

# *Collective flows in heavy-ion collisions from AGS to RHIC*

*Akira Ohnishi @ Hokkaido Univ.  
in Collaboration with*

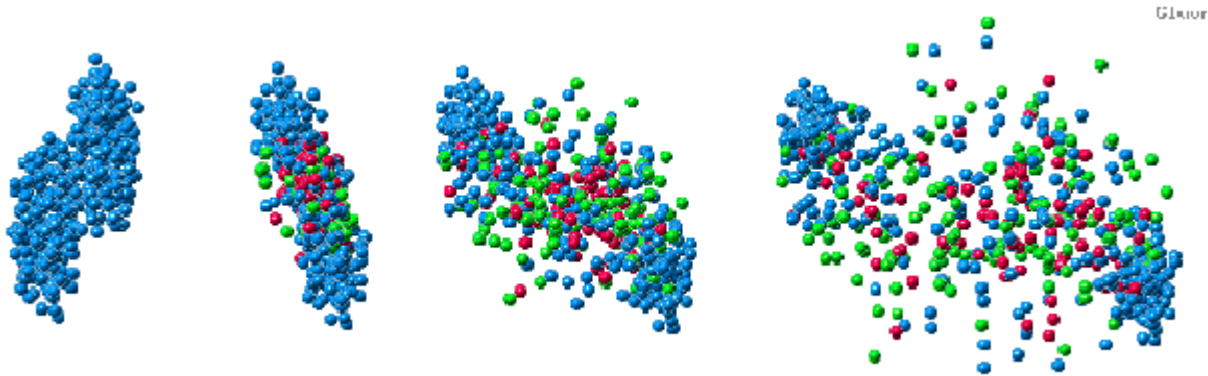
*K. Yoshino (Hokkaido U.), M. Isse (Hokkaido U. → Osaka U.),  
T. Hirano (U-Tokyo), Y. Nara (Frankfurt), P. K. Sahu (IOP, India)*

- **Collective Flows from AGS to SPS Energies**  
*Isse, AO, Otuka, Sahu, Nara, Phys. Rev. C 72 (2005), 064908*
- **Hydro. vs Cascade Comparison at RHIC**  
*Hirano, Isse, Nara, AO, Yoshino, Phys. Rev. C 72 (2005), 041901*  
*Sahu, Isse, Otuka, AO, Pramana, 2006.*  
*Isse, Ph.D Thesis*
- **Jet Fluid-String formation and decay at RHIC**  
*Isse, Hirano, Mizukawa, Nara, AO, 2007.*

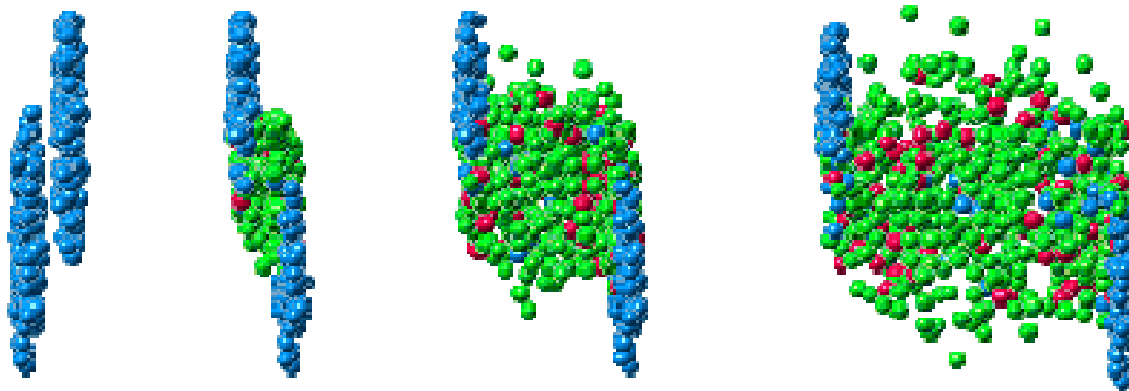
# Heavy-Ion Collisions at $E_{\text{inc}} \sim (1-100) A \text{ GeV}$

- Study of Hot and Dense Hadronic Matter  
→ Particle Yield, Collective Dynamics (Flow), EOS, .....

AGS



SPS



JAMming on the Web, linked from <http://www.jcprg.org/>

# *Collective Flow and EOS: Old Problem ?*

- **1970's-1980's: First Suggestions and Measurement**
  - ◆ **Hydrodynamics suggested the Existence of Flow.**
  - ◆ **Strong Collective Flow suggests Hard EOS**
- **1980's-1990's: Deeper Discussions in Wider  $E_{inc}$  Range**
  - ◆ **Momentum Dep. Pot. can generate Strong Flows.**
  - ◆  **$E_{inc}$  deps. implies the importance of Momentum Deps.**
  - ◆ **Flow Measurement up to AGS Energies.**
- **2000's: Extention to SPS and RHIC Energies**
  - ◆ **EOS is determined with Mom. AND Density Dep. Pot. ?**

***Old but New (Continuing) Problem !***

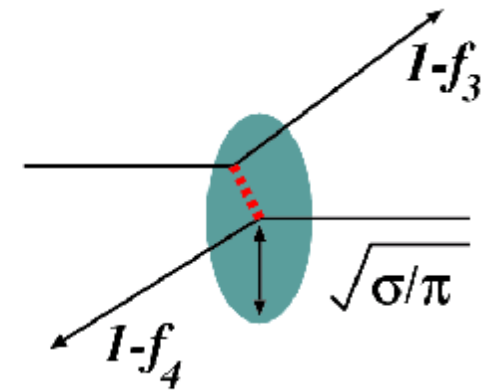
# Mean Field Dynamics + Two-Body Collision

- **BUU Equation** (Bertsch and Das Gupta, Phys. Rept. 160( 88), 190)
  - ◆ Time-Dependent Hartree-Fock Eq. を Wigner 変換 → Vlasov Eq.
  - ◆ Pauli blocking を導入した Boltzmann 方程式の衝突項を導入
    - 低エネルギーで重要な平均場理論と  
高エネルギー(>100 A MeV) で支配的なカスケード過程を統合
    - どのような粒子自由度を導入すべきか？

$$\frac{\partial f}{\partial t} + v \cdot \nabla_r f - \nabla U \cdot \nabla_p f = I_{coll}[f]$$

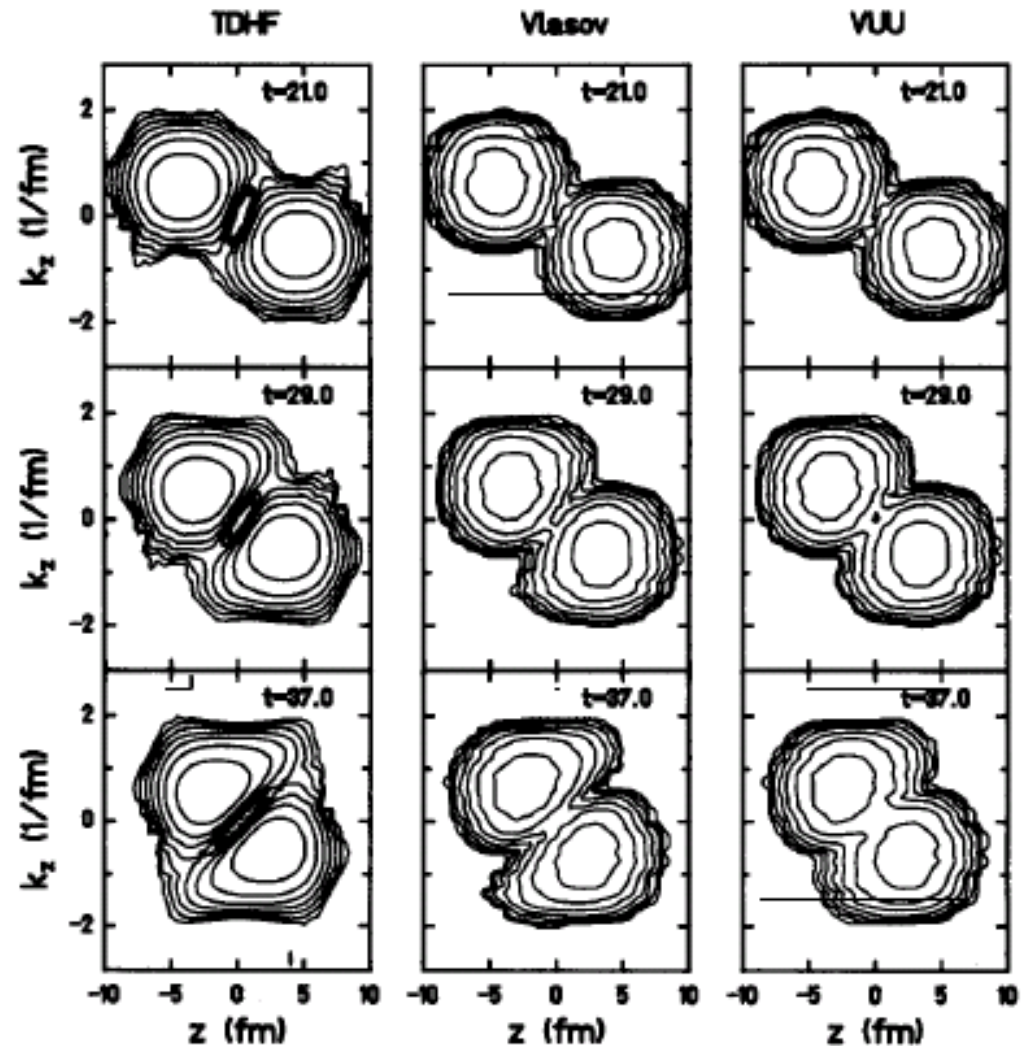
$$I_{coll}[f] = -\frac{1}{2} \int \frac{d^3 p_2 d\Omega}{(2\pi\hbar)^3} v_{12} \frac{d\sigma}{d\Omega}$$

$$\times [f f_2 (1-f_3)(1-f_4) - f_3 f_4 (1-f)(1-f_2)]$$



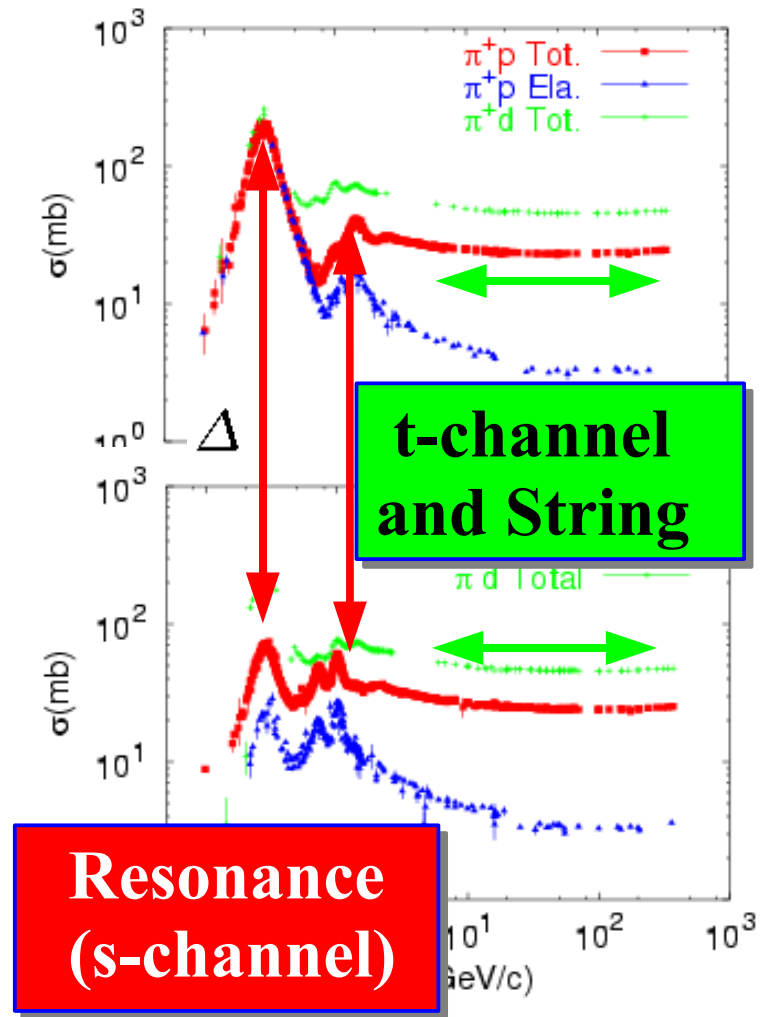
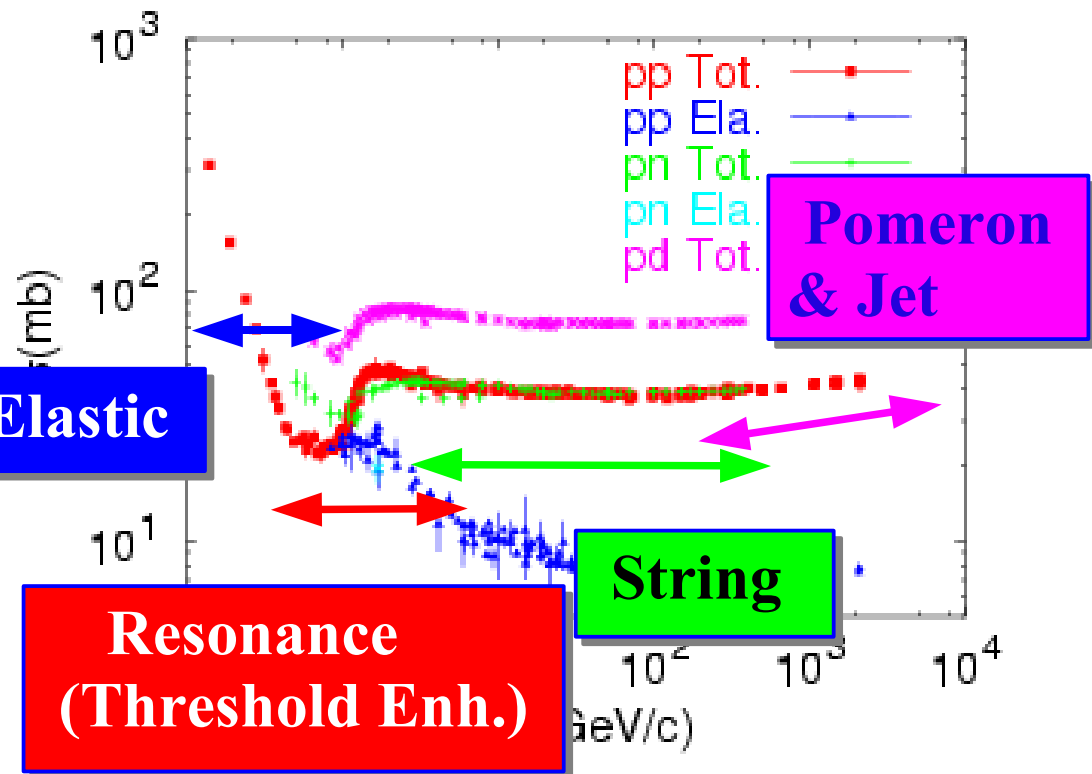
# Comparison of TDHF, Vlasov and BUU(VUU)

- Ca+Ca, 40 A MeV  
(Cassing-Metag-Mosel-Niita, Phys. Rep. 188 (1990) 363).



# Hadron-Hadron Cross Sections

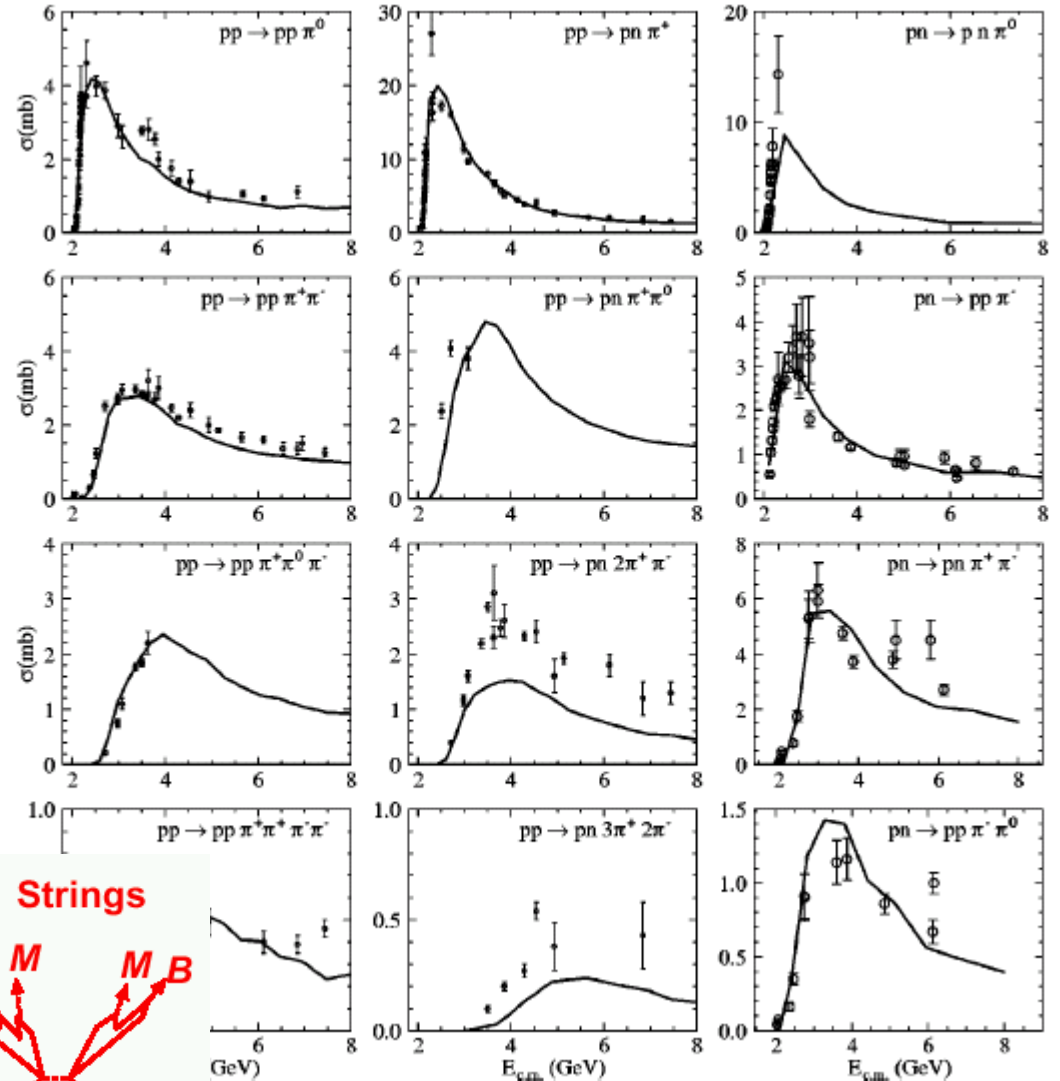
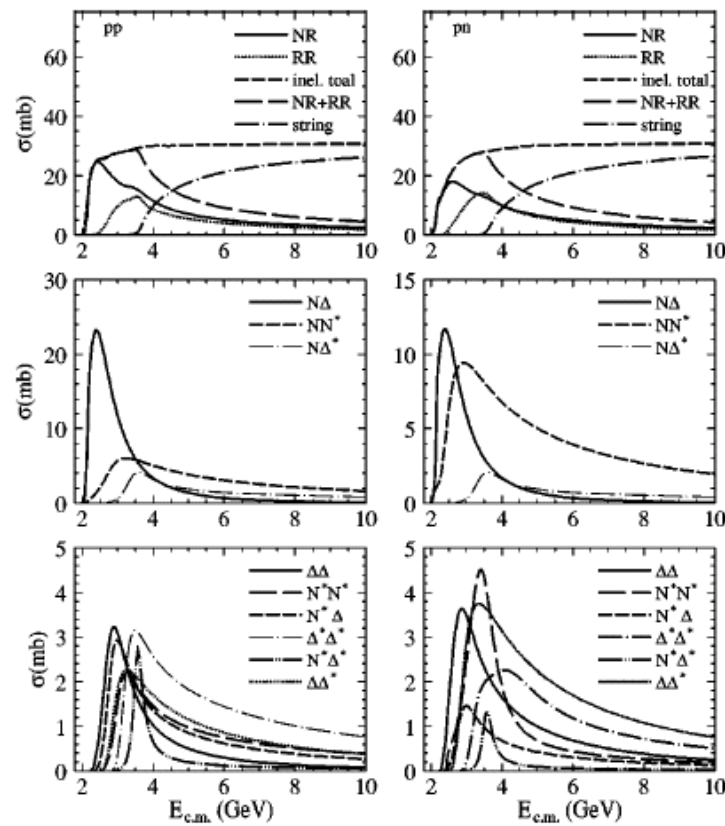
From Particle Data Group



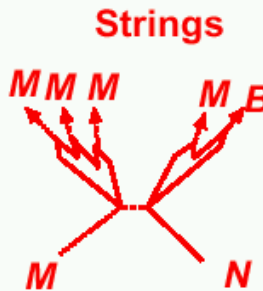
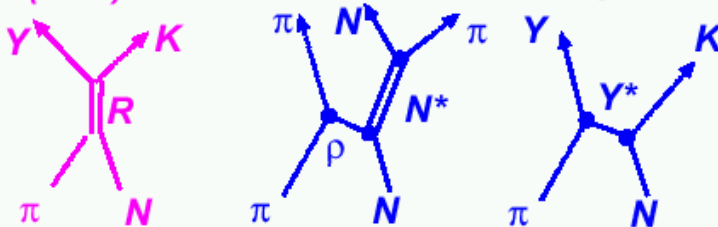
# Exclusive Cross Sections in JAM

Nara, Otuka, AO, Niita, Chiba (JAM), PRC 61 (2000), 024901.

## Ground State Hadrons, Resonances, and Strings



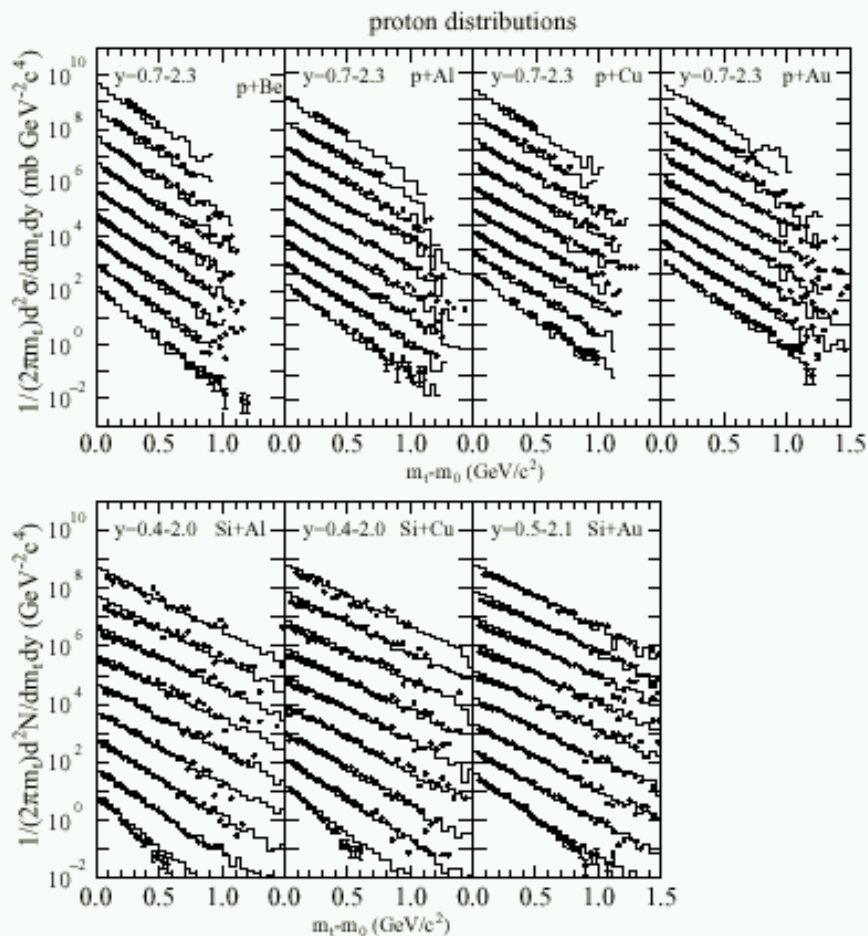
**s-channel** R (or S) Form.      **t-channel** Reggeon Exch.      **u-channel** Baryon Exch.



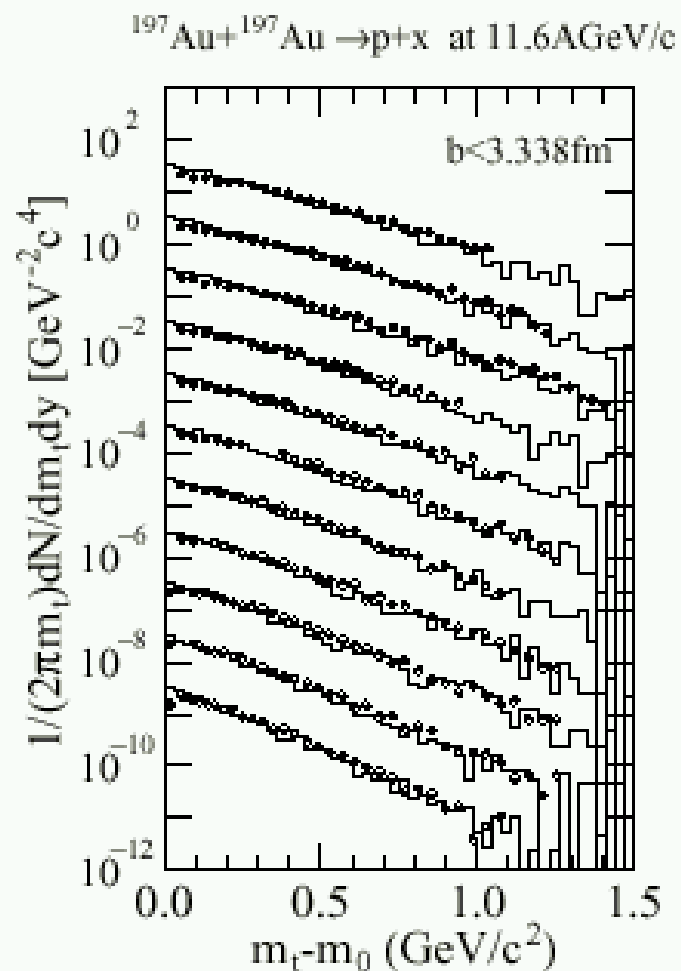


# JAM Results @ AGS Energy

## • p-A collisions



## • Au+Au Collision



**JAM explains AA collisions as well as pA collisions:  
→ Good Elementary Cross Sections for MM, MN and NN**



# Mean Field and Particle DOF Effects @ AGS

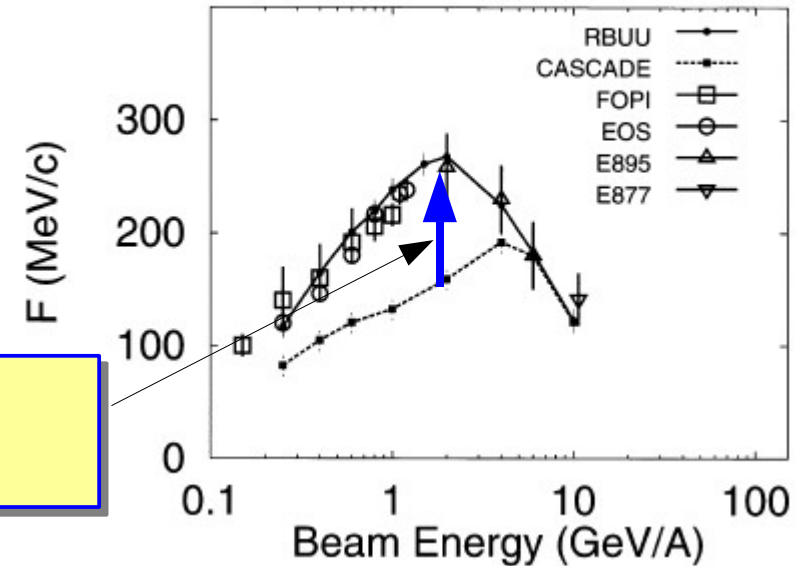
- Mean Field Effects at AGS  
→ Visible but small for  $p_T$  spectrum

Sahu, Cassing, Mosel, Ohnishi, 2000

P.K. Sahu et al. / Nuclear Physics A 672 (2000) 376–386

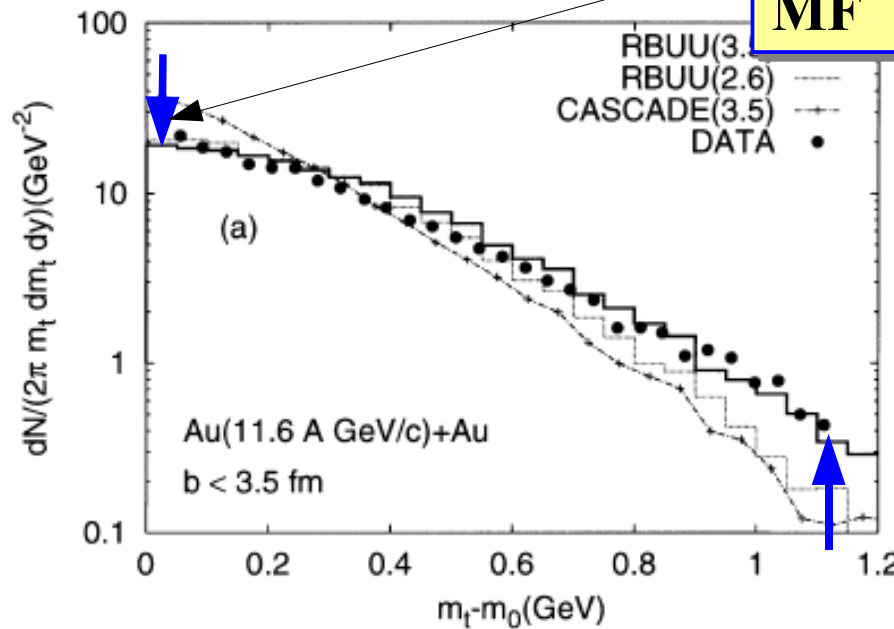
Essential for Flow

- Particle DOF Effects  
→ Seen at high  $p_T$



P.K. Sahu et al. / Nuclear Physics A 672 (2000) 376–386

Repulsive MF



Switching  $\sqrt{s} = 3.5$  GeV (JAM fit)

Switching  $\sqrt{s} = 2.6$  GeV (HSD default)

---

*Relativistic Quantum Molecular Dynamics  
(RQMD)*

*-- Tool to attack collective flows at high E. --*

# *Relativistic QMD/Simplified (RQMD/S)*

- **RQMD = Constraint Hamiltonian Dynamics**  
*(Sorge, Stocker, Greiner, Ann. of Phys. 192 (1989), 266.)*
- **Constraints:  $\varphi \approx 0$  (Satisfied on the realized trajectory, by Dirac)**
  - Variables in Covariant Dynamics =  $8N$  phase space:  $q_\mu, p_\mu$
  - Variables in EOM =  $6N$  phase space  
→ We need  $2N$  constraints to get EOM

- **On Mass-Shell Constraints**

$$H_i \equiv p_i^2 - m_i^2 - 2m_i V_i \approx 0$$

- **Time-Fixation in RQMD/S**

$$\chi_i \equiv \hat{a} \cdot (q_i - q_N) \approx 0 \quad (i = 1, \sim N-1) \quad , \quad \chi_N \equiv \hat{a} \cdot q_N - \tau \approx 0$$

$\hat{a} = \text{Time-like unit vector in the Calculation Frame}$

*(Tomoyuki Maruyama et al., Prog. Theor. Phys. 96(1996), 263.)*

# RQMD/S (cont.)

- **Hamiltonian is made of constraints**

$$H = \sum_i u_i \phi_i \quad (\phi_i = H_i (i=1 \sim N), \chi_{i-N} (i=N+1 \sim 2N))$$

- **Time Development**  $\frac{d f}{d \tau} = \frac{\partial f}{\partial \tau} + \{f, H\}$  ,  $\{q_\mu, p_\nu\} = g_{\mu\nu}$

- **Lagrange multipliers are determined to keep constraints**

→ *We can solve obtain the multipliers analytically in RQMD/S*

$$\frac{d \phi_i}{d \tau} \approx 0 \rightarrow \delta_{i,2N} + \sum_j u_j \{\phi_i, \phi_j\} \approx 0$$

- **Equations of Motion**

$$H = \sum_i (p_i^2 - m_i^2 - 2m_i V_i) / 2p_i^0 \quad , \quad p_i^0 = E_i = \sqrt{\vec{p}_i^2 + m_i^2 + 2m_i V_i}$$
$$\frac{d \vec{r}_i}{d \tau} \approx -\frac{\partial H}{\partial \vec{p}_i} = \frac{\vec{p}_i}{p_i^0} + \sum_j \frac{m_j}{p_j^0} \frac{\partial V_j}{\partial \vec{p}_i} \quad , \quad \frac{d \vec{p}_i}{d \tau} \approx \frac{\partial H}{\partial \vec{r}_i} = -\sum_j \frac{m_j}{p_j^0} \frac{\partial V_j}{\partial \vec{r}_i}$$

**We can include MF in an almost covariant way in molecular dynamics**

Particle “DISTANCE”

$$r_{Tij}^2 \equiv r_\mu r^\mu - \left( r_\mu P_{ij}^\mu \right)^2 / P_{ij}^2 = \vec{r}^2 \quad (\text{in } CM)$$

$$P_{ij} \equiv p_i + p_j \quad , \quad r \equiv r_i - r_j$$

Particle “Momentum Difference”

$$p_{Tij}^2 \equiv p_\mu p^\mu - \left( p_\mu P_{ij}^\mu \right)^2 / P_{ij}^2 = \vec{p}^2 \quad (\text{in } CM)$$

$$p \equiv p_i - p_j$$

**Lorentz Invariant, and Becomes Normal Distance in CM !**

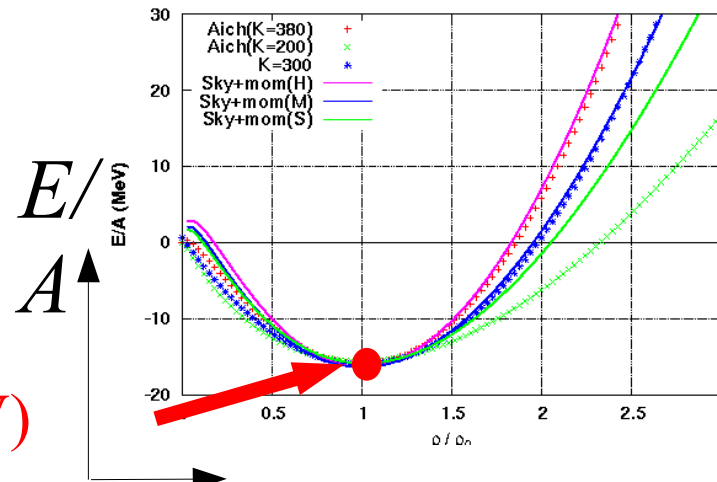
---

*Nuclear Mean Field for HIC*  
*--- Density and Momentum Deps. ---*

# Nuclear Mean Field

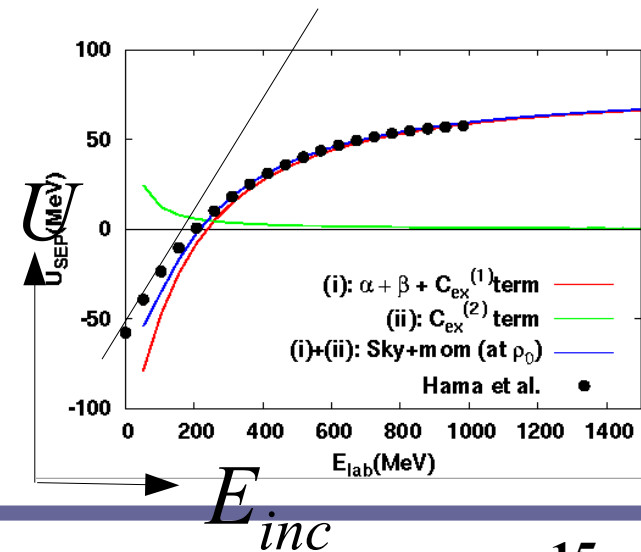
- MF has on both of  $\rho$  and  $p$ -deps.
  - $\rho$  dep.:  $(\rho_0, E/A) = (0.15 \text{ fm}^{-3}, -16.3 \text{ MeV})$  is known  
Stiffness is not known well
  - $p$  dep.: Global potential up to  $E=1 \text{ GeV}$  is known from  $pA$  scattering  
 $U(\rho_0, E) = U(\rho_0, E=0) + 0.3 E$
- Ab initio Approach; LQCD, GFMC, DBHF, G-matrix, ...  
→ Not easy to handle, Not satisfactory for phen. purposes
- Effective Interactions (or Energy Functionals):  
Skyrme HF, RMF, ...

$(\rho_0, E/A)$   
 $= (0.15 \text{ fm}^{-3}, -16.3 \text{ MeV})$



高エネルギー  $\rho$  重イオン反応

$$U(E) = U(0) + 0.3E$$





# Skyrme Hartree-Fock

See Ring-Schuck for details

- **Zero-Range Two- and Three-Body Interaction**

$$v_{ij} = t_0 \delta(r_i - r_j) + \frac{1}{2} \left[ \delta(r_i - r_j) k^2 + k^2 \delta(r_i - r_j) \right] \\ + t_2 k \delta(r_i - r_j) k + i W_0 [\sigma_i + \sigma_j] \times \delta(r_i - r_j) k \\ k = \frac{1}{2i} (\nabla_i - \nabla_j) \\ v_{ijk} = t_3 \delta(r_i - r_j) \delta(r_j - r_k)$$

- **Energy Density (Even-Even, N=Z)**

$$H(r) = \frac{\hbar^2}{2m^*(\rho)} \tau + \frac{3}{8} t_0 \rho^2 + \frac{1}{16} t_3 \rho^3 + \text{Deriv. terms} \rightarrow \rho \left[ \frac{3}{5} \frac{\hbar^2 k_F^2}{2m^*(\rho)} + \frac{3}{8} t_0 \rho + \frac{1}{16} t_3 \rho^2 \right] \\ \tau = \sum_i |\nabla \phi_i|^2, \quad \frac{\hbar^2}{2m^*(\rho)} = \frac{\hbar^2}{2m} + \frac{1}{16} (3t_1 + 5t_2) \rho$$

**Problems in Skyrme HF (in Dense Nuclear Matter/High Energy)**  
**Repulsive Zero-Range 3-body Int.: → Ferromagnetism**  
**Energy Dep. = Linear ( $m^*$  term) → Too Repulsive at High E**

# Relativistic Mean Field (I)

*Serot-Walecka, Walecka text book.*

- Describe nuclear energy functional in meson and baryon fields
  - ◆ Fit B.E. of Stable as well as Unstable (n-rich) Nuclei
  - ◆ Has been successfully applied to Supernova Explosion
  - ◆ Three Mesons ( $\sigma, \omega, \rho$ ) are included
  - ◆ Meson Self-Energy Term ( $\sigma, \omega$ )

$$\begin{aligned} \mathcal{L} = & \bar{\psi}_N (i\partial - M - g_\sigma \sigma - g_\omega \psi - g_\rho \tau^a \rho^a) \psi_N \\ & + \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 \\ & - \frac{1}{4} W^{\mu\nu} W_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4} R^{a\mu\nu} R_{\mu\nu}^a + \frac{1}{2} m_\rho^2 \rho^{a\mu} \rho_\mu^a + \frac{1}{4} c_3 (\omega_\mu \omega^\mu)^2 \\ & + \bar{\psi}_e (i\partial - m_e) \psi_e + \bar{\psi}_\nu i\partial \psi_\nu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} , \end{aligned}$$

$$W_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu ,$$

$$R_{\mu\nu}^a = \partial_\mu \rho_\nu^a - \partial_\nu \rho_\mu^a + g_\rho \epsilon^{abc} \rho^{b\mu} \rho^{c\nu} ,$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu .$$

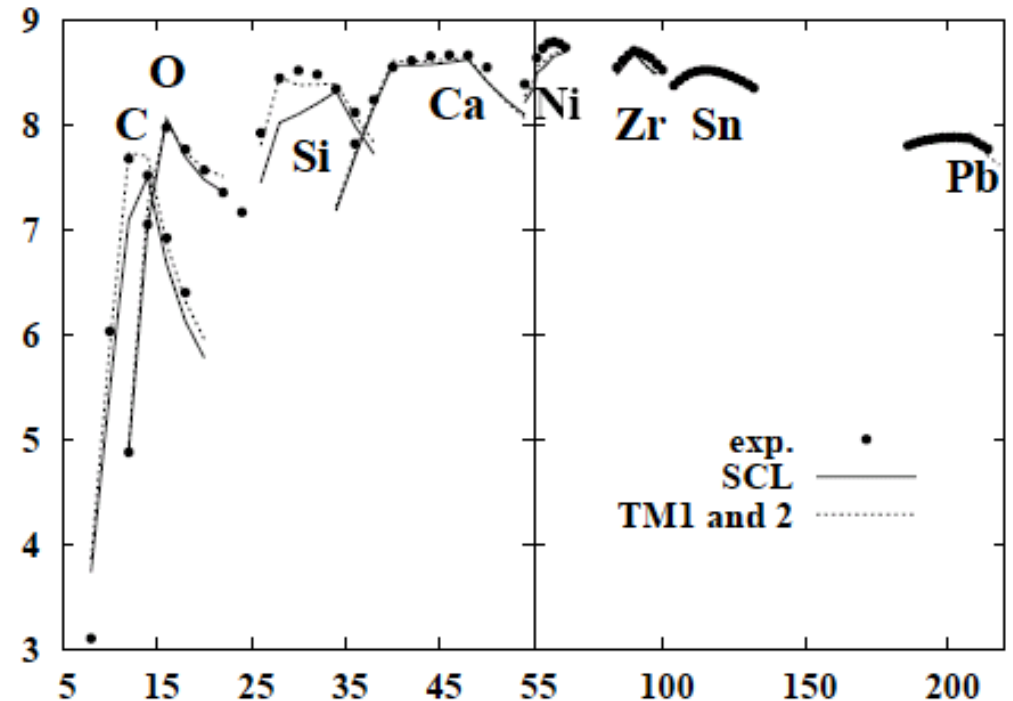
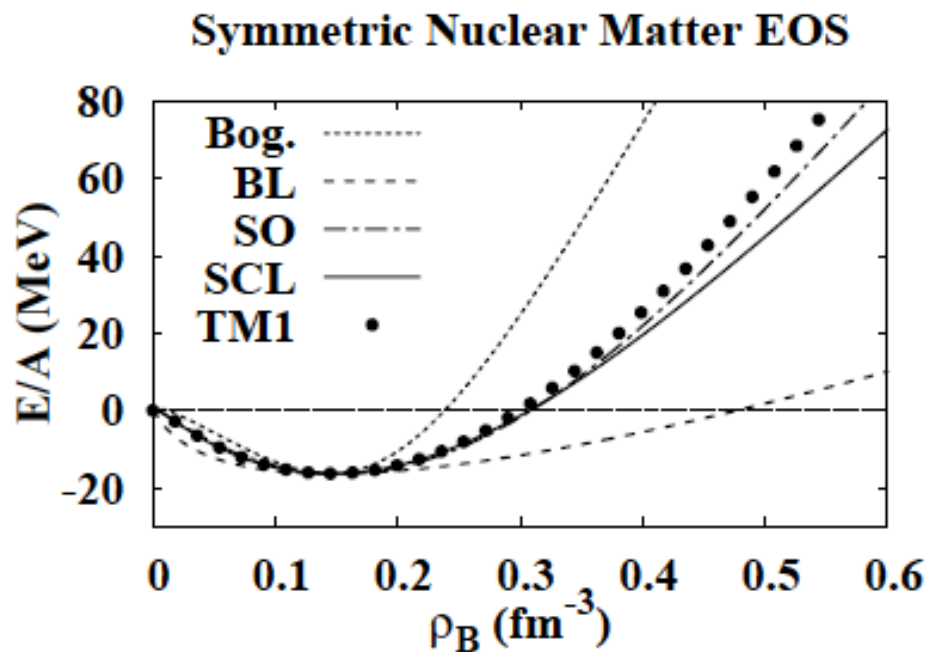
(2)

# Nuclear Matter EOS and Nuclear Binding E in TM

*Sugahara-Toki, NPA579 (1994), 557.*

- **Example: TM1 parameter set**

- ◆ **Nuclear Matter:  $\sigma_4$  and  $\omega_4$  terms soften EOS ( $K \sim 280$  MeV)**
- ◆ **Finite nuclei: Explains B.E. from C to Pb isotopes**



*c.f. SCL=Chiral RMF with  $\log \sigma$  term.*

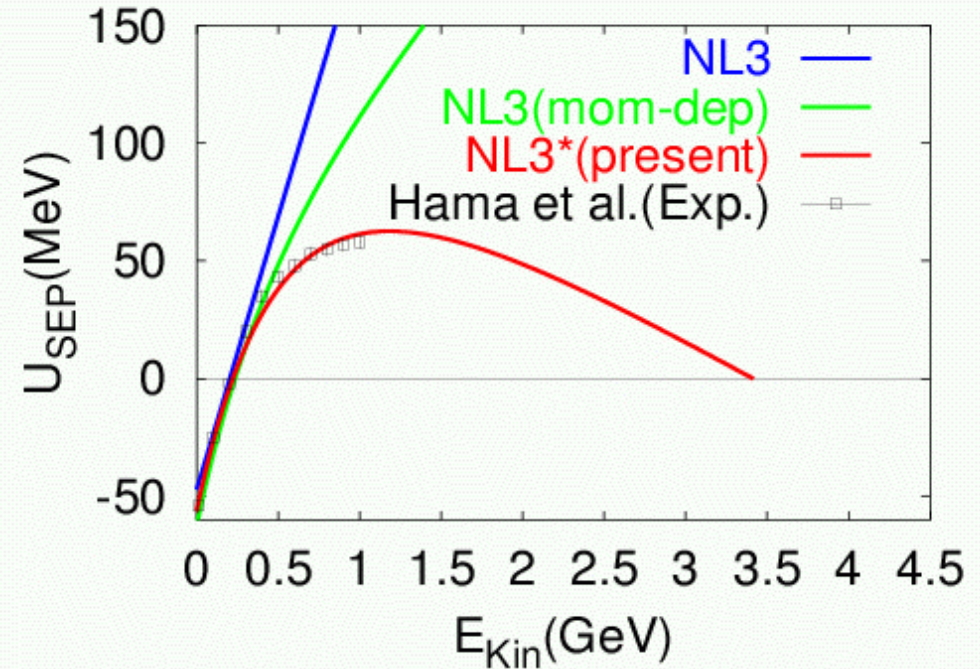
(K. Tsubakihara and AO, 2007)

# Relativistic Mean Field (II)

- **Dirac Equation**  $(i\gamma\partial - \gamma^0 U_v - M - U_s)\psi = 0$  ,  $U_v = g_\omega \omega$  ,  $U_s = -g_\sigma \sigma$
- **Schroedinger Equivalent Potential**

$$\begin{pmatrix} E - U_v - M - U_s & -i\sigma \cdot \nabla \\ i\sigma \cdot \nabla & -E + U_v - M - U_s \end{pmatrix} \begin{pmatrix} f \\ g \end{pmatrix} = 0$$

$$\begin{aligned} U_{sep} &\sim U_s + \frac{E}{m} U_v = -g_\sigma \sigma + \frac{E}{m} g_\omega \omega \\ &= -\frac{g_\sigma^2}{m_\sigma^2} \rho_s + \frac{E}{m} \frac{g_\omega^2}{m_\omega^2} \rho_B \end{aligned}$$



**Saturation: -Scalar+Baryon Density**

**Linear Energy Dependence: Good at Low Energies,**

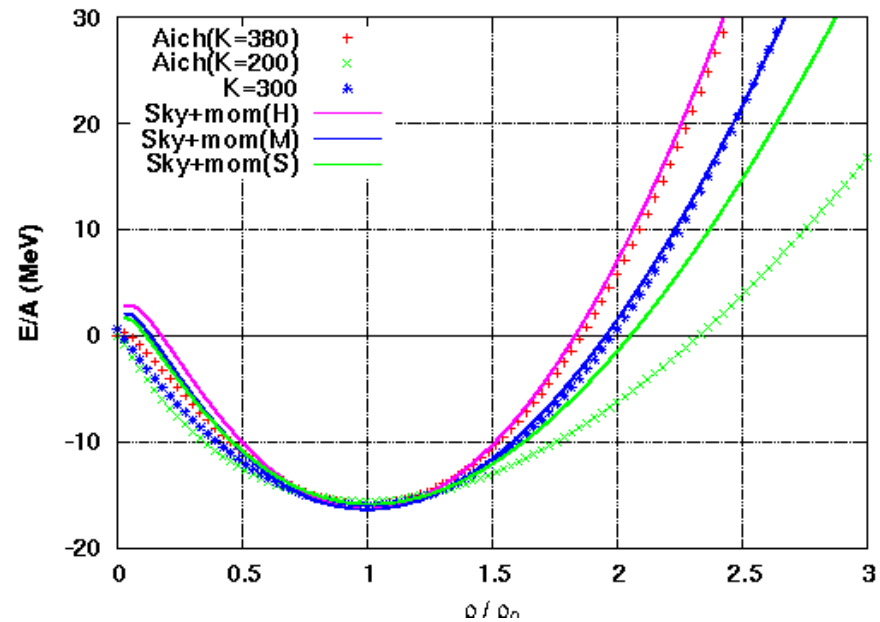
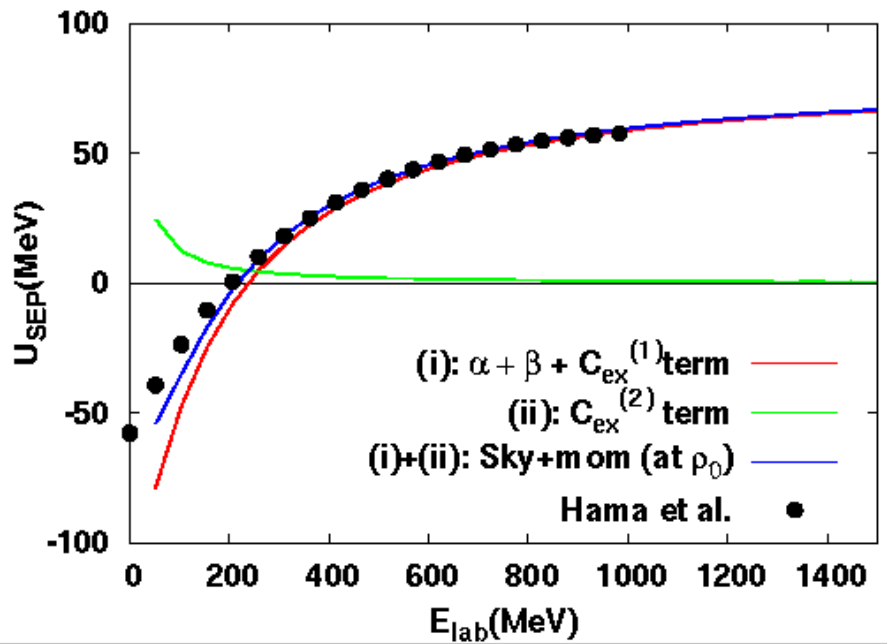
**Bad at High Energies (We need cut off !)**

(Sahu, Cassing, Mosel, AO, Nucl. Phys. A672 (2000), 376.)

# Phenomenological Mean Field

- Skyrme type  $\rho$ -Dep. + Lorentzian  $p$ -Dep. Potential

$$V = \sum_i V_i = \int d^3 r \left[ \frac{\alpha}{2} \left( \frac{\rho}{\rho_0} \right)^2 + \frac{\beta}{\gamma+1} \left( \frac{\rho}{\rho_0} \right)^{\gamma+1} \right] \\ + \sum_k \int d^3 r d^3 p d^3 p' \frac{C_{ex}^{(k)}}{2\rho_0} \frac{f(r, p) f(r, p')}{1 + (\mathbf{p} - \mathbf{p}')^2 / \mu_k^2}$$



Isse, AO, Otuka, Sahu, Nara, *Phys.Rev. C* 72 (2005), 064908

## *Exercise (4)*

---

- **Prove that the single particle potential with Skyrme interaction has a linear dependence on energy. From NA elastic scattering, the energy dependence is found to be**

$$U(\rho_0, E) \sim U(\rho_0, E=0) + 0.3 E$$

**at low energies. Obtain the value of  $m^*/m$  which explains the above energy dependence.**

- **Obtain the form of the Schrodinger equivalent potential in RMF. You will find that the spin-orbit potential appears as a sum of scalar and vector potential.**

---

*Collective Flows  
at AGS and SPS Energies*



# What is Collective Flow ?

## (Directed) Flow ( $dP_x/dY$ )

Stiffness (Low E)  
+ Time Scale (High E)

## Elliptic Flow ( $V_2$ )

Thermalization  
& Pressure Gradient

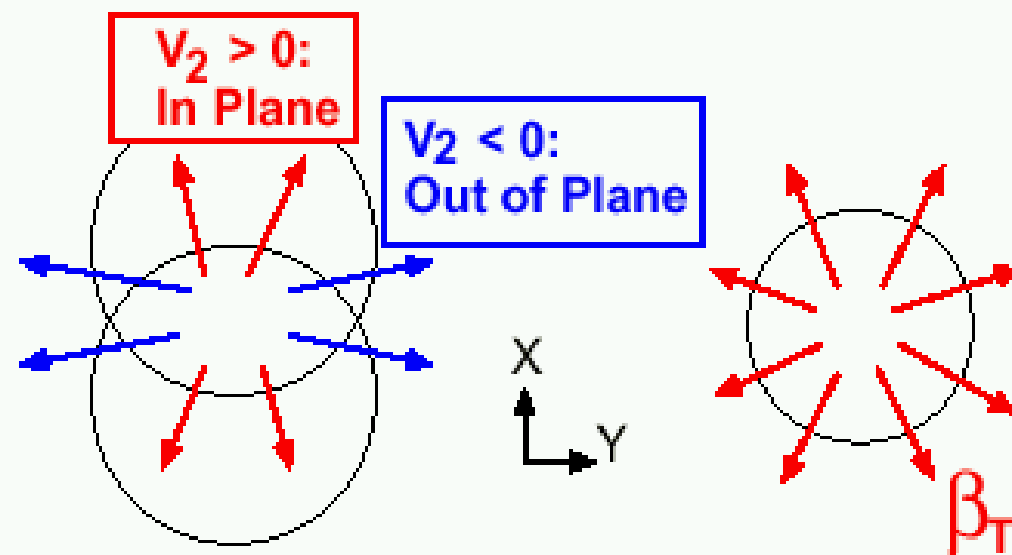
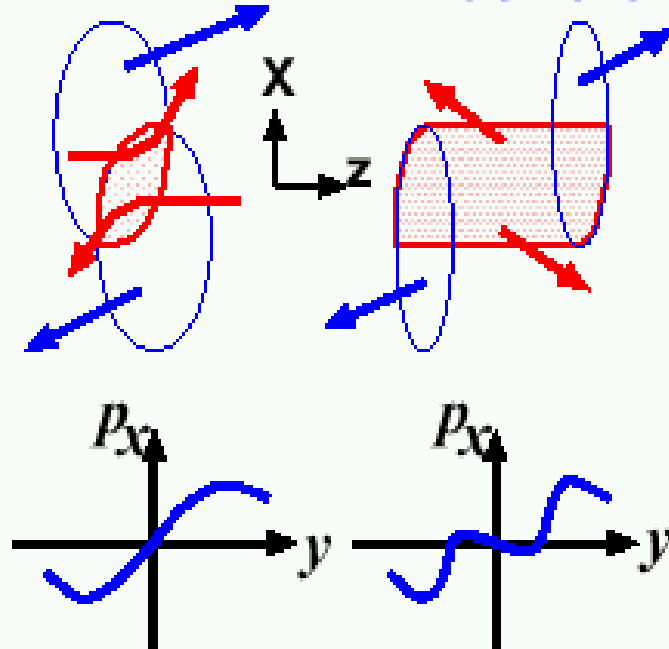
## Radial Flow ( $\beta_T$ )

Pressure History

$$\epsilon \frac{DV}{Dt} = -\nabla P$$

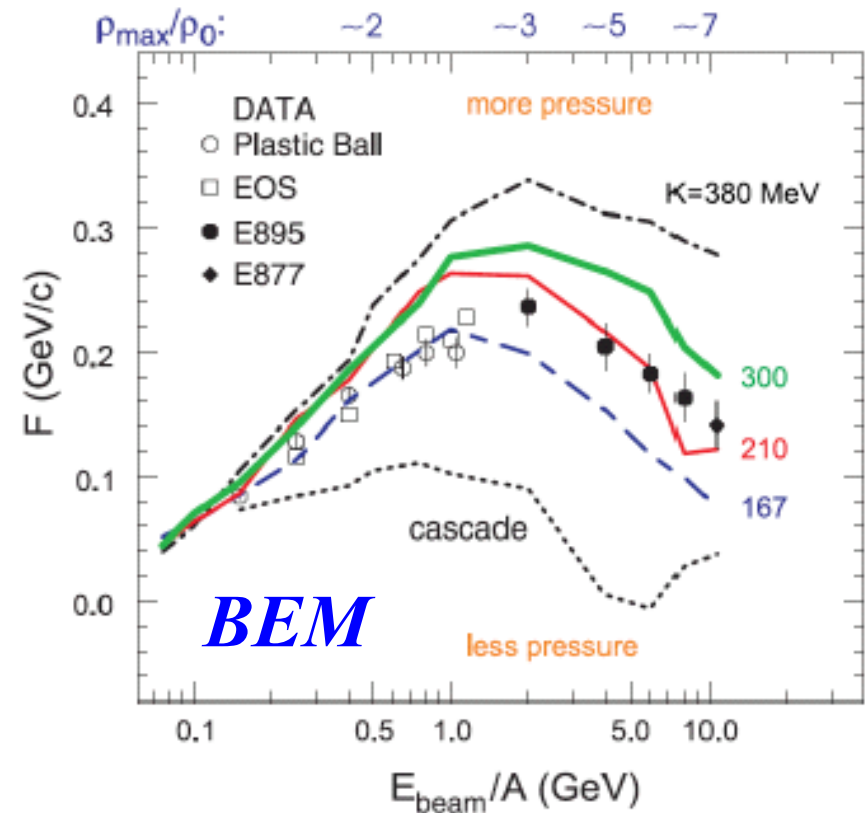
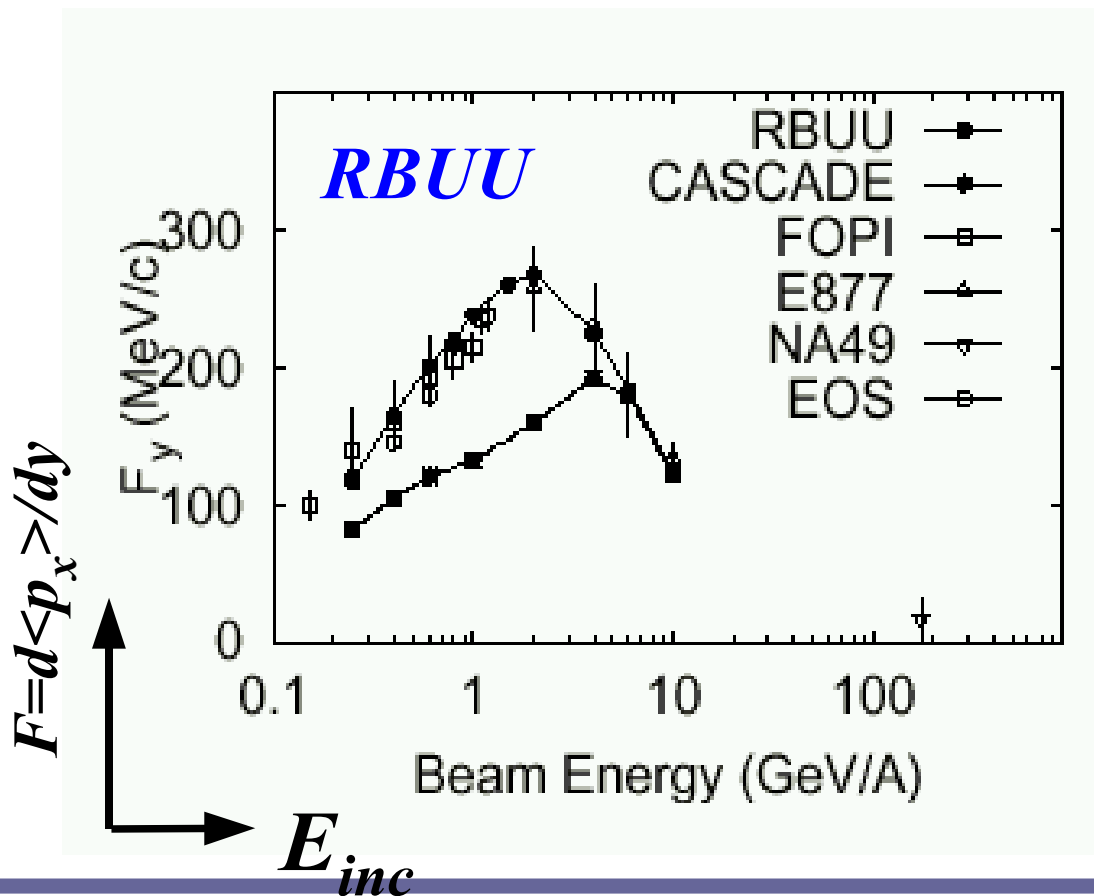
$$\rightarrow V = \int_{path} \frac{-\nabla P dt}{\epsilon}$$

**Until AGS**      **Above SPS**



# Side Flow at AGS Energies

- Relativistic BUU (RBUU) model:  $K \sim 300 \text{ MeV}$   
(Sahu, Cassing, Mosel, AO, Nucl. Phys. A672 (2000), 376.)
- Boltzmann Equation Model (BEM):  $K=167\sim 210 \text{ MeV}$   
(P. Danielewicz, R. Lacey, W.G. Lynch, Science 298(2002), 1592.)

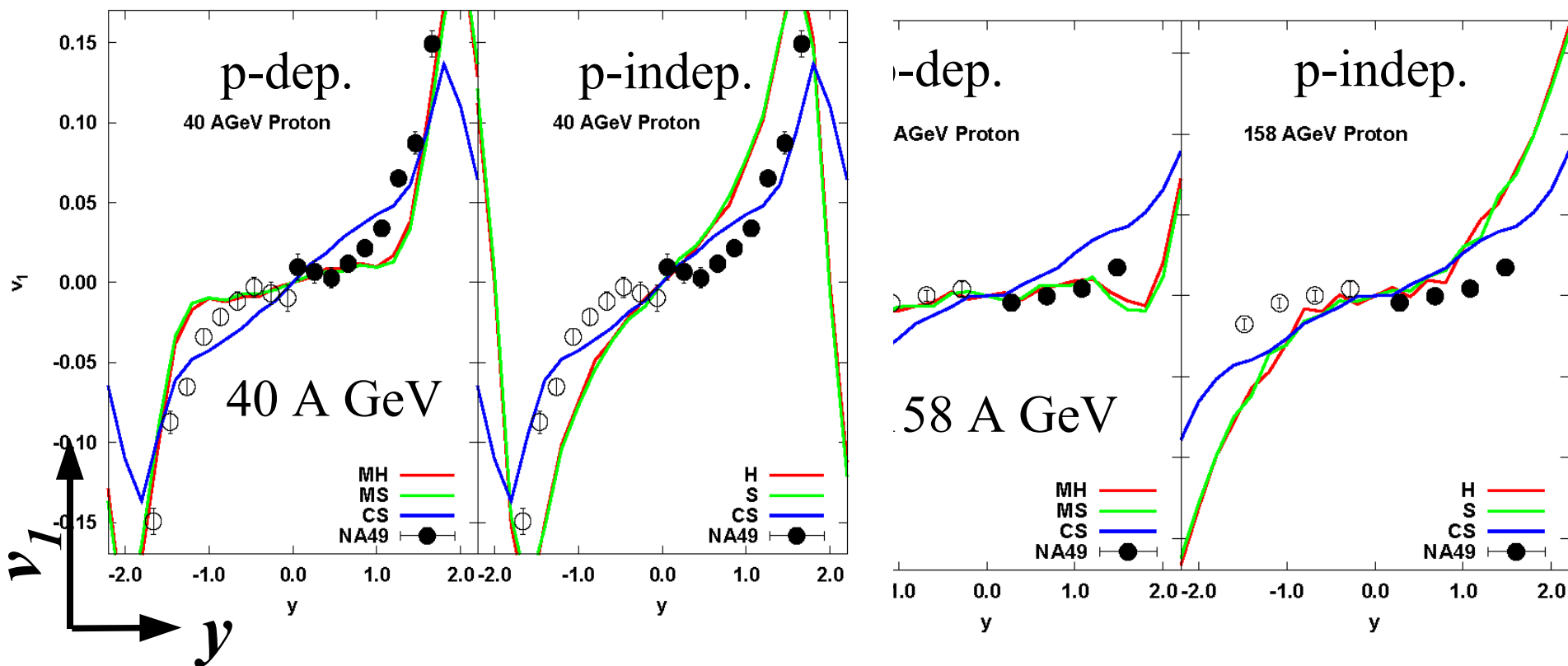


# Directed flow $v_1$ at SPS

Isse, AO, Otuka, Sahu, Nara, PRC 72 (2005), 064908

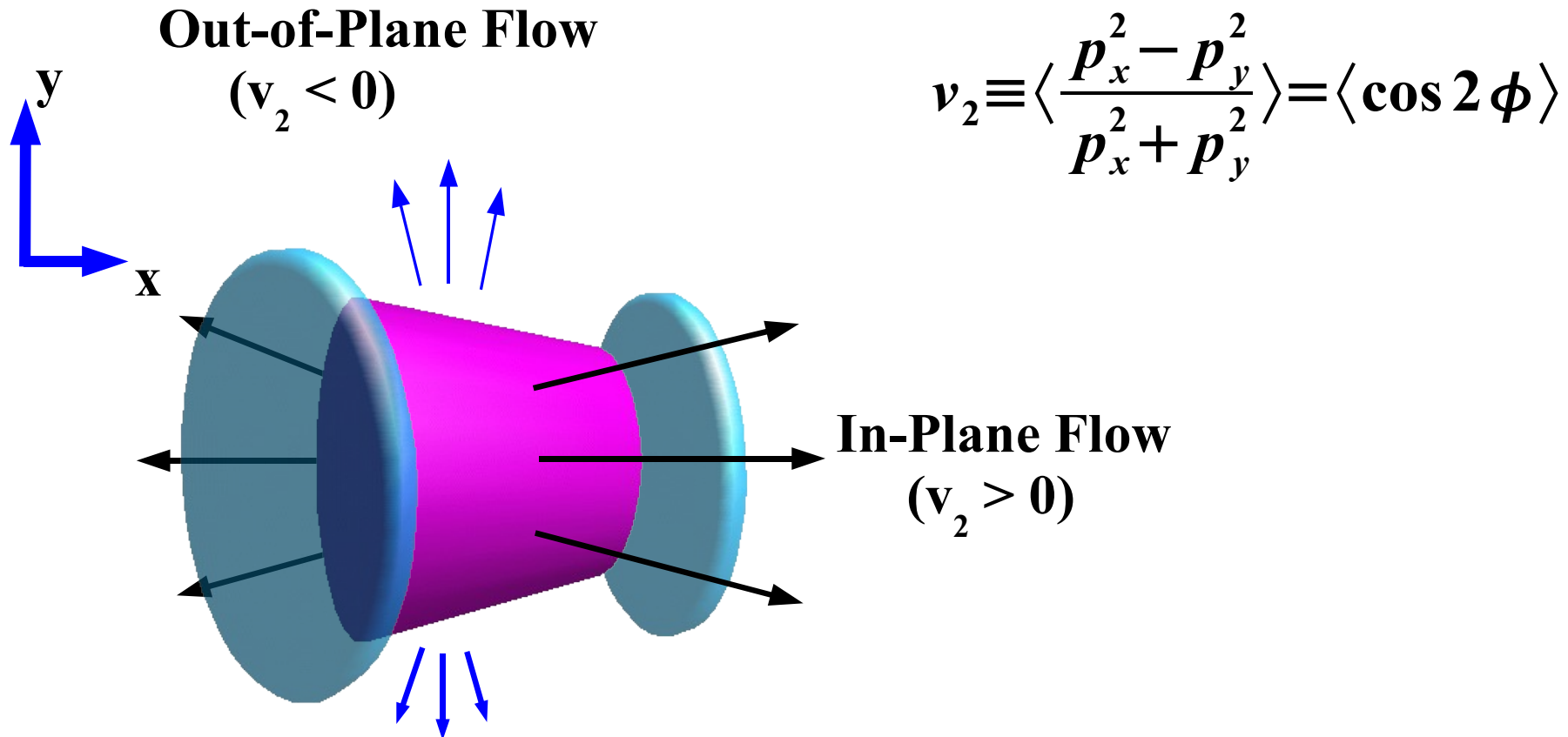
## • JAM-RQMD/S

- ◆ p-dep. (indep.) MF suppresses (enhances)  $v_1$ .  $v_1 = \langle \cos \phi \rangle = \langle p_x / p_T \rangle$
- ◆ “Wiggle” behavior appears with p-dep. MF at 158 A GeV.



# *Elliptic Flow*

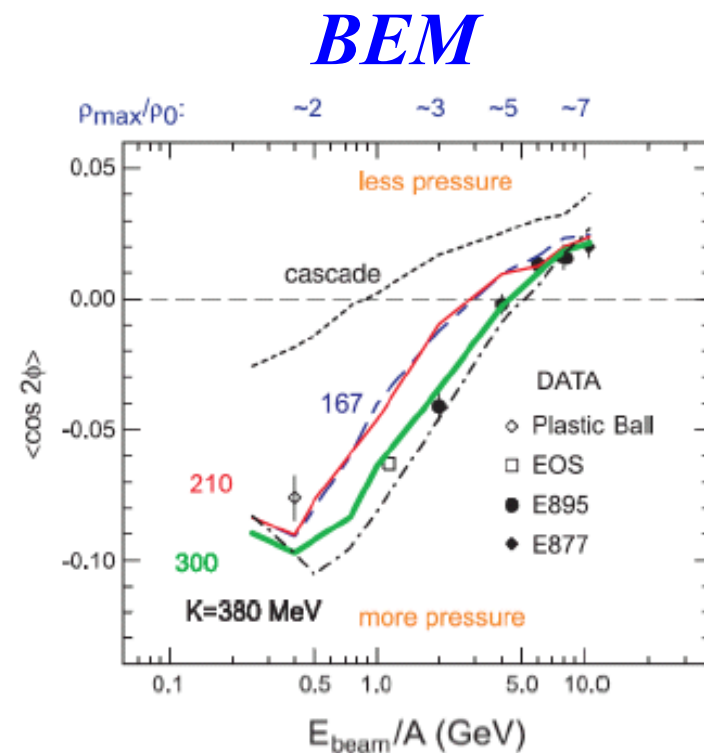
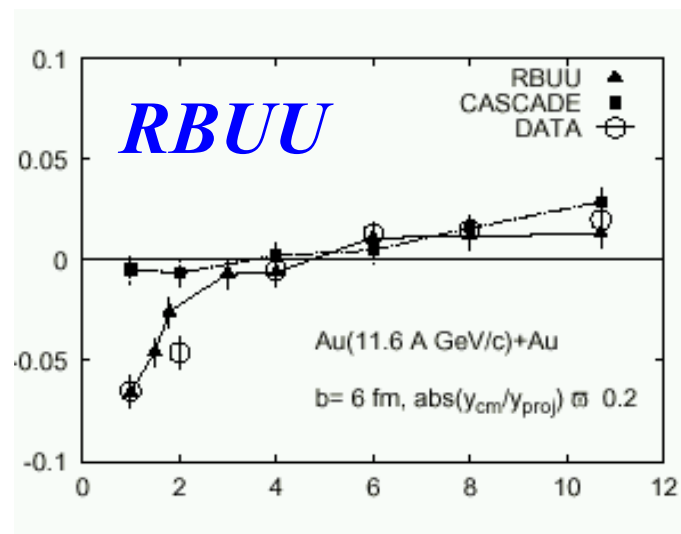
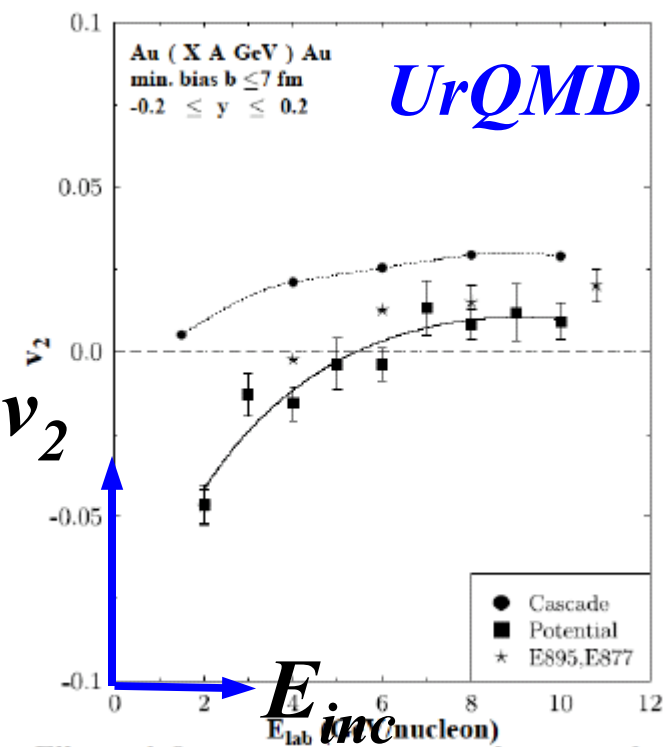
- **What is Elliptic Flow ? → Anisotropy in P space**
- **Hydrodynamical Picture**
  - ◆ **Sensitive to the Pressure Anisotropy in the Early Stage**
  - ◆ **Early Thermalization is Required for Large  $v_2$**



# Elliptic Flow at AGS

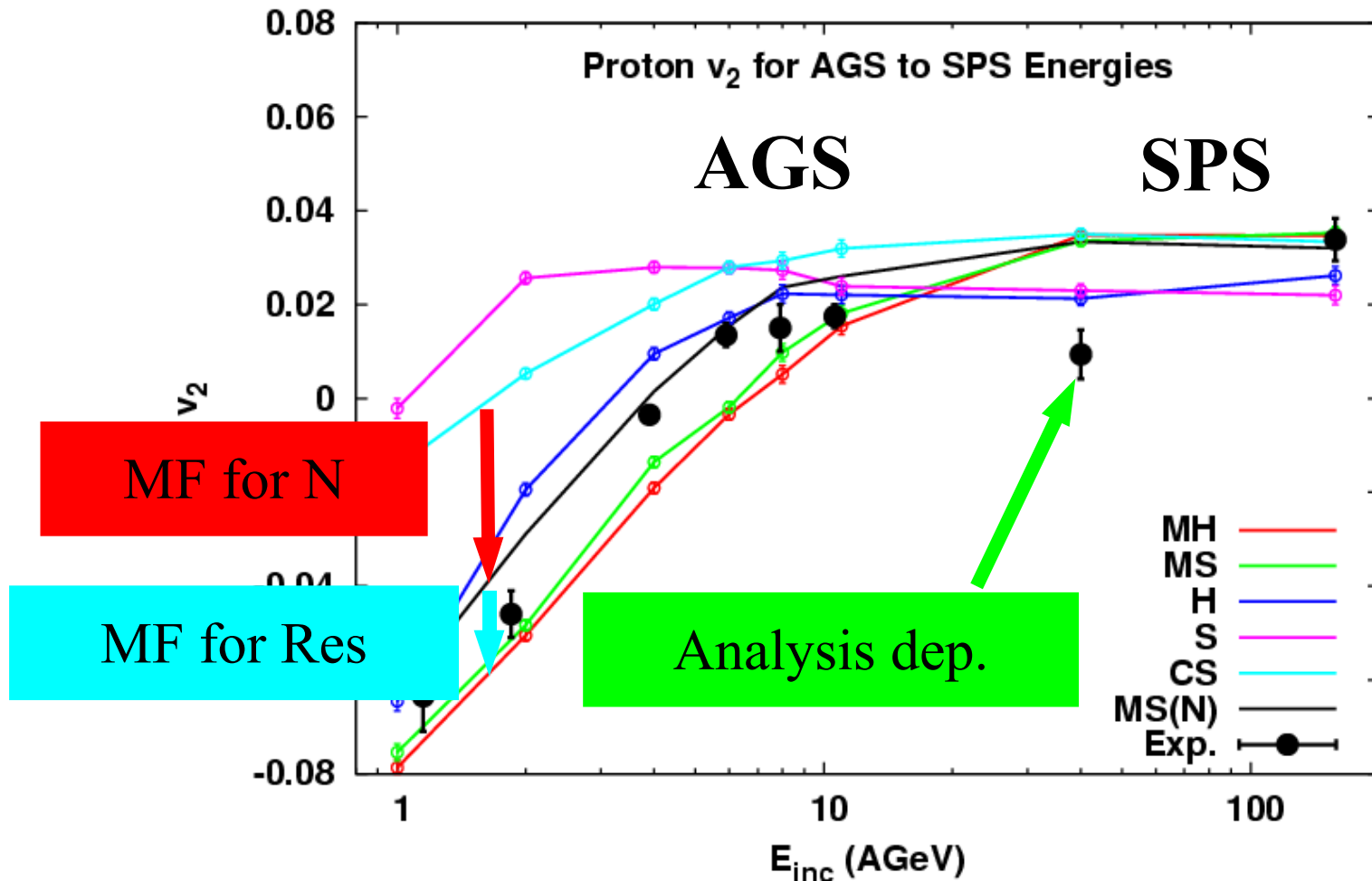
## Strong Squeezing Effects at low E (2-4 A GeV)

- ◆ **UrQMD: Hard EOS (S.Soff et al., nucl-th/9903061)**
- ◆ **RBUU (Sahu-Cassing-Mosel-AO, 2000):  $K \sim 300$  MeV**
- ◆ **BEM(Danielewicz2002):  $K = 167 \rightarrow 300$  MeV**



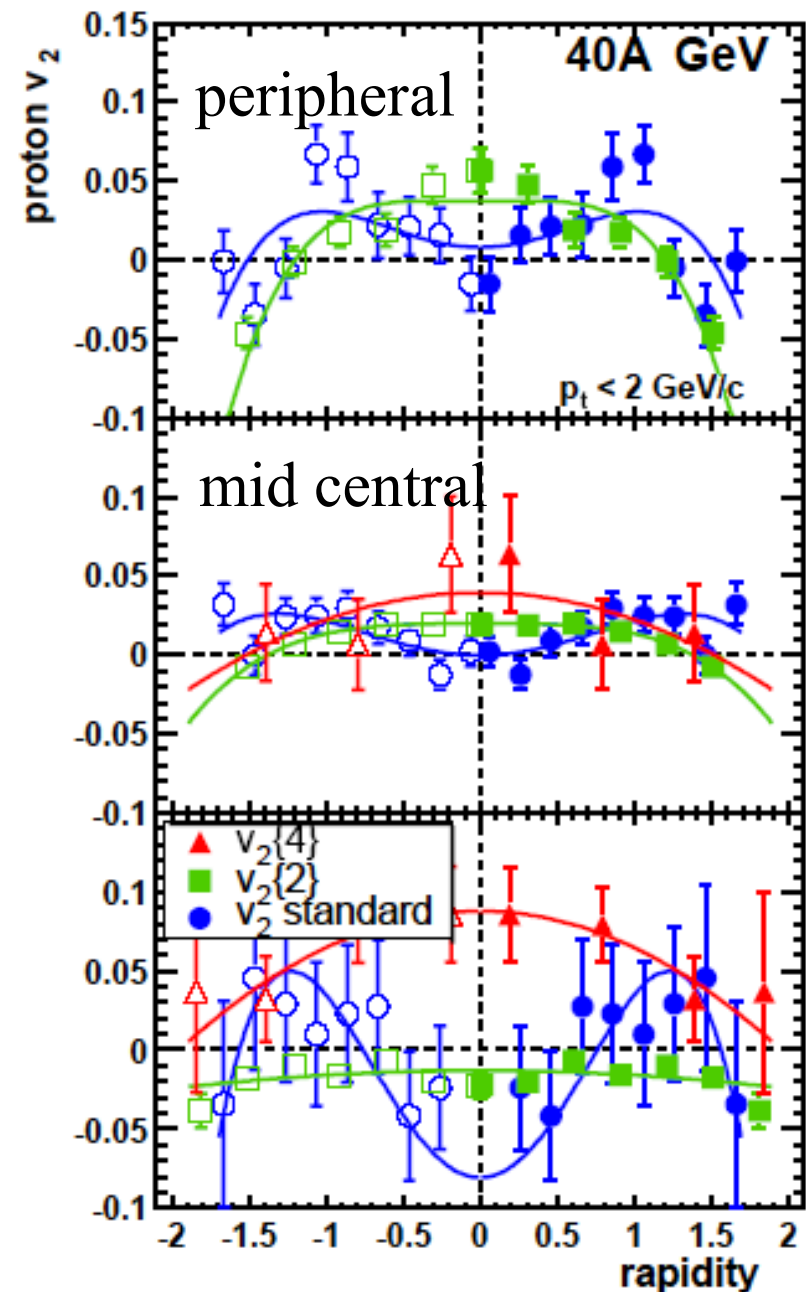
# Elliptic Flow from AGS to SPS

- JAM-MF with p dep. MF explains proton  $v_2$  at 1-158 A GeV
  - $v_2$  is not very sensitive to K (incompressibility)
  - Data lies between MS(B) and MS(N)



# Dip of $V_2$ at 40 A GeV: Phase Transition ?

- Dip of  $V_2$  at 40 A GeV may be a signal of QCD phase transition at high baryon density. (Cassing et al.)
- However, the data is too sensitive to the way of the analysis (reaction plane/two particle correlation).
- We have to wait for better data.





# Flow and EOS; to be continued

- In addition to the ambiguities in in-medium cross sections, Res.-Res. cross sections, we have model dependence.

- ◆ **RBUU** (*e.g. Sahu, Cassing, Mosel, AO, Nucl. Phys. A672 (2000), 376.*)

- In RMF, Strong cut-off for meson-N coupling in RMF  
→ Smaller EOS dep.

- ◆ **Scalar potential interpretation in BUU**

*Larionov, Cassing, Greiner, Mosel, PRC62,064611('00), Danielewicz, NPA673,375('00)*

$$\varepsilon(\mathbf{p}, \rho) = \sqrt{[m + U_s(\mathbf{p}, \rho)]^2 + \mathbf{p}^2} = \sqrt{m^2 + \mathbf{p}^2} + U(\mathbf{p}, \rho)$$

- Due to the Scalar potential nature, EOS dependence is smaller.

- ◆ **Scalar/Vector Combination** *Danielewicz, Lacey, Lynch, Science 298('02), 1592*

$$\varepsilon(p, \rho) = m + \int_0^p dp' v^*(p', \rho) + \tilde{U}(\rho), \quad v^*(p, \rho) = \frac{p}{\sqrt{p^2 + [m^*(p, \rho)]^2}}$$

- Relatively Strong EOS dependence even at high energy

- ◆ **JAM-RQMD/S** *Isse, AO, Otuka, Sahu, Nara, PRC 72 (2005), 064908*

- Similar to the Scalar model BUU

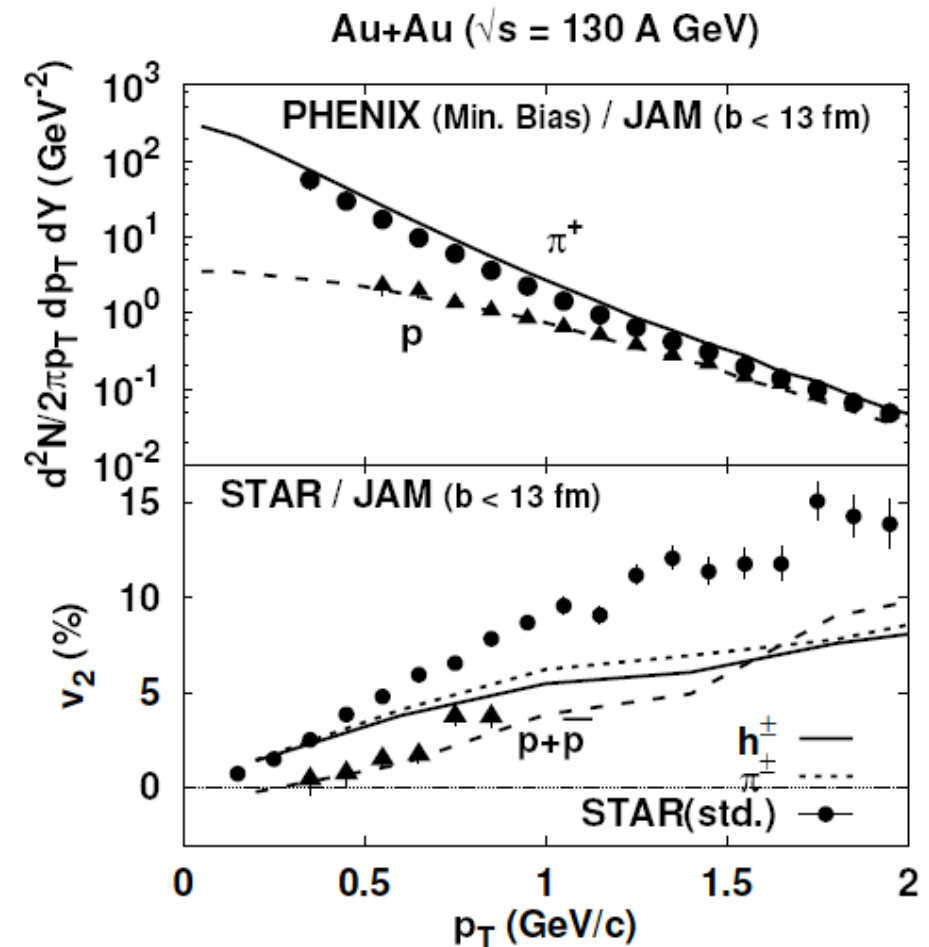
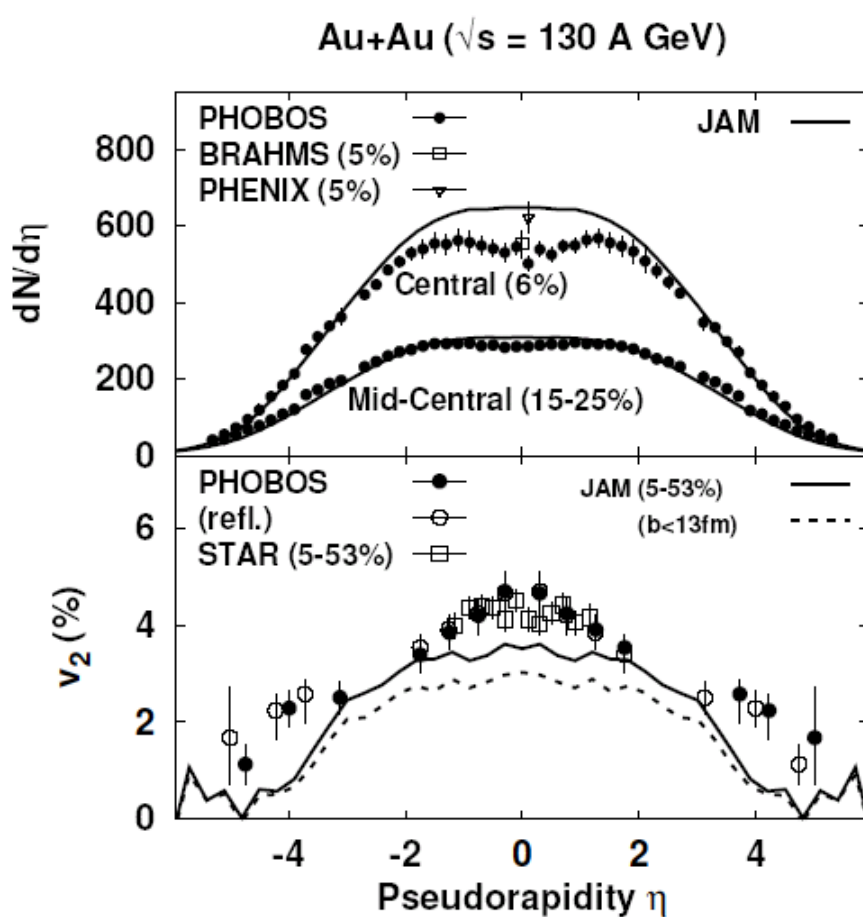
---

*Elliptic Flow @ RHIC*

# Elliptic Flow in Hadron-String Cascade (I)

## Hadron-String Cascade (JAM) @ RHIC

- Hadron Yield is reasonably explained up to 2 GeV/c (10-20 % error)
- $v_2$  is underestimated (20-30 % (integrated), 50 % ( $p_T > 1$  GeV))



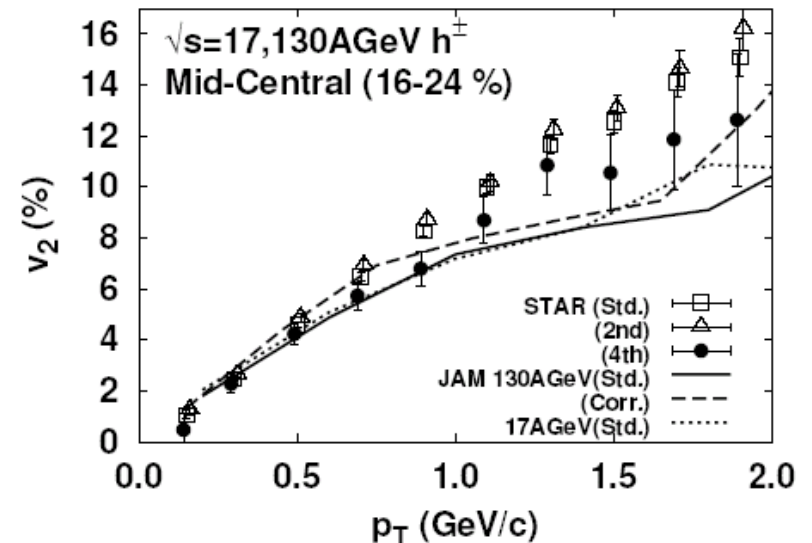
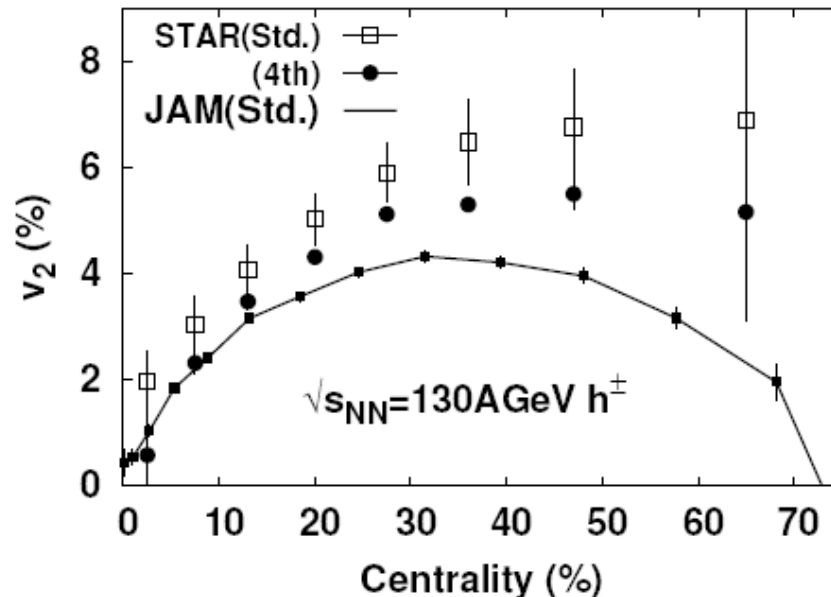
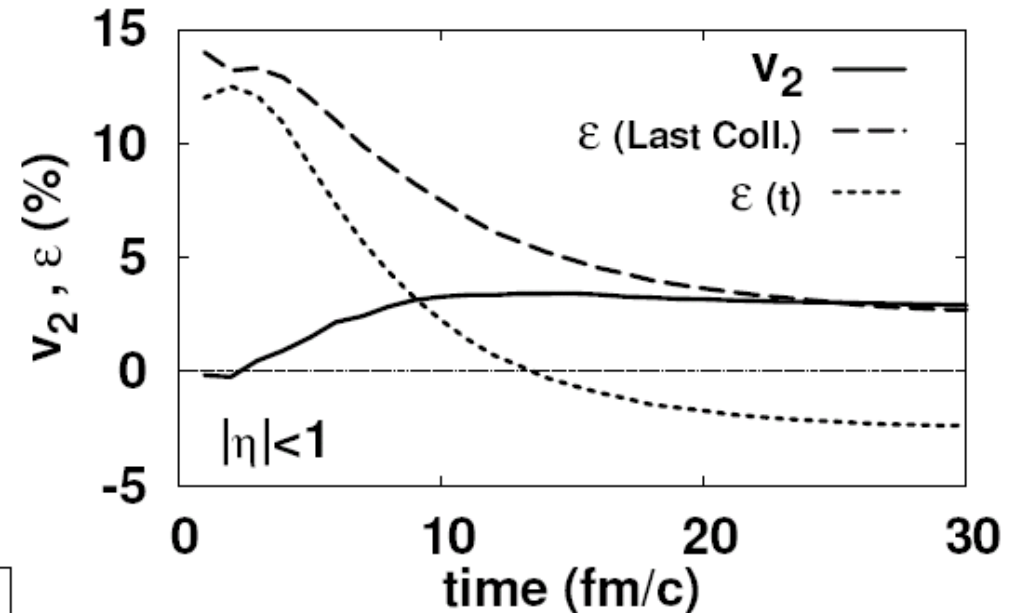
# Elliptic Flow in Hadron-String Cascade (II)

## Why do we underestimate $v_2$ in Hadron-String Cascade ?

- $v_2$  growth time is long ( $\sim 10$  fm/c), due to hadron formation time ( $\tau \sim 1$  fm/c).  
 $\rightarrow$  much longer than hydro

Sahu-Isse-AO-Otuka-Phatak 2006

Au+Au,  $\sqrt{s_{NN}} = 130$  GeV,  $b < 13$  fm



# Results of Parton Cascade

- Unexpectedly high parton cross sections of  $\sigma = 5-6$  mb have to be assumed in parton cascades in order to reproduce the elliptic flow.

ZI-WEI LIN AND C. M. KO

PHYSICAL REVIEW C 65 034904

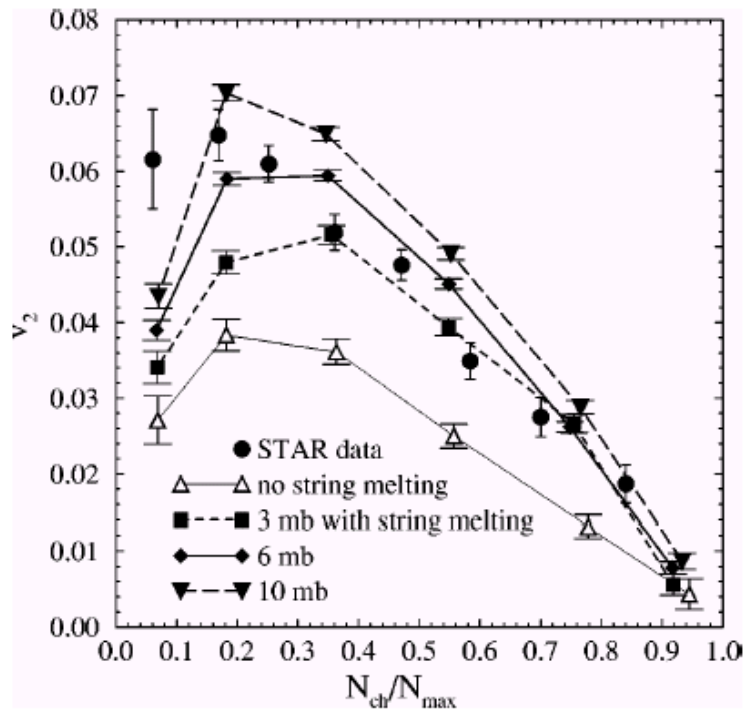


FIG. 3. Impact parameter dependence of elliptic flow at 130 A GeV. The data from the STAR collaboration [7] are shown by filled circles, while the theoretical results for different partonic dynamics are given by curves.

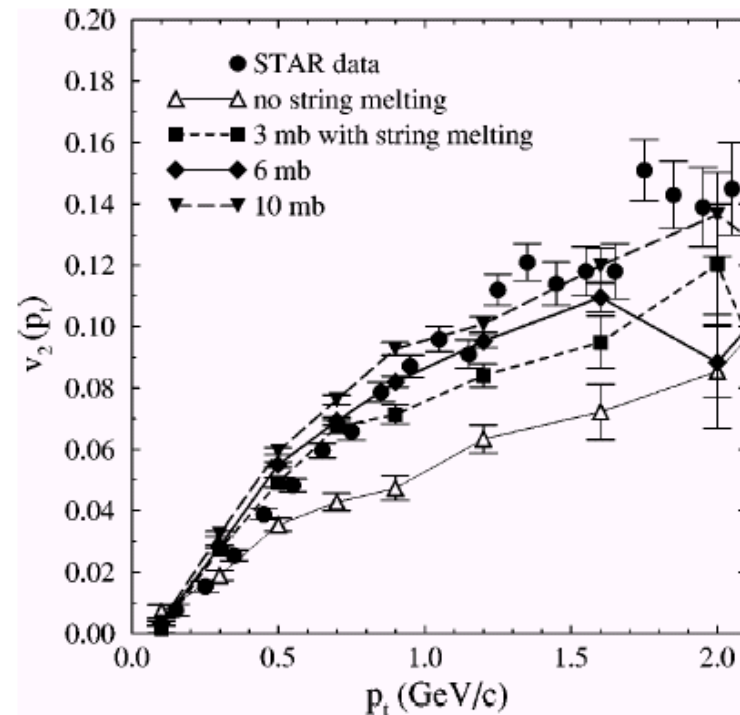


FIG. 4. Transverse momentum dependence of elliptic flow at 130 A GeV. Circles are the STAR data for minimum-bias Au+Au collisions [7], and curves represent the minimum-bias results for charged particles within  $\eta \in (-1.3, 1.3)$  from the AMPT model.

# Initial Conditions in Hydro

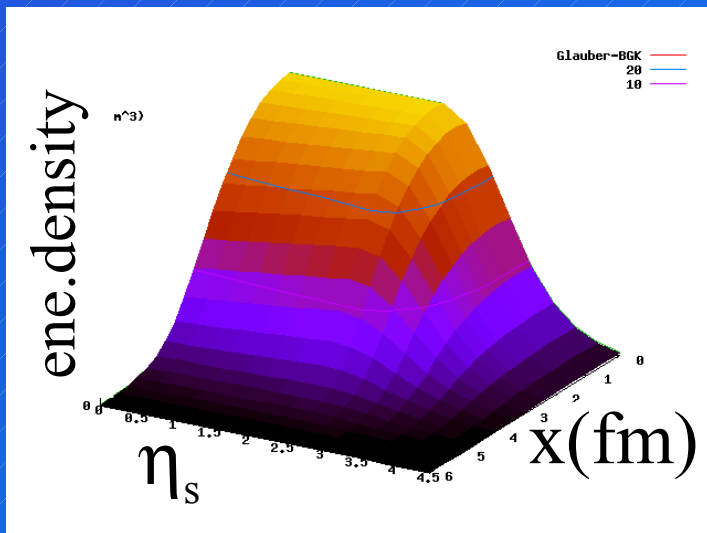
## Glauber-BGK type

[Reference Initial Condition]  
Transverse profile:

Entropy density  
 $\propto a\rho_{\text{part}} + b\rho_{\text{coll}}$

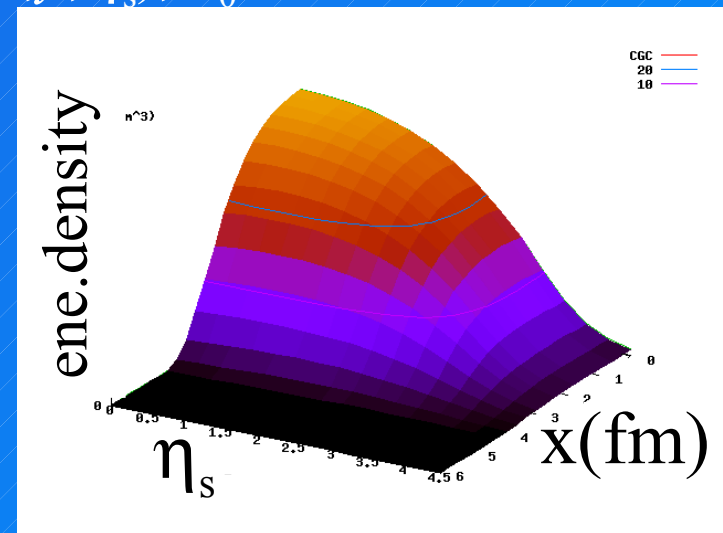
Longitudinal Profile:

Brodsky-Gunion-Kuhn triangle



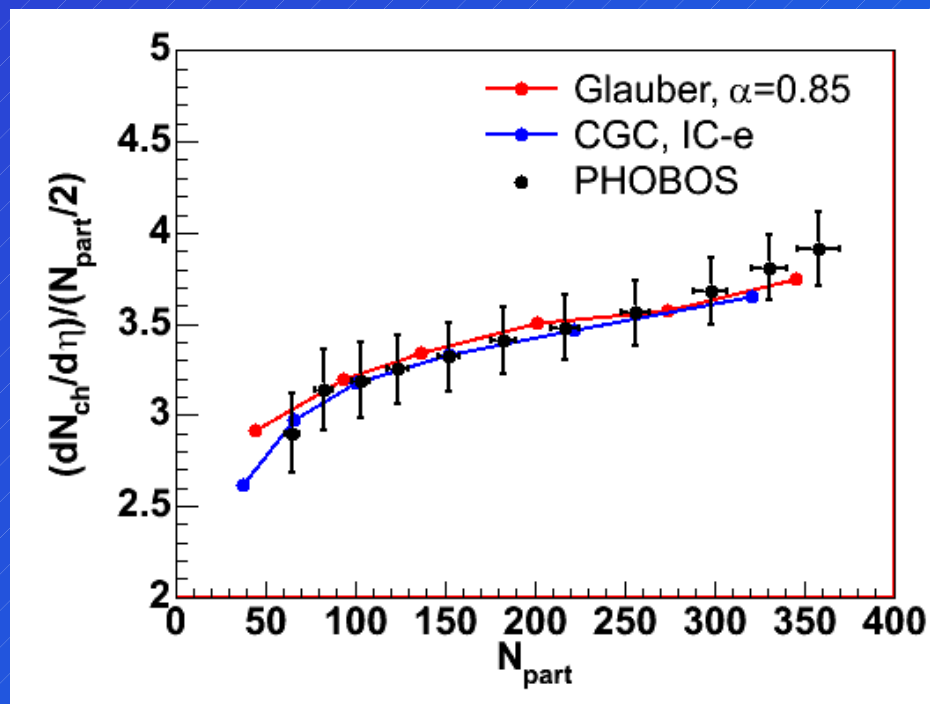
## Color Glass Condensate

- Unintegrated gluon distribution a.l.a. Kharzeev, Levin, and Nardi
- Gluon production via  $k_T$  factorization formula
- Count deposited energy in  $dV$  at  $(\tau_0, x, y, \eta_s)$ ,  $\tau_0 = 0.6\text{fm}/c$

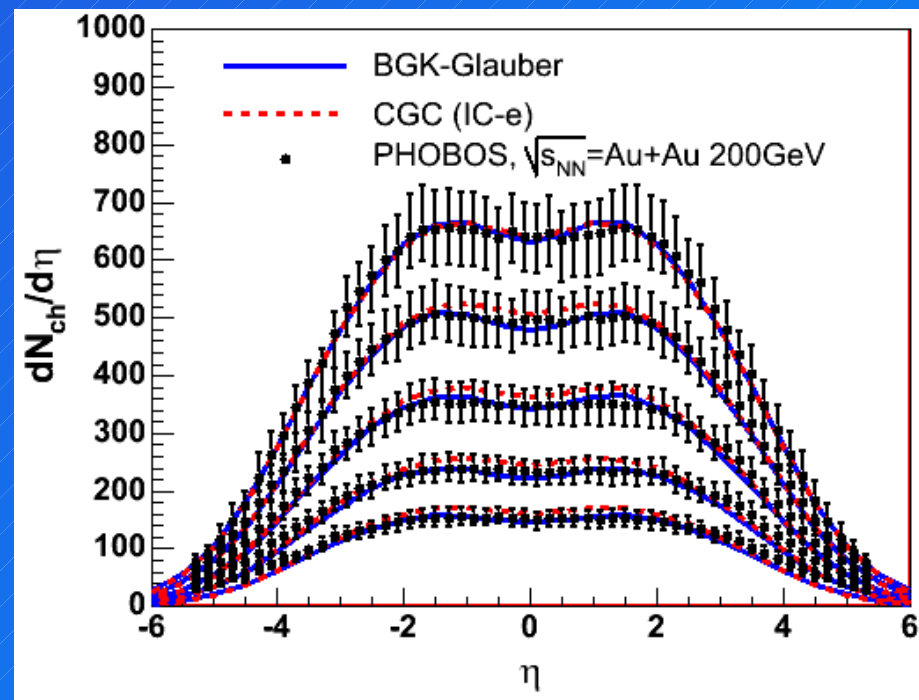


# Two Hydro Initial Conditions Which Clear the “First Hurdle”

Centrality dependence



Rapidity dependence



1. CGC model

Khazeev, Levin, and Nardi

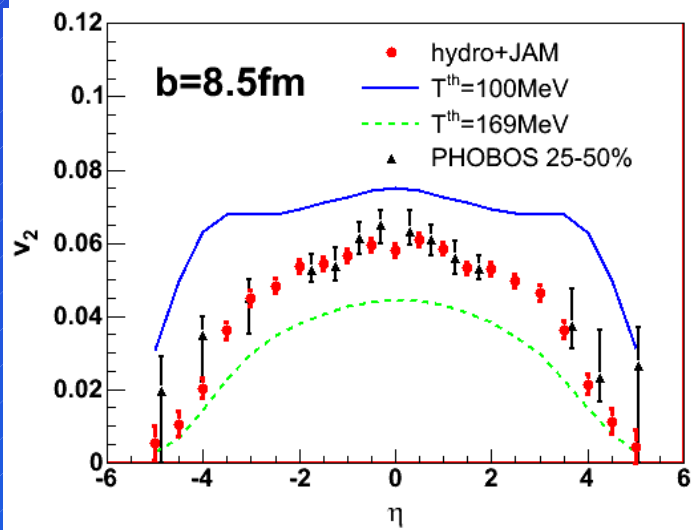
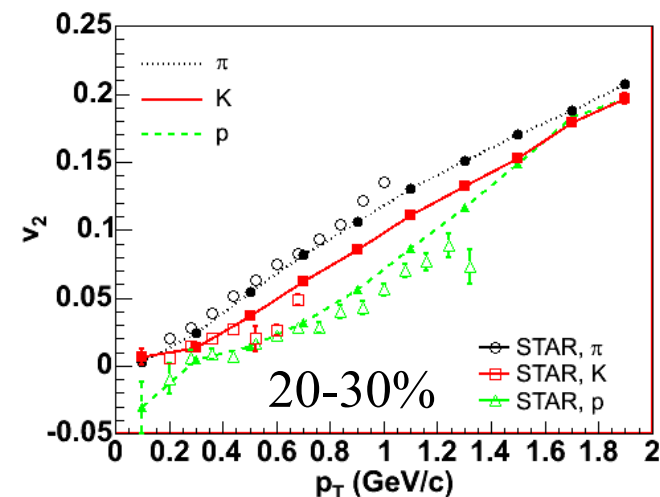
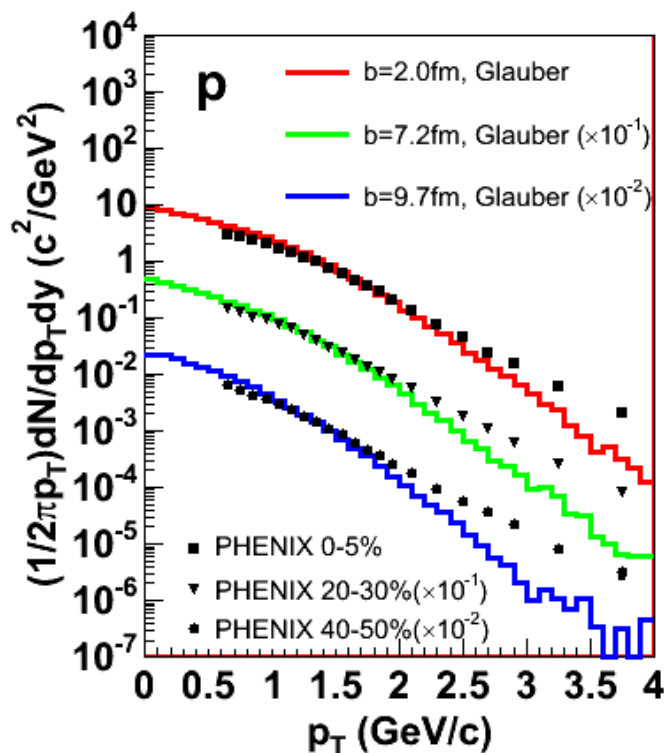
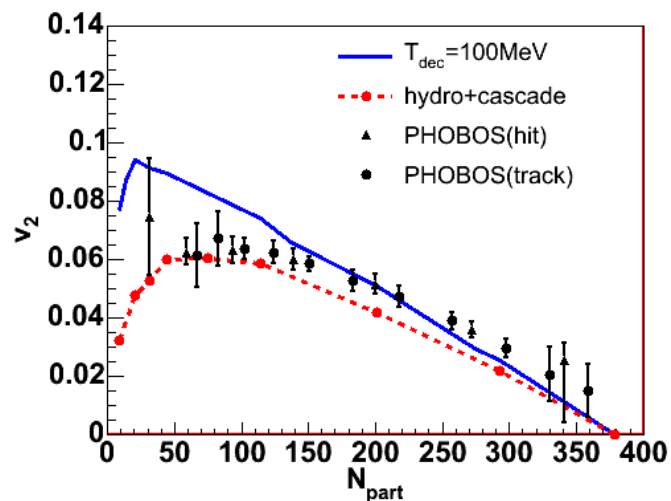
Implemented in hydro by TH and Nara  
Matching I.C. via  $e(x,y,\eta)$

2. Glauber model (as a reference)

$$N_{part} : N_{coll} = 85\% : 15\%$$



# Highlights from Glauber + QGP Fluid + Hadron Gas Model



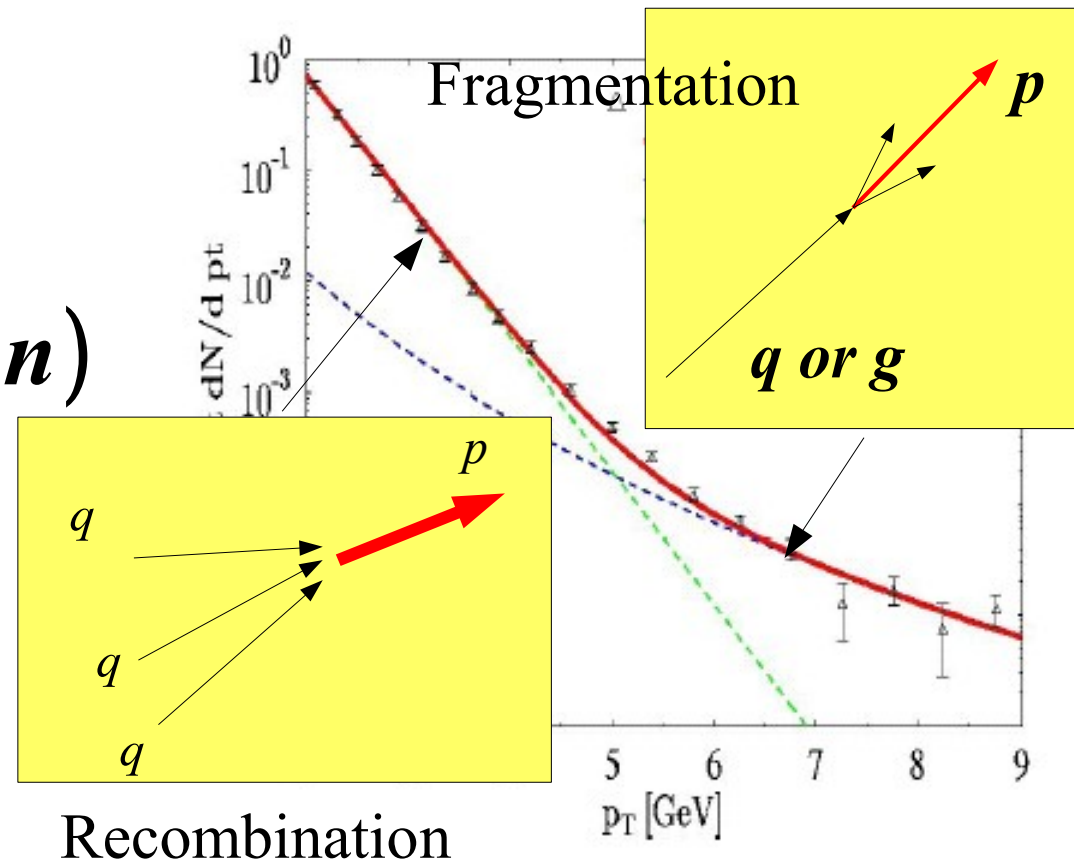
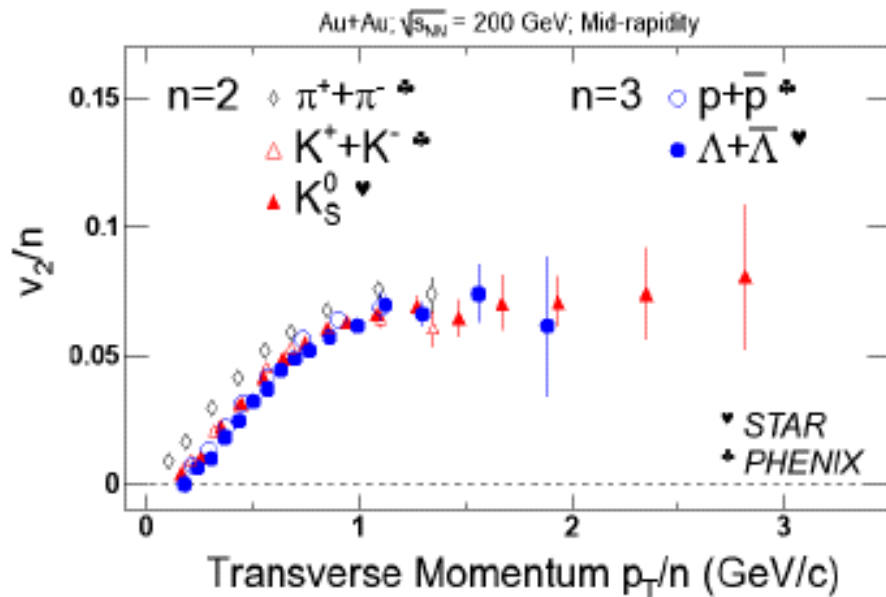
Good agreement for bulk  
( $p_T < \sim 1.5\text{GeV}/c$ )

→ What happens to the CGC case?

# QGP Signals: Quark Number Scaling

- When  $n$  quarks recombines to a hadron,  $v_2$  is enhanced by  $n$  times.

$$v_2^{Hadron}(P_T) = n v_2^{Parton}(P_T/n)$$



Fries et al. PRL 90 (2003), 202303  
Nonaka et al., nucl-th/0308051

**Recombination Picture seems to work well  
... Parton Elliptic Flow**

# Recombination and Fragmentation

Fries, Muller, Nonaka, Bass, *PRL*90, 202303(2003); *PRC*68,044902 (2003)

- **Successes: quark number scaling, baryon/meson ratio**  
→  $v_2 \sim 0.10$  at high- $p_T$  .

$$f(p, \varphi) = (1 + 2 v_2(p/2) \cos \varphi) \times (1 + 2 v_2(p/2) \cos \varphi)$$
$$\approx 1 + 2 \times 2 v_2(p/2) \cos \varphi$$

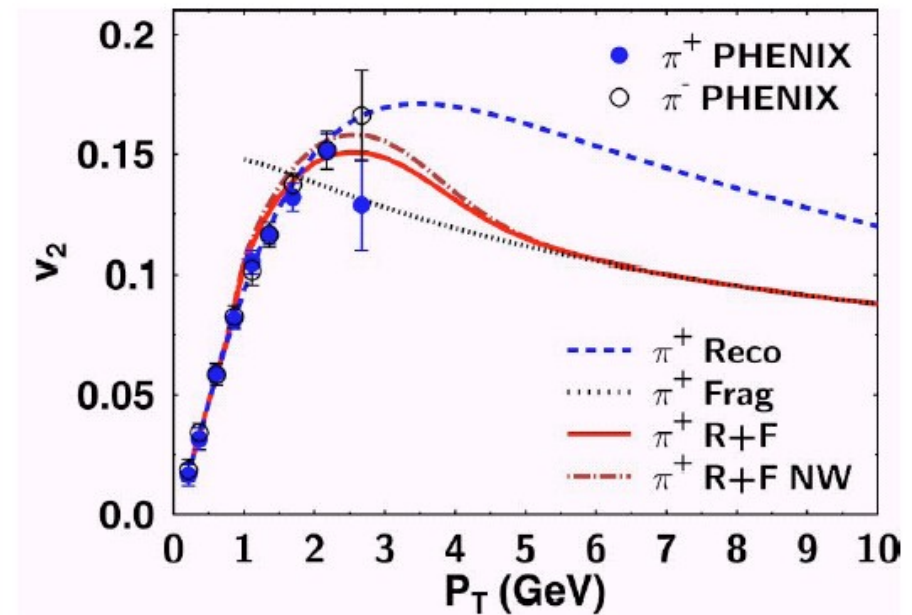
- **Problems: Sharply edged density dist. (Hard Sphere)**

$$\ell(b) = \sqrt{R_A^2 - (b/2)^2}$$

→  $E\text{-loss} \propto \ell \rightarrow v_2 \sim 0.10$

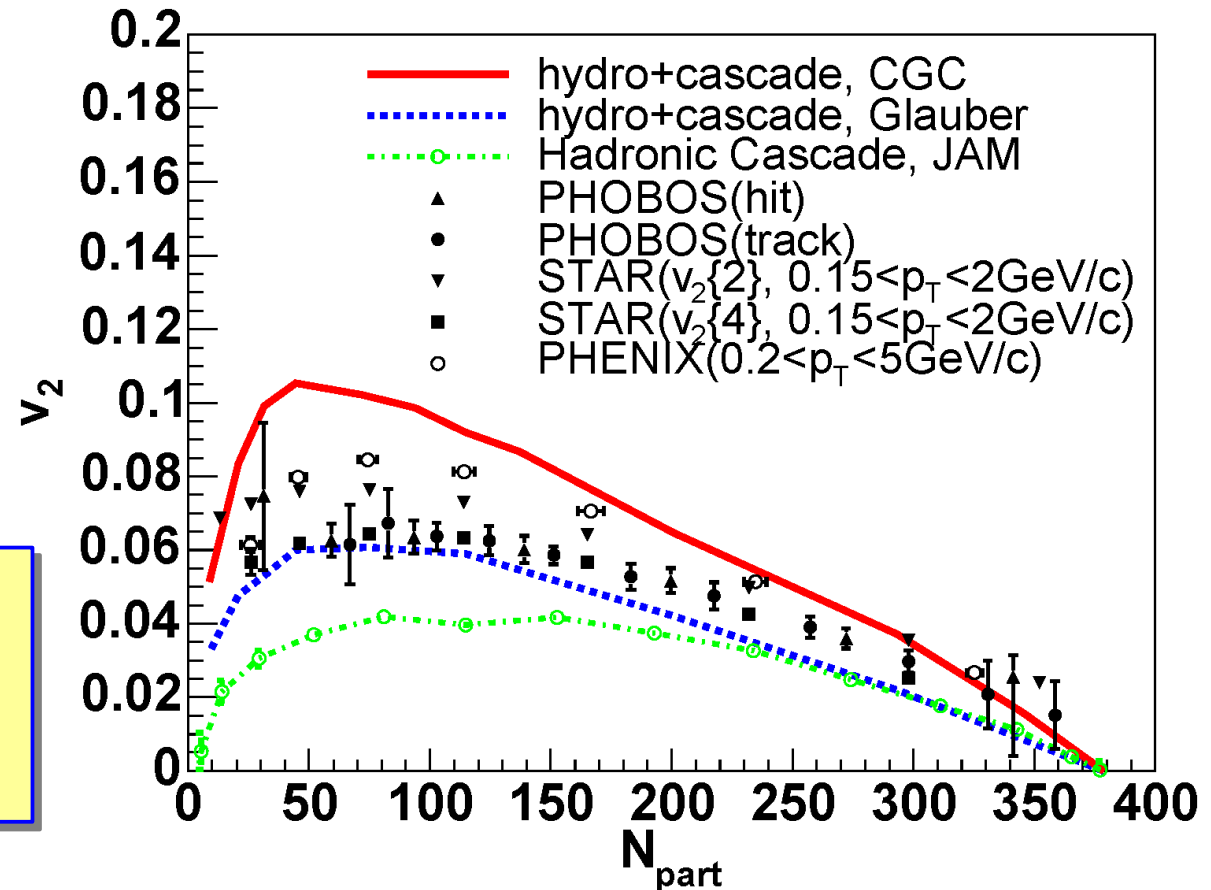
- ◆ **Woods-Saxon density distribution**

→  $v_2 \sim 0.05$  : Half of H.S.



# Cascade vs Hydro @ RHIC: Au+Au

- Comparison of  $v_2$  as a function of  $N_{part}$ 
  - Cascade predict smaller  $v_2$  in peripheral collisions
  - Data lies between hydro results with two different initial condition CGC (Color Glass Condensate) and Glauber type initial condition.



Hydro is better,  
CGC may be realized  
in central collisions.

# *When and where is QGP formed ?*

## ● Incident Energy

- ◆ AGS: Strangeness Enh. (High baryon  $\rho$  effect ?)
- ◆ SPS:  
J/ $\psi$  suppression (QGP?), Low mass dilepton enh. (chiral sym.)  
Hydro overestimate  $v_2$  data
- ◆ RHIC:  
Jet quenching, Strong  $v_2$ , Quark number scaling of  $v_2$ , ...  
Hadronic Cascade underestimate  $v_2$  data  
→ Bulk QGP formation seems to start between SPS and RHIC

## ● Proj./Targ. Mass dependence

- ◆ Au+Au:  $v_2(\text{Casc.}) < v_2(\text{hydro}) \sim v_2(\text{data})$
- ◆ Cu+Cu: Recently Measured

# Predictions of Cu+Cu Collisions @ RHIC (I)

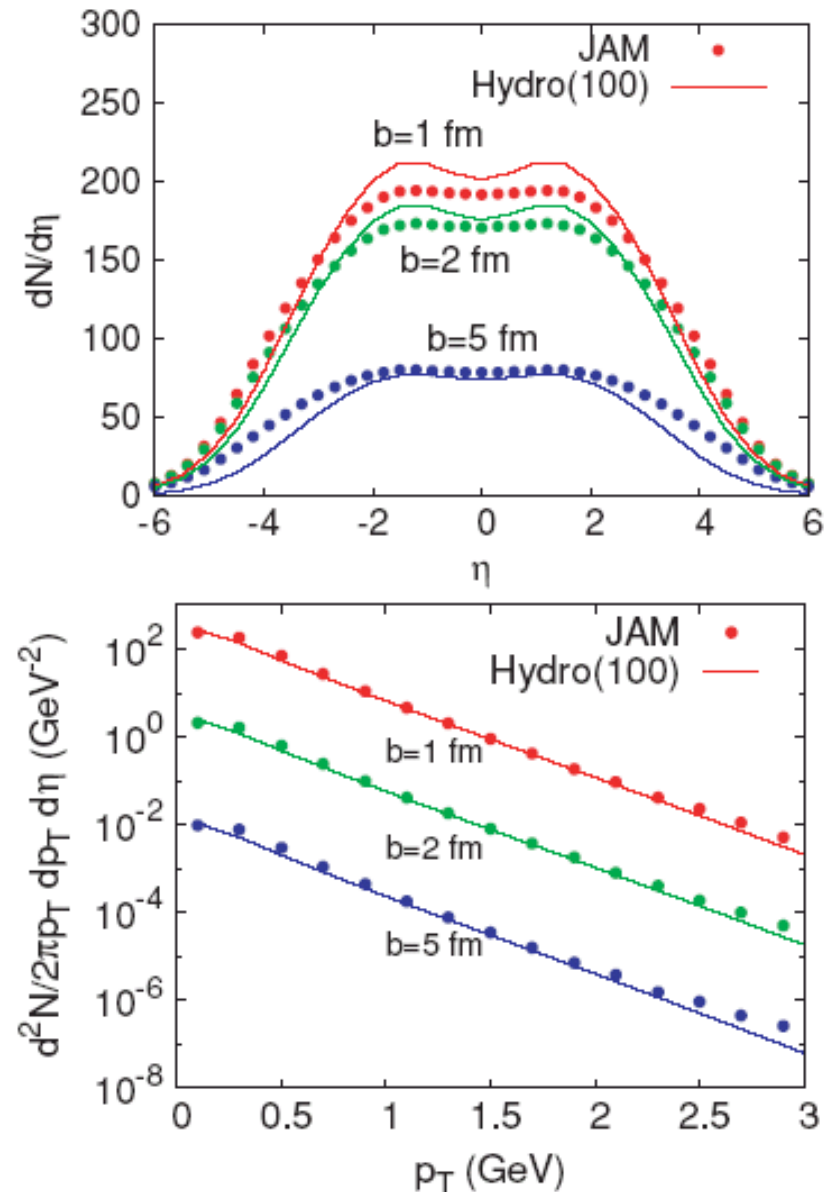
## • Single particle spectra

- Cascade (JAM) and Hydro predict almost the same single particle spectra

$$dN/d\eta, d^2N/p_T dp_T d\eta$$

## • Surprising ?

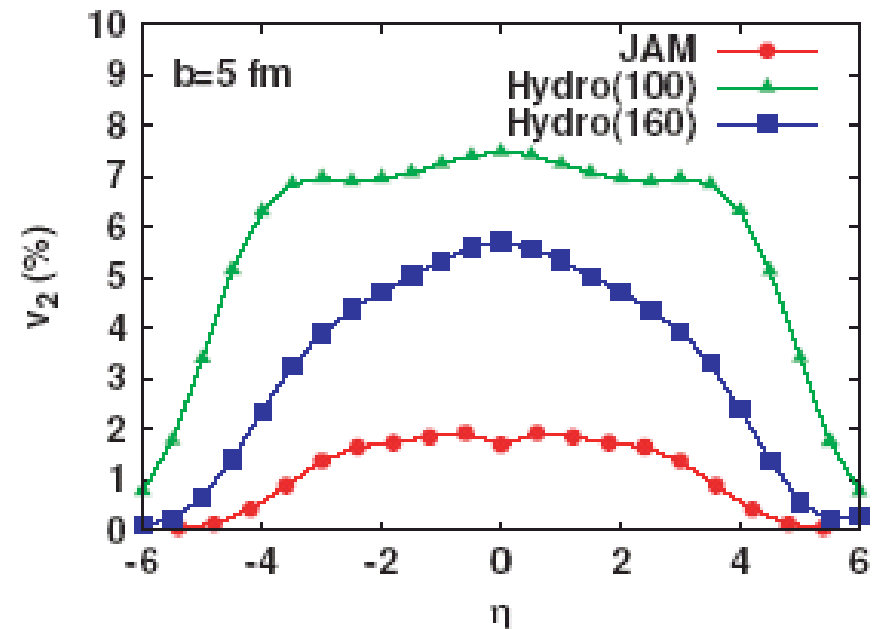
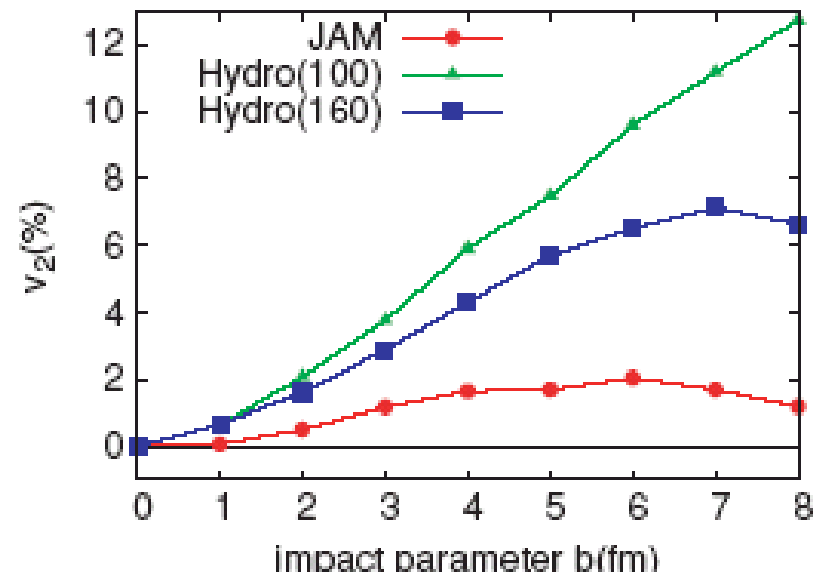
- Initial Cond. of Hydro is tuned to fit  $dN/d\eta$  (~ Energy per rapidity)
- Cascade use fitted  $\sigma_{NN}$
- Thermalization is expected at Low  $p_T$  (long time before particle production)  
→ Coincidence may not be surprising



*Hirano, Isse, Nara, AO, Yoshino, Phys. Rev. C 72(2005), 041901*

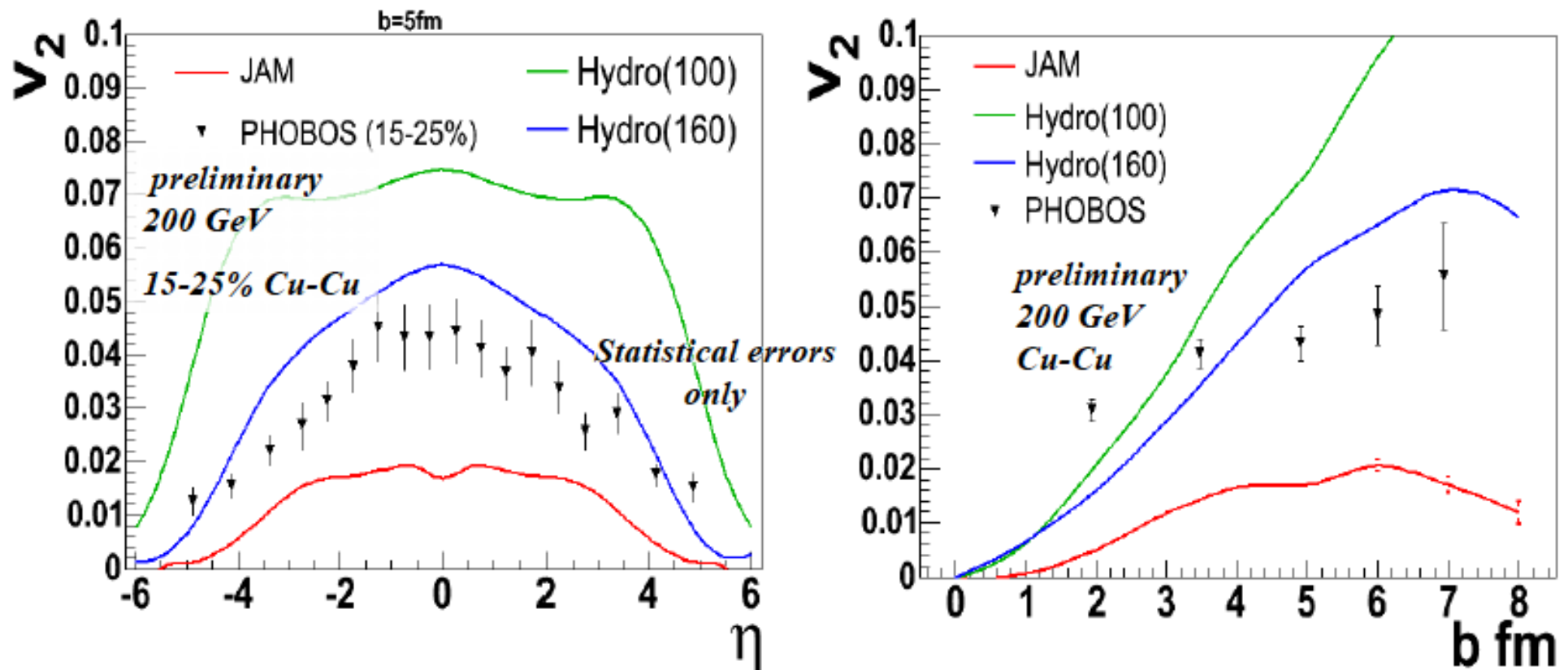
# Predictions of Cu+Cu Collisions @ RHIC (II)

- Calculations were done **BEFORE** the data are opened to public.
- Cascade and Hydro predict very different Elliptic Flow !
  - ◆ Cascade: small  $v_2$   
→ Small int. in the early stage
  - ◆ Hydro: large  $v_2$   
→ Strong int. after  $\tau = \tau_0 \sim 0.6$  fm/c
- $T^{th}$  dependence
  - ◆  $T^{th} = 160$  MeV  $\sim T_c = 170$  MeV  
→ short time of expansion in the hadron phase
  - ◆  $T^{th} = 100$  MeV  $< T_c = 170$  MeV  
→ long time of expansion





# Compared to JAM Model



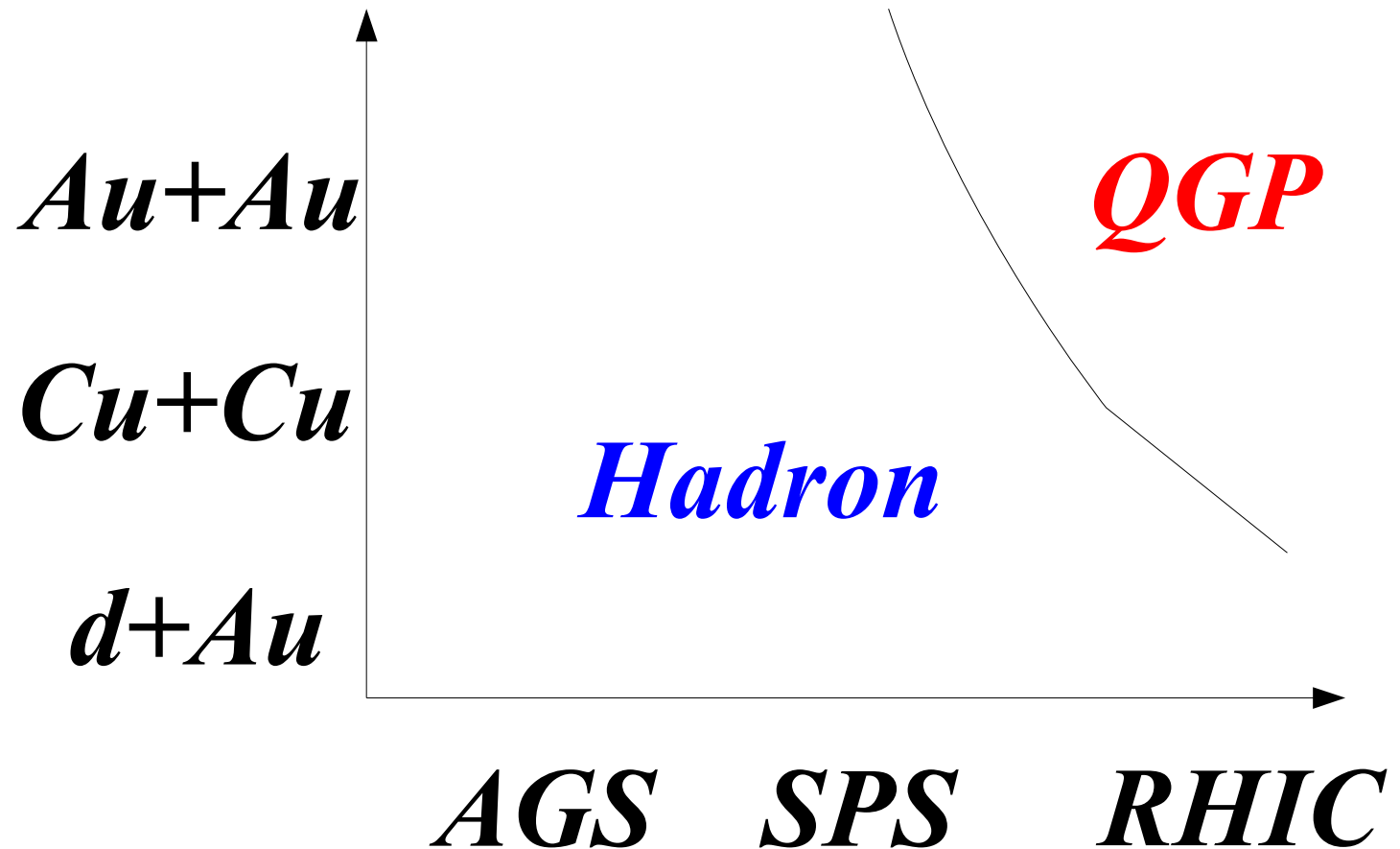
**Cu-Cu more like Hydro than JAM hadron string cascade model**

*Here JAM uses a 1 fm/c formation time. Hydro (160) has kinetic freezeout temperature at 160 MeV*



# *After Data are opened, ....*

- Hydro wins Cascade at RHIC even for Cu+Cu collisions in the initial stage evolution.....
- “Reaction Phase Diagram” seems to be .....



# Lessons from AGS and SPS Energy HIC

- 粒子生成の主要過程
  - ◆ 1 A GeV Energy → 共鳴粒子生成 + 共鳴崩壊
  - ◆ 10 A GeV Energy (AGS) → 2共鳴ハドロン生成、ストリング生成+崩壊
  - ◆ 100 A GeV Energy (SPS) → ストリング生成+崩壊
- 必要な自由度の輸送を取り入れる必要性
  - ◆ pT スペクトルに直接的に影響を与える。
  - ◆ あらわな自由度を取り入れない場合  
→ formation time 等を導入して相互作用の強さ(圧力)を調整  
(量子論的な「漸近領域に達する時間」だけではないだろう)
- RHICエネルギーでのハドロン輸送モデルの失敗
  - ◆ SPS エネルギーまでで成功している formation time を使うと、  
反応初期での相互作用が小さすぎる  
→ 遅い熱平衡化時間、小さな楕円フロー(特に high pT)

---

*Hadron Correlation at RHIC  
in Jet-Fluid String Model*

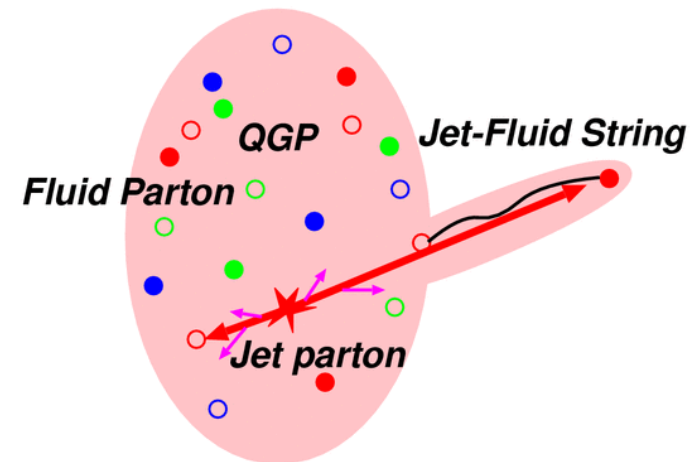
# Jet-Fluid String 模型における粒子相関

東大理、阪大理<sup>A</sup>、北大理<sup>B</sup>、フランクフルト大<sup>C</sup>  
平野哲文、一瀬昌嗣<sup>A</sup>、水川零<sup>B</sup>、奈良寧<sup>C</sup>、大西明<sup>B</sup>、吉野公二<sup>B</sup>

## *Hadron correlations in Jet-Fluid String model*

T.Hirano, M.Isse<sup>A</sup>, R. Mizukawa<sup>B</sup>, Y.Nara<sup>C</sup>, A. Ohnishi<sup>B</sup>, K.Yoshino<sup>B</sup>  
U. Tokyo, Osaka U.<sup>A</sup>, Hokkaido U.<sup>B</sup>, Frankfurt U.<sup>C</sup>

- Introduction
- High  $p_T$  でのハドロン化模型
- JFSでのハドロン相関
- まとめ



*Isse, Hirano, Mizukawa, AO, Yoshino, Nara, nucl-th/0702068*

# High $p_T$ ハドロン生成

- GSI, AGS, SPS → 共鳴ハドロン、ストリング生成と破砕

Nara et al., PRC61('00),024901; Isse et al., PRC72('05),064908.

- RHICでの標準描像= pQCD+E-loss+独立破砕

$$\begin{aligned} \frac{dN^{AA}(b)}{dy d^2 p_T} &= \int d\mathbf{r}_T t_A(\mathbf{r}_T - b/2) t_B(\mathbf{r}_T + b/2) && \text{Geometry} \\ &\times K \sum_{abcd} \int dx_a dx_b d^2 k_a d^2 k_b f_{a/A} f_{b/B} \frac{d\sigma^{ab \rightarrow cd}}{d\hat{t}} && \text{pQCD} \times \text{K-fac.} \\ &\times D(E_c - \Delta E_c(\mathbf{r}_T); c \rightarrow h) && \text{E-loss} + \text{Indep. Frag.} \end{aligned}$$

→ しかし問題は残っている (high  $p_T$  での  $v_2$  など)

→ RHICではストリング破砕は必要ないのか？

Hirano et al., PLB636('06)299 (afterburner improves  $v_2$  in Hydro+Jet)

Sahu et al. Pramana 67 ('07)257 (cascade → low  $p_T$  data except for  $v_2$ )

Parton Cascade (Kinder-Geiger) (Parton cluster → hadrons)

# Hadronization Mechanism at RHIC

- **High  $p_T$ : Indep. Frag. of Jet Partons** (E.g. Hirano-Nara)
  - Explains  $p_T$  spectrum when E-loss is included.
  - ✗ Elliptic Flow  $v_2$  is small at high  $p_T$  ← This Talk
- **Medium  $p_T$ : Recombination** (E.g. Duke-Osaka-Nagoya)
  - Explains Baryon Puzzle and Quark Number Scaling of  $v_2$
  - ✗ Hard sphere density profile is implicitly assumed
- **Low  $p_T$ : Equil. Fluid Hadronization** (E.g. Hirano-Gyulassy)
  - Explains  $p_T$  spec. and  $v_2$  at low  $p_T$
  - ✗ Results depends on the Freeze-Out Conditions

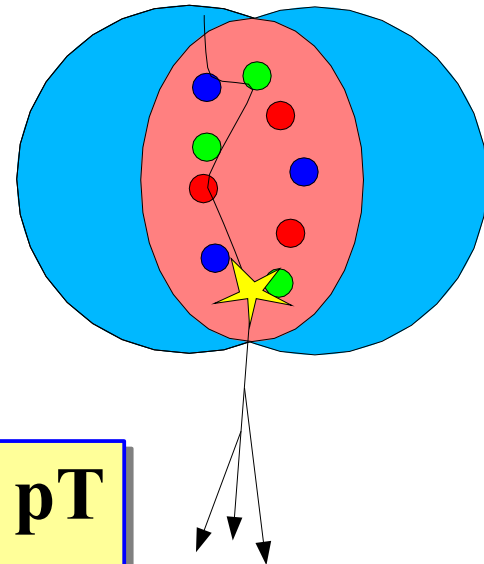
QGP Signals are understood separately,  
and they are not necessarily consistent.  
→ Further Ideas are required !

# How can we get large $v_2$ at high $p_T$ ?

- Quark Recombination → Combined Objects have larger  $v_2$

$$f(p, \varphi) = (1 + 2 v_2(p/2) \cos \varphi) \times (1 + 2 v_2(p/2) \cos \varphi) \\ \approx 1 + 2 \times 2 v_2(p/2) \cos \varphi$$

- Energy Loss in QGP generates  $v_2$ 
  - Large/Small suppression in  $y/x$  directions



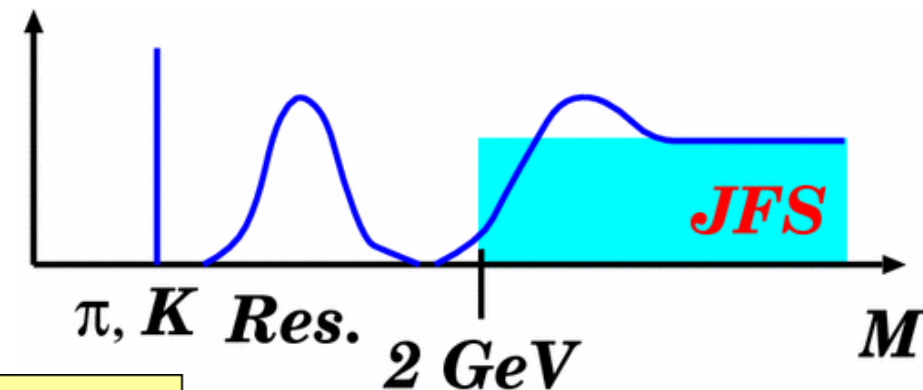
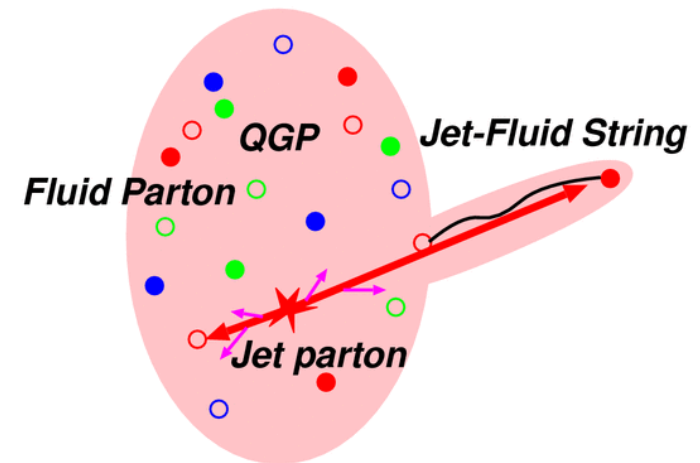
**Plausible Hadronization giving large  $v_2$  at high  $p_T$**

- Combination of several partons
- Large Energy Loss
  - Jet parton picks up Fluid parton and forms a string (Jet-Fluid String)

# Jet-Fluid String formation and decay: Model

Isse, Hirano, Mizukawa, AO, Yoshino, Nara, nucl-th/0702068

- ミニジェット生成=pQCD (PYTHIA 6.4)
- QGP中のパートン伝播
  - ◆ 3次元流体模型  
Hirano-Nara, PRL91('03), 082301;  
PRC69('04),034908  
Hirano,Tsuda, PRC66('02),054905
  - ◆ GLV エネルギー損失× factor (C)  
Gyulassy-Levai-Vitev, PRL85('00), 5535.
- スtring生成・破砕
  - ◆ "スペクトル"関数  $\Theta$  ( $\sqrt{s} - 2 \text{ GeV}$ )



$$D(j \rightarrow h) = \int d^3 k_f f_f(k_f, T, u_\mu, \mathbf{x}(\tau_f)) \\ \times S(s = (k_j + k_f)^2) D(\text{String}(\sqrt{s}, k_j, k_f) \rightarrow h)$$

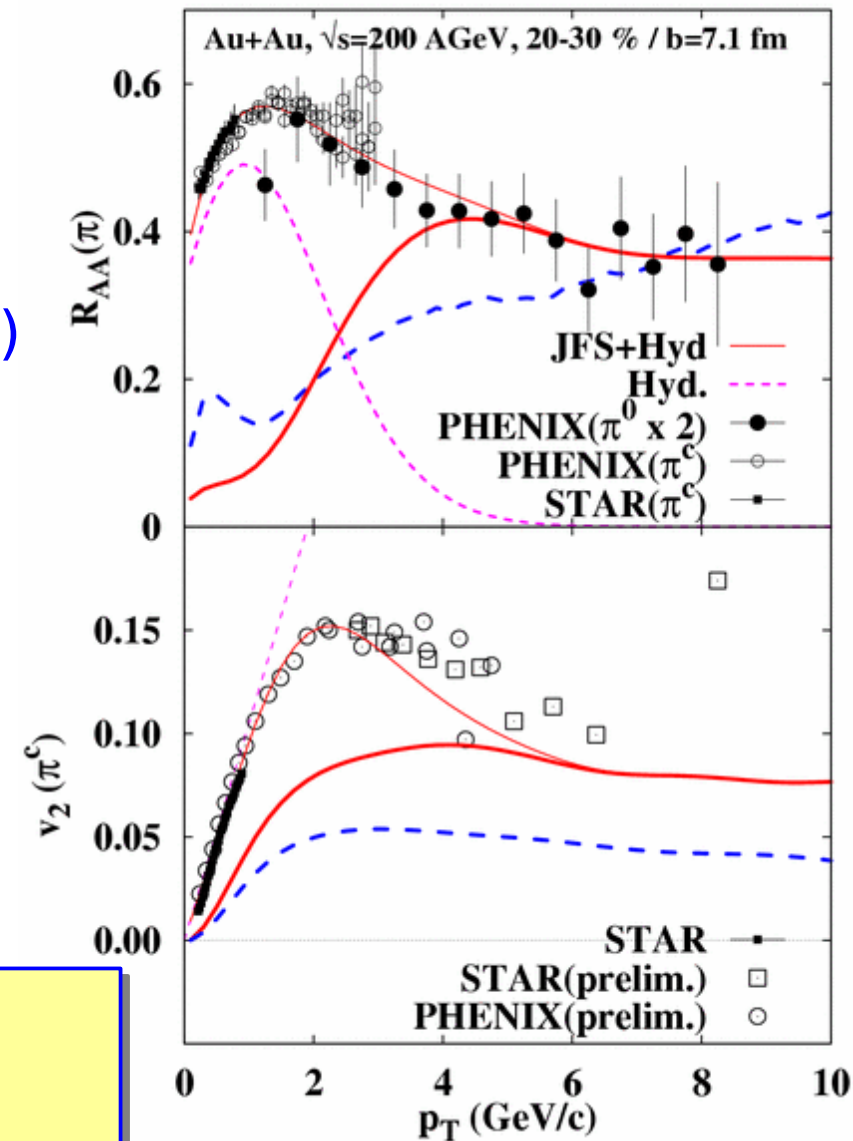


# Jet-Fluid String formation and decay: Results

Isse, Hirano, Mizukawa, AO, Yoshino, Nara, nucl-th/0702068

- 高い high  $p_T$  ハドロン生成率  
→ 大きなエネルギー損失が必要
  - ◆  $R_{AA}$  fit → E-loss fac.  $C = (6-8)$   
 $C = (2-3)$  in Hydro+Jet (Hirano-Nara)
- 大きなエネルギー損失  
+ 流体パートンの  $v_2$   
→ high  $p_T$  での大きな  $v_2$ 
  - ◆  $v_2 \sim 8\%$  @  $p_T > 6$  GeV/c  
 $v_2 \sim (3-5)\%$  in Indep. Frag.

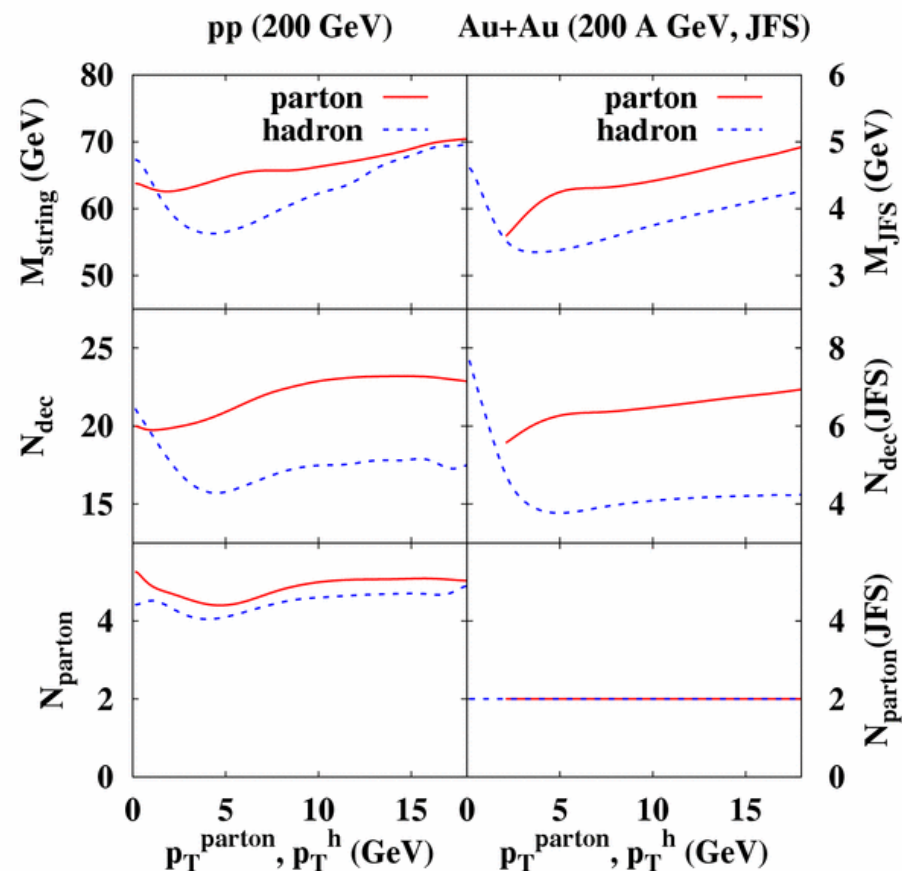
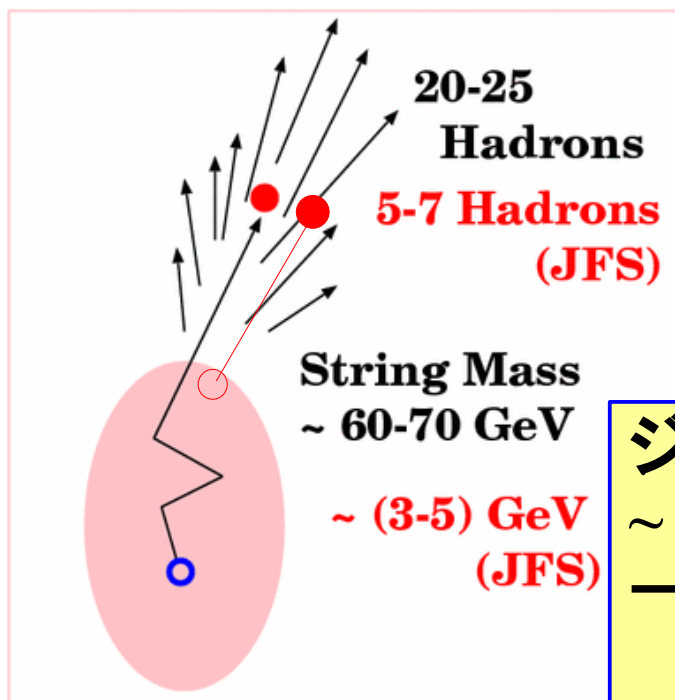
High  $p_T$  で RAA を説明する E-loss で  
 $v_2$  データをほぼ説明  
→ なぜ high  $p_T$  ハドロンが作られやすい？



# 独立破碎模型との比較

## 独立破碎模型(IF)

- ppでは IF ~ スtring 破碎  
→ 重いString (60-70 GeV) が  
多くのハドロン(20-25) に崩壊
- AAでの IF @  $p_T$  (After E-loss)  
~ pp でのString 破碎 @  $p_T$

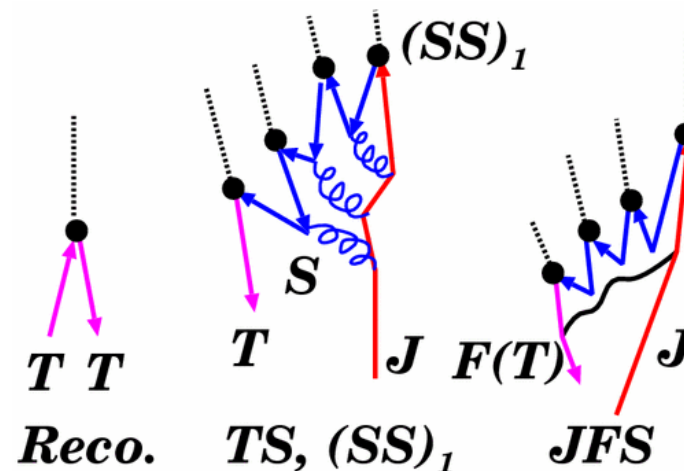


ジェットパートンの独立破碎  
~ 重いStringの崩壊  
→ AA衝突のジェット破碎段階で  
これほど重いStringが作られるか？

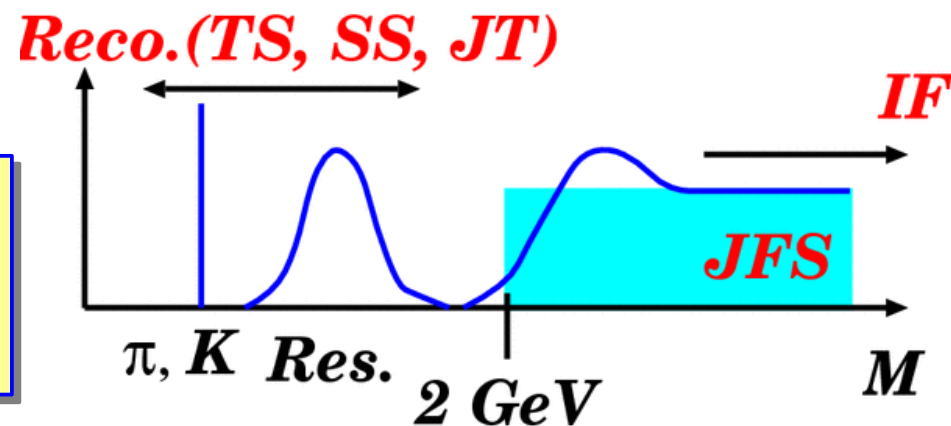
# High $p_T$ 領域での再結合模型との比較

- **TT(T)  $\rightarrow$  med. pT**  
Nonaka et al., PRC69('04),031902
- **JT  $\rightarrow$  med. pT (soft-hard)**  
Greco-Ko-Levai, PRC68('03),034904
- **TS  $\rightarrow$  med. pT,  $(SS)_1 \rightarrow$  high pT**  
Hwa-Yang, PRC70('04)024905
- **TT(T)  $\rightarrow$  Res.  $\rightarrow$  med./low pT**  
Greco-Ko, PRC70('04)024901

T: Thermal (Fluid) parton  
J: Jet parton  
S: Shower parton

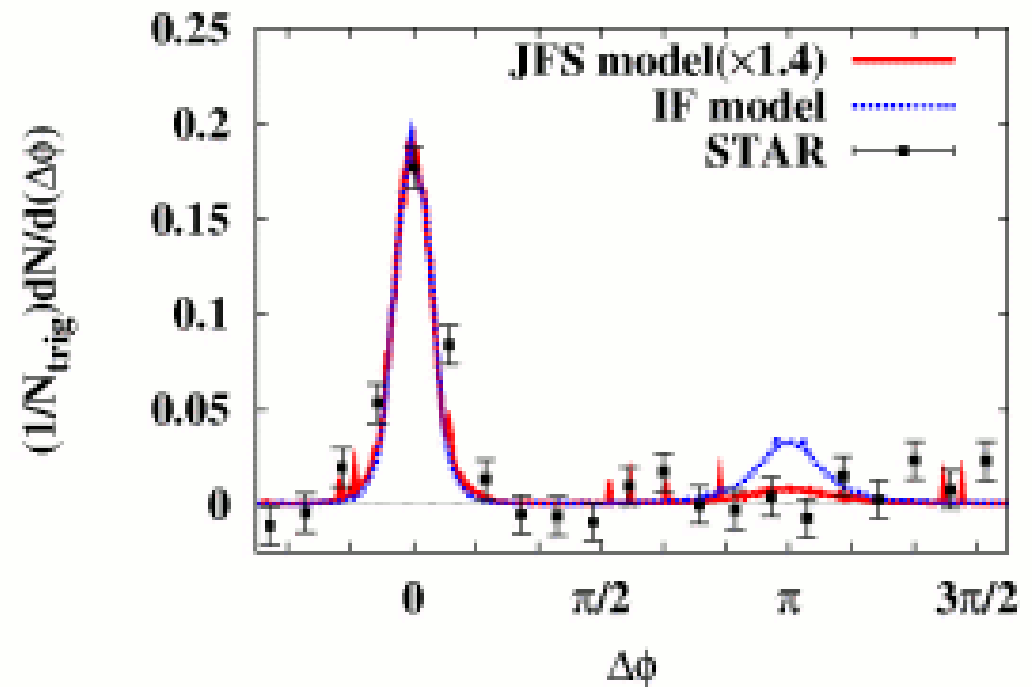


- T+”part of Jet”によるハドロン生成
- $\pi$  の直接生成より共鳴崩壊が有利  
 $\rightarrow$  JFSの描像と無矛盾

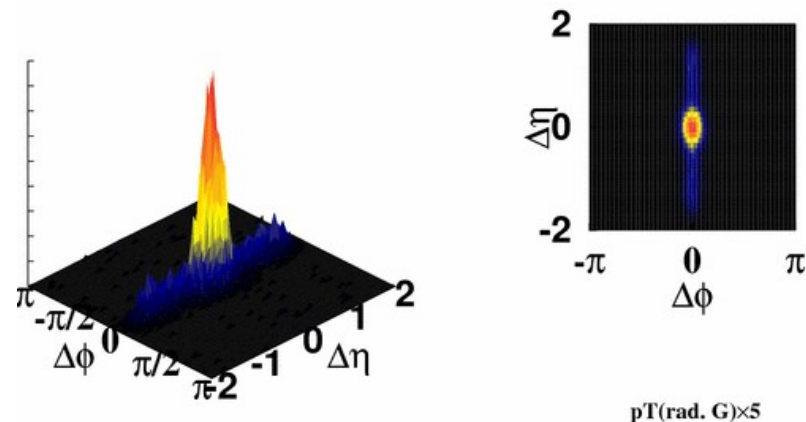
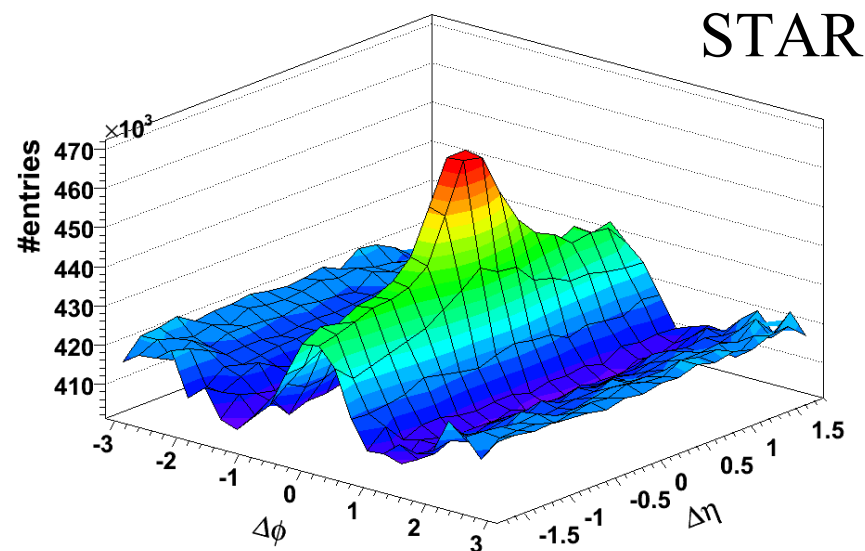


# ハドロンの方位角相関

- Au+Au衝突での後方相関の消失 = QGP生成の強いシグナル
  - ◆ Hydro + Jet model Hirano & Nara, PRL (2003)
    - $R_{AA}$  を説明する E-loss では後方相関の抑制が十分でない
    - ジェットの変化する散乱効果を取り入れると説明可能
  - ◆ Jet “absorption” model Drees, Feng, Jia, PRC71('05),034909
    - ジェットパートンが  $\exp(-\alpha L)$  の確率で「吸収」
    - エネルギー損失との関係は？
  - ◆ JFS
    - $R_{AA}$  を説明する E-loss により、後方相関は消失



- Jet-Ridge structure  
→ 横に狭く、縦に広い相関  
(narrow  $\Delta\phi$  + wide  $\Delta\eta$  corr.)
  - ◆ JFS では見えない
  - ◆ Chiu and Hwa: 早い時点で Jet により熱せられた広い  $\eta$  の範囲の流体 parton が coalesce
  - ◆ Shuryak: エネルギーを失った jet parton が流体中の「非平衡成分」として現れる
  - ◆ Wong: Jet のエネルギー損失時に放出された radiation が平衡に達する前の広い  $\eta$  分布を持つ bulk parton に大きな  $p_T$  を与えてハドロン化  
→ radiation の  $p_T$  を拡大すると、弱いが ridge 構造を出す



水川

# *Lessons from RHIC physics (until now...)*

- 分かったこと → jet parton が局所熱平衡に達した bulk quark-gluon matter 中をエネルギーを失いながら伝播し、parton が再結合してハドロンを作る。
  - ◆ jet parton は媒質中でエネルギーを失ってからハドロン化
  - ◆ bulk 部分が完全流体によりうまく記述できる (SPS energy までは、流体モデルは「アイデア」を与えるが、定量的には粒子シミュレーションに勝てなかった。)
  - ◆ 中間 pT 領域で baryon が meson より多く、 $v_2$  も大きい。
- 分かりつつあること → QCD 物性
  - ◆ jet は流体への feed back や流体 parton との再結合により、より低い pT ハドロンの分布にも影響を与える。
- よく分かっていないこと
  - ◆ 初期条件、速い熱平衡化、jet の失われたエネルギーの行き先、jet 起源で十分な量の baryon を作る方法、viscous hydro の輸送係数.....

# Summary

- **Heavy-ion collisions up to SPS** energies seems to be reasonably described by using **hadron-string cascade** such as JAM model, while **HIC at RHIC** requires **earlier thermalization** (larger anisotropic pressure) even in lighter nuclear collisions such as Cu+Cu collisions.
- There are many things to do in high-energy heavy-ion collision physics.
  - ◆ **AGS-FAIR-SPS energies**  
Nuclear matter EOS, Baryon rich QGP, Strangeness enh., ...
  - ◆ **RHIC-LHC energies**  
Detailed studies of QGP properties have just started  
→ Consistent understandings are not yet achieved,  
and we still have many puzzles

---

# *Backups*



---

*Jet-Fluid String Formation  
and Decay at RHIC*

Hirano, Isse, Mizukawa, Nara, AO, Yoshino, in preparation

# Jet-Fluid String Formation and Decay

*Jet production*: pQCD(LO)  $\times$  K-factor (PYTHIA6.3, K=1.8,  $pp$  fit)

$$\sigma_{jet} = K \sigma_{jet}^{pQCD(LO)}$$

*Jet propagation in QGP*

3D Hydro + Simplified GLV 1st order formula  $\times C$

(Hirano-Nara, NPA743('04)305, Hirano-Tsuda, PRC 66('02)054905. Web version!

Gylassy-Levai-Vitev, PRL85('00)5535)

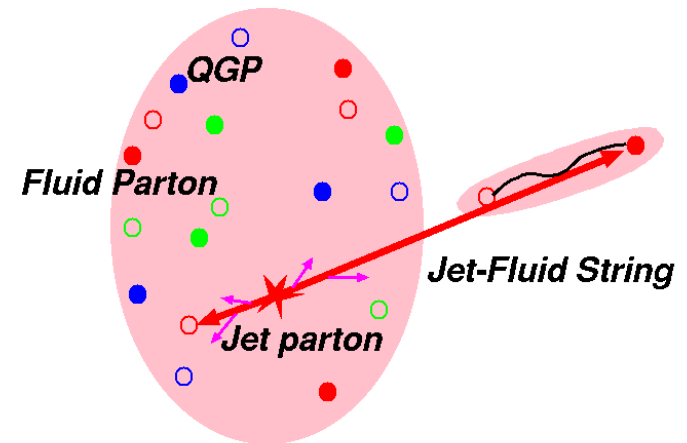
$$\Delta E = C \times 9 \pi \frac{\alpha_s^3}{4} C_R \int d\tau (\tau - \tau_0) \rho_{\text{eff}} \log\left(\frac{2E_0}{\mu^2 L}\right)$$

*Jet-Fluid String formation*

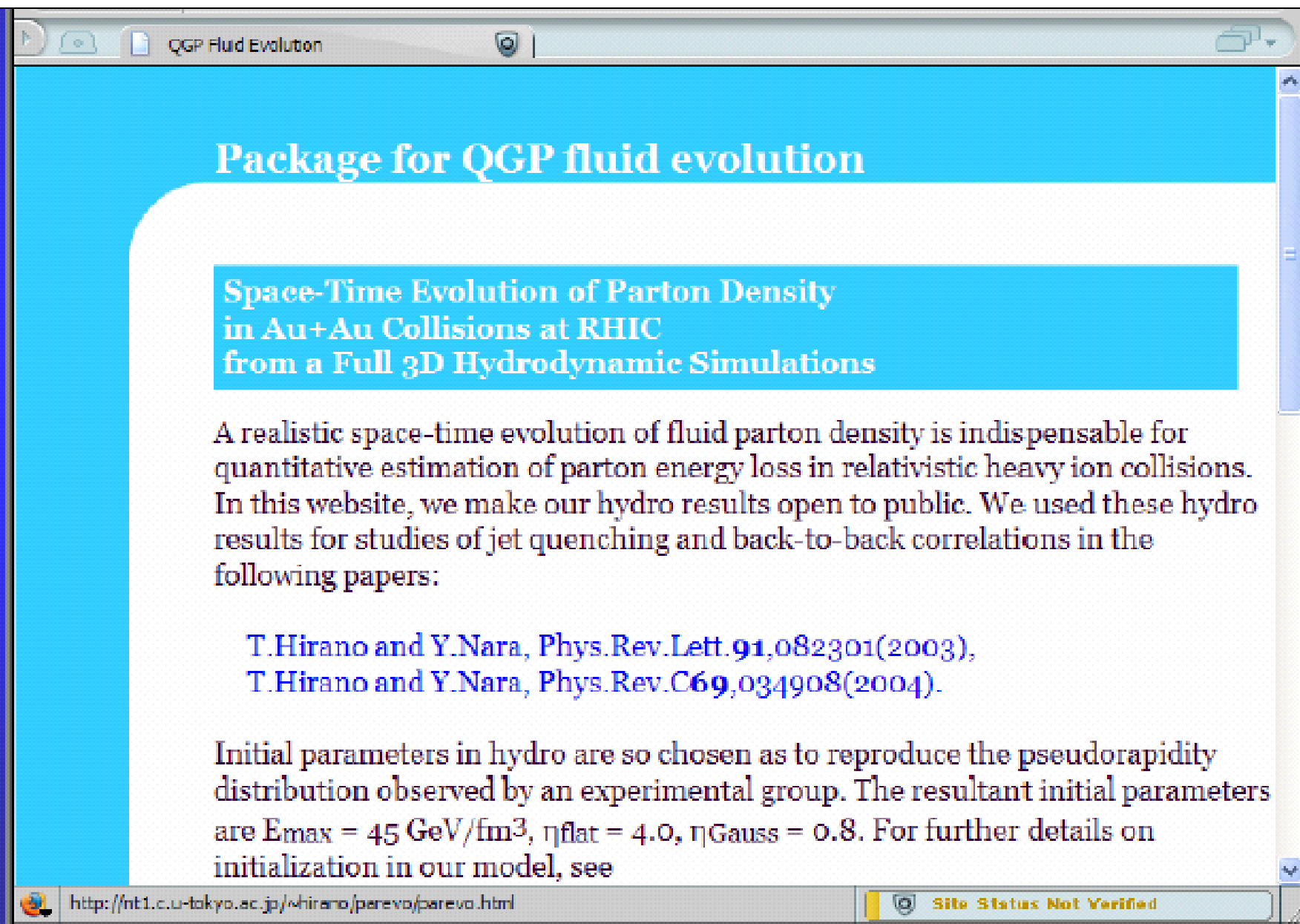
**Fluid parton breaks color flux,  
according to string spectral func.**

$$P(\sqrt{s}) \propto \Theta(\sqrt{s} - \sqrt{s_0}) \quad (\sqrt{s_0} = 2 \text{ GeV})$$

**Only g and light q (qbar) are considered.**



<http://nt1.c.u-tokyo.ac.jp/~hirano/parevo/parevo.html>



**QGP Fluid Evolution**

## Package for QGP fluid evolution

### Space-Time Evolution of Parton Density in Au+Au Collisions at RHIC from a Full 3D Hydrodynamic Simulations

A realistic space-time evolution of fluid parton density is indispensable for quantitative estimation of parton energy loss in relativistic heavy ion collisions. In this website, we make our hydro results open to public. We used these hydro results for studies of jet quenching and back-to-back correlations in the following papers:

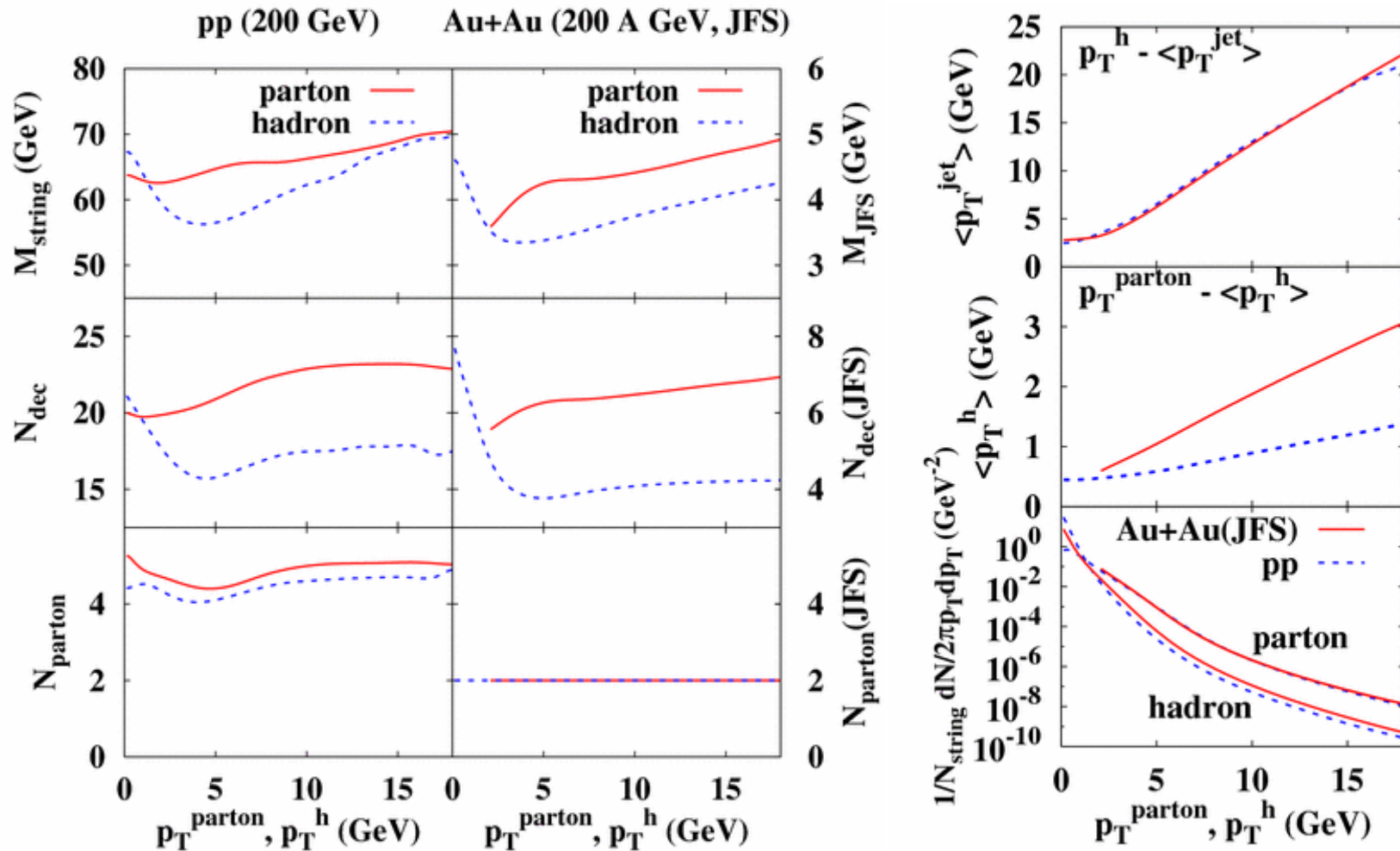
T.Hirano and Y.Nara, *Phys.Rev.Lett.* **91**,082301(2003),  
T.Hirano and Y.Nara, *Phys.Rev.* **C69**,034908(2004).

Initial parameters in hydro are so chosen as to reproduce the pseudorapidity distribution observed by an experimental group. The resultant initial parameters are  $E_{\text{max}} = 45 \text{ GeV}/\text{fm}^3$ ,  $\eta_{\text{flat}} = 4.0$ ,  $\eta_{\text{Gauss}} = 0.8$ . For further details on initialization in our model, see

<http://nt1.c.u-tokyo.ac.jp/~hirano/parevo/parevo.html> Site Status Not Verified

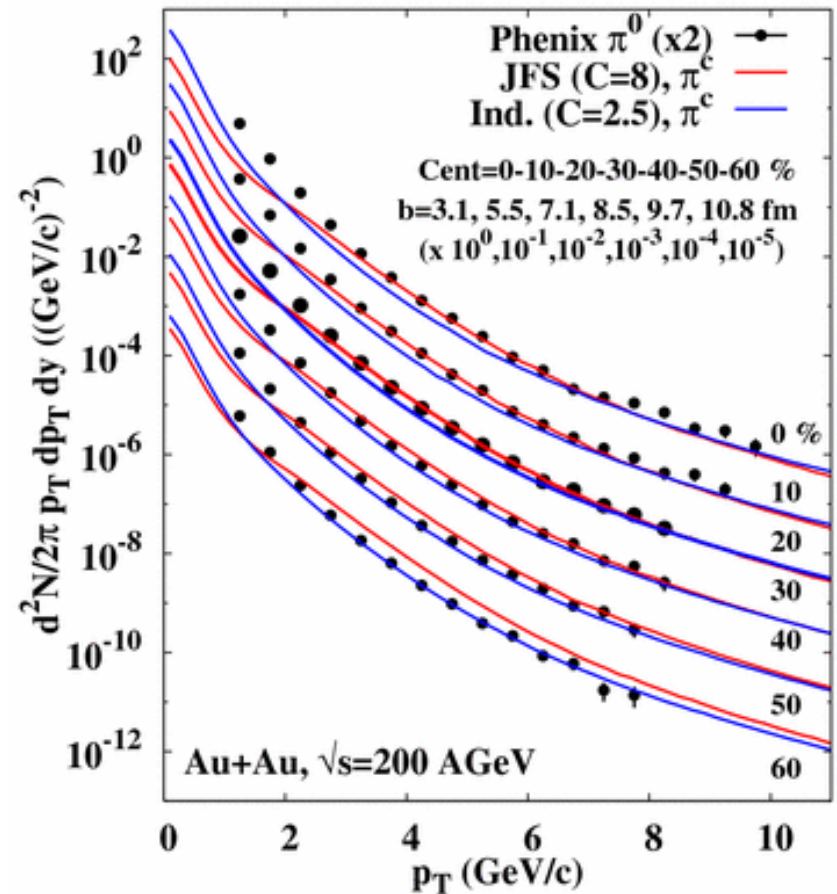
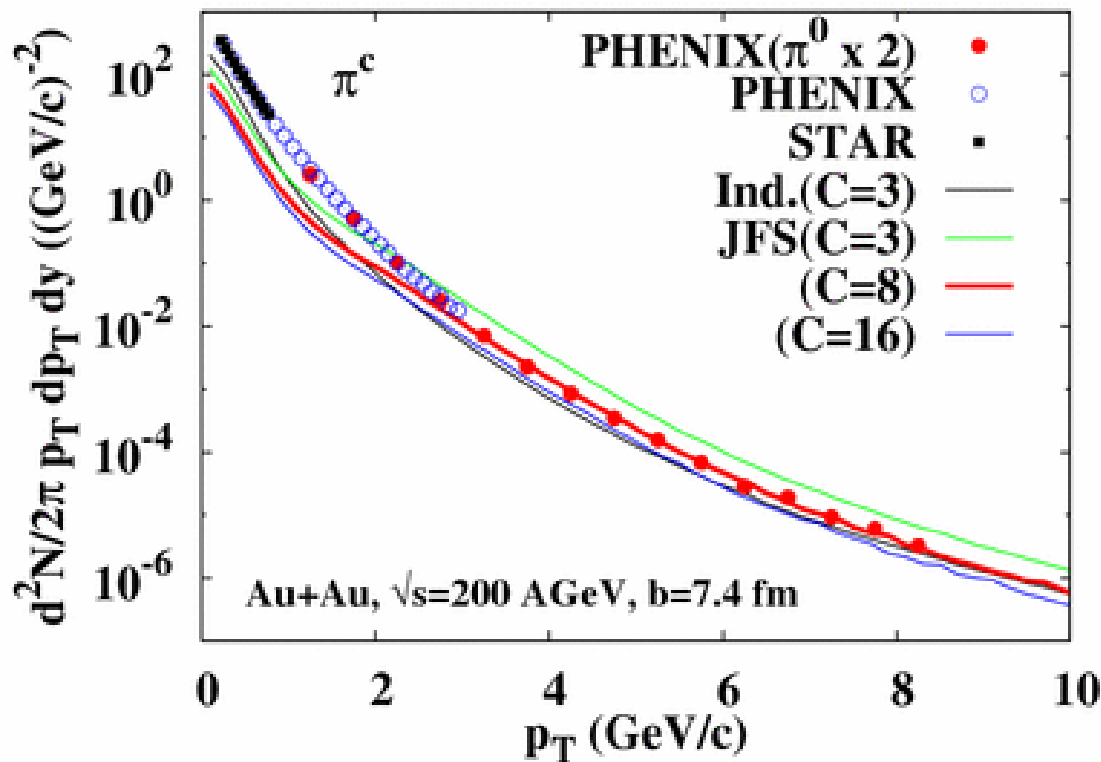
# String Mass and $p_T$ in JFS

- Compared to pp collisions (and thus to Ind. Frag.), JFS has much smaller mass and decays into fewer hadrons.
  - high  $p_T$  hadrons are enhanced → Larger E-loss is required



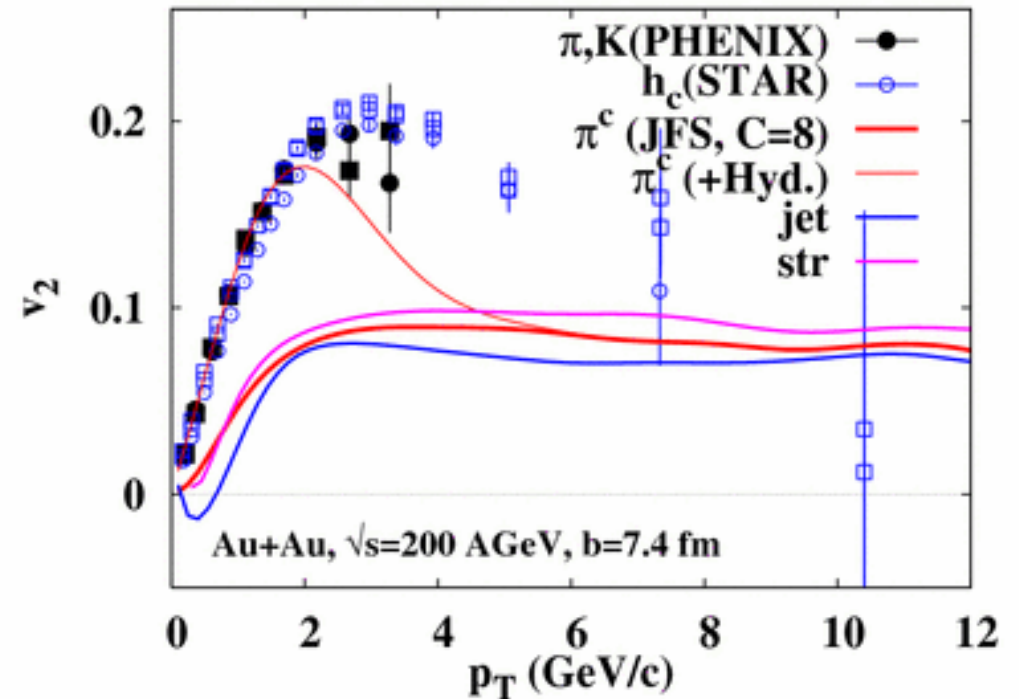
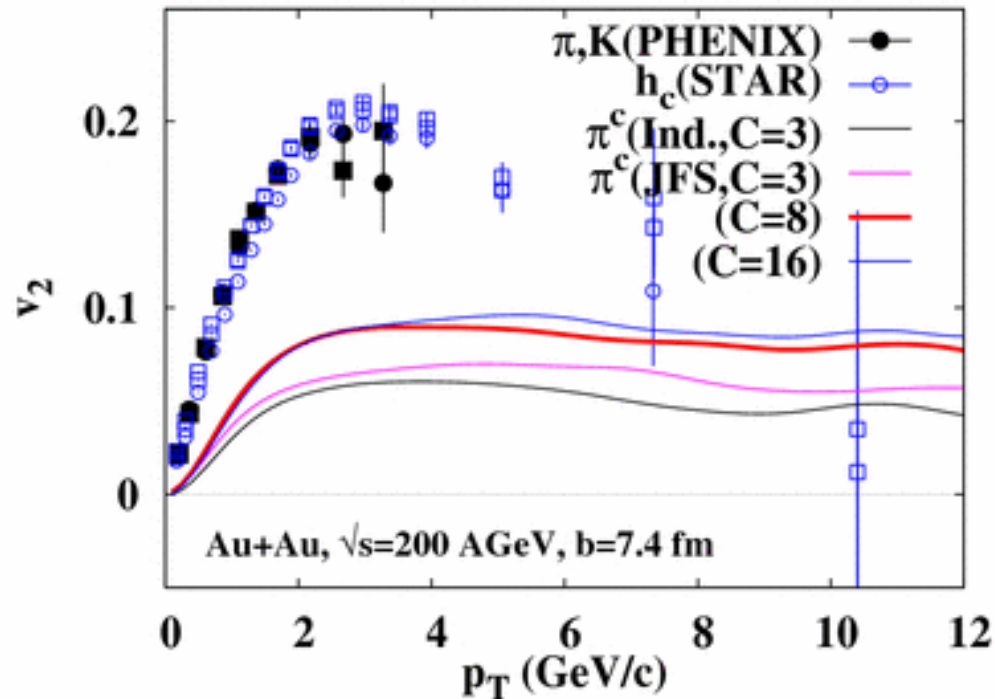
# Energy Loss Factor $C$ : $p_T$ Spectrum Fit

- For the same  $C \rightarrow dN_{JFS}(\text{high } p_T) > dN_{Ind}(\text{high } p_T)$
- $p_T$  spec. fit  $\rightarrow$  Ind. Frag.:  $C \approx (2.5-3)$ , JFS:  $C \approx 8$   
 $\rightarrow$  *Large Energy Loss is necessary / allowed in JFS*



# Elliptic Flow: $p_T$ Deps.

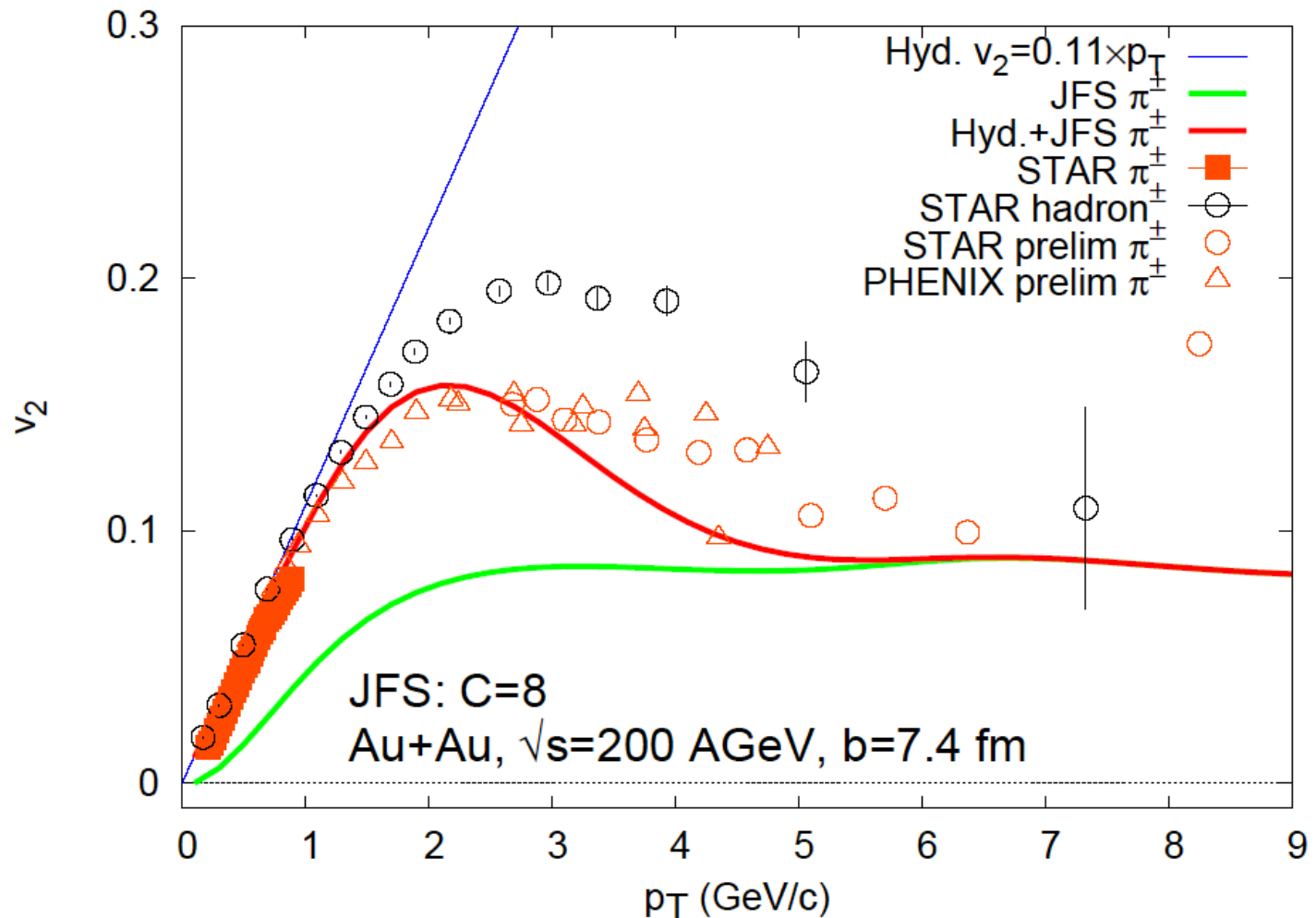
- High  $p_T$   $v_2$ :  $\sim 5\%$  in Ind. ( $C=3$ )  $\leftrightarrow$   $\sim 8\%$  in JFS ( $C=8$ )



Origin of Large  $v_2$  = Large E-loss factor  $C$  + Fluid parton  $v_2$

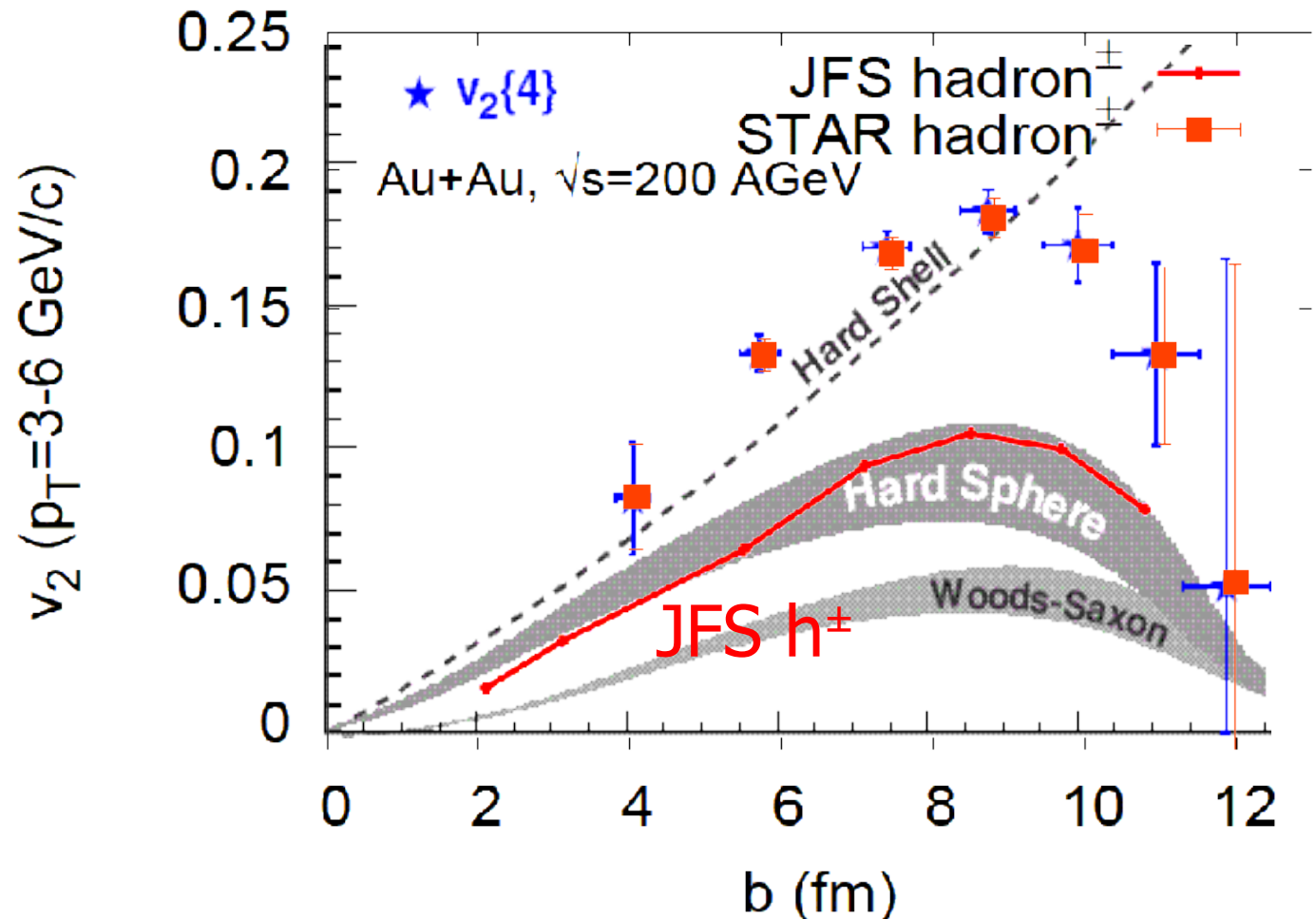
# Elliptic Flow of pions

- Observed pion  $v_2$  at  $p_T > 5 \text{ GeV}/c \sim 10 \%$   
 $\leftrightarrow v_2(\text{JFS}, p_T > 5 \text{ GeV}/c) \sim 8 \%$



# Impact Parameter Dependence

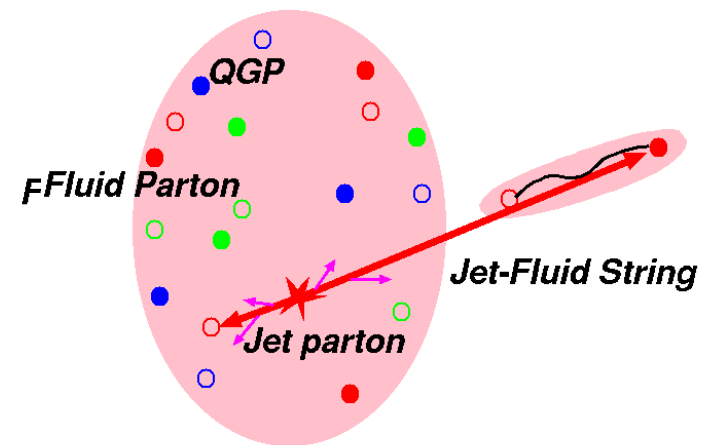
- Mid- $p_T$   $v_2$  ( $3 < p_T < 6$  GeV/c) in JFS is larger than the “Strong E-loss Limit” with Woods-Saxon profile in Independent Fragmentation, but still smaller than Data.





# JFS Summary

- **Jet-Fluid String (JFS) formation and decay** is proposed as a mechanism to produce high  $p_T$  hadrons.
  - Effective to produce high  $p_T$  hadrons
  - Event-by-Event Energy-Mom. conservation  $\leftrightarrow$  Ind. Frag.
  - Simple and small mass strings decaying into a few hadrons  $\leftrightarrow$  Ind. Frag.
- When we FIT  $p_T$  spectrum, **large  $v_2$  emerges at high  $p_T$** 
  - Large E-loss+fluid parton  $v_2$
- Problems and Homeworks
  - Mechanism of large E-loss
  - d+Au fit  $\rightarrow$  Cronin Effects
  - s-quarks, string spectral func.



---

# *Backups*

# *Comparison with Previous Works*

---

- J. Casalderrey-Solana, E.V. Shuryak, hep-ph/0305160
  - Quarks, diquarks and gluons in QGP cut color flux ( $\sim$  JFS).
  - Large E-loss is generated by “phaleron”
  - *Large E-loss leads “surface emission”  $\rightarrow$  large  $v_2$*
- Recombination (Duke-Osaka-(Minnesota)-Nagoya)
  - Predicts large  $v_2$  ( $\sim 10\%$ ) at high-pT
    - Sharply edged density dist.  $\rightarrow$  E-loss  $\propto L \rightarrow v_2 \approx 10\%$
    - Woods-Saxon density dist.  $\rightarrow v_2 \approx 5\%$
  - Entropy problem:  $S(\text{QGP}) \approx S(\text{H})$  requires Res. and Strings
  - *Spectral Func.:  $\delta$  func.  $\leftrightarrow$   $\theta$  func. in JFS*

# *K-factor*

- **K-factor** → absolute value of  $\sigma_{\text{jet}}$
- **Experimental Data:**  $pp \rightarrow \pi^0$  @  $\sqrt{s_{\text{NN}}} = 200 \text{ GeV}$  (PHENIX)

$$\frac{1}{\sigma^{\text{exp}}} \frac{d^2 \sigma^{\text{exp}}}{2\pi p_T d p_T dy} = K \frac{\sigma^{\text{pQCD}(1st)}}{\sigma^{\text{exp}}} \frac{d^2 N^{\text{pQCD}(1st)}}{2\pi p_T d p_T dy} \quad A = K \frac{\sigma^{\text{pQCD}(1st)}}{\sigma^{\text{exp}}}$$

$\sigma^{\text{Exp.}} = 21.8 \text{ mb}$  (trigger)

$\sigma^{\text{pQCD}(1st)} = 9.9 \text{ mb}$

- **pythia6.3 fit:**

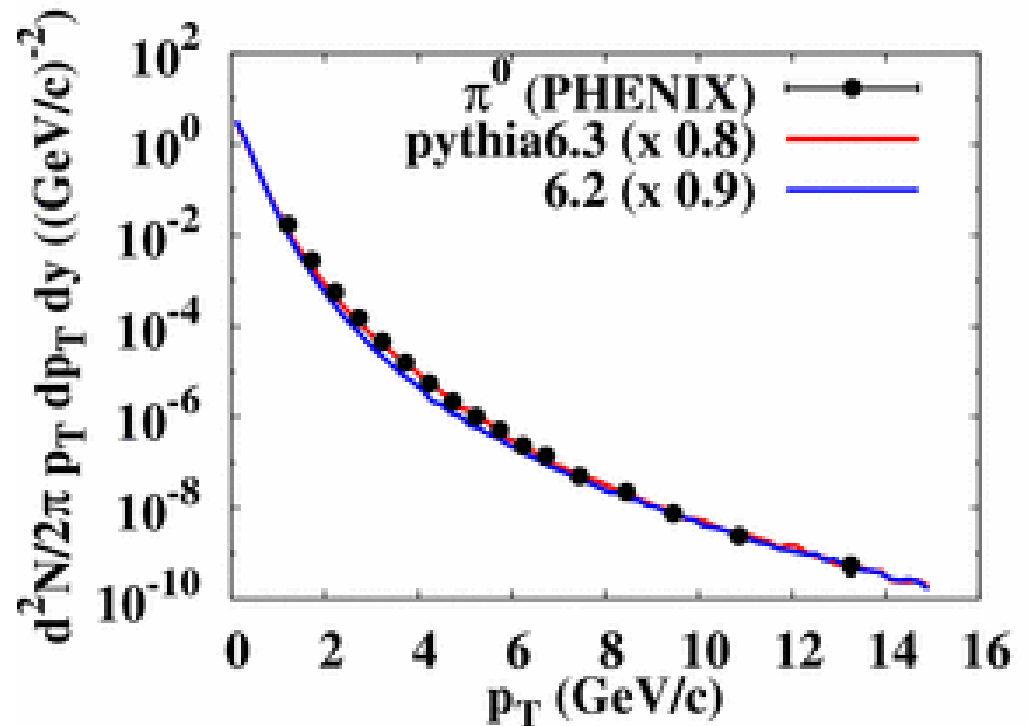
$A \approx 0.8 \rightarrow K = 1.8$

$(\sigma_{\text{jet}} (p_T^{\text{hard}} > 2 \text{ GeV}/c) \approx 17.5 \text{ mb})$

- **pythia6.2 fit:**

$A \approx 0.9 \rightarrow K = 2.0$

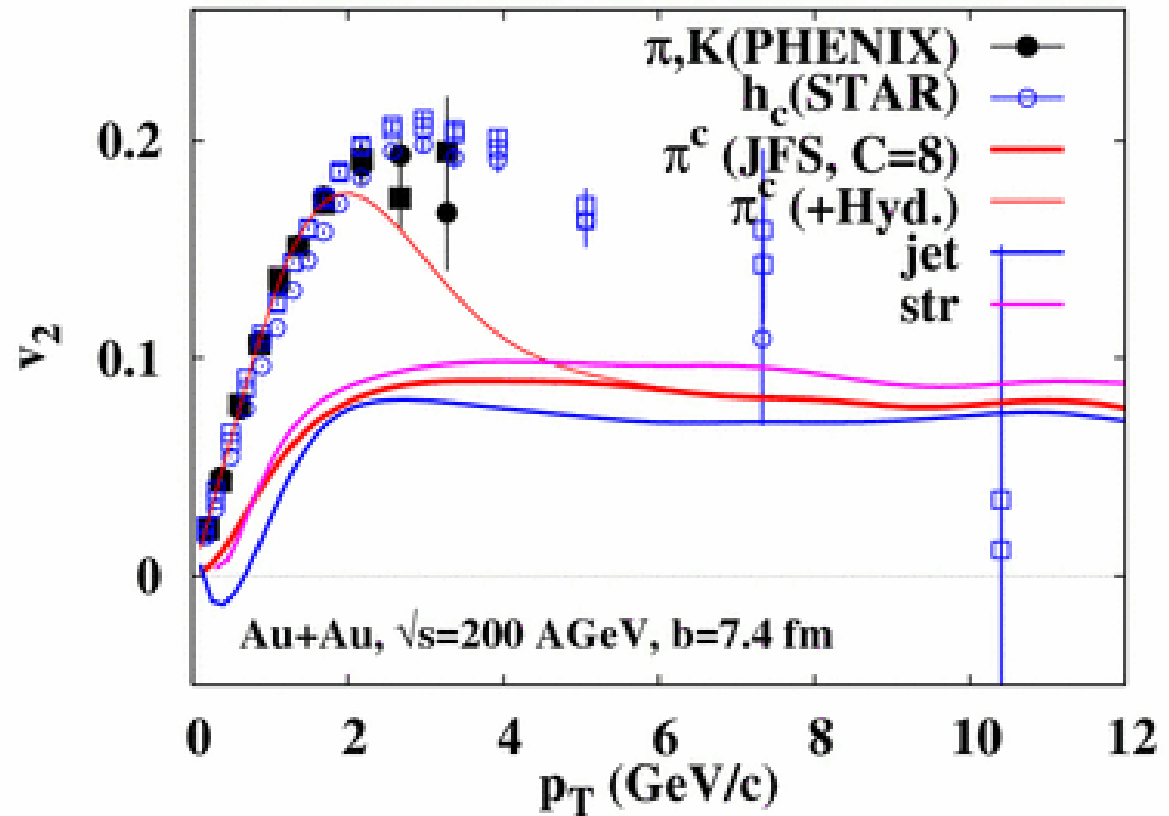
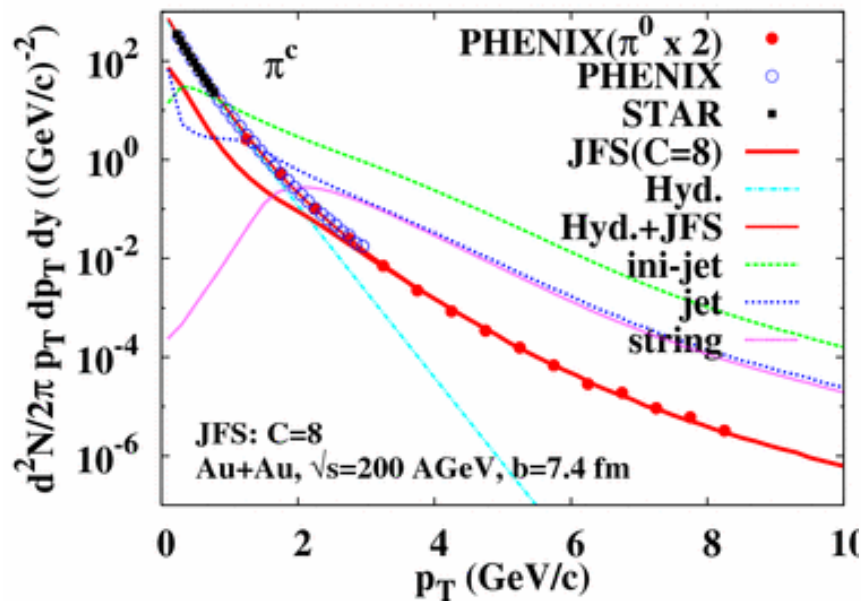
$(\sigma_{\text{jet}} \approx 19.6 \text{ mb})$



# Combined with Low $p_T$ spectrum

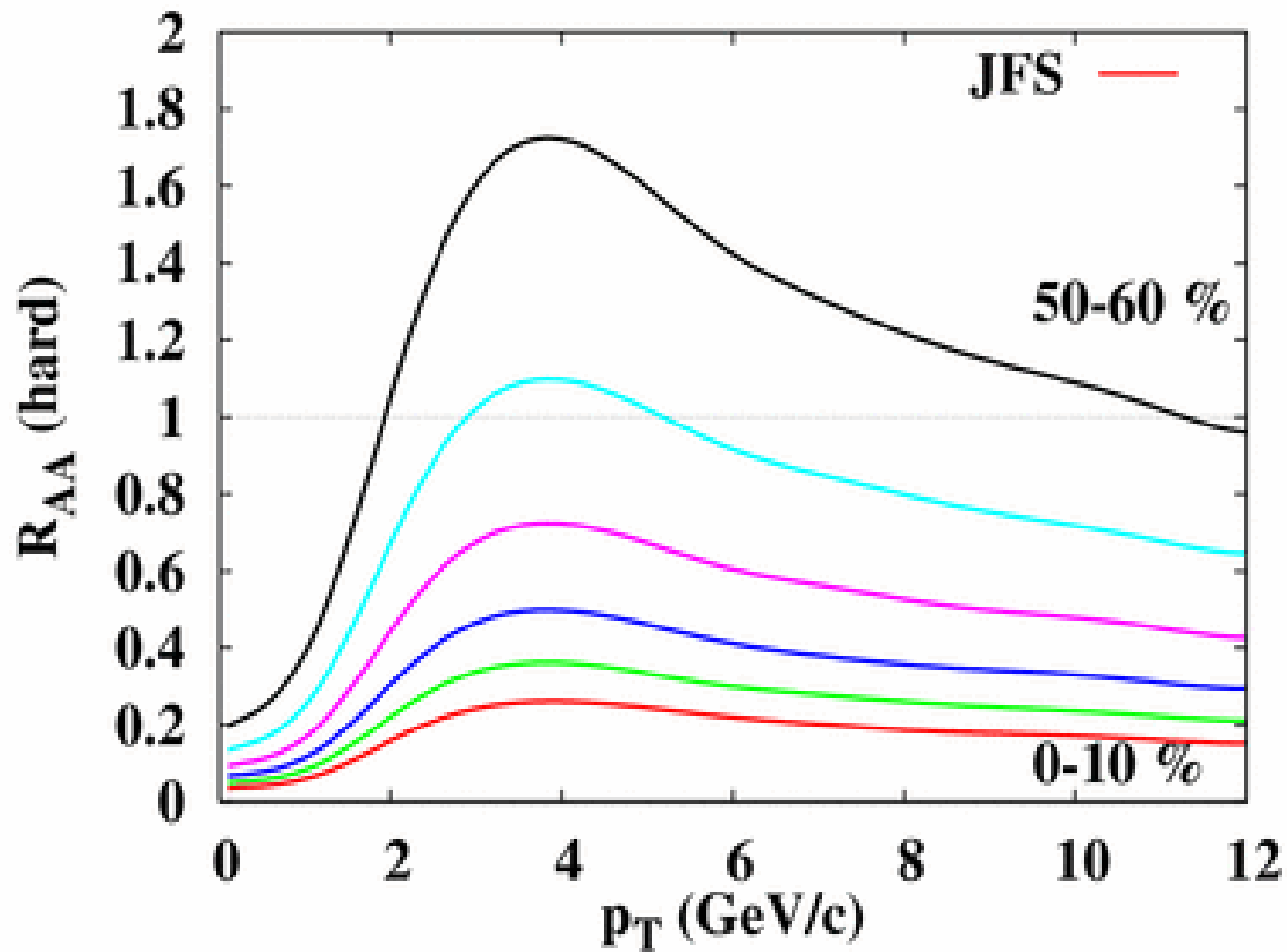
- Low  $p_T$  spectrum is assumed and combined.

$$E \frac{d^3 N_{Hyd}}{dp^3}(p_T) = A \exp(-p_T/T) (1 + B / (1 + (p_T/p_0)^8)) \quad v_2^{Hyd}(p_T) = 0.14 p_T$$



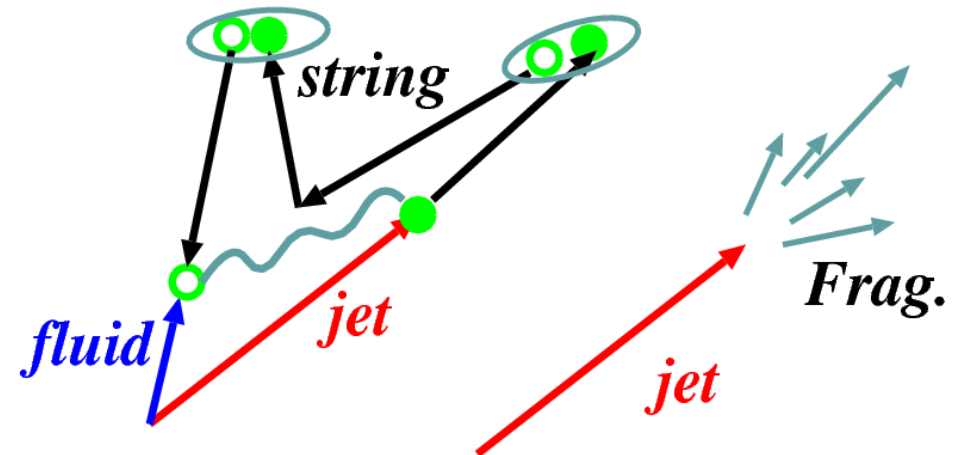
# *Nuclear Modification Factor*

## ■ $p_T$ Deps.



# Discussion

- Mechanism to produce high  $p_T$  hadrons in JFS
  - String Decay from Lorenz boosted fluid
  - Relative momentum is relatively small
    - Smaller number of hadrons with high  $p_T$  are formed
- ↔ Independent Frag. (Large no. of Low  $p_T$  hadrons)



# Energy Loss Factor

- Additional Factor for Energy Loss → High  $p_T$  hadron yield
- Exp. Data:  $p_T$  spectra of  $\pi$  in Au+Au (PHENIX, STAR)

$$\frac{d^2 N^{Exp.}}{2\pi p_T d p_T dy} = N_{jet} \frac{1}{N_{jet}} \frac{d^2 N^{JFS}(C)}{2\pi p_T d p_T dy}$$

→ Determining  $N_{jet}$  is important !

$N_{coll} = 373$  @  $b=7.4$  fm (PHENIX estimate)

$\sigma_{jet}^{NN} = 17.5$  mb (pp fit pythia 6.3),  $\sigma_{tot}^{NN} = 47.4$  mb (JAM)

$$N_{jet} = \sigma_{jet}^{NN} \int d^2r T_A(r_T + b/2) T_B(r_T - b/2) = \frac{\sigma_{jet}^{NN}}{\sigma_{tot}^{NN}} N_{coll}$$

$$T_A(r_T) = \int dz \rho(r_T, z)$$



# *Further Problems*

---

- Very large energy loss is required to explain  $p_T$  spectrum.
  - $C \approx 8$  in JFS  $\leftrightarrow C \approx 2.7$  in Hydro+Jet model (Hirano-Nara)

Is it possible to justify this large energy loss ?

- Elliptic flow at medium  $p_T$  is underestimated.  
→ Fluid-Fluid String would be necessary to consider.
- Large baryon yield at medium  $p_T$  may not be explained.  
→ Three parton string ? (Jet-Fluid-Fluid, Fluid-Fluid-Fluid)
- String formation probability should be evaluated  
in pQCD matrix element + string level density.
- Strange hadrons