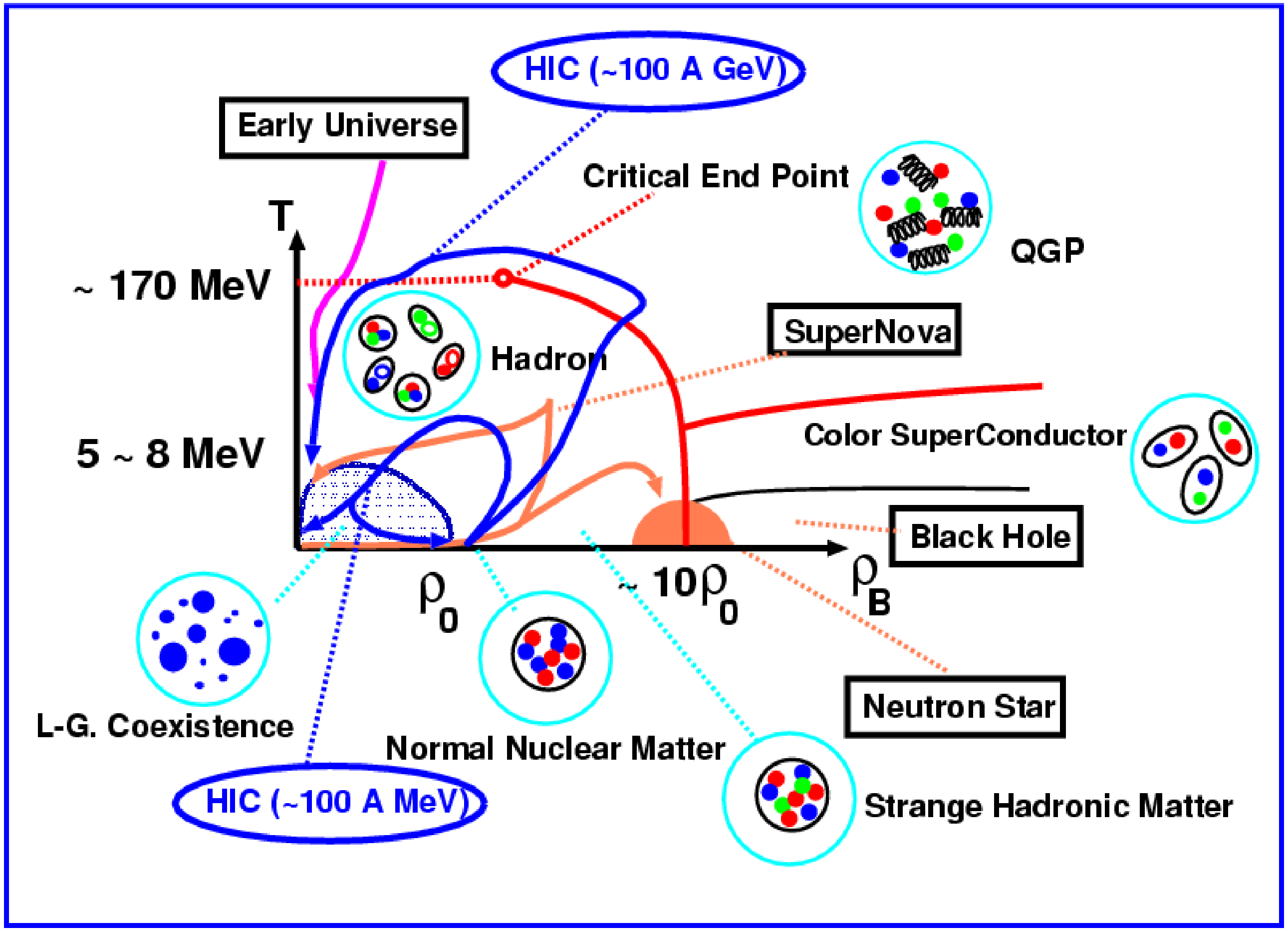
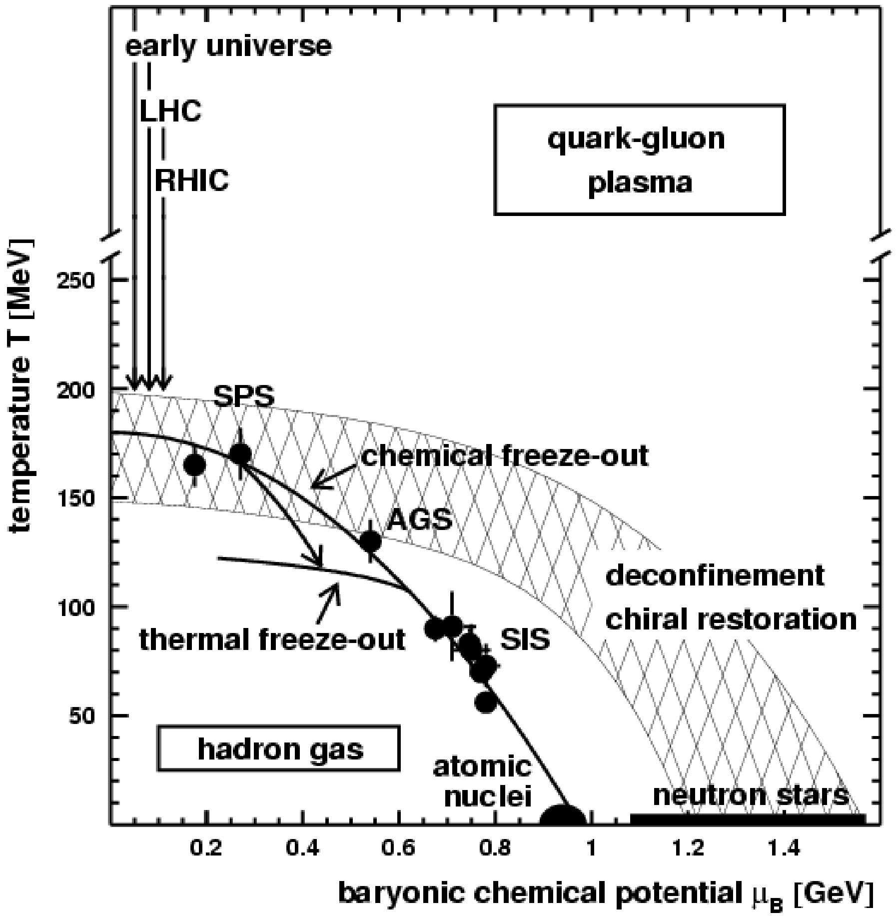
Basic Ingredients in Transport Models of Heavy-Ion Collisions

Hadronic Matter Phase Diagram

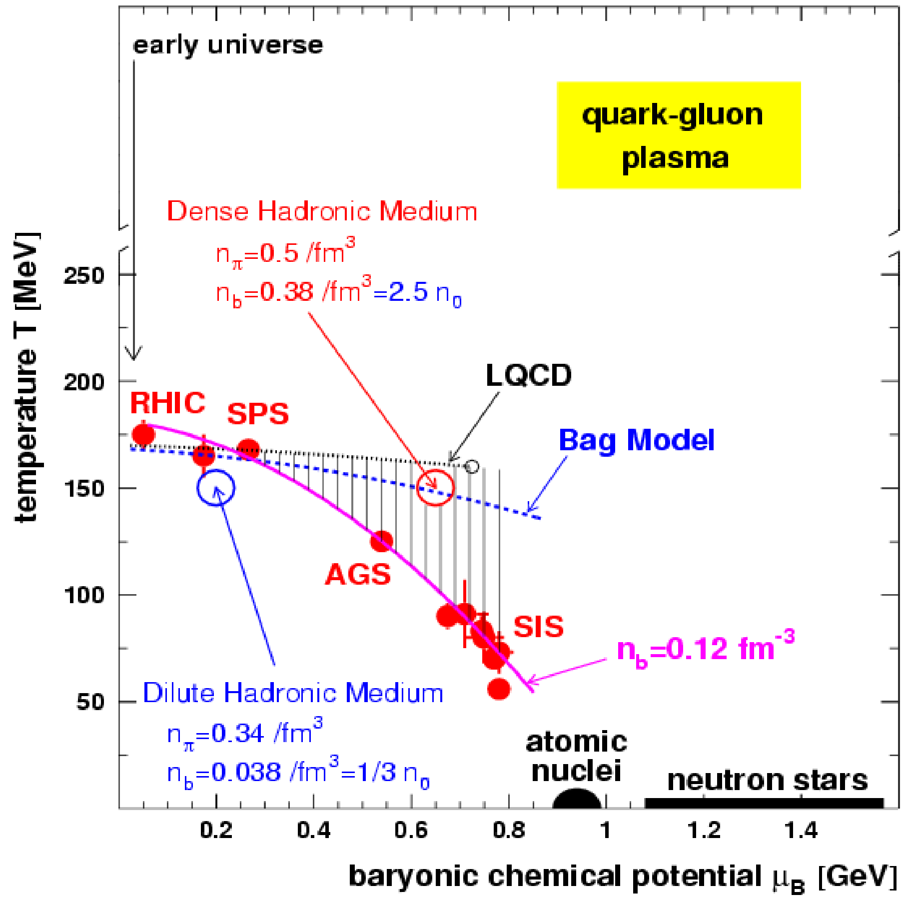


HIC (~ A few 100 A MeV) = Little Supernova
HIC (100+100 A GeV) = Little Big Bang

Experimentally Estimated Phase Diagram



J. Stachel et al., 1998

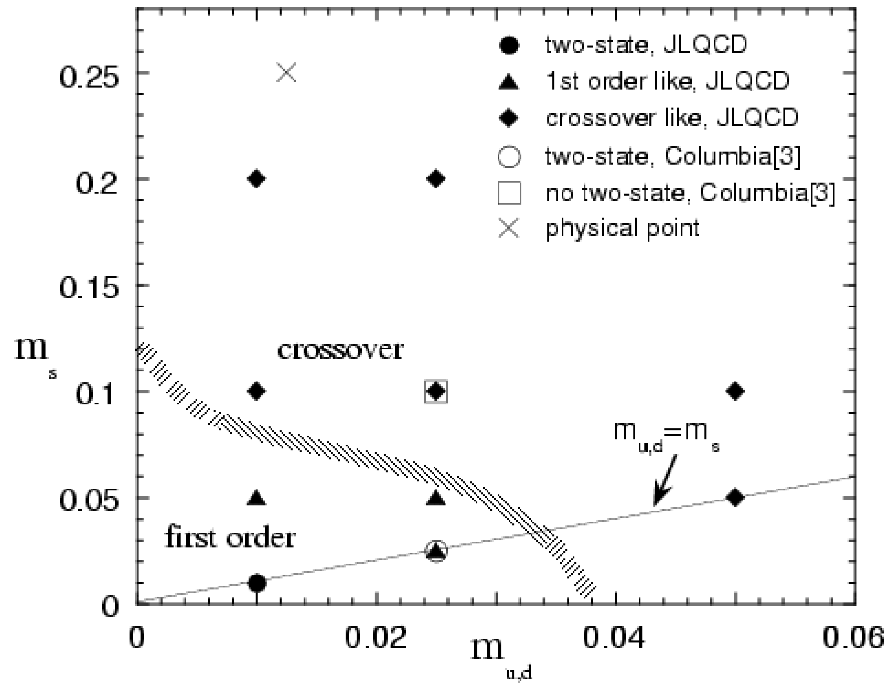


Braun-Munzinger et al., 2002

Chem. Freeze-Out Points are very Close to Expected QCD Phase Transition Boundary

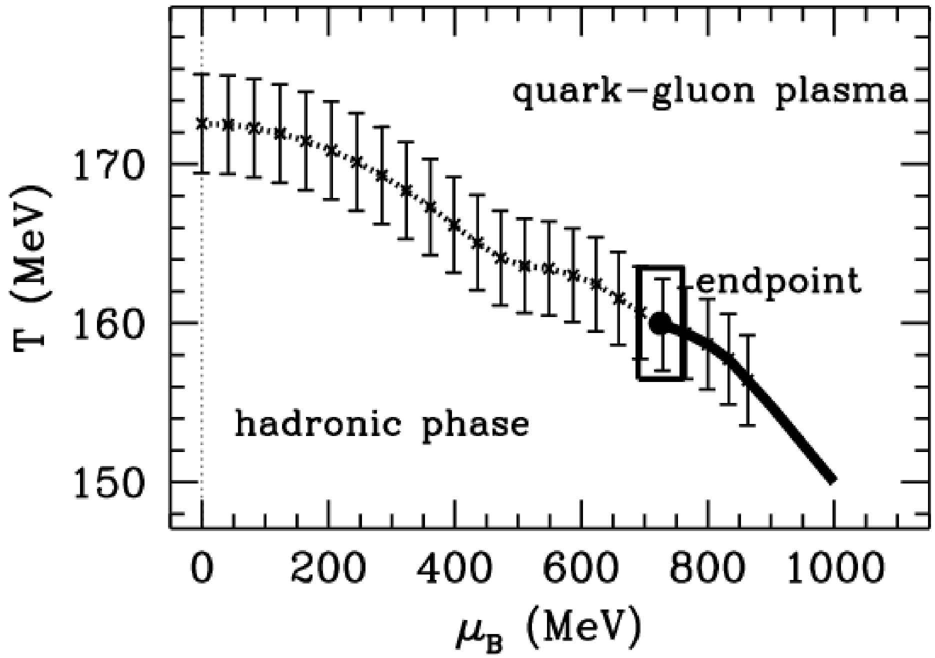
Theoretically Expected QCD Phase Diagram

Zero Chem. Pot.



JLQCD Collab. (S. Aoki et al.),
Nucl. Phys. Proc. Suppl. 73 (1999) 459.

Finite Chem. Pot.



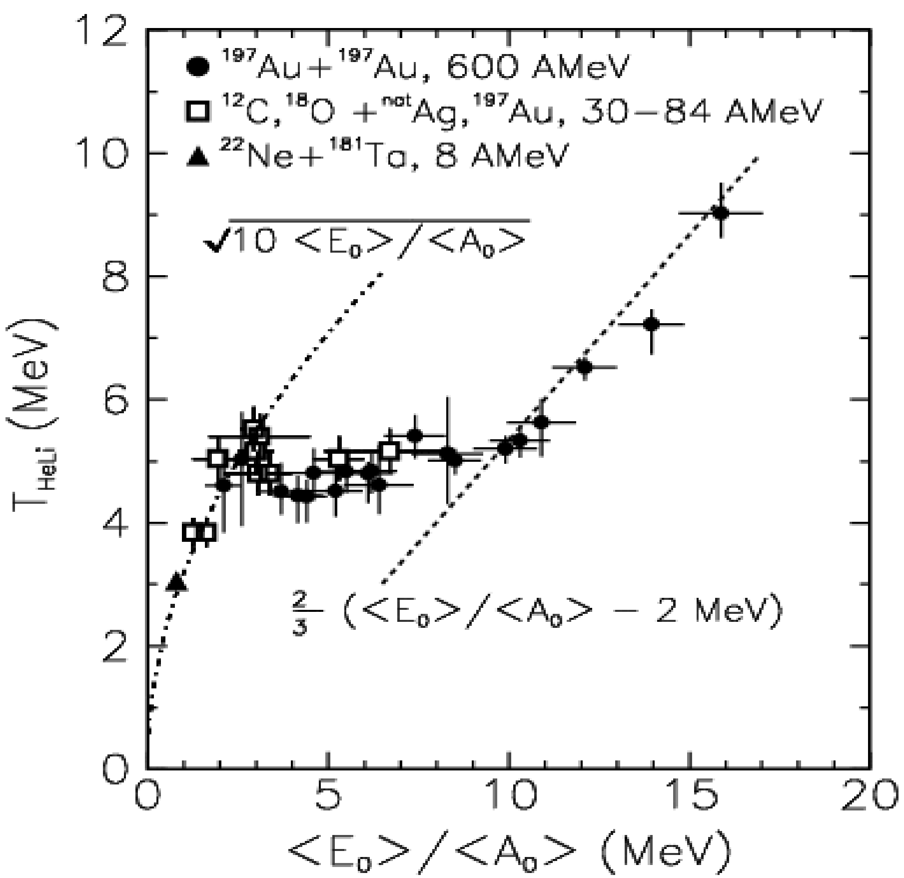
Finite μ: Fodor & Katz,
JHEP 0203 (2002), 014.

Zero Chem. Pot. : *Cross Over*
Finite Chem. Pot.: *Critical End Point*

Nuclear Caloric Curve

J. Pochadzalla et al (GSI-ALLADIN collab.) ., PRL 75 (1995) 1040.

Boiling Temperature is Clearly Seen



Fragment Yields are assumed
to follow Equilibrium Statistics

$$Y_f \propto g_f \exp((B_f + Z\mu_p + N\mu_n)/T)$$

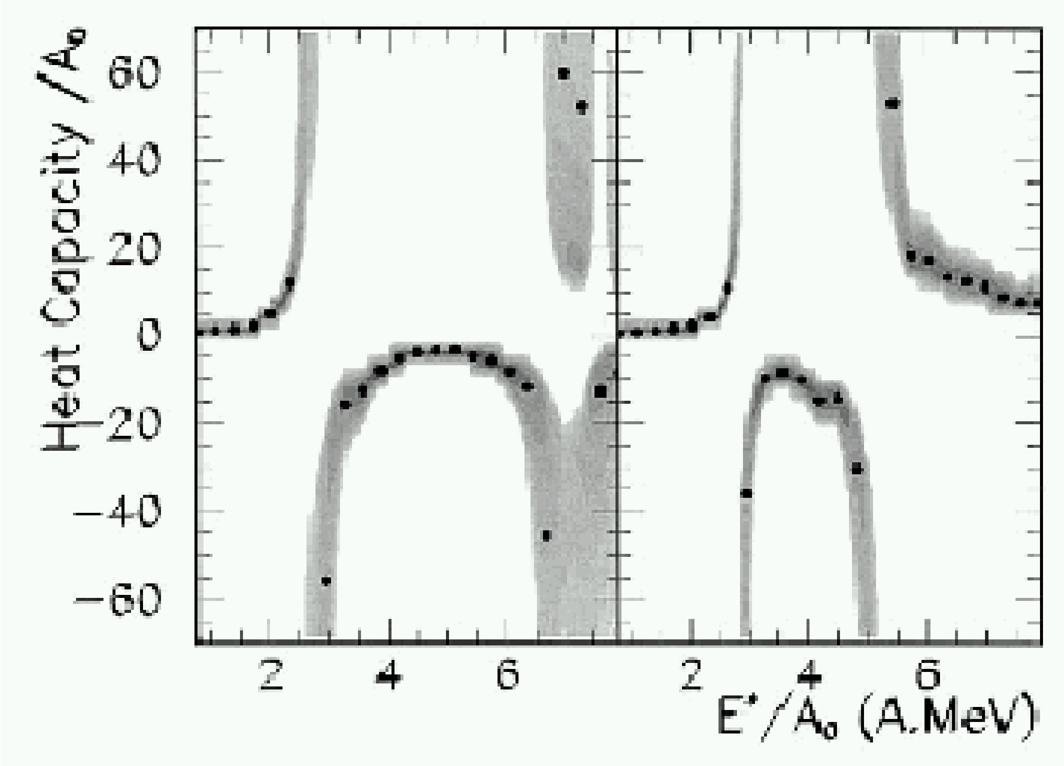
$$\rightarrow \frac{Y(^4He)/Y(^3He)}{Y(^7Li)/Y(^6Li)} \propto \exp(\Delta B/T)$$

Negative Heat Capacity

M. D Agostino et al.,(MSU Exp./INFN-IN2P3 Collab.) PLB 473 (2000) 219.

- **Negative Heat Capacity**
  → Evidence of the First Order Phase Transition

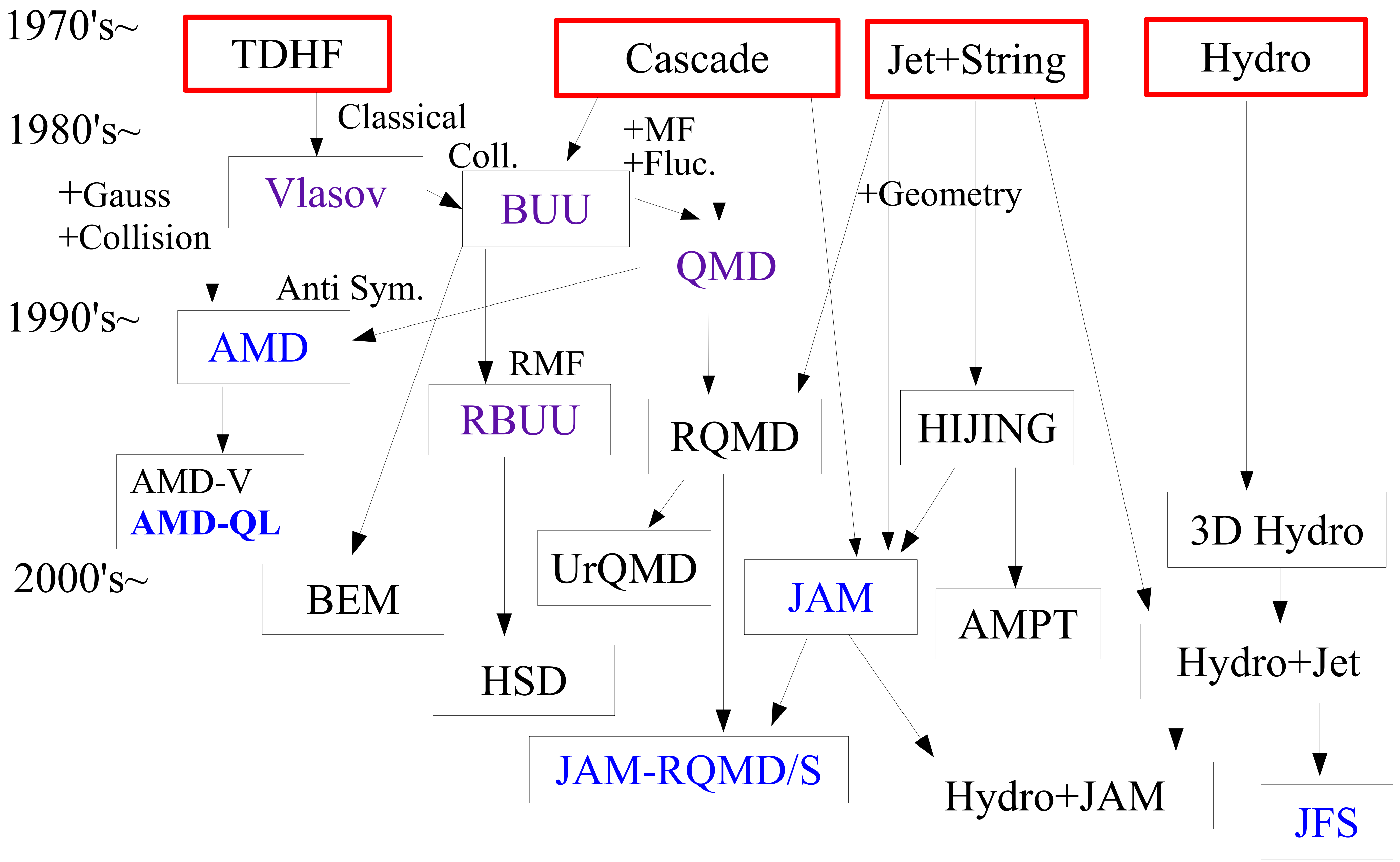
- ***T* and *E** are determined**
  from Fragment Multiplicity and Kinetic Energy
  based on Theoretical Model



HIC Transport Models: Major Four Origins

- *Nuclear Mean Field Dynamics*
  - **Basic Element of Low Energy Nuclear Physics, and Critically Determines High Density EOS / Collective Flows**
  - **TDHF → Vlasov → BUU**
- *NN two-body (residual) interaction*
  - **Main Source of Particle Production**
  - **Intranuclear Cascade Models**
- *Partonic Interaction and String Decay*
  - **Main Source of high pT Particles at Collider Energies**
  - **JETSET + (previous) PYTHIA (Lund model) → (new) PYTHIA**
- *Relativistic Hydrodynamics*
  - **Most Successful Picture at RHIC**

HIC Models: History



Nuclear Mean Field Models for Heavy-Ion Collisions

TDHF and Vlasov Equation

- **Time-Dependent Mean Field Theory (e.g., TDHF)** $i\hbar \frac{\partial \phi_i}{\partial t} = h \phi_i$
- **Density Matrix**

$$\rho(r, r') = \sum_i^{Occ} \phi_i(r) \phi_i^*(r') \quad \rightarrow \quad \rho_W = f \text{ (phase space density)}$$

- **TDHF for Density Matrix**

$$i\hbar \frac{\partial \rho}{\partial t} = [h, \rho] \quad \rightarrow \quad \frac{\partial f}{\partial t} = \{h_W, f\}_{P.B.} + O(\hbar^2)$$

- **Wigner Transformation and Wigner-Kirkwood Expansion**
  (Ref.: Ring-Schuck)

$$O_W(r, p) \equiv \int d^3 s \exp(-i p \cdot s / \hbar) < r + s/2 | O | r - s/2 >$$

$$(AB)_W = A_W \exp(i\hbar \Lambda) B_W \quad \Lambda \equiv \nabla'_r \cdot \nabla_p - \nabla'_p \cdot \nabla_r \quad (\nabla' \text{ acts on the left})$$

$$[A, B]_W = 2\mathrm{i}\, A_W \sin(\hbar \Lambda / 2) B_W = i\hbar \{A_W, B_W\}_{P.B.} + O(\hbar^3)$$

Test Particle Method

- **Vlasov Equation**

$$\frac{\partial f}{\partial t} - \{h_W, f\}_{P.B.} = \frac{\partial f}{\partial t} + v \cdot \nabla_r f - \nabla U \cdot \nabla_p f = 0$$

- **Classical Hamiltonian**

$$h_W(r, p) = \frac{p^2}{2\mathrm{m}} + U(r, p)$$

- **Test Particle Method (C. Y. Wong, 1982)**

$$f(r, p) = \frac{1}{N_0} \sum_i^{AN_0} \delta(r - r_i)\,\delta(p - p_i) \quad \rightarrow \quad \frac{dr_i}{dt} = \nabla_p h_w, \quad \frac{dp_i}{dt} = -\nabla_r h_w,$$

Mean Field Evolution can be simulated by Classical Test Particles
→ Opened a possibility to Simulate High Energy HIC including Two-Body Collisions in Cascade

BUU (Boltzmann-Uehling-Uhlenbeck) Equation

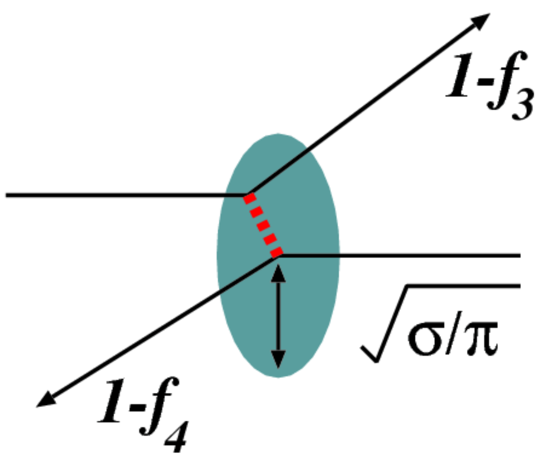
- **BUU Equation** **(Bertsch and Das Gupta, Phys. Rept. 160(88), 190)**

$$\frac{\partial f}{\partial t} + v \cdot \nabla_r f - \nabla U \cdot \nabla_p f = I_{coll}[f]$$

$$I_{coll}[f] = -\frac{1}{2} \int \frac{d^3 p_2 d\Omega}{(2\pi\hbar)^3} v_{12} \frac{d\sigma}{d\Omega}$$

$$\times \quad [f f_2 (1-f_3)(1-f_4) - f_3 f_4 (1-f)(1-f_2)]$$

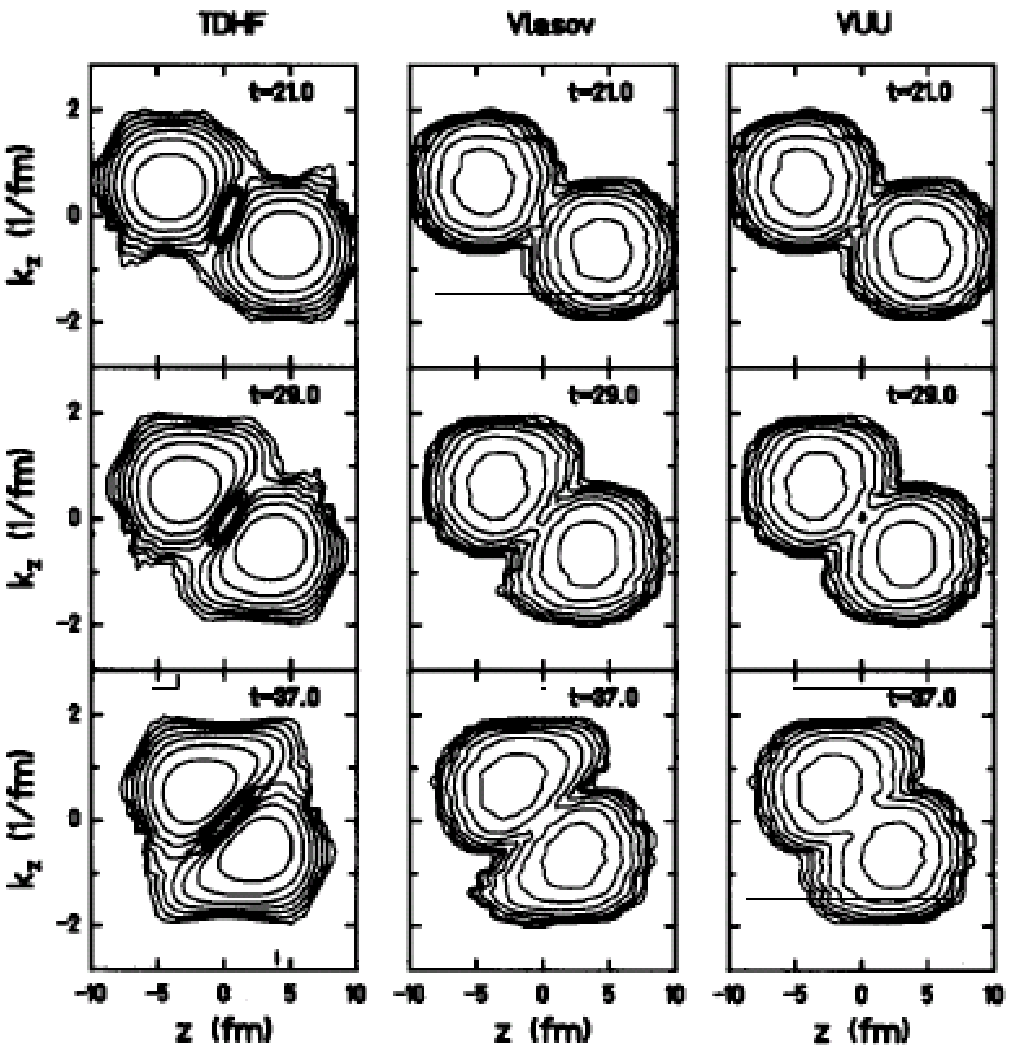
- **Incorporated Physics in BUU**
  - **Mean Field Evolution**
  - **(Incoherent) Two-Body Collisions**
  - **Pauli Blocking in Two-Body Collisions**



O One-Body Observables (Particle Spectra, Collective Flow, ..)
X Event-by-Event Fluctuation (Fragment, Intermittency, ...)

Comarison of TDHF, Vlasov and BUU(VUU)

- **Ca+Ca, 40 A MeV (Cassing-Metag-Mosel-Niita, Phys. Rep. 188 (1990) 363).**



Exercise (1)

- **Prove that the spatial integral of the Wigner function *f(x,p)* gives a momentum distribution of nucleons.**
- **Prove that the Wigner function with test particles satisfy the Vlasov equation when the test particle follows the classical EOM.**
- **Prove that the collision term becomes zero (i.e. gain and loss terms cancel) in equilibrium.**
- **Derive the collision term for bosons, which disappears in equilibrium.**
- ***(ADVANCED)* Prove the relation of the commutator and Poisson bracket. (It takes a long time ....)**
- ***(ADVANCED)* Prove that the Wigner function can be negative. (Therefore, the probability interpretation is not always possible.)**

Relativistic QMD/Simplified (RQMD/S)

- RQMD = Constraint Hamiltonian Dynamics
  (Sorge, Stocker, Greiner, Ann. of Phys. 192 (1989), 266.)
- Constraints: φ ≈ 0 (Satisfied on the realized trajectory, by Dirac)
  - Variables in Covariant Dynamics = 8N phase space: q_μ, p_μ
  - Variables in EOM = 6N phase space
    → We need 2N constraints to get EOM
- On Mass-Shell Constraints

$$H_i \equiv p_i^2 - m_i^2 - 2m_i V_i \approx 0$$

- Time-Fixation in RQMD/S

$$\chi_i \equiv \hat{a}\cdot(q_i - q_N) \approx 0 \; (i = 1, \sim N-1) \quad , \quad \chi_N \equiv \hat{a}\cdot q_N - \tau \approx 0$$

$$\hat{a} = \text{Time-like unit vector in the Calculation Frame}$$

(Tomoyuki Maruyama et al., Prog. Theor. Phys. 96(1996), 263.)

RQMD/S (cont.)

- **Hamiltonian is made of constraints**

$$H=\sum_i u_i \phi_i \quad (\phi_i = H_i (i=1\sim N), \chi_{i-N} (i=N+1\sim 2N))$$

- **Time Development** $\quad \frac{df}{d\tau}=\frac{\partial f}{\partial \tau}+\{f, H\} \quad , \quad \{q_\mu, p_\nu\}=g_{\mu\nu}$

- **Lagrange multipliers are determined to keep constraints**
  → ***We can solve obtain the multipliers analytically in RQMD/S***

$$\frac{d\phi_i}{d\tau}\approx 0 \;\rightarrow\; \delta_{i,2N}+\sum_j u_j \{\phi_i, \phi_j\}\approx 0$$

- **Equations of Motion**

$$H=\sum_i (p_i^2-m_i^2-2m_i V_i)/2p_i^0 \quad , \quad p_i^0=E_i=\sqrt{\vec{p}_i^{\,2}+m_i^2+2m_i V_i}$$

$$\frac{d\vec{r}_i}{d\tau}\approx -\frac{\partial H}{\partial \vec{p}_i}=\frac{\vec{p}}{p_i^0}+\sum_j \frac{m_j}{p_j^0}\frac{\partial V_j}{\partial \vec{p}_i} \quad , \quad \frac{d\vec{p}_i}{d\tau}\approx \frac{\partial H}{\partial \vec{r}_i}=-\sum_j \frac{m_j}{p_j^0}\frac{\partial V_j}{\partial \vec{r}_i}$$

We can include MF in an almost covariant way in molecular dynamics

Particle “DISTANCE”

$$r_{Tij}^{\ 2} \equiv r_\mu r^\mu - \left(r_\mu P_{ij}^\mu\right)^2 / P_{ij}^2 = \vec{r}^{\,2} \qquad (in\ CM)$$

$$P_{ij} \equiv p_i + p_j \quad , \quad r \equiv r_i - r_j$$

Particle “Momentum Difference”

$$p_{Tij}^{\ 2} \equiv p_\mu p^\mu - \left(p_\mu P_{ij}^\mu\right)^2 / P_{ij}^2 = \vec{p}^{\,2} \qquad (in\ CM)$$

$$p \equiv p_i - p_j$$

Lorentz Invariant, and Becomes Normal Distance in CM !

AMD (Antisymmetrized Molecular Dynamics)

Ono-Horiuchi-Maruyama-AO, 1992

- **Gaussian Approximation for single particle wave function**

$$|\Psi\rangle = A\prod|\psi_i\rangle \quad , \quad \psi_i = \phi(r;Z_i)\chi(\sigma,\tau) \quad , \quad Z = \sqrt{\nu}\,D + \frac{i}{2\hbar\sqrt{\nu}}K$$

$$\phi(r;Z) = \left(\frac{2\nu}{\pi}\right)^{3/4} \exp\left(-\nu(r - Z/\sqrt{\nu})^2 + Z^2/2\right) \propto \exp\left(-\nu(r-D)^2 + i\,K\cdot(r-D)/\hbar\right)$$

- **Time-dependent Variational Principle → Equations of Motion**

$$L = \frac{\langle\Psi|i\hbar\partial/\partial t - H|\Psi\rangle}{\langle\Psi|\Psi\rangle} \quad , \quad \frac{d}{dt}\frac{\partial L}{\partial(d\bar{Z}_i/dt)} - \frac{\partial L}{\partial\bar{Z}_i} = 0 \rightarrow \quad i\hbar C_{i\alpha,j\beta}\frac{dZ_i}{dt} = \frac{\partial H}{\partial\bar{Z}_i}$$

- **Ignoring Antisymmetrization**
  → Quantum Molecular Dynamics EOM (= Classical EOM)

$$C = \delta \quad \rightarrow \quad \frac{dD_i}{dt} = \frac{\partial H}{\partial K_i} \quad , \quad \frac{dK_i}{dt} = -\frac{\partial H}{\partial D_i}$$

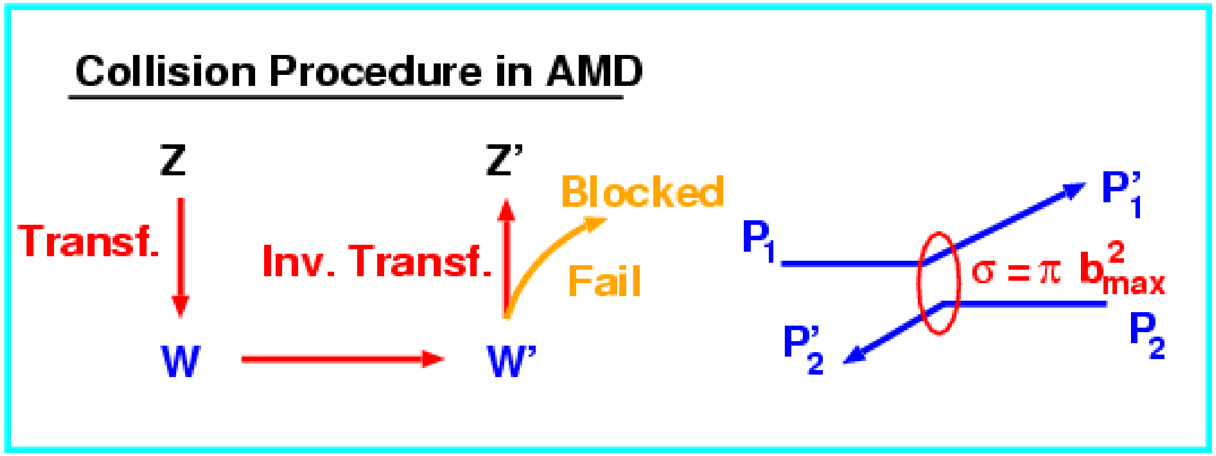
Classical-type EOM is obtained through Gaussian + TDVP

Collision Term in AMD

- **Approximate Canonical Variables**

$$W_i = \sqrt{Q}_{ij} Z_j = \sqrt{\nu} R_i + \frac{i}{\sqrt{\nu}\hbar} P_i \ , \quad Q_{ij} \equiv B_{ij} \quad B_{ij}^{-1} \ , \quad B_{ij} = \langle \psi_i | \psi_j \rangle$$

$$Example \quad \langle \mathbf{L} \rangle = \sum_{ij} B_{ji}^{-1} B_{ij} \frac{1}{i} \overline{Z}_i \times Z_j = \sum_i \overline{W}_i \times W_i$$



Physics included in AMD
Time Evolution of Anti-Symmetrized Wave Function
Collision Term = "Canonical" Variable + Classical Analogy
Event-by-Event Fluctuation
Problems: Non-Rela., Classical Analogy of Collision term,CPU cost

Exercise (2)

- **Prove that the TDVP (time-dependent variational principle) gives the Schrodinger equation when the wave function is not restricted.**
- ***(ADVANCED)*** **Prove that the AMD wave function is equivalent to harmonic oscillator shell model wave function when all Z's goes to zero. (This tells you why the Slater determinant of (s-wave) Gaussians can describe nuclei above s-shell.)**
- ***(ADVANCED)*** **Obtain the Lagrange multiplier in RQMD/S.**

Cascade Model
Hadron-Hadron Collisions

AA collisions at High E.

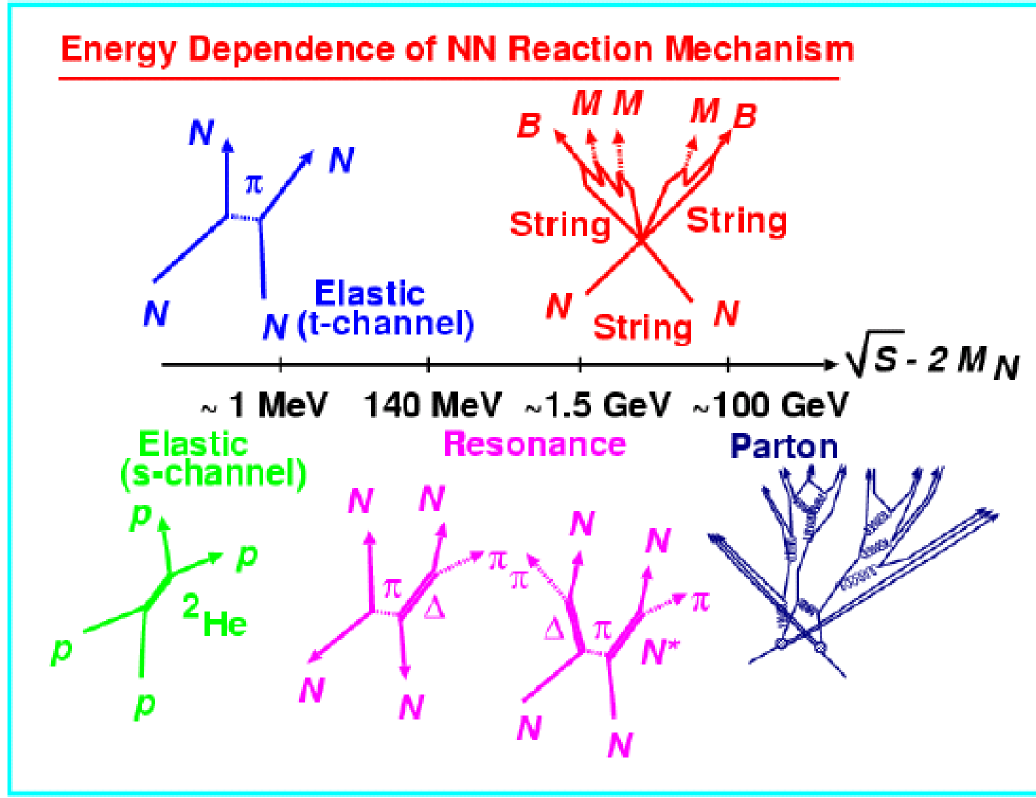
~ Sum of (Multistep) NN collisions (Cascade)

+ Interesting Physics

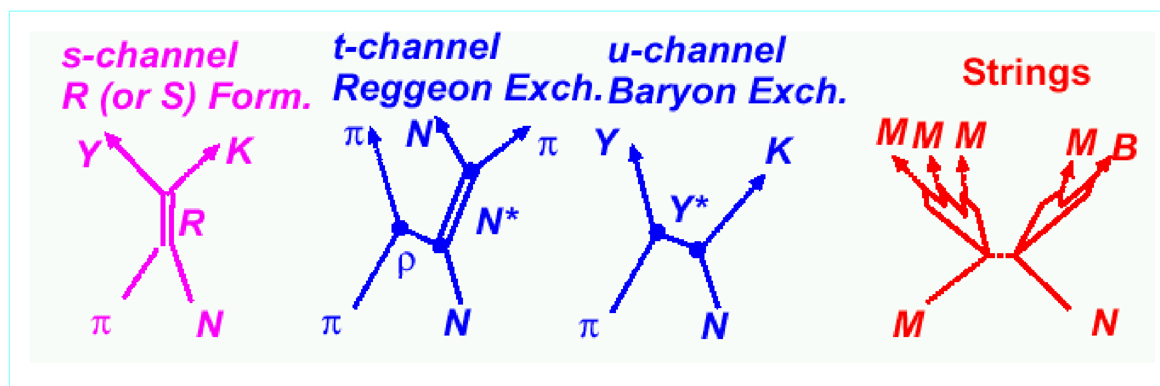
→ Cascade gives the "baseline" of evaluation !

Baryon-Baryon and Meson-Baryon Collisions

- NN collision mechanism
  Elastic
  → Resonance
  → String
  → Jet

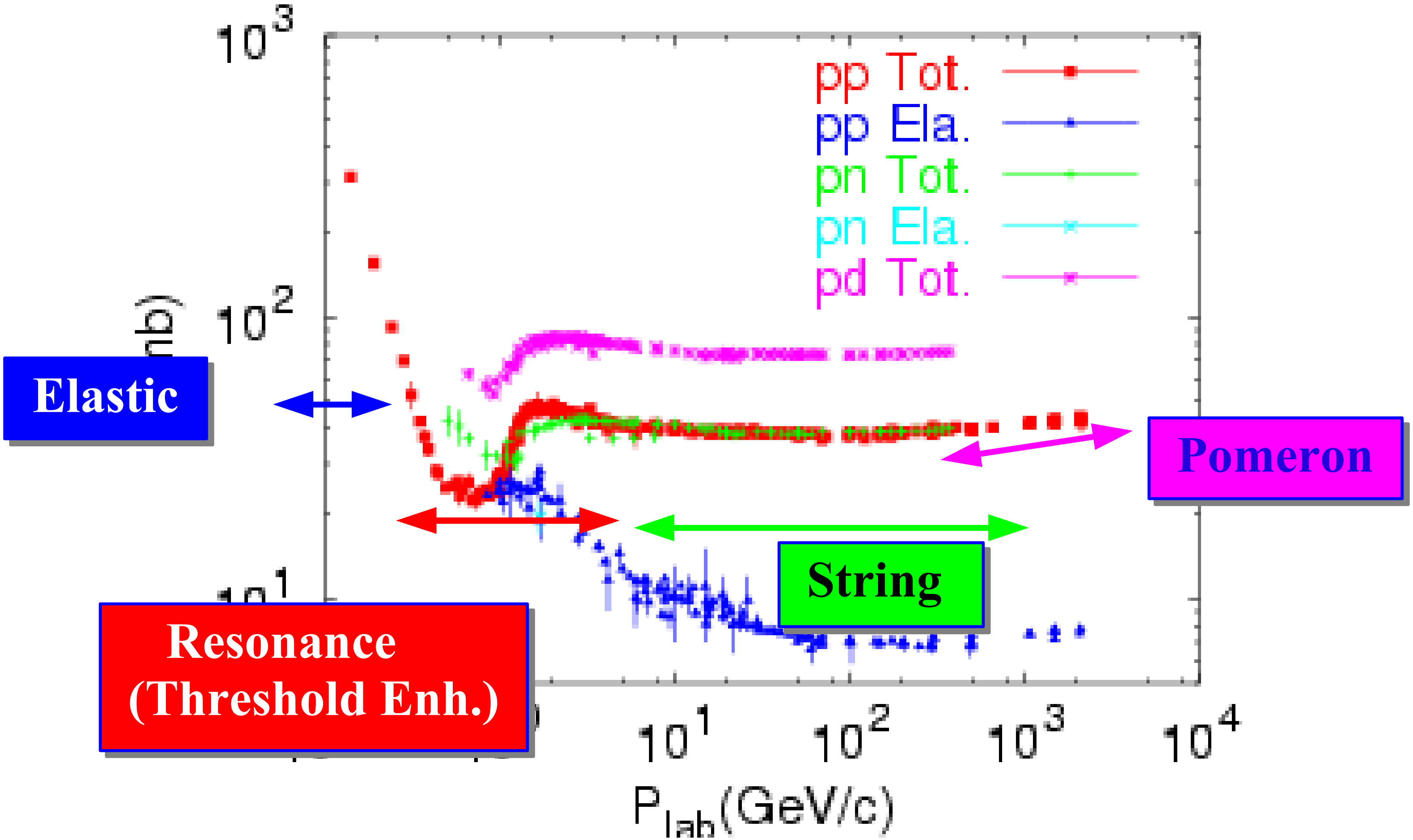


- Meson-Nucleon Collision
  → s-channel Resonance
  → t-(u-) channel Res.
  → String formation

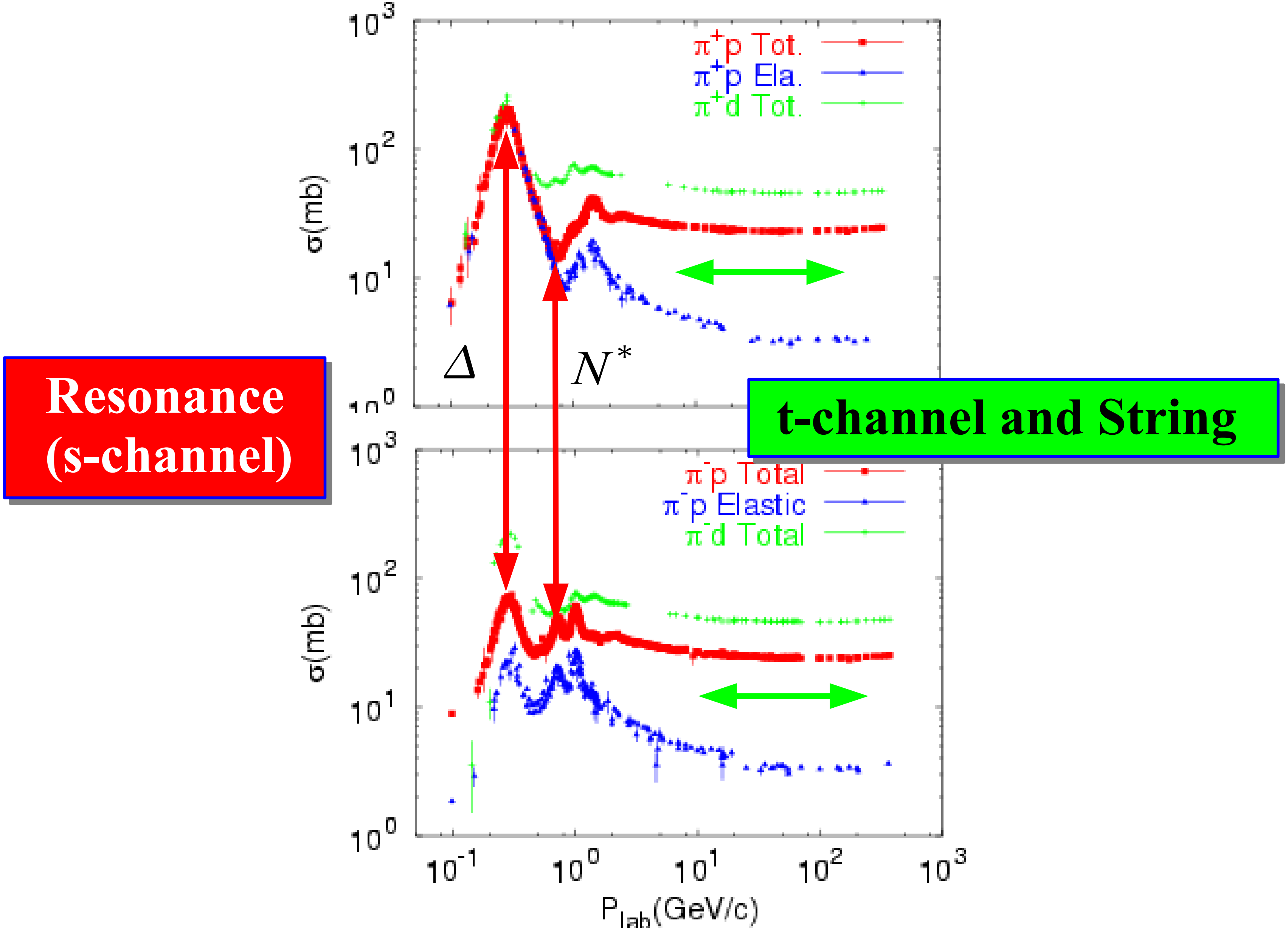


NN Cross Sections

From Particle Data Group



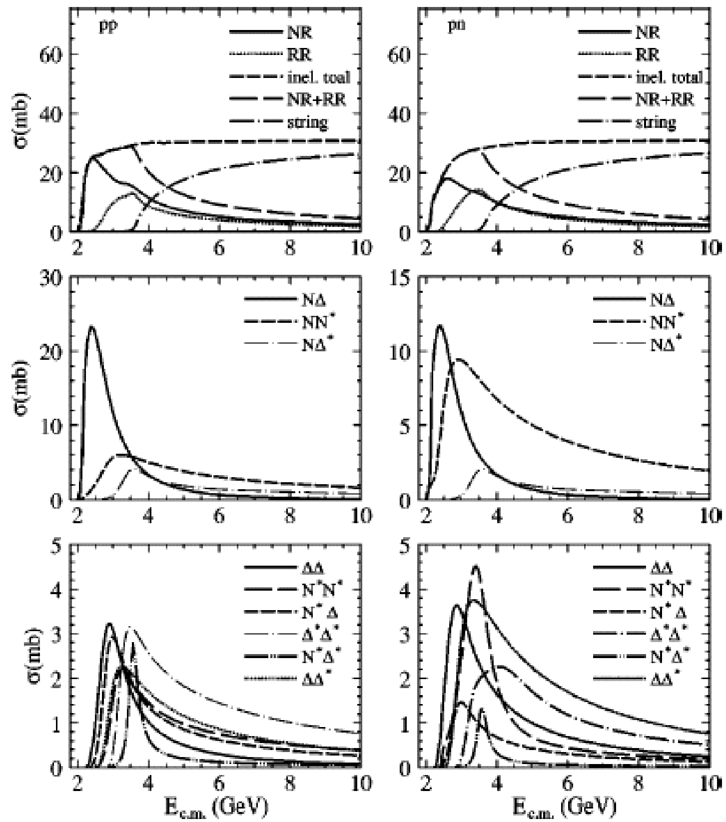
Meson-Baryon Cross Section

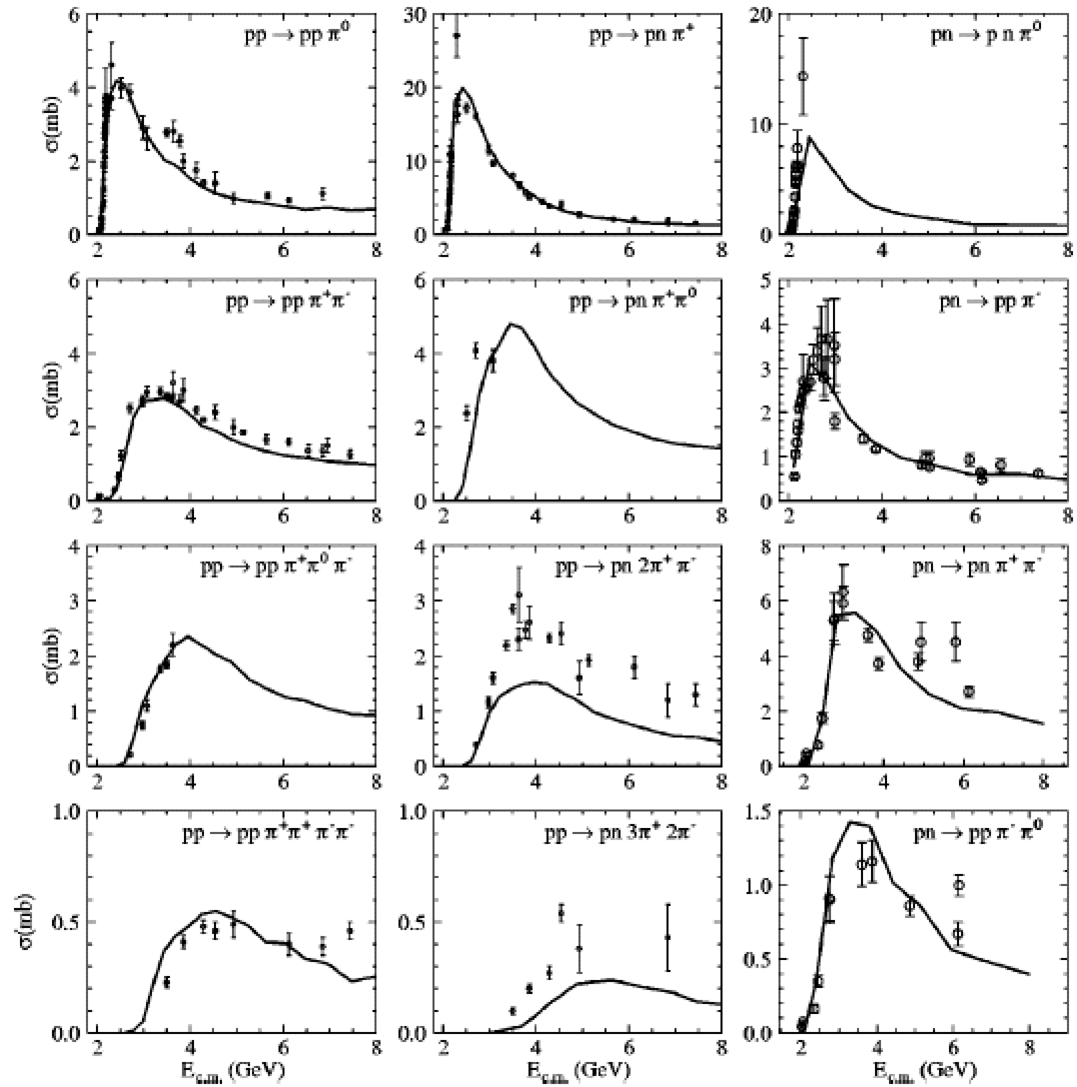


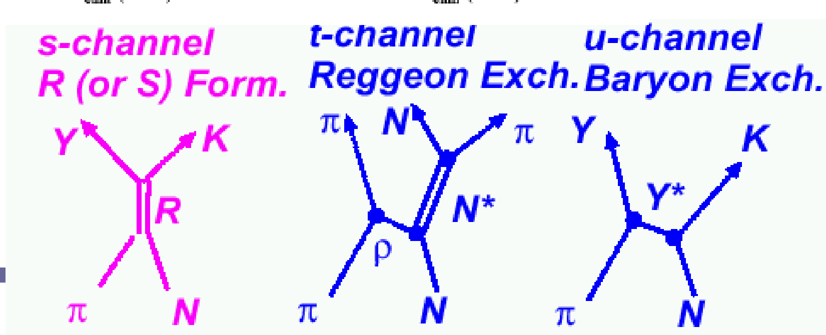
Exclusive Cross Sections

Nara, Otuka, AO, Niita, Chiba (JAM), PRC 61 (2000), 024901.

- **We need not only Total and Elastic Cross Sections, but also Cross Sections for *Each Channel* !**







Regge, String, and Jet --- High Energy hh Collisions ---

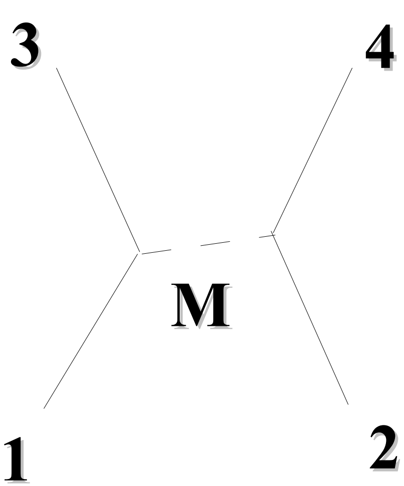
Reggeon Exchange

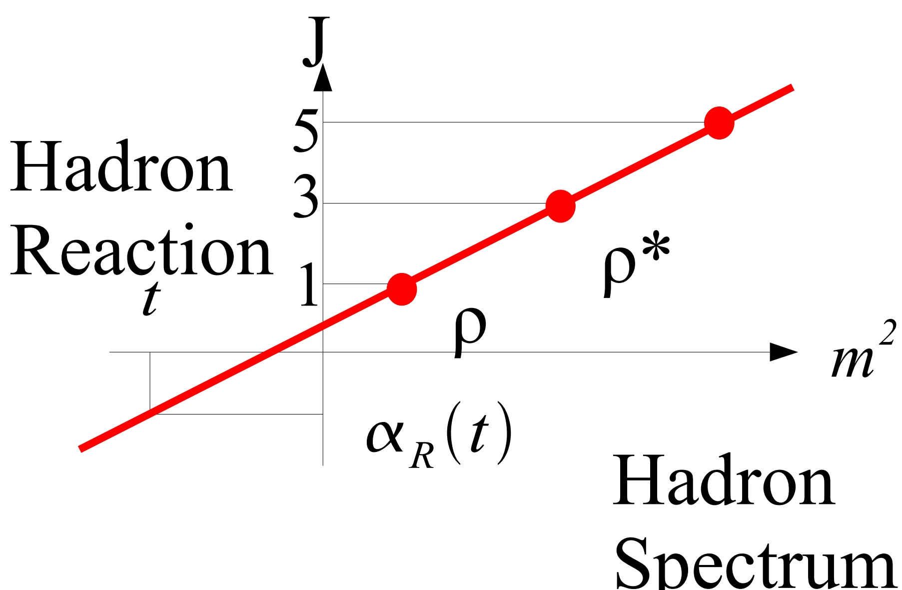
Barger and Cline (Benjamin, 1969), H. Sorge (RQMD), PRC (1995)

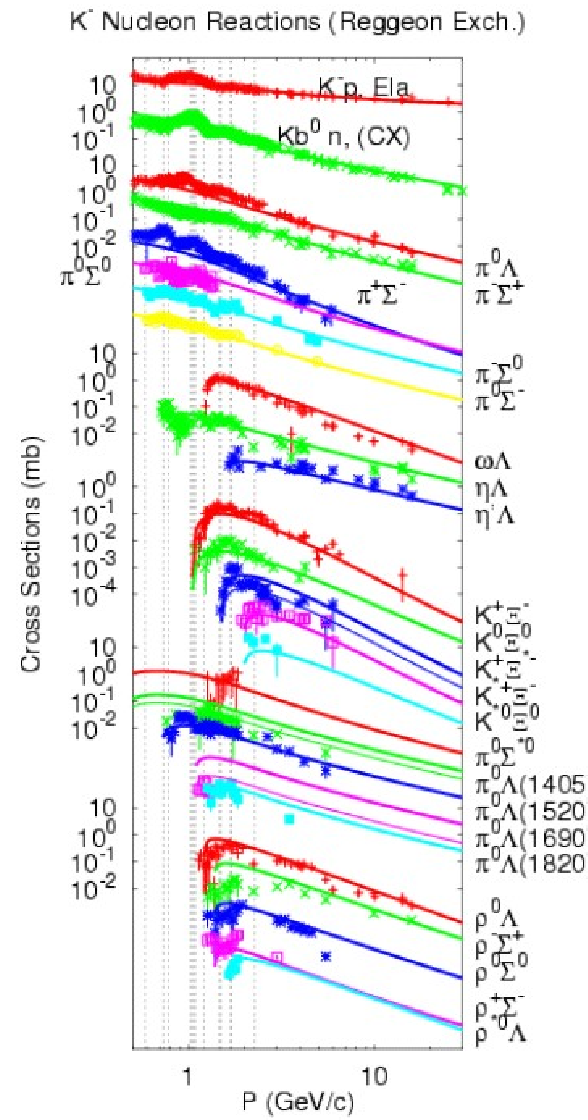
- **Regge Trajectory** $J = \alpha_R(t) \sim \alpha_R(0) + \alpha'_R(0)\,t$
- **2 to 2 Cross Section**

$$\frac{d\sigma}{d\Omega} = \frac{p_f}{64\pi s\, p_i} |M(s,t)|^2$$

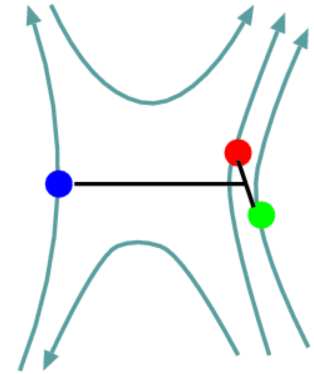
$$M(s,t) \sim \sum_R \frac{(p_i p_f)^J}{t - M_R} \sim F(t) \exp\left[\alpha_R(t) \log(s/s_0)\right]$$







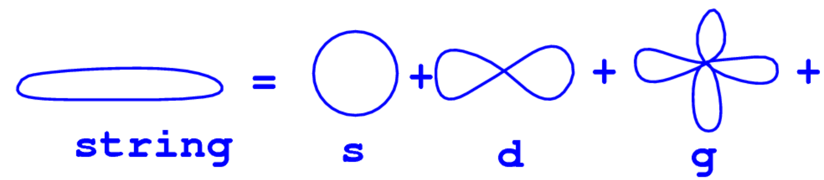
String formation and decay



- **What does the regge trajectory suggest ?**
  → Existence of (color- or hadron-)String !

$$M = 2\int_0^R \frac{\kappa\, dr}{\sqrt{1-(r/R)^2}} = \pi\kappa R \quad , \quad J = 2\int_0^R r \times \frac{\kappa\, dr}{\sqrt{1-(r/R)^2}}\frac{r}{R} = \frac{\pi\kappa R^2}{2}\pi$$

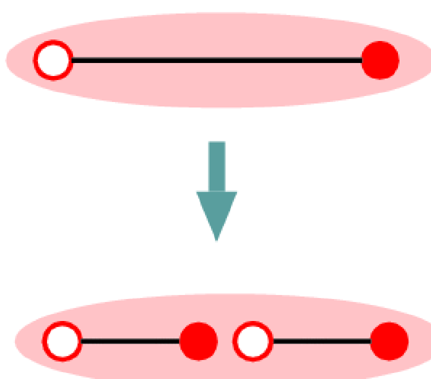
$$\rightarrow \quad J = \frac{M^2}{2\pi\kappa}$$



- **String Tension**

$$\frac{1}{2\pi\kappa} = \alpha'_R(0) \approx 0.9\, GeV \quad \rightarrow \quad \kappa \approx 1\, GeV/fm$$

- **String decay**
  Extended String
  → Large E stored
  → q qbar pair creation (Schwinger mech.)



String = Coherent superposition of hadron resonances with various J

High p_T ハドロン生成 @ RHIC

- **GSI, AGS, SPS → 共鳴ハドロン、ストリング生成と破砕**
  Nara et al., PRC61('00),024901; Isse et al., PRC72('05),064908.
- **RHICでの標準描像= pQCD+E-loss+独立破砕**

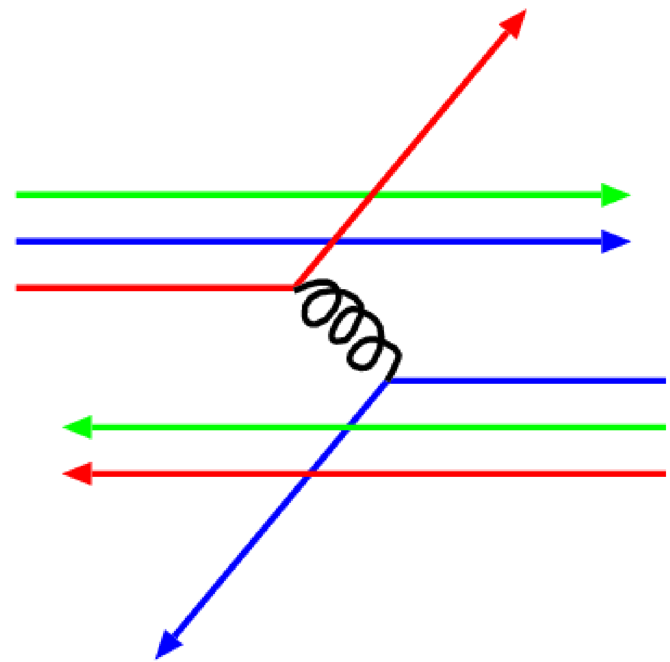
$$\frac{dN^{AA}(b)}{dy\,d^2p_T} = \int d\boldsymbol{r}_T\, t_A(\boldsymbol{r}_T - \boldsymbol{b}/2)\, t_B(\boldsymbol{r}_T + \boldsymbol{b}/2)$$ Geometry

$$\times K \sum_{abcd} \int dx_a dx_b d^2k_a d^2k_b f_{a/A} f_{b/B} \frac{d\sigma^{ab\to cd}}{d\hat{t}}$$ pQCD × K-fac.

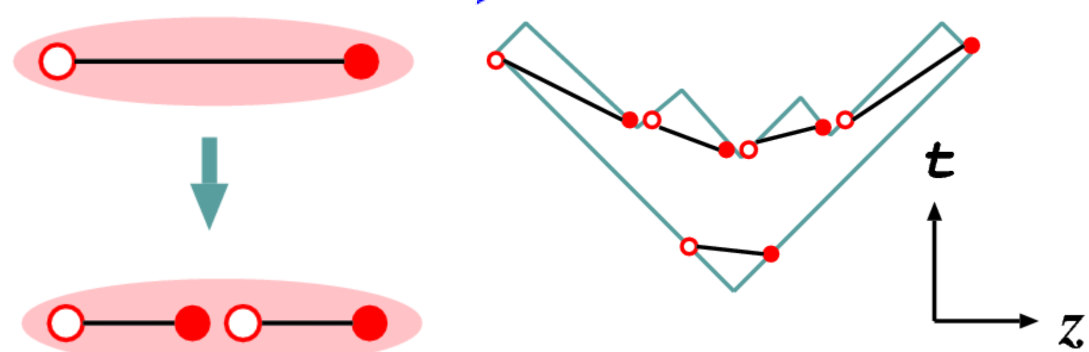
$$\times D(E_c - \Delta E_c(\boldsymbol{r}_T); c \to h)$$ E-loss + Indep. Frag.

Jet Production

- **Elastic Scattering of Partons (mainly) with One Gluon Exch.**
- **Color Exch. between Hadrons**
  → Complex color flux starting from leading partons
  → many hadron production
  → Jet production



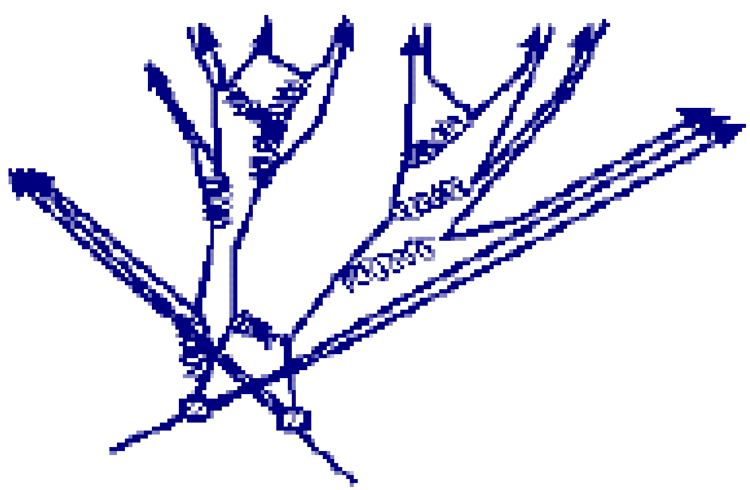
- ***PYTHIA***
  - **Event Generator of High Energy Reactions**
    → Jet production +String decay
    for QCD processes

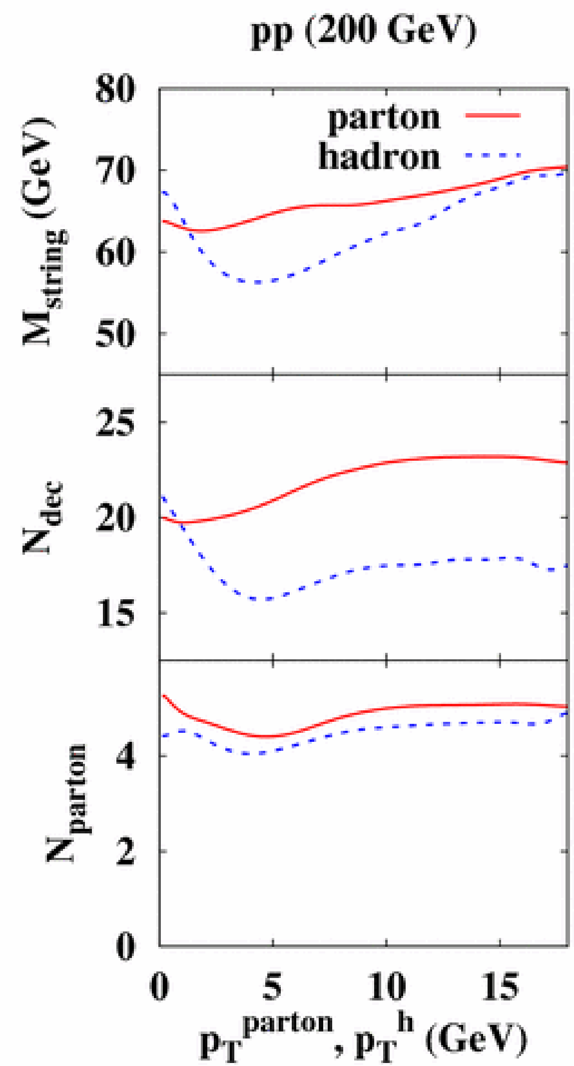


(T. Sjostrand et al., Comput. Phys. Commun. 135 (2001), 238.)

String Mass and p_T in Jet

- **In average, Jet Strings**
  have 60-70 GeV masses,
  contain around 4-5 partons (q-g-g-g-qbar, …).
  and
  decay into 20-25 hadrons.
  - **Complex color flux starting from leading partons make strings heavy !**

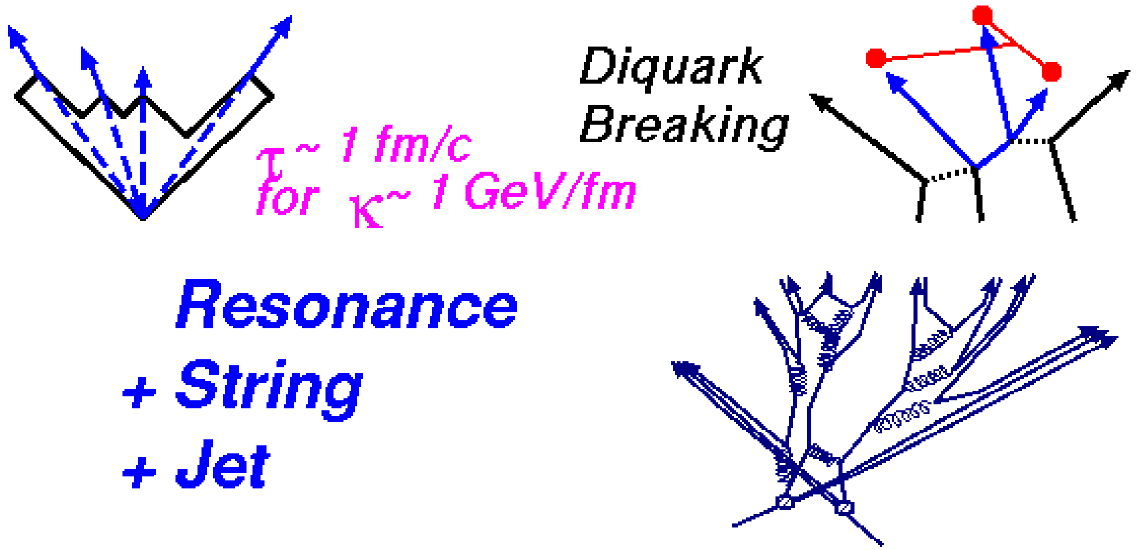




JAM (Jet AA Microscopic transport model)

Nara, Otuka, AO, Niita, Chiba, PRC 61 (2000), 024901.

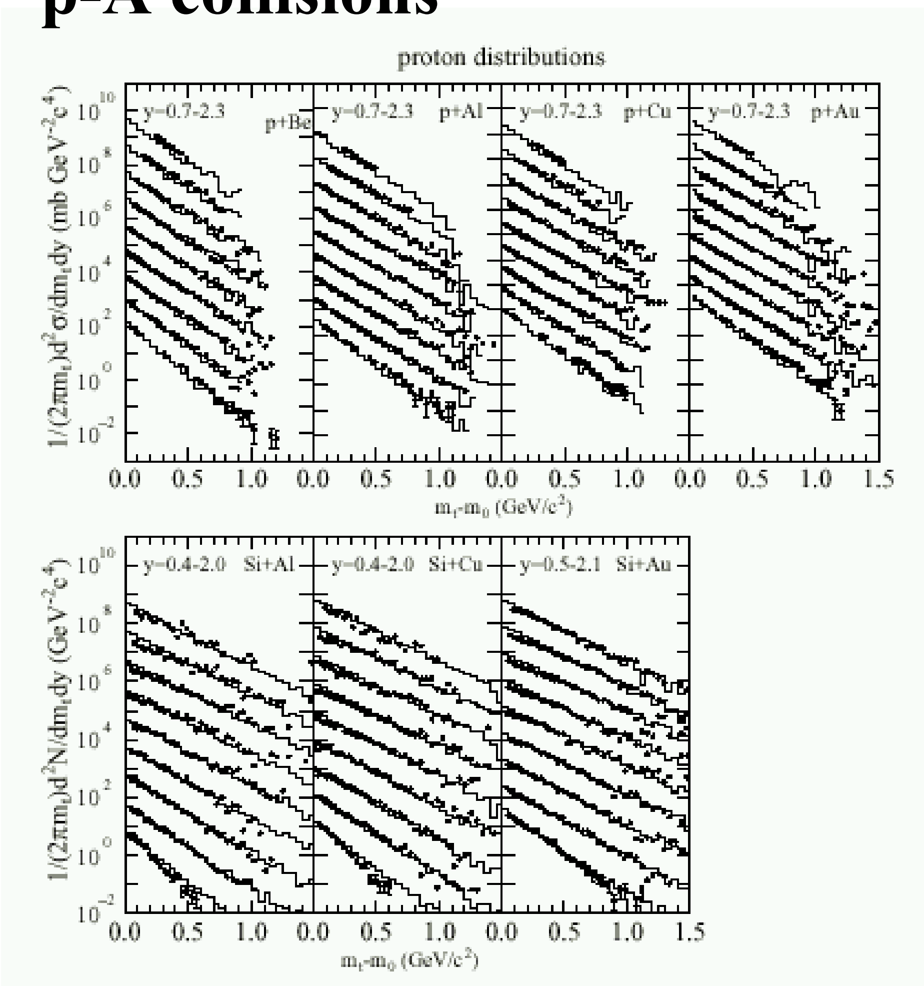
- **Hadron-String Cascade with Jet production**
  - **hh collision with Res. up to m < 2 GeV (3.5 GeV) for M (B)**
  - **String excitation and decay**
  - **String-Hadron collisions are simulated by hh collisions in the formation time.**
  - **jet production is incl. using PYTHIA**
  - **Secondary partonic int.: NOT incl.**
  - **Color transparency: NOT taken care of**



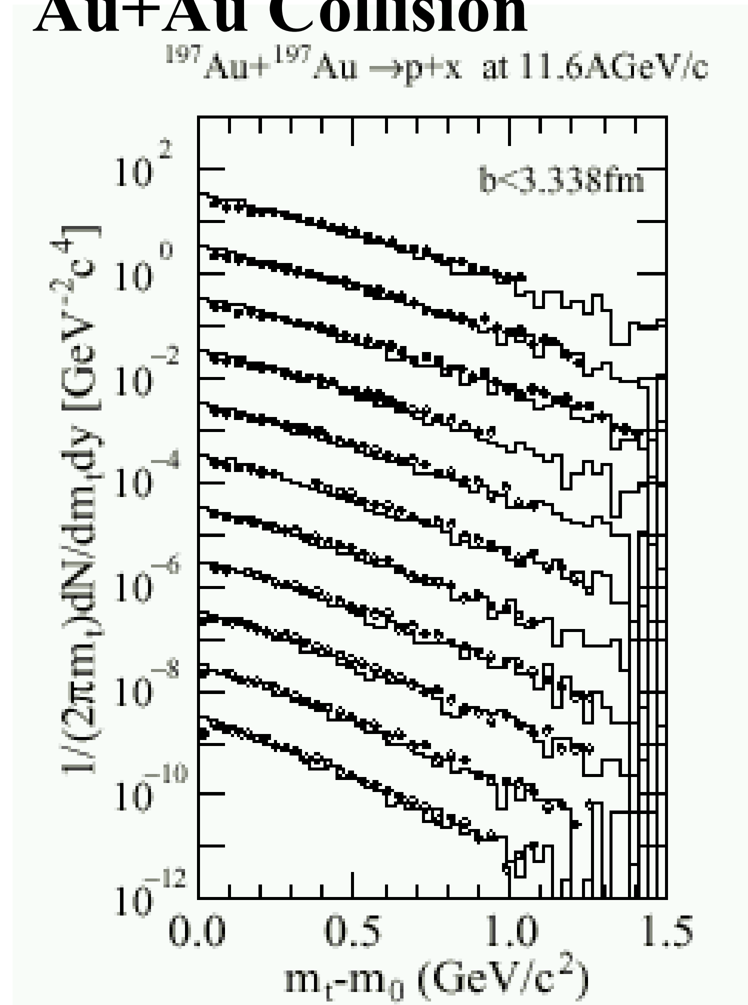
JAM Results @ AGS Energy

Nara, Otuka, AO, Niita, Chiba, PRC 61 (2000), 024901.

- **p-A collisions**



- **Au+Au Collision**



JAM explains AA collisions as well as pA collisions:
→ Good Elementary Cross Sections for MM, MN and NN

Exercise (3)

- **Prove that the sum of Mandelstam variables becomes a constant.**
  $s = (p_1+p_2)^2,\ \ t = (p_1-p_3)^2,\ \ u = (p_1-p_4)^2,$
  in 1+2 → 3+4 reaction.

- **Draw the Feynman diagram of $K^-+p \rightarrow \pi^+ + \Sigma^-$.**
  You will see that the angular distribution becomes backward peaked due to the u-channel dominance.

- **Explain why we have peak structures in MB collisions and we do not see peaks in BB collisions.**

- ***(If you already learned pQCD,)*** **Obtain the squared Feynman amplitude of qq → qq in the tree level averaged over the color and spin. (You can ignore quark mass.)**
  You will see the cross section is divergent at forward angle.
  Explain why we do not see this divergence in NN collisions.

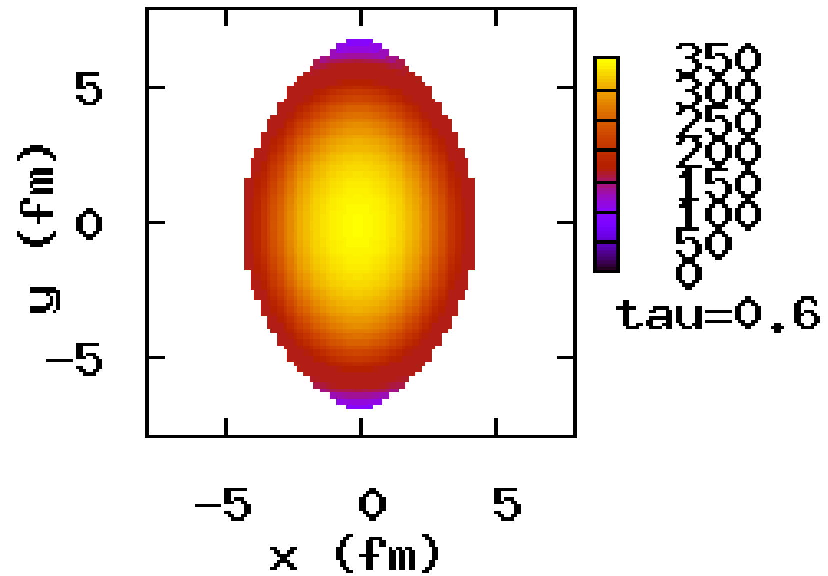
Relativistic Hydrodynamics

- **EOM: Conservation Laws**

$\partial_\mu T^{\mu\nu} = 0$ Energy Momentum Conservation
$\partial_\mu (n_i u^\mu) = 0$ Conservation of Charge (Baryon, Strangeness, ...)

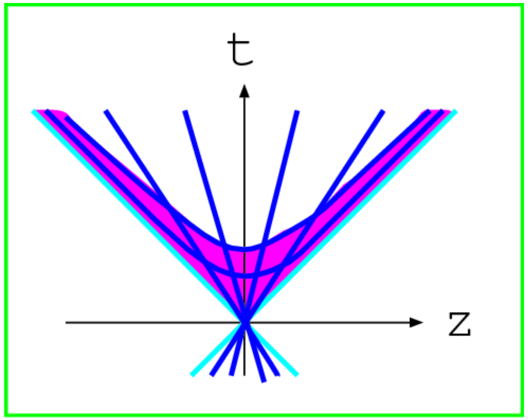
$$T^{\mu\nu} = (e + P)u^\mu u^\nu - P g^{\mu\nu}$$

e **: energy density, *P: pressure,***
u^μ: :four velocity $\gamma(1,v)$, n_i: :number density



T. Hirano, Y. Nara,   NPA743, 305 (2004)
T. Hirano, K. Tsuda, PRC 66, 054905(2002)

Relativistic Hydrodynamics (II)



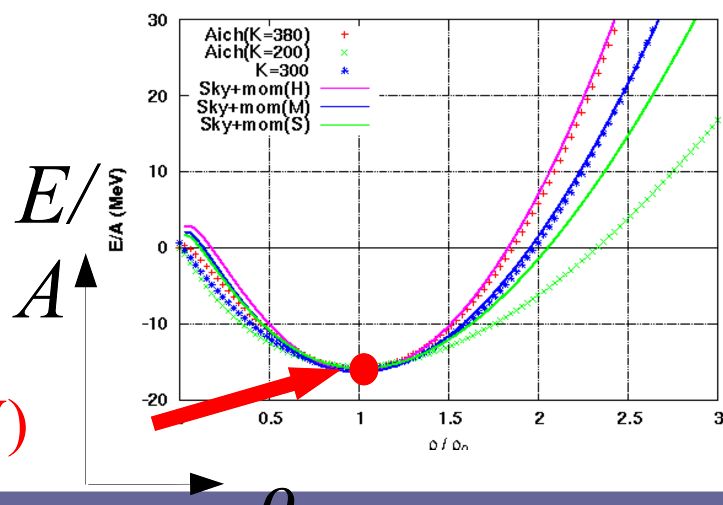
- **One more condition is necessary**
  → *Equation of State $P = P(e, n_i)$ is needed*
  - **Independent Variables: e, P, v, n_i → 6**
  - **Independent Equations: 4+1 =5**
- **Solve Hydro. in Bjorken Variables (τ, η_s, x, y) → Save CPU a lot !**
  - **Most of the Dynamics is govered by τ during τ < 10 fm/c**
  - **η_s approximately corresponds to η, and fixed by inc. E.**
- **Parameters**
  - **τ_0 (thermalization time), T^{ch} (chem. F.O.) → Au+Au *$dN/d\eta$* fit**
  - **T^{th}: Free Parameter**
- **Initial Condition: Glauber type / Color Glass Condensate**

Nuclear Mean Field for HIC --- Density and Momentum Deps. ---

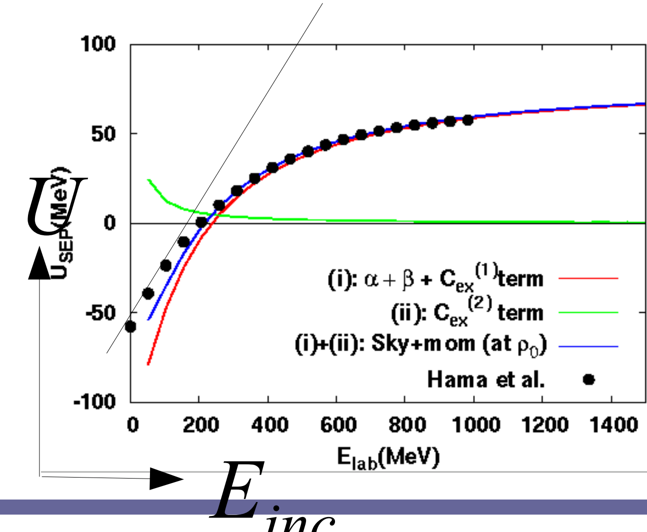
Nuclear Mean Field

- **MF has on both of ρ and p-deps.**
  - **ρ dep.: (ρ_0, *E/A*) = (0.15 fm^{-3}, -16.3 MeV) is known**
    Stiffness is not known well
  - **p dep.: Global potential up to E=1 GeV is known from pA scattering**
    $U(\rho_0, E) = U(\rho_0, E{=}0) + 0.3\,E$
- **Ab initio Approach; LQCD, GFMC, DBHF, G-matrix, ...**
  → Not easy to handle, Not satisfactory for phen. purposes
- **Effective Interactions (or Energy Functionals):**
  Skyrme HF, RMF, ...

$(\rho_0, E/A)$
$=(0.15\ \mathrm{fm}^{-3}, -16.3\ \mathrm{MeV})$



U(E)=U(0)+0.3E



Skyrme Hartree-Fock

See Ring-Schuck for details

- **Zero-Range Two- and Three-Body Interaction**

$$v_{ij} = t_0 \delta(r_i - r_j) + \frac{1}{2}\left[\delta(r_i - r_j) k^2 + k^2 \delta(r_i - r_j)\right]$$

$$+ t_2 k \delta(r_i - r_j) k + i W_0 \left[\sigma_i + \sigma_j\right] \times \delta(r_i - r_j) k$$

$$k = \frac{1}{2i}\left(\nabla_i - \nabla_j\right)$$

$$v_{ijk} = t_3 \delta(r_i - r_j) \delta(r_j - r_k)$$

- **Energy Density (Even-Even, N=Z)**

$$H(r) = \frac{\hbar^2}{2m^*(\rho)} \tau + \frac{3}{8} t_0 \rho^2 + \frac{1}{16} t_3 \rho^3 + \mathtt{Deriv.\ terms} \rightarrow \rho \left[\frac{3}{5} \frac{\hbar^2 k_F^2}{2m^*(\rho)} + \frac{3}{8} t_0 \rho + \frac{1}{16} t_3 \rho^2\right]$$

$$\tau = \sum_i |\nabla \phi_i|^2 \ , \quad \frac{\hbar^2}{2m^*(\rho)} = \frac{\hbar^2}{2m} + \frac{1}{16}(3 t_1 + 5 t_2) \rho$$

Problems in Skyrme HF (in Dense Nuclear Matter/High Energy)
Repulsive Zero-Range 3-body Int.: → Ferromagnetism
Energy Dep. = Linear (m* term) → Too Repulsive at High E

Relativistic Mean Field (I)

Serot-Walecka, Walecka text book.

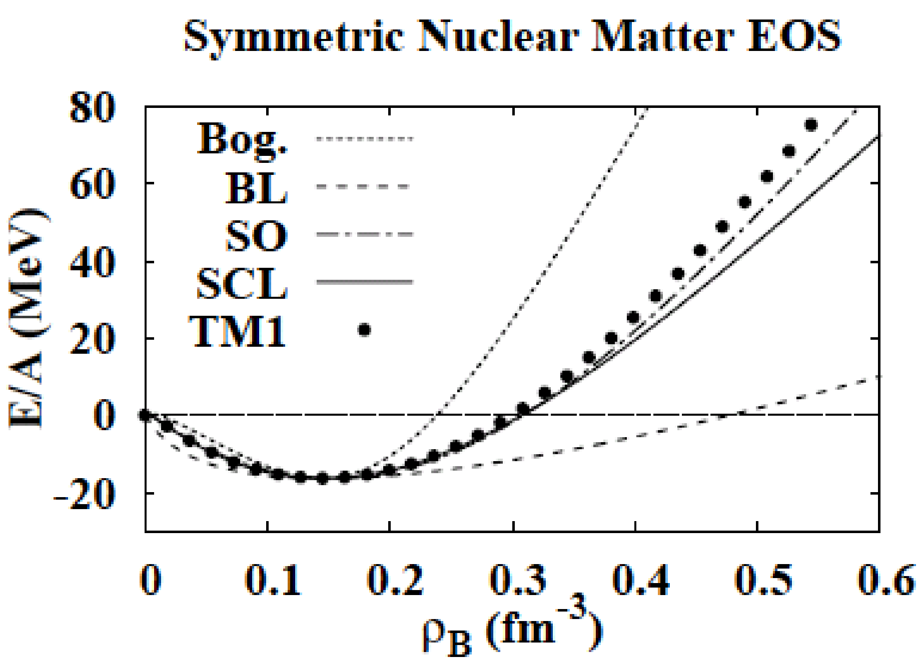
- **Describe nuclear energy functional in meson and baryon fields**
  - **Fit B.E. of Stable as well as Unstable (n-rich) Nuclei**
  - **Has been successfully applied to Supernova Explosion**
  - **Three Mesons (σ,ω,ρ) are included**
  - **Meson Self-Energy Term (σ,ω)**

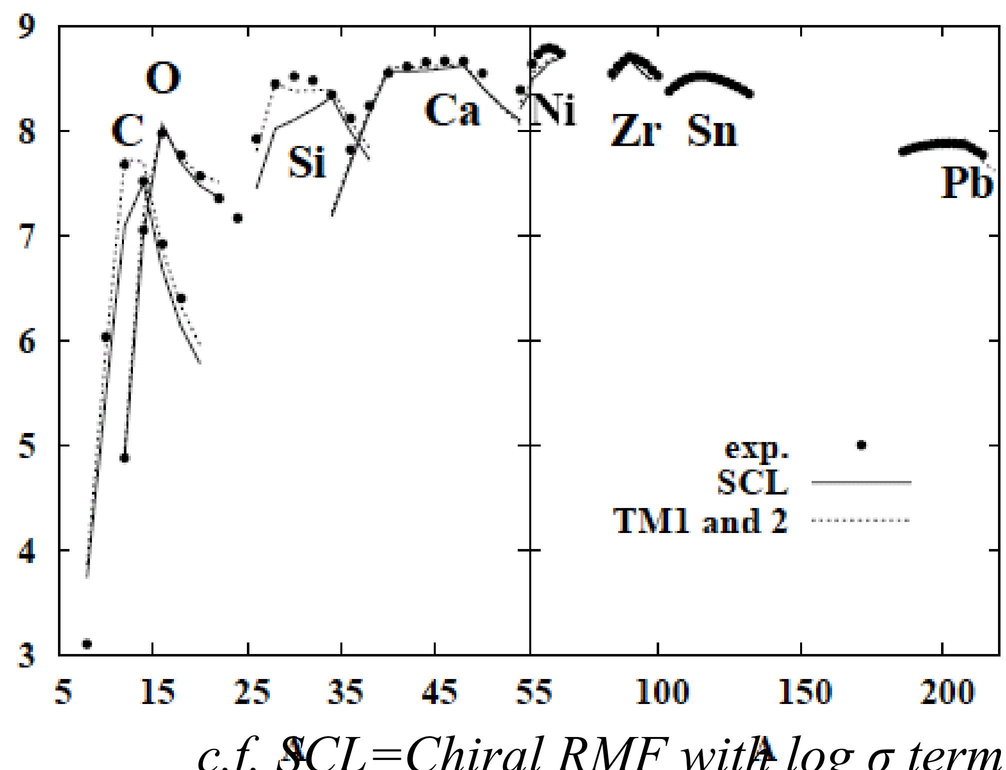
$$
\begin{aligned}
\mathcal{L} =& \bar{\psi}_N \left(i\partial\!\!\!/ - M - g_\sigma \sigma - g_\omega \omega\!\!\!/ - g_\rho \tau^a \rho\!\!\!/^a \right) \psi_N \\
&+ \frac{1}{2}\partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 \\
&- \frac{1}{4} W^{\mu\nu} W_{\mu\nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4} R^{a\mu\nu} R^a_{\mu\nu} + \frac{1}{2} m_\rho^2 \rho^{a\mu} \rho^a_\mu + \frac{1}{4} c_3 \left(\omega_\mu \omega^\mu \right)^2 \\
&+ \bar{\psi}_e \left(i\partial\!\!\!/ - m_e \right) \psi_e + \bar{\psi}_\nu i\partial\!\!\!/ \psi_\nu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \ , \\
W_{\mu\nu} =& \partial_\mu \omega_\nu - \partial_\nu \omega_\mu \ , \\
R^a_{\mu\nu} =& \partial_\mu \rho^a_\nu - \partial_\nu \rho^a_\mu + g_\rho \epsilon^{abc} \rho^{b\mu} \rho^{c\nu} \ , \\
F_{\mu\nu} =& \partial_\mu A_\nu - \partial_\nu A_\mu \ . \qquad (2)
\end{aligned}
$$

Nuclear Matter EOS and Nuclear Binding E in TM

Sugahara-Toki, NPA579 (1994), 557.

- **Example: TM1 parameter set**
  - **Nuclear Matter: σ4 and ω4 terms soften EOS (K ~ 280 MeV)**
  - **Finite nuclei: Explains B.E. from C to Pb isotopes**





c.f. SCL=Chiral RMF with log σ term.

(K. Tsubakihara and AO, 2007)

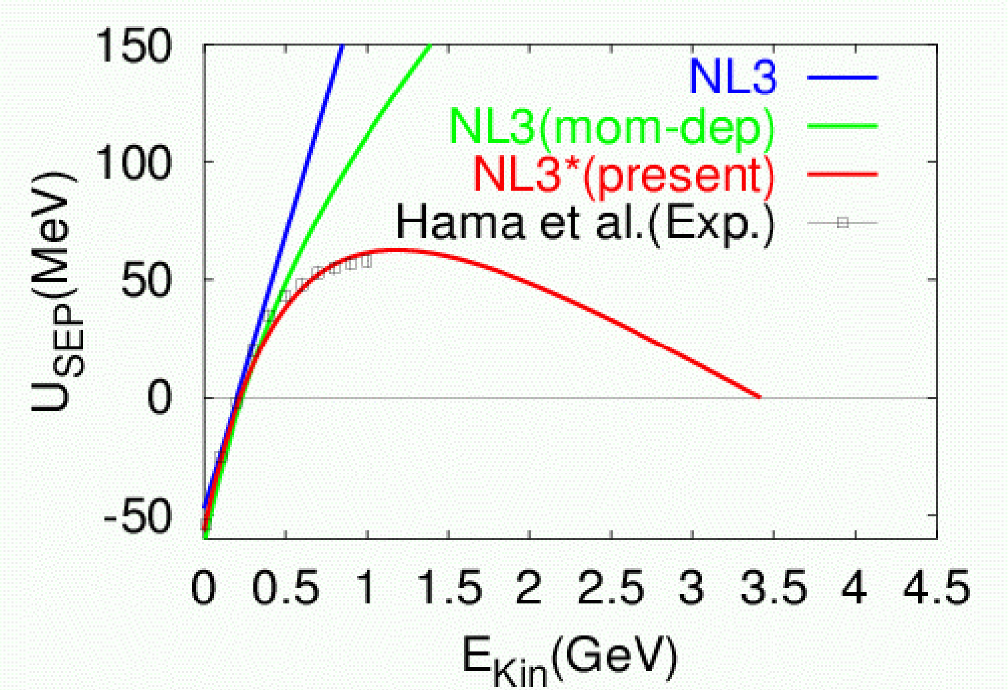
Relativistic Mean Field (II)

- **Dirac Equation** $\left(i\gamma\partial-\gamma^0 U_v - M - U_s\right)\psi=0\ ,\quad U_v=g_\omega\omega\ ,\quad U_s=-g_\sigma\sigma$
- **Schroedinger Equivalent Potential**

$$\begin{pmatrix} E-U_v-M-U_s & -i\sigma\cdot\nabla \\ i\sigma\cdot\nabla & -E+U_v-M-U_s \end{pmatrix}\begin{pmatrix} f \\ g \end{pmatrix}=0$$

$$U_{sep}\sim U_S+\frac{E}{m}U_V=-g_\sigma\sigma+\frac{E}{m}g_\omega\omega$$

$$=-\frac{g_\sigma^2}{m_\sigma^2}\rho_s+\frac{E}{m}\frac{g_\omega^2}{m_\omega^2}\rho_B$$



Saturation: -Scalar+Baryon Density
Linear Energy Dependence: Good at Low Energies,
Bad at High Energies (We need cut off !)

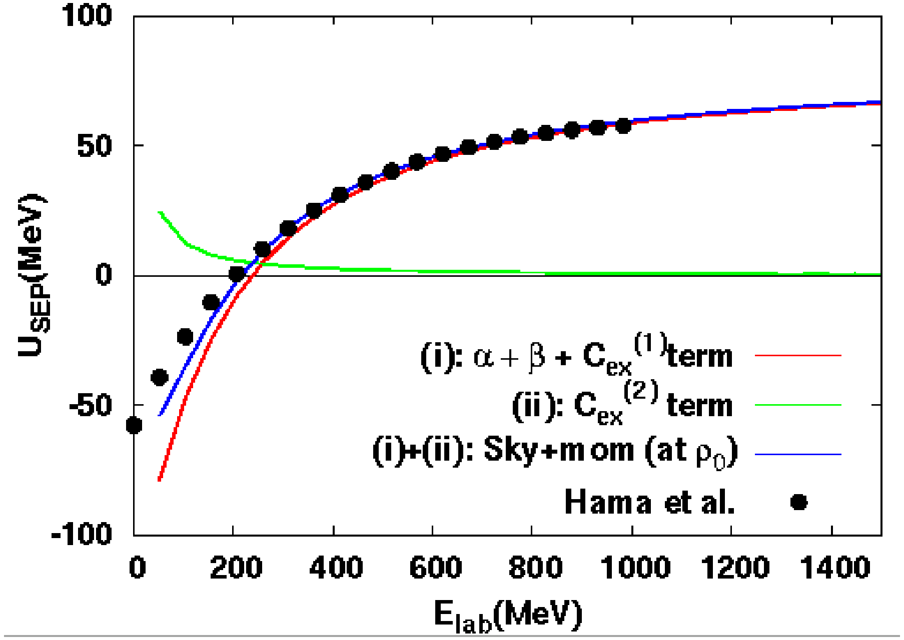
(Sahu, Cassing, Mosel, AO, Nucl. Phys. A672 (2000), 376.)

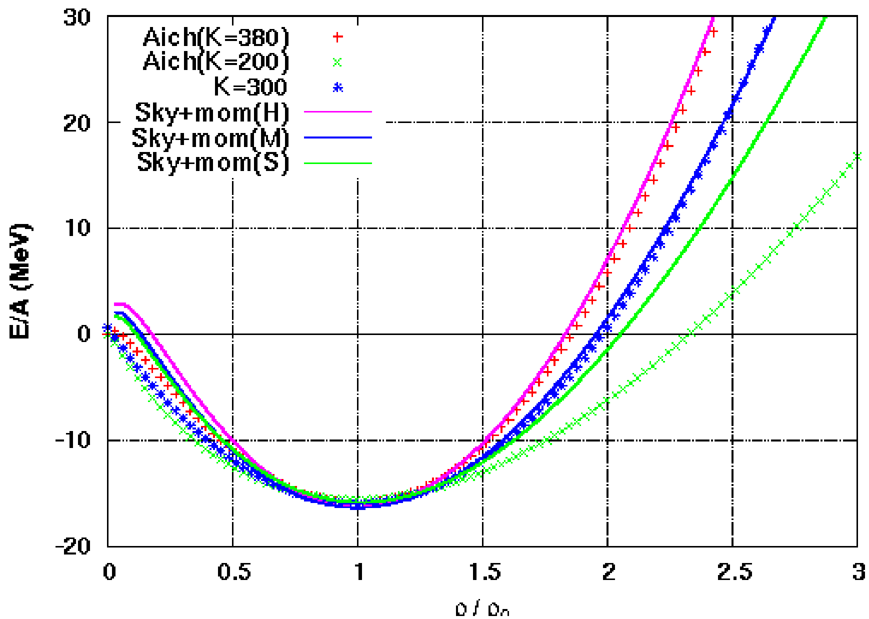
Phenomenological Mean Field

- **Skyrme type ρ-Dep. + Lorentzian p-Dep. Potential**

$$V = \sum_i V_i = \int d^3 r \left[\frac{\alpha}{2} \left(\frac{\rho}{\rho_0} \right)^2 + \frac{\beta}{\gamma+1} \left(\frac{\rho}{\rho_0} \right)^{\gamma+1} \right]$$

$$+ \sum_k \int d^3 r\, d^3 p\, d^3 p' \frac{C_{ex}^{(k)}}{2\rho_0} \frac{f(r,p) f(r,p')}{1+(p-p')^2/\mu_k^2}$$





Isse, AO, Otuka, Sahu, Nara, Phys.Rev. C 72 (2005), 064908

Exercise (4)

- **Prove that the single particle potential with Skyrme interaction has a linear dependence on energy. From NA elastic scattering, the energy dependence is found to be**

  $$U(\rho_0, E) \sim U(\rho_0, E=0) + 0.3\, E$$

  at low energies. Obtain the value of m*/m which explains the above energy dependence.
- **Obtain the form of the Schrodinger equivalent potential in RMF. You will find that the spin-orbit potential appears as a sum of scalar and vector potential.**

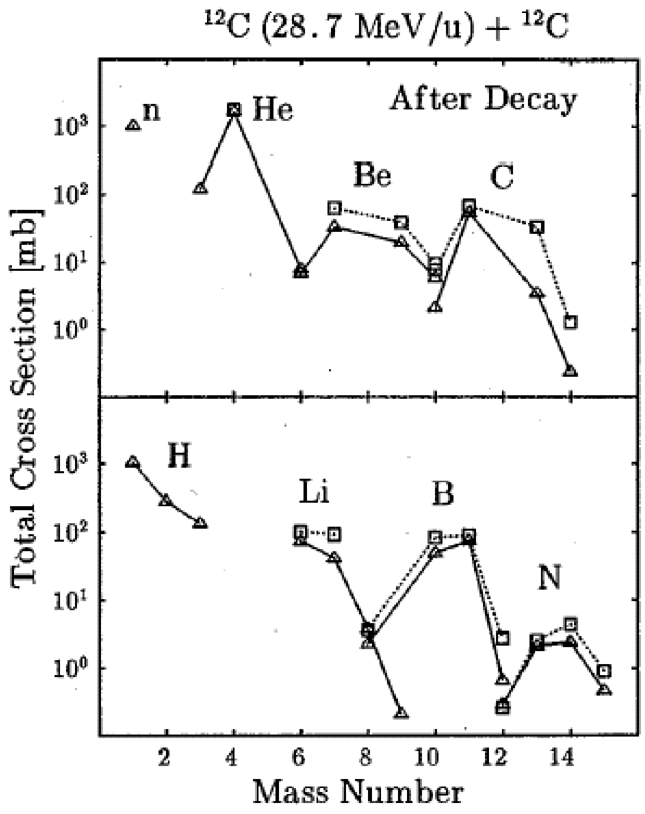
Summary

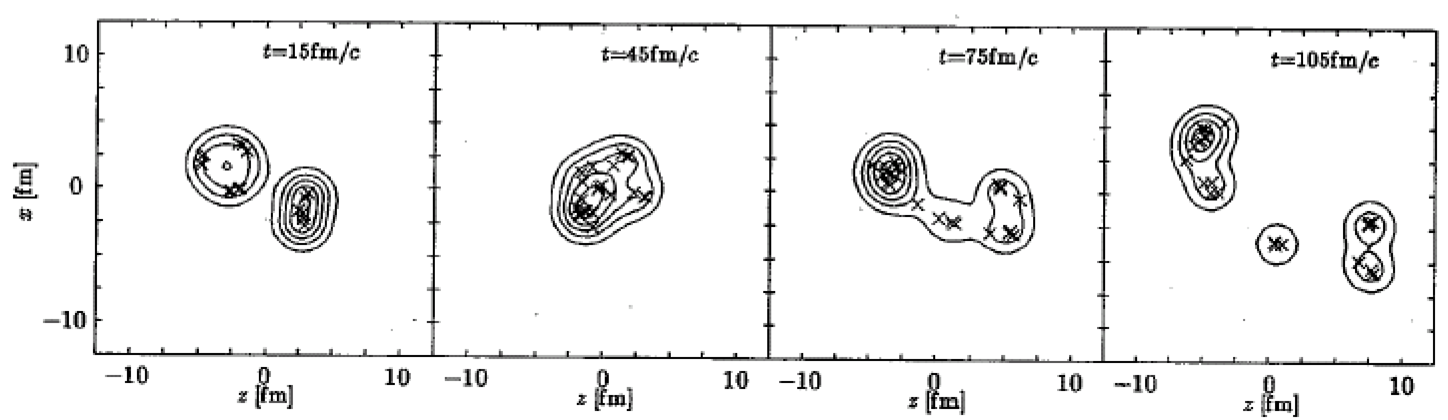
- **Basic ingredients in HIC models are explained.**
  - **Mean field dynamics**
  - **Two-body hadron-hadron collisions**
  - **String formation and Jet production**
  - **Hydrodynamics**
- **While nuclear MF at low energies are well investigated, it is not trivial how to apply these MFs to higher energy reactions.**
  At present, phenomenologically parametrized potentials are frequently used.
- **Students interested in HIC up to 1 A GeV should understand mean-field dynamics and NN cross sections (and π productions).**
  Students interested in RHIC physics should understand parton dynamics and strings, and hydrodynamics.

Backups

Fragment Formation in AMD

- ***Fluctuation is ESSENTIAL*** **to Fragment Formation !**
  - **Initial Orientation of Deformed Nuclei**
  - **Stochastic Two-Body Collision Term**
- **Fragment Formation is well described in *AMD + Statistical Decay***
  - **Exception: ^{13}C (Compared to Mirror ^{13}N, σ(^{13}C) is around 10 times larger in Data)**





Fragmentation: Low T and Low Density Matter

- **Experimental Evidence on the First Order LG Phase Transition**
  - **Two indep. exp. on two indep. Observables show**
  - **We need models, which describes both of**
    Nuclear Reactions and Statistics
    having Quantum Statistical nature $E \propto T^2$ in nuclei at low T
- **Molecular Dynamics with Quantum Fluctuation**
  - **AO & Randrup: Quantum Langevin Model**
    NPA565(1993), 474; PRL 75(1995), 596; Ann.Phys.253 (1997), 279; PLB394(1997), 260; PRA55(1997), 3315R
  - **Hirata, Nara, AO, Harada, Randrup; AMD-QL (PTP 102 (1999), 89)**
  - **Ono-Horiuchi: AMD-V (E.g., PRC53 (1996), 2958)**
  - **Sugawa-Horiuchi-Ono: AMD-MF (PRC60 (1999) 064607)**

Quantum Langevin Model

Wave Packet Statistics

- **Partition Function**

$$\mathcal{Z}_\beta \equiv \mathrm{Tr}\left(\exp(-\beta\hat{H})\right) = \int d\Gamma\ \mathcal{W}_\beta(\mathbf{Z})$$
$$\mathcal{W}_\beta(\mathbf{Z}) \equiv \langle\mathbf{Z}|\exp(-\beta\hat{H})|\mathbf{Z}\rangle \neq \exp(-\beta\langle\hat{H}\rangle)$$

- **Thermal Average**

$$\prec\hat{O}\succ_\beta \equiv \frac{1}{\mathcal{Z}_\beta}\mathrm{Tr}\left(\hat{O}\exp(-\beta\hat{H})\right) = \frac{1}{\mathcal{Z}_\beta}\int d\Gamma\,\mathcal{W}_\beta(\mathbf{Z})\ \mathcal{O}_\beta(\mathbf{Z})$$
$$\mathcal{O}_\beta(\mathbf{Z}) \equiv \frac{\langle\mathbf{Z}_{\beta/2}|\hat{O}|\mathbf{Z}_{\beta/2}\rangle}{\langle\mathbf{Z}_{\beta/2}|\mathbf{Z}_{\beta/2}\rangle} \neq \langle\hat{O}\rangle$$
$$|\mathbf{Z}_{\beta/2}\rangle \equiv \exp(-\beta\hat{H}/2)|\mathbf{Z}\rangle \neq |\mathbf{Z}\rangle$$

- **Harmonic Approximation**

$$\mathcal{W}_\beta(\mathbf{Z}) \approx \exp\left[-\frac{\mathcal{H}}{D}\left(1-e^{-\beta D}\right)\right] = \exp(-\beta\mathcal{H} + \beta^2\sigma_E^2/2 + \cdots)$$
$$D(\mathbf{Z}) \equiv \sigma_E^2/\mathcal{H}$$
$$\mathcal{H}_\beta(\mathbf{Z}) \equiv -\frac{\partial\log\mathcal{W}_\beta(\mathbf{Z})}{\partial\beta} \approx \mathcal{H}(\mathbf{Z})\ e^{-\beta D}$$

$\rightarrow$ Improved β Expansion

Wave Packet Dynamics

Fokker-Planck Eq.

$$\frac{D\phi}{Dt} = \left[-\sum_i\frac{\partial}{\partial q_i}V_i + \sum_{ij}\frac{\partial}{\partial q_i}M_{ij}\frac{\partial}{\partial q_j}\right]\phi\ ,$$
$$V_i = -\sum_j M_{ij}\frac{\partial\mathcal{F}_\beta}{\partial q_j}\ ,$$

$$V_i = -\alpha\beta\sum_j M_{ij}\frac{\partial\mathcal{H}}{\partial q_j}\ ,\qquad \alpha = \frac{1-\exp(-\beta D)}{\beta D}$$

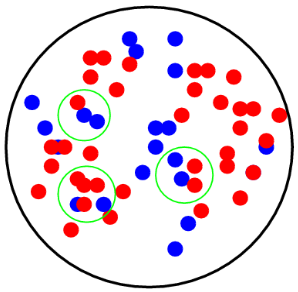
Langevin Equation

$$\dot{\boldsymbol{p}} = \boldsymbol{f} - \alpha\beta\boldsymbol{M}^p\cdot\boldsymbol{v} + \boldsymbol{g}^p\cdot\boldsymbol{\xi}^p\ ,$$
$$\dot{\boldsymbol{r}} = \boldsymbol{v} + \alpha\beta\boldsymbol{M}^r\cdot\boldsymbol{f} + \boldsymbol{g}^r\cdot\boldsymbol{\xi}^r\ ,$$
$$\boldsymbol{v} = \frac{\partial\mathcal{H}}{\partial\boldsymbol{p}}\ ,\quad \boldsymbol{f} = -\frac{\partial\mathcal{H}}{\partial\boldsymbol{r}}\ ,$$
$$\boldsymbol{M}^p = \boldsymbol{g}^p\cdot\boldsymbol{g}^p\ ,\quad \boldsymbol{M}^r = \boldsymbol{g}^r\cdot\boldsymbol{g}^r\ .$$

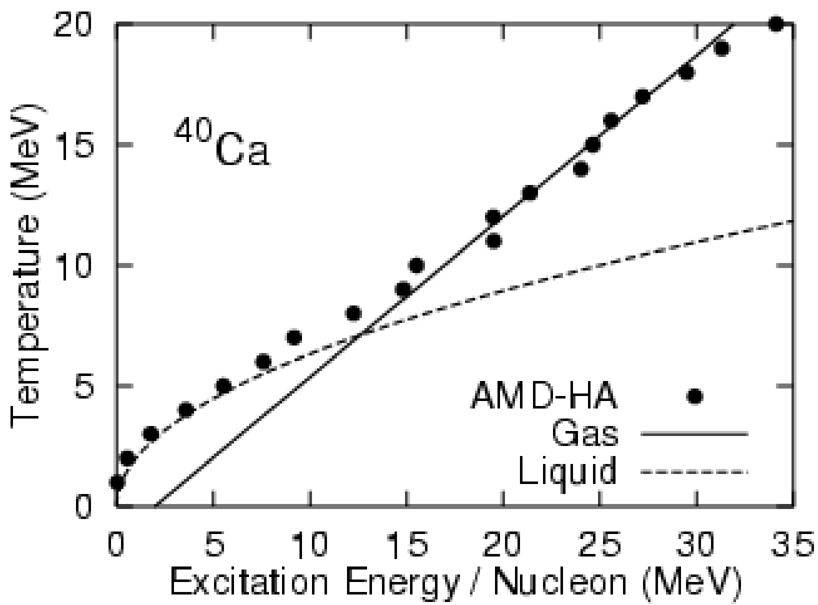
Sampled State ≠ Observing State
Dual State Structure make it possible
to Simulate Quantum Statistics in Molecular Dynamics

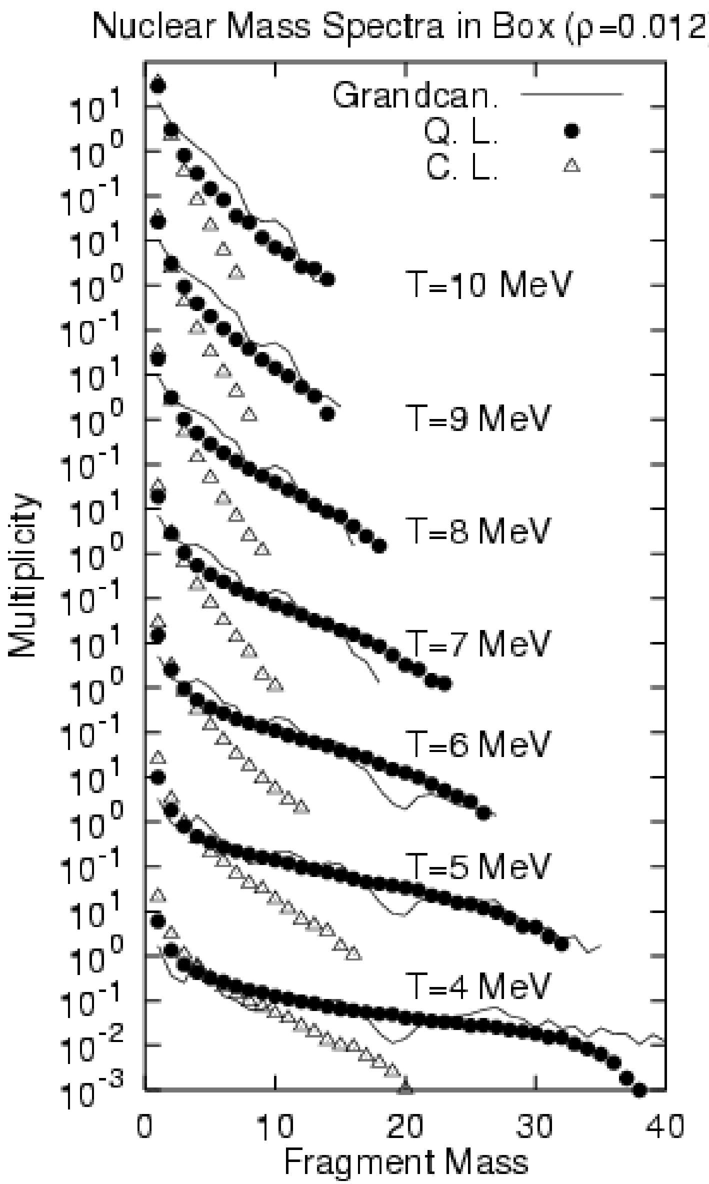
Wave Packet Statistics

Mass Distribution

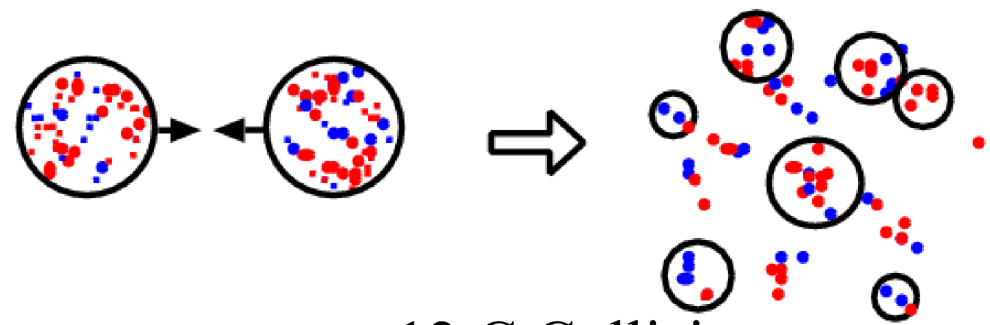


Caloric Curve



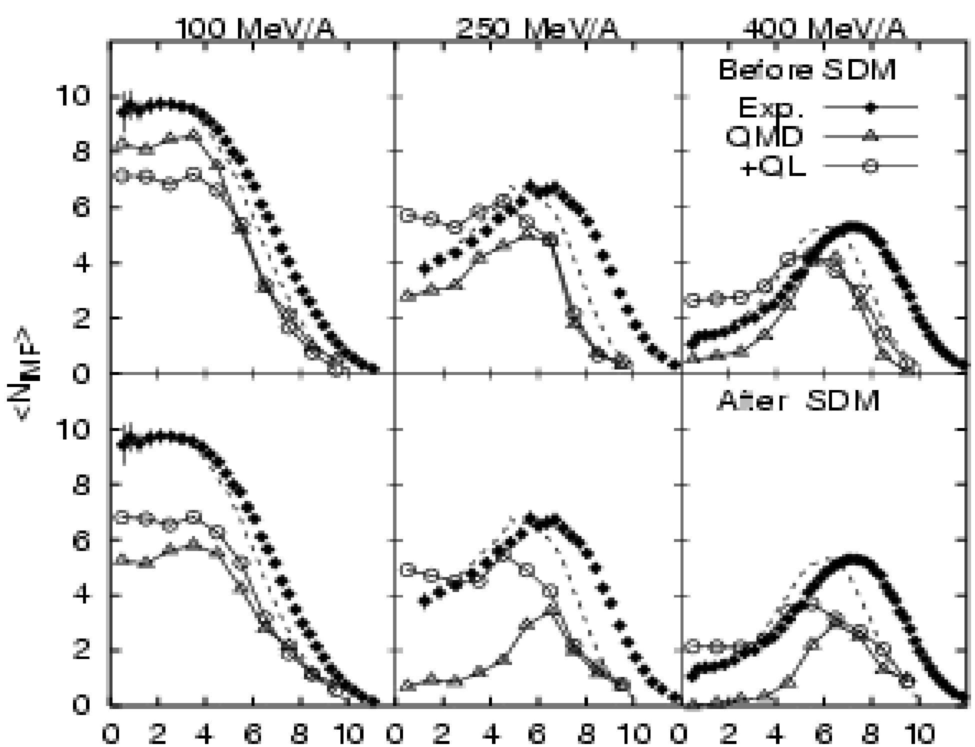


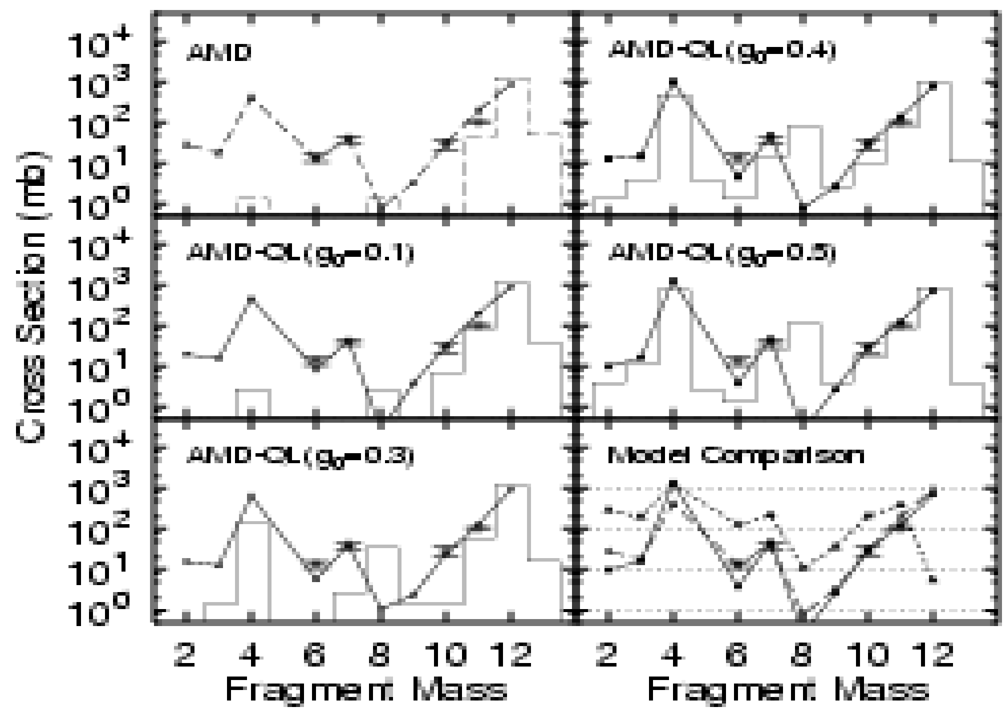
Wave Packet Dynamics



Au + Au Collisions

p + 12 C Collisions





AO-Randrup, 1997

Hirata et al. PTP102(99),89

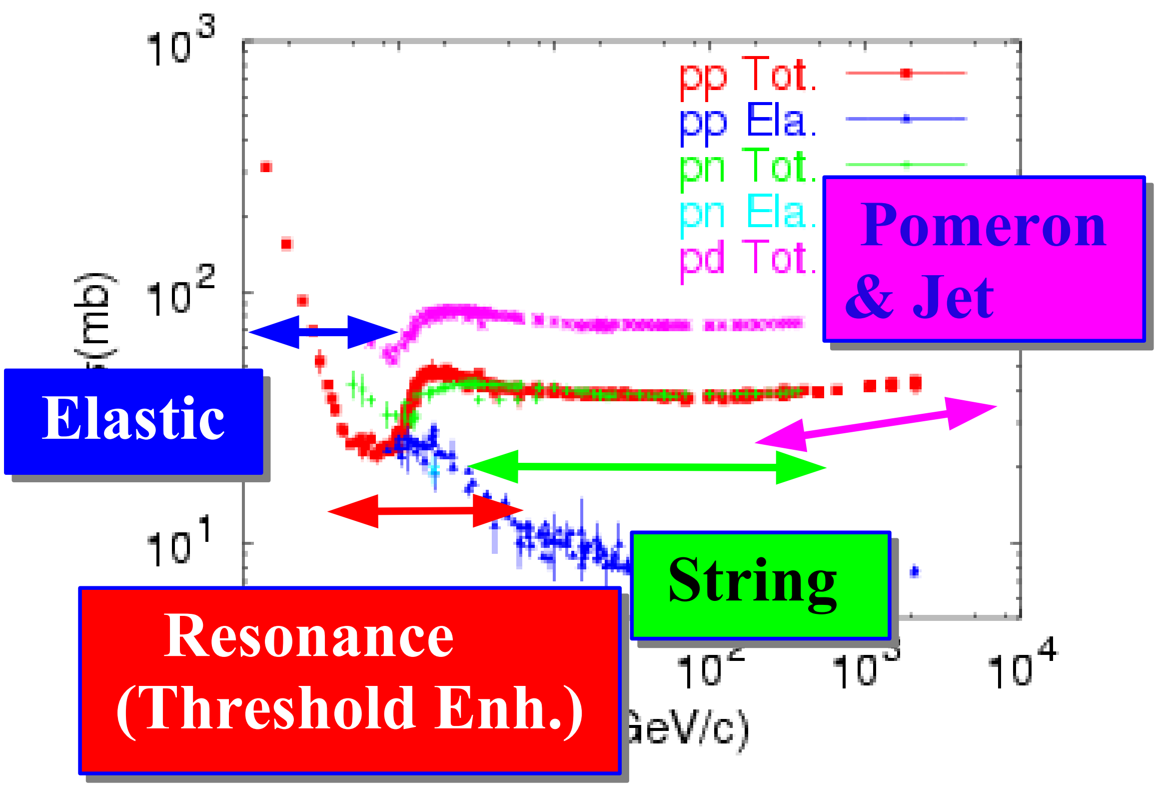
What is Understood ?

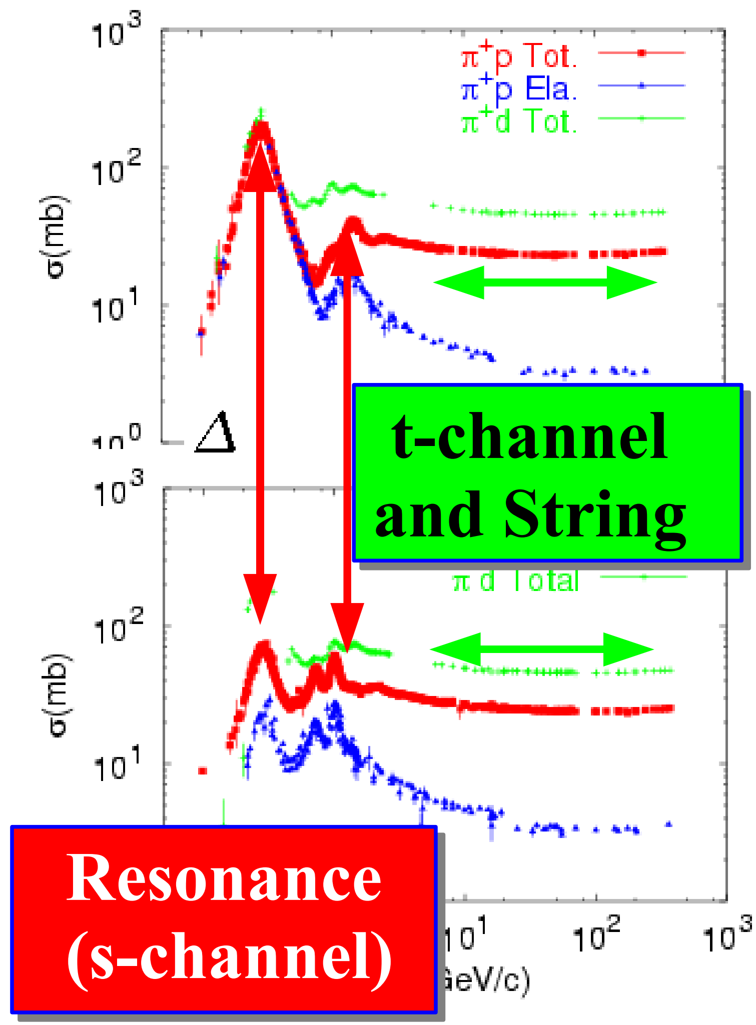
- **What is understood ?**
  - **LG Phase Transition is of First Order (Exp.).**
  - **It can be understood in Microscopic MD qualitatively, e.g. Fragment Yield.**
- **What is NOT Understood ?**
  - **Direct Relation between Fragment Formation and the Properties of Nuclear Matter**
  - **Are Fragments Produced through LG Phase Transition ?**
  - **"Initial" Condition of Fragmenation At Which T and ρ Fragments are Formed ?**
  - **Is Equilibrium Reached in Heavy-Ion Collisions ?**

Simpler Cases: pA Reaction & Supernova Explosion !

Hadron-Hadron Cross Sections

From Particle Data Group



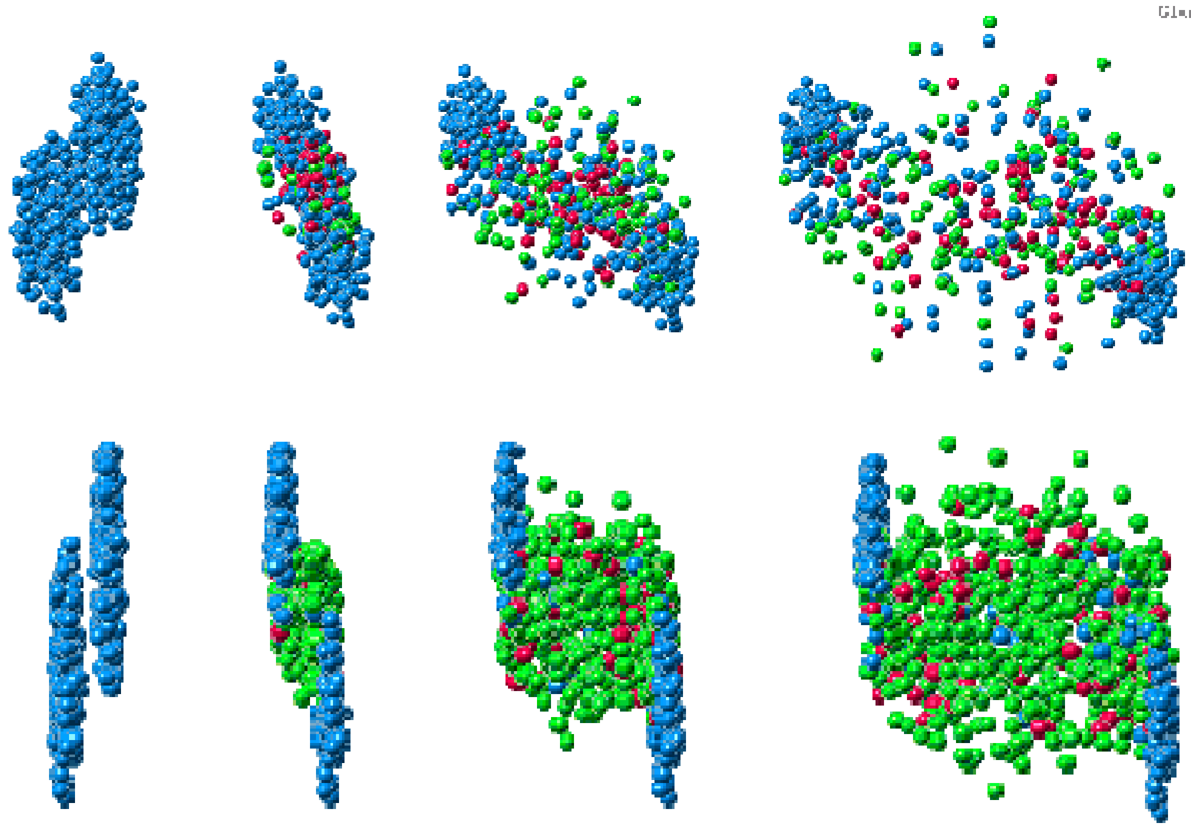


Heavy-Ion Collisions at Einc ~ (1-100) A GeV

- **Study of Hot and Dense Hadronic Matter**
  → Particle Yield, Collective Dynamics (Flow), EOS, ...

AGS

SPS



JAMming on the Web, linked from http://www.jcprg.org/

Collective Flow and EOS: Old Problem ?

- **1970's-1980's: First Suggestions and Measurement**
  - **Hydrodynamics suggested the Exsitence of Flow.**
  - **Strong Collective Flow suggests Hard EOS**
- **1980's-1990's: Deeper Discussions in Wider E_{inc} Range**
  - **Momentum Dep. Pot. can generate Strong Flows.**
  - **Einc deps. implies the importance of Momentum Deps.**
  - **Flow Measurement up to AGS Energies.**
- **2000's: Extention to SPS and RHIC Energies**
  - **EOS is determined with Mom. AND Density Dep. Pot. ?**

Old but New (Continuing) Problem !

Mean Field Dynamics + Two-Body Collision

- **BUU Equation　(Bertsch and Das Gupta, Phys. Rept. 160(88), 190)**
  - **Time-Dependent Hartree-Fock Eq. を Wigner 変換→ Vlasov Eq.**
  - **Pauli blocking を導入した Boltzmann 方程式の衝突項を導入**

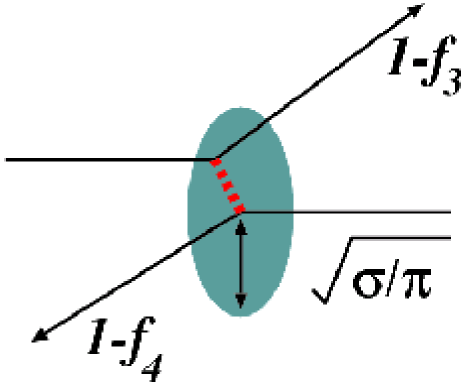
→ 低エネルギーで重要な平均場理論と
高エネルギー(>100 A MeV) で支配的なカスケード過程を統合

→ どのような粒子自由度を導入すべきか？

$$\frac{\partial f}{\partial t} + v \cdot \nabla_r f - \nabla U \cdot \nabla_p f = I_{coll}[f]$$

$$I_{coll}[f] = -\frac{1}{2} \int \frac{d^3 p_2 d\Omega}{(2\pi\hbar)^3} v_{12} \frac{d\sigma}{d\Omega}$$

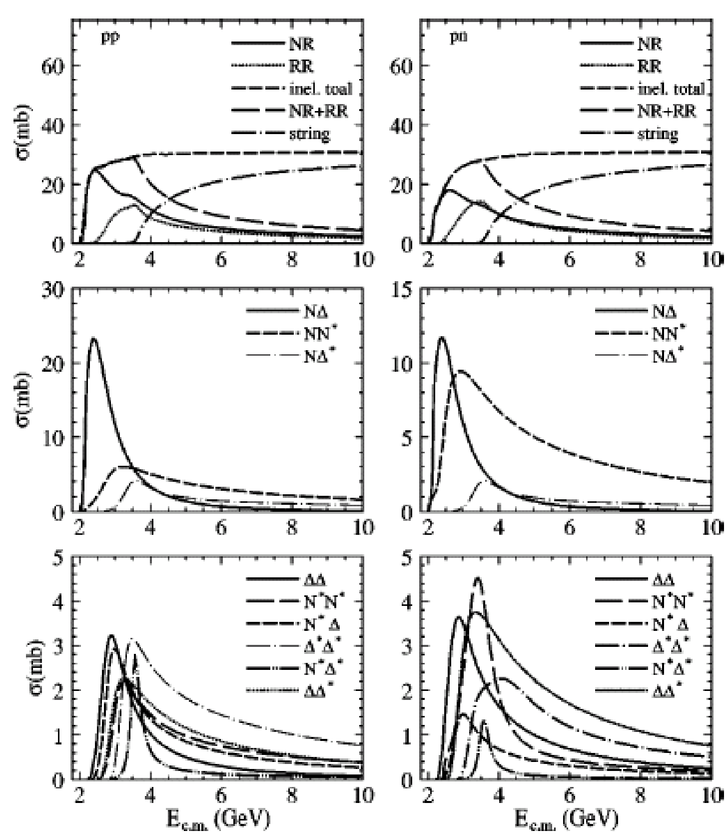
$$\times \left[f f_2 (1-f_3)(1-f_4) - f_3 f_4 (1-f)(1-f_2) \right]$$

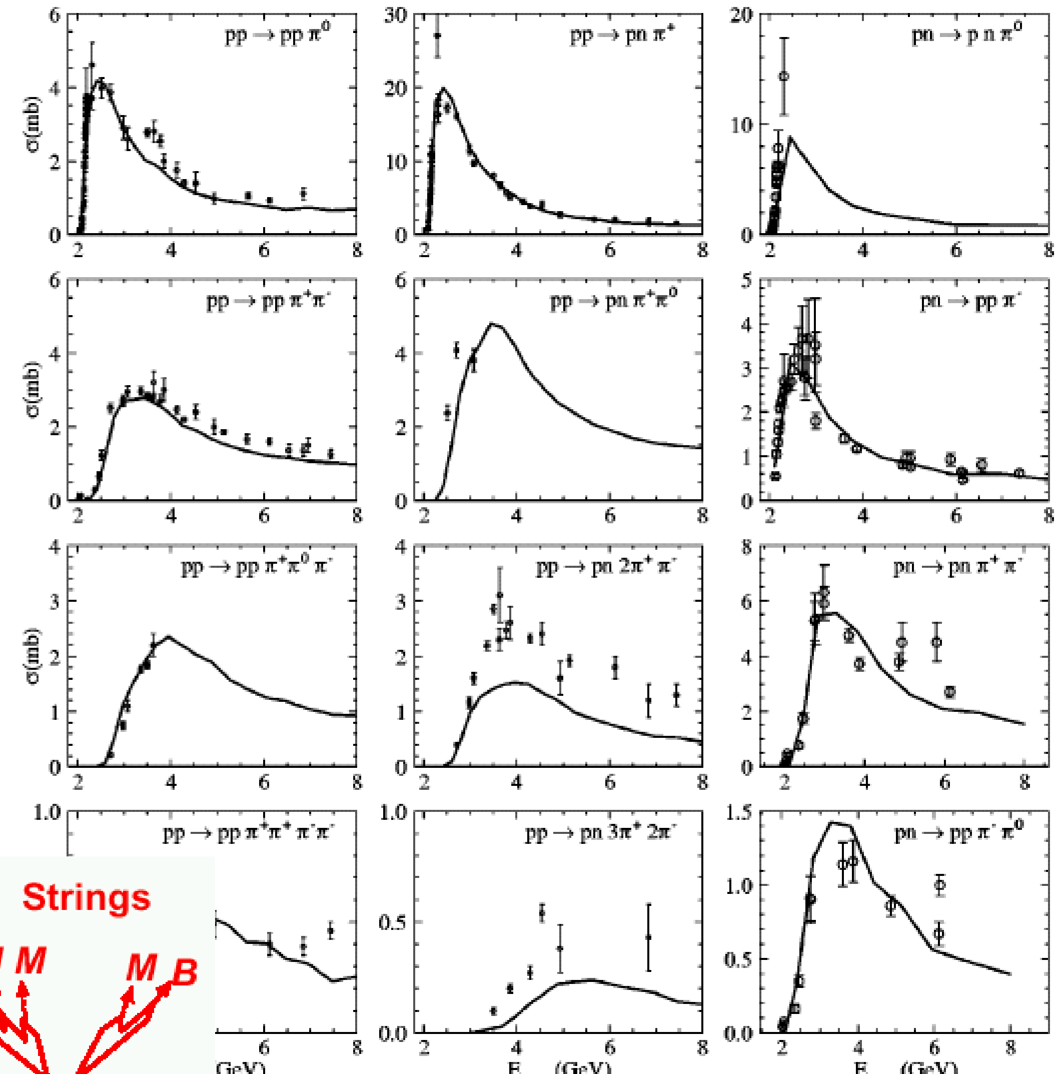


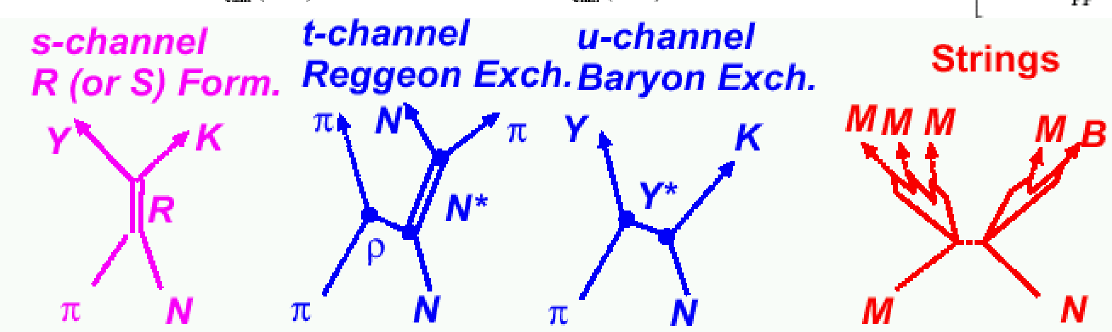
Exclusive Cross Sections in JAM

Nara, Otuka, AO, Niita, Chiba (JAM), PRC 61 (2000), 024901.

- **Ground State Hadrons, Resonances, and Strings**



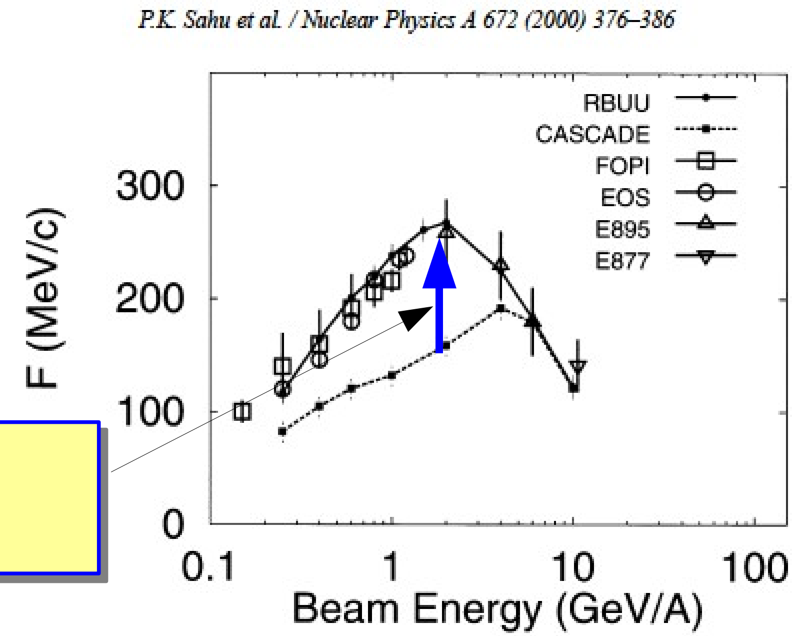


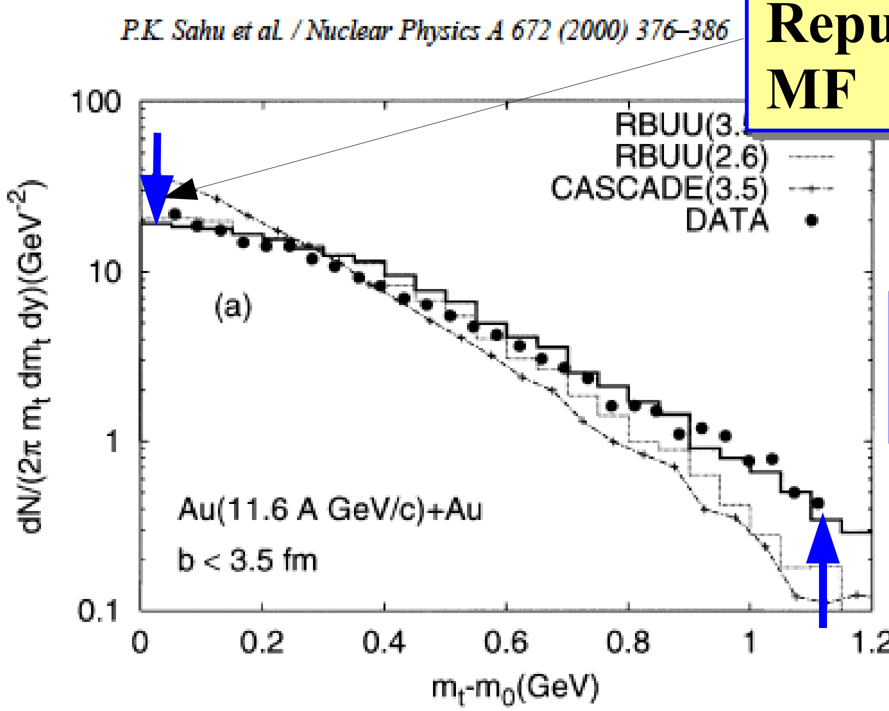


Mean Field and Particle DOF Effects @ AGS

- **Mean Field Effects at AGS**
  → Visible but small for p_T spectrum
  Essential for Flow
- **Particle DOF Effects**
  → Seen at high p_T

Sahu, Cassing, Mosel, Ohnishi, 2000





Repulsive MF

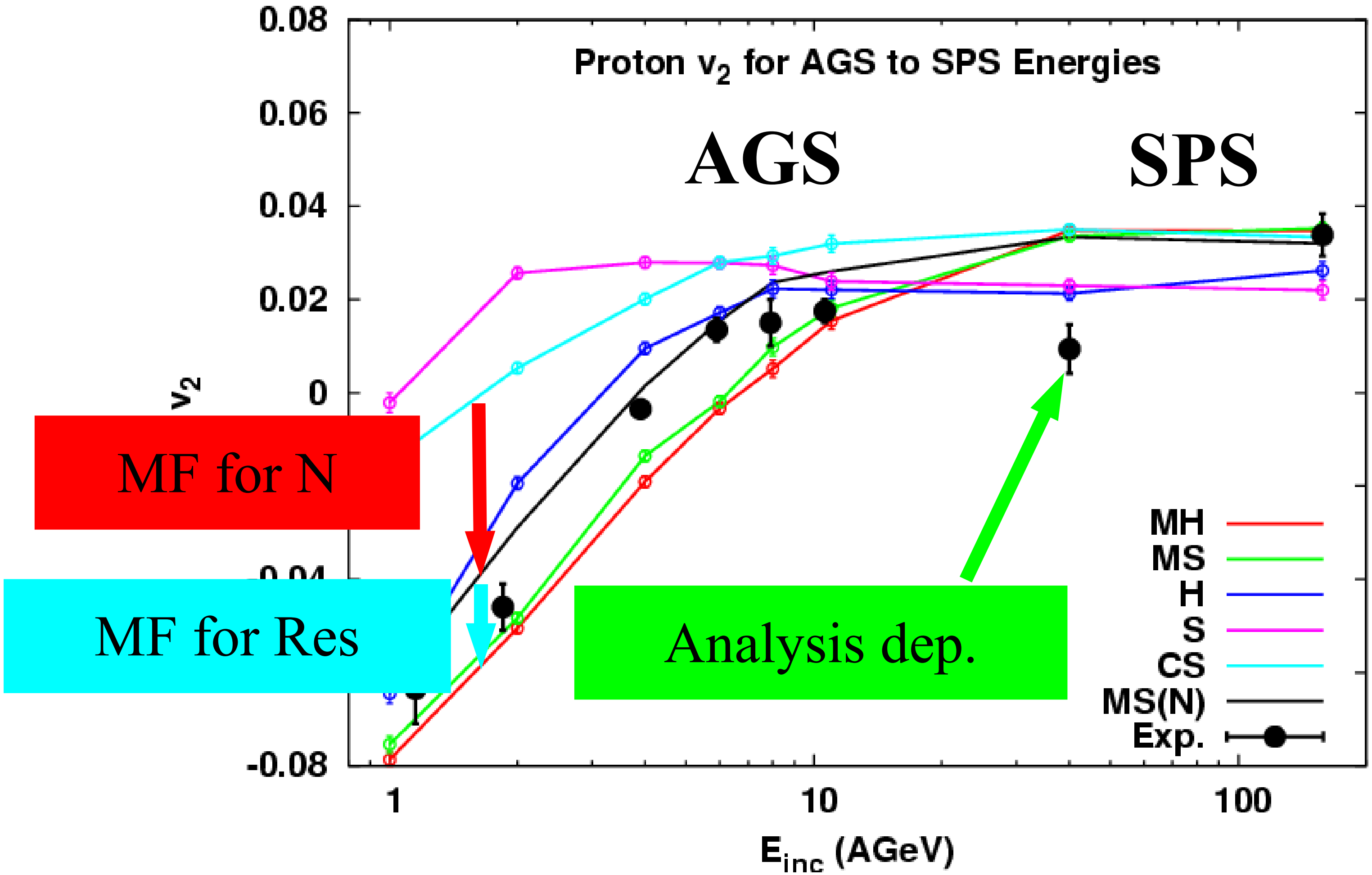
Switching √s = 3.5 GeV (JAM fit)

Switching √s = 2.6 GeV (HSD default)

Elliptic Flow from AGS to SPS

Isse, AO, Otuka, Sahu, Nara, PRC 72 (2005), 064908

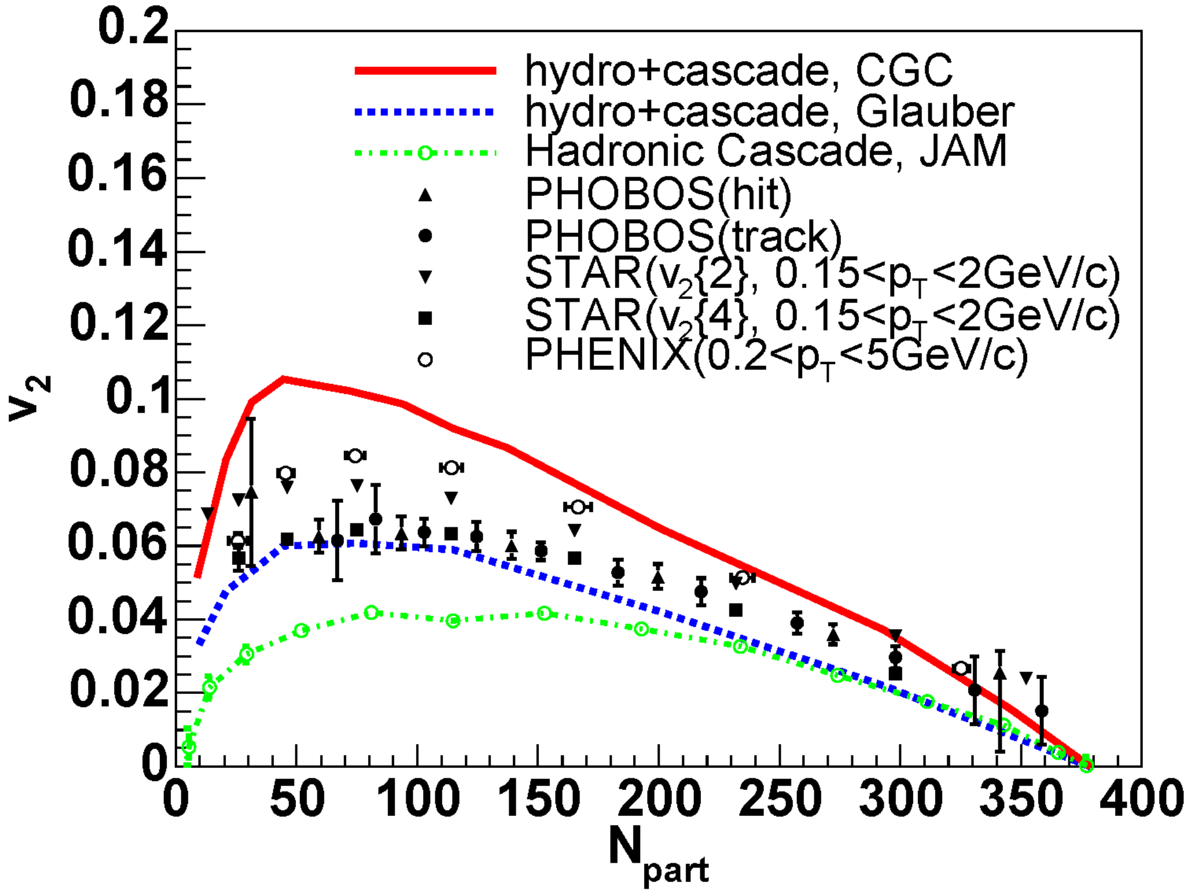
- **JAM-MF with p dep. MF explains proton v2 at 1-158 A GeV**
  - **v2 is not very sensitive to K (incompressibility)**
  - **Data lies between MS(B) and MS(N)**



Cascade vs Hydro @ RHIC: Au+Au

- **Comparison of v2 as a function of N_{part}**
  - **Cascade predict smaller v2 in peripheral collisions**
  - **Data lies between hydro results with two different initial condition CGC (Color Glass Condensate) and Glauber type initial condition.**

**Hydro is better,
CGC may be realized
in central collisions.**

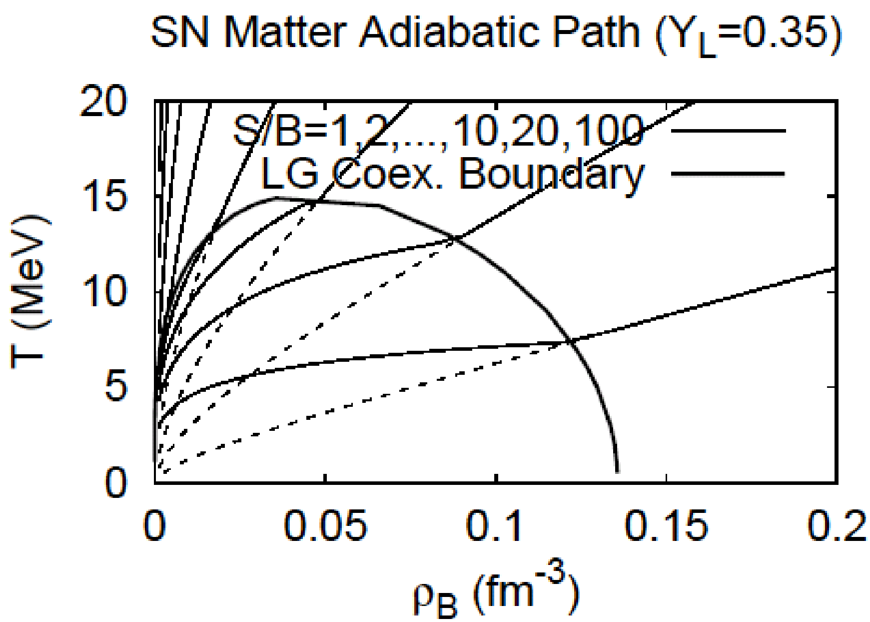


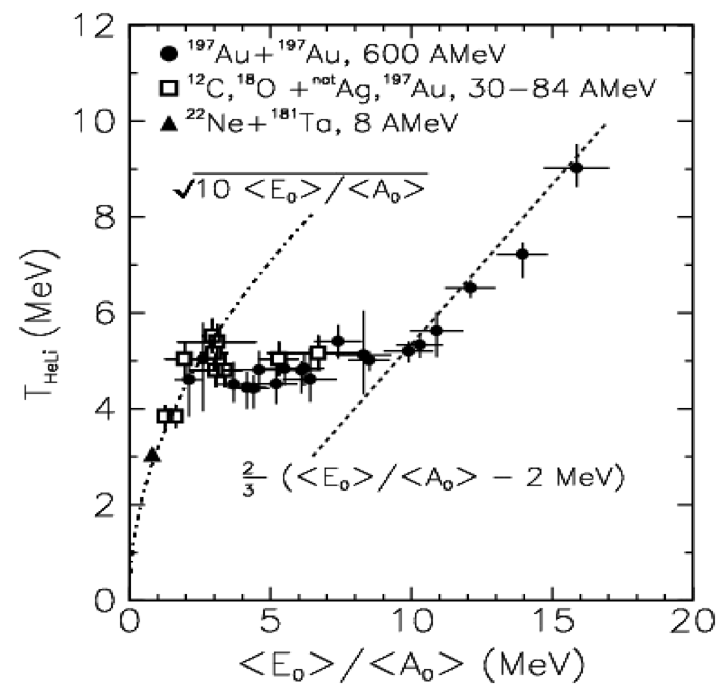
Lessons from AGS and SPS Energy HIC

- 粒子生成の主要過程
  - 1 A GeV Energy → 共鳴粒子生成 + 共鳴崩壊
  - 10 A GeV Energy (AGS) → 2共鳴ハドロン生成、ストリング生成+崩壊
  - 100 A GeV Energy (SPS) → ストリング生成+崩壊
- 必要な自由度の輸送を取り入れる必要性
  - pT スペクトルに直接的に影響を与える。
  - あらわな自由度を取り入れない場合
→ formation time 等を導入して相互作用の強さ(圧力)を調整
(量子論的な「漸近領域に達する時間」だけではないだろう)
- RHICエネルギーでのハドロン輸送模型の失敗
  - SPS エネルギーまでで成功している formation time を使うと、
反応初期での相互作用が小さすぎる
→ 遅い熱平衡化時間、小さな楕円フロー(特に high pT)

HIC at E_{inc} ~ a few 100 MeV/A

- **Physics of Fragmentation and Liquid-Gas Phase transition**
  - **低温では $\rho_B \sim (0.3\text{-}0.6)\ \rho_0$ において、$dP/d\rho_B < 0$ (spinodal region)**
    → 一様な核物質は不安定となり、液相と気相が共存する。





→ 重イオン反応という
非平衡・非一様な状況で
「相」の情報はみえるのか？

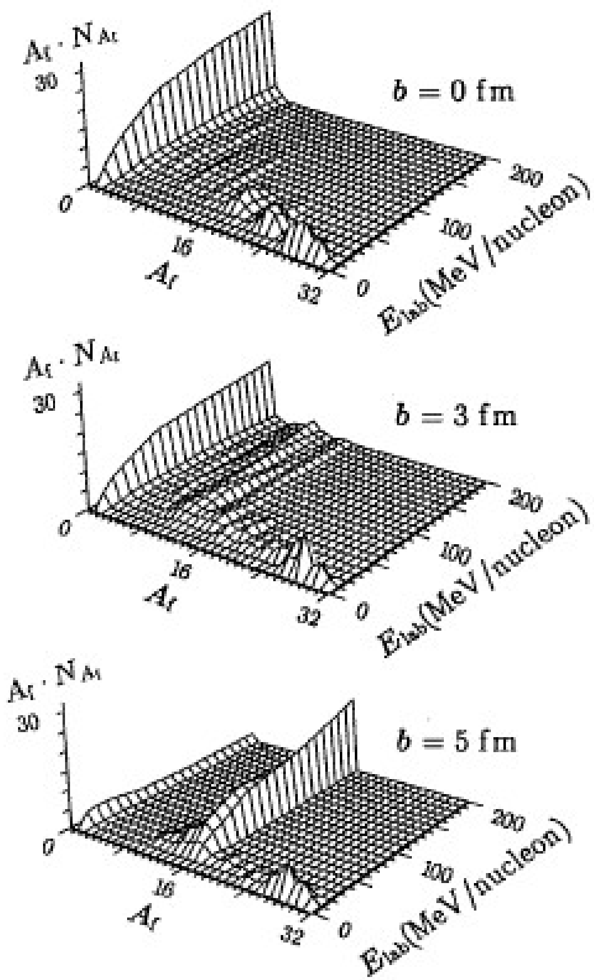
J. Pochadzalla et al (GSI-ALLADIN collab.), PRL 75 (1995) 1040.

$$Y_f \propto g_f \exp((B_f + Z\mu_p + N\mu_n)/T)$$

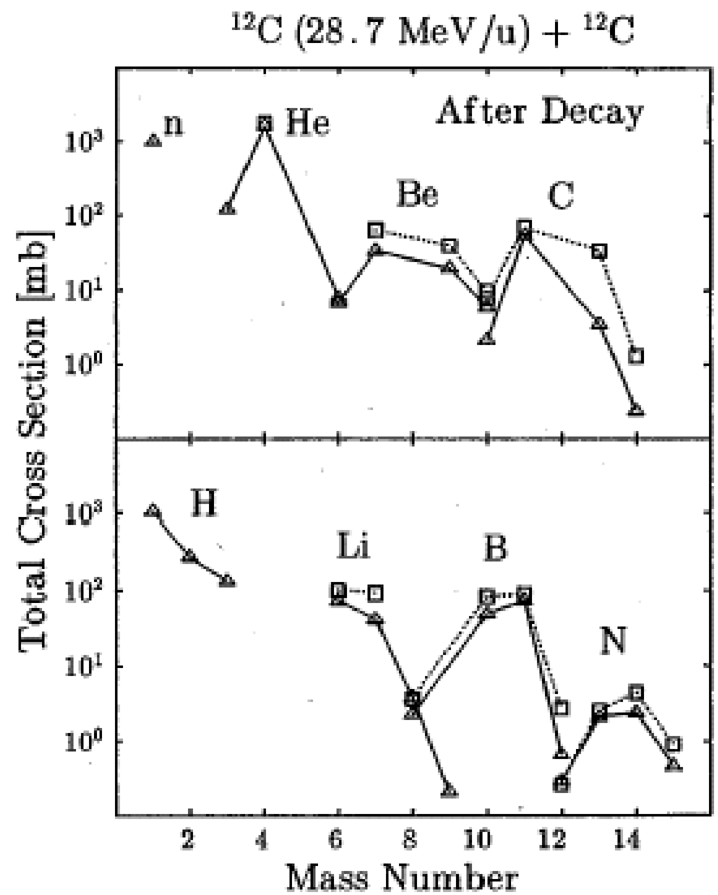
$$\rightarrow \frac{Y(^4He)/Y(^3He)}{Y(^7Li)/Y(^6Li)} \propto \exp(\Delta B/T)$$

Molecular Dynamics Study of Fragmentation

- **Quantum Molecular Dynamics (QMD)**
  & Antisymmetrized Molecular Dynamics (AMD)
  → Reaction Dynamics, Mass & Isotope Dist.



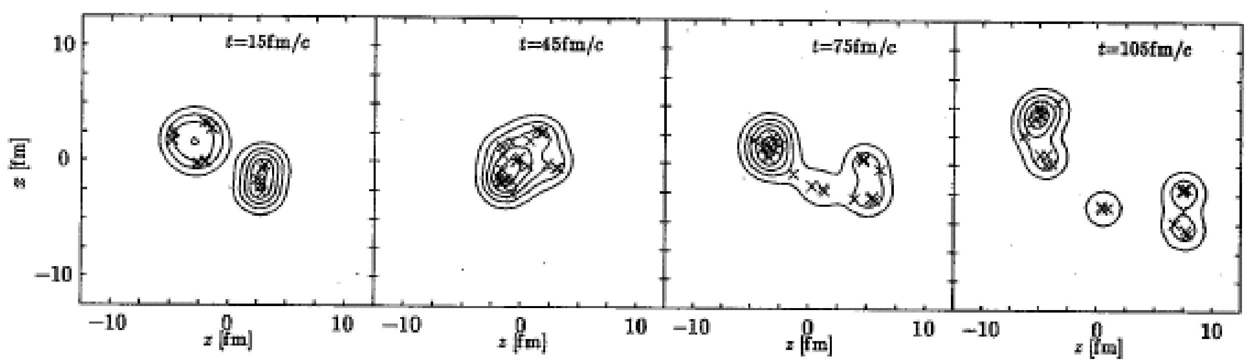
Maruyama, Ohnishi, Horiuchi, PRC45('92)2355

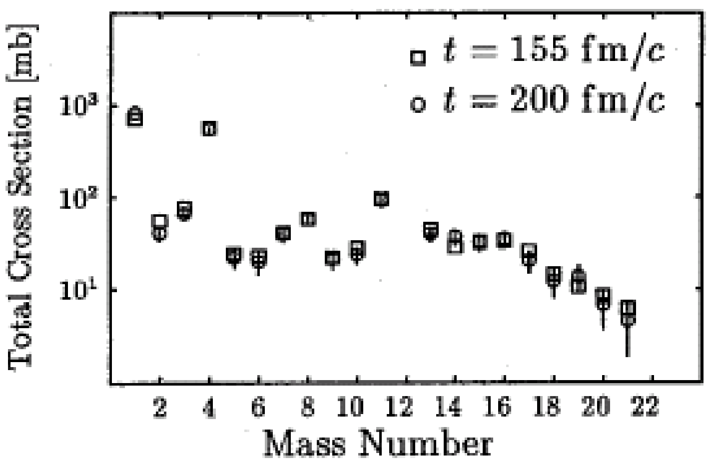


Ono, Horiuchi, Maruyama, Ohnishi, PTP87('92)1185

Molecular Dynamics Study of Fragmentation

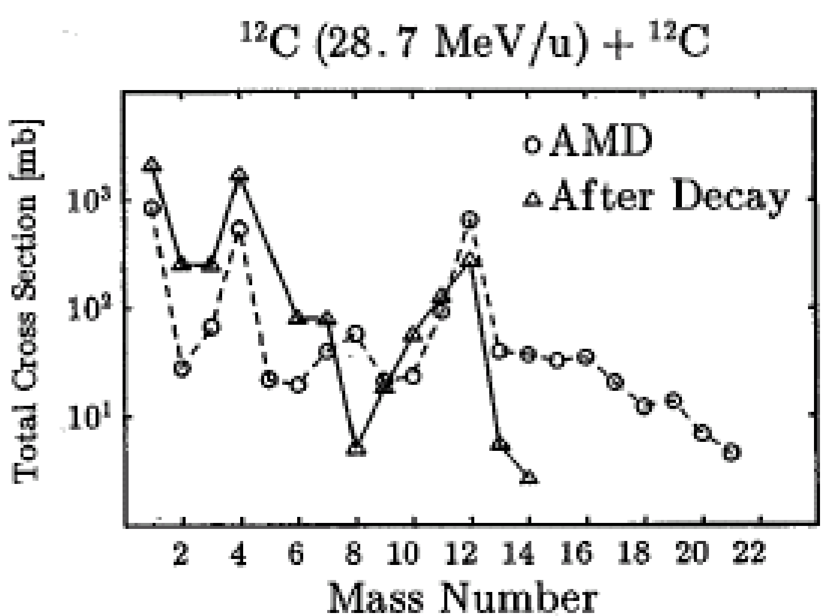
- フラグメント分布を求める手続き
  - Equation of Motion
    Gauss 波束の(反対称化)積波動関数+ 時間依存変分原理
    → (半古典的な)運動方程式
  - クラスター判定
    終状態で位相空間の近い核子の集合を原子核と判定
  - 統計崩壊
    原子核の励起エネルギーと角運動量から統計的に崩壊させる
    (after burner)



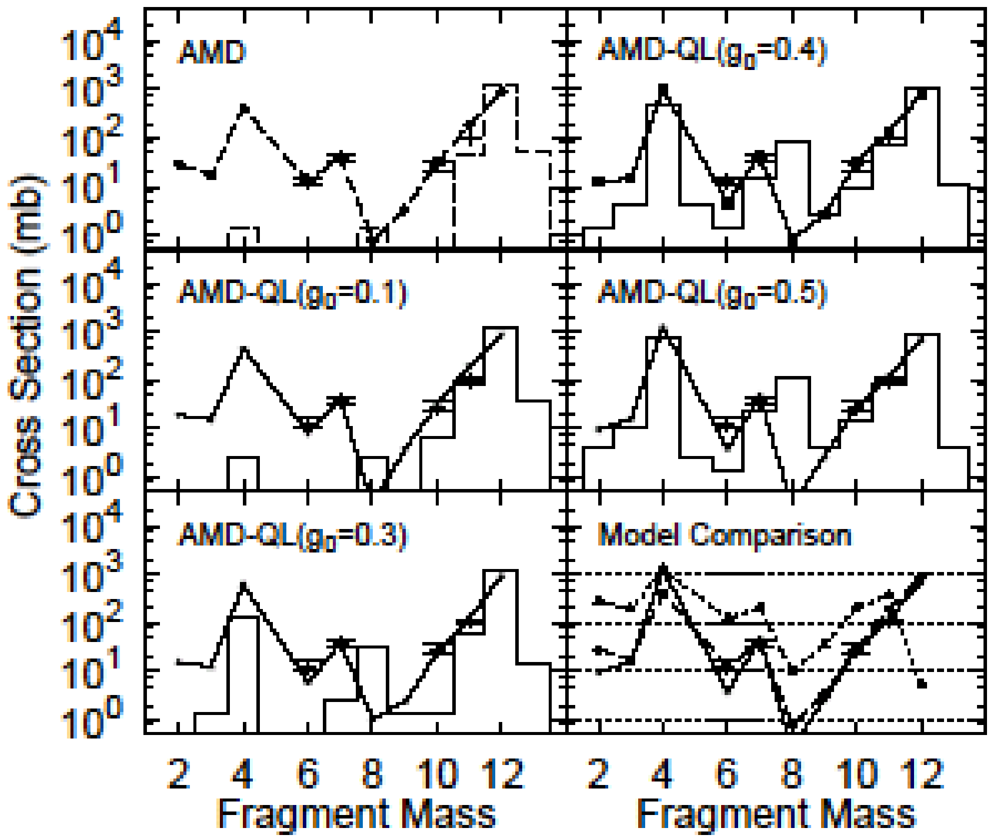


Molecular Dynamics Study of Fragmentation

- 粒子シミュレーションでの時間スケール
  - 統計崩壊模型で与えられる時間スケールよりも長い
    → 古典的シミュレーションの問題点



Ono et al., 1992



p+12C (45 MeV)
Hirata,Nara,Ohnishi,Harada,Randrup, 1999

Lessons from HIC @ 100 A MeV

- フラグメントの起源
  → まず質量数の大きな励起した原子核が作られ、
  その「統計的」崩壊により多くの原子核が作られる。
  - 「大きな原子核の高い励起状態の状態数 ~ 位相空間」
    → 短い時間スケールでの系の励起エネルギー分布は
    古典的なシミュレーションでよく説明可能であろう。
  - 基底状態フラグメント放出、低エネルギーでの核子放出には、
    エネルギー集中が必要→ 統計崩壊模型との組み合わせが必要。
- 反応・崩壊の時間スケールが近い場合の問題点
  - 軽イオン(d, t, 3He, ...)、中間質量片(IMF)生成は
    比較的短時間でおこる
    → 量子揺らぎ、Coalescence 過程、軽イオン自体の輸送等を
    あらわに取り入れる必要がある。
  - 時間スケールをそのまま信頼できないだろう例
    → 超新星での Pasta 状態生成時間
    (おそらく古典シミュレーションより短い時間でおこる。
    結論は問題なし)