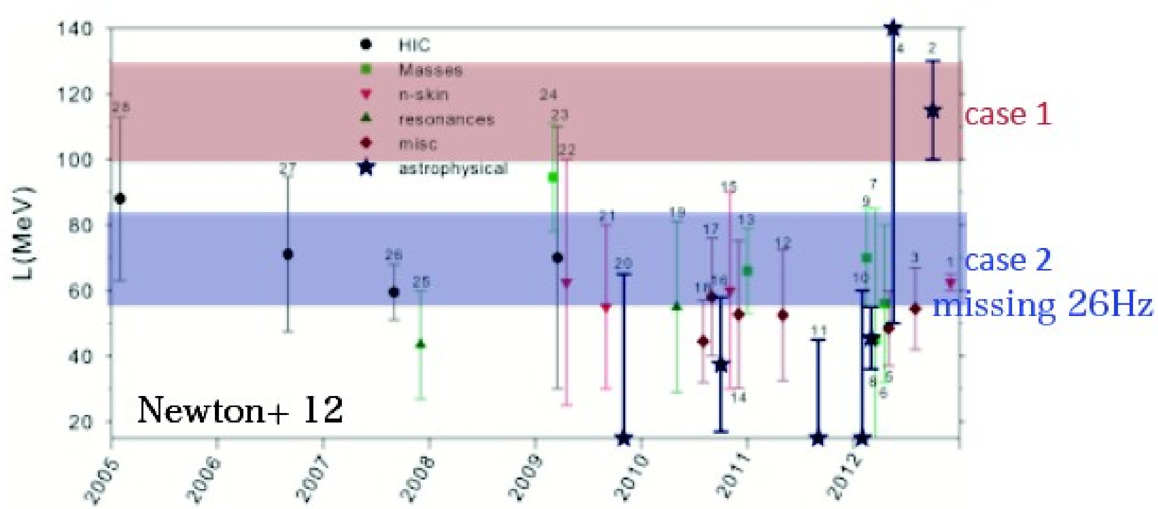
$$ \mu = 0.1194 \frac{n_i (Ze)^2}{a} $$

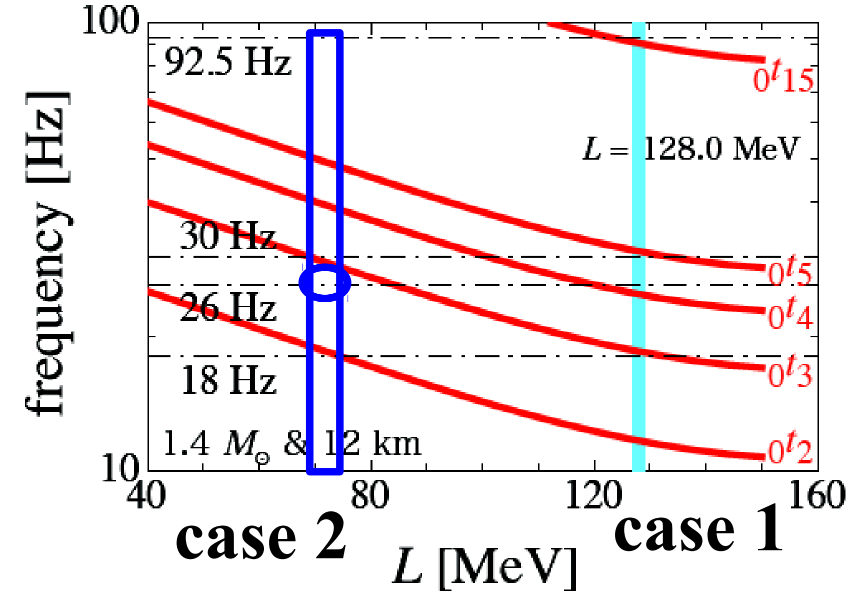
n_i : number density of quark droplet
Z : charge of quark droplet
a : Wigner-Seitz radius

QPO and Symmetry Energy

- **For a given set of (M,R), we can solve TOV equation from the surface.**
- **Torsional oscillation frequencies are calculated using EOSs with various (L,K).**
  Oyamatsu, Iida ('03,'07)
- **Compared with observed freq.**
  → constrains on L (small dep. on K)







Large L or missing 26 Hz

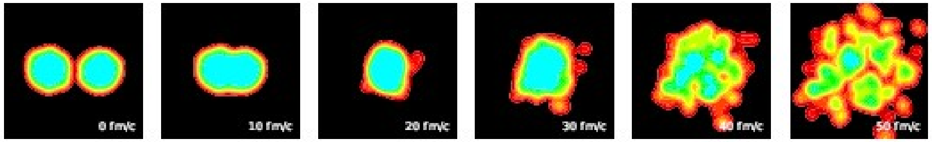
Sotani+ '13

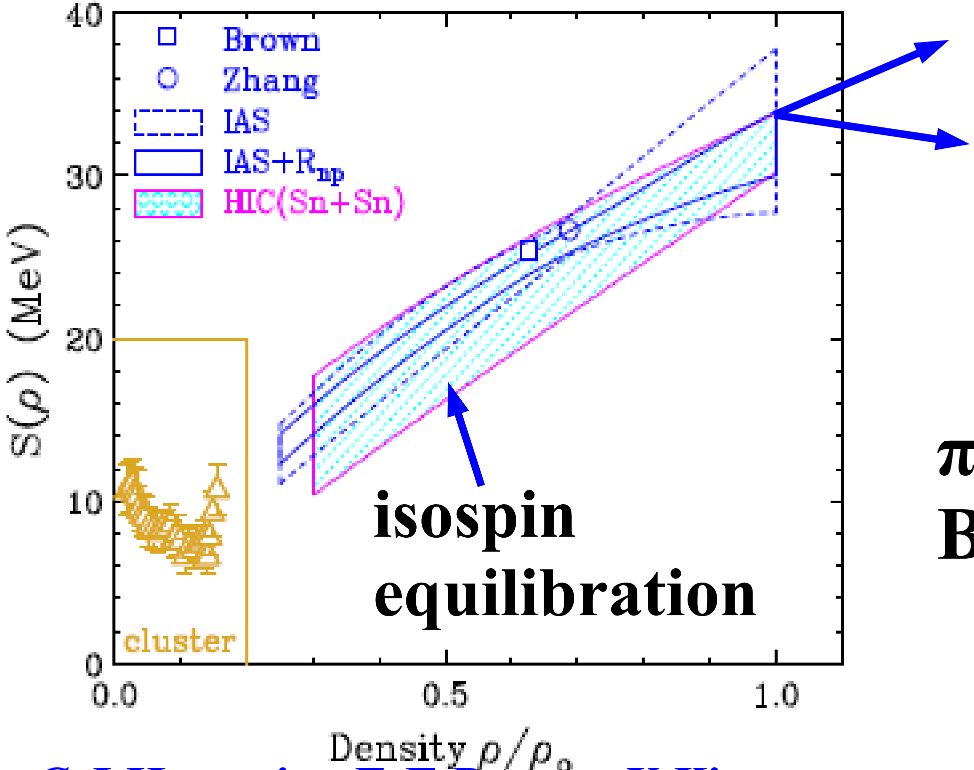
High Density Symmetry Energy

- Symmetry energy at ρ=(2-4) ρ_0 dominantly determines NS radius.

  → Central Heavy-Ion collisions at a few 100 MeV !

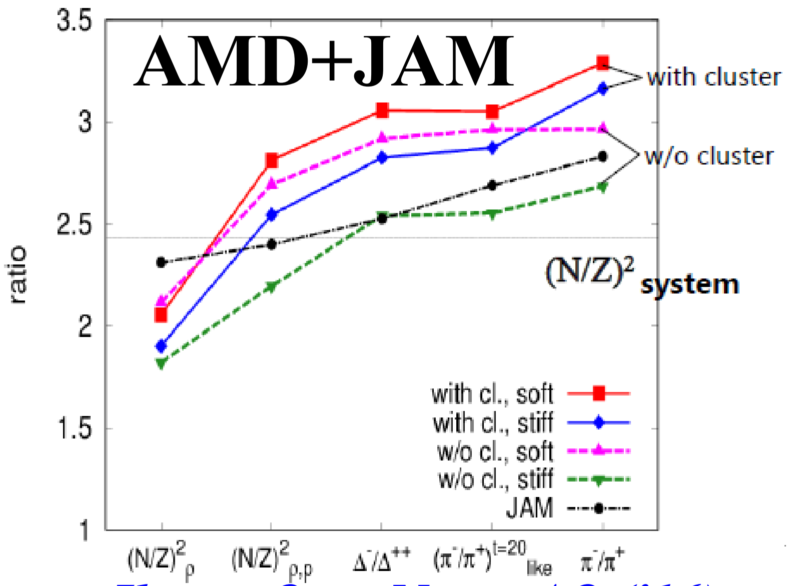
  (Li, Trautmann, Murakami, Ono)





Let's wait for SπRIT B01 results (Murakami) ! More theor. work needed.

π^-/π^+
B.A.Li

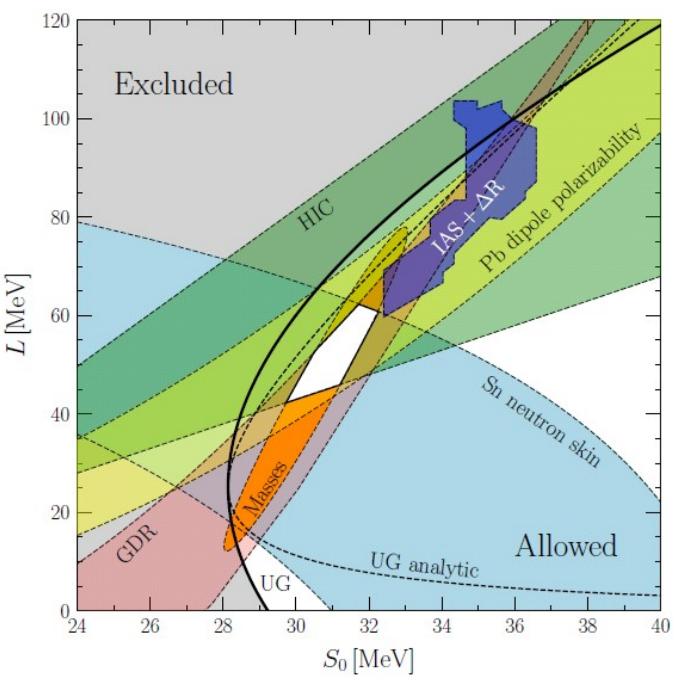


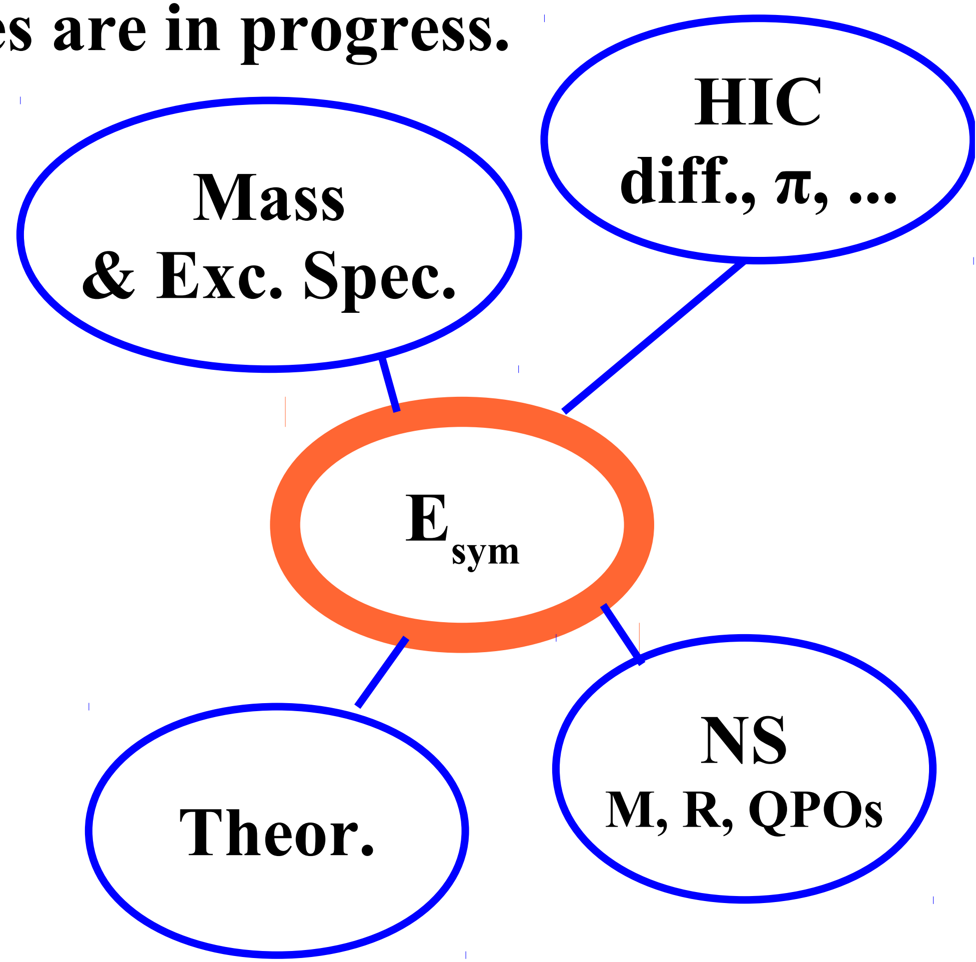
Ikeno, Ono, Nara, AO ('16)

C.J.Horowitz, E.F.Brown, Y.Kim, W.G.Lynch, R.Michaels, A. Ono, J. Piekarewicz, M. B. Tsang, H.H.Wolter (NuSYM13), JPG41('14) 093001

Summary

- **Symmetry energy is decisive in neutron star matter EOS, and is related to various properties of nuclei.**
- **Experimental & Theoretical studies are in progress.**
- **If you have a new idea to determine Esym, please propose as soon as possible.**

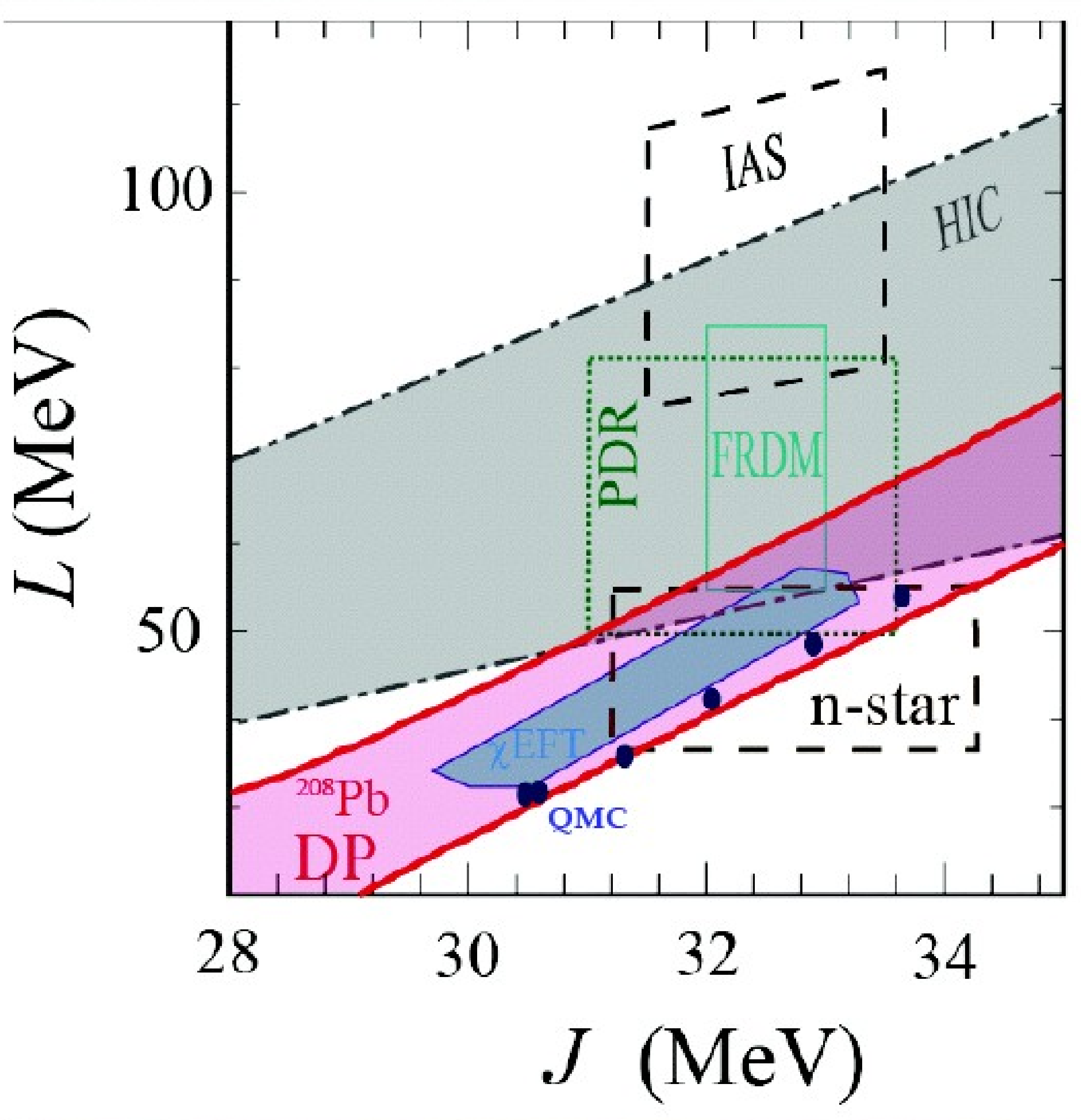




Thank you !

Constraints on J and L

by Tamii



AT *et al.*, to be published in EPJA.

M.B. Tsang *et al.*, PRC86, 015803 (2012).

I. Tews et al., PRL110, 032504 (2013)

DP: Dipole Polarizability (this work)
HIC: Heavy Ion Collision
PDR: Pygmy Dipole Resonance
IAS: Isobaric Analogue State
FRDM: Finite Range Droplet Model (nuclear mass analysis)
n-star: Neutron Star Observation (A.W. Steiner et al.,)
cEFT: Chiral Effective Field Theory
QMC: Quantum Monte-Carlo Calc.